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Research Article

Cyclic Mappings and Further Results in B-Metric-Like Spaces

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Fixed point problem of many mappings has been widely studied in the research work of fixed point theory. The generalized metric space is one of the research objects of fixed point theory. B-metric-like space is one of the generalized metric spaces; in fact, the research work in B-metric-like spaces is attractive. The intention of this paper is to introduce the concept of other cyclic mappings, named as $L\beta$-type cyclic mappings in the setting of B-metric-like space, study the existence and uniqueness of fixed point problem of $L\beta$-type cyclic mapping, and obtain some new results in B-metric-like spaces. Furthermore, the main results in this paper are illustrated by a concrete example. The work of this paper extend and promote the previous results in B-metric-like spaces.

1. Introduction

The fixed point theory is well known, and it is a very important branch of mathematics. In fact, the research of fixed point theory plays an important role in many aspects, especially in nonlinear analysis. In general, nonlinear analysis is a basic theory and method to deal with nonlinear problems from the perspective of mathematics. The nonlinear analysis is based on Banach space, Hilbert space, Sobolev space, paracompact space and unit decomposition, the properties of $\triangle$-Laplacian operator, the regularization theory of elliptic equation, Bochner integrability and vector value distribution, etc. It mainly includes topological degree theory and its application, convex analysis and optimization, monotone operator theory, variation and critical point theory, and branching theory. The category of fixed point theory is extensive. It is well known that, in 1922, Banach [1] published an essay, which became a classic work. Under the frame of metric space, the existence and uniqueness of a fixed point in terms of self-contractive mapping are studied. The Banach contraction principle is a classic result. It is an important tool in metric space theory. There are many extension of metric spaces, such as B-metric space [2], metric-like space [3], extended B-metric space [4], controlled metric-type space [5], double controlled metric-type space [6], double controlled metric-like space [7], probabilistic B-metric space [8], S-metric space [9, 10], and partial B-metric space [11]. In 2013, Alghamdi et al. [12] generalized the notion of three metric spaces, that is, B-metric space, metric-like space, and partial metric space. Based on this research work, they introduced the concept of an other special metric space and named it as B-metric-like space; furthermore, they established the theorems of existence and uniqueness of fixed points in this space. In 2018, Jleli and Samet [13] introduced the concept of F-metric space, which generalized the notion of metric spaces. A natural topology τF was defined in F-metric spaces, and they studied their topological properties. At the same time, they established a new version of Banach contraction principle in the setting of F-metric spaces. In order to illustrate their research, several examples were presented.

Definition 1 (see [13]). Let X be a nonempty set. Suppose that $d\colon X \times X \longrightarrow [0, +\infty)$ is a given mapping. Assume that there exist $(f, \alpha) \in tFn \times q[0, +\infty)$ such that the following three properties are satisfied:

(d1) $(x, y) \in X \times X$, $d(x, y) = 0 \Longleftrightarrow x = y$.
(d2) $d(x, y) = d(y, x)$ for all $(x, y) \in X \times X$.

(d3) For each $(x, y) \in X \times X$ and for every $N \in \mathbb{N}$, $N \geq 2$, and for each $(u_i)_i^N \subset X$, with $(u_1, u_N) = (x, y)$, we get

$$d(x, y) > 0 \Longrightarrow f(d(x, y)) \leq f\left(\sum_{i=1}^{N-1} D(x_i, x_{i+1})\right) + \alpha. \quad (1)$$

D is said to be an F metric on X, and the pair (X, D) is called an F-metric space.

From Definition 1, assume that F denotes the set of continuous function f, and the function $f\colon (0, +\infty) \longrightarrow R$ which satisfies some conditions is concretely given as follows:

(F1) f is nondecreasing; in detail, $0 < s < t \Longrightarrow f(s) \leq f(t)$.

(F2) $\lim_{n \longrightarrow \infty} \alpha_n = 0$ if and only if $\lim_{n \longrightarrow \infty} f(\alpha_n) = -\infty$ for every sequence $\{t_n\} \subseteq R^+$.

In fact, for the certain basic properties for the metric space, such as its third properties, it was modified with the set of continuous function F, for example, (d3) in Definition 1. Generalization work of metric spaces is important for fixed point theory. On the other hand, the extension of many mappings also is an important work. A generation of Banach contraction mapping, which appeared in F-metric spaces, was proposed and some coincident fixed point results were established as follows.

Theorem 1. *Assume that (X, D) is an F-metric space, and let $F\colon X \longrightarrow X$ be a given mapping that satisfies the following conditions:*

(i) The pair (X, D) is F-complete.

(ii) There exists $k \in (0, 1)$ which implies that

$$D(F(x), F(y)) \leq \mathrm{k}D(x, y). \quad (2)$$

Then, F has a unique fixed point $x^* \in X$. Moreover, for any $x_0 \in X$, $n \in \mathbb{N}$, the sequence $\{x_n\} \subset X$ defined by

$$x_{n+1} = F(x_n) \quad (3)$$

is F-convergent to x^*.

Likely, the concept of $\alpha - \psi$-contraction, which appeared in F-metric spaces [14], was considered; in addition, an interesting and important fixed point result was obtained as follows.

Theorem 2. *Assume that (X, D) is an F-metric space, and let the symbol $F\colon X \longrightarrow X$ be a β-admissible mapping and the following conditions are satisfied:*

(i) The pair (X, D) is F-complete.

(ii) There exist two functions $\beta\colon X \times X \longrightarrow [0, +\infty)$ and $\psi \in \Psi$ such that

$$\beta(x, y) D(F(x), F(y)) \leq \psi(M(x, y)), \quad (4)$$

where

$$M(x, y) = \max\{D(x, y), D(x, F(x)), D(y, F(y))\}, \quad (5)$$

for $x, y \in X$.

(iii) There exist $x_0 \in X$ such that $\beta(x_0, F(x_0))$.

Then, the mapping F contains a unique fixed point $x^* \in X$.

In 1969, Kannan [15] published an article, and some important results were obtained; the concept of Kannan mapping is introduced in this article. In 2003, Kirk et al. published a paper, and the intention of this paper is to introduce an important notion, that is, cyclic contraction mapping [16]. In 2010, Karapinar and Erhan [17] published an article, and they proposed a new concept and named it as Kannan-type cyclic mapping; then, they established some fixed point theorems. In 2018, Weng et al. published some articles, and the concept of LW-type Lipschitz cyclic mapping was proposed in the setting of B-metric-like space, and they got some fixed point results under certain suitable conditions. In 2019, Weng et al. published a paper, and a new cyclic mapping concept was proposed, that is, the general LW-type cyclic mapping, and some interesting results were obtained in some proper conditions. In 2020, Weng et al. also published an article, and in this article, a new kind of cyclic mapping was given and some meaningful results were obtained under some suitable conditions. In 2021, a new cyclic mapping was studied by Weng et al. and they called it as φ-type cyclic mapping [18]; some fixed point results from 2018 to 2021 can also be found in [18]. On the other hand, Huang et al. [19] in 2021 introduced the concept of F-contraction and established some fixed point theorems for such contractions in B-metric spaces. Thus, it can be seen that the generalization of research work of metric space and the extended research work of many kinds of mappings in different generalized metric spaces are simultaneous. In particular, it is essential to verify the applicability of Banach contraction mapping in every generalized metric spaces.

Through the continuous efforts of mathematical researchers in mathematical research, from the perspective of history, we can see further development of fixed point theory. In fact, it is a fascinating research hot spot to discuss and study the fixed point problem of new mapping in generalized metric space. Many researchers continue to explore the three basic properties of metric space, and they focus on nonnegativity, symmetry, and triangle inequality of metric spaces, take some examples of specific functions in some (two or three) dimensional spaces to show the reasonability of newly defined generalized metric spaces, and extend some classical conclusions to these newly defined generalized metric spaces. It is even very attractive for us to propose more new content according to the structure of some of these generalized metric spaces.

The research work for cyclic mappings is continuous and a series of works. The research content of this paper also belongs to the continuation of this series. In fact, the research work involving cyclic mapping is important, and it is not over yet. There is some research work that can be done. Inspired and encouraged by earlier research work

[4, 8, 11, 20, 21], another new cyclic mapping concept is proposed in this paper, that is, $L\beta$-type cyclic mapping (Definition 7 of Section 2). Moreover, the main results of this paper will be presented.

2. Preliminaries

The concept of B-metric-like space has been mentioned in Section 1; now the details of its definition will be presented as follows.

Definition 2 (see [12]). A B-metric-like space on a nonempty set X is a function $r\colon X \times X \longrightarrow [0, +\infty)$ such that for all $x, y, z \in X$ and a constant $s \geq 1$, the following three conditions hold true:

(**r**1) If $r(x, y) = 0 \Longrightarrow x = y$.

(**r**2) $r(x, y) = r(y, x)$.

(**r**3) $r(x, y) \leq s(r(x, z) + r(z, y))$.

The pair (X, r) is called a B-metric-like space.

In fact, let $X = [0, +\infty)$, suppose that the function $r\colon X \times X \longrightarrow [0, \infty)$ is the metric function, it can be defined concretely by $r(x, y) = (x + y)^4$, and it is easy to show the fact that conditions (r1) and (r2) also are satisfied. On the other hand, for arbitrary $x, y, z \in X$,

$$\begin{aligned}
r(x, y) &= (x + y)^4 \\
&\leq (x + z + z + y)^4 \\
&= \left[(x + z)^2 + (z + y)^2 + 2(x + z)(z + y)\right]^2 \\
&= (x + z)^4 + (z + y)^4 + 6(x + z)^2 (z + y)^2 \\
&\quad + 4\left((x + z)^2 + (z + y)^2\right)(x + z)(z + y) \\
&\leq (x + z)^4 + (z + y)^4 + 3\left[(x + z)^4 + (z + y)^4\right] \\
&\quad + 2\left[(x + z)^2 + (z + y)^2\right)\right] \\
&\leq 6(x + z)^4 + 6(z + y)^4 + 4(x + z)^2 (z + y)^2 \\
&\leq 8\left((x + z)^4 + (z + y)^4\right) \\
&= 8(r(x, z) + r(z + y)),
\end{aligned} \tag{6}$$

so (r3) also holds. Then, it is obvious that (X, r) is a B-metric-like space with the parameter $s = 8$. Next, the definition of convergence, completeness, Cauchy sequence, and so on is given in a B-metric-like space.

Definition 3 (see [12]). Let (X, r) be a B-metric-like space. Suppose that $\{x_n\}$ is a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{n\longrightarrow\infty} r(x, x_n) = r(x, x)$, and we say that the sequence $\{x_n\}$ is convergent to x and denote it by $x_n \longrightarrow x$ as $n \longrightarrow \infty$.

Definition 4 (see [12]). Let (X, r) be a B-metric-like space. A sequence $\{x_n\}$ is called Cauchy if and only if $\lim_{n\longrightarrow\infty} r(x_n, x_m)$ exists and is finite. A B-metric-like space (X, r) is said to be complete if and only if every Cauchy sequence $\{x_n\}$ in X converges to $x \in X$ so that $\lim_{m,n\longrightarrow\infty} r(x_n, x_m) = r(x, x) = \lim_{n\longrightarrow\infty} r(x_n, x)$.

Definition 5 (see [12]). Let G_1, G_2 be nonempty sets of metric space; if $B(G_1) \subset G_2$ and $S(G_2) \subset G_1$, then the mapping $(B, S)\colon G_1 \times G_2 \longrightarrow G_2 \times G_1$ is called as a pair semicyclic mapping, where B is said to be a lower semicyclic mapping and S is said to be a upper semicyclic mapping. If $B = S$, then B is said to be a cyclic mapping.

Definition 6 (see [22]). If $\prec$ is partially ordered in B-metric-like spaces (X, r), then $(X, r, \prec)$ is a partially ordered B-metric-like space.

The definition of $L\beta$-type cyclic mapping is given as follows.

Definition 7. Let (X, r) be a B-metric-like space. Assume that G_1 and G_2 are nonempty closed two subsets of X. If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$, there exists γ, δ, L, β, and γ, δ, L, β are certain real nonnegative numbers, and $L \leq \beta$, $\gamma + \delta = \beta$, $\beta \in (0, 1]$ such that for all $x \in G_1$, $y \in G_2$, the following inequality is satisfied:

$$r(\mathrm{Bx}, \mathrm{Sy}) \leq L[\beta r(x, y) - M(x, y)], \tag{7}$$

where $M(x, y) = \max\{\delta r(y, \mathrm{Sy}), \gamma r(x, \mathrm{Bx})\}$. Thus, (B, S) is called a $L\beta$-type cyclic mapping.

What calls for special attention is that the $L\beta$-type cyclic mapping is a W-type cyclic mapping while $\beta = 1$, and the expression of $L\beta$-type cyclic mapping is different from the W-type cyclic mapping, and the expression of $L\beta$-type cyclic mapping is further promotion of W-type cyclic mapping; then, it is innovative.

3. Fixed Point Results

The main content of this article is to research the fixed point problem of a new mapping. In this section, some main results for $L\beta$-type cyclic mapping are given under some suitable conditions.

Theorem 3. *Assume that (X, r) is a B-metric-like space and (X, r) is complete. Suppose that (B, S) is a $L\beta$-type cyclic mapping, and G_1 and G_2 are nonempty closed two subsets of X, $G_1 \cap G_2 \neq \varnothing$. Let $p = \max\{L\beta/(1 + L\delta), L\delta, L\gamma, L\beta/(1 + L\gamma)\}$ and $p < 1/s$. Then, there exists a unique z and $z \in G_1 \cap G_2$ such that $Bz = z = Sz$. In other words, the mappings B and S have a unique common fixed point.*

Proof. Let the sequence $\{z_n\}$ be defined in the following manner:

$$z_0 \in G_1,\ z_1 = \mathrm{Bz}_0,\ z_2 = \mathrm{Sz}_1,\ z_3 = \mathrm{Bz}_2,\ z_4 = \mathrm{Sz}_3, \ldots, z_{2n+1} = \mathrm{Bz}_{2n}, z_{2n+2} = \mathrm{Sz}_{2n+1}, \ldots,\ n \geq 0. \tag{8}$$

□

Step 1. The first step is to show that $\{z_n\}$ is a Cauchy sequence. Through the Definition 7, this shows that

$$L[\beta r(z_0,z_1) - M(z_0,z_1)] \geq r(\mathrm{Bz}_0, \mathrm{Sz}_1), \tag{9}$$

that is,

$$L[\beta r(z_0,z_1) - M(z_0,z_1)] \geq r(z_1,z_2). \tag{10}$$

Because $M(x,y) = \max\{\delta r(y,\mathrm{Sy}), \gamma r(x,\mathrm{Bx})\}$, this is obvious, and it needs to be discussed case by case.

While $M(z_0,z_1) = \gamma r(z_0,\mathrm{Bz}_0) = \gamma r(z_0,z_1)$, it shows by (10) that

$$L[\beta r(z_0,z_1) - \gamma r(z_0,z_1)] \geq r(z_1,z_2), \tag{11}$$

that is,

$$L(\beta-\gamma) r(z_0,z_1) \geq r(z_1,z_2). \tag{12}$$

In fact, $\gamma+\delta=\beta$, and this implies that

$$r(z_1,z_2) \leq L\delta r(z_0,z_1). \tag{13}$$

While $M(z_0,z_1) = \delta r(z_1,\mathrm{Sz}_1) = \delta r(z_1,z_2)$, this show that

$$L[\beta r(z_0,z_1) - \delta r(z_1,z_2)] \geq r(z_1,z_2), \tag{14}$$

after appropriate transformation can be obtained, that is,

$$r(z_1,z_2) \leq \frac{L\beta}{1+L\delta} r(z_0,z_1). \tag{15}$$

Because $L[\beta r(z_2,z_1) - M(z_2,z_1)] \geq r(\mathrm{Bz}_2,\mathrm{Sz}_1) = r(z_3,z_2)$, it follows the above discussion.

While $M(z_2,z_1) = \gamma r(z_2,\mathrm{Bz}_2) = \gamma r(z_2,z_3)$, it shows that

$$L[\beta r(z_2,z_1) - \gamma r(z_2,z_3)] \geq r(z_3,z_2), \tag{16}$$

and after appropriate transformation, the following can be obtained:

$$r(z_3,z_2) = r(z_2,z_3) \leq \frac{L\beta}{1+L\gamma} r(z_2,z_1) = \frac{L\beta}{1+L\gamma} r(z_1,z_2), \tag{17}$$

that is,

$$r(z_2,z_3) \leq \frac{L\beta}{1+L\gamma} r(z_1,z_2). \tag{18}$$

While $M(z_2,z_1) = \delta r(z_1,\mathrm{Sz}_1) = \delta r(z_1,z_2)$, this implies that

$$L[\beta r(z_2,z_1) - \delta r(z_1,z_2)] \geq r(z_3,z_2), \tag{19}$$

and after appropriate transformation can be obtained, by $\gamma+\delta=\beta$, we can get

$$r(z_2,z_3) \leq L\gamma r(z_1,z_2). \tag{20}$$

Since $L\leq\beta$, $\gamma+\delta=\beta$, $\beta\in(0,1]$,

$$0<L\delta<1, 0<L\gamma<1, 0<\frac{L\beta}{1+L\gamma}<1, 0<\frac{L\beta}{1+L\delta}<1. \tag{21}$$

Because $p = \max\{L\delta, L\gamma, (L\beta/(1+L\gamma)), (L\beta/(1+L\delta))\}$, it shows that $p\in(0,1)$. By virtue of (13)–(20), this implies that

$$r(z_2,z_3) \leq \mathrm{pr}(z_1,z_2) \leq p^2 r(z_0,z_1). \tag{22}$$

Repeat the above work over and over again, so as to get

$$r(z_n,z_{n+1}) \leq p^n r(z_0,z_1),\ \forall n\in N. \tag{23}$$

Furthermore, for arbitrary $m,n\in N$, and $n\leq m$, we have

$$\begin{aligned} r(z_n,z_m) &\leq \mathrm{sr}(z_n,z_{n+1}) + \mathrm{sr}(z_{n+1},z_m) \\ &\leq \mathrm{sr}(z_n,z_{n+1}) + s^2 r(z_{n+1},z_{n+2}) + s^2 r(z_{n+2},z_m) \\ &\leq s(p)^n r(z_0,z_1) + s^2(p)^{n+1} r(z_0,z_1) + \cdots + s^{m-n} \\ &\quad\cdot (p)^{m-1} r(z_0,z_1) \\ &\leq \left[s(p)^n + s^2(p)^{n+1} + \cdots + s^{m-n}(p)^{m-1}\right] r(z_0,z_1) \\ &= \frac{1-(\mathrm{sp})^{m-n}}{1-\mathrm{sp}} \mathrm{sp}^n r(z_0,z_1) \\ &\leq \frac{1}{1-\mathrm{sp}} (p)^{n-1} r(z_0,z_1). \end{aligned} \tag{24}$$

Since $p<1/s$, from (24) and letting $n\longrightarrow\infty$, we get that

$$\lim_{n\longrightarrow\infty} r(z_n,z_m) = 0. \tag{25}$$

By (25), it shows that $\{z_n\}$ is a Cauchy sequence.

Step 2. Since $G_1\cap G_2\neq\varnothing$, (X,r) is complete, so it is easy to know that there exists a point $z\in X$ such that

$$z_n \longrightarrow z\ (n\longrightarrow\infty). \tag{26}$$

Then, it shows that

$$z_{2n}\longrightarrow z;\ z_{2n+1}\longrightarrow z\ (n\longrightarrow\infty). \tag{27}$$

Because $\{z_{2n}\}\subset G_1$, $\{z_{2n+1}\}\subset G_2$, and G_1, G_2 are closed, then

$$z\in G_1\cap G_2. \tag{28}$$

Step 3. Next, the fact that a point z is a common fixed point of the mapping S and B will be proved in this step. Since (B,S) is a $L\beta$-type cyclic mapping, the following inequality is easy to obtain:

$$r(\mathrm{Bz},\mathrm{Sz}) \leq L[\beta r(z,z) - M(z,z)]. \tag{29}$$

Now, it needs to be discussed case by case.

While $M(z,z) = \gamma r(z,\mathrm{Bz})$, it shows that

$$r(\mathrm{Bz},\mathrm{Sz}) \leq L[\beta r(z,z) - \gamma r(z,\mathrm{Bz})], \tag{30}$$

that is,

$$r(\mathrm{Bz},\mathrm{Sz}) + L\gamma r(z,\mathrm{Bz}) \leq L\beta r(z,z), \tag{31}$$

Based on the equality $\lim_{n,m\longrightarrow\infty} r(z_n, z_m) = r(z,z)$ and making use of (25) and (31), we can get

$$r(Bz, z) = 0. \quad (32)$$

Conditions of Definition 7 are considered, and it shows that

$$z = \mathrm{Bz}. \quad (33)$$

While $M(z,z) = \delta r(z, \mathrm{Sz})$, this shows that

$$r(\mathrm{Bz}, \mathrm{Sz}) \leq L[\beta r(z,z) - \delta r(z, \mathrm{Sz})], \quad (34)$$

that is,

$$r(\mathrm{Bz}, \mathrm{Sz}) + L\delta r(z, \mathrm{Sz}) \leq L\beta r(z,z). \quad (35)$$

Since $\lim_{n,m\longrightarrow\infty} r(z_n, z_m) = r(z,z)$ and making use of equality (25) and inequality (35), it implies that

$$r(\mathrm{Sz}, z) = 0. \quad (36)$$

By virtue of the condition of Definition 7, it shows that

$$z = \mathrm{Sz}. \quad (37)$$

By (33) and (37), it implies that

$$\mathrm{Bz} = z = \mathrm{Sz}. \quad (38)$$

Step 4. In this step, the fact that B and S have a unique common fixed point will be fully proved. Now, suppose that $z^*, z \in X$ are two different common fixed points for the pair B and S in (X, r). So, making use of Definition 7 and (38), we have

$$r(\mathrm{Bz}, \mathrm{Sz}^*) \leq L[\beta r(z, z^*) - M(z, z^*)]. \quad (39)$$

Discussion is presented next.

While $M(z, z^*) = \gamma r(z, \mathrm{Bz}) = \gamma r(z,z)$, it shows that

$$r(\mathrm{Bz}, \mathrm{Sz}^*) \leq L[\beta r(z, z^*) - \gamma r(z, \mathrm{Bz})], \quad (40)$$

$$r(z, z^*) \leq L[\beta r(z, z^*) - \gamma r(z,z)], \quad (41)$$

$$r(z, z^*) \leq L\beta r(z, z^*) - L\gamma r(z,z), \quad (42)$$

$$r(z, z^*) + L\gamma r(z,z) \leq L\beta r(z, z^*). \quad (43)$$

While $M(z, z^*) = \delta r(z^*, \mathrm{Bz}^*) = \delta r(z^*, z^*)$, it shows that

$$r(\mathrm{Bz}, \mathrm{Sz}^*) \leq L[\beta r(z, z^*) - \delta r(z^*, z^*)], \quad (44)$$

$$r(z, z^*) \leq L[\beta r(z, z^*) - \delta r(z^*, z^*)], \quad (45)$$

$$r(z, z^*) \leq L\beta r(z, z^*) - L\delta r(z^*, z^*), \quad (46)$$

$$r(z, z^*) + L\gamma r(z^*, z^*) \leq L\beta r(z, z^*). \quad (47)$$

From Definition 7 and formulas (25), (43), and (47), these show that

$$r(z, z^*) \leq L\beta r(z, z^*). \quad (48)$$

Since $L \leq \beta$, $\beta \in (0,1]$, L, β are nonnegative real constants, it shows that $L\beta \leq 1$. In fact, through inequality (48), it is easy to know that $L\beta \geq 1$, and this is a contradiction. Thus,

$$r(z, z^*) = 0. \quad (49)$$

That is,

$$z = z^*. \quad (50)$$

This completes the proof.

In Section 2, the definition of the partially ordered B-metric-like space is given, and then the result of Theorem 3 is extended to this partially ordered B-metric-like space, and thus the following corollary can be obtained.

Corollary 1. *Assume that (X, r) is a partially ordered B-metric-like space and (X, r) is complete. Suppose that (B, S) is a $L\beta$-type cyclic mapping, and G_1 and G_2 are nonempty closed two subsets of X, $G_1 \cap G_2 \neq \varnothing$. Let $p = \max\{L\beta/(1 + L\delta), L\delta, L\gamma, L\beta/(1 + L\gamma)\}$ and $p < 1/s$. Then, there exists a unique $z \in G_1 \cap G_2$ such that $Bz = z = Sz$. In other words, the mappings B and S have a unique common fixed point.*

Remark 1. From Definition 7 and Theorem 3, when $\beta = 1$, a $L\beta$-type cyclic mapping can be seen as a W-type cyclic mapping. In fact, the condition $\gamma + \delta = 1$ is replaced by $\gamma + \delta = \beta$. Thus, a W-type cyclic mapping is a special case of $L\beta$-type cyclic mapping. It is obvious that the $L\beta$-type cyclic mapping is more extensive than a W-type cyclic mapping. The research extension work involving cyclic mappings and B-metric-like space is important. This set of results will become the part of the basis for discovering more properties in B-metric-like spaces. In order to show the effectiveness of the $L\beta$-type cyclic mapping, a concrete example is given as follows.

4. Example

In this section, a concrete example is given to illustrate the effectiveness of the $L\beta$-type cyclic mapping and show the rationality of the obtained theorems.

4.1. Part One. In this part, a concrete B-metric-like space is presented; at the same time, the mappings B and S are also defined in this part. Based on these preparations, the validity of the definition of the $L\beta$-type cyclic mapping and the correctness of Theorem 3 are verified.

Consider the set $X = \{0, 1, 2\}$ and let the metric function $r: X \times X \longrightarrow [0, +\infty)$ be defined by

$$\begin{aligned} r(0,0) &= 0, r(1,1) = 0,\ r(2,2) \\ &= 1,\ r(0,1) = 4, r(1,0) = 4, \end{aligned} \quad (51)$$

$$r(0,2) = 2, r(2,0) = 2,\ r(1,2) = 1, r(2,1) = 1. \quad (52)$$

It is clear that (X, r) is a B-metric-like complete space with the constant $s = 1.4$ (see [23]).

Recall $L\beta$-type cyclic mapping concept. Assume that G_1 and G_2 are nonempty closed two subsets of X. If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$, there exists γ, δ, L, β, γ, δ, L, β are some real nonnegative numbers, and $L \leq \beta$, $\gamma + \delta = \beta$, $\beta \in (0, 1]$ such that for all $x \in G_1$, $y \in G_2$, the following inequality is satisfied:

$$r(\mathrm{Bx}, \mathrm{Sy}) \leq L[\beta r(x, y) - M(x, y)], \tag{53}$$

and we should need to pay attention to the symbol $M(x, y)$, where $M(x, y) = \max\{\gamma r(x, \mathrm{Bx}), \delta r(y, \mathrm{Sy})\}$. Thus, (B, S) is called as the $L\beta$-type cyclic mapping. Then, the mappings B and S are defined by the following way. Let

$$G_1 = \{0, 1\},\ G_2 = \{1, 2\}, G_1, G_2 \in X, \tag{54}$$

$$B(0) = 2, B(1) = 1, S(1) = 1, S(2) = 1. \tag{55}$$

Next, we explore the existence and uniqueness of fixed points for B and S under the corresponding conditions of Definition 7 and Theorem 3 in a concrete, complete B-metric-like space. In fact, when the mappings B and S are concretely defined, it can get four points, that is, $r(x, y)$ should be $r(0, 1), r(0, 2), r(1, 1), r(1, 2)$.

Thus, some concrete calculation will be shown as follows.

Situation 1. If we choose $r(x, y) = r(0, 1) = 4$, then

$$r(B(0), S(1)) \leq L[\beta r(0, 1) - M(0, 1)], \tag{56}$$

that is,

$$r(2, 1) \leq L[\beta r(0, 1) - M(0, 1)]. \tag{57}$$

In fact,

$$1 \leq L[4\beta - M(0, 1)], \tag{58}$$

where

$$\begin{aligned} M(0, 1) &= \max\{\gamma r(0, B(0)), \delta r(1, S(1))\} \\ &= \max\{\gamma r(0, 2), \delta r(1, 1)\} \\ &= \max\{2\gamma, 0\}, \end{aligned} \tag{59}$$

and thus it should be discussed case by case.

While $M(0, 1) = 2\gamma$, it can get the inequality

$$1 \leq L(4\beta - 2\gamma), \tag{60}$$

that is,

$$1 \leq 2L\beta + L\delta. \tag{61}$$

While $M(0, 1) = 0$, it can get the inequality

$$1 \leq L(4\beta - 0), \tag{62}$$

that is,

$$1 \leq 4L\beta. \tag{63}$$

Situation 2. If we choose $r(x, y) = r(0, 2) = 2$, then

$$r(B(0), S(2)) \leq L[\beta r(0, 2) - M(0, 2)], \tag{64}$$

that is,

$$r(2, 1) \leq L[\beta r(0, 2) - M(0, 2)]. \tag{65}$$

In fact,

$$1 \leq L[2\beta - M(0, 2)], \tag{66}$$

where

$$\begin{aligned} M(0, 2) &= \max\{\gamma r(0, B(0)), \delta r(2, S(2))\} \\ &= \max\{\gamma r(0, 2), \delta r(2, 1)\} \\ &= \max\{2\gamma, \delta\}, \end{aligned} \tag{67}$$

and thus it should be discussed case by case.

While $M(0, 2) = 2\gamma$, it can get the inequality

$$1 \leq L(2\beta - 2\gamma), \tag{68}$$

that is,

$$1 \leq 2L\delta. \tag{69}$$

While $M(0, 2) = \delta$, it can get the inequality

$$1 \leq L(2\beta - \delta), \tag{70}$$

that is,

$$1 \leq L\beta + L\gamma. \tag{71}$$

Situation 3. If we choose $r(x, y) = r(1, 1) = 0$, then

$$r(B(1), S(1)) \leq L[\beta r(1, 1) - M(1, 1)], \tag{72}$$

that is,

$$0 \leq L[0 - M(1, 1)], \tag{73}$$

where

$$\begin{aligned} M(1, 1) &= \max\{\gamma r(1, B(1)), \delta r(1, S(1))\} \\ &= \max\{\gamma r(1, 1), \delta r(1, 1)\} \\ &= \max\{0, 0\} \\ &= 0, \end{aligned} \tag{74}$$

and thus it should be discussed case by case. In fact, arbitrary value of parameters γ, δ, β, L is true in this situation.

Situation 4. If we choose $r(x, y) = r(1, 2) = 1$, then

$$r(B(1), S(2)) \leq L[\beta r(1, 2) - M(1, 2)], \tag{75}$$

that is,

$$r(1, 1) \leq L[\beta r(1, 2) - M(1, 2)]. \tag{76}$$

In fact,

$$0 \leq L[\beta - M(1, 2)], \tag{77}$$

where

$$M(1,2) = \max\{\gamma r(1, B(1)), \delta r(2, S(2))\} = \max\{\gamma r(1,1), \delta r(1,1)\} = \max\{0, \delta\}, \quad (78)$$

and thus it should be discussed case by case.

While $M(1,2) = 0$, it can get the inequality

$$0 \leq L\beta. \quad (79)$$

While $M(1,2) = \delta$, it can get the inequality

$$0 \leq L(\beta - \delta), \quad (80)$$

that is,

$$0 \leq L\gamma. \quad (81)$$

4.2. Part Two. In this part, the validity of the conditions of the theorem is verified. Furthermore, three sets of data are used to verify the existence and the rationality of the theorem conditions.

Let $\gamma = \delta = 1/2, \beta = 1, L = 1$, and take this set of data into the above inequalities of situations 1-4 for verification, and all the inequalities will be true. Because $p = \max\{L\beta/(1 + L\delta), L\delta, L\gamma, L\beta/(1 + L\gamma)\}$, then $p = \max\{1/1.5, 0.5, 0.5, 1/1.5\} = 0.6667$. So, $sp = 1.4 \times 0.6667 = 0.9334 < 1$. It is easy to know that the condition $s < 1/p$ is satisfied.

Let $\gamma = 0.47, \delta = 0.52, \beta = 0.99, L = 0.98$, and take this set of data into the above inequalities of situations 1-4 for verification, and all the inequalities will be true. Because $p = \max\{L\beta/(1 + L\delta), L\delta, L\gamma, L\beta/(1 + L\gamma)\}$, then $p = \max\{0.98 \times 0.99/(1 + 0.98 \times 0.52), 0.98 \times 0.52, 0.98 \times 0.47, 0.98 \times 0.99/(1 + 0.98 \times 0.47)\} = 0.6642$. So, $sp = 1.4 \times 0.6642 = 0.9299 < 1$. It is obvious that the condition $s < 1/p$ is satisfied.

Let $\gamma = 0.45, \delta = 0.53, \beta = 0.98, L = 0.96$, and take this set of data into the above inequalities of situations I–4 for verification, and all the inequalities will be true. Because $p = \max\{L\beta/(1 + L\delta), L\delta, L\gamma, L\beta/(1 + L\gamma)\}$, then $p = \max\{0.96 \times 0.98/(1 + 0.96 \times 0.53), 0.96 \times 0.53, 0.96 \times 0.45, 0.96 \times 0.98/(1 + 0.96 \times 0.45)\} = 0.65698$. So, $sp = 1.4 \times 0.65698 = 0.9198 < 1$. It is easy to verify the fact that the condition $s < 1/p$ is satisfied.

Overall, after concrete data validation above, because G_1 and G_2 are two nonempty closed subsets in (X, r), B and S have a unique common fixed point. Therefore, it is verified by an example to show that the theorem holds.

5. Conclusions

A special note is needed. In fact, B-metric-like space is one of the generalized metric spaces, and certain related research work in different generalized metric spaces for promotion is very important. Some research work about cyclic mappings under the B-metric-like space frame is presented in this paper. The $L\beta$-type cyclic mapping proposed in this paper is discussed and researched, and the corresponding fixed point theorems are obtained. The correctness of the theoretical results is verified by a concrete real example. The correctness of the main results in this paper has been verified and is correct. Actually, $L\beta$-type cyclic mappings are not difficult to find to become a W-type cyclic mapping while $\beta = 1$. Thus, the $L\beta$-type cyclic mapping is more extensive than a W-type cyclic mapping. Before this, phi-type cyclic mapping, W-type cyclic mapping [21], and some other cyclic mappings have been proposed and studied by us under the frame of B-metric-like spaces; when these cyclic mappings were further studied and extended, some fixed point theorems were obtained. In fact, these research works in B-metric spaces are beneficial to the further enrichment of B-metric-like space theory Of course, the current research work is not deep; in fact, some of the conditions presented in these research papers may be able to be further adjusted and optimized; as for the follow-up research, we will do our best. Also, it is interesting to apply our results to network systems [24, 25] and stochastic differential equation [26].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to this study.

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References

[1] S. Banach, "Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales," *Fundamenta Mathematicae*, vol. 3, no. 1, pp. 133–181, 1922.
[2] S. Czerwik, "Contraction mappings in B-metric spaces," *Acta Mathematica et Informatica Universitatis Ostraviensis*, vol. 1, no. 1, pp. 5–11, 1993.
[3] A. Amini Harandi, "Metric-like spaces, partial metric spaces and fixed points," *Journal of Fixed Point Theory and Applli-cation*, vol. 2012, Article ID 204, 2012.
[4] T. Kamran, M. Samreen, and Q. UlAin, "A generalization of B-metric space and some fixed point theorems," *Mathematics*, vol. 5, Article ID 19, 2017.
[5] N. Mlaiki, H. Aydi, N. Souayah, and T. Abdeljawad, "Controlled metric type spaces and the related contraction principle," *Mathematics*, vol. 6, no. 10, Article ID 194, 2018.
[6] T. Abdeljawad, N. Mlaiki, H. Aydi, and N. Souayah, "Double controlled metric type spaces and some fixed point results," *Mathematics*, vol. 6, Article ID 320, 2018, https://doi.org/10.3390/math6120320.
[7] N. Mlaiki, "Double controlled metric-like spaces," *Journal of Inequalities and Applications*, vol. 2020, Article ID 189, 2020, https://doi.org/10.1186/s13660-020-02456-z.

[8] A. Mbarki and R. Oubrahim, “Probabilistic B-metric spaces and nonlinear contractions,” *Journal of Fixed Point Theory and Application*, vol. 29, p. 15, 2017.

[9] S. Sedghi, N. Shobeand, and T. Doenovic, “Fixed point results in S-metric spaces,” *Nonlinear Functional Analysis and Applications*, vol. 20, no. 1, pp. 55–67, 2015.

[10] A. Singh and N. Hooda, “Coupled fixed point theorems in S-metric spaces,” *Inter. J. Math. Stat. Invent*, vol. 2, pp. 33–39, 2014.

[11] S. Shukla, “Partial B-metric spaces and fixed point theorems,” *Mediterranean Journal of Mathematics*, vol. 11, no. 2, pp. 703–711, 2014.

[12] M. A. Alghamdi, N. Hussain, and P. Salimi, “Fixed point and coupled fixed point theorems on B-metric-like spaces,” *Journal of Inequalities and Applications*, vol. 2013, no. 1, Article ID 402, 2013.

[13] M. Jleli and B. Samet, “On a new generalization of Metric Spaces,” *Journal of fixed point theory and Application*, vol. 20, p. 20, 2018.

[14] A. Hussain and T. kanwal, “Existence and uniqueness for a neutral differential problem with unbounded delay via fixed point results,” *Transactions of A. Razmadze Mathematical Institute*, vol. 172, no. 3, pp. 481–490, 2018.

[15] R. Kannan, “Some results on fixed points-II,” *The American Mathematical Monthly*, vol. 76, no. 4, pp. 405–408, 1969.

[16] W. A. Kirk, P. S. Srinivasan, and P. Veeramani, “Fixed points for mappings satisfying cyclical contractive conditions,” *Fixed Point Theory*, vol. 4, pp. 79–89, 2003.

[17] E. Karapinar and I. M. Erhan, “Best proximity on different type contractions,” *Appl.Math.Inf.Sci*, vol. 25, no. 5, pp. 558–569, 2010.

[18] S. Q. Weng and Q. X. Zhu, “Some fixed point theorems on generalized cyclic mappings in B-metric-like spaces,” *Complexity*, vol. 2021, Article ID 9042402, 7 pages, 2021.

[19] H. Huang, Z. D. Mitrović, K. Zoto, S. Radenović, and R. Stojan, “On convex F-contraction in B-metric spaces,” *Axioms*, vol. 10, no. 2, p. 71, 2021, https://doi.org/10.3390/axioms10020071.

[20] T. Dosenovic, M. De La Sen, L. Paunovic, R. Dusan, and S. Radenovic, “Some new observations on generalized contractive mappings and related results in B-metric-like spaces,” *Journal of Mathematics*, vol. 2021, Article ID 6634822, 9 pages, 2021.

[21] S. Q. Weng and L. Liang, “A fxied-point theorem of cyclic mappings in complete B-metric-like spaces,” *Journal of Yibin University*, vol. 21, no. 6, pp. 77–80, 2021.

[22] A. HAydi, S. Felhi, and Sahmim, “On common fixed points for (α, ψ)-contractions and generalized cyclic contractions in B-metric-like spaces and consequences,” *The Journal of Nonlinear Science and Applications*, vol. 9, no. 5, pp. 2492–2510, 2016.

[23] H. Nashine and Z. Kadelburg, “Existence of solutions of cantilever beam problem via (α-β-FG)-contractions in B-metric-like spaces,” *Filomat*, vol. 31, no. 11, pp. 3057–3074, 2017.

[24] H. Su, Y. Sun, and Z. Zeng, “Semiglobal observer-based nonnegative edge consensus of networked systems with actuator saturation,” *IEEE Transactions on Cybernetics*, vol. 50, no. 6, pp. 2827–2836, 2020.

[25] F. Kong and Q. Zhu, “New fixed-time synchronization control of discontinuous inertial neural networks via indefinite Lyapunov-Krasovskii functional method,” *International Journal of Robust and Nonlinear Control*, vol. 31, no. 2, pp. 471–495, 2021.

[26] X. Zhang, S. P. He, V. Stojanovic, X. Luan, and F. Liu, “Finite-time asynchronous dissipative filtering of conic-type nonlinear Markov jump systems,” *Science China Information Sciences*, vol. 64, no. 5, Article ID 152206, 2021.