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Research Article

Extended Model on Structural Stability and Robustness to Bounded Rationality

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In this article, we focus on an extended model $\overline{M}$ of bounded rationality. Based on a rationality function with lower semi-continuity, we analyze the relationship between structural stability and robustness of $\overline{\Omega}$. To further demonstrate the applicability of our theory, we introduce a model $\overline{\Omega}_0$ containing an abstract rationality function and generalize abstract fuzzy economies. We demonstrate the structural stability of the extended model $\overline{\Omega}_0$ at $(\overline{\xi}, \varepsilon)$. That is to say, $\overline{\Omega}_0$ is robust to the $(\overline{\xi}, \varepsilon)$-equilibria.

1. Introduction

Arrow and Debreu [1] studied the existence of equilibrium for a social competitive economy assuming that all participants are perfectly rational in the game in 1952. However, in reality, perfect rationality does not often occur. Hence, the assumption of perfect rationality restricted the application of the model. On the other hand, it is well known that the stability analysis is one of the significant topics in economic models. Anderlini and Canning [2] constructed an abstract framework Ω and derived necessary and sufficient conditions for the model being robust to ϵ-equilibria. Yu and Yu [3] extended the results in [2] under weaker conditions and proved the stability and robustness of the model. Wang et al. [4] generalized the model Ω [2] with abstract fuzzy economies. These authors studied the stability and showed its robustness. Miyazaki and Azuma [5] investigated the structural stability and robustness of the model introduced in [2]. They proved that if a system is (ξ, ε)-stable, then it is (ξ, ε)-robust. Loi and Matta [6] extended the model Ω introduced in [2] to a more general model $\overline{\Omega}$. They investigated a pure exchange economy using the abstract construction. Yu et al. [3] studied (ξ, ε)-stability and (ξ, ε)-robustness of model Ω introduced in [2]. In particular, they considered their relationship. Then, these results on stability and robustness in control and economy were further studied during the last decades [7–11].

Zadeh [12] initiated fuzzy set theory to describe scenarios with imprecise parameters. Kim and Lee [13] studied a fuzzy game and obtained corresponding equilibrium existence theorem. Huang [14, 15] studied a generalized abstract fuzzy model of economics and considered the existence of its equilibrium. Patriche [16, 17] presented a Bayesian abstract fuzzy economic model with a measure space of agents and demonstrated the existence of equilibrium of the constructed model. More recently, Cui et al. [18] studied the loss aversion level of a bimatrix game with payoff function described by fuzzy variables. These authors obtained synchronization conditions for the fuzzy stochastic complex networks.

Motivated by these existing studies, in this work, we study the extended model $\overline{\Omega}_0$ with generalized abstract fuzzy economies and an abstract rationality function. We are particularly interested in whether small deviations in the additional rationality of the extended model $\overline{\Omega}_0$ will cause only minor changes in bounded rational equilibria. When we extend the model Ω to a complex model $\overline{\Omega}$, it is difficult to obtain its structural stability. It is essential to derive the relationship between the extended model $\overline{\Omega}$ and its structural stability. We mainly focus on the structure of Ω and how to extend Ω to $\overline{\Omega}$ in a natural way.

The rest of the paper is organized as follows. In section 2, we recall the notion of economic model Ω, structural stability, robustness to bounded rationality, and their connection. Moreover, we extend the model Ω to a complex model $\overline{\Omega}$. The relationship between $(\overline{\xi}, \varepsilon)$-stability and $(\overline{\xi}, \varepsilon)$-robustness of $\overline{\Omega}$ is discussed. In section 3, we prove a new theorem about the existence of equilibrium in $L\Gamma$-spaces for the generalized abstract fuzzy economic model. We further prove the structural stability of the extended model $\overline{\Omega}_0$ with a category of generalized abstract fuzzy economies at $(\overline{\xi}, \varepsilon)$. Finally, we present the conclusions of this article in section 4.

2. The Extended Model $\overline{\Omega}$

In this section, we introduce the following notation. Suppose that X and Y are topological spaces. For a subset D of topological space X, we use 2^D and $\langle D\rangle$ to denote, respectively, the set of all subsets of D and the family of all nonempty finite subset of D. If $G(x)$ is compact for all $x \in X$, we then say that $G\colon X \longrightarrow 2^Y$ is a compact valued correspondence. Assume X and Y are metric spaces. If for any $x_n \longrightarrow x$ and $y_n \in G(x_n)$, $y_n \longrightarrow y$, we have $y \in G(x)$, then G is upper semicontinuous at $x \in X$. If for any $x_n \longrightarrow x$ and $y \in G(x)$, we have $y_n \in G(x_n)$ such that $y_n \longrightarrow y$, then G is lower semicontinuous at x. If for any $x_n \longrightarrow x, h(G(x_n), G(x)) \longrightarrow 0$, where h is the Hausdorff distance defined on Y, then G is continuous at x.

Anderlini and Canning [2] studied an economic model of bounded rationality. The model Ω with a parameter space given by quadruple (Ψ, X, F, R): (Ψ, ϱ) is a parameter space and (X, d) is an action space, in which ϱ and d are metrics. Here, $F\colon \Psi \times X \longrightarrow 2^X$ represents the feasibility correspondence inducing a further correspondence $f(\xi) = \{x \in X\colon x \in F(\xi, x)\}$, for all $\xi \in \Psi$. The graph of f is denoted by $\mathrm{Graph}(f) = \{(\xi, x) \in \Psi \times X\colon x \in f(\xi)\}$ and $R\colon \mathrm{Graph}(f) \longrightarrow R^+$ is a rational function. When $R(\xi, x) = 0$, we say that the full rationality is realized. Given a model Ω, for all $\xi \in \Psi$, we define the ε-equilibria set at ξ as follows:

$$E(\xi, \varepsilon) = \{x \in f(\xi)\colon R(\xi, x) \le \varepsilon\}, \quad \forall \varepsilon \ge 0. \tag{1}$$

We use $E(\xi)$ to denote all equilibria set at ξ as

$$E(\xi) = E(\xi, 0) = \{x \in f(\xi)\colon R(\xi, x) = 0\}. \tag{2}$$

Here, we extend the model Ω proposed in [2] to a more complex model $\overline{\Omega}$. Using a rationality function with lower semicontinuity, we prove that $\overline{\Omega}$ is $(\overline{\xi}, \varepsilon)$-stable, which implies that $\overline{\Omega}$ is robust to $(\overline{\xi}, \varepsilon)$-equilibria.

Loi and Matta [6] studied the extended model $\overline{\Omega}$ as follows.

Definition 1 (see [6]). For a model $\Omega = \{\Psi, X, F, R\}$, its extended model $\overline{\Omega}$ is defined as a quadruple $\overline{\Omega} = \{\overline{\Psi}, \overline{X}, \overline{F}, \overline{R}\}$ satisfying the following:

(1) $(\overline{X}, \overline{d}) = (X, d)$;

(2) $\overline{\Psi} = (K(\Psi), h_\varrho)$, where $K(\Psi)$ is to be the set of all compact subsets of Ψ with the Hausdorff distance h_ϱ related to the metric ϱ of Ψ;

(3) $\overline{F}\colon \overline{\Psi} \times X \longrightarrow 2^X$ is denoted by $\overline{F} = \cup_{\mu \in \overline{\lambda}} F(\mu, x)$. We thus have $\overline{f}\colon \overline{\Psi} \longrightarrow 2^X, \overline{f}(\overline{\lambda}) = \{x \in X | x \in \overline{F}(\overline{\xi}, x)\}$;

(4) $\overline{R}\colon \mathrm{Graph}(\overline{f}) \longrightarrow R^+$ is an extended R. That is to say, for all $\xi \in \Psi, \overline{R}(\{\xi\}, x) = R(\xi, x)$ that satisfies

$$\overline{R}(\overline{\xi}, x) = \{0\} \Longleftrightarrow R(\xi, x) = 0, \quad \forall \xi \in \overline{\xi}. \tag{3}$$

Definition 2. Given an extended model $\overline{\Omega}$, for any $\overline{\xi} \in \overline{\Psi}$, the ϵ-equilibria set at $\overline{\xi}$ is given by

$$\overline{E}(\overline{\xi}, \varepsilon) = \{x \in \overline{f}(\overline{\xi}) | \overline{R}(\overline{\xi}, x) \subset [0, \varepsilon]\}, \quad \forall \varepsilon \ge 0. \tag{4}$$

We use $\overline{E}(\overline{\xi})$ to denote all equilibria set at $\overline{\xi}$ as

$$\overline{E}(\overline{\xi}) = \overline{E}(\overline{\xi}, 0) = \{x \in \overline{f}(\overline{\xi}) | \overline{R}(\overline{\xi}, x) = \{0\}\}. \tag{5}$$

Definition 3. The extended model $\overline{\Omega}$ is structurally stable at $(\overline{\xi}, \varepsilon)$ if $\overline{E}\colon \overline{\Psi} \times R^+ \longrightarrow 2^X$ is continuous at $(\overline{\xi}, \varepsilon)$.

Definition 4. The extended model $\overline{\Omega}$ is robust to $(\overline{\xi}, \varepsilon)$-equilibria if for all $\delta > 0$, we can find an $\overline{\varepsilon} > 0$ satisfying, for all $\varepsilon' > 0$ with $|\varepsilon - \varepsilon'| < \overline{\varepsilon}$ and for all $\xi' \in \overline{\Psi}$ with $h_\varrho(\xi, \xi') < \overline{\varepsilon}$ and $h_d(\overline{E}(\overline{\xi'}, \varepsilon), \overline{E}(\overline{\xi'}, \varepsilon')) < \delta$, where h_d is the Hausdorff distance defined on X.

Theorem 1. *Given a model $\Omega = (\Psi, X, F, R)$ and $\overline{\Omega} = \{\overline{\Psi}, \overline{X}, \overline{F}, \overline{R}\}$ is its corresponding extended model. If the model Ω satisfies the following assumptions:*

(1) (Ψ, ϱ) is a complete metric space and X is a compact metric space,

(2) $f\colon \Psi \longrightarrow 2^X$ is nonempty compact valued and upper semicontinuous,

(3) R: Gragh$(f) \longrightarrow R^+$ is lower semicontinuous,

(4) for all $\xi \in \Psi$, $E(\xi) = \{x \in f(\xi)\colon R(\xi, x)\} \neq \varnothing$,

then $\overline{\Omega}$ being $(\overline{\xi}, \varepsilon)$-stable guarantees that $\overline{\Omega}$ is robust to $(\overline{\xi}, \varepsilon)$-equilibria.

Proof. Obviously, if (Ψ, ϱ) is a complete metric space, then it follows from Theorem 3.3 in Henrikson [19] that $(\overline{\Psi}, h_\varrho)$ is a complete metric space.

Suppose model $\overline{\Omega}$ is not robust to $(\overline{\xi}, \varepsilon)$-equilibria. Consequently, there exist $\overline{\xi} \longrightarrow \xi, \varepsilon_n \longrightarrow \varepsilon$, and $h_d(\overline{E}(\overline{\xi}_n, \varepsilon_n), \overline{E}(\overline{\xi}_n, \varepsilon)) \ge \delta_0$, where $\delta_0 > 0$, $\{\varepsilon_n\} \subset R^+$, and $\{\overline{\xi}\}$ is a sequence of compact subsets Ψ. Take the subsequence $\{\varepsilon_{n_k}\}$ of $\{\varepsilon_n\}$ such that $\varepsilon_{n_k} \ge \varepsilon_{n_{k+1}}$, $\varepsilon_{n_k} \ge \varepsilon$ or $\varepsilon_{n_k} \le \varepsilon_{n_{k+1}}$, $\varepsilon_{n_k} \le \varepsilon$.

First, consider the subsequence $\{\varepsilon_{n_k}\}$ satisfying $\varepsilon_{n_k} \ge \varepsilon_{n_{k+1}}$ and $\varepsilon_{n_k} \ge \varepsilon$. Then, we have $\overline{E}(\overline{\xi}_{n_k}, \varepsilon) \subset \overline{E}(\overline{\xi}_{n_k}, \varepsilon_{n_k})$. Thus, we can choose $x_{n_k} \in \overline{E}(\overline{\xi}_{n_k}, \varepsilon_{n_k})$ such that

$$\min_{y\in\overline{E}\left(\overline{\xi}_{n_k},\varepsilon\right)} d\left(x_{n_k}, y\right) > \frac{\delta_0}{2}. \tag{6}$$

Since $x_{n_k} \in \overline{E}(\overline{\xi}_{n_k}, \varepsilon_{n_k})$, then $x_{n_k} \in \overline{f}(\overline{\xi}_{n_k})$, and it follows from Definition 1 that $\overline{R}(\overline{\xi}_{n_k}, \varepsilon_{n_k}) \subset [0, \varepsilon_{n_k}]$. Moreover, since $f: \Psi \longrightarrow 2^X$ is nonempty compact valued and upper semicontinuous, it follows from Lemma 3.7 in [6] that the correspondence $\overline{f}: \overline{\Psi} \longrightarrow 2^X$ is nonempty compact valued and upper semicontinuous. Since $\xi_{n_k} \longrightarrow \xi$, without loss of generality, by Lemma 2.1 in [3], we can assume $x_{n_k} \longrightarrow x \in \overline{f}(\overline{\xi})$.

Because of the lower semicontinuity of $\overline{R}$, we obtain that $\overline{R}(\overline{\xi}_{n_k}, \varepsilon_{n_k}) \subset [0, \varepsilon_{n_k}], \varepsilon_{n_k} \longrightarrow \varepsilon$ which implies that $\overline{R}(\overline{\xi}, \varepsilon) \subset [0, \varepsilon]$. Therefore, $x \in \overline{E}(\overline{\xi}, \varepsilon)$. Since the extended model $\overline{\Omega}$ is $(\overline{\xi}, \varepsilon)$-stable, we have that $h_d(\overline{E}(\overline{\xi}_n, \varepsilon_n), \overline{E}(\overline{\xi}, \varepsilon)) \longrightarrow 0$. It follows from Lemma 2.5 in [20] and (6) that

$$\min_{y\in\overline{E}\left(\overline{\xi}_{n_k},\varepsilon\right)} d\left(x_{n_k}, y\right) \geq \frac{\delta_0}{2}. \tag{7}$$

However, this is in contradiction with $x \in \overline{E}(\overline{\xi}, \varepsilon)$.

Next, consider the subsequence $\left\{\varepsilon_{n_k}\right\}$ satisfying $\varepsilon_{n_k} \leq \varepsilon_{n_{k+1}}$ and $\varepsilon_{n_k} \leq \varepsilon$. We then have $\overline{E}(\overline{\xi}_{n_k}, \varepsilon_{n_k}) \subset \overline{E}(\overline{\xi}_{n_k}, \varepsilon)$. Therefore, we can choose $x_{n_k} \in \overline{E}(\overline{\xi}_{n_k}, \varepsilon)$ such that

$$\min_{y\in\overline{E}\left(\overline{\xi}_{n_k},\varepsilon_{n_k}\right)} d\left(x_{n_k}, y\right) > \frac{\delta_0}{2}. \tag{8}$$

Since $\overline{f}: \overline{\Psi} \longrightarrow 2^X$ is nonempty compact valued and upper semicontinuous and $\xi_{n_k} \longrightarrow \xi$, without loss of generality, it follows from Lemma 2.1 in [3] that we can assume $x_{n_k} \longrightarrow x \in \overline{f}(\overline{\xi})$.

Because of the lower semicontinuity of $\overline{R}$, we have that $\overline{R}(\overline{\xi}_{n_k}, \varepsilon_{n_k}) \subset [0, \varepsilon_{n_k}], \varepsilon_{n_k} \longrightarrow \varepsilon$. That is to say, $\overline{R}(\overline{\xi}, \varepsilon) \subset [0, \varepsilon]$. Therefore, $x \in \overline{E}(\overline{\xi}, \varepsilon)$. Since the extended model $\overline{\Omega}$ is $(\overline{\xi}, \varepsilon)$-stable, we have that $h_d(\overline{E}(\overline{\xi}_n, \varepsilon_n), \overline{E}(\overline{\xi}, \varepsilon)) \longrightarrow 0$. It follows from Lemma 2.5 in [20] and (8) that

$$\min_{y\in\overline{E}\left(\overline{\xi}_{n_k},\varepsilon\right)} d\left(x_{n_k}, y\right) \geq \frac{\delta_0}{2}. \tag{9}$$

However, this is in contradiction with $x \in \overline{E}(\overline{\xi}, \varepsilon)$, which completes the proof. □

Remark 1. Theorem 1 improves Theorem 4.6 in [5] and Theorem 3.4 in [3], where the extended model $\overline{\Omega}$ was extended. We also generalize Theorem 3.10 in [6] where the structurally stability at ξ and robustness to ϵ-equilibria are extended to the structurally stability at $(\overline{\xi}, \varepsilon)$ and the robustness to $(\overline{\xi}, \varepsilon)$-equilibria, respectively.

3. Application

Here, we propose a category of new generalized abstract fuzzy economies in an $L\Gamma$-space.

We then study the equilibrium existence theorem for the models. Furthermore, we establish an extended model $\overline{\Omega}_0$ with an abstract rationality function and a category of generalized abstract fuzzy economies. We demonstrate that $\overline{\Omega}_0$ is structurally stable at $(\overline{\xi}, \varepsilon)$. In other words, $\overline{\Omega}_0$ is robust to the $(\overline{\xi}, \varepsilon)$-equilibria.

3.1. The Equilibria of Generalized Abstract Fuzzy Economic Model in an $L\Gamma$-Space. The notions of abstract convex space and $L\Gamma$-space were introduced by Park [21, 22].

Definition 5 (see [21]). An abstract convex space $(E, D; \Gamma)$ consists of a topological space E, a nonempty set D, and a map $\Gamma: \langle D\rangle \longrightarrow E$ with nonempty values.

For any $A \in \langle D\rangle$, we denote $\Gamma_A = \Gamma(A)$. Let $co_\Gamma D' = \bigcup\{\Gamma_A | A \in \langle D\rangle\} \subset E$ be the Γ-convex hull of D', where $D' \subset D$. If for any $N \in \langle D\rangle$, we have $\Gamma_N \subset X$, i.e., $co_\Gamma D' \subset X$, then X is said to be a Γ-convex subset of $(E, D; \Gamma)$ related to D'. If $D \subset E$, the space is defined as $(E^{D;\Gamma})$. Here, if $co_\Gamma(X \cap D) \subset X$, then X is called Γ-convex. That is to say, X is Γ-convex related to $D' = X \cap D$. When $E = D$, we let $(E; \Gamma) = (E, E; \Gamma)$.

Definition 6 (see [22]). If an abstract convex space $(E, D; \Gamma)$ has a basis $\mathscr{B}$ of a uniformity of E, then it is said to be an abstract convex uniform space $(E, D; \Gamma; \mathscr{B})$. If D is dense in E and for any $U \in \mathscr{B}$ and for any Γ-convex subset $A \subset E$, the set $\{x \in E: A \cap U[x] \neq \varnothing\}$ is Γ-convex, then an abstract convex uniform space $(E^{D;\Gamma;\mathscr{B}})$ is said to be an $L\Gamma$-space.

Based on Proposition 5.1 and Theorem 8.5 in Park [22], we can obtain Lemma 1.

Lemma 1. *Let $(X, D; \Gamma)$ be a $L\Gamma$-space. Suppose that $G: X \longrightarrow 2^X$ is nonempty closed Γ_i-convex valued and compact upper semicontinuous. Then, we can find an $\widehat{x} \in X$ satisfying $\widehat{x} \in G(\widehat{x})$.*

Lemma 2. *Let $(X_i, D_i; \Gamma_i)_{i\in I}$ be a family of $L\Gamma$-spaces. For each $i \in I$, suppose that $G_i: X = \prod X_i \longrightarrow 2^{X_i}$ is nonempty closed Γ_i-convex valued and compact upper semicontinuous. Thus, for each $i \in I$, we can find an $\widehat{x} = (\widehat{x}_i)_{i\in I} \in X$ satisfying $\widehat{x}_i \in G_i(\widehat{x})$.*

Proof. Let $X = \prod_{i\in I} X_i$ and $D = \prod_{i\in I} D_i$. For each $i \in I$, let $\pi_i: X \longrightarrow X_i$ be the projection of X onto X_i. Then, for any $x \in X$, $G: X \longrightarrow 2^X$ is defined by $G(x) = \prod_{i\in I} G_i(x)$. We denote $\Gamma: \langle D\rangle \longrightarrow 2^X\backslash\{\varnothing\}$ as $\Gamma(A) = \prod_{i\in I}\Gamma_i(\pi_i(A)), \forall A \in \langle D\rangle$. It follows from Lemma 2 in [23] that (X, D, Γ) is a $L\Gamma$-space. Since each G_i is nonempty closed Γ_i-convex valued and compact upper semicontinuous, it follows from Lemma 3 of Fan [24] that G is also nonempty closed Γ_i-convex valued and compact upper semicontinuous. Next, we prove that, for any $x \in X$, $G(x)$ is Γ-convex. For every $A \in G(x)$ and $A \subset D$, we derive that for every $i \in I, \pi(A) \subset G_i(x) \cap D_i$. Since every $G_i(x)$ is Γ_i-convex, we have that $\Gamma_i(\pi_i(A)) \subset G_i(x)$ for each $i \in I$. Therefore, $\Gamma(A) = \prod_{i\in I}\Gamma_i(\pi_i(A)) \subset \prod_{i\in I} G_i(x) = G(x)$. Thus, we obtain that $G(x)$ is Γ-convex and $G: X \longrightarrow 2^X$ is nonempty closed Γ_i-convex valued and compact upper semicontinuous. It follows from Lemma 1, for each $i \in I$, we can find an $\widehat{x} = (\widehat{x}_i)_{i\in I} \in X$ satisfying $\widehat{x} \in G(\widehat{x}) = \prod_{i\in I} G_i(\widehat{x})$, i.e., $\widehat{x}_i \in G_i(\widehat{x})$.

Let X and Y be two nonempty convex subsets of a Hausdorff topological vector space. In this article, we use $\mathrm{F}(X)$ to denote all the fuzzy sets on X. F: $X \longrightarrow \mathrm{F}(Y)$ is said to be a fuzzy mapping. Thus, we have that for each $x \in X, F(x)$(denote by F_x in the sequel) is a fuzzy set in $\mathrm{F}(Y)$. $F_x(y)$ is defined as the degree of membership of point y in F_x.

For every $q \in [0,1]$, we use $(A)_q = \{x \in X\colon A(x) \geq q\}$ to denote the q-cut set of $A \in \mathrm{F}(X)$. If I is a set of agents (finite or infinite), X_i is a nonempty topological space (a choice set), A_i, B_i: $X = \prod_{i\in I} X_i \longrightarrow \mathrm{F}(X_i)$ are fuzzy constraint mappings (fuzzy constraint correspondences), and P_i: $X \longrightarrow \mathrm{F}(X_i)$ is a fuzzy preference mapping (fuzzy preference correspondence), then $\Upsilon = (X_i, A_i, B_i, P_i)_{i\in I}$ is said to be a generalized abstract fuzzy economy. If, for every $i \in I, \widehat{x}_i \in (B_{i\widehat{x}})_{b_i(\widehat{x})}$ and $(A_{i\widehat{x}})_{a_i(\widehat{x})} \cap (P_{i\widehat{x}})_{p_i(\widehat{x})} = \varnothing$, where a_i, b_i, p_i: $X \longrightarrow (0,1]$, then the point $\widehat{x} \in X$ is said to be an equilibrium of Υ. □

Theorem 2. *Suppose that $\Upsilon = (X_i, A_i, B_i, P_i)_{i\in I}$ is an abstract fuzzy model of economics, $X = \prod_{i\in I} X_i$, and a_i, b_i, p_i: $X \longrightarrow (0,1]$. Suppose for every $i \in I$, $\Upsilon = (X_i, A_i, B_i, P_i)_{i\in I}$ satisfies the following conditions:*

(1) $(X_i, D_i; \Gamma_i)$ is a compact $L\Gamma$-space

(2) for every $x \in X, x \mapsto (A_{ix})_{a_i(x)}$: $X \longrightarrow 2^{X_i}$ and $x \mapsto (B_{ix})_{b_i(x)}$: $X \longrightarrow 2^{X_i}$ are compact and nonempty closed Γ_i-convex valued

(3) for every $x \in X, x \mapsto (P_{ix})_{p_i(x)}$: $X \longrightarrow 2^{X_i}$ is closed Γ_i-convex valued

(4) $E_i = \left\{x \in X\colon (A_{ix})_{a_i(x)} \cap (P_{ix})_{p_i(x)} \neq \varnothing\right\}$ is open in X

(5) for every $x \in X, x_i \notin (A_{ix})_{a_i(x)} \cap (P_{ix})_{p_i(x)}$

Then, for each $i \in I$, we can find an $\widehat{x} \in X$ satisfying $\widehat{x}_i \in (B_{i\widehat{x}})_{b_i(\widehat{x})}$ and $(A_{i\widehat{x}})_{a_i(\widehat{x})} \cap (P_{i\widehat{x}})_{p_i(\widehat{x})} = \varnothing$.

Proof. Let $X = \prod_{i\in I} X_i, D = \prod_{i\in I} D_i$, and $\Gamma(A) = \prod_{i\in I} \Gamma_i$ (π_i (A)), where π_i is a projection of X onto X_i. By Lemma 2 in [23], we have that $(X, \mathcal{U}, \{\varphi_N\})$ is an $L\Gamma$-space. For each $i \in I, G_i$: $X \longrightarrow 2^{X_i}$ is defined as

$$G_i(x) = \begin{cases} (A_{ix})_{a_i(x)} \cap (P_{ix})_{p_i(x)}, & \text{if } x \in E_i, \\ (B_{ix})_{b_i(x)}, & \text{if } x \notin E_i. \end{cases} \tag{10}$$

From conditions (2) and (3) and Theorem 3.18 in [25], we have that $x \longrightarrow (A_{ix})_{a_i(x)} \cap (P_{ix})_{p_i(x)}$ is nonempty closed valued and compact upper semicontinuous. On the other hand, from conditions (2)–(4), Lemma 3 in [24], and Lemma 1 in [26], we have that G_i is nonempty closed Γ_i-convex valued and compact upper semicontinuous. From Lemma 2, for each $i \in I$, we can find an $\widehat{x} \in X$ satisfying $\widehat{x}_i \in G_i(\widehat{x})$. If, for some $j \in I, \widehat{x} \in E_j$, we obtain that $\widehat{x}_j \in (A_{j\widehat{x}})_{a_j(\widehat{x})} \cap (P_{j\widehat{x}})_{p_j(\widehat{x})}$. However, this is in contradiction with condition (5). Therefore, for each $i \in I$, $\widehat{x} \notin E_i$. By the definition of G_i, we have that for each $i \in I, \widehat{x}_i \in (B_{i\widehat{x}})_{b_i(\widehat{x})}$, and $(A_{i\widehat{x}})_{a_i(\widehat{x})} \cap (P_{i\widehat{x}})_{p_i(\widehat{x})} = \varnothing$. The proof is complete. □

Remark 2. Theorem 2 improves Theorem 2 in [14] and Theorem 2 in [15]. We also generalize the related results in [4] from generalized convex space to $L\Gamma$-space. Theorem 2 can be considered as an extended variant of Theorem 2 in [27] with respect to $L\Gamma$-spaces. We note that $L\Gamma$-spaces include a variety of topological spaces such as the LC-spaces and the LG-spaces (see [21, 22] and references therein).

3.2. Structural Stability and Robustness to Generalized Abstract Fuzzy Economies. Let $\Upsilon = (X_i, A_i, B_i, P_i)_{i\in I}$ be a generalized abstract fuzzy economic model satisfying all the conditions of Theorem 2. We introduce the definition $\Psi = \{\xi = \{G_i\}_{i\in I}\}$, where

$$G_i(x) = \begin{cases} (A_{ix})_{a_i(x)} \cap (P_{ix})_{p_i(x)}, & \text{if } x \in E_i, \\ (B_{ix})_{b_i(x)}, & \text{if } x \notin E_i. \end{cases} \tag{11}$$

By the proof of Theorem 2, we have that G_i: $X = \prod_{i\in I} X_i \longrightarrow 2^{X_i}$ is compact upper semicontinuous on X and there exists a point $x \in X$ such that $x_i \in G_i(x)$.

Let X_i be a metric space induced by a metric d_i. For each $\xi_1 = \{G_{1i}\}_{i\in I} \in \Psi$ and $\xi_2 = \{G_{2i}\}_{i\in I} \in \Psi$, we define

$$h_\varrho(\xi_1, \xi_2) = \sum_{i\in I} \sup_{x\in X} h_i(G_{1i}(x), G_{2i}(x)), \tag{12}$$

where h_i is the Hausdorff metric induced by d_i on X_i, for each $i \in I$.

Next, we consider the model $\Omega_0 = (\Psi, X, F, R)$, where $F(\xi, x) = G(x) = \prod_{i\in I} G_i(x)$, for all $\xi = \{G_i\}_{i\in I} \in \Psi$ and for any $x \in X$. Here, we denote f: $\Psi \longrightarrow 2^X$ by $f(\xi) = \{x \in X\colon x \in F(\xi, x)\} = \{x \in X\colon x_i \in G_i(x)\}$ and R: Gragh$(f) \longrightarrow R^+$ is a rationality function, which is defined as

$$R(\xi, x) = \max_{i\in I} d_i(x_i, G_i(x)) = \max_{i\in I} \min_{y_i \in G_i(x)} d_i(x_i, y_i), \tag{13}$$

for any $x \in f(\xi)$. Here, for each $i \in I$, d_i is the distance on X_i.

For every $\xi \in \Psi$, the ϵ-equilibria set of the generalized abstract fuzzy model of economics Υ at ξ is defined as follows:

$$E(\xi, \varepsilon) = \{x \in f(\xi)\colon R(\xi, x) \leq \varepsilon\}. \tag{14}$$

We use $E(\xi)$ to denote all equilibria set of the generalized abstract fuzzy model of economics Υ at ξ as

$$E(\xi) = E(\xi, 0) = \{x \in f(\xi)\colon R(\xi, x) = 0\}. \tag{15}$$

We notice that for all $(\xi, \varepsilon) \in \Psi \times R^+$, $E(\xi, \varepsilon)$ is compact if R is lower semicontinuous. It follows from Theorem 1 that $E(\xi) \neq \varnothing$.

Obviously, if and only if for each $i \in I$, $x_i \in G_i(x)$, and $x \in E(\xi)$, $R(\xi, x) \geq 0$ and $R(\xi, x) = 0$.

Theorem 3. *Let $\overline{\Omega}_0 = \{\overline{\Psi}, \overline{X}, \overline{F}, \overline{R}\}$ be the extended model of Ω_0; if $\overline{\Omega}_0$ is structurally stable at $(\overline{\xi}, \varepsilon)$, then $\overline{\Omega}_0$ is robust to $(\overline{\xi}, \varepsilon)$-equilibria.*

Proof. First, we prove that (Ψ, ϱ) is a complete metric space. Let $\{\xi_m\}$ be any Cauchy sequence in Ψ. Consequently, for any $\varepsilon > 0$, we can find a positive integer $P(\varepsilon)$ such that

$$\varrho\left(\xi_m, \xi_p\right) = \sum_{i\in I} \sup_{x\in X} h_i\left(G_{mi}(x), G_{pi}(x)\right) < \varepsilon, \quad \forall m, p \geq P(\varepsilon). \tag{16}$$

Since X_i is compact, X_i is complete. As a result, $X = \prod_{i\in I} X_i$ is compact. Obviously, under the Hausdorff distance, the family of compact subsets of X_i is a complete metric space. We notice that $G_{mi}: X \longrightarrow 2^{X_i}$ is compact upper semicontinuous. Based on Proposition 3.1.11 in [25], for each $i \in I$, we can always find a compact set $G_i(x) \subset X_i$ satisfying $\lim_{p\longrightarrow\infty} G_{pi}(x) = G_i(x)$. From (16), we obtain that

$$\sum_{i\in I} \sup_{x\in X} h_i\left(G_{mi}(x), G_i(x)\right) \leq \varepsilon, \quad \forall m \geq P(\varepsilon). \tag{17}$$

Since $\xi_m = \{G_{mi}\}_{i\in I} \in \Psi$, we can always find an $x_m \in X$, such that $x_{mi} \in G_{mi}(x_m)$, for each $i \in I$. Because of the compactness of X, we can let $x_m \longrightarrow x$. Since G_i is upper semicontinuous on X, we can find a positive integer $m_0 \geq P(\varepsilon)$ satisfying

$$G_i(x_m) \subset U(\varepsilon, G_i(x)), \quad \forall m \geq m_0. \tag{18}$$

Therefore, we get

$$x_{mi} \in G_{mi}(x_m) \subset U(2\varepsilon, G_i(x_m)) \subset U(3\varepsilon, G_i(x)), \quad \forall m \geq m_0. \tag{19}$$

Let $m \longrightarrow \infty$, we thus have that $d_i(x_i, G_i(x)) \leq 3\varepsilon$. Because of the arbitrariness of ϵ, we obtain that $d_i(x_i, G_i(x)) = 0$ and as such $x_i \in G_i(x)$. That is to say, $\xi = \{G_i\}_{i\in I} \in \Psi$. Thus, (Ψ, ϱ) is a complete metric space.

In what follows, we aim to prove that $R(\xi, x)$ is lower semicontinuous at (ξ, x).

For every $\xi_n = \{G_{ni}\}_{i\in I} \in \Psi$ with $\varrho(\xi_n, \xi) \longrightarrow 0$, in which $\xi = \{G_i\}_{i\in I} \in \Psi$, and for any $x_n \in X$ with $x_n \longrightarrow x$, we obtain that

$$R(\xi_n, x_n) > R(\xi, x) - \varepsilon, \quad \forall \varepsilon > 0. \tag{20}$$

Since $G_i: X \longrightarrow 2^{X_i}$ is compact upper semicontinuous, it follows from Proposition 3.1.19 in [25] that $d_i(x_i, G_i(x)) = \min_{y_i\in G_i(x)} d_i(x_i, y_i)$ is a lower semicontinuous mapping at x_i. Hence, we can find a positive integer N_1 satisfying, for all $n \geq N_1$,

$$\min_{y_i\in G_i(x_n)} d_i(x_{ni}, y_i) > \min_{y_i\in G_i(x)} d_i(x_i, y_i) - \frac{\varepsilon}{2}, \quad \forall i \in I. \tag{21}$$

For any $n = 1, 2, \ldots$, since G_{ni} is compact, then we can find a $y_{ni} \in G_{ni}(x_n)$ satisfying

$$d_i(x_{ni}, y_{ni}) = \min_{y_i\in G_{ni}(x_n)} d_i(x_{ni}, y_i), \quad \forall i \in I. \tag{22}$$

From $h_i(G_{ni}(x_n), G_i(x_n)) \longrightarrow 0$ and $y_{ni} \in G_{ni}(x_n)$, there is always a positive integer N_2 such that, for all $n \geq N_2$, there exists $y'_{ni} \in G_i(x_n)$ with $d(y'_{ni}, y_{ni}) < (\varepsilon/2)$ for every $i \in I$. Let $N = \max\{N_1, N_2\}$. Then, for all $n \in N$, we have

$$\begin{aligned} R(\xi_n, x_n) &= \max_{i\in I} \min_{y_i\in G_{ni}(x_n)} d_i(x_{ni}, y_i) \\ &= \max_{i\in I} d_i(x_{ni}, y_{ni}) \\ &\geq \max_{i\in I}\left(d_i(x_{ni}, y'_{ni}) - d_i(y'_{ni}, y_{ni})\right) \\ &\geq \max_{i\in I} d_i(x_{ni}, y'_{ni}) - \max_{i\in I} d_i(y'_{ni}, y_{ni}) \\ &\geq \max_{i\in I} \min_{y_i\in G_i(x_n)} d_i(x_{ni}, y_i) - \frac{\varepsilon}{2} \\ &> \max_{i\in I} \min_{y_i\in G_i(x)} d_i(x_i, y_i) - \varepsilon \\ &= R(\xi, x) - \varepsilon. \end{aligned} \tag{23}$$

Now, we prove that $R(\xi, x)$ is lower semicontinuous at (ξ, x).

Consider the correspondence

$$f(\xi) = \{x \in X: x \in F(\xi, x)\} = \{x \in X: x_i \in G_i(x)\}. \tag{24}$$

It follows from Lemma 6 in [28] that $f: \Psi \longrightarrow 2^X$ is an upper semicontinuous mapping. Therefore, according to Theorem 2, we know that if $\overline{\Omega}_0$ is structurally stable at $(\overline{\xi}, \varepsilon)$, then $\overline{\Omega}_0$ is robust to $(\overline{\xi}, \varepsilon)$-equilibria. Thus, the proof is complete. □

Remark 3. Theorem 3 improves Theorem 2 of [2]. First, the compactness of Ψ is dropped. Second, f is upper semicontinuous weakened compared with the continuity. Third, F is lower semicontinuity weakened than the continuity. Compared with Theorem 2 in [4], we introduce a more general model in Theorem 3. The structural stability at ξ and robustness to ε-equilibria are extended to the structural stability at $(\overline{\xi}, \varepsilon)$ and the robustness to $(\overline{\xi}, \varepsilon)$-equilibria, respectively.

4. Conclusions

In this paper, we discuss the relationship between $(\overline{\xi}, \varepsilon)$-stability and $(\overline{\xi}, \varepsilon)$-robustness of $\overline{\Omega}$ using a lower semicontinuous rationality function. As an application, we introduce the model $\overline{\Omega}_0$ consisting of a category of the parameterized generalized abstract fuzzy economic model with related abstract rationality function. We prove that the model $\overline{\Omega}_0$ is structurally stable at $(\overline{\xi}, \varepsilon)$. Therefore, we can conclude the robustness of $\overline{\Omega}_0$ to $(\overline{\xi}, \varepsilon)$-equilibria. In short, our work generalizes the results in the extant literature.

For economic models, stability analysis is one of the most significant issues. Our assumptions and methods employed in this research can be adopted to solve other economic models. In summary, the results derived in this paper can help solve a variety of economic and financial problems. The conclusions drawn in this article are also helpful in solving optimal control problems, financial optimization problems, and Nash equilibrium problems. In our future research, we will use the results obtained in this article to study various economic models.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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