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Research Article

Complex q-Rung Orthopair Fuzzy N-Soft Sets: A New Model with Applications

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This research article expands the formal representation of human thinking to a most generalized hybrid theory, namely, complex q-rung orthopair fuzzy N-soft set. It is able to capture a great deal of graded imprecision and vagueness, which so often appear together in human interpretations. This model renders a parameterized mathematical tool for the ranking-based fuzzy modeling of two-dimensional paradoxical data. To that purpose, the proposed theory integrates complex q-rung orthopair fuzzy sets with the parametric structure of N-soft sets. The framework that arises captures information beyond the confined space of complex intuitionistic fuzzy N-soft sets and complex Pythagorean fuzzy N-soft sets, with the assistance of a parameter q. We establish the basic set-theoretical operations of this model and prove some of its fundamental properties. The Einstein and other elementary algebraic operations on complex q-rung orthopair fuzzy N-soft values shall be introduced to broaden the mathematical toolbox of this field. Its relationships with contemporary approaches shall demonstrate its outstanding flexibility. Moreover, we establish two competent multicriteria decision-making algorithms that capture the nuances of periodical inconsistent data. Their feasibility shall be demonstrated with an explicit application to the selection of optimum aerospace technology required for the economic development of the Mexican space agency. A comparative analysis of both strategies with the prevailing techniques substantiates their rationality. In addition, we illustrate this comparative study with an explicative bar chart that shows the compatibility of their outcomes. Finally, we examine the functionality of the proposed model and compare it with alternative theories.

1. Introduction

Decision-making plays a substantial role in almost every discipline of actual life, from engineering to the social sciences or medicine. It is an analytical process whereby real-world problems are modeled and their best solutions are identified after careful examination of all rational alternatives. In this competitive era, decision-making has become a quite challenging task due to the presence of multiple criteria for the appraisement of the alternatives. This has resulted into a growing interest in multiple criteria decision-making (MCDM) as a thriving branch of operation research. In recent decades, MCDM methodologies have gained prestige in numerous territories, including information technology, robotics, automotive industries, social sciences, business management, and various other disciplines of science and technology. In classical MCDM techniques, the assessments of the performance of the alternatives regarding their characteristics are entirely precise and exact in nature. However, most of the actual-world decision-making problems cannot be modeled with such clear-cut interpretations because of inherent vagueness and ambiguity of human perceptions. The limitations that this imprecision impose on the traditional approaches has lead the researchers to establish novel theories with more flexible structures. Their target is the incorporation of ample forms of inconsistent knowledge into the umbrella of MCDM.

Zadeh [1] initially put forward the pioneer solution of such difficulty by establishing the foundations of fuzzy sets (FSs). In a fuzzy set, each element is endowed with a

membership degree lying within the unit interval $[0, 1]$. Fuzzy set theory provides a simple but powerful tool for coping with certain types of imprecision of inexact information. Atanassov [2] extended the idea of FS into intuitionistic fuzzy set (IFS) by adding a nonmembership degree (Θ) to the membership degree (Ψ) of FS. For reasons of consistency, he imposed that that the sum of both degrees should not exceed 1. Later on, Yager [3] introduced the notion of Pythagorean fuzzy set (PFS) which relaxes the aforementioned constraint condition of IFSs to $\Theta^2 + \Psi^2 \leq 1$. Although both FSs and its extension by IFSs have succeeding in producing remarkable applications, these theories restrict the opinions of the decision-makers since they must be confined to a fixed boundary. To overcome this complexity, Yager [4] proposed the idea of q-rung orthopair fuzzy set (q-ROFS) in which the sum of the q-th power of both membership and nonmembership degrees lies between 0 and 1. Nowadays, the q-ROFS is regarded as a most generalized model of the partial membership approach to uncertainty. Concerning decision-making, Akram and Shahzadi [5] developed a hybrid decision-making technique based on q-rung orthopair fuzzy soft information and illustrated its application in the field of medical diagnosis. Shaheen et al. [6] proposed an alternative algorithm by employing relative frequency distributions to generate membership and nonmembership grades of q-ROFSs. Recently, Yang et al. [7, 8] have discussed new decision-making methods in this context.

These traditional and recent models that expand fuzzy set theory are limited to address nonperiodic information. To address periodic phenomena, Ramot et al. [9] remodeled the notion of fuzzy sets by introducing the novel theory of complex fuzzy sets (CFSs) in which the range of membership function is extended from the unit interval $[0, 1]$ to the complex unit disc $\{w \mid w \in \mathbb{C}, |w| \leq 1\}$. Alkouri and Salleh [10] developed the concept of complex intuitionistic fuzzy set (CIFS) by proposing the complex-valued membership $(\Theta e^{i\Omega})$ and nonmembership $(\Psi e^{i\Upsilon})$ degrees. Ullah et al. [11] presented the idea of complex Pythagorean fuzzy sets (CPFSs) and improved the restricted conditions of CIFSs.

Liu et al. [12] have established the most generalized extension of complex fuzzy sets, namely, the theory of complex q-rung orthopair fuzzy sets (C_q-ROFSs) which significantly relaxed the constraints in terms of a parameter q. The C_q-ROFS provides an effective tool for capturing both vagueness and periodicity within the semantics of human assessments. The remarkable potentialities of C_q-ROFS have sparked the attention of many researchers who have implemented this outstanding theory in various scientific areas, including image processing, electromagnetism, pattern recognition, fluid dynamics, networking, relativity, and thermodynamics. Mahmood and Ali [13] developed a novel TOPSIS method that uses the correlation coefficient of C_q-ROFS and showed its applications for the evaluation of firewall productions and security assessment of computer systems. Garg et al. [14] successfully established AHP and TOPSIS methods for complex interval-valued q-rung orthopair fuzzy sets (CIV_q-ROFSs). Jana et al. [15] developed a remarkable MCDM method based on some novel aggregation operators of single-valued trapezoidal neutrosophic (SVTN) numbers for the prioritization of the best commercial software systems.

Despite the benefits and applications of the models explained above, including FSs [1], CFSs [9], CIFSs [10], CPFSs [11], and C_q-ROFSs [12], one major common limitation is their inadequacy for parameterized descriptions. An entirely novel escape was provided by the theory of soft sets (SSs) which presents a competent mathematical tool integrating parametrization for the description of the alternatives [16] and their comparisons [17]. The idea of soft set theory was further hybridized with other fuzzy mathematical structures to develop new models such as fuzzy soft sets (FSSs) [18], intuitionistic fuzzy soft sets (IFSSs) [19], Pythagorean fuzzy soft sets (PFSSs) [20], and q-rung orthopair fuzzy soft sets (q-ROFSSs) [21]. Altogether they produce a rich variety of environments for the fuzzy modeling of parameterized noncrisp data. Later on, Chinram et al. [22] established a competent multicriteria decision-making technique by employing novel geometric aggregation operators to diagnose the ailments of patients. Garg et al. [23] presented the concepts of generalized dice similarity measures for C_q-ROFSs and demonstrated its empirical application in pattern recognition. Wang et al. [24] developed an advanced MABAC method under q-rung orthopair fuzzy information for selection of the best construction project. Yang and Pang [25] proposed a novel q-rung orthopair hesitant fuzzy decision-making technique based on TOPSIS and linear programming for the inspection of self-service book sterilizer problems. Jana et al. [26] employed novel Dombi aggregation operators of q-rung orthopair fuzzy numbers for the production of an advanced decision-making strategy. Furthermore, they demonstrated its potential applicability in the emerging field of information technology. Wang et al. [27] established the Pythagorean fuzzy interactive Hamacher power aggregation operators along with a novel MCDM technique. This procedure was applied to the evaluation of the quality of an express service.

A routine inspection of the research work on soft-set-inspired theories will show that they tended to prioritize the use of binary interpretations (either 0 or 1) or else real numbers lying within the unit interval $[0, 1]$. In spite of this fact, numerous decision-making settings are equipped with multinary, discrete-type data. Therefore, a generic modelization of soft set theory was required for coping with such daily-life parameterized ranking-based systems. All these practical concerns lead Fatimah et al. [28] to introduce the now thriving theory of N-soft sets (NSSs) along with appropriate decision-making techniques. Real examples were used to highlight the importance of ordered grades and ranking-based annotations in practical applications. Akram et al. [29, 30] soon launched hybrid theories such as hesitant N-soft set (HNSS) and fuzzy N-soft set (FNSS) by amalgamating the idea of N-soft set theory with specific mathematical declinations such as hesitancy and fuzziness, respectively. Beyond these first attempts, Akram et al. [31] established another hybrid theory of intuitionistic fuzzy N-soft set (IFNSS) and proposed decision-making algorithms under such a versatile environment. Then, Akram et al. [32] developed the more advanced idea of complex

Pythagorean fuzzy N-soft sets (CPFNSSs) which integrated N-soft sets with the very general complex Pythagorean fuzzy sets. The literature on NSS theory was further extended after analyzing its rich potentiality for empirical applications in various directions such as optimization theory, data analysis, forecasting, algebraic structures, information systems, and mathematical analysis and in sundry other decision-making complications. Shahzadi et al. [33] proposed an innovative MCDM strategy based on novel Hamacher interactive aggregation operators of Fermatean fuzzy numbers to properly classify the air quality (AQ) of Guangzhou for 16th Asian Olympic Games. Feng et al. [34] proposed Minkowski score function and a novel decision-making algorithm for the appraisement of sophisticated benchmark problems. Lin et al. [35] developed the linguistic q-rung orthopair fuzzy sets ($L_{\mathbf{q}}$-ROFSs) and their interactional partitioned Heronian mean aggregation operators in order to evaluate the credibility of cloud service productions. For the other notation and application, the reader are referred to [36–55]

The following arguments and evidences condense the motivation and significance of the theory that shall be studied in this work:

(i) The $C_{\mathbf{q}}$-ROFS theory has succeeded to model both the uncertainty and periodicity of source data at the same time. However, it has some limitations because of its inadequacy in situations with graded parametrizations, which cannot be related with this theory as it stands. Our proposed theory is designed to address such parameterized fuzziness of 2-dimensional vague data, which makes it superior to the $C_{\mathbf{q}}$-ROFS model since its inception.

(ii) The spirit of the NSS model is exclusively concerned with the grading-based uncertainty of the inspected universe of objects, but it has no abilities to cope with other sources of inexactness. In this regard, our proposed $C_{\mathbf{q}}$-ROFNSS theory renders a more generalized and constructive mathematical framework that allows for parameterized modeling of periodical and fuzzy data.

(iii) Although the existing CIFSS and CPFSS theories have great abilities to deal with the parameterized vagueness of 2-dimensional problems, their boundary ranges are restricted by some rigid conditions. Our developed model relaxes them, and at the same time, it enables us to capture the graded imprecision embodied in some parameterized uncertain environments.

(iv) From another position, also the $\mathbf{q}$-ROFNSS model provides a proficient mathematical structure for addressing the uncertainty of a parameterized dataset. However, it is constrained by a one-dimensional purpose. Our theory generalizes the $\mathbf{q}$-ROFNSS model by the inclusion of periodical fuzzy interpretations of inconsistent information.

In conjunction with the previous point, both advantages demonstrate the robust generalization ability of the proposed concept.

Motivated by all these facts, this research article establishes an advanced hybrid model, namely, complex q-rung orthopair fuzzy N-soft set ($C_{\mathbf{q}}$-ROFNSS). To do so, it merges the advantageous features of $C_{\mathbf{q}}$-ROFS with outstanding mathematical theory of NSS. In this way, the proposed mathematical framework is designed to capture the graded evaluations of parameterized decision-making problems. The $C_{\mathbf{q}}$-ROFNSS model is highly competent to express a great deal of two-dimensional ambiguous human evaluations. It is a robust generalization of CIFNSSs and CPFNSSs that widens their restricted boundary space by relaxing their constraint conditions in terms of an adjustable parameter q. Moreover, the fundamental set-theoretic operations and mathematical properties of this theory are elaborated. The remarkable flexibility of the proposed model is briefly illustrated with a description of its relationship with existing competing approaches. Then, the Einstein operators and some other elementary algebraic operations of $C_{\mathbf{q}}$-ROFNSVs are properly formulated. We develop two advanced decision-making algorithms and demonstrate their effectivity by investigating a heuristic application for the prioritization of aerospace technologies of the Mexican space agency. At the end, a comparative analysis along with an explicative bar chart is presented to vindicate the reliability and functionality of the proposed strategies.

In a nutshell, the paramount contributions of this article can be summarized as follows:

(i) This research article systematically expands the literature by introducing a multiskilled and most generalized hybrid model $C_{\mathbf{q}}$-ROFNSS. It is designed for the correct modelization of ranking-based fuzzy modeling of two-dimensional parameterized inconsistent data.

(ii) The rationality and accountability of the proposed techniques are substantiated by an empirical application to the discipline of heavy aerospace industries.

(iii) We present a comparative analysis with existing MCDM techniques based on $\mathbf{q}$-ROFS$_{ft}$WG, $C_{\mathbf{q}}$-ROFWA and $C_{\mathbf{q}}$-ROFWG operators. It proves both the feasibility of the strategies developed in this paper and the compatibility of their final outcomes.

(iv) The merits and potentiality of the MCDM methodologies here formulated shed light on their flexibility, competency, and prominence over the contemporary decision-making techniques.

The remainder of this article is organized as follows: Section 2 briefly recalls some fundamental preliminary concepts and terminologies preceding the target theory. Section 3 establishes the framework and basic set-theoretic operations of the proposed $C_{\mathbf{q}}$-ROFNSS model. Section 4 demonstrates the Einstein and other algebraic operational laws on $C_{\mathbf{q}}$-ROFNSVs. Section 5 introduces two decision-making algorithms and illustrates their applicability by means of a potential application for the selection of aerospace technology. Furthermore, Section 6 substantiates the versatility of the proposed strategies by conducting a

comparative study with existing MCDM techniques based on $\mathbf{q}$-ROFS$_{ft}$WG, C$_{\mathbf{q}}$-ROFWA, and C$_{\mathbf{q}}$-ROFWG operators. Section 7 expounds the merits and superiority of the developed C$_{\mathbf{q}}$-ROFNSS theory over other contemporary decision-making models. Finally, Section 8 elaborates the concluding remarks of the article and raises some future research questions. The entire contribution of this research article is graphically interpreted in Figure 1.

2. Preliminaries

In this section, we briefly explicate some basic rudimentary concepts and terminologies required for the proficient modeling of elementary ideas of coming sections.

Definition 1 (see [2]). Let W be a universe of discourse. An intuitionistic fuzzy set T over the universe W is an object having the following form:

$$T = \{(w, \Theta_T(w), \Psi_T(w))\} \mid w \in W\}, \tag{1}$$

where real-valued functions $\Theta_T: W \longrightarrow [0,1]$ and $\Psi_T: W \longrightarrow [0,1]$ specify the membership and nonmembership degrees of an element $w \in W$, satisfying the constraint condition $0 \le \Theta_T(w) + \Psi_T(w) \le 1$. For all $w \in W$, $\mathfrak{A}_T(w) = (1 - \Theta_T(w) - \Psi_T(w))$ represents the degree of indeterminacy. The pair of membership and nonmembership degrees $(\Theta_T(w), \Psi_T(w))$ can be referred as an intuitionistic fuzzy number (IFN).

Definition 2 (see [4]). Let W be a universe of discourse. A q-rung orthopair fuzzy set T over the universe W can be characterized as

$$T = \{(w, \Theta_T(w), \Psi_T(w))\} \mid w \in W\}, \tag{2}$$

where real-valued functions $\Theta_T: W \longrightarrow [0,1]$ and $\Psi_T: W \longrightarrow [0,1]$ indicate membership and nonmembership degrees of an element $w \in W$, subjected to the condition $0 \le \Theta_T^q(w) + \Psi_T^q(w) \le 1$. For all $w \in W$, $\mathfrak{A}_T(w) = \sqrt[q]{1 - \Theta_T^q(w) - \Psi_T^q(w)}$ represents the degree of indeterminacy. The pair of membership and nonmembership degrees $(\Theta_T(w), \Psi_T(w))$ is called a q-rung orthopair fuzzy number ($\mathbf{q}$-ROFN).

Definition 3 (see [12]). Let W be a universe of discourse. A complex q-rung orthopair fuzzy set T over the universe W can be elaborated as follows:

$$T = \left\{\left(w, \Theta_T(w)e^{i\Omega_T(w)}, \Psi_T(w)e^{i\Upsilon_T(w)}\right) \mid w \in W\right\}, \tag{3}$$

where $i = \sqrt{-1}$, and the amplitude terms $\Theta_T(w), \Psi_T(w) \in [0,1]$ and phase terms $\Omega_T(w), \Upsilon_T(w) \in [0, 2\pi]$ subjected to the conditions $0 \le \Theta_T^q(w) + \Psi_T^q(w) \le 1$ and $0 \le (\Omega_T(w)/2\pi)^q + (\Upsilon_T(w)/\ 2\pi)^q \le 1$. For all $w \in W$, $\mathfrak{A}_T(w) = \sqrt[q]{1 - \Theta_T^q(w) - \Psi_T^q(w)}e^{i2\pi\sqrt[q]{1-(\Omega_T(w)/2\pi)^q-(\Upsilon_T(w)/2\pi)^q}}$ represents the degree of indeterminacy. The pair of membership and nonmembership degrees $(\Theta_T(w)e^{i\Omega_T(w)}, \Psi_T(w)e^{i\Upsilon_T(w)})$ can be referred as a complex q-rung orthopair fuzzy number (C$_{\mathbf{q}}$-ROFN).

Definition 4 (see [21]). Let W be a universe of discourse and J be a collection of parameters, $M \subseteq J$. A pair $T = (\mathfrak{F}, M)$ is said to be a q-rung orthopair fuzzy soft set over W if $\mathfrak{F}: M \longrightarrow \mathbf{q} - \mathscr{ROF}(W)$, where $\mathbf{q} - \mathscr{ROF}(W)$ is the family of all $\mathbf{q}$-ROFSs over the universe W. A $\mathbf{q}$-ROFSS T over the universe W can be represented as follows:

$$T = \{(m_\alpha, \mathfrak{F}(m_\alpha)) \mid m_\alpha \in M, \mathfrak{F}(m_\alpha) \in \mathbf{q} - \mathscr{ROF}(W)\}, \tag{4}$$

where $\mathfrak{F}(m_\alpha) = \{(w_\eta, \Theta_{\eta\alpha}(w_\eta), \Psi_{\eta\alpha}(w_\eta)) \mid w_\eta \in W\}$ represents the $\mathbf{q}$-ROFS over the universe W. The membership $\Theta_{\eta\alpha}(w_\eta)$ and nonmembership $\Psi_{\eta\alpha}(w_\eta)$ degrees belong to the unit interval $[0,1]$ satisfying the condition $0 \le \Theta_{\eta\alpha}^q(w_\eta) + \Psi_{\eta\alpha}^q(w_\eta) \le 1$. The $\mathfrak{A}_{\eta\alpha}(w) = \sqrt[q]{1 - \Theta_{\eta\alpha}^q(w) - \Psi_{\eta\alpha}^q(w)}$ represents the degree of indeterminacy, for all $w \in W$. The pair of membership and nonmembership degrees $(\Theta_{\eta\alpha}(w), \Psi_{\eta\alpha}(w))$ is called a q-rung orthopair fuzzy soft number ($\mathbf{q}$-ROFSN).

Definition 5 (see [21]). The score function for any $\mathbf{q}$-ROFSN, $\mathfrak{B}_{\eta\alpha} = (\Theta_{\eta\alpha}, \Psi_{\eta\alpha})$ can be characterized as follows:

$$\mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) = \Theta_\eta^q - \Psi_\eta^q + \left(\frac{e^{\Theta_\eta^q - \Psi_\eta^q}}{e^{\Theta_\eta^q - \Psi_\eta^q} + 1} - \frac{1}{2}\right)\pi_\eta^q, \quad \text{for } q \ge 1, \tag{5}$$

where $\mathfrak{L}$ represents the score function of $\mathbf{q}$-ROFSN and $\mathfrak{L}(\mathfrak{B}_{\eta\alpha})$ lies inside the closed interval $[-1,1]$.

Definition 6 (see [21]). Let $\mathfrak{B}_{1\alpha} = (\Theta_{1\alpha}, \Psi_{1\alpha})(\alpha = 1,2)$ and $\mathfrak{B} = (\Theta, \Psi)$ be any three $\mathbf{q}$-ROFSNs and $\mathfrak{H} > 0$ be any real number. Then, the operations based on these three $\mathbf{q}$-ROFSNs can be explicated as follows:

(1) $\mathfrak{B}_{11} \oplus \mathfrak{B}_{12} = (\sqrt[q]{\Theta_{11}^q + \Theta_{12}^q - \Theta_{11}^q\Theta_{12}^q}, \Psi_{11}\Psi_{12})$

(2) $\mathfrak{B}_{11} \otimes \mathfrak{B}_{12} = (\Theta_{11}\Theta_{12}, \sqrt[q]{\Psi_{11}^q + \Psi_{12}^q - \Psi_{11}^q\Psi_{12}^q})$

(3) $\mathfrak{H}\mathfrak{B} = (\sqrt[q]{1 - (1 - \Theta^q)^{\mathfrak{H}}}, \Psi^{\mathfrak{H}})$

(4) $\mathfrak{B}^{\mathfrak{H}} = (\Theta^{\mathfrak{H}}, \sqrt[q]{1 - (1 - \Psi^q)^{\mathfrak{H}}})$

Definition 7 (see [28]). Let W be a universe of discourse and J be a collection of attributes. Let $M \subseteq J$ and $\mathscr{K} = \{0, 1, \ldots, N-1\}$ be a set of ordered grades, where $N \in \{2, 3, \ldots\}$. A triplet $T = (\wp, M, N)$ is said to be a N-soft set over the universe W if $\wp: M \longrightarrow 2^{W \times \mathscr{K}}$ having property that for each $m \in M$, there exist an unique dyad $(w, k_m) \in W \times \mathscr{K}$ such that $(w, k_m) \in \wp(m)$, $w \in W$, and $k_m \in \mathscr{K}$. The N-soft set T over the universe W can be explicated as follows:

$$T = \left\{(m_\alpha, \wp(m_\alpha)) \mid m_\alpha \in M, \wp(m_\alpha) \in 2^{W \times \mathscr{K}}\right\}. \tag{6}$$

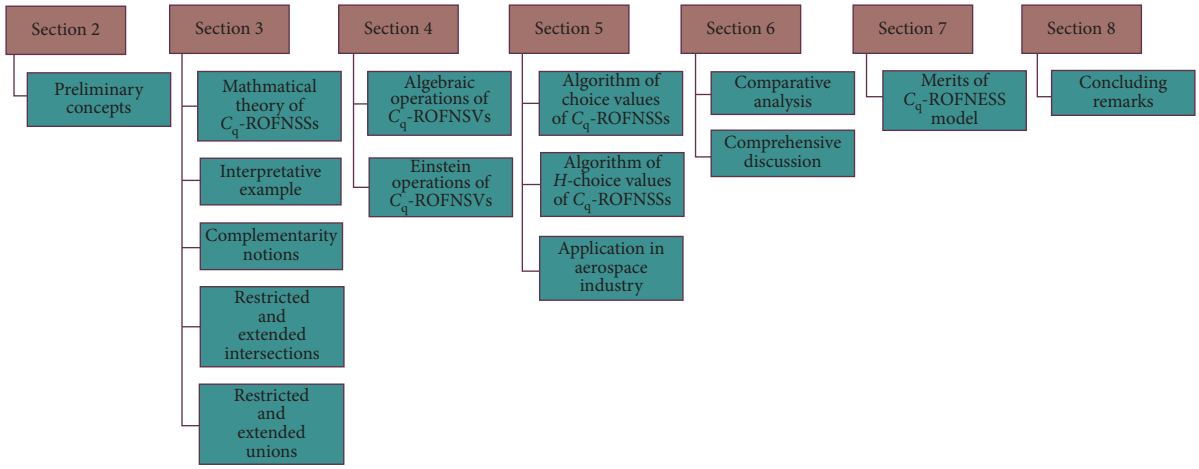

Figure 1: Graphical demonstration of proposed work.

3. Complex q-Rung Orthopair Fuzzy N-Soft Sets

Definition 8. Let W be a universe of discourse and J be a collection of attributes. Let $M \subseteq J$ and $K = \{0, 1, \ldots, N-1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triplet $\widetilde{T} = (p_q, Y, N)$ is said to be a complex q-rung orthopair fuzzy N-soft set on W, when $Y = (P, M, N)$ is NSS on the universe W and p_q: $M \longrightarrow \mathscr{C}_{\mathbf{q}} - \mathscr{R}\mathscr{O}\mathscr{F}^{W\times K}$, where $\mathscr{C}_{\mathbf{q}} - \mathscr{R}\mathscr{O}\mathscr{F}^{W\times K}$ is the collection of all complex q-rung orthopair fuzzy sets over $W \times K$. The $C_{\mathbf{q}}$-ROFNSS (p_q, Y, N) can be characterized in the following decorum:

$$\widetilde{T} = \left\{\left\langle m_\alpha, p_q(m_\alpha)\right\rangle \mid m_\alpha \in M, p_q(m_\alpha) \in \mathscr{C}_{\mathbf{q}} - \mathscr{R}\mathscr{O}\mathscr{F}^{W\times K}\right\}, \tag{7}$$

where $p_q(m_\alpha) = \{((w_\eta, k_{\eta\alpha}), \Theta_{\eta\alpha}(w_\eta, k_{\eta\alpha})e^{i\Omega_{\eta\alpha}(w_\eta,k_{\eta\alpha})}, \Psi_{\eta\alpha}(w_\eta, k_{\eta\alpha})e^{i\Upsilon_{\eta\alpha}(w_\eta,k_{\eta\alpha})}) \mid (w_\eta, k_{\eta\alpha}) \in W \times K\}$ represents the $C_{\mathbf{q}}$-ROFS over $W \times K$. The amplitude terms $\Theta_{\eta\alpha}(w_\eta, k_{\eta\alpha})$, $\Psi_{\eta\alpha}(w_\eta, k_{\eta\alpha})$ belong to unit interval $[0, 1]$ satisfying the condition

$$0 \le \Theta^q_{\eta\alpha}\left(w_\eta, k_{\eta\alpha}\right) + \Psi^q_{\eta\alpha}\left(w_\eta, k_{\eta\alpha}\right) \le 1, \tag{8}$$

and phase terms $\Omega_{\eta\alpha}(w_\eta, k_{\eta\alpha})$, $\Upsilon_{\eta\alpha}(w_\eta, k_{\eta\alpha})$ belong to closed interval $[0, 2\pi]$ subjected to the condition

$$0 \le \left(\frac{\Omega_{\eta\alpha}\left(w_\eta, k_{\eta\alpha}\right)}{2\pi}\right)^q + \left(\frac{\Upsilon_{\eta\alpha}\left(w_\eta, k_{\eta\alpha}\right)}{2\pi}\right)^q \le 1, \tag{9}$$

where $i = \sqrt{-1}$. For all $(w_\eta, k_{\eta\alpha}) \in W \times K$, the degree of indeterminacy can be explicated as follows:

$$\mathfrak{A}_{\eta\alpha}\left(w_\eta, k_{\eta\alpha}\right) = \left(1 - \Theta^q_{\eta\alpha}\left(w_\eta, k_{\eta\alpha}\right) - \Psi^q_{\eta\alpha}\left(w_\eta, k_{\eta\alpha}\right)\right)^{1/q} e^{i2\pi\left(1-\left(\Omega_{\eta\alpha}\left(w_\eta,k_{\eta\alpha}\right)/2\pi\right)^q-\left(\Upsilon_{\eta\alpha}\left(w_\eta,k_{\eta\alpha}\right)/2\pi\right)^q\right)^{1/q}}. \tag{10}$$

In other words, $C_{\mathbf{q}}$-ROFNSS is a ranking-based parameterized family of complex q-rung orthopair fuzzy sets of $W \times K$; that is, the mapping p_q assigns each parameter $m_\alpha \in M$ to a complex q-rung orthopair fuzzy set $p_q(m_\alpha)$ of $W \times K$.

Now, we present tabular representation of general complex q-rung orthopair fuzzy N-soft set:

Let W be a collection of $\wp$ objects and $\mathbb{M}$ be a set of $\hbar$ attributes, then the tabular characterization of $C_{\mathbf{q}}$-ROFNSS is illustrated in Table 1.

Definition 9. The $C_{\mathbf{q}}$-ROFNSS can be interpreted as $\wp \times \hbar$ table, where $\wp = |W|$, $\hbar = |M|$ whose $\eta\alpha$ th element is called a complex q-rung orthopair fuzzy N-soft value ($C_{\mathbf{q}}$-ROFNSV), and it has the representative form, which is interpreted as follows:

$$\mathfrak{B}_{\eta\alpha} = \left\langle k_{\eta\alpha}, \left(\Theta_{\eta\alpha}e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha}e^{i\Upsilon_{\eta\alpha}}\right)\right\rangle, \tag{11}$$

where $\eta = 1, 2, \ldots, \wp$; $\alpha = 1, 2, \ldots, \hbar$.

Definition 10. The score function $\mathfrak{L}$ for any $C_{\mathbf{q}}$-ROFNSV, $\mathfrak{B}_{\eta\alpha} = \left\langle k_{\eta\alpha}, (\Theta_{\eta\alpha}e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha}e^{i\Upsilon_{\eta\alpha}})\right\rangle$ can be demarcated as follows:

$$\mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) = \left(\Theta^q_{\eta\alpha} - \Psi^q_{\eta\alpha}\right) + \left(\left(\frac{\Omega_{\eta\alpha}}{2\pi}\right)^q - \left(\frac{\Upsilon_{\eta\alpha}}{2\pi}\right)^q\right) + \left(\frac{k_{\eta\alpha}}{N-1}\right)^q, \tag{12}$$

where $\mathfrak{L}(\mathfrak{B}_{\eta\alpha}) \in [-2, 3]$.

Definition 11. The accuracy function $\mathfrak{Q}$ for any $C_{\mathbf{q}}$-ROFNSV, $\mathfrak{B}_{\eta\alpha} = \left\langle k_{\eta\alpha}, (\Theta_{\eta\alpha}e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha}e^{i\Upsilon_{\eta\alpha}})\right\rangle$ can be explicated as follows:

Table 1: Tabular representation of C_q-ROFNSS.

(p_q, Y, N)	m_1	m_2	$\cdots$	m_h
w_1	$\lvert k_{11}, (\Theta_{11}e^{i\Omega_{11}}, \Psi_{11}e^{iY_{11}})\rvert$	$\langle k_{12}, (\Theta_{12}e^{i\Omega_{12}}, \Psi_{12}e^{iY_{12}})\rangle$	$\cdots$	$\langle k_{1h}, (\Theta_{1h}e^{i\Omega_{1h}}, \Psi_{1h}e^{iY_{1h}})\rangle$
w_2	$\langle k_{21}, (\Theta_{21}e^{i\Omega_{21}}, \Psi_{21}e^{iY_{21}})\rangle$	$\langle k_{22}, (\Theta_{22}e^{i\Omega_{22}}, \Psi_{22}e^{iY_{22}})\rangle$	$\cdots$	$\langle k_{2h}, (\Theta_{2h}e^{i\Omega_{2h}}, \Psi_{2h}e^{iY_{2h}})\rangle$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$w_\wp$	$\langle k_{\wp 1}, (\Theta_{\wp 1}e^{i\Omega_{\wp 1}}, \Psi_{\wp 1}e^{iY_{\wp 1}})\rangle$	$\langle k_{\wp 2}, (\Theta_{\wp 2}e^{i\Omega_{\wp 2}}, \Psi_{\wp 2}e^{iY_{\wp 2}})\rangle$	$\cdots$	$\langle k_{\wp h}, (\Theta_{\wp h}e^{i\Omega_{\wp h}}, \Psi_{\wp h}e^{iY_{\wp h}})\rangle$

$$\mathfrak{Q}\left(\mathfrak{B}_{\eta\alpha}\right) = \left(\Theta_{\eta\alpha}^{q} + \Psi_{\eta\alpha}^{q}\right) + \left(\left(\frac{\Omega_{\eta\alpha}}{2\pi}\right)^{q} + \left(\frac{Y_{\eta\alpha}}{2\pi}\right)^{q}\right) + \left(\frac{k_{\eta\alpha}}{N-1}\right)^{q}, \tag{13}$$

where $\mathfrak{Q}(\mathfrak{B}_{\eta\alpha}) \in [0, 5]$.

Definition 12. Let $\mathfrak{B}_{11} = \langle k_{11}, (\Theta_{11}e^{i\Omega_{11}}, \Psi_{11}e^{iY_{11}})\rangle$ and $\mathfrak{B}_{12} = \langle k_{12}, (\Theta_{12}e^{i\Omega_{12}}, \Psi_{12}e^{iY_{12}})\rangle$ be any two C_q-ROFNSVs, then the comparison between $\mathfrak{B}_{11}$ and $\mathfrak{B}_{12}$ can be determined as follows:

(1) If $\mathfrak{L}(\mathfrak{B}_{11}) < \mathfrak{L}(\mathfrak{B}_{12})$, then $\mathfrak{B}_{11} \prec \mathfrak{B}_{12}$ ($\mathfrak{B}_{11}$ is inferior to $\mathfrak{B}_{12}$)

(2) If $\mathfrak{L}(\mathfrak{B}_{11}) > \mathfrak{L}(\mathfrak{B}_{12})$, then $\mathfrak{B}_{11} \succ \mathfrak{B}_{12}$ ($\mathfrak{B}_{11}$ is superior to $\mathfrak{B}_{12}$)

(3) If $\mathfrak{L}(\mathfrak{B}_{11}) = \mathfrak{L}(\mathfrak{B}_{12})$, then

  (i) if $\mathfrak{Q}(\mathfrak{B}_{11}) < \mathfrak{Q}(\mathfrak{B}_{12})$, then $\mathfrak{B}_{11} \prec \mathfrak{B}_{12}$ ($\mathfrak{B}_{11}$ is inferior to $\mathfrak{B}_{12}$)

  (ii) if $\mathfrak{Q}(\mathfrak{B}_{11}) > \mathfrak{Q}(\mathfrak{B}_{12})$, then $\mathfrak{B}_{11} \succ \mathfrak{B}_{12}$ ($\mathfrak{B}_{11}$ is superior to $\mathfrak{B}_{12}$)

  (iii) if $\mathfrak{Q}(\mathfrak{B}_{11}) = \mathfrak{Q}(\mathfrak{B}_{12})$, then $\mathfrak{B}_{11} \sim \mathfrak{B}_{12}$ ($\mathfrak{B}_{11}$ is equivalent to $\mathfrak{B}_{12}$)

The motivation and prominence of this newly established model is clearly demonstrated by the following real-life practical application. In order to develop intuition for this advanced theory, this application has been kept comparatively simple. This empirical application interprets the deficiencies of the existing q-rung orthopair fuzzy N-soft model, proposed by Zhang et al. [36] and authenticates the remarkable effectuality of the proposed model in the current sophisticated decision-making problems.

Example 1. Spillways are the hydraulic structures constructed on the hydroelectric dams to provide a safe pathway for surplus floodwater to discharge from a reservoir to a downstream area. These are the most important part of dams that significantly protects the infrastructural integrity of the dam. Nowadays, the selection of the most efficient spillway is one of the most crucial decision-making problems. Suppose that a spillway has to be constructed on the rockfill dam in the north of Greece for the irrigation purpose. An appropriate spillway is determined by star rankings and ratings awarded by the designated experts of the selection board. These ratings are based on the proficiency of spillways in the last 5 years. Let $W = \{w_1 = \text{Chute spillway}, w_2 = \text{Shaft spillway}, w_3 = \text{Ogee spillway}, w_4 = \text{Siphon spillway}, \text{and } w_5 = \text{Labyrinth spillway}\}$ be a collection of five spillways and $M = \{m_1 = \text{Topography}, m_2 = \text{Construction cost}, m_3 = \text{Discharge rate, and } m_4 = \text{Reservoir capacity}\}$ be a set of four parameters which are employed to assign grades to these spillways.

A 5-soft set can easily be obtained from Table 2, where

(i) Four stars represent "Excellent"

(ii) Three stars represent "Superb"

(iii) Two stars represent "Good"

(iv) One star represents "Average"

(v) Big inverted delta represents "Poor"

The grades $K = \{0, 1, 2, 3, 4\}$ can be definitely associated with the ranking-based evaluation carried out by stars as follows:

(i) 0 stands for "▼"

(ii) 1 stands for "★"

(iii) 2 stands for "★★"

(iv) 3 stands for "★★★"

(v) 4 stands for "★★★★"

The appointed experts of the panel rank each spillway on the basis of their overall competency which is further presented in Table 2.

Now, the tabular representation of its corresponding identified 5-soft set is provided in Table 3.

The selection panel comprehensively evaluate all the identified spillways to determine their ratings by virtue of both membership $(\Theta_{\eta\alpha})$ and nonmembership degrees $(\Psi_{\eta\alpha})$. So, we extract a 3-rung orthopair fuzzy 5-soft set (3-ROF5SS) by following a certain grading criteria which is given as follows.

For all $w_\eta \in W$ and $q = 3$, $0 \le \Theta_{\eta\alpha}^3 + \Psi_{\eta\alpha}^3 \le 1$ with

$$\begin{aligned} &0.0 \le \Theta_{\eta\alpha}(w_\eta) < 0.2 \text{ when } k_{\eta\alpha} = 0,\\ &0.2 \le \Theta_{\eta\alpha}(w_\eta) < 0.4 \text{ when } k_{\eta\alpha} = 1,\\ &0.4 \le \Theta_{\eta\alpha}(w_\eta) < 0.6 \text{ when } k_{\eta\alpha} = 2,\\ &0.6 \le \Theta_{\eta\alpha}(w_\eta) < 0.8 \text{ when } k_{\eta\alpha} = 3,\\ &0.8 \le \Theta_{\eta\alpha}(w_\eta) \le 1.0 \text{ when } k_{\eta\alpha} = 4. \end{aligned} \tag{14}$$

Then, **3**-ROF5SS can be explicated as follows:

$$(p_3, Y, 5) = \{(m_1, p_3(m_1)), (m_2, p_3(m_2))), (m_3, p_3(m_3)), (m_4, p_3(m_4))\}, \tag{15}$$

where

Table 2: Ranking of spillways.

W/M	m_1	m_2	m_3	m_4
w_1	★★★★	★★★	▼	★★
w_2	★★	★	★★★★	★★★
w_3	▼	★★★	★★	▼
w_4	★★★	★★★★	▼	★★
w_5	★★	▼	★★★	★★★★

Table 3: Associated 5-soft set.

$(P, M, 5)$	m_1	m_2	m_3	m_4
w_1	4	3	0	2
w_2	2	1	4	3
w_3	0	3	2	0
w_4	3	4	1	2
w_5	2	0	3	4

$$\begin{aligned}
p_3(m_1) &= \{\langle(w_1,4),0.96,0.01\rangle, \langle(w_2,2),0.68,0.74\rangle, \langle(w_3,0),0.03,0.97\rangle,\\
&\quad \langle(w_4,3),0.86,0.19\rangle\}, \langle(w_5,2),0.16,0.72\rangle\},\\
p_3(m_2) &= \{\langle(w_1,3),0.45,0.14\rangle, \langle(w_2,1),0.45,0.57\rangle, \langle(w_3,3),0.68,0.11\rangle,\\
&\quad \langle(w_4,4),0.97,0.03\rangle\}, \langle(w_5,0),0.04,0.97\rangle\},\\
p_3(m_3) &= \{\langle(w_1,0),0.01,0.99\rangle, \langle(w_2,4),1.00,0.02\rangle, \langle(w_3,2),0.75,0.81\rangle,\\
&\quad \langle(w_4,1),0.43,0.55\rangle\}, \langle(w_5,3),0.84,0.16\rangle\},\\
p_3(m_4) &= \{\langle(w_1,2),0.56,0.63\rangle, \langle(w_2,3),0.76,0.26\rangle, \langle(w_3,0),0.05,0.98\rangle,\\
&\quad \langle(w_4,2),0.57,0.48\rangle\}, \langle(w_5,4),0.95,0.35\rangle\}.
\end{aligned} \tag{16}$$

The tabular representation of **3**-ROF5SS is demonstrated in Table 4.

The experts of the selection panel analyze all the inspected spillways and assign membership and nonmembership degrees $\Theta_{\eta\alpha}(w_\eta, k_{\eta\alpha})$, $\Psi_{\eta\alpha}(w_\eta, k_{\eta\alpha})$, respectively, to each spillway. Now, suppose an expert interprets that "The discharge rate of a spillway w_1 is very low in the first 3 years but dramatically increases in the last 2 years because of rapid climate change and global warming." Then, the values of $\Theta_{11}(w_1,4) = 0.96$ and $\Psi_{11}(w_1,4) = 0.01$ are paradoxical and all the information regarding the time frame of discharge rate would be lost which may badly collapse the infrastructure of spillway as well as corresponding dam also. To avoid such possible disaster, it is better to assign complex-valued membership and nonmembership $\Psi_{11}(w_1,4)$ degrees for the instant elaboration of all the information scrutinized by the expert. Hence, we establish complex 3-rung orthopair fuzzy 5-soft set (C_3-ROF5SS) rather than 3-rung orthopair fuzzy 5-soft set (3-ROF5SS) for the proper settlement of such deficiency.Now, the following values may be allocated to $\Theta_{11}(w_1,4)$ and $\Psi_{11}(w_1,4)$ in the light of redefined grading criteria:

$$\begin{aligned}
\Theta_{11}(w_1,4) &= 0.96e^{i1.98\pi},\\
\Psi_{11}(w_1,4) &= 0.01e^{i0.02\pi}.
\end{aligned} \tag{17}$$

Here, phase term illustrates all the time-dependent information regarding discharge rate of spillway under consideration. Therefore, C_3-ROF5SS $(p_3, Y, 5)$ is proposed by integrating the peculiarities of both 3-ROFS and 5-soft set theories for the two-dimensional ranking-based assessment of these investigated spillways. Now, we reshape the grading criteria on the basis of score degrees of C_3-ROF5SVs $\mathfrak{L}(\mathfrak{V}_{\eta\alpha})$, where $\mathfrak{V}_{\eta\alpha} = \left\langle k_{\eta\alpha}, (\Theta_{\eta\alpha}e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha}e^{iY_{\eta\alpha}})\right\rangle$, which can be defined as follows:

$$\begin{aligned}
-2.0 \le \mathfrak{L}\left(\mathfrak{V}_{\eta\alpha}\right) < -1.0 \text{ when } k_{\eta\alpha} = 0,\\
-1.0 \le \mathfrak{L}\left(\mathfrak{V}_{\eta\alpha}\right) < 0.0 \text{ when } k_{\eta\alpha} = 1,\\
0.0 \le \mathfrak{L}\left(\mathfrak{V}_{\eta\alpha}\right) < 1.0 \text{ when } k_{\eta\alpha} = 2,\\
1.0 \le \mathfrak{L}\left(\mathfrak{V}_{\eta\alpha}\right) < 2.0 \text{ when } k_{\eta\alpha} = 3,\\
2.0 \le \mathfrak{L}\left(\mathfrak{V}_{\eta\alpha}\right) \le 3.0 \text{ when } k_{\eta\alpha} = 4.
\end{aligned} \tag{18}$$

Finally, an advanced C_3-ROF5SS can be interpreted as follows:

$$\begin{aligned}
(p_3, Y, 5) = \{&(m_1, p_3(m_1)), (m_2, p_3(m_2))), (m_3, p_3(m_3)),\\
&(m_4, p_3(m_4))\},
\end{aligned} \tag{19}$$

where

Table 4: Tabular representation of 3-ROF5SS.

$(p_3, Y, 5)$	m_1	m_2	m_3	m_4
w_1	$\langle 4, (0.96, 0.01)\rangle$	$\langle 3, (0.45, 0.14)\rangle$	$\langle 0, (0.01, 0.99)\rangle$	$\langle 2, (0.56, 0.63)\rangle$
w_2	$\langle 2, (0.68, 0.74)\rangle$	$\langle 1, (0.45, 0.57)\rangle$	$\langle 4, (1.00, 0.02)\rangle$	$\langle 3, (0.76, 0.26)\rangle$
w_3	$\langle 0, (0.03, 0.97)\rangle$	$\langle 3, (0.68, 0.11)\rangle$	$\langle 2, (0.75, 0.81)\rangle$	$\langle 0, (0.05, 0.98)\rangle$
w_4	$\langle 3, (0.86, 0.19)\rangle$	$\langle 4, (0.97, 0.03)\rangle$	$\langle 1, (0.43, 0.55)\rangle$	$\langle 2, (0.57, 0.48)\rangle$
w_5	$\langle 2, (0.16, 0.72)\rangle$	$\langle 0, (0.04, 0.97)\rangle$	$\langle 3, (0.84, 0.16)\rangle$	$\langle 4, (0.95, 0.35)\rangle$

$$
\begin{aligned}
p_3(m_1) &= \left\{\left\langle (w_1, 4), 0.96e^{i1.98\pi}, 0.01e^{i0.02\pi}\right\rangle, \left\langle (w_2, 2), 0.68e^{i1.56\pi}, 0.74e^{i1.47\pi}\right\rangle, \left\langle (w_3, 0), 0.03e^{i0.02\pi}, 0.97e^{i1.94\pi}\right\rangle,\right.\\
&\quad \left.\left\langle (w_4, 3), 0.86e^{i1.75\pi}, 0.19e^{i0.07\pi}\right\rangle\right\}, \left\langle (w_5, 2), 0.16e^{i1.89\pi}, 0.72e^{i1.66\pi}\right\rangle\Big\},\\
p_3(m_2) &= \left\{\left\langle (w_1, 3), 0.45e^{i1.84\pi}, 0.14e^{i1.17\pi}\right\rangle, \left\langle (w_2, 1), 0.45e^{i0.34\pi}, 0.57e^{i0.45\pi}\right\rangle, \left\langle (w_3, 3), 0.68e^{i1.84\pi}, 0.11e^{i0.12\pi}\right\rangle,\right.\\
&\quad \left.\left\langle (w_4, 4), 0.97e^{i1.96\pi}, 0.03e^{i0.04\pi}\right\rangle\right\}, \left\langle (w_5, 0), 0.04e^{i0.02\pi}, 0.97e^{i1.99\pi}\right\rangle\Big\},\\
p_3(m_3) &= \left\{\left\langle (w_1, 0), 0.01e^{i0.04\pi}, 0.99e^{i1.95\pi}\right\rangle, \left\langle (w_2, 4), 1.00e^{i1.94\pi}, 0.02e^{i0.01\pi}\right\rangle, \left\langle (w_3, 2), 0.75e^{i1.45\pi}, 0.81e^{i1.37\pi}\right\rangle,\right.\\
&\quad \left.\left\langle (w_4, 1), 0.43e^{i0.42\pi}, 0.55e^{i0.56\pi}\right\rangle\right\}, \left\langle (w_5, 3), 0.84e^{i1.72\pi}, 0.16e^{i0.08\pi}\right\rangle\Big\},\\
p_3(m_4) &= \left\{\left\langle (w_1, 2), 0.56e^{i1.85\pi}, 0.63e^{i1.76\pi}\right\rangle, \left\langle (w_2, 3), 0.76e^{i1.79\pi}, 0.26e^{i0.04\pi}\right\rangle, \left\langle (w_3, 0), 0.05e^{i0.03\pi}, 0.98e^{i1.96\pi}\right\rangle,\right.\\
&\quad \left.\left\langle (w_4, 2), 0.57e^{i1.67\pi}, 0.48e^{i1.60\pi}\right\rangle\right\}, \left\langle (w_5, 4), 0.95e^{i1.98\pi}, 0.35e^{i0.08\pi}\right\rangle\Big\}.
\end{aligned}
\tag{20}
$$

Evidently, C_3-ROF5SS can be easily demonstrated in tabular form by Table 5.

Remark 1

(1) Any $C_{\mathbf{q}}$-ROF2SS $(p_q, Y, 2)$ can be definitely associated with complex q-rung orthopair fuzzy soft set ($C_{\mathbf{q}}$-ROFSS). We can generally identify the $C_{\mathbf{q}}$-ROF2SS p_q: $M \longrightarrow \mathscr{C}_{\mathbf{q}} - \mathscr{ROF}^{W\times\{0,1\}}$ through $C_{\mathbf{q}}$-ROFSS $\widetilde{p}$: $M \longrightarrow \mathscr{C}_{\mathbf{q}} - \mathscr{ROF}(W)$, which is characterized as follows:

$$
\widetilde{p}(m_\alpha) = \left\{\left(w_\eta, \Theta_{\eta\alpha}(w_\eta)e^{i\Omega_{\eta\alpha}(w_\eta)}, \Psi_{\eta\alpha}(w_\eta)e^{i\Upsilon_{\eta\alpha}(w_\eta)}\right) \mid \left\langle (w_\eta, 1), \Theta_{\eta\alpha}(w_\eta, 1)e^{i\Omega_{\eta\alpha}(w_\eta,1)}, \Psi_{\eta\alpha}(w_\eta, 1)e^{i\Upsilon_{\eta\alpha}(w_\eta,1)}\right\rangle \in p_q(m_\alpha)\right\},
\tag{21}
$$

for every $m_\alpha \in M$.

(2) A $C_{\mathbf{q}}$-ROFNSS (p_q, Y, N) over the universe W is said to be efficient if $Y = (P, M, N)$ is NSS on W and $\left\langle (w_\eta, N-1), \Theta(w_\eta, N-1)e^{i\Omega(w_\eta,N-1)}, \Psi(w_\eta, N-1)e^{i\Upsilon(w_\eta,N-1)}\right\rangle \in p_q(m_\alpha)$, for some $m_\alpha \in M$, $w_\eta \in W$.

(3) Grade $0 \in K$ in Definition 8 indicates the lowest score. It does not interpret that there is insufficient information or lack of proper evaluations.

We now proceed to elaborate the various fundamental notions of complementarity in the advanced framework of complex q-rung orthopair fuzzy N-soft sets.

Definition 13. Let (p_q, Y, N) be a $C_{\mathbf{q}}$-ROFNSS over the universe W, where $Y = (P, M, N)$ be its corresponding NSS on W. Then, the complex q-rung orthopair fuzzy complement $(p_q^{\mathfrak{c}}, Y, N)$ of (p_q, Y, N) can be explicated as

$$
\left(p_q^{\mathfrak{c}}, Y, N\right) = \left\{\left\langle m_\alpha, p_q^{\mathfrak{c}}(m_\alpha)\right\rangle \mid m_\alpha \in M, p_q^{\mathfrak{c}}(m_\alpha) \in \mathscr{C}_{\mathbf{q}} - \mathscr{ROF}^{W\times K}\right\},
\tag{22}
$$

where

$$
p_q^{\mathfrak{c}}(m_\alpha) = \left\{\left\langle (w_\eta, k_{\eta\alpha}), \Psi(w_\eta, k_{\eta\alpha})e^{i\Upsilon(w_\eta,k_{\eta\alpha})}, \Theta(w_\eta, k_{\eta\alpha})e^{i\Omega(w_\eta,k_{\eta\alpha})}\right\rangle \mid (w_\eta, k_{\eta\alpha}) \in W \times K\right\}.
\tag{23}
$$

Definition 14. Let (p_q, Y, N) be a $C_{\mathbf{q}}$-ROFNSS over the universe W, where $Y = (P, M, N)$ be its respective NSS on W. Then, the weak complex q-rung orthopair fuzzy complement $(p_q^{\mathfrak{c}}, Y^{\mathfrak{c}}, N)$ of (p_q, Y, N) is defined as

Table 5: Tabular representation of C_3-ROF5SS.

$(p_3, Y, 5)$	m_1	m_2	m_3	m_4
w_1	$\langle 4, (0.96e^{i1.98\pi}, 0.01e^{i0.02\pi})\rangle$	$\langle 3, (0.45e^{i1.84\pi}, 0.14e^{i1.17\pi})\rangle$	$\langle 0, (0.01e^{i0.04\pi}, 0.99e^{i1.95\pi})\rangle$	$\langle 2, (0.56e^{i1.85\pi}, 0.63e^{i1.76\pi})\rangle$
w_2	$\langle 2, (0.68e^{i1.56\pi}, 0.74e^{i1.47\pi})\rangle$	$\langle 1, (0.45e^{i0.34\pi}, 0.57e^{i0.45\pi})\rangle$	$\langle 4, (1.00e^{i1.94\pi}, 0.02e^{i0.01\pi})\rangle$	$\langle 3, (0.76e^{i1.79\pi}, 0.26e^{i0.04\pi})\rangle$
w_3	$\langle 0, (0.03e^{i0.02\pi}, 0.97e^{i1.94\pi})\rangle$	$\langle 3, (0.68e^{i1.84\pi}, 0.11e^{i0.12\pi})\rangle$	$\langle 2, (0.75e^{i1.45\pi}, 0.81e^{i1.37\pi})\rangle$	$\langle 0, (0.05e^{i0.03\pi}, 0.98e^{i1.96\pi})\rangle$
w_4	$\langle 3, (0.86e^{i1.75\pi}, 0.19e^{i0.07\pi})\rangle$	$\langle 4, (0.97e^{i1.96\pi}, 0.03e^{i0.04\pi})\rangle$	$\langle 1, (0.43e^{i0.42\pi}, 0.55e^{i0.56\pi})\rangle$	$\langle 2, (0.57e^{i1.67\pi}, 0.48e^{i1.60\pi})\rangle$
w_5	$\langle 2, (0.16e^{i1.89\pi}, 0.72e^{i1.66\pi})\rangle$	$\langle 0, (0.04e^{i0.02\pi}, 0.97e^{i1.99\pi})\rangle$	$\langle 3, (0.84e^{i1.72\pi}, 0.16e^{i0.08\pi})\rangle$	$\langle 4, (0.95e^{i1.98\pi}, 0.35e^{i0.08\pi})\rangle$

$$\left(p_q^{\mathfrak{c}}, Y^{\mathfrak{c}}, N\right) = \left\{\left\langle m_\alpha, p_q^{\mathfrak{c}}(m_\alpha)\right\rangle \mid m_\alpha \in M, p_q^{\mathfrak{c}}(m_\alpha) \in \mathscr{C}_{\mathbf{q}} - \mathscr{ROF}^{W\times K}\right\}, \tag{24}$$

where $p_q^{\mathfrak{c}}(m_\alpha) = \{\langle (w_\eta, k_{\eta\alpha}), \Psi(w_\eta, k_{\eta\alpha})e^{i\Upsilon(w_\eta,k_{\eta\alpha})}, \Theta(w_\eta, k_{\eta\alpha})e^{i(\Omega w_\eta,k_{\eta\alpha})}\rangle \mid (w_\eta, k_{\eta\alpha}) \in W \times K\}$ and for all $m_\alpha \in M$, $P^{\mathfrak{c}}(m_\alpha) \cap P(m_\alpha) = \varnothing$.

Example 2. Consider the C_3-ROF5SS as illustrated in Example 1. Then, its complex 3-rung orthopair fuzzy complement $(p_3^{\mathfrak{c}}, Y, 5)$ and weak complex 3-rung orthopair fuzzy complement $(p_3^{\mathfrak{c}}, Y^{\mathfrak{c}}, 5)$ are provided in Tables 6 and 7, respectively.

Definition 15. Let (p_q, Y, N) be a $C_{\mathbf{q}}$-ROFNSS over the universe W, where $Y = (P, M, N)$ be the corresponding NSS on W. Then, its top complex q-rung orthopair fuzzy weak complement, represented by $(p_q^{\mathfrak{t}}, Y^{\mathfrak{t}}, N)$, can be demarcated as

$$\left(p_q^{\mathfrak{t}}, Y^{\mathfrak{t}}, N\right) = \begin{cases} p_q^{\mathfrak{t}}(m_\alpha) = \left\langle (w_\eta, N-1), \Psi(w_\eta, k_{\eta\alpha})e^{i\Upsilon(w_\eta,k_{\eta\alpha})}, \Theta(w_\eta, k_{\eta\alpha})e^{i\Omega(w_\eta,k_{\eta\alpha})}\right\rangle, & \text{if } k_{\eta\alpha} < N-1, \\ p_q^{\mathfrak{t}}(m_\alpha) = \left\langle (w_\eta, 0), \Psi(w_\eta, k_{\eta\alpha})e^{i\Upsilon(w_\eta,k_{\eta\alpha})}, \Theta(w_\eta, k_{\eta\alpha})e^{i\Omega(w_\eta,k_{\eta\alpha})}\right\rangle, & \text{if } k_{\eta\alpha} = N-1. \end{cases} \tag{25}$$

Definition 16. Let (p_q, Y, N) be a $C_{\mathbf{q}}$-ROFNSS over the universe W, where $Y = (P, M, N)$is the respective NSS on W. Then, its bottom complex q-rung orthopair fuzzy weak complement, represented by $(p_q^{\mathfrak{b}}, Y^{\mathfrak{b}}, N)$, can be interpreted as

$$\left(p_q^{\mathfrak{b}}, Y^{\mathfrak{b}}, N\right) = \begin{cases} p_q^{\mathfrak{b}}(m_\alpha) = \left\langle (w_\eta, 0), \Psi(w_\eta, k_{\eta\alpha})e^{i\Upsilon(w_\eta,k_{\eta\alpha})}, \Theta(w_\eta, k_{\eta\alpha})e^{i\Omega(w_\eta,k_{\eta\alpha})}\right\rangle, & \text{if } k_{\eta\alpha} > 0, \\ p_q^{\mathfrak{b}}(m_\alpha) = \left\langle (w_\eta, N-1), \Psi(w_\eta, k_{\eta\alpha})e^{i\Upsilon(w_\eta,k_{\eta\alpha})}, \Theta(w_\eta, k_{\eta\alpha})e^{i\Omega(w_\eta,k_{\eta\alpha})}\right\rangle, & \text{if } k_{\eta\alpha} = 0. \end{cases} \tag{26}$$

Example 3. Consider the C_3-ROF5SS as provided in Example 1. Then, its top complex 3-rung orthopair fuzzy weak complement $(p_3^{\mathfrak{t}}, Y^{\mathfrak{t}}, 5)$ and bottom complex 3-rung orthopair fuzzy weak complement $(p_3^{\mathfrak{b}}, Y^{\mathfrak{b}}, 5)$ are summarized in Tables 8 and 9, respectively.

We now interpret two significant practicable approaches for the union and intersection of novel complex q-rung orthopair fuzzy N-soft sets.

Definition 17. Let (p_q^a, Y^a, N^a) and (p_q^b, Y^b, N^b) be two $C_{\mathbf{q}}$-ROFNSS over the universe of discourse W, where $Y^a = (P^a, F, N^a)$ and $Y^b = (P^b, G, N^b)$ be corresponding NSSs on W. Then, their restricted intersection symbolized by $(p_q^a, Y^a, N^a) \cap_{\mathscr{R}} (p_q^b, Y^b, N^b) = (z_q, Y^a \cap_{\mathrm{R}} Y^b, \min(N^a, N^b))$, where $Y^a \cap_{\mathrm{R}} Y^b = (Z, F \cap G, \min(N^a, N^b))$ can be characterized as follows:

$\forall m_\alpha \in F \cap G, w_\eta \in W$, $\langle (w_\eta, k_{\eta\alpha}), \mathrm{X}, \mathrm{Y}\rangle \in z_q(m_\alpha) \Leftrightarrow k_{\eta\alpha} = \min(k_{\eta\alpha}^a, k_{\eta\alpha}^b)$, $\mathrm{X} = \min(\Theta_S(w_\eta, k_{\eta\alpha}^a), \Theta_T(w_\eta, k_{\eta\alpha}^b))e^{i\min(\Omega_S(w_\eta,k_{\eta\alpha}^a),\Omega_T(w_\eta,k_{\eta\alpha}^b))}$, $\mathrm{Y} = \max(\Psi_S(w_\eta, k_{\eta\alpha}^a), \Psi_T(w_\eta, k_{\eta\alpha}^b))e^{i\max(\Upsilon_S(w_\eta,k_{\eta\alpha}^a),\Upsilon_T(w_\eta,k_{\eta\alpha}^b))}$, if $\langle (w_\eta, k_{\eta\alpha}^a), \Theta_S(w_\eta, k_{\eta\alpha}^a)e^{i\Omega_S(w_\eta,k_{\eta\alpha}^a)}, \Psi_S(w_\eta, k_{\eta\alpha}^a)e^{i\Upsilon_S(w_\eta,k_{\eta\alpha}^a)}\rangle \in p_q^a(m_\alpha)$ and $\langle (w_\eta, k_{\eta\alpha}^b), \Theta_T(w_\eta, k_{\eta\alpha}^b)e^{i\Omega_T(w_\eta,k_{\eta\alpha}^b)}, \Psi_T(w_\eta, k_{\eta\alpha}^b)e^{i\Upsilon_T(w_\eta,k_{\eta\alpha}^b)}\rangle \in p_q^b(m_\alpha)$, and S and T are $C_{\mathbf{q}}$-ROFSs on $p_q^a(m_\alpha)$ and $p_q^b(m_\alpha)$, respectively.

Definition 18. Let (p_q^a, Y^a, N^a) and (p_q^b, Y^b, N^b) be two $C_{\mathbf{q}}$-ROFNSSs over the universe of discourse W, where $Y^a = (P^a, F, N^a)$ and $Y^b = (P^b, G, N^b)$ are respective NSSs on W. Then, their extended intersection represented by $(p_q^a, Y^a, N^a) \cap_{\mathscr{E}} (p_q^b, Y^b, N^b) = (d_q, Y^a \cap_{\mathrm{E}} Y^b, \max(N^a,$

Table 6: Complex 3-rung orthopair fuzzy complement.

$(p_3^{\mathfrak{c}}, Y, 5)$	m_1	m_2	m_3	m_4
w_1	$\langle 4, (0.01e^{i0.02\pi}, 0.96e^{i1.98\pi})\rangle$	$\langle 3, (0.14e^{i1.17\pi}, 0.45e^{i1.84\pi})\rangle$	$\langle 0, (0.99e^{i1.95\pi}, 0.01e^{i0.04\pi})\rangle$	$\langle 2, (0.63e^{i1.76\pi}, 0.56e^{i1.85\pi})\rangle$
w_2	$\langle 2, (0.74e^{i1.47\pi}, 0.68e^{i1.56\pi})\rangle$	$\langle 1, (0.57e^{i0.45\pi}, 0.45e^{i0.34\pi})\rangle$	$\langle 4, (0.02e^{i0.01\pi}, 1.00e^{i1.94\pi})\rangle$	$\langle 3, (0.26e^{i0.04\pi}, 0.76e^{i1.79\pi})\rangle$
w_3	$\langle 0, (0.97e^{i1.94\pi}, 0.03e^{i0.02\pi})\rangle$	$\langle 3, (0.11e^{i0.12\pi}, 0.68e^{i1.84\pi})\rangle$	$\langle 2, (0.81e^{i1.37\pi}, 0.75e^{i1.45\pi})\rangle$	$\langle 0, (0.98e^{i1.96\pi}, 0.05e^{i0.03\pi})\rangle$
w_4	$\langle 3, (0.19e^{i0.07\pi}, 0.86e^{i1.75\pi})\rangle$	$\langle 4, (0.03e^{i0.04\pi}, 0.97e^{i1.96\pi})\rangle$	$\langle 1, (0.55e^{i0.56\pi}, 0.43e^{i0.42\pi})\rangle$	$\langle 2, (0.48e^{i1.60\pi}, 0.57e^{i1.67\pi})\rangle$
w_5	$\langle 2, (0.72e^{i1.66\pi}, 0.16e^{i1.89\pi})\rangle$	$\langle 0, (0.97e^{i1.99\pi}, 0.04e^{i0.02\pi})\rangle$	$\langle 3, (0.16e^{i0.08\pi}, 0.84e^{i1.72\pi})\rangle$	$\langle 4, (0.35e^{i0.08\pi}, 0.95e^{i1.98\pi})\rangle$

Table 7: Weak complex 3-rung orthopair fuzzy complement.

$(p_3^{\mathfrak{c}}, Y^{\mathfrak{c}}, 5)$	m_1	m_2	m_3	m_4
w_1	$\langle 1, (0.01e^{i0.02\pi}, 0.96e^{i1.98\pi})\rangle$	$\langle 2, (0.14e^{i1.17\pi}, 0.45e^{i1.84\pi})\rangle$	$\langle 1, (0.99e^{i1.95\pi}, 0.01e^{i0.04\pi})\rangle$	$\langle 3, (0.63e^{i1.76\pi}, 0.56e^{i1.85\pi})\rangle$
w_2	$\langle 3, (0.74e^{i1.47\pi}, 0.68e^{i1.56\pi})\rangle$	$\langle 2, (0.57e^{i0.45\pi}, 0.45e^{i0.34\pi})\rangle$	$\langle 3, (0.02e^{i0.01\pi}, 1.00e^{i1.94\pi})\rangle$	$\langle 4, (0.26e^{i0.04\pi}, 0.76e^{i1.79\pi})\rangle$
w_3	$\langle 2, (0.97e^{i1.94\pi}, 0.03e^{i0.02\pi})\rangle$	$\langle 4, (0.11e^{i0.12\pi}, 0.68e^{i1.84\pi})\rangle$	$\langle 0, (0.81e^{i1.37\pi}, 0.75e^{i1.45\pi})\rangle$	$\langle 1, (0.98e^{i1.96\pi}, 0.05e^{i0.03\pi})\rangle$
w_4	$\langle 4, (0.19e^{i0.07\pi}, 0.86e^{i1.75\pi})\rangle$	$\langle 3, (0.03e^{i0.04\pi}, 0.97e^{i1.96\pi})\rangle$	$\langle 2, (0.55e^{i0.56\pi}, 0.43e^{i0.42\pi})\rangle$	$\langle 0, (0.48e^{i1.60\pi}, 0.57e^{i1.67\pi})\rangle$
w_5	$\langle 3, (0.72e^{i1.66\pi}, 0.16e^{i1.89\pi})\rangle$	$\langle 1, (0.97e^{i1.99\pi}, 0.04e^{i0.02\pi})\rangle$	$\langle 4, (0.16e^{i0.08\pi}, 0.84e^{i1.72\pi})\rangle$	$\langle 3, (0.35e^{i0.08\pi}, 0.95e^{i1.98\pi})\rangle$

Table 8: Top complex 3-rung orthopair fuzzy weak complement.

$(p_3^{\mathfrak{t}}, Y^{\mathfrak{t}}, 5)$	m_1	m_2	m_3	m_4
w_1	$\langle 0, (0.01e^{i0.02\pi}, 0.96e^{i1.98\pi})\rangle$	$\langle 4, (0.14e^{i1.17\pi}, 0.45e^{i1.84\pi})\rangle$	$\langle 4, (0.99e^{i1.95\pi}, 0.01e^{i0.04\pi})\rangle$	$\langle 4, (0.63e^{i1.76\pi}, 0.56e^{i1.85\pi})\rangle$
w_2	$\langle 4, (0.74e^{i1.47\pi}, 0.68e^{i1.56\pi})\rangle$	$\langle 4, (0.57e^{i0.45\pi}, 0.45e^{i0.34\pi})\rangle$	$\langle 0, (0.02e^{i0.01\pi}, 1.00e^{i1.94\pi})\rangle$	$\langle 4, (0.26e^{i0.04\pi}, 0.76e^{i1.79\pi})\rangle$
w_3	$\langle 4, (0.97e^{i1.94\pi}, 0.03e^{i0.02\pi})\rangle$	$\langle 4, (0.11e^{i0.12\pi}, 0.68e^{i1.84\pi})\rangle$	$\langle 4, (0.81e^{i1.37\pi}, 0.75e^{i1.45\pi})\rangle$	$\langle 4, (0.98e^{i1.96\pi}, 0.05e^{i0.03\pi})\rangle$
w_4	$\langle 4, (0.19e^{i0.07\pi}, 0.86e^{i1.75\pi})\rangle$	$\langle 0, (0.03e^{i0.04\pi}, 0.97e^{i1.96\pi})\rangle$	$\langle 4, (0.55e^{i0.56\pi}, 0.43e^{i0.42\pi})\rangle$	$\langle 4, (0.48e^{i1.60\pi}, 0.57e^{i1.67\pi})\rangle$
w_5	$\langle 4, (0.72e^{i1.66\pi}, 0.16e^{i1.89\pi})\rangle$	$\langle 4, (0.97e^{i1.99\pi}, 0.04e^{i0.02\pi})\rangle$	$\langle 4, (0.16e^{i0.08\pi}, 0.84e^{i1.72\pi})\rangle$	$\langle 0, (0.35e^{i0.08\pi}, 0.95e^{i1.98\pi})\rangle$

Table 9: Bottom complex 3-rung orthopair fuzzy weak complement.

$(p_3^{\mathfrak{b}}, Y^{\mathfrak{b}}, 5)$	m_1	m_2	m_3	m_4
w_1	$\langle 0, (0.01e^{i0.02\pi}, 0.96e^{i1.98\pi})\rangle$	$\langle 0, (0.14e^{i1.17\pi}, 0.45e^{i1.84\pi})\rangle$	$\langle 4, (0.99e^{i1.95\pi}, 0.01e^{i0.04\pi})\rangle$	$\langle 0, (0.63e^{i1.76\pi}, 0.56e^{i1.85\pi})\rangle$
w_2	$\langle 0, (0.74e^{i1.47\pi}, 0.68e^{i1.56\pi})\rangle$	$\langle 0, (0.57e^{i0.45\pi}, 0.45e^{i0.34\pi})\rangle$	$\langle 0, (0.02e^{i0.01\pi}, 1.00e^{i1.94\pi})\rangle$	$\langle 0, (0.26e^{i0.04\pi}, 0.76e^{i1.79\pi})\rangle$
w_3	$\langle 4, (0.97e^{i1.94\pi}, 0.03e^{i0.02\pi})\rangle$	$\langle 0, (0.11e^{i0.12\pi}, 0.68e^{i1.84\pi})\rangle$	$\langle 0, (0.81e^{i1.37\pi}, 0.75e^{i1.45\pi})\rangle$	$\langle 4, (0.98e^{i1.96\pi}, 0.05e^{i0.03\pi})\rangle$
w_4	$\langle 0, (0.19e^{i0.07\pi}, 0.86e^{i1.75\pi})\rangle$	$\langle 0, (0.03e^{i0.04\pi}, 0.97e^{i1.96\pi})\rangle$	$\langle 0, (0.55e^{i0.56\pi}, 0.43e^{i0.42\pi})\rangle$	$\langle 0, (0.48e^{i1.60\pi}, 0.57e^{i1.67\pi})\rangle$
w_5	$\langle 0, (0.72e^{i1.66\pi}, 0.16e^{i1.89\pi})\rangle$	$\langle 4, (0.97e^{i1.99\pi}, 0.04e^{i0.02\pi})\rangle$	$\langle 0, (0.16e^{i0.08\pi}, 0.84e^{i1.72\pi})\rangle$	$\langle 0, (0.35e^{i0.08\pi}, 0.95e^{i1.98\pi})\rangle$

$N^b))$, where $Y^a \cap_{\mathrm{E}} Y^b = (D, F \cup G, \max(N^a, N^b))$ can be delineated as

$$
d_q(m_\alpha) = \begin{cases} p_q^a(m_\alpha), & \text{if } m_\alpha \in F - G, \\ p_q^b(m_\alpha), & \text{if } m_\alpha \in G - F, \\ \left\langle \left(w_\eta, k_{\eta\alpha}\right), \mathcal{X}, \mathcal{Y} \right\rangle, & \begin{array}{l} \text{such that } k_{\eta\alpha} = \min\left(k_{\eta\alpha}^a, k_{\eta\alpha}^b\right), \\ \mathcal{X} = \min\left(\Theta_S\left(w_\eta, k_{\eta\alpha}^a\right), \Theta_T\left(w_\eta, k_{\eta\alpha}^b\right)\right) e^{i \min\left(\Omega_S\left(w_\eta, k_{\eta\alpha}^a\right), \Omega_T\left(w_\eta, k_{\eta\alpha}^b\right)\right)}, \\ \mathcal{Y} = \max\left(\Psi_S\left(w_\eta, k_{\eta\alpha}^a\right), \Psi_T\left(w_\eta, k_{\eta\alpha}^b\right)\right) e^{i \max\left(\Upsilon_S\left(w_\eta, k_{\eta\alpha}^a\right), \Upsilon_T\left(w_\eta, k_{\eta\alpha}^b\right)\right)}, \\ \text{where } \left\langle \left(w_\eta, k_{\eta\alpha}^a\right), \Theta_S\left(w_\eta, k_{\eta\alpha}^a\right) e^{i\Omega_S\left(w_\eta, k_{\eta\alpha}^a\right)}, \Psi_S\left(w_\eta, k_{\eta\alpha}^a\right) e^{i\Upsilon_S\left(w_\eta, k_{\eta\alpha}^a\right)} \right\rangle \in p_q^a(m_\alpha), \\ \text{and } \left\langle \left(w_\eta, k_{\eta\alpha}^b\right), \Theta_T\left(w_\eta, k_{\eta\alpha}^b\right) e^{i\Omega_T\left(w_\eta, k_{\eta\alpha}^b\right)}, \Psi_T\left(w_\eta, k_{\eta\alpha}^b\right) e^{i\Upsilon_T\left(w_\eta, k_{\eta\alpha}^b\right)} \right\rangle \in p_q^b(m_\alpha), \\ S \text{ and } T \text{ are } \mathrm{C}_{\mathbf{q}} - \mathrm{ROFSs} \text{ on } p_q^a(m_\alpha) \text{ and } p_q^b(m_\alpha), \text{ respectively.} \end{array} \end{cases} \tag{27}
$$

Example 4. Consider the tabular form of C_2-ROF6SS $(p_2^a, Y^a, 6)$ and C_2-ROF4SS $(p_2^b, Y^b, 4)$ as presented in Tables 10 and 11, respectively, where $Y^a = (P^a, F, 6)$ and $Y^b = (P^b, G, 4)$ are 6-soft set and 4-soft set over the universe W, respectively. Then, their restricted intersection $(z_2, Y^a \cap_R Y^b, 4)$ and extended intersection $(d_2, Y^a \cap_E Y^b, 6)$ are encapsulated in Tables 12 and 13, respectively.

Definition 19. Let (p_q^a, Y^a, N^a) and (p_q^b, Y^b, N^b) be two $C_\mathbf{q}$-ROFNSSs over the universe of discourse W, where $Y^a = (P^a, F, N^a)$ and $Y^b = (P^b, G, N^b)$ be corresponding NSSs on W. Then, their restricted union symbolized by $(p_q^a, Y^a, N^a) \cup_{\mathscr{R}} (p_q^b, Y^b, N^b) = (z_q, Y^a \cup_R Y^b, \max(N^a, N^b))$, where $Y^a \cup_R Y^b = (Z, F \cap G, \max(N^a, N^b))$ can be explicated as follows:

$\forall m_\alpha \in F \cap G, w_\eta \in W, \ \langle (w_\eta, k_{\eta\alpha}), \mathrm{X}, \mathrm{Y} \rangle \in z_q(m_\alpha) \Leftrightarrow k_{\eta\alpha} = \max(k_{\eta\alpha}^a, k_{\eta\alpha}^b), \quad \mathrm{X} = \max(\Theta_S(w_\eta, k_{\eta\alpha}^a), \Theta_T(w_\eta, k_{\eta\alpha}^b))$, $e^{i\max(\Omega_S(w_\eta, k_{\eta\alpha}^a), \Omega_T(w_\eta, k_{\eta\alpha}^b))}$, $\mathrm{Y} = \min(\Psi_S(w_\eta, k_{\eta\alpha}^a), \Psi_T(w_\eta, k_{\eta\alpha}^b))$ $e^{i\min(\Upsilon_S(w_\eta, k_{\eta\alpha}^a), \Upsilon_T(w_\eta, k_{\eta\alpha}^b))}$, if $\langle (w_\eta, k_{\eta\alpha}^a), \Theta_S(w_\eta, k_{\eta\alpha}^a) e^{i\Omega_S(w_\eta, k_{\eta\alpha}^a)}, \Psi_S(w_\eta, k_{\eta\alpha}^a) e^{i\Upsilon_S(w_\eta, k_{\eta\alpha}^a)} \rangle \in p_q^a(m_\alpha)$ and $\langle (w_\eta, k_{\eta\alpha}^b), \Theta_T(w_\eta, k_{\eta\alpha}^b) e^{i\Omega_T(w_\eta, k_{\eta\alpha}^b)}, \Psi_T(w_\eta, k_{\eta\alpha}^b) e^{i\Upsilon_T(w_\eta, k_{\eta\alpha}^b)} \rangle \in p_q^b(m_\alpha)$, and S and T are $C_\mathbf{q}$-ROFSs on $p_q^a(m_\alpha)$ and $p_q^b(m_\alpha)$, respectively.

Definition 20. Let (p_q^a, Y^a, N^a) and (p_q^b, Y^b, N^b) be two $C_\mathbf{q}$-ROFNSSs over the universe of discourse W, where $Y^a = (P^a, F, N^a)$ and $Y^b = (P^b, G, N^b)$ are respective NSSs on W. Then, their extended union represented by $(p_q^a, Y^a, N^a) \cup_{\mathscr{E}} (p_q^b, Y^b, N^b) = (d_q, Y^a \cup_E Y^b, \max(N^a, N^b))$, where $Y^a \cup_E Y^b = (D, F \cup G, \max(N^a, N^b))$ is interpreted as follows:

$$d_q(m_\alpha) = \begin{cases} p_q^a(m_\alpha), & \text{if } m_\alpha \in F - G, \\ p_q^b(m_\alpha), & \text{if } m_\alpha \in G - F, \\ \langle (w_\eta, k_{\eta\alpha}), \mathscr{X}, \mathscr{Y} \rangle, & \begin{array}{l} \text{such that } k_{\eta\alpha} = \max\left(k_{\eta\alpha}^a, k_{\eta\alpha}^b\right), \\ \mathscr{X} = \max\left(\Theta_S\left(w_\eta, k_{\eta\alpha}^a\right), \Theta_T\left(w_\eta, k_{\eta\alpha}^b\right)\right) e^{i\max\left(\Omega_S\left(w_\eta, k_{\eta\alpha}^a\right), \Omega_T\left(w_\eta, k_{\eta\alpha}^b\right)\right)}, \\ \mathscr{Y} = \min\left(\Psi_S\left(w_\eta, k_{\eta\alpha}^a\right), \Psi_T\left(w_\eta, k_{\eta\alpha}^b\right)\right) e^{i\min\left(\Upsilon_S\left(w_\eta, k_{\eta\alpha}^a\right), \Upsilon_T\left(w_\eta, k_{\eta\alpha}^b\right)\right)}, \\ \text{where } \left\langle \left(w_\eta, k_{\eta\alpha}^a\right), \Theta_S\left(w_\eta, k_{\eta\alpha}^a\right) e^{i\Omega_S\left(w_\eta, k_{\eta\alpha}^a\right)}, \Psi_S\left(w_\eta, k_{\eta\alpha}^a\right) e^{i\Upsilon_S\left(w_\eta, k_{\eta\alpha}^a\right)} \right\rangle \in p_q^a(m_\alpha) \\ \text{and } \left\langle \left(w_\eta, k_{\eta\alpha}^b\right), \Theta_T\left(w_\eta, k_{\eta\alpha}^b\right) e^{i\Omega_T\left(w_\eta, k_{\eta\alpha}^b\right)}, \Psi_T\left(w_\eta, k_{\eta\alpha}^b\right) e^{i\Upsilon_T\left(w_\eta, k_{\eta\alpha}^b\right)} \right\rangle \in p_q^b(m_\alpha), \\ S \text{ and } T \text{ are } C_\mathbf{q} - \text{ROFSs on } p_q^a(m_\alpha) \text{ and } p_q^b(m_\alpha), \text{respectively.} \end{array} \end{cases} \tag{28}$$

Example 5. Consider the C_2-ROF6SS and C_2-ROF4SS as defined in Example 4. Then, their restricted union $(z_2, Y^a \cup_R Y^b, 6)$ and extended union $(d_2, Y^a \cup_E Y^b, 6)$ are illustrated in Tables 14 and 15, respectively.

Remark 2. The Definitions 17–20 of extended (or restricted) union and intersection will suffice the defined grading criteria if and only if $N^a = N^b$.

The notion of $C_\mathbf{q}$-ROFNSS can be easily identified with contemporary theories including complex q-rung orthopair fuzzy soft set ($C_\mathbf{q}$-ROFSS), q-rung orthopair fuzzy N-soft set, N-soft set, and soft set. Now, we are going to derive $C_\mathbf{q}$-ROFSS and SS from the proposed $C_\mathbf{q}$-ROFNSS by employing the following definitions:

Definition 21. Let (p_q, Y, N) be a $C_\mathbf{q}$-ROFNSS over a universe of discourse W, where $Y = (P, M, N)$ be its respective NSS on W. Let $0 < H < N$ be a threshold. Then, $C_\mathbf{q}$-ROFSS over the universe W associated with (p_q, Y, N) and H is (p_q^H, M), which can be interpreted as follows:

$$p_q^H(m_\alpha) = \begin{cases} \left\langle \Theta_{\eta\alpha} e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha} e^{i\Upsilon_{\eta\alpha}} \right\rangle, & \text{if } \left\langle k_{\eta\alpha}, \left(\Theta_{\eta\alpha} e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha} e^{i\Upsilon_{\eta\alpha}}\right) \right\rangle \in p_q(m_\alpha) \text{ and } k_{\eta\alpha} \geq H, \\ \begin{cases} \left\langle 0.00 e^{i0.00\pi}, 0.50 e^{i1.00\pi} \right\rangle, & \text{if } \dfrac{k_{\eta\alpha}}{N} \geq 0.50, \\ \left\langle 0.00 e^{i0.00\pi}, 1.00 e^{i2.00\pi} \right\rangle, & \text{if } \dfrac{k_{\eta\alpha}}{N} < 0.50. \end{cases} \end{cases} \tag{29}$$

Table 10: C_2-ROF6SS.

$(p_2^a, Y^a, 6)$	m_1	m_2	m_3
w_1	$\langle 4, (0.96e^{i1.84\pi}, 0.15e^{i0.12\pi})\rangle$	$\langle 3, (0.74e^{i1.45\pi}, 0.36e^{i0.69\pi})\rangle$	$\langle 2, (0.53e^{i1.32\pi}, 0.46e^{i0.83\pi})\rangle$
w_2	$\langle 5, (0.98e^{i1.97\pi}, 0.01e^{i0.02\pi})\rangle$	$\langle 1, (0.15e^{i0.28\pi}, 0.92e^{i1.78\pi})\rangle$	$\langle 0, (0.01e^{i0.03\pi}, 0.97e^{i1.95\pi})\rangle$
w_3	$\langle 3, (0.82e^{i1.44\pi}, 0.38e^{i0.62\pi})\rangle$	$\langle 0, (0.02e^{i0.01\pi}, 0.99e^{i1.96\pi})\rangle$	$\langle 3, (0.79e^{i1.46\pi}, 0.42e^{i0.57\pi})\rangle$
w_4	$\langle 2, (0.58e^{i1.25\pi}, 0.43e^{i0.76\pi})\rangle$	$\langle 2, (0.65e^{i1.33\pi}, 0.48e^{i0.86\pi})\rangle$	$\langle 5, (0.95e^{i1.99\pi}, 0.02e^{i0.03\pi})\rangle$

Table 11: C_2-ROF4SS.

$(p_2^b, Y^b, 4)$	m_1	m_2	m_4
w_1	$\langle 2, (0.56e^{i1.99\pi}, 0.12e^{i0.26\pi})\rangle$	$\langle 1, (0.12e^{i0.22\pi}, 0.88e^{i0.97\pi})\rangle$	$\langle 3, (0.83e^{i1.69\pi}, 0.24e^{i0.14\pi})\rangle$
w_2	$\langle 3, (0.91e^{i1.85\pi}, 0.32e^{i0.06\pi})\rangle$	$\langle 2, (0.68e^{i1.95\pi}, 0.05e^{i0.24\pi})\rangle$	$\langle 0, (0.04e^{i0.01\pi}, 0.97e^{i1.98\pi})\rangle$
w_3	$\langle 0, (0.11e^{i0.04\pi}, 0.96e^{i1.99\pi})\rangle$	$\langle 2, (0.93e^{i0.97\pi}, 0.52e^{i0.04\pi})\rangle$	$\langle 2, (0.79e^{i1.78\pi}, 0.13e^{i0.23\pi})\rangle$
w_4	$\langle 1, (0.23e^{i0.15\pi}, 0.96e^{i1.86\pi})\rangle$	$\langle 3, (0.88e^{i1.93\pi}, 0.16e^{i0.12\pi})\rangle$	$\langle 1, (0.25e^{i0.13\pi}, 0.91e^{i1.89\pi})\rangle$

Table 12: Restricted intersection.

$(z_2, Y^a \cap_R Y^b, 4)$	m_1	m_2
w_1	$\langle 2, (0.56e^{i1.84\pi}, 0.15e^{i0.26\pi})\rangle$	$\langle 1, (0.12e^{i0.22\pi}, 0.88e^{i0.97\pi})\rangle$
w_2	$\langle 3, (0.91e^{i1.85\pi}, 0.32e^{i0.06\pi})\rangle$	$\langle 1, (0.15e^{i0.28\pi}, 0.92e^{i1.78\pi})\rangle$
w_3	$\langle 0, (0.11e^{i0.04\pi}, 0.96e^{i1.99\pi})\rangle$	$\langle 0, (0.02e^{i0.01\pi}, 0.99e^{i1.96\pi})\rangle$
w_4	$\langle 1, (0.23e^{i0.15\pi}, 0.96e^{i1.86\pi})\rangle$	$\langle 2, (0.65e^{i1.33\pi}, 0.48e^{i0.86\pi})\rangle$

Table 13: Extended intersection.

$(d_2, Y^a \cap_E Y^b, 6)$	m_1	m_2	m_3	m_4
w_1	$\langle 2, (0.56e^{i1.84\pi}, 0.15e^{i0.26\pi})\rangle$	$\langle 1, (0.12e^{i0.22\pi}, 0.88e^{i0.97\pi})\rangle$	$\langle 2, (0.53e^{i1.32\pi}, 0.46e^{i0.83\pi})\rangle$	$\langle 3, (0.83e^{i1.69\pi}, 0.24e^{i0.14\pi})\rangle$
w_2	$\langle 3, (0.91e^{i1.85\pi}, 0.32e^{i0.06\pi})\rangle$	$\langle 1, (0.15e^{i0.28\pi}, 0.92e^{i1.78\pi})\rangle$	$\langle 0, (0.01e^{i0.03\pi}, 0.97e^{i1.95\pi})\rangle$	$\langle 0, (0.04e^{i0.01\pi}, 0.97e^{i1.98\pi})\rangle$
w_3	$\langle 0, (0.11e^{i0.04\pi}, 0.96e^{i1.99\pi})\rangle$	$\langle 0, (0.02e^{i0.01\pi}, 0.99e^{i1.96\pi})\rangle$	$\langle 3, (0.79e^{i1.46\pi}, 0.42e^{i0.57\pi})\rangle$	$\langle 2, (0.79e^{i1.78\pi}, 0.13e^{i0.23\pi})\rangle$
w_4	$\langle 1, (0.23e^{i0.15\pi}, 0.96e^{i1.86\pi})\rangle$	$\langle 2, (0.65e^{i1.33\pi}, 0.48e^{i0.86\pi})\rangle$	$\langle 5, (0.95e^{i1.99\pi}, 0.02e^{i0.03\pi})\rangle$	$\langle 1, (0.25e^{i0.13\pi}, 0.91e^{i1.89\pi})\rangle$

Table 14: Restricted union.

$(z_2, Y^a \cup_R Y^b, 6)$	m_1	m_2
w_1	$\langle 4, (0.96e^{i1.99\pi}, 0.12e^{i0.12\pi})\rangle$	$\langle 3, (0.74e^{i1.45\pi}, 0.36e^{i0.69\pi})\rangle$
w_2	$\langle 5, (0.98e^{i1.97\pi}, 0.01e^{i0.02\pi})\rangle$	$\langle 2, (0.68e^{i1.95\pi}, 0.05e^{i0.24\pi})\rangle$
w_3	$\langle 3, (0.82e^{i1.44\pi}, 0.38e^{i0.62\pi})\rangle$	$\langle 2, (0.93e^{i0.97\pi}, 0.52e^{i0.04\pi})\rangle$
w_4	$\langle 2, (0.58e^{i1.25\pi}, 0.43e^{i0.76\pi})\rangle$	$\langle 3, (0.88e^{i1.93\pi}, 0.16e^{i0.12\pi})\rangle$

Table 15: Extended union.

$(d_2, Y^a \cup_E Y^b, 6)$	m_1	m_2	m_3	m_4
w_1	$\langle 4, (0.96e^{i1.99\pi}, 0.12e^{i0.12\pi})\rangle$	$\langle 3, (0.74e^{i1.45\pi}, 0.36e^{i0.69\pi})\rangle$	$\langle 2, (0.53e^{i1.32\pi}, 0.46e^{i0.83\pi})\rangle$	$\langle 3, (0.83e^{i1.69\pi}, 0.24e^{i0.14\pi})\rangle$
w_2	$\langle 5, (0.98e^{i1.97\pi}, 0.01e^{i0.02\pi})\rangle$	$\langle 2, (0.68e^{i1.95\pi}, 0.05e^{i0.24\pi})\rangle$	$\langle 0, (0.01e^{i0.03\pi}, 0.97e^{i1.95\pi})\rangle$	$\langle 0, (0.04e^{i0.01\pi}, 0.97e^{i1.98\pi})\rangle$
w_3	$\langle 3, (0.82e^{i1.44\pi}, 0.38e^{i0.62\pi})\rangle$	$\langle 2, (0.93e^{i0.97\pi}, 0.52e^{i0.04\pi})\rangle$	$\langle 3, (0.79e^{i1.46\pi}, 0.42e^{i0.57\pi})\rangle$	$\langle 2, (0.79e^{i1.78\pi}, 0.13e^{i0.23\pi})\rangle$
w_4	$\langle 2, (0.58e^{i1.25\pi}, 0.43e^{i0.76\pi})\rangle$	$\langle 3, (0.88e^{i1.93\pi}, 0.16e^{i0.12\pi})\rangle$	$\langle 5, (0.95e^{i1.99\pi}, 0.02e^{i0.03\pi})\rangle$	$\langle 1, (0.25e^{i0.13\pi}, 0.91e^{i1.89\pi})\rangle$

Specifically, (p_q^1, M) is said to be the bottom $C_\mathbf{q}$-ROFSS and (p_q^{N-1}, M) is said to be the top $C_\mathbf{q}$-ROFSS relative to $C_\mathbf{q}$-ROFNSS.

Definition 22. Let $0 < H < N$ and $\sigma \in [-2, 3]$ be thresholds. The soft set over the universe W corresponding to (p_q, Y, N) and (H, σ) is represented by $(p_q^{(H,\sigma)}, M)$ and can be explicated by the assignment:

$$p_q^{(H,\sigma)}(m_\alpha) = \left\{w \in W\colon \mathfrak{L}\left(p_q^H(m_\alpha)\right) > \sigma\right\}, \quad \text{for each } m_\alpha \in M, \tag{30}$$

where $\mathfrak{L}(p_q^H(m_\alpha))$ is the score function of $p_q^H(m_\alpha) = \left\langle \Theta_{\eta\alpha}^H e^{i\Omega_{\eta\alpha}^H}, \Psi_{\eta\alpha}^H e^{i\Upsilon_{\eta\alpha}^H} \right\rangle$.

Example 6. Consider the C_3-ROF5SS $(p_3, Y, 5)$ as represented by Table 5. We have $0 < H < 5$, from Definition 21. The C_3-ROFSS associated with feasible threshold $H = 3$ is summarized in Table 16. Meanwhile, while taking $(H, \sigma) = (3, 1.5)$, we can define the soft set $(p_q^{(3,1.5)}, M)$ which is illustrated in Table 17.

In view of above analysis, it is contemplated that $C_\mathbf{q}$-ROFNSSs can be transformed into $C_\mathbf{q}$-ROFSSs and SSs under specific conditions. In other words, $C_\mathbf{q}$-ROFNSSs are the potent generalization of $C_\mathbf{q}$-ROFSSs and SSs.

Remark 3

(1) Let (p_q, Y, N) be a $C_\mathbf{q}$-ROFNSS over the universe W and J be a set of parameters with $M \subseteq J$. Then, the NSS interrelated with $C_\mathbf{q}$-ROFNSS (p_q, Y, N) is Y, where $Y = (P, M, N)$. This simple assignment illustrates that $C_\mathbf{q}$-ROFNSS generalizes NSS as well as SS too.

(2) Every complex intuitionistic fuzzy N-soft set (CIFNSS) and complex Pythagorean fuzzy N-soft set (CPFNSS) are also a $C_\mathbf{q}$-ROFNSS for all $q \geq 3$, but converse is not true, because

(i) If $0 \leq \Theta + \Psi \leq 1$ or $0 \leq \Theta^2 + \Psi^2 \leq 1$, then $0 \leq \Theta^q + \Psi^q \leq 1$, for all $\Theta, \Psi \in [0, 1]$ and $q \geq 3$

(ii) If $0 \leq (\Omega/2\pi) + (\Upsilon/2\pi) \leq 1$ or $0 \leq (\Omega/2\pi)^2 + (\Upsilon/2\pi)^2 \leq 1$, then $0 \leq (\Omega/2\pi)^q + (\Upsilon/2\pi)^q \leq 1$, for all $\Omega, \Upsilon \in [0, 2\pi]$ and $q \geq 3$

This implies that $C_\mathbf{q}$-ROFNSS significantly extrapolates the ideal concepts of CIFNSS and CPFNSS.

(3) $C_\mathbf{q}$-ROFNSS particularly becomes a q-rung orthopair fuzzy N-soft set by taking phase terms of both membership and nonmembership degrees equivalent to zero; that is, when $\Omega_{\eta\alpha} = 0 = \Upsilon_{\eta\alpha}$ in Definition 8, we grasp the definition of q-ROFNSS which is elucidated as follows:

$$\left(p_q, Y, N\right) = \left\{\left\langle m_\alpha, p_q(m_\alpha) \,|\, m_\alpha \in M, p_q(m_\alpha) \in q\text{-}\mathcal{ROF}^{W\times K}\right\rangle\right\}, \tag{31}$$

where $p_q(m_\alpha) = \left\{((w_\eta, k_{\eta\alpha}), \Theta_{\eta\alpha}(w_\eta, k_{\eta\alpha}), \Psi_{\eta\alpha}(w_\eta, k_{\eta\alpha}) \,|\, (w_\eta, k_{\eta\alpha}) \in W \times K\right\}$ represents the q-ROFS and $\mathbf{q}\text{-}\mathcal{ROF}^{W\times K}$ denotes the family of all q-rung orthopair fuzzy sets over $W \times K$.

Also, it is understood that q-ROFNSSs generalize the PFNSSs, IFNSSs, FNSSs, NSSs, PFSSs, IFSSs, and SSs as well. Therefore, we can also claim that $C_\mathbf{q}$-ROFNSSs admirably generalizes all these extant models.

4. Operations

We now elaborate some rudimentary operational laws on complex q-rung orthopair fuzzy N-soft values.

Definition 23. Consider any three $C_\mathbf{q}$-ROFNSVs, $\mathfrak{B} = \langle k, (\Theta e^{i\Omega}, \Psi e^{i\Upsilon})\rangle$, $\mathfrak{B}_{1\alpha} = \langle k_{1\alpha}, (\Theta_{1\alpha} e^{i\Omega_{1\alpha}}, \Psi_{1\alpha} e^{i\Upsilon_{1\alpha}})\rangle$ $(\alpha = 1, 2)$ and $\mathfrak{H} > 0$ be any real number. Then, the elementary algebraic operations relative to these $C_\mathbf{q}$-ROFNSVs can be defined as follows:

(1) $\mathfrak{B}_{11} \oplus \mathfrak{B}_{12} = \Big\langle \max(k_{11}, k_{12}), \big(\sqrt[q]{\Theta_{11}^q + \Theta_{12}^q - \Theta_{11}^q\Theta_{12}^q}$ $e^{i2\pi\sqrt[q]{(\Omega_{11}/2\pi)^q + (\Omega_{12}/2\pi)^q - (\Omega_{11}/2\pi)^q(\Omega_{12}/2\pi)^q}}, \Psi_{11}\Psi_{12}$ $e^{i2\pi((\Upsilon_{11}/2\pi)(\Upsilon_{12}/2\pi))}\big)\Big\rangle$

(2) $\mathfrak{B}_{11} \otimes \mathfrak{B}_{12} = \big\langle \min(k_{11}, k_{12}), (\Theta_{11}\Theta_{12}$ $e^{i2\pi((\Omega_{11}/2\pi)(\Omega_{12}/2\pi))}, \sqrt[q]{\Psi_{11}^q + \Psi_{12}^q - \Psi_{11}^q\Psi_{12}^q}$ $e^{i2\pi\sqrt[q]{(\Upsilon_{11}/2\pi)^q + (\Upsilon_{12}/2\pi)^q - (\Upsilon_{11}/2\pi)^q(\Upsilon_{12}/2\pi)^q}}\big)\big\rangle$

(3) $\mathfrak{H}\mathfrak{B} = \Big\langle k, \big(\sqrt[q]{1 - (1-\Theta^q)^\mathfrak{H}} e^{i2\pi\sqrt[q]{1-(1-(\Omega/2\pi)^q)^\mathfrak{H}}}$, $\Psi^\mathfrak{H} e^{i2\pi(\Upsilon/2\pi)^\mathfrak{H}}\big)\Big\rangle$

(4) $\mathfrak{B}^\mathfrak{H} = \Big\langle k, \big((\Theta^\mathfrak{H} e^{i2\pi(\Omega/2\pi)^\mathfrak{H}}, \sqrt[q]{1 - (1-\Psi^q)^\mathfrak{H}}$ $e^{i2\pi\sqrt[q]{1-(1-(\Upsilon/2\pi)^q)^\mathfrak{H}}}\big)\Big\rangle$

Definition 24. Let $\mathfrak{B}_{1\alpha} = \langle k_{1\alpha}, (\Theta_{1\alpha} e^{i\Omega_{1\alpha}}, \Psi_{1\alpha} e^{i\Upsilon_{1\alpha}})\rangle$ $(\alpha = 1, 2)$ and $\mathfrak{B} = \langle k, (\Theta e^{i\Omega}, \Psi e^{i\Upsilon})\rangle$ be any three $C_\mathbf{q}$-ROFNSVs and $\mathfrak{H} > 0$ be any real number. Then, the Einstein operations corresponding to these $C_\mathbf{q}$-ROFNSVs can be defined as follows:

(1) $\mathfrak{B}_{11} \oplus_e \mathfrak{B}_{12} = \Big\langle \max(k_{11}, k_{12}),$ $\left(\begin{array}{c} \sqrt[q]{(\Theta_{11}^q + \Theta_{12}^q/1 + \Theta_{11}^q\Theta_{12}^q)} e^{i2\pi\sqrt[q]{((\Omega_{11}/2\pi)^q + (\Omega_{12}/2\pi)^q/1+(\Omega_{11}/2\pi)^q(\Omega_{12}/2\pi)^q)}}, \\ (\Psi_{11}\Psi_{12}/\sqrt[q]{1 + (1-\Psi_{11}^q)(1-\Psi_{12}^q)}) e^{i2\pi((\Upsilon_{11}/2\pi)(\Upsilon_{12}/2\pi)/\sqrt[q]{1+(1-(\Upsilon_{11}/2\pi)^q)(1-(\Upsilon_{12}/2\pi)^q)})} \end{array}\right)\Big\rangle$

(2) $\mathfrak{B}_{11} \otimes_e \mathfrak{B}_{12} = \big\langle \min(k_{11}, k_{12}),$ $\left(\begin{array}{c} (\Theta_{11}\Theta_{12}/\sqrt[q]{1 + (1-\Theta_{11}^q)(1-\Theta_{12}^q)}) e^{i2\pi((\Omega_{11}/2\pi)(\Omega_{12}/2\pi)/\sqrt[q]{1+(1-(\Omega_{11}/2\pi)^q)(1-(\Omega_{12}/2\pi)^q)})} \\ , \sqrt[q]{(\Psi_{11}^q + \Psi_{12}^q/1 + \Psi_{11}^q\Psi_{12}^q)} e^{i2\pi\sqrt[q]{((\Upsilon_{11}/2\pi)^q + (\Upsilon_{12}/2\pi)^q/1+(\Upsilon_{11}/2\pi)^q(\Upsilon_{12}/2\pi)^q)}} \end{array}\right)\big\rangle$

(3) $\mathfrak{H}_{.e}\mathfrak{B} = \langle k,$ $\left(\begin{array}{c} \sqrt[q]{((1+\Theta^q)^\mathfrak{H} - (1-\Theta^q)^\mathfrak{H}/(1+\Theta^q)^\mathfrak{H} + (1-\Theta^q)^\mathfrak{H})} e^{i2\pi\sqrt[q]{((1+(\Omega/2\pi)^q)^\mathfrak{H} - (1-(\Omega/2\pi)^q)^\mathfrak{H}/(1+(\Omega/2\pi)^q)^\mathfrak{H} + (1-(\Omega/2\pi)^q)^\mathfrak{H})}}, \\ (\sqrt[q]{2}\Psi^\mathfrak{H}/\sqrt[q]{(2-\Psi^q)^\mathfrak{H} + (\Psi^q)^\mathfrak{H}}) e^{i2\pi(\sqrt{[q]}2(\Upsilon/2\pi)^\mathfrak{H}/\sqrt{[q](2-(\Upsilon/2\pi)^q)^\mathfrak{H} + ((\Upsilon/2\pi)^q)^\mathfrak{H})}} \end{array}\right)\big\rangle$

(4) $\mathfrak{B}^\mathfrak{H} = \langle k,$ $\left(\begin{array}{c} (\sqrt[q]{2}\Theta^\mathfrak{H}/\sqrt[q]{(2-\Theta^q)^\mathfrak{H} + (\Theta^q)^\mathfrak{H}}) e^{i2\pi(\sqrt[q]{2}\cdot(\Omega/2\pi)^\mathfrak{H}/\sqrt[q]{(2-(\Omega/2\pi)^q)^\mathfrak{H} + ((\Omega/2\pi)^q)^\mathfrak{H})}}, \\ \sqrt[q]{((1+\Psi^q)^\mathfrak{H} - (1-\Psi^q)^\mathfrak{H}/(1+\Psi^q)^\mathfrak{H} + (1-\Psi^q)^\mathfrak{H})} e^{i2\pi\sqrt[q]{((1+(\Upsilon/2\pi)^q)^\mathfrak{H} - (1-(\Upsilon/2\pi)^q)^\mathfrak{H}/(1+(\Upsilon/2\pi)^q)^\mathfrak{H} + (1-(\Upsilon/2\pi)^q)^\mathfrak{H})}} \end{array}\right)\big\rangle$

These operations of $C_\mathbf{q}$-ROFNSVs are highly advantageous and expedient in decision-making as compared to other algebraic operations of $C_\mathbf{q}$-ROFNSVs because they are formulated on the basis of incredible Einstein t-norm and t-conorm to provide end-results that demonstrate remarkable accuracy and precision.

Theorem 1. *Let* $\mathfrak{B} = \langle k, (\Theta e^{i\Omega}, \Psi e^{i\Upsilon})\rangle$, $\mathfrak{B}_{11} = \langle k_{11}, (\Theta_{11} e^{i\Omega_{11}}, \Psi_{11} e^{i\Upsilon_{11}})\rangle$, *and* $\mathfrak{B}_{12} = \langle k_{12}, (\Theta_{12} e^{i\Omega_{12}}, \Psi_{12} e^{i\Upsilon_{12}})\rangle$ *be any three* $C_\mathbf{q}$*-ROFNSVs and* $\mathfrak{H}, \mathfrak{H}_1, \mathfrak{H}_2 > 0$ *be any three real numbers, then*

(1) $\mathfrak{B}_{11} \oplus_e \mathfrak{B}_{12} = \mathfrak{B}_{12} \oplus_e \mathfrak{B}_{11}$

(2) $\mathfrak{B}_{11} \otimes_e \mathfrak{B}_{12} = \mathfrak{B}_{12} \otimes_e \mathfrak{B}_{11}$

(3) $\mathfrak{H}_{.e}(\mathfrak{B}_{11} \oplus_e \mathfrak{B}_{12}) = \mathfrak{H}_{.e}\mathfrak{B}_{11} \oplus_e \mathfrak{H}_{.e}\mathfrak{B}_{12}$

(4) $(\mathfrak{B}_{11} \otimes_e \mathfrak{B}_{12})^\mathfrak{H} = (\mathfrak{B}_{11})^\mathfrak{H} \otimes_e (\mathfrak{B}_{12})^\mathfrak{H}$

(5) $(\mathfrak{H}_{1.e} \oplus_e \mathfrak{H}_{2.e})\mathfrak{B} = \mathfrak{H}_{1.e}\mathfrak{B} \oplus_e \mathfrak{H}_{2.e}\mathfrak{B}$

(6) $\mathfrak{B}^{\mathfrak{H}_1} \otimes_e \mathfrak{B}^{\mathfrak{H}_2} = \mathfrak{B}^{\mathfrak{H}_1 + \mathfrak{H}_2}$

Table 16: Identified C_3-ROFSS.

(p_3^3, M)	m_1	m_2	m_3	m_4
w_1	$\langle 0.96e^{i1.98\pi}, 0.01e^{i0.02\pi}\rangle$	$\langle 0.45e^{i1.84\pi}, 0.14e^{i1.17\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 0.50e^{i1.00\pi}\rangle$
w_2	$\langle 0.00e^{i0.00\pi}, 0.50e^{i1.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 1.00e^{i1.94\pi}, 0.02e^{i0.01\pi}\rangle$	$\langle 0.76e^{i1.79\pi}, 0.26e^{i0.04\pi}\rangle$
w_3	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.68e^{i1.84\pi}, 0.11e^{i0.12\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 0.50e^{i1.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$
w_4	$\langle 0.86e^{i1.75\pi}, 0.19e^{i0.07\pi}\rangle$	$\langle 0.97e^{i1.96\pi}, 0.03e^{i0.04\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 0.50e^{i1.00\pi}\rangle$
w_5	$\langle 0.00e^{i0.00\pi}, 0.50e^{i1.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.84e^{i1.72\pi}, 0.16e^{i0.08\pi}\rangle$	$\langle 0.95e^{i1.98\pi}, 0.35e^{i0.08\pi}\rangle$

Table 17: Identified SS.

$(p_3^{(3,1.5)}, M)$	m_1	m_2	m_3	m_4
w_1	1	1	0	0
w_2	0	0	1	1
w_3	0	1	0	0
w_4	0	1	0	0
w_5	0	0	0	1

Proof

(1) $\mathfrak{B}_{11}\oplus_e\mathfrak{B}_{12} = \langle \max(k_{11}, k_{12}),$
$\left(\sqrt[q]{(\Theta_{11}^q + \Theta_{12}^q/1 + \Theta_{11}^q\Theta_{12}^q)}\, e^{i2\pi\sqrt[q]{((\Omega_{11}/2\pi)^q + (\Omega_{12}/2\pi)^q/1+(\Omega_{11}/2\pi)^q(\Omega_{12}/2\pi)^q)}},\right.$
$\left.(\Psi_{11}\Psi_{12}/\sqrt[q]{1 + (1-\Psi_{11}^q)(1-\Psi_{12}^q)})\right)$
$e^{i2\pi((\Upsilon_{11}/2\pi)(\Upsilon_{12}/2\pi)/\sqrt[q]{1+(1-(\Upsilon_{11}/2\pi)^q)(1-(\Upsilon_{12}/2\pi)^q)})}\rangle =$
$\left\langle \max(k_{12}, k_{11}), \left(\sqrt[q]{(\Theta_{12}^q + \Theta_{11}^q/1 + \Theta_{12}^q\Theta_{11}^q)}\right.\right.$
$e^{i2\pi((\Omega_{12}/2\pi)^q + (\Omega_{11}/2\pi)^q/\sqrt[q]{1+(\Omega_{12}/2\pi)^q(\Omega_{11}/2\pi)^q})},$
$\left.\left.(\Psi_{12}\Psi_{11}/\sqrt[q]{1 + (1-\Psi_{12}^q)(1-\Psi_{11}^q)})\right)\right\rangle =$
$e^{i2\pi((\Upsilon_{12}/2\pi)(\Upsilon_{11}/2\pi)/\sqrt[q]{1+(1-(\Upsilon_{12}/2\pi)^q)(1-(\Upsilon_{11}/2\pi)^q)})}\rangle =$
$\mathfrak{B}_{12}\oplus_e\mathfrak{B}_{11}$.

Analogously, we can prove (2) in the similar fashion.

(2) First, consider the L.H.S, $\mathfrak{B}_{11}\oplus_e\mathfrak{B}_{12} = \langle \max(k_{11}, k_{12}), \left(\frac{\sqrt[q]{(\Theta_{11}^q + \Theta_{12}^q/1 + \Theta_{11}^q\Theta_{12}^q)}\, e^{i2\pi\sqrt[q]{((\Omega_{11}/2\pi)^q + (\Omega_{12}/2\pi)^q/1+(\Omega_{11}/2\pi)^q(\Omega_{12}/2\pi)^q)}},}{(\Psi_{11}\Psi_{12}/\sqrt[q]{1 + (1-\Psi_{11}^q)(1-\Psi_{12}^q)})\, e^{i2\pi((\Upsilon_{11}/2\pi)(\Upsilon_{12}/2\pi)/\sqrt[q]{1+(1-(\Upsilon_{11}/2\pi)^q)(1-(\Upsilon_{12}/2\pi)^q)})}}\right)\rangle$. Above equation can be transformed into the following mathematicalexpression:

$$\mathfrak{B}_{11}\oplus_e\mathfrak{B}_{12} = \left\langle \max(k_{11}, k_{12}), \left(\sqrt[q]{\frac{(1+\Theta_{11}^q)(1+\Theta_{12}^q) - (1-\Theta_{11}^q)(1-\Theta_{12}^q)}{(1+\Theta_{11}^q)(1+\Theta_{12}^q) + (1-\Theta_{11}^q)(1-\Theta_{12}^q)}} \right.\right.$$
$$\cdot e^{i2\pi\sqrt[q]{\left((1+(\Omega_{11}/2\pi)^q)(1+(\Omega_{12}/2\pi)^q) - (1-(\Omega_{11}/2\pi)^q)(1-(\Omega_{12}/2\pi)^q)/(1+(\Omega_{11}/2\pi)^q)(1+(\Omega_{12}/2\pi)^q) + (1-(\Omega_{11}/2\pi)^q)(1-(\Omega_{12}/2\pi)^q)\right)}},$$
$$\left.\left. \cdot \frac{\sqrt[q]{2}\Psi_{11}\Psi_{12}}{\sqrt[q]{(2-\Psi_{11}^q)(2-\Psi_{12}^q) + \Psi_{11}^q\Psi_{12}^q}} e^{i2\pi\left(\sqrt[q]{2}(\Upsilon_{11}/2\pi)(\Upsilon_{12}/2\pi)/\sqrt[q]{(2-(\Upsilon_{11}/2\pi)^q)(2-(\Upsilon_{12}/2\pi)^q) + (\Upsilon_{11}/2\pi)^q(\Upsilon_{12}/2\pi)^q)}\right)} \right)\right\rangle. \tag{32}$$

Let us consider

$$d = (1+\Theta_{11}^q)(1+\Theta_{12}^q),$$
$$f = (1-\Theta_{11}^q)(1-\Theta_{12}^q),$$
$$g = \left(1+\left(\frac{\Omega_{11}}{2\pi}\right)^q\right)\left(1+\left(\frac{\Omega_{12}}{2\pi}\right)^q\right),$$
$$h = \left(1-\left(\frac{\Omega_{11}}{2\pi}\right)^q\right)\left(1-\left(\frac{\Omega_{12}}{2\pi}\right)^q\right),$$
$$m = \Psi_{11}^q\Psi_{12}^q,$$
$$p = (2-\Psi_{11}^q)(2-\Psi_{12}^q),$$
$$s = \left(\frac{\Upsilon_{11}}{2\pi}\right)^q\left(\frac{\Upsilon_{12}}{2\pi}\right)^q,$$
$$r = \left(2-\left(\frac{\Upsilon_{11}}{2\pi}\right)^q\right)\left(2-\left(\frac{\Upsilon_{12}}{2\pi}\right)^q\right). \tag{33}$$

Then,

$$\begin{aligned}
\mathfrak{B}_{11}\oplus_e\mathfrak{B}_{12} &= \left\langle \max(k_{11},k_{12}),\left(\sqrt[q]{\frac{d-f}{d+f}}e^{i2\pi\sqrt[q]{(g-h/g+h)}},\sqrt[q]{\frac{2m}{p+m}}e^{i2\pi\sqrt[q]{(2s/s+r)}}\right)\right\rangle,\\
\mathfrak{H}_{.e}(\mathfrak{B}_{11}\oplus_e\mathfrak{B}_{12}) &= \left\langle \max(k_{11},k_{12}),\left(\sqrt[q]{\frac{(1+(d-f/d+f))^{\mathfrak{H}}-(1-(d-f/d+f))^{\mathfrak{H}}}{(1+(d-f/d+f))^{\mathfrak{H}}+(1-(d-f/d+f))^{\mathfrak{H}}}}\right.\right.\\
&\quad\cdot e^{i2\pi\sqrt[q]{\left((1+(g-h/g+h))^{\mathfrak{H}}-(1-(g-h/g+h))^{\mathfrak{H}}/(1+(g-h/g+h))^{\mathfrak{H}}+(1-(g-h/g+h))^{\mathfrak{H}}\right)}},\\
&\quad\cdot\frac{\sqrt[q]{2}\left(\sqrt[q]{(2m/p+m)}\right)^{\mathfrak{H}}}{\sqrt[q]{(2-(2m/p+m))^{\mathfrak{H}}+(2m/p+m)^{\mathfrak{H}}}}e^{i2\pi\left(\sqrt[q]{2}\left(\sqrt[q]{(2s/r+s)}\right)^{\mathfrak{H}}/\sqrt[q]{(2-(2s/r+s))^{\mathfrak{H}}+(2s/r+s)^{\mathfrak{H}}}\right)}\left.\right)\Big\rangle\\
&= \left\langle \max(k_{11},k_{12}),\left(\sqrt[q]{\frac{d^{\mathfrak{H}}-f^{\mathfrak{H}}}{d^{\mathfrak{H}}+f^{\mathfrak{H}}}}e^{i2\pi\sqrt[q]{(g^{\mathfrak{H}}-h^{\mathfrak{H}}/g^{\mathfrak{H}}+h^{\mathfrak{H}})}},\sqrt[q]{\frac{2m^{\mathfrak{H}}}{p^{\mathfrak{H}}+m^{\mathfrak{H}}}}e^{i2\pi\sqrt[q]{(2s^{\mathfrak{H}}/r^{\mathfrak{H}}+s^{\mathfrak{H}})}}\right)\right\rangle\\
&= \left\langle \max(k_{11},k_{12}),\left(\sqrt[q]{\frac{(1+\Theta_{11}^q)^{\mathfrak{H}}(1+\Theta_{12}^q)^{\mathfrak{H}}-(1-\Theta_{11}^q)^{\mathfrak{H}}(1-\Theta_{12}^q)^{\mathfrak{H}}}{(1+\Theta_{11}^q)^{\mathfrak{H}}(1+\Theta_{12}^q)^{\mathfrak{H}}+(1-\Theta_{11}^q)^{\mathfrak{H}}(1-\Theta_{12}^q)^{\mathfrak{H}}}}\right.\right.\\
&\quad\cdot e^{i2\pi\sqrt[q]{\left((1+(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}(1+(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}-(1-(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}(1-(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}/(1+(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}(1+(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}+(1-(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}(1-(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}\right)}},\\
&\quad\cdot\frac{\sqrt[q]{2}\Psi_{11}^{\mathfrak{H}}\Psi_{12}^{\mathfrak{H}}}{\sqrt[q]{(2-\Psi_{11}^q)^{\mathfrak{H}}(2-\Psi_{12}^q)^{\mathfrak{H}}+(\Psi_{11}^q)^{\mathfrak{H}}(\Psi_{12}^q)^{\mathfrak{H}}}}\\
&\quad\cdot e^{i2\pi\left(\sqrt[q]{2}(\Upsilon_{11}/2\pi)^{\mathfrak{H}}(\Upsilon_{12}/2\pi)^{\mathfrak{H}}/\sqrt[q]{(2-(\Upsilon_{11}/2\pi)^q)^{\mathfrak{H}}(2-(\Upsilon_{12}/2\pi)^q)^{\mathfrak{H}}+((\Upsilon_{11}/2\pi)^q)^{\mathfrak{H}}((\Upsilon_{12}/2\pi)^q)^{\mathfrak{H}}}\right)}\left.\right)\Big\rangle.
\end{aligned}\tag{34}$$

Now, assume the R.H.S,

$$\begin{aligned}
\mathfrak{H}_{.e}\mathfrak{B}_{11} &= \left\langle k_{11},\left(\sqrt[q]{\frac{(1+\Theta_{11}^q)^{\mathfrak{H}}-(1-\Theta_{11}^q)^{\mathfrak{H}}}{(1+\Theta_{11}^q)^{\mathfrak{H}}+(1-\Theta_{11}^q)^{\mathfrak{H}}}}\right.\right.\\
&\quad\cdot e^{i2\pi\sqrt[q]{\left((1+(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}-(1-(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}/(1+(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}+(1-(\Omega_{11}/2\pi)^q)^{\mathfrak{H}}\right)}},\frac{\sqrt[q]{2}\Psi_{11}^{\mathfrak{H}}}{\sqrt[q]{(2-\Psi_{11}^q)^{\mathfrak{H}}+(\Psi_{11}^q)^{\mathfrak{H}}}}\\
&\quad\cdot e^{i2\pi\left(\sqrt[q]{2}(\Upsilon_{11}/2\pi)^{\mathfrak{H}}/\sqrt[q]{(2-(\Upsilon_{11}/2\pi)^q)^{\mathfrak{H}}+((\Upsilon_{11}/2\pi)^q)^{\mathfrak{H}}}\right)}\left.\right)\Big\rangle,\\
\mathfrak{H}_{.e}\mathfrak{B}_{12} &= \left\langle k_{12},\left(\sqrt[q]{\frac{(1+\Theta_{12}^q)^{\mathfrak{H}}-(1-\Theta_{12}^q)^{\mathfrak{H}}}{(1+\Theta_{12}^q)^{\mathfrak{H}}+(1-\Theta_{12}^q)^{\mathfrak{H}}}}\right.\right.\\
&\quad\cdot e^{i2\pi\sqrt[q]{\left((1+(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}-(1-(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}/(1+(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}+(1-(\Omega_{12}/2\pi)^q)^{\mathfrak{H}}\right)}},\frac{\sqrt[q]{2}\Psi_{12}^{\mathfrak{H}}}{\sqrt[q]{(2-\Psi_{12}^q)^{\mathfrak{H}}+(\Psi_{12}^q)^{\mathfrak{H}}}}\\
&\quad\cdot e^{i2\pi\left(\sqrt[q]{2}(\Upsilon_{12}/2\pi)^{\mathfrak{H}}/\sqrt[q]{(2-(\Upsilon_{12}/2\pi)^q)^{\mathfrak{H}}+((\Upsilon_{12}/2\pi)^q)^{\mathfrak{H}}}\right)}\left.\right)\Big\rangle.
\end{aligned}\tag{35}$$

Let us consider

$$
\begin{aligned}
d_1 &= \left(1+\Theta_{11}^{q}\right)^{\mathfrak{H}},\\
f_1 &= \left(1-\Theta_{11}^{q}\right)^{\mathfrak{H}},\\
g_1 &= \left(1+\left(\frac{\Omega_{11}}{2\pi}\right)^{q}\right)^{\mathfrak{H}},\\
h_1 &= \left(1-\left(\frac{\Omega_{11}}{2\pi}\right)^{q}\right)^{\mathfrak{H}},\\
s_1 &= \left(\Psi_{11}^{q}\right)^{\mathfrak{H}},\\
r_1 &= \left(2-\Psi_{11}^{q}\right)^{\mathfrak{H}},\\
m_1 &= \left(\left(\frac{\Upsilon_{11}}{2\pi}\right)^{q}\right)^{\mathfrak{H}},\\
p_1 &= \left(2-\left(\frac{\Upsilon_{11}}{2\pi}\right)^{q}\right)^{\mathfrak{H}},\\
d_2 &= \left(1+\Theta_{12}^{q}\right)^{\mathfrak{H}},\\
f_2 &= \left(1-\Theta_{12}^{q}\right)^{\mathfrak{H}},\\
g_2 &= \left(1+\left(\frac{\Omega_{12}}{2\pi}\right)^{q}\right)^{\mathfrak{H}},\\
h_2 &= \left(1-\left(\frac{\Omega_{12}}{2\pi}\right)^{q}\right)^{\mathfrak{H}},\\
s_2 &= \left(\Psi_{12}^{q}\right)^{\mathfrak{H}},\\
r_2 &= \left(2-\Psi_{12}^{q}\right)^{\mathfrak{H}},\\
m_2 &= \left(\left(\frac{\Upsilon_{12}}{2\pi}\right)^{q}\right)^{\mathfrak{H}},\\
p_2 &= \left(2-\left(\frac{\Upsilon_{12}}{2\pi}\right)^{q}\right)^{\mathfrak{H}}.
\end{aligned}
\tag{36}
$$

Then, the above equations can be modified as follows:

$$
\begin{aligned}
\mathfrak{H}_{.e}\mathfrak{B}_{11} &= \left\langle k_{11}, \left(\sqrt[q]{\frac{d_1-f_1}{d_1+f_1}}\, e^{i2\pi\sqrt[q]{\left(g_1-h_1/g_1+h_1\right)}}, \sqrt[q]{\frac{2s_1}{r_1+s_1}}\, e^{i2\pi\sqrt[q]{\left(2m_1/p_1+m_1\right)}} \right) \right\rangle,\\
\mathfrak{H}_{.e}\mathfrak{B}_{12} &= \left\langle k_{12}, \left(\sqrt[q]{\frac{d_2-f_2}{d_2+f_2}}\, e^{i2\pi\sqrt[q]{\left(g_2-h_2/g_2+h_2\right)}}, \sqrt[q]{\frac{2s_2}{r_2+s_2}}\, e^{i2\pi\sqrt[q]{\left(2m_2/p_2+m_2\right)}} \right) \right\rangle.
\end{aligned}
\tag{37}
$$

Now,

$$
\begin{aligned}
\mathfrak{H}_{.e}\mathfrak{B}_{11} \oplus_e \mathfrak{H}_{.e}\mathfrak{B}_{12} &= \left\langle \max\left(k_{11},k_{12}\right), \left(\sqrt[q]{\frac{\left(d_1-f_1/d_1+f_1\right)+\left(d_2-f_2/d_2+f_2\right)}{1+\left(d_1-f_1/d_1+f_1\right)\left(d_2-f_2/d_2+f_2\right)}} \right.\right.\\
&\quad \cdot e^{i2\pi\sqrt[q]{\left(\left(g_1-h_1/g_1+h_1\right)+\left(g_2-h_2/g_2+h_2\right)/1+\left(g_1-h_1/g_1+h_1\right)\left(g_2-h_2/g_2+h_2\right)\right)}},\\
&\quad \left.\left. \cdot \sqrt{q\frac{\left(4s_1s_2/\left(r_1+s_1\right)\left(r_2+s_2\right)\right)}{1+\left(1-\left(2s_1/r_1+s_1\right)\right)\left(1-\left(2s_2/r_2+s_2\right)\right)}}\, e^{i2\pi\sqrt[q]{\left(\left(4m_1m_2/\left(p_1+m_1\right)\left(p_2+m_2\right)\right)/1+\left(1-\left(2m_1/p_1+m_1\right)\right)\left(1-\left(2m_2/p_2+m_2\right)\right)\right)}} \right) \right\rangle\\
&= \left\langle \max\left(k_{11},k_{12}\right), \left(\sqrt[q]{\frac{d_1d_2-f_1f_2}{d_1d_2+f_1f_2}}\, e^{i2\pi\sqrt[q]{\left(g_1g_2-h_1h_2/g_1g_2+h_1h_2\right)}}, \sqrt[q]{\frac{2s_1s_2}{r_1r_2+s_1s_2}}\, e^{i2\pi\sqrt[q]{\left(2m_1m_2/p_1p_2+m_1m_2\right)}} \right) \right\rangle\\
&= \left\langle \max\left(k_{11},k_{12}\right), \left(\sqrt[q]{\frac{\left(1+\Theta_{11}^{q}\right)^{\mathfrak{H}}\left(1+\Theta_{12}^{q}\right)^{\mathfrak{H}}-\left(1-\Theta_{11}^{q}\right)^{\mathfrak{H}}\left(1-\Theta_{12}^{q}\right)^{\mathfrak{H}}}{\left(1+\Theta_{11}^{q}\right)^{\mathfrak{H}}\left(1+\Theta_{12}^{q}\right)^{\mathfrak{H}}+\left(1-\Theta_{11}^{q}\right)^{\mathfrak{H}}\left(1-\Theta_{12}^{q}\right)^{\mathfrak{H}}}} \right.\right.
\end{aligned}
$$

$$\cdot e^{i2\pi\sqrt[q]{\left(\left(1+\left(\Omega_{11}/2\pi\right)^{q}\right)^{\mathfrak{H}}\left(1+\left(\Omega_{12}/2\pi\right)^{q}\right)^{\mathfrak{H}}-\left(1-\left(\Omega_{11}/2\pi\right)^{q}\right)^{\mathfrak{H}}\left(1-\left(\Omega_{12}/2\pi\right)^{q}\right)^{\mathfrak{H}}/\left(1+\left(\Omega_{11}/2\pi\right)^{q}\right)^{\mathfrak{H}}\left(1+\left(\Omega_{12}/2\pi\right)^{q}\right)^{\mathfrak{H}}+\left(1-\left(\Omega_{11}/2\pi\right)^{q}\right)^{\mathfrak{H}}\left(1-\left(\Omega_{12}/2\pi\right)^{q}\right)^{\mathfrak{H}}\right)}},$$

$$\cdot\frac{\sqrt[q]{2}\Psi_{11}^{\mathfrak{H}}\Psi_{12}^{\mathfrak{H}}}{\sqrt[q]{\left(2-\Psi_{11}^{q}\right)^{\mathfrak{H}}\left(2-\Psi_{12}^{q}\right)^{\mathfrak{H}}+\left(\Psi_{11}^{q}\right)^{\mathfrak{H}}\left(\Psi_{12}^{q}\right)^{\mathfrak{H}}}}$$

$$\cdot e^{i2\pi\left(\sqrt[q]{2}\left(\Upsilon_{11}/2\pi\right)^{\mathfrak{H}}\left(\Upsilon_{12}/2\pi\right)^{\mathfrak{H}}/\sqrt[q]{\left(2-\left(\Upsilon_{11}/2\pi\right)^{q}\right)^{\mathfrak{H}}\left(2-\left(\Upsilon_{12}/2\pi\right)^{q}\right)^{\mathfrak{H}}+\left(\left(\Upsilon_{11}/2\pi\right)^{q}\right)^{\mathfrak{H}}\left(\left(\Upsilon_{12}/2\pi\right)^{q}\right)^{\mathfrak{H}}}}\right)}\Bigg\rangle. \tag{38}$$

Consequently, verify that $\mathfrak{H}_{.e}(\mathfrak{V}_{11}\oplus_e\mathfrak{V}_{12})=\mathfrak{H}_{.e}\mathfrak{V}_{11}\oplus_e\mathfrak{H}_{.e}\mathfrak{V}_{12}$.

Similarly, we can prove (4) in the analogous manner. (5) Assume that $\mathfrak{H}_1,\ \mathfrak{H}_2>0$:

$$(\mathfrak{H}_{1.e}\oplus_e\mathfrak{H}_{2.e})\mathfrak{V}==\Bigg\langle k,\Bigg(\sqrt{\frac{\left(1+\Theta^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}-\left(1-\Theta^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}}{\left(1+\Theta^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}+\left(1-\Theta^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}}}$$

$$\cdot e^{i2\pi\sqrt[q]{\left(\left(1+(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}-\left(1-(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}/\left(1+(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}+\left(1-(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}\right)}}, \tag{39}$$

$$\cdot\frac{\sqrt[q]{2}\Psi^{\mathfrak{H}_1+\mathfrak{H}_2}}{\sqrt[q]{\left(2-\Psi^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}+\left(\Psi^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}}}e^{i2\pi\left(\sqrt[q]{2}\left(\Upsilon/2\pi\right)^{\mathfrak{H}_1+\mathfrak{H}_2}/\sqrt[q]{\left(2-\left(\Upsilon/2\pi\right)^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}+\left(\left(\Upsilon/2\pi\right)^{q}\right)^{\mathfrak{H}_1+\mathfrak{H}_2}}}\right)}\Bigg\rangle.$$

Now, considering R.H.S, we have

$$\mathfrak{H}_{1.e}\mathfrak{V}=\Bigg\langle k,\Bigg(\sqrt{\frac{\left(1+\Theta^{q}\right)^{\mathfrak{H}_1}-\left(1-\Theta^{q}\right)^{\mathfrak{H}_1}}{\left(1+\Theta^{q}\right)^{\mathfrak{H}_1}+\left(1-\Theta^{q}\right)^{\mathfrak{H}_1}}}e^{i2\pi\sqrt[q]{\left(\left(1+(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1}-\left(1-(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1}/\left(1+(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1}+\left(1-(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_1}\right)}},$$

$$\cdot\frac{\sqrt[q]{2}\Psi^{\mathfrak{H}_1}}{\sqrt[q]{\left(2-\Psi^{q}\right)^{\mathfrak{H}_1}+\left(\Psi^{q}\right)^{\mathfrak{H}_1}}}e^{i2\pi\left(\sqrt[q]{2}\left(\Upsilon/2\pi\right)^{\mathfrak{H}_1}/\sqrt[q]{\left(2-\left(\Upsilon/2\pi\right)^{q}\right)^{\mathfrak{H}_1}+\left(\left(\Upsilon/2\pi\right)^{q}\right)^{\mathfrak{H}_1}}}\right)}\Bigg\rangle,$$

$$\mathfrak{H}_{2.e}\mathfrak{V}=\Bigg\langle k,\Bigg(\sqrt{\frac{\left(1+\Theta^{q}\right)^{\mathfrak{H}_2}-\left(1-\Theta^{q}\right)^{\mathfrak{H}_2}}{\left(1+\Theta^{q}\right)^{\mathfrak{H}_2}+\left(1-\Theta^{q}\right)^{\mathfrak{H}_2}}}e^{i2\pi\sqrt[q]{\left(\left(1+(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_2}-\left(1-(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_2}/\left(1+(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_2}+\left(1-(\Omega/2\pi)^{q}\right)^{\mathfrak{H}_2}\right)}}, \tag{40}$$

$$\cdot\frac{\sqrt[q]{2}\Psi^{\mathfrak{H}_2}}{\sqrt[q]{\left(2-\Psi^{q}\right)^{\mathfrak{H}_2}+\left(\Psi^{q}\right)^{\mathfrak{H}_2}}}e^{i2\pi\left(\sqrt[q]{2}\left(\Upsilon/2\pi\right)^{\mathfrak{H}_2}/\sqrt[q]{\left(2-\left(\Upsilon/2\pi\right)^{q}\right)^{\mathfrak{H}_2}+\left(\left(\Upsilon/2\pi\right)^{q}\right)^{\mathfrak{H}_2}}}\right)}\Bigg\rangle.$$

Let us consider, for $k = 1, 2$,

$$
\begin{aligned}
d_k &= \left(1 + \Theta^q\right)^{\mathfrak{H}_k},\\
f_k &= \left(1 - \Theta^q\right)^{\mathfrak{H}_k},\\
g_k &= \left(1 + \left(\frac{\Omega}{2\pi}\right)^q\right)^{\mathfrak{H}_k},\\
h_k &= \left(1 - \left(\frac{\Omega}{2\pi}\right)^q\right)^{\mathfrak{H}_k},\\
s_k &= \left(\Psi^q\right)^{\mathfrak{H}_k},\\
r_k &= \left(2 - \Psi^q\right)^{\mathfrak{H}_k},\\
m_k &= \left(\left(\frac{\Upsilon}{2\pi}\right)^q\right)^{\mathfrak{H}_k},\\
p_k &= \left(2 - \left(\frac{\Upsilon}{2\pi}\right)^q\right)^{\mathfrak{H}_k},\\
\mathfrak{H}_{1.e}\mathfrak{B} \oplus_e \mathfrak{H}_{2.e}\mathfrak{B} &= \left\langle \max\left(k_{11}, k_{12}\right), \left(\sqrt[q]{\frac{\left(d_1 - f_1/d_1 + f_1\right) + \left(d_2 - f_2/d_2 + f_2\right)}{1 + \left(d_1 - f_1/d_1 + f_1\right)\left(d_2 - f_2/d_2 + f_2\right)}} \right.\right.\\
&\quad \cdot e^{i2\pi\sqrt[q]{\left(\left(g_1 - h_1/g_1 + h_1\right) + \left(g_2 - h_2/g_2 + h_2\right)/1 + \left(g_1 - h_1/g_1 + h_1\right)\left(g_2 - h_2/g_2 + h_2\right)\right)}},\\
&\quad \cdot \sqrt{q\frac{\left(4s_1s_2/\left(r_1 + s_1\right)\left(r_2 + s_2\right)\right)}{1 + \left(1 - \left(2s_1/r_1 + s_1\right)\right)\left(1 - \left(2s_2/r_2 + s_2\right)\right)}}\\
&\quad \left.\left. \cdot e^{i2\pi\sqrt[q]{\left(\left(4m_1m_2/\left(p_1 + m_1\right)\left(p_2 + m_2\right)\right)/1 + \left(1 - \left(2m_1/p_1 + m_1\right)\right)\left(1 - \left(2m_2/p_2 + m_2\right)\right)\right)}} \right) \right\rangle\\
&= \left\langle \max\left(k_{11}, k_{12}\right), \left(\sqrt[q]{\frac{d_1d_2 - f_1f_2}{d_1d_2 + f_1f_2}} \right.\right.\\
&\quad \left.\left. \cdot e^{i2\pi\sqrt[q]{\left(g_1g_2 - h_1h_2/g_1g_2 + h_1h_2\right)}}, \sqrt[q]{\frac{2s_1s_2}{r_1r_2 + s_1s_2}} e^{i2\pi\sqrt[q]{\left(2m_1m_2/p_1p_2 + m_1m_2\right)}} \right) \right\rangle\\
&= \left\langle k, \left(\sqrt[q]{\frac{\left(1 + \Theta^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2} - \left(1 - \Theta^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2}}{\left(1 + \Theta^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2} + \left(1 - \Theta^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2}}} \right.\right.\\
&\quad \cdot e^{i2\pi\sqrt[q]{\left(\left(1 + (\Omega/2\pi)^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2} - \left(1 - (\Omega/2\pi)^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2}/\left(1 + (\Omega/2\pi)^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2} + \left(1 - (\Omega/2\pi)^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2}\right)}},\\
&\quad \left.\left. \cdot \frac{\sqrt[q]{2}\Psi^{\mathfrak{H}_1 + \mathfrak{H}_2}}{\sqrt[q]{\left(2 - \Psi^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2} + \left(\Psi^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2}}} e^{i2\pi\left(\sqrt{2}\left(\Upsilon/2\pi\right)^{\mathfrak{H}_1 + \mathfrak{H}_2}/\sqrt[q]{\left(2 - (\Upsilon/2\pi)^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2} + \left((\Upsilon/2\pi)^q\right)^{\mathfrak{H}_1 + \mathfrak{H}_2}}\right)} \right) \right\rangle.
\end{aligned}
\tag{41}
$$

Hence, verify that $(\mathfrak{H}_{1.e} \oplus_e \mathfrak{H}_{2.e})\mathfrak{V} = \mathfrak{H}_{1.e}\mathfrak{V} \oplus_e \mathfrak{H}_{2.e}\mathfrak{V}$.

Analogously, we can prove (6) in the similar decorum. □

5. Decision-Making Algorithms

In this section, we develop two eloquent MCDM algorithms that implemented on the proposed multiskilled C_q-ROFNSS model. These strategies are especially designed for the identification of the optimal alternative of real-life heuristic problems that are characterized by C_q-ROFNSSs. The underlying methodology of these established decision-making algorithms can be interpreted as follows (Algorithms 1 and 2).

5.1. Application. In this section, we inspect an empirical MCDM problem related to selection of the most proficient industrial robot for automation, by executing our newly formulated decision-making algorithms in order to demonstrate the compatibility and functionality of the proposed strategies.

5.2. Selection of Most Proficient Aerospace Technology. Aerospace industry is a multidisciplinary branch of science and technology that incredibly overlaps aeronautical and astronautical engineering for the manufacturing of high standard military aircrafts, cosmic shuttles, gliders, jets, space stations, and all other related units. Its fascinating technological progress has revolutionized the entire world by accomplishing their iconic achievements such as landing astronauts on the Moon, remodelling of advanced transportation systems and generation of technical spinoffs. The rapid advancement of this fabulous industry has sparked the attention of many practitioners and space enthusiasts to reveal more about galaxies, stars, planets, and other astronomical phenomena. Nowadays, spacecraft are one of the most important aerospace technologies designed to make significant contributions in telecommunication, navigation systems, space colonization, meteorology, planetary exploration, and the manipulation of information-transfer networks. Its remarkable technical capabilities play a substantial role in the prestigious economic and political development of a sovereign country. Acquisition of competent spacecraft is one of the most tricky decision-making problems because of its massive capital investment and availability of a wide range of such amazing technologies with various dominant specifications in global markets. To accomplish the current need, suppose that a Mexican space agency is intended to launch an outstanding spacecraft for the exclusive operation of large-scale global missions. The collection of seven spacecraft $W = \{w_1, w_2, \ldots, w_7\}$ is considered after scrutinizing their versatile potentialities, where

w_1 is the penetrator spacecraft: this high-quality spacecraft is a bullet-shaped projectile that remarkably penetrates the surface of a moon, planet, or any other such comet in space. The penetrator spacecraft monitors the inherent peculiarities of the penetrated surface to determine whether life exists on other cosmic planets of the solar system.

w_2 is the orbiter spacecraft: this spacecraft is designed for traveling to a distant celestial body with considerable propulsive capacity in order to decelerate it at the right moment for proper insertion of spacecraft in their orbits. The orbiter spacecraft can significantly analyze the same planet from different heights without landing on its surface.

w_3 is the flyby spacecraft: this ideal spacecraft has potential to spin continuously and utilize thrusters or reaction wheels to be stabilized in 3 axes. These high-speed flyby spacecraft productively utilized their onboard optical instruments to critically observe the apparent motion of its target at the highest possible temporal resolution.

w_4 is the lander spacecraft: the lander spacecraft reaches the surface of an astronomical body and remains long enough to measure the electrical, thermal, and mechanical properties of its inner surface. It has an incredible capability to perform geological and meteorological experiments and telemeter such precise information back to the Earth.

w_5 is the atmospheric spacecraft: these competent spacecraft are especially designed for comparatively short-term exploration missions to take accurate measurements of temperature, clouds content, lightning, pressure, density, and atmospheric composition of a planet or satellite with the assistance of their advanced scientific instruments.

w_6 is the rover spacecraft: this special type of spacecraft can move across planetary celestial bodies and other solid surfaces of cosmic planets. This surface exploration device is mainly developed to gather detailed information about the terrain of astronomical objects and to take their different crust specimens such as rocks, soil, dust, and even liquids.

w_7 is the observatory spacecraft: an observatory spacecraft is launched to provide amazing capabilities for the clearer observation of distant galaxies, planets, and other important terrestrial events. Such spacecraft competently perform their operations twenty-four hours a day at low temperature, and their observing tasks are free from blurring and obscuring impacts of the Earth's atmosphere.

All the aforementioned spacecraft are analyzed on the basis of four identified decisive factors $\mathscr{M} = \{m_1, m_2, m_3, m_4\}$ that strongly influence the selection process of spacecraft will serve as decision-criteria for this inspected MCDM problem, where

m_1 is the propulsion system: this significant feature is employed to accelerate the speed of spacecraft by providing required propelling force. While launching a spacecraft from the Earth, a competent propulsion system continually changes the velocity of the spacecraft and overcomes the high gravitational pull of Earth.

(1) Input $W = \{w_1, w_2, \ldots, w_\wp\}$ as a universe of $\wp$ objects.
(2) Input $M = \{m_1, m_2, \ldots, m_\hbar\}$ as a collection of $\hbar$ attributes.
(3) Input the N-soft set (P, M, N) with $K = \{0, 1, \ldots, N-1\}$, where $N \in \{2, 3, \ldots\}$. Then, for each $w_\eta \in W$, $m_\alpha \in M$, there exist unique $k_{\eta\alpha} \in K$.
(4) Input $C_{\mathbf{q}}$-ROFNSS (p_q, Y, N), where p_q: $M \longrightarrow \mathscr{C}_{\mathbf{q}} - \mathscr{R}\mathscr{O}\mathscr{F}^{W\times K}$ and $Y = (P, M, N)$.
(5) Calculate $\mathfrak{U}_\eta = \sum_{\alpha=1}^{\hbar} \mathfrak{B}_{\eta\alpha}$, where $\mathfrak{B}_{\eta\alpha} = \left\langle k_{\eta\alpha}, (\Theta_{\eta\alpha} e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha} e^{iY_{\eta\alpha}}) \right\rangle$ and the Einstein addition of two $\mathfrak{B}_{\eta\alpha}$ is interpreted as follows:

$$\mathfrak{B}_{11} \oplus_e \mathfrak{B}_{12} = \left\langle \max(k_{11}, k_{12}), \begin{pmatrix} \sqrt[q]{(\Theta_{11}^q + \Theta_{12}^q / 1 + \Theta_{11}^q \Theta_{12}^q)}\, e^{i2\pi \sqrt[q]{((\Omega_{11}/2\pi)^q + (\Omega_{12}/2\pi)^q / 1 + (\Omega_{11}/2\pi)^q (\Omega_{12}/2\pi)^q)}}, \\ (\Psi_{11}\Psi_{12} / \sqrt[q]{1 + (1 - \Psi_{11}^q)(1 - \Psi_{12}^q)})\, e^{i2\pi((\Upsilon_{11}/2\pi)(\Upsilon_{12}/2\pi) / \sqrt[q]{1 + (1-(\Upsilon_{11}/2\pi)^q)(1-(\Upsilon_{12}/2\pi)^q)})} \end{pmatrix} \right\rangle.$$

(6) Evaluate its choice value $\mathfrak{L}_\eta(\mathfrak{U}_\eta) = (\Theta_\eta^q - \Psi_\eta^q) + ((\Omega_\eta/2\pi)^q - (\Upsilon_\eta/2\pi)^q) + (k_\eta/N - 1)^q$, for each $w_\eta \in W$ and $\eta = 1, 2, \ldots, \wp$.
(7) Compute all the indices ξ_j for which $\mathfrak{L}_{\xi_j} = \max_\eta \mathfrak{L}_\eta$.
(8) **if** $\mathfrak{L}_{\xi_1} = \mathfrak{L}_{\xi_2}$ for some $\xi_1, \xi_2 \in \{1, 2, \ldots, \wp\}$, then
Evaluate its accuracy degree $\mathfrak{Q}_\eta(\mathfrak{U}_\eta) = (\Theta_\eta^q + \Psi_\eta^q) + ((\Omega_\eta/2\pi)^q + (\Upsilon_\eta/2\pi)^q) + (k_\eta/N - 1)^q$, for each $w_\eta \in W$, and choose the alternative having maximum accuracy degree.
else
Identify the alternatives w_{ξ_j} for which $\mathfrak{L}_{\xi_j} = \max_\eta \mathfrak{L}_\eta$.
(9) The alternative with maximum score or accuracy degree will be selected.

Algorithm 1: The algorithm of choice values of $C_{\mathbf{q}}$-ROFNSS.

(1) Input $W = \{w_1, w_2, \ldots, w_\wp\}$ as a universe of $\wp$ objects.
(2) Input $M = \{m_1, m_2, \ldots, m_\hbar\}$ as a collection of $\hbar$ attributes.
(3) Input the N-soft set (P, M, N) with $K = \{0, 1, \ldots, N-1\}$, where $N \in \{2, 3, \ldots\}$ Then, for each $w_\eta \in W$, $m_\alpha \in M$, there exist unique $k_{\eta\alpha} \in K$.
(4) Input $C_{\mathbf{q}}$-ROFNSS (p_q, Y, N), where p_q: $M \longrightarrow \mathscr{C}_{\mathbf{q}} - \mathscr{R}\mathscr{O}\mathscr{F}^{W\times K}$ and $Y = (P, M, N)$.
(5) Input H threshold.
(6) Compute $p_q^H(m_\alpha)(w_\eta) = \begin{cases} \left\langle \Theta_{\eta\alpha} e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha} e^{iY_{\eta\alpha}} \right\rangle, & \text{if } \left\langle k_{\eta\alpha}, (\Theta_{\eta\alpha} e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha} e^{iY_{\eta\alpha}}) \right\rangle \in p_q(m_\alpha) \text{ and } k_{\eta\alpha} \geq H \\ \begin{cases} \left\langle 0.00e^{i0.00\pi}, 0.50e^{i1.00\pi} \right\rangle & \text{if } (k_{\eta\alpha}/N) \geq 0.50 \\ \left\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi} \right\rangle & \text{if } (k_{\eta\alpha}/N) < 0.50 \end{cases} \end{cases}$
(7) Calculate $\mathfrak{U}_\eta^H = \sum_{\alpha=1}^{\hbar} \mathfrak{B}_{\eta\alpha}^H$, where $\mathfrak{B}_{\eta\alpha}^H = \left\langle \Theta_{\eta\alpha}^H e^{i\Omega_{\eta\alpha}^H}, \Psi_{\eta\alpha}^H e^{iY_{\eta\alpha}^H} \right\rangle$ and Einstein addition of two $\mathfrak{B}_{\eta\alpha}^H$ is presented as follows:

$$\mathfrak{B}_{11}^H \oplus_e \mathfrak{B}_{12}^H = \left\langle \begin{pmatrix} \sqrt[q]{((\Theta_{11}^H)^q + (\Theta_{12}^H)^q / 1 + (\Theta_{11}^H)^q (\Theta_{12}^H)^q)}\, e^{i2\pi \sqrt[q]{((\Omega_{11}^H/2\pi)^q + (\Omega_{12}^H/2\pi)^q / 1 + (\Omega_{11}^H/2\pi)^q (\Omega_{12}^H/2\pi)^q)}}, \\ (\Psi_{11}^H \Psi_{12}^H / \sqrt[q]{1 + (1 - (\Psi_{11}^H)^q)(1 - (\Psi_{12}^H)^q)})\, e^{i2\pi((\Upsilon_{11}^H/2\pi)(\Upsilon_{12}^H/2\pi) / \sqrt[q]{1 + (1-(\Upsilon_{11}^H/2\pi)^q)(1-(\Upsilon_{12}^H/2\pi)^q)})} \end{pmatrix} \right\rangle.$$

(8) Evaluate its H-choice value $\mathfrak{L}_\eta^H(\mathfrak{U}_\eta^H) = ((\Theta_\eta^H)^q - (\Psi_\eta^H)^q) + ((\Omega_\eta^H/2\pi)^q - (\Upsilon_\eta^H/2\pi)^q)$, for each w_η, and $\eta = 1, 2, \ldots, \wp$.
(9) Compute all the indices ξ_j for which $\mathfrak{L}_{\xi_j}^H = \max_\eta \mathfrak{L}_\eta^H$.
(10) **if** $\mathfrak{L}_{\xi_1}^H = \mathfrak{L}_{\xi_2}^H$ for some $\xi_1, \xi_2 \in \{1, 2, \ldots, \wp\}$, then
Evaluate its accuracy degree $\mathfrak{Q}_\eta^H(\mathfrak{U}_\eta^H) = ((\Theta_\eta^H)^q + (\Psi_\eta^H)^q) + ((\Omega_\eta^H/2\pi)^q + (\Upsilon_\eta^H/2\pi)^q)$, for each $w_\eta \in W$ and choose the alternative having maximum accuracy degree.
else
Identify the alternatives w_{ξ_j} for which $\mathfrak{L}_{\xi_j} = \max_\eta \mathfrak{L}_\eta$.
(11) The alternative with maximum score or accuracy degree will be selected.

Algorithm 2: The algorithm of H-choice values of $C_{\mathbf{q}}$-ROFNSS.

m_2 is the communication and navigation: all spacecraft require a good communication and navigation system to avoid constraints on future space missions. These potent capabilities of spacecraft provide a secure environment for constructive global telecommunication and ensure the accurate tracking of their trajectories.

m_3 is the payload capacity: the payload capacity is the most important consideration in designing the infrastructure of spacecraft. This measurable parameter usually quantifies the carrying capacity of spacecraft which may include cargo, flight crew, munitions, and other scientific equipment by analyzing the nature of their flight missions.

m_4 is the cosmic radiation protection: a broad spectrum of ionizing radiation can penetrate spacesuits, solar equipment, and spacecraft and can badly harm the biological health of astronauts. The cosmic radiation protection plays an indispensable role in the proper

execution of large-scale space missions. The adequate shielding of spacecraft can minimize the risk of the exposure of such harmful radiation.

The main purpose of this analysis is the determination of the best spacecraft for the remarkable strategic and economic advancement of Mexican space agency. The characteristic selection of ideal spacecraft with mind-blowing features will provide space-based solutions to the complicated Earth-bound problems by the accomplishment of ground-breaking scientific discoveries and plays a leading role in exploration of various indispensable cosmological phenomena which in turn dramatically optimizes the long-term profit of aerospace industries. The framework for the identification of the scrutinized MCDM problem is demonstrated in Figure 2.

Now, an identified 7-soft set can easily be obtained from Table 18, where

(i) six-spade suit represents "Outstanding"
(ii) five-spade suit represents "Superb"
(iii) four-spade suit represents "Excellent"
(iv) three-spade suit represents "Very Good"
(v) two-spade suit represents "Good"
(vi) one-spade suit represents "Average"
(vii) big lozenge represents "Poor"

The grades $K = \{0, 1, 2, 3, 4, 5, 6\}$ can be definitely interrelated with the ranking-based evaluations carried out by the allocated spade suit as follows:

(i) 0 serves as "♦"
(ii) 1 serves as "♠"
(iii) 2 serves as "♠♠"
(iv) 3 serves as "♠♠♠"
(v) 4 serves as "♠♠♠♠"
(vi) 5 serves as "♠♠♠♠♠"
(vii) 6 serves as "♠♠♠♠♠♠"

Each space exploration technology is ranked on the basis of their overall competency which is further presented in Table 18.

Now, the tabular representation of its corresponding specified 7-soft set is illustrated in Table 19.

In this respect, the grading criteria based on the score degrees of C_5-ROF7SVs are defined as follows:

$$\begin{aligned}
-2.00 \le \mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) < -1.29 \text{ when } k_{\eta\alpha} = 0,\\
-1.29 \le \mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) < -0.58 \text{ when } k_{\eta\alpha} = 1,\\
-0.58 \le \mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) < 0.13 \text{ when } k_{\eta\alpha} = 2,\\
0.13 \le \mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) < 0.84 \text{ when } k_{\eta\alpha} = 3,\\
0.84 \le \mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) < 1.55 \text{ when } k_{\eta\alpha} = 4,\\
1.55 \le \mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) < 2.26 \text{ when } k_{\eta\alpha} = 5,\\
2.26 \le \mathfrak{L}\left(\mathfrak{B}_{\eta\alpha}\right) \le 3.00 \text{ when } k_{\eta\alpha} = 6,
\end{aligned} \tag{42}$$

where $\mathfrak{L}(\mathfrak{B}_{\eta\alpha}) = (\Theta_{\eta\alpha}^{q} - \Psi_{\eta\alpha}^{q}) + ((\Omega_{\eta\alpha}/2\pi)^{q} - (\Upsilon_{\eta\alpha}/2\pi)^{q}) + (k_{\eta\alpha}/N - 1)^{q}$ and $\mathfrak{B}_{\eta\alpha} = \left\langle k_{\eta\alpha}, (\Theta_{\eta\alpha}e^{i\Omega_{\eta\alpha}}, \Psi_{\eta\alpha}e^{i\Upsilon_{\eta\alpha}})\right\rangle$; $\eta = 1, 2, 3, 4, 5, 6, 7$; $\alpha = 1, 2, 3, 4$.

Therefore, by Definition 8, C_5-ROF7SS can be interpreted as follows:

$$(p_5, Y, 7) = \{(m_1, p_5(m_1)), (m_2, p_5(m_2))), (m_3, p_5(m_3)), (m_4, p_5(m_4))\}, \tag{43}$$

where

$$\begin{aligned}
p_5(m_1) = &\left\{\left\langle (w_1, 4), 0.78e^{i1.89\pi}, 0.44e^{i1.55\pi}\right\rangle, \left\langle (w_2, 1), 0.14e^{i0.12\pi}, 0.77e^{i1.82\pi}\right\rangle, \left\langle (w_3, 3), 0.45e^{i1.68\pi}, 0.29e^{i1.57\pi}\right\rangle,\right.\\
&\cdot \left\langle (w_4, 6), 0.95e^{i1.95\pi}, 0.13e^{i0.09\pi}\right\rangle, \left\langle (w_5, 2), 0.54e^{i1.96\pi}, 0.99e^{i1.24\pi}\right\rangle\}, \left\langle (w_6, 3), 0.34e^{i1.64\pi}, 0.22e^{i1.53\pi}\right\rangle,\\
&\cdot \left\langle (w_7, 5), 0.89e^{i1.86\pi}, 0.21e^{i0.02\pi}\right\rangle\},\\
p_5(m_2) = &\left\{\left\langle (w_1, 3), 0.49e^{i1.77\pi}, 0.35e^{i1.62\pi}\right\rangle, \left\langle (w_2, 2), 0.98e^{i1.08\pi}, 0.62e^{i1.97\pi}\right\rangle, \left\langle (w_3, 5), 0.81e^{i1.93\pi}, 0.20e^{i1.03\pi}\right\rangle,\right.\\
&\cdot \left\langle (w_4, 4), 0.75e^{i1.78\pi}, 0.16e^{i1.18\pi}\right\rangle, \left\langle (w_5, 0), 0.07e^{i0.05\pi}, 0.89e^{i1.92\pi}\right\rangle\}, \left\langle (w_6, 1), 0.12e^{i0.09\pi}, 0.74e^{i1.75\pi}\right\rangle,\\
&\cdot \left\langle (w_7, 2), 0.68e^{i1.34\pi}, 0.82e^{i1.63\pi}\right\rangle\},\\
&\cdot p_5(m_3) = \left\{\left\langle (w_1, 2), 0.78e^{i1.55\pi}, 0.89e^{i1.67\pi}\right\rangle, \left\langle (w_2, 1), 0.18e^{i0.11\pi}, 0.82e^{i1.76\pi}\right\rangle, \left\langle (w_3, 0), 0.04e^{i0.06\pi}, 0.96e^{i1.89\pi}\right\rangle,\right.\\
&\cdot \left\langle (w_4, 6), 0.93e^{i1.97\pi}, 0.05e^{i0.04\pi}\right\rangle, \left\langle (w_5, 4), 0.76e^{i1.86\pi}, 0.11e^{i1.12\pi}\right\rangle\}, \left\langle (w_6, 5), 0.85e^{i1.89\pi}, 0.01e^{i0.05\pi}\right\rangle,\\
&\cdot \left\langle (w_7, 2), 0.53e^{i1.45\pi}, 0.64e^{i1.56\pi}\right\rangle\},\\
p_5(m_4) = &\left\{\left\langle (w_1, 1), 0.17e^{i0.13\pi}, 0.84e^{i1.85\pi}\right\rangle, \left\langle (w_2, 6), 0.94e^{i1.96\pi}, 0.03e^{i0.12\pi}\right\rangle, \left\langle (w_3, 2), 0.61e^{i1.13\pi}, 0.85e^{i1.54\pi}\right\rangle,\right.\\
&\cdot \left\langle (w_4, 5), 0.87e^{i1.87\pi}, 0.23e^{i0.04\pi}\right\rangle, \left\langle (w_5, 2), 0.55e^{i1.23\pi}, 0.67e^{i1.57\pi}\right\rangle\}, \left\langle (w_6, 0), 0.01e^{i0.03\pi}, 0.99e^{i1.96\pi}\right\rangle,\\
&\cdot \left\langle (w_7, 3), 0.39e^{i1.81\pi}, 0.27e^{i1.72\pi}\right\rangle\}.
\end{aligned} \tag{44}$$

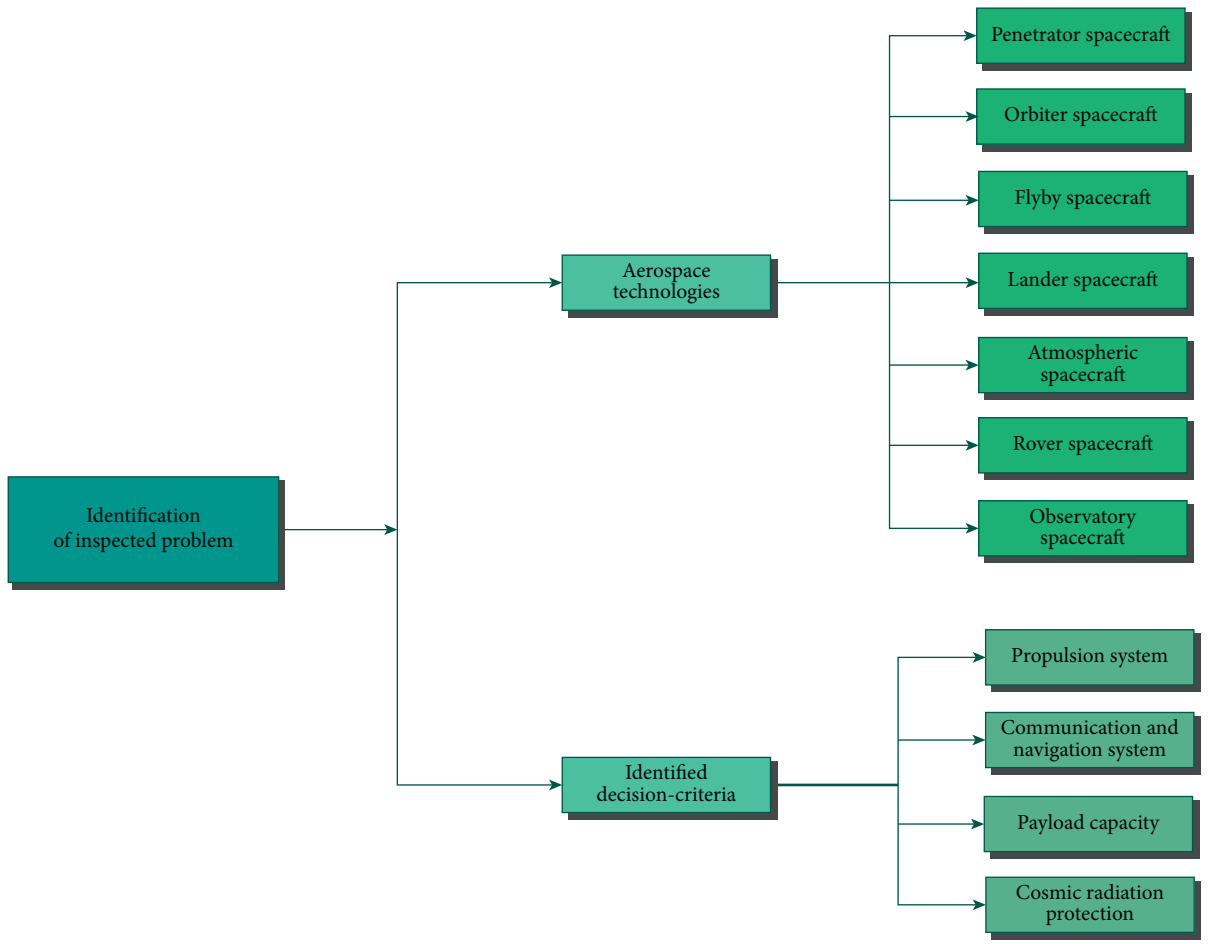

Figure 2: Specification of investigated problem.

Table 18: Rating of spacecraft.

$(P, M, 7)$	m_1	m_2	m_3	m_4
w_1				
w_2				
w_3				
w_4				
w_5				
w_6				
w_7				

Table 19: Identified 7-soft set.

$(P, M, 7)$	m_1	m_2	m_3	m_4
w_1	4	3	2	1
w_2	1	2	1	6
w_3	3	5	0	2
w_4	6	4	6	5
w_5	2	0	4	2
w_6	3	1	5	0
w_7	5	2	2	3

Now, the C_5-ROF7SS can be easily demonstrated in tabular form by Table 20.

5.3. Choice Values of C_5-ROF7SS. The quantified choice values of C_5-ROF7SS for the determination of best spacecraft are summarized in Table 21.

The spacecraft are further categorized in an ascending order of their respective choice values. The hierarchical ranking of inspected space exploration technologies is highlighted in Table 22.

Consequently, we infer that the lander spacecraft (w_4) will be selected as the most competent space exploration technology for the strategic and economic development of Mexican space agency.

5.4. H-Choice Values of C_5-ROF7SS. The calibrated H-choice values of C_5-ROF7SS are encapsulated in Table 23, where we select $H = 3$.

The spacecraft are eventually organized in an ascending order of their respective H-choice values, where $H = 3$. The hierarchical ranking of inspected aerospace technologies is clearly demonstrated in Table 24.

Hence, we conclude that the lander spacecraft (w_4) will be selected as the most ideal spacecraft for the exploration of various cosmological phenomena.

6. Comparative Analysis

In this section, a comparative study of our proposed decision-making techniques with contemporary MCDM technique based on the q-rung orthopair fuzzy soft weighted geometric (q-ROFS$_{ft}$WG) operator is presented. We investigate the pragmatic application named "selection of the most proficient aerospace technology" by applying existing technique to authenticate the flexibility and sustainability of our established strategies.

6.1. q-ROFS$_{ft}$WG Operator-Based MCDM Strategy. We now simplify the practical application 5.2 by implementing the systematic methodology of q-ROFS$_{ft}$WG operator-based MCDM technique, proposed by Chinram et al. [22].

Step 1. The first step is to construct the 5-rung orthopair fuzzy soft decision matrix (5-ROFS$_{ft}$DM) by scrutinizing the credibility of each aerospace technology relative to specific decision-criteria. The 5-rung orthopair fuzzy soft entries of 5-ROFS$_{ft}$DM $\mathfrak{F} = (\mathbb{F}_{\eta\alpha})_{7\times4}$ are obtained by omitting the grades and phase terms of all C_5-ROF7SVs in Table 20. The calibrated 5-ROFS$_{ft}$-entries $\mathbb{F}_{\eta\alpha} = \langle\Theta, \Psi\rangle$ of 5-ROFS$_{ft}$DM regarding proficiency of spacecraft are summarized in Table 25.

Step 2. Normalize the evaluated 5-ROFS$_{ft}$DM $\mathfrak{F} = (\mathbb{F}_{\eta\alpha})_{7\times4}$ according to the nature and specification of each identified decision-criterion by utilizing the following expression:

$$\mathbb{E}_{\eta\alpha} = \begin{cases} \mathbb{F}^{c}_{\eta\alpha}, & \text{for cost type parameter,} \\ \mathbb{F}_{\eta\alpha}, & \text{for benefit type parameter,} \end{cases} \tag{45}$$

where $\mathbb{F}^{c}_{\eta\alpha} = \langle\Psi, \Theta\rangle$ represents the complement of $\mathbb{F}_{\eta\alpha} = \langle\Theta, \Psi\rangle$. Here, the normalized 5-rung orthopair fuzzy soft decision matrix $\mathfrak{E} = (\mathbb{E}_{\eta\alpha})_{7\times4}$ is the same as presented in Table 25 because all the specified decisive parameters of inspected problem are of the same benefit type.

Step 3. Aggregate the evaluated interpretations of each aerospace technology corresponding to identified decision-criterion by employing the q-ROFS$_{ft}$WG operator, which is defined as follows:

$$\begin{aligned} \widetilde{\mathbb{E}}_{\eta} &= q\text{–ROFS}_{\text{ft}}\text{WG}_{\mathfrak{H}}\left(\mathbb{E}^{(1)}_{\eta 1}, \mathbb{E}^{(2)}_{\eta 2}, \mathbb{E}^{(3)}_{\eta 3}, \ldots, \mathbb{E}^{(\tau)}_{\eta\hbar}\right) \\ &= \bigotimes_{\alpha=1}^{\hbar}\left(\bigotimes_{\zeta=1}^{\tau}\left(\mathbb{E}^{(\zeta)}_{\eta\alpha}\right)^{k_\zeta}\right)^{z_\alpha} \\ &= \left\langle \prod_{\alpha=1}^{\hbar}\left(\prod_{\zeta=1}^{\tau}\left(\Theta^{(\zeta)}_{\eta\alpha}\right)^{k_\zeta}\right)^{z_\alpha}, \sqrt[q]{1-\prod_{\alpha=1}^{\hbar}\left(\prod_{\zeta=1}^{\tau}\left(1-\Psi^{(\zeta)}_{\eta\alpha}\right)^{k_\zeta}\right)^{z_\alpha}}\right\rangle, \end{aligned} \tag{46}$$

where k_ζ indicates the weights of decision-makers and z_α denotes the relative weights of decisive parameters satisfying the conditions that $k_\zeta, z_\alpha \in [0, 1]$ with $\sum_{\zeta=1}^{\tau} k_\zeta$ and $\sum_{\alpha=1}^{\hbar} z_\alpha$. Now, the appointed expert assigns weights $\mathbf{z} = (0.32, 0.20, 0.33, 0.15)^{\mathbf{T}}$ to the identified decision-criteria. The aggregated 5-ROFS$_{ft}$Ns of each spacecraft corresponding to decisive criteria are presented as follows:

$$\begin{aligned} \widetilde{\mathbb{E}}_1 &= \langle 0.56554, 0.78625\rangle, \\ \widetilde{\mathbb{E}}_2 &= \langle 0.29868, 0.75104\rangle, \\ \widetilde{\mathbb{E}}_3 &= \langle 0.23834, 0.86201\rangle, \\ \widetilde{\mathbb{E}}_4 &= \langle 0.88799, 0.16690\rangle, \\ \widetilde{\mathbb{E}}_5 &= \langle 0.40281, 0.92670\rangle, \\ \widetilde{\mathbb{E}}_6 &= \langle 0.22009, 0.83050\rangle, \\ \widetilde{\mathbb{E}}_7 &= \langle 0.62802, 0.65689\rangle. \end{aligned} \tag{47}$$

Step 4. The next step is to evaluate the score degrees $(\mathfrak{L}_\eta)$ of aggregated 5-ROFS$_{ft}$Ns $\widetilde{\mathbb{E}}_\eta = \left\langle\widetilde{\Theta}_\eta, \widetilde{\Psi}_\eta\right\rangle$ of each aerospace technology which is interpreted as follows:

$$\mathfrak{L}\left(\widetilde{\mathbb{E}}_\eta\right) = \Theta^q_\eta - \Psi^q_\eta + \left(\frac{e^{\Theta^q_\eta - \Psi^q_\eta}}{e^{\Theta^q_\eta - \Psi^q_\eta} + 1} - \frac{1}{2}\right)\pi^q_\eta, \quad \text{for } q \geq 1, \tag{48}$$

where $\eta = 1, 2, \ldots, 7$ and $\pi_\eta = \sqrt[q]{1 - ((\Theta)^q + (\Psi)^q)}$ represents the hesitancy degree of q-ROFS$_{ft}$Ns. The quantified score degrees of $\widetilde{\mathbb{E}}_\eta$ for each spacecraft are clearly illustrated in Table 26.

Step 5. All the aerospace technologies of the inspected MCDM problem are hierarchically ranked in an ascending

Table 20: Tabular representation of C_5-ROF7SS.

$(p_5, Y, 7)$	m_1	m_2	m_3	m_4
w_1	$\langle 4, (0.78e^{i1.89\pi}, 0.44e^{i1.55\pi})\rangle$	$\langle 3, (0.49e^{i1.77\pi}, 0.35e^{i1.62\pi})\rangle$	$\langle 2, (0.78e^{i1.55\pi}, 0.89e^{i1.67\pi})\rangle$	$\langle 1, (0.17e^{i0.13\pi}, 0.84e^{i1.85\pi})\rangle$
w_2	$\langle 1, (0.14e^{i0.12\pi}, 0.77e^{i1.82\pi})\rangle$	$\langle 2, (0.98e^{i1.08\pi}, 0.62e^{i1.97\pi})\rangle$	$\langle 1, (0.18e^{i0.11\pi}, 0.82e^{i1.76\pi})\rangle$	$\langle 6, (0.94e^{i1.96\pi}, 0.03e^{i0.12\pi})\rangle$
w_3	$\langle 3, (0.45e^{i1.68\pi}, 0.29e^{i1.57\pi})\rangle$	$\langle 5, (0.81e^{i1.93\pi}, 0.20e^{i1.03\pi})\rangle$	$\langle 0, (0.04e^{i0.06\pi}, 0.96e^{i1.89\pi})\rangle$	$\langle 2, (0.61e^{i1.13\pi}, 0.85e^{i1.54\pi})\rangle$
w_4	$\langle 6, (0.95e^{i1.95\pi}, 0.13e^{i0.09\pi})\rangle$	$\langle 4, (0.75e^{i1.78\pi}, 0.16e^{i1.18\pi})\rangle$	$\langle 6, (0.93e^{i1.97\pi}, 0.05e^{i0.04\pi})\rangle$	$\langle 5, (0.87e^{i1.87\pi}, 0.23e^{i0.04\pi})\rangle$
w_5	$\langle 2, (0.54e^{i1.96\pi}, 0.99e^{i1.24\pi})\rangle$	$\langle 0, (0.07e^{i0.05\pi}, 0.89e^{i1.92\pi})\rangle$	$\langle 4, (0.76e^{i1.86\pi}, 0.11e^{i1.12\pi})\rangle$	$\langle 2, (0.55e^{i1.23\pi}, 0.67e^{i1.57\pi})\rangle$
w_6	$\langle 3, (0.34e^{i1.64\pi}, 0.22e^{i1.53\pi})\rangle$	$\langle 1, (0.12e^{i0.09\pi}, 0.74e^{i1.75\pi})\rangle$	$\langle 5, (0.85e^{i1.89\pi}, 0.01e^{i0.05\pi})\rangle$	$\langle 0, (0.01e^{i0.03\pi}, 0.99e^{i1.96\pi})\rangle$
w_7	$\langle 5, (0.89e^{i1.86\pi}, 0.21e^{i0.02\pi})\rangle$	$\langle 2, (0.68e^{i1.34\pi}, 0.82e^{i1.63\pi})\rangle$	$\langle 2, (0.53e^{i1.45\pi}, 0.64e^{i1.56\pi})\rangle$	$\langle 3, (0.39e^{i1.81\pi}, 0.27e^{i1.72\pi})\rangle$

order corresponding to their calibrated score degrees and further demonstrated their systematic ranking in Table 27.

Hence, we conclude that lander spacecraft (w_4) will be selected as the most proficient aerospace technology having maximum score degree for the accomplishment of large-scale global missions.

6.2. Comprehensive Discussion

(1) We exhibit a comparison of developed methodologies with contemporary MCDM techniques based on $\mathbf{q}$-ROFS$_{ft}$WG, $C_\mathbf{q}$-ROFWA, and $C_\mathbf{q}$-ROFWG operators to substantiate the accountability and feasibility of the proposed strategies. The methodological implications of both developed and compared strategies are persuasively spotlighted in Table 28.

(2) All the proposed and compared strategies prioritize lander spacecraft (w_4) as the most productive aerospace technology for the prestigious economic advancement of Mexican space agency which demonstrate the compatibility and enforceability of remarkable decision-making skills of our established techniques in the sophisticated MCDM problems.

(3) An explicative bar chart is illustrated in Figure 3 in order to visualize the comparative final outcomes of both proposed and compared MCDM techniques regarding hierarchical ordering of aerospace technologies, which in turn authenticate the rationality and plausibility of our developed methodologies.

(4) The hierarchical ordering of inspected aerospace technologies differ slightly in compared and proposed strategies which is due to the appraisement of distinct fuzzy environments as proposed methodologies significantly overcome all the deficiencies of compared strategies by capturing the graded obscurity of two-dimensional parameterized information. In spite of all these, all the compared and proposed approaches interpret the similar end-results which illuminate the effectuality of the proposed strategies.

(5) The $\mathbf{q}$-ROFS$_{ft}$WG operator is not capable enough to deal with $C_\mathbf{q}$-ROFNS information because of its limitations that arise due to the inadequacy of multinary assessment grades. However, our proposed systematic techniques have convincingly resolved these shortcomings of compared technique by capturing the graded evaluations of such paradoxical and uncertain data. This remarkable flexibility of our proposed strategies makes them more powerful as compared to the existing techniques.

(6) Our developed strategies have an edge over the compared techniques that are directly based on $C_\mathbf{q}$-ROFWA and $C_\mathbf{q}$-ROFWG operators as it fantastically addresses the vagueness and periodicity of imprecise data at the same time, but the compared techniques are constrained to model the ambiguity of nonperiodic information only, which may lead to the inconsistent and illogical decisions. This incredible peculiarity of our proposed methodologies stimulates them as the most generalized and sustainable decision-making strategies for real-world MCDM complications.

7. Merits of the $C_\mathbf{q}$-ROFNSS Model

The next bullet points summarize the main advantages of the model proposed in this paper:

(1) We have proposed a $C_\mathbf{q}$-ROFNSS model that provides a multiskilled and most proficient mathematical framework for the fuzzy representation of parameterized vague data. The underlying features of both NSSs and $C_\mathbf{q}$-ROFSs are inherited by this extension. In fact, we believe that the theory arising from this model has significantly resolved all the flaws and limitations of existing models for the representation of ambiguous knowledge.

(2) A paramount advantage of the proposed model is its powerful ability to model two-dimensional imprecise information. It can address the periodicity and vagueness of parameterized data at the same time in a convincing manner. However, not only is the idea of the proposed model restricted to the two-dimensional case, but also it can handle one-dimensional uncertain data with similar accuracy and precision.

(3) In the modern digital era, grading systems became massively popular as they are standard tools for collecting users' inputs about their opinions on electronic services, web pages, online products, marketing sites, movies, and countless other online applications. Our theory is especially crafted with an aim to capture the graded assessments of a variety of informational sources in order to meet future challenges.

Table 21: Choice values of C_5-ROF7SS.

$(p_5, Y, 7)$	m_1	m_2	m_3	m_4	$\mathfrak{U}_\eta$	$\mathfrak{L}_\eta$
w_1	$\langle 4, (0.78e^{i1.89\pi}, 0.44e^{i1.55\pi})\rangle$	$\langle 3, (0.49e^{i1.77\pi}, 0.35e^{i1.62\pi})\rangle$	$\langle 2, (0.78e^{i1.55\pi}, 0.89e^{i1.67\pi})\rangle$	$\langle 1, (0.17e^{i0.13\pi}, 0.84e^{i1.85\pi})\rangle$	$\langle 4, (0.88e^{i1.98\pi}, 0.08e^{i0.77\pi})\rangle$	1.6300
w_2	$\langle 1, (0.14e^{i0.12\pi}, 0.77e^{i1.82\pi})\rangle$	$\langle 2, (0.98e^{i1.08\pi}, 0.62e^{i1.97\pi})\rangle$	$\langle 1, (0.18e^{i0.11\pi}, 0.82e^{i1.76\pi})\rangle$	$\langle 6, (0.94e^{i1.96\pi}, 0.03e^{i0.12\pi})\rangle$	$\langle 6, (0.99e^{i1.96\pi}, 0.01e^{i0.08\pi})\rangle$	2.8966
w_3	$\langle 3, (0.45e^{i1.68\pi}, 0.29e^{i1.57\pi})\rangle$	$\langle 5, (0.81e^{i1.93\pi}, 0.20e^{i1.03\pi})\rangle$	$\langle 0, (0.04e^{i0.06\pi}, 0.96e^{i1.89\pi})\rangle$	$\langle 2, (0.61e^{i1.13\pi}, 0.85e^{i1.54\pi})\rangle$	$\langle 5, (0.84e^{i1.97\pi}, 0.03e^{i0.45\pi})\rangle$	1.7741
w_4	$\langle 6, (0.95e^{i1.95\pi}, 0.13e^{i0.09\pi})\rangle$	$\langle 4, (0.75e^{i1.78\pi}, 0.16e^{i1.18\pi})\rangle$	$\langle 6, (0.93e^{i1.97\pi}, 0.05e^{i0.04\pi})\rangle$	$\langle 5, (0.87e^{i1.87\pi}, 0.23e^{i0.04\pi})\rangle$	$\langle 6, (0.99e^{i1.99\pi}, 0.00e^{i0.00\pi})\rangle$	2.9262
w_5	$\langle 2, (0.54e^{i1.96\pi}, 0.99e^{i1.24\pi})\rangle$	$\langle 0, (0.07e^{i0.05\pi}, 0.89e^{i1.92\pi})\rangle$	$\langle 4, (0.76e^{i1.86\pi}, 0.11e^{i1.12\pi})\rangle$	$\langle 2, (0.55e^{i1.23\pi}, 0.67e^{i1.57\pi})\rangle$	$\langle 4, (0.81e^{i1.99\pi}, 0.05e^{i0.40\pi})\rangle$	1.4575
w_6	$\langle 3, (0.34e^{i1.64\pi}, 0.22e^{i1.53\pi})\rangle$	$\langle 1, (0.12e^{i0.09\pi}, 0.74e^{i1.75\pi})\rangle$	$\langle 5, (0.85e^{i1.89\pi}, 0.01e^{i0.05\pi})\rangle$	$\langle 0, (0.01e^{i0.03\pi}, 0.99e^{i1.96\pi})\rangle$	$\langle 5, (0.85e^{i1.94\pi}, 0.001e^{i0.02\pi})\rangle$	1.7280
w_7	$\langle 5, (0.89e^{i1.86\pi}, 0.21e^{i0.02\pi})\rangle$	$\langle 2, (0.68e^{i1.34\pi}, 0.82e^{i1.63\pi})\rangle$	$\langle 2, (0.53e^{i1.45\pi}, 0.64e^{i1.56\pi})\rangle$	$\langle 3, (0.39e^{i1.81\pi}, 0.27e^{i1.72\pi})\rangle$	$\langle 5, (0.92e^{i1.98\pi}, 0.02e^{i0.01\pi})\rangle$	2.0375

Table 22: Hierarchical ranking of spacecraft.

Spacecraft	w_1	w_2	w_3	w_4	w_5	w_6	w_7
Ranking	6	2	4	1	7	5	3

Table 23: 3-choice values of C_5-ROF7SS.

(p_5^3, M)	m_1	m_2	m_3	m_4	$\mathfrak{U}_\eta^3$	$\mathfrak{L}_\eta^3$
w_1	$\langle 0.78e^{i1.89\pi}, 0.44e^{i1.55\pi}\rangle$	$\langle 0.49e^{i1.77\pi}, 0.35e^{i1.62\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.79e^{i1.96\pi}, 0.13e^{i1.16\pi}\rangle$	1.1680
w_2	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.94e^{i1.96\pi}, 0.03e^{i0.12\pi}\rangle$	$\langle 0.94e^{i1.96\pi}, 0.03e^{i0.12\pi}\rangle$	1.6378
w_3	$\langle 0.45e^{i1.68\pi}, 0.29e^{i1.57\pi}\rangle$	$\langle 0.81e^{i1.93\pi}, 0.20e^{i1.03\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.81e^{i1.97\pi}, 0.05e^{i0.72\pi}\rangle$	1.2880
w_4	$\langle 0.95e^{i1.95\pi}, 0.13e^{i0.09\pi}\rangle$	$\langle 0.75e^{i1.78\pi}, 0.16e^{i1.18\pi}\rangle$	$\langle 0.93e^{i1.97\pi}, 0.05e^{i0.04\pi}\rangle$	$\langle 0.87e^{i1.87\pi}, 0.23e^{i0.04\pi}\rangle$	$\langle 0.99e^{i1.99\pi}, 0.00e^{i0.00\pi}\rangle$	1.9262
w_5	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.76e^{i1.86\pi}, 0.11e^{i1.12\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.76e^{i1.86\pi}, 0.11e^{i1.12\pi}\rangle$	0.8941
w_6	$\langle 0.34e^{i1.64\pi}, 0.22e^{i1.53\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.85e^{i1.89\pi}, 0.01e^{i0.05\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.85e^{i1.94\pi}, 0.001e^{i0.03\pi}\rangle$	1.3261
w_7	$\langle 0.89e^{i1.86\pi}, 0.21e^{i0.02\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.00e^{i0.00\pi}, 1.00e^{i2.00\pi}\rangle$	$\langle 0.39e^{i1.81\pi}, 0.27e^{i1.72\pi}\rangle$	$\langle 0.89e^{i1.96\pi}, 0.04e^{i0.01\pi}\rangle$	1.4805

Table 24: Hierarchical ranking of spacecraft.

Spacecraft	w_1	w_2	w_3	w_4	w_5	w_6	w_7
Ranking	6	2	5	1	7	4	3

Table 25: Tabular representation of 5-ROFS$_{ft}$DM.

$\mathfrak{T}$	m_1	m_2	m_3	m_4
w_1	$\langle 0.78, 0.44\rangle$	$\langle 0.49, 0.35\rangle$	$\langle 0.78, 0.89\rangle$	$\langle 0.17, 0.84\rangle$
w_2	$\langle 0.14, 0.77\rangle$	$\langle 0.98, 0.62\rangle$	$\langle 0.18, 0.82\rangle$	$\langle 0.94, 0.03\rangle$
w_3	$\langle 0.45, 0.29\rangle$	$\langle 0.81, 0.20\rangle$	$\langle 0.04, 0.96\rangle$	$\langle 0.61, 0.85\rangle$
w_4	$\langle 0.95, 0.13\rangle$	$\langle 0.75, 0.16\rangle$	$\langle 0.93, 0.05\rangle$	$\langle 0.87, 0.23\rangle$
w_5	$\langle 0.54, 0.99\rangle$	$\langle 0.07, 0.89\rangle$	$\langle 0.76, 0.11\rangle$	$\langle 0.55, 0.67\rangle$
w_6	$\langle 0.34, 0.22\rangle$	$\langle 0.12, 0.74\rangle$	$\langle 0.85, 0.01\rangle$	$\langle 0.01, 0.99\rangle$
w_7	$\langle 0.89, 0.21\rangle$	$\langle 0.68, 0.82\rangle$	$\langle 0.53, 0.64\rangle$	$\langle 0.39, 0.27\rangle$

Table 26: Score degrees of $\widetilde{\mathbb{E}}_\eta$

$\mathfrak{L}_\eta/w_\eta$	w_1	w_2	w_3	w_4	w_5	w_6	w_7
$\mathfrak{L}_\eta$	−0.29785	−0.29230	−0.57764	0.66663	−0.80079	−0.48265	−0.03048

Table 27: Hierarchical ranking of spacecraft.

Spacecraft	w_1	w_2	w_3	w_4	w_5	w_6	w_7
Ranking	4	3	6	1	7	5	2

Table 28: Comparative analysis.

Methods	Ranking	Best spacecraft
Proposed choice values of $C_\mathbf{q}$-ROFNSS method	$w_4 \succ w_2 \succ w_7 \succ w_3 \succ w_6 \succ w_1 \succ w_5$	w_4
Proposed H-choice values of $C_\mathbf{q}$-ROFNSS method	$w_4 \succ w_2 \succ w_7 \succ w_6 \succ w_3 \succ w_1 \succ w_5$	w_4
$\mathbf{q}$-ROFS$_{ft}$WG operator-based MCDM method [22]	$w_4 \succ w_7 \succ w_2 \succ w_1 \succ w_6 \succ w_3 \succ w_5$	w_4
$C_\mathbf{q}$-ROFWA operator-based MADM method [12]	$w_4 \succ w_7 \succ w_2 \succ w_5 \succ w_6 \succ w_1 \succ w_3$	w_4
$C_\mathbf{q}$-ROFWG operator-based MADM method [12]	$w_4 \succ w_5 \succ w_3 \succ w_2 \succ w_6 \succ w_1 \succ w_7$	w_4

(4) The remarkable decision-making skills of $C_\mathbf{q}$-ROFNSS excellently increase the modeler's ability to express their two-dimensional ambiguous assessments. The outstanding generic structure of our proposed model broadens the confined space of traditional fuzzy models by relaxing their constraint conditions. The proposed hybrid model also renders the most generalized tool for ranking-based competent modeling of parameterized inexact data of periodic and nonperiodic nature as well.

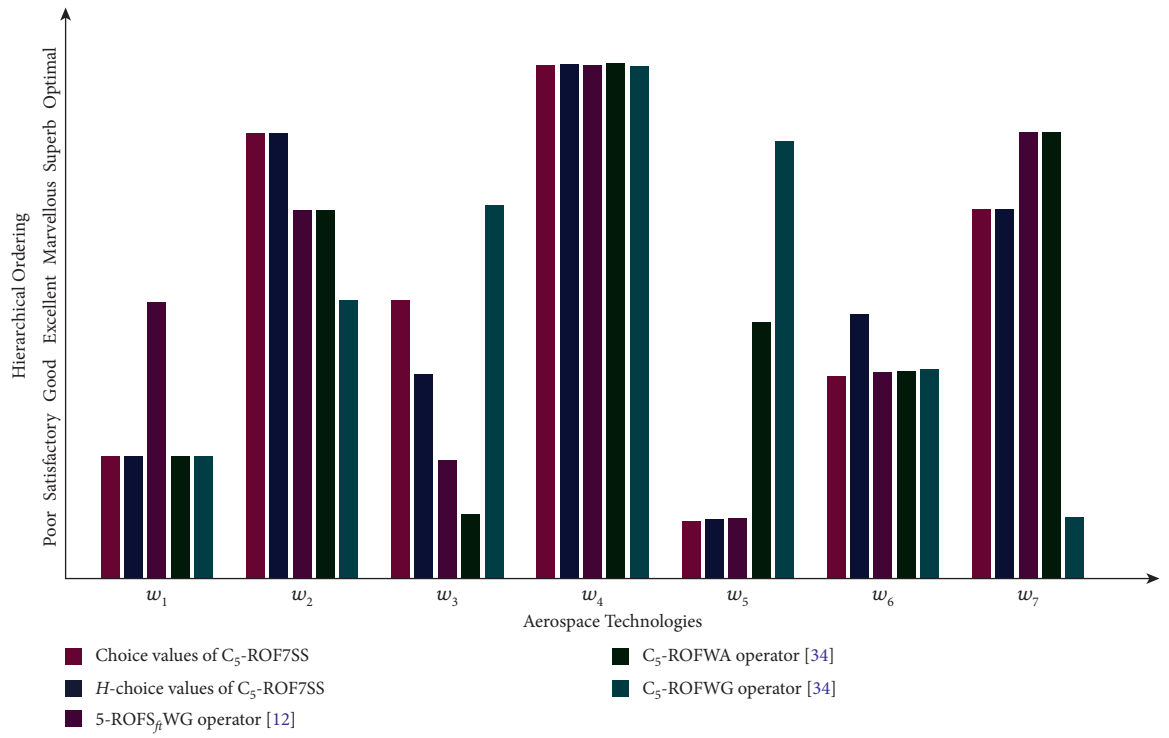

Figure 3: Comparative analysis.

8. Conclusion

In this research article, we have accomplished our goal by establishing the foundations of a multiskilled hybrid model called C_q-ROFNSS. It can capture a wide range of imprecision embedded in human cognition. This highly competent and most flexible theory sets up a mathematical framework for the representation of two-dimensional inexact information. It has extended several contemporary models by the integration of the expertise of C_q-ROFSs and the remarkable parametric nature of NSSs. This theory means a powerful generalization of both CIFNSSs and CPFNSSs. It broadens their limited boundary space by relaxing their constraint conditions in terms of an adjustable parameter q. To this purpose, we have elaborated the formal definition of C_q-ROFNSS and its basic fundamental set-theoretic operations. We have proposed the Einstein and some other algebraic operations of C_q-ROFNSVs. We have demonstrated the rationality of the proposed model with a concise analysis of its relationships with existing theories.

Another remarkable contribution of this study is the development of two systematic MCDM algorithms for the identification of a most favorable alternative on the basis of their intrinsic characteristics. We have tested their feasibility with a real-life application for the evaluation of the most productive aerospace technology. We have conducted a comparative analysis with contemporary MCDM techniques to justify that our strategies are reasonable extensions. We have accompanied the comparative study with an interpretative bar chart in order to illustrate the compatibility and veracity of the final outcomes. Finally, we have investigated the dynamic features of the proposed model in order to throw light on its merits and preeminence over existing decision-making theories.

Although we can argue that the developed model has an edge over the contemporary approaches, this theory does not lack for limitations. Its structure cannot capture the abstinence and refusal aspect of inexact human expressions. Also, the strategies that we used to solve MCDM problems may be computationally costly, as they involve rather cumbersome and tedious calculations. Thus, in the future, we are committed to reduce the massive calculations of these MCDM techniques with the assistance of some computer programming and to develop graphical representations of the proposed model in order to elaborate this proficient concept more effectively. Additionally, we are planning to extend our research work by establishing more advanced MCGDM strategies, including the C_q-ROFNS-PROMETHEE method, C_q-ROFNS-VIKOR method, C_q-ROFNS-AHP method, C_q-ROFNS-TOPSIS method, and C_q-ROFNS-ELECTRE method. We intend to explore the scope of the potential applications under the versatile environment of the C_q-ROFNS model.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[3] R. R. Yager, "Pythagorean fuzzy subsets," in *Proceedings of the Joint IFSA World Congress and NAFIPS Annual Meeting*, (IFSA/NAFIPS) Edmonton, Canada, June 2013.

[4] R. R. Yager, "Generalized orthopair fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1222–1230, 2017.

[5] M. Akram and G. Shahzadi, "A hybrid decision-making model under$q$-rung orthopair fuzzy yager aggregation operators," *Granular Computing*, vol. 4, 2020.

[6] T. Shaheen, M. I. Ali, and H. Toor, "Why do we need$q$-rung orthopair fuzzy sets? Some evidence established via mass assignment," *International Journal of Intelligence Systems*, 2021.

[7] Z. Yang, X. Li, H. Garg, and M. Qi, "A cognitive information-based decision-making algorithm using interval-valued q-rung picture fuzzy numbers and heronian mean operators," *Cognitive Computation*, vol. 13, no. 2, pp. 357–380, 2021.

[8] Z. Yang, H. Garg, J. Li et al., "Investigation of multiple heterogeneous relationships using aq-rung orthopair fuzzy multi-criteria decision algorithm," *Neural Computing & Applications*, 2020.

[9] D. Ramot, R. Milo, M. Friedman, and A. Kandel, "Complex fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp. 171–186, 2002.

[10] A. S. Alkouri and A. R. Salleh, "Complex intuitionistic fuzzy sets," *AIP Conference Proceedings*, vol. 1482, no. 1, pp. 464–470, 2012.

[11] K. Ullah, T. Mahmood, Z. Ali, and N. Jan, "On some distance measures of complex pythagorean fuzzy sets and their applications in pattern recognition," *Complex & Intelligent Systems*, vol. 6, no. 1, pp. 15–27, 2020.

[12] P. Liu, T. Mahmood, and Z. Ali, "Complex q-rung orthopair fuzzy aggregation operators and their applications in multi-attribute group decision making," *Information*, vol. 11, no. 1, p. 5, 2020.

[13] T. Mahmood and Z. Ali, "Entropy measure and TOPSIS method based on correlation coefficient using complex$q$-rung orthopair fuzzy information and its application to multi-attribute decision making," *Soft Computing*, vol. 25, pp. 1249–1275, 2021.

[14] H. Garg, Z. Ali, and T. Mahmood, "Algorithms for complex interval-valued$q$-rung orthopair fuzzy sets in decision making based on aggregation operators," *AHP, and TOPSIS, Expert Systems*, vol. 3, no. 1, 2021.

[15] C. Jana, G. Muhiuddin, and M. Pal, "Multiple-attribute decision making problems based on SVTNH methods," *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, no. 9, pp. 3717–3733, 2020.

[16] D. Molodtsov, "Soft set theory-first results," *Computers and Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[17] J. C. R. Alcantud, "Softarisons: theory and practice," *Neural Computing And Applications*, vol. 4, 2021.

[18] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *The Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589–602, 2001.

[19] P. K. Maji, R. Biswas, and A. R. Roy, "Intuitionistic fuzzy soft sets," *The Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 677–692, 2001.

[20] X. Peng, Y. Yang, and J. Song, "Pythagorean fuzzy soft set and its application," *Computer Engineering*, vol. 41, no. 7, pp. 224–229, 2015.

[21] A. Hussain, M. I. Ali, T. Mahmood, and M. Munir, "q-Rung orthopair fuzzy soft average aggregation operators and their application in multicriteria decision-making," *International Journal of Intelligent Systems*, vol. 35, no. 4, pp. 571–599, 2020.

[22] R. Chinram, A. Hussain, M. I. Ali, and T. Mahmood, "Some geometric aggregation operators under q-rung orthopair fuzzy soft information with their applications in multi-criteria decision making," *IEEE Access*, vol. 9, pp. 31975–31993, 2021.

[23] H. Garg, Z. Ali, and T. Mahmood, "Generalized dice similarity measures for complex q-rung orthopair fuzzy sets and its application," *Complex and Intelligent Systems*, vol. 7, pp. 667–686, 2021.

[24] J. Wang, G. Wei, C. Wei, and Y. Wei, "MABAC method for multiple attribute group decision making under q-rung orthopair fuzzy environment," *Defence Technology*, vol. 16, no. 1, pp. 208–216, 2020.

[25] W. Yang and Y. Pang, "New q-rung orthopair hesitant fuzzy decision making based on linear programming and TOPSIS," *IEEE Access*, vol. 8, pp. 221299–221311, 2020.

[26] C. Jana, G. Muhiuddin, and M. Pal, "Some Dombi aggregation of q-rung orthopair fuzzy numbers in multiple-attribute decision making," *International Journal of Intelligent Systems*, vol. 34, no. 12, pp. 3220–3240, 2019.

[27] L. Wang, H. Garg, and N. Li, "Pythagorean fuzzy interactive Hamacher power aggregation operators for assessment of express service quality with entropy weight," *Soft Computing*, vol. 25, pp. 973–993, 2021.

[28] F. Fatimah, D. Rosadi, R. B. F. Hakim, and J. C. R. Alcantud, "N-soft sets and their decision making algorithms," *Soft Computing*, vol. 22, no. 12, pp. 3829–3842, 2018.

[29] M. Akram, A. Adeel, and J. C. R. Alcantud, "Group decision-making methods based on hesitant N-soft sets," *Expert Systems with Applications*, vol. 115, pp. 95–105, 2019.

[30] M. Akram, A. Adeel, and J. C. R. Alcantud, "Fuzzy N-soft sets: a novel model with applications," *Journal of Intelligent & Fuzzy Systems*, vol. 35, no. 4, pp. 4757–4771, 2018.

[31] M. Akram, G. Ali, and J. C. R. Alcantud, "New decision-making hybrid model: intuitionistic fuzzy N-soft rough sets," *Soft Computing*, vol. 23, no. 20, pp. 9853–9868, 2019.

[32] M. Akram, F. Wasim, and A. N. Al-Kenani, "A hybrid decision-making approach under complex pythagorean fuzzy N-soft sets," *International Journal of Computational Intelligence Systems*, vol. 14, no. 1, pp. 1263–1291, 2021.

[33] G. Shahzadi, G. Muhiuddin, M. A. Butt, and A. Ashraf, "Hamacher interactive hybrid weighted averaging operators under Fermatean fuzzy numbers," *Journal of Mathematics*, vol. 2021, Article ID 5556017, 17 pages, 2021.

[34] F. Feng, Y. Zheng, B. Sun, and M. Akram, "Novel score functions of generalized orthopair fuzzy membership grades

with application to multiple attribute decision making," *Granular Computing*, vol. 143, 2021.

[35] M. Lin, X. Li, and L. Chen, "Linguistic q-rung orthopair fuzzy sets and their interactional partitioned Heronian mean aggregation operators," *International Journal of Intelligent Systems*, vol. 35, no. 2, pp. 217–249, 2020.

[36] H. Zhang, T. Nan, and Y. He, "*q*-rung orthopair fuzzy N-soft aggregation operators and corresponding applications to multiple-attribute group decision making," *Soft Computing*, vol. 1, 2021.

[37] M. Akram, A. Adeel, and J. C. R. Alcantud, "Hesitant fuzzy *N*-soft sets: a new model with applications in decision-making," *Journal of Intelligent & Fuzzy Systems*, vol. 36, no. 6, pp. 6113–6127, 2019.

[38] M. Akram, A. Adeel, A. N. Al-Kenani, and J. C. R. Alcantud, "Hesitant fuzzy N-soft ELECTRE-II model: a new framework for decision-making," *Neural Computing and Applications*, vol. 33, no. 13, pp. 7505–7520, 2021.

[39] J. C. R. Alcantud, F. Feng, and R. R. Yager, "An *N*-soft set approach to rough sets," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 11, pp. 2996–3007, 2020.

[40] A. P. Darko and D. Liang, "Some q-rung orthopair fuzzy Hamacher aggregation operators and their application to multiple attribute group decision making with modified EDAS method," *Engineering Applications of Artificial Intelligence*, vol. 87, Article ID 103259, 2020.

[41] F. Fatimah and J. C. R. Alcantud, "The multi-fuzzy *N*-soft set and its applications to decision-making," *Neural Computing and Applications*, vol. 177, 2021.

[42] F. Feng, M. Akram, B. Davvaz, and V. Leoreanu-Fotea, "Attribute analysis of information systems based on elementary soft implications," *Knowledge-Based Systems*, vol. 70, pp. 281–292, 2014.

[43] L. Huang, X. Yuan, and Y. Ren, "The q-rung orthopair hesitant fuzzy uncertain linguistic aggregation operators and their application in multi-attribute decision making," *IEEE Access*, vol. 8, pp. 187084–187113, 2020.

[44] M. J. Khan, P. Kumam, and M. Shutaywi, "Knowledge measure for the q-rung orthopair fuzzy sets," *International Journal of Intelligent Systems*, vol. 36, no. 2, pp. 628–655, 2021.

[45] T. Kumar and R. K. Bajaj, "On complex intuitionistic fuzzy soft sets with distance measures and entropies," *Journal of Mathematics*, vol. 2014, Article ID 972198, 12 pages, 2014.

[46] Y. Liu, J. C. R. Alcantud, R. M. Rodríguez, K. Qin, and L. Martínez, "Intertemporal hesitant fuzzy soft sets: application to group decision making," *International Journal of Fuzzy Systems*, vol. 22, no. 2, pp. 619–635, 2020.

[47] D. Liu, X. Chen, and D. Peng, "Some cosine similarity measures and distance measures between *q*-rung orthopair fuzzy sets," *International Journal of Intelligent Systems*, vol. 34, no. 7, pp. 1572–1587, 2019.

[48] P. Liu, S. M. Chen, and P. Wang, "Multiple-attribute group decision-making based on *q*-rung orthopair fuzzy power Maclaurin symmetric mean operators," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 10, pp. 3741–3756, 2020.

[49] Y. Liu, R. M. Rodríguez, J. C. R. Alcantud, K. Qin, and L. Martínez, "Hesitant linguistic expression soft sets: application to group decision making," *Computers & Industrial Engineering*, vol. 136, pp. 575–590, 2019.

[50] P. Liu and P. Wang, "Multiple-attribute decision-making based on archimedean bonferroni operators of q-rung orthopair fuzzy numbers," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 5, pp. 834–848, 2019.

[51] A. Luqman, M. Akram, and A. Al-Kenani, "Q-rung orthopair fuzzy hypergraphs with applications," *Mathematics*, vol. 7, no. 3, p. 260, 2019.

[52] A. Pinar and F. E. Boran, "A q-rung orthopair fuzzy multi-criteria group decision making method for supplier selection based on a novel distance measure," *International Journal of Machine Learning and Cybernetics*, vol. 11, no. 8, pp. 1749–1780, 2020.

[53] P. Thirunavukarasu, R. Suresh, and V. Ashokkumar, "Theory of complex fuzzy soft set and its applications," *International Journal for Innovative Research in Science and Technology*, vol. 3, no. 10, pp. 13–18, 2017.

[54] J. Wang, X. Shang, K. Bai, and Y. Xu, "A new approach to cubic q-rung orthopair fuzzy multiple attribute group decision-making based on power Muirhead mean," *Neural Computing and Applications*, vol. 32, no. 17, pp. 14087–14112, 2020.

[55] P. Wang, J. Wang, G. Wei, and C. Wei, "Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications," *Mathematics*, vol. 7, no. 4, p. 340, 2019.