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Research Article

Integrating an Extended Outranking-TOPSIS Method with Probabilistic Linguistic Term Sets for Multiattribute Group Decision-Making

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Group decision-making is a common activity in organizational management and economic conditions. In practice, the opinions of experts may be fuzzy. This paper proposes integrating an extended outranking-TOPSIS method with probabilistic linguistic term sets for multiattribute group decision-making, which is used to solve the real-world public-private partnership (PPP) project selection problem. First, an extended outranking method based on probabilistic linguistic term sets is proposed, and each expert's ranking of alternatives is obtained according to this method. After the individual ranking is completed, the large-scale expert group is clustered by the K-means clustering method, and then the improved consensus mechanism is used to study the degree of consensus of the expert group. If the consensus of the group is not up to the standard, then, for clusters with a lower degree of consensus with the group, the feedback mechanism is used to adjust the weight between different clusters so that the group consensus can be improved. After achieving the target group consensus, an improved technique for order preference by similarity to an ideal solution (TOPSIS) method is used to synthesize expert opinions, and the ranking results are obtained. Finally, there are cases used to demonstrate the feasibility and rationality of the method.

1. Introduction

Research on decision-making has been conducted for many years, and it has long been recognized by enterprises and other entities. In the decision-making problem, the language of the decision-maker and its expression are worthy of being studied. The decision-maker, similar to the decision-making problem, faces many constraints. As specific events cannot be accurately quantified and each decision-maker's personal educational background, living environment, abilities, and so on are different, ambiguities are present in the decision-making process. Therefore, it is necessary to convert qualitative expressions into quantitative expressions. Zadeh [1] proposed fuzzy linguistics and the use of linguistic variables to represent decision-making. However, linguistic variables are not sufficient to accurately reflect opinions. Torra [2, 3] proposed the concept of hesitant fuzzy sets and pointed out that its membership degree exists in a subset of [0, 1]. Rodriguez et al. [4] proposed a hesitant fuzzy linguistic term set to obtain a contiguous subset of a set of linguistic terms to describe linguistic variables. The hesitant fuzzy linguistic term set assigns the same weight to each linguistic variable, which is not sufficient to reflect the probability difference between different linguistic variables assigned by the decision-maker. In this context, Pang et al. [5] proposed a new probabilistic linguistic term set (PLTS) that can fully quantify the linguistic scores of decision-makers and reflect the quantitative differences between the linguistic variables. In this manner, comprehensive and accurate preference information of decision-makers can be obtained.

There have been many studies on PLTS in recent years. Zhang and She [6] used it for multicriteria decision-making (MCDM) problems. Liao et al. [7] also proposed a method based on PLTS to solve the multicriteria decision-making problems. Lin et al. [8] constructed an IoT evaluation system and introduced the concept of probabilistic linguistic term set to express the group preference information of the IoT platform with respect to the criteria. Yao et al. [9] proposed a probabilistic linguistic term envelopment analysis and studied the optimization method of the allocation efficiency of PM2.5 emission rights. Bai et al. [10] proposed an interval-valued probabilistic linguistic term set and studied the related operations and comparison laws of the theory to solve the multicriteria group decision-making (MCGDM) problem. Yu et al. [11] proposed probabilistic linguistic weight average (PLWA) and probabilistic linguistic order weight average (PLOWA) operators and studied their properties, and they proposed a multicriteria decision-making method based on the proposed operators. Gou and Xu [12] proposed some new operation laws for hesitant fuzzy linguistic elements and probabilistic linguistic term sets based on two equivalent transformation functions. By using PLTS for calculations, the probability information can be kept complete.

With the development of the economy and technology, the importance of decision science is increasing [13, 14]. Multiattribute group decision-making is an important part of modern decision science, and its theories and methods are widely used in the fields of economy, management, and military strategy. Different experts or decision-makers are needed to evaluate alternatives with multiple attributes and finally give a ranking of the options approved by the decision-making group. Liu et al. [15] first determined the percentage distribution of the assessment by each group of each alternative and then aggregated the subjective weights provided by the organizer and the objective weights determined by the level of consensus between the participant evaluations to obtain the decision weights for each group of each option and then rank by comparing the advantages between the programs. Wu et al. [16] used linguistic principal component analysis to reduce the attribute dimension. Shen et al. [17] used a new intuitionistic fuzzy ordering method to solve related multiattribute group decision problems. Rodríguez et al. [18] combined hesitant fuzzy linguistic and group decision-making to expand the scope of hesitant fuzzy linguistic method. The Score-HeDLiSF proposed by Liao et al. [19] has advantages in dealing with balanced and unbalanced language information with hesitant and linguistic scale functions. Lin et al. [20] proposed an aggregation-based technology to sort alternatives to solve the problem of multiattribute group decision-making. Gou et al. [21] proposed a similarity-based clustering method and a double hierarchy information entropy-based weighting method and consensus metric. Lin et al. [22] proposed a new PDOWA operator to address multiattribute group decision-making problems and gave an example to illustrate. Yu et al. [23] extended the classic TODIM method to develop a new MCGDM method based on unbalanced hesitant fuzzy linguistic term sets (HFLTS).

In many situations, a large number of experts participate in the group decision-making effort. Due to factors such as the observation angle of the experts and personal abilities, the degree of decision-making consensus is often not high enough. At the same time, the increase in the number of group decision-making participants may make some models no longer applicable. This is the problem of large-scale group decision-making. The problem of large-scale group decision-making requires increasing the degree of consensus among experts and unifying expert opinions. Wu et al. [24] combined the interval type-2 fuzzy method with the TOPSIS method to solve the large-scale multiattribute group decision problem. Xu et al. [25] proposed some new concepts, including collective adjustment proposals and rationality to solve large-scale group decision-making problems. Du et al. [26] proposed a new large-scale group decision-making method considering the expert knowledge structure and proposed an information extraction mechanism that provides three kinds of reasoning methods: single-attribute reasoning, local integral reasoning, and global integral reasoning. Liu et al. [27] proposed a dynamic weight penalty mechanism to increase the degree of consensus for overconfident decision-makers in large-scale group decision-making problems. Liu et al. [28] aimed at large-scale group decision-making in the social network environment and detected and reduced conflicts among decision-makers in the three processes of trust propagation, conflict detection and elimination, and selection.

Clustering is a common method to solve group decision-making problems and improve the degree of consensus. It groups experts with similar opinions, so as to effectively adjust the opinions between experts or adjust the weights attributed by experts. Ma et al. [29] applied a fuzzy clustering approach to create expert clusters based on expert similarity and phase and attribute weights. Kamis et al. [30] suggested three steps, namely, the identification of experts who contribute less to consensus, identification of leaders in the network, and generation of recommendations to achieve clustering mechanisms. Yoon et al. [31] proposed a mediation group decision-making method based on preference clustering to minimize subjectivity issues. Xu et al. [32] also constructed a group membership clustering algorithm to cluster large groups and then obtained the best alternative algorithm by comparing the exact functions of the score function and interval intuitionistic fuzzy numbers. Wu and Liu [33] used interval type-2 fuzzy equivalence clustering to classify decision-makers. In addition to this, there are other methods to optimize the consensus of group decision problems. For example, Wu et al. [34] used equivalent integer linear programming to optimize the size of the change, the number of modifications, and the individuals who need to modify their preferences. To improve the acceptability of the proposed preferences, an interactive consistency process and an interactive consensus process based on the multistage model were also designed to illustrate the developed method. Wu and Xu [35] proposed a process of direct consensus to solve the HFLPR consensus problem. The consensus arrival process has a salient feature that the feedback system is directly based on the degree of consensus, thereby effectively reducing the proximity measure calculations. Some existing consensus improvement mechanisms have practices that completely ignore the opinions of marginal expert groups, such as transferring all of their opinions directly to other expert groups, which is not in line with the idea of group decision-making. At present, it is necessary to propose a new consensus improvement mechanism to increase consensus on the basis of respecting the opinions of marginal expert groups.

This article has the following innovations and contributions:

(1) This work proposes a new extended outranking method based on probabilistic linguistic term sets that can effectively rank the alternatives. Then, clustering is based on the proposed extended outranking relation, and the matrix used is an asymmetric matrix. Compared with the traditional symmetric matrix based on probabilistic linguistic term sets, such as the Euclidean distance matrix between ordinary probabilistic linguistic term sets, the information is more abundant and complete, and the clustered expert group opinions are closer.

(2) This work uses an improved consensus improvement mechanism. The feedback mechanism respects the opinions of marginal expert groups and respects their weight adjustments. This helps to improve the persuasiveness and acceptability of decision-making, and it can also effectively improve the overall consensus degree. By setting the threshold of the consensus mechanism, a good improvement effect can be obtained, thereby making the conclusion more accurate and reliable.

(3) This work proposes an improved TOPSIS ranking method based on net credibility and weight adjustment. This method constructs the initial matrix by using net credibility as relative closeness and introduces the adjusted weights of decision-makers to obtain the group closeness matrix, which can effectively solve the problem of large-scale multi-attribute group decision-making.

The rest of the article is organized as follows: Section 2introduces the basic concepts related to the probabilistic linguistic term set, and Section 3introduces the extended outranking method based on probabilistic linguistic term sets, clustering mechanism, consensus mechanism and feedback mechanism, and the improved TOPSIS method. Section 4introduces the specific examples to illustrate the method of this work. Section 5compares and discusses related literature. Section 6gives the conclusions. The Appendix section provides the original data of this article.

2. Basic Concepts and Theories and Their Relationship

In this section, we introduce the basic concepts related to the probabilistic linguistic term set (PLTS).

Definition 1 (see [5]). Consider $S = \{S_i | i = 0, 1, 2, \ldots, g\}$as a set of linguistic terms, then the PLTS can be defined as follows:

$$L(p) = \left\{ L^{(k)}\left(p^{(k)}\right) | L^{(k)} \in S, p^{(k)} \geq 0, k = 1, 2, \ldots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1 \right\}, \tag{1}$$

where $L^{(k)}\left(p^{(k)}\right)$is the linguistic term $L^{(k)}$associated with probability $p^{(k)}$and $\#L(p)$is the number of all different linguistic terms in $L^{(p)}$.

The advantage of the probabilistic linguistic term set is that it can reflect the complete probabilistic distribution in linguistic terms. Thus, in a complex decision-making environment, decision-makers can selectively assign several linguistic terms and their probabilities, which is convenient for decision-makers to use linguistic information to effectively articulate decision-making views and to express linguistic information flexibly. Therefore, it is more in line with the inner thinking of decision-makers.

To effectively compare the two probabilistic linguistic term sets, it is necessary to introduce the concept of score function, using the score function based on concentration degree for probabilistic linguistic term sets score function proposed by Lin et al.

Definition 2 (see [36]). Let $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$be a PLTS, and let $r^{(k)}$be the subscript for the linguistic term $L^{(k)}$. Thus, the score function $F(L(p))$of $L(p)$can be expressed as follows:

$$F(L(p)) = s_{\overline{\alpha} \times cd(L(P))}. \tag{2}$$

In (2), $\overline{\alpha} = \sum_{l=1}^{L} I(s^{(l)}) p^{(l)} / \sum_{l=1}^{L} p^{(l)}$. The concentration degree of$L(p)$is $c\,d(L(P)) = 1 + \sum_{l=1}^{L} p^{(l)} \log_2 (1 - (|I(s^{(l)}) - I(E(L(P)))| / I(d_{lts})))$. The expected value of $L(P)$is $E(L(P))$, $I(s^{(l)})$is the subscript of the linguistic term $s^{(l)}$, $I(d_{\text{lts}})$is the subscript of the linguistic term which is the difference value between the maximum linguistic term and the minimum linguistic term in the LTS S, and $I(E(L(P)))$is the subscript of the expectation value of $L(P)$. It considers hesitance and uncertainty degree in the concentration degree.

The score function is composed of expectation value and concentration degree, effectively processing the probability information contained in probabilistic linguistic term sets and achieving a comparison of probabilistic linguistic term sets: for two probabilistic linguistic term sets $L_{P_1}$and L_{P_2}, if $F(L_{P_1}) < F(L_{P_2})$, then $L_{P_1} < L_{P_2}$. If $F(L_{P_1}) = F(L_{P_2})$, then $L_{P_1} = L_{P_2}$.

3. A Multiattribute Group Decision-Making Method with Probabilistic Linguistic Term Sets

3.1. An Extended Outranking Method Based on Probabilistic Linguistic Term Sets. This article proposes introducing the score function $F$of the probabilistic linguistic term sets as the attribute score of the scheme into the ELECTRE_IIIalgorithm as a new extended outranking method based on probabilistic linguistic term sets. The superior and inferior relationships between the schemes are obtained according to this method. To effectively study the multiattribute group decision problem in this article, the following symbols are adopted: the attribute set $G = \{G_1, G_2, \ldots, G_m\}$, scheme set $A = \{A_1, A_2, \ldots, A_n\}$, expert set $X = \{x_1, x_2, \ldots, x_q\}$, expert weight set $\omega_x = \{\omega_{x_1}, \omega_{x_2}, \ldots, \omega_{x_q}\}$, and the score $F_j(A_i)$, where $F$is the function defined in Definition 2above, $A_i$is the i-th scheme in the scheme set A, and $j$is the j-th attribute in the attribute set G.

In the extended outranking relation based on probabilistic linguistic term sets, the attribute threshold is used to determine the scheme level difference and the attribute threshold is divided into three thresholds: indifference threshold q, preference threshold p, and veto threshold v. The indifference threshold $q$occurs when schemes $A_i$and $A_k$are compared on a certain attribute G_j; if the score difference is less than q, that is, $A_i - A_k < q$, it can be considered that there is no difference between the two schemes on this attribute. The preference threshold $p$occurs when schemes $A_i$and $A_k$are compared on a certain attribute G_j; if the score difference is greater than p, that is, $A_i - A_k > p$, it can be considered that $A_i$strictly takes precedence over $A_k$on this attribute. The veto threshold $v$occurs when schemes $A_i$and $A_k$are compared on a certain attribute G_j; if the score difference is greater than or equal to v, that is, $A_i - A_k > v$, it can be considered that the attribute is higher than $A_k$on the $A_i$level. At the same time, multiattribute decision-making research needs to analyze the level relationship of the program and introduce the harmony degree, rejection degree, and credibility index to determine the merits and demerits of each attribute.

Definition 3. The harmony degree based on probabilistic linguistic term sets indicates the degree to which "scheme $A_i$is higher than A_k," and the calculation method is given by the following equation:

$$R(A_i, A_k) = \frac{1}{\sum_{j=1}^{n} w_j} \sum_{j=1}^{n} w_j r_j(A_i, A_k), \tag{3}$$

where for the degree that scheme $A_i$is better than $A_k$on attribute G_j, it is represented by $r_j(A_i, A_k)$, and the calculation method is given by the following equation:

$$r_j(A_i, A_k) = \begin{cases} 0, & \text{if } F_j(A_i) + p \le F_j(A_k), \\ 1, & \text{if } F_j(A_i) + q \ge F_j(A_k), \\ \dfrac{F_j(A_i) + p - F_j(A_k)}{p - q}, & \text{others.} \end{cases} \tag{4}$$

Definition 4. For the rejection degree of scheme $A_i$is better than that of $A_k$on attribute G_j, it is represented by $t_j(A_i, A_k)$, which is calculated by the following equation:

$$t_j(A_i, A_k) = \begin{cases} 0, & \text{if } F_j(A_i) + p \ge F_j(A_k), \\ 1, & \text{if } F_j(A_i) + v \le F_j(A_k), \\ \dfrac{F_j(A_k) - F_j(A_i) - p}{v - p}, & \text{others.} \end{cases} \tag{5}$$

Definition 5. The credibility index based on probabilistic linguistic term sets is expressed as the degree of trust that scheme $A_i$is higher than $A_k$in all attributes. It is necessary to comprehensively consider the degree that scheme $A_i$is better than $A_k$on attribute $G_j$and the rejection degree of scheme $A_i$is better than that of $A_k$on attribute G_j, and its size is defined by the following equation:

$$U(A_i, A_k) = \begin{cases} R(A_i, A_k), & \text{if } H = \varnothing, \\ R(A_i, A_k) * \prod_{j \in H} \dfrac{1 - t_j(A_i, A_k)}{1 - R(A_i, A_k)}, & \text{if } H \ne \varnothing, \end{cases} \tag{6}$$

where$H = \{j \mid t_j(A_i, A_k) > R(A_i, A_k)\}$and $t_j(A_i, A_k)$is obtained by (5).

3.2. Expert Consensus Improvement Mechanism Based on Clustering Improvement

3.2.1. Clustering Mechanism. The K-means clustering method was introduced to effectively adjust the weight assigned by each expert and improve the consensus of the program. Given that some experts may be too vague and the overall consensus level could be too low, the K-means clustering method is used to optimize the whole group decision-making to increase the group decision-making consensus. Some previous works have used this clustering method [37–39]. This work clusters the credibility given by the extended outranking relation based on probabilistic linguistic term sets. According to the reliability given by this method, $U(A_i, A_k)$and $U(A_k, A_i)$are not necessarily equal, which can effectively reflect the extended outranking relation between the two schemes. General clustering based on linguistic term sets uses a symmetric matrix. Symmetry means that there is only a unique absolute mathematical relationship between the two schemes, which cannot effectively reflect the fuzziness of each scheme on a certain attribute. Using the asymmetric matrix of credibility given by the extended outranking relation based on probabilistic linguistic term sets to cluster, the information contained in it is richer, and the opinions of the clustered expert group have more reference value.

First, the overall degree of ambiguity and consensus of the opinions given by experts are measured and Euclidean is used to measure the distance between the credibility matrices of each expert. As the elements of the matrix obtained by the extended outranking relation based on probabilistic linguistic term sets are located between 0 and 1, it is not necessary to standardize and can be directly substituted into the K-means clustering method.

In the K-means clustering method, the standardized Euclidean distance between the two is given by the following equation:

$$\mathrm{d}E(x, y) = \sqrt{\sum_{k=1}^{n} (x_k - y_k)^2}. \tag{7}$$

For fuzzy preference relationships, consensus measures can be given at three different levels: pairs of alternatives, alternatives, and relations [40–43].

For each expert's credibility index matrix $(U(A_i, A_k))_{n\times n}$, calculate the Euclidean distance between each other, and cluster the credibility index matrices as follows:

Step 1: as the n elements $U(A_i, A_i)$on the main diagonal of the obtained matrix are necessarily 1, it has no meaning for the group decision consensus calculation and is eliminated. To facilitate the operation of the K-means clustering method, for the matrix of the δ thexpert, the remaining matrix elements are successively listed as a row vector as shown in the following equation:

$$U_\delta = \{U_\delta(A_1, A_2), U_\delta(A_1, A_3), \ldots, U_\delta(A_1, A_n), U_\delta(A_2, A_1), \ldots, U_\delta(A_n, A_{n-1})\}. \tag{8}$$

The row vector has a total of $n \times (n-1)$elements. Let $\varphi = n \times (n-1)$, let ξdenote the ξ thelement of the vector, and let $U_{\delta_1} = U_\delta(A_1, A_2), U_{\delta_2} = U_\delta(A_1, A_3), \ldots, U_{\delta_\varphi} = U_\delta(A_n, A_{n-1})$.

Step 2: set the value of Kin the K-means clustering method based on experience. Then, Kcluster centroid points are randomly generated, and the t-th cluster centroid points are expressed as shown in the following equation:

$$\Omega_t^0 = \left\{\Omega_{t_1}^0, \Omega_{t_2}^0, \ldots, \Omega_{t_\varphi}^0\right\}. \tag{9}$$

Step 3: calculate the distance from each point in the data set to the centroid of the cluster in which it is located and assign the data points to the closest cluster. For the t-th cluster, the normalized Euclidean distance formula for the δth expert to the centroid in the cluster (where $\eta$is the number of updates) is given by the following equation:

$$\mathrm{d}E(U_\delta, \Omega_t^\eta) = \sqrt{\sum_{l=1}^{\varphi} \left(U_{\delta_l} - \Omega_{t_l}^\eta\right)^2}. \tag{10}$$

Step 4: for each cluster, calculate the mean value of all points in the cluster. According to the calculation result, the mean value is taken as the new cluster centroid $\Omega_t^\eta = \left\{\Omega_{t_1}^\eta, \Omega_{t_2}^\eta, \ldots, \Omega_{t_\varphi}^\eta\right\}$, update η, and repeat step 3 until a stable cluster is generated.

Step 5: output of the final clustered result is given by the following equation:

$$\theta = \{\theta_1, \theta_2, \ldots, \theta_K\} \tag{11}$$

Step 6: end of process.

3.2.2. Consensus Mechanism. The consensus mechanism is to determine whether the overall consensus of the expert group is up to the standard, so as to determine whether the expert opinion or expert weights need to be adjusted. Through the linkage with the feedback mechanism, the overall consensus of the expert group can be improved, and the opinions of the group decision-making are more consistent so that the final program can be ranked. Set the cluster set as $\{\theta_1, \theta_2, \ldots, \theta_K\}$.

Step 1: for the clustering results obtained, a consensus mechanism is needed to obtain the overall consensus degree, and then it is determined whether the overall consensus degree meets the requirements. First, the expert weight vector is standardized:

For a finally obtained cluster θ_1, there are a total of ρexperts. The weight of the entire cluster is $\omega_{\theta_1} = \sum_{i=1}^{\rho} \omega_{x_i}^{\theta_1}$; the normalized result of the weight vector of expert $i$is given by $\omega_{x_i}^{\theta_1} = (\omega_{x_i}^{\theta_1}/\omega_{\theta_1})$.

Example 1. Suppose that there are 10 experts $\{x_1, x_2, \ldots, x_{10}\}$, and the weights are $\{0.10.0.05, 0.2, 0.06, 0.03, 0.04, 0.17, 0.15, 0.09, 0.11\}$. Now cluster $\{x_2, x_4, x_6, x_8\}$into a cluster θ_1. Then, within the cluster, the weight of $x_2$is $\omega_{x_2}^{\theta_1} = (0.05/0.05 + 0.06 + 0.04 + 0.15) = (1/6)$.

Step 2: the research consensus is divided into three stages to obtain the overall consensus [44, 45]:

Step 2.1: obtain the intracluster consensus matrix.

For each cluster, find a weighted average of the credibility of the two schemes within its cluster as shown in the following equation:

$$\overline{cd_{\theta_1}^l} = \frac{\sum_{i=1}^{\rho} \omega_{x_i}^{\theta_1} U_{x_{il}}}{\rho}. \tag{12}$$

From the above, the cluster consensus degree matrix can be obtained as follows:

$$cd_{\theta_1} = \begin{pmatrix} - & \overline{cd_{\theta_1}^1} & & \overline{cd_{\theta_1}^{n-2}} & \overline{cd_{\theta_1}^{n-1}} \\ \overline{cd_{\theta_1}^n} & - & \cdots & \overline{cd_{\theta_1}^{2n-3}} & \overline{cd_{\theta_1}^{2n-2}} \\ \vdots & & \ddots & \vdots & \\ \overline{cd_{\theta_1}^{n^2-3n+3}} & \overline{cd_{\theta_1}^{n^2-3n+4}} & & - & \overline{cd_{\theta_1}^{(n-1)(n-1)}} \\ \overline{cd_{\theta_1}^{n^2-2n+2}} & \overline{cd_{\theta_1}^{n^2-2n+3}} & \cdots & \overline{cd_{\theta_1}^{n(n-1)}} & - \end{pmatrix}. \tag{13}$$

Step 2.2: obtain the intercluster similarity matrix and aggregation.

For the two clusters $\theta_\alpha$and θ_β, the intercluster similarity matrix element is calculated as given in [45] and shown in the following equation:

$$sm_{\alpha\beta}^{ik} = 1 - \left| cd_{\theta_\alpha}^{ik} - cd_{\theta_\beta}^{ik} \right|. \tag{14}$$

As $\mathrm{d}E(cd_{\theta_\alpha}^{ik}, cd_{\theta_\beta}^{ik}) = \mathrm{d}E(cd_{\theta_\beta}^{ik}, cd_{\theta_\alpha}^{ik})$, there are $(K(K-1)/2)$intercluster similarity matrices. For the convenience of identification, take the matrices of $\alpha < \beta$.

For the obtained $(K(K-1)/2)$intercluster similarity matrix, a specific aggregation function $\Gamma$is used to aggregate it as given in [45] and shown in the following equation:

$$cm_{ik} = \Gamma\left(sm_{\alpha\beta}^{ik}\right). \tag{15}$$

In general, the aggregation method is a weighted average, and the obtained elements $cm_{ik}$are arranged in a matrix to obtain an overall consensus degree matrix, that is, cm.

Step 2.3: obtain a general consensus degree matrix.

Level 1: the consensus degree between any two schemes: for the consensus degree relationship between any two schemes (A_i, A_k), cp_{ik}, directly take the corresponding positional element $cm_{ik}$from matrix cm[44] as shown in the following equation:

$$cp_{ik} = cm_{ik}. \tag{16}$$

Level 2: the consensus level for a scheme: for a scheme A_i, its consensus degree is expressed by ca_i, which is defined as in [44] and shown by the following equation:

$$ca_i = \frac{\sum_{k=1, k \neq i}^{n} cp_{ik}}{n-1}. \tag{17}$$

Level 3: the overall consensus degree: it is expressed in terms of ocd, which is used to measure the degree of consensus of the entire group. The calculation method is shown in [44] and according to the following equation:

$$\mathrm{ocd} = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{18}$$

3.2.3. Feedback Mechanism

Step 1: the use of the aforementioned consensus mechanism can effectively obtain the consensus within the cluster and the overall consensus. Assume that the overall consensus degree of the presupposition is $\overline{\mathrm{ocd}}$. If the obtained $\mathrm{ocd} \geq \overline{\mathrm{ocd}}$, then it is the situation where the overall consensus degree meets the expected requirements; then go directly to the next step of the method. For $\mathrm{ocd} < \overline{\mathrm{ocd}}$, as a situation where the expected requirements are not met, a feedback mechanism is required. This feedback mechanism has been inspired by the literature [38] and has been improved.

Through the definition of the overall consensus degree $\overline{cd^l} = \sum_{t=1}^{K} \omega_{\theta_t} \overline{cd_{\theta_t}^l}$, it can be seen that, in all clusters for positions unchanged, lowering the expert weight of the clusters with farther distances can effectively improve the overall consensus.

Step 2: obtain the overall average consensus matrix.

Obtain the weighted average value of the elements of the cluster consensus matrix as shown in the following equation:

$$\overline{cd^l} = \sum_{t=1}^{K} \omega_{\theta_t} \overline{cd_{\theta_t}^l}. \tag{19}$$

Rank them in a matrix to obtain the overall average consensus matrix as given by the following equation:

$$cd = \begin{pmatrix} - & \overline{cd^1} & \cdots & \overline{cd^{n-2}} & \overline{cd^{n-1}} \\ \overline{cd^n} & - & \cdots & \overline{cd^{2n-3}} & \overline{cd^{2n-2}} \\ \vdots & & \ddots & & \vdots \\ \overline{cd^{n^2-3n+3}} & \overline{cd^{n^2-3n+4}} & & - & \overline{cd^{n^2-3n+4}} \\ \overline{cd^{n^2-2n+2}} & \overline{cd^{n^2-2n+3}} & \cdots & \overline{cd^{n(n-1)}} & - \end{pmatrix}. \quad (20)$$

As the overall average consensus matrix is the centroid of the entire cluster, for a series of clusters $\{\theta_1, \theta_2, \ldots, \theta_K\}$, a way can be adopted, similar to the consensus mechanism, to identify the distance from the centroid of the cluster, thus providing a theoretical basis to adjust the weight of each cluster.

Level 1: the degree of consensus between any two schemes obtained by the consensus relationship between any two schemes (A_i, A_k)between the group centroid and the cluster $\theta_t$centroid $gp_{ik}^t$is given in [45] and shown in the following equation:

$$gp_{ik}^t = 1 - \left|cd^{ik}, cd_{\theta_t}^{ik}\right|. \quad (21)$$

Level 2: the consensus degree for a scheme: for a scheme A_i, the degree of consensus between the group centroid and the cluster $\theta_t$centroid is expressed by pa_i^t, which is defined in [45] and shown in the following equation:

$$pa_i^t = \frac{\sum_{k=1,k\neq i}^n gp_{ik}^t}{n-1}. \quad (22)$$

Level 3: the overall consensus degree: it is expressed in g, and $g^{\theta_t}$is used to measure the degree of consensus between the entire group and the entire cluster θ_t. The calculation method is given in [45] and shown in the following equation:

$$g^{\theta_t} = \frac{\sum_{i=1}^n pa_i^t}{n}. \quad (23)$$

Step 3: after obtaining the degree of consensus between each cluster and the group, the mechanism for adjusting the weight can be enabled for the clusters with too low consensus level, and part of the weight $\omega_{\theta_t}$of the experts is distributed to other cluster experts.

The weight adjustment system should be based on the opinion of the adjusted expert group that chooses which one or several expert clusters to obtain the weight of the adjustment part, $\Delta\omega_{\theta_t}(0 < \Delta\omega_{\theta_t} < \omega_{\theta_t})$. The existing weight of the expert cluster $p$that obtained the new weight is given in [45] and shown in the following equation:

$$\omega_{\theta_p}^{r+1} = \omega_{\theta_p}^r + \mu \times \Delta\omega_{\theta_t}, \quad (24)$$

where $\mu$is the ratio of the weighted portion $\Delta\omega_{\theta_t}$to the expert cluster and $r$is the number of times the weight is adjusted.

At the same time, although there are some differences between these experts and the group opinions, there may be merits in their opinions. The expert group cannot completely disperse the weight of this part of the experts. The expert group should coordinate and set a weighted upper limit, such as 80%; that is, $\Delta\omega_{\theta_t} \leq 0.8\omega_{\theta_t}$, so that some opinions of the cluster experts can be retained. After the first adjustment, the existing weights of the clusters with the worst consensus are obtained as in the following equation:

$$\omega_{\theta_t}^1 = \omega_{\theta_t}^0 - \Delta\omega_{\theta_t}^0. \quad (25)$$

The cluster expert weight will not be affected by the subsequent weight adjustments.

After the first weight adjustment is completed, the degree of consensus is recalculated. If the standard is still not met, the weight of the expert group with the second lowest degree of consensus is adjusted, and it is assigned to other expert clusters (except for the clusters whose degree of consensus is worse). The above steps are repeated until the target consensus degree is reached.

After the r-th adjustment reaches the target consensus degree, for the expert δbelonging to the cluster θ_t, his expert weight at this time is given by the following equation:

$$\omega_\delta^r = \omega_{\theta_t}^r. \quad (26)$$

The resulting expert cluster weight vector is obtained from the following equation:

$$\omega = \left\{\omega_{\theta_1}^r, \omega_{\theta_2}^r, \ldots, \omega_{\theta_K}^r\right\}. \quad (27)$$

3.3. Improved TOPSIS Ranking Method Based on the Net Credibility and Weight Adjustment. After solving the expert consensus degree problem through clustering and adjusting the expert weight, the credibility index based on probabilistic linguistic term sets should be compared. This requires the introduction of the concepts of consistent credibility, inconsistent credibility, and net credibility to achieve a comparison of the merits and demerits of a scheme as compared with all other schemes.

Definition 6. The consistent credibility $\Phi^+(A_i)$is used to describe the total extent of scheme $A_i$over other schemes. The calculation formula is given by the following equation:

$$\Phi^+(A_i) = \sum_{k\neq i} U(A_i, A_k). \tag{28}$$

Definition 7. Inconsistent credibility $\Phi^-(A_i)$is used to describe the total degree of other schemes better than scheme A_i. The calculation formula is given by the following equation:

$$\Phi^-(A_i) = \sum_{k\neq i} U(A_k, A_i). \tag{29}$$

Definition 8. The net credibility $\Phi(A_i)$represents the difference between the scheme and other schemes under this attribute. The calculation formula is given by the following equation:

$$\Phi(A_i) = \Phi^+(A_i) - \Phi^-(A_i). \tag{30}$$

The larger $\Phi(A_i)$, the better scheme $A_i$as compared with other schemes, and the higher the ranking.

TOPSIS is a method that uses virtual "positive ideal target points" and "negative ideal target points" to achieve program ordering. There are many works in the literature which use TOPSIS and its improved methods to solve group decision problems [46–49]. The vector formed by all positive ideal target points is the positive ideal solution, and the vector formed by all negative ideal target points is the negative ideal solution. Take the deviation squared form to measure the square of the distance. After the square root is obtained, calculate the relative closeness degree to rank the schemes. The core principle of TOPSIS improvement in this work is to construct the initial matrix with net credibility as the relative closeness degree and introduce the adjusted decision-maker weights to obtain the group closeness matrix.

Step 1: First, use the net credibility as the relative closeness degree to build the initial matrix as given in the following equation:

$$X = \begin{bmatrix} \Phi_1(A_1) & \Phi_2(A_1) & \cdots & \Phi_q(A_1) \\ \Phi_1(A_2) & \Phi_2(A_2) & \cdots & \Phi_q(A_2) \\ \cdots & \cdots & \cdots & \cdots \\ \Phi_1(A_n) & \Phi_2(A_n) & \cdots & \Phi_q(A_n) \end{bmatrix}. \tag{31}$$

Step 2: only normalized values can be guaranteed to lie in $[-1, 1]$and thus are comparable. If the initial matrix does not lie in $[-1, 1]$, the initial matrix values are normalized to obtain a group relative closeness normalization matrix. The standardization method is to mark $\Phi_s(A_i)$with x_{is}, $y_{is} = (x_{is}/\sqrt{\sum_{i=1}^{n} x_{is}^2})$as given by the following equation:

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1q} \\ y_{21} & y_{22} & \cdots & y_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{nq} \end{bmatrix}. \tag{32}$$

Step 3: in decision-making, in general, the experience, level, and status of different decision-makers are different, so the weight of their decision-making is generally different. This is to introduce the final decision-maker's decision weight vector $\omega = \{\omega^r_{x_1}, \omega^r_{x_2}, \ldots, \omega^r_{x_q}\}$to obtain the group closeness degree matrix as shown in the following equation:

$$Z = \begin{bmatrix} \omega^r_{x_1} y_{11} & \omega^r_{x_2} y_{12} & \cdots & \omega^r_{x_q} y_{1q} \\ \omega^r_{x_1} y_{21} & \omega^r_{x_2} y_{22} & \cdots & \omega^r_{x_q} y_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ \omega^r_{x_1} y_{n1} & \omega^r_{x_2} y_{n2} & \cdots & \omega^r_{x_q} y_{nq} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1q} \\ z_{21} & z_{22} & \cdots & z_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ z_{n1} & z_{n2} & \cdots & z_{nq} \end{bmatrix}. \tag{33}$$

Step 4: according to the obtained group closeness matrix, the values of the positive and negative ideal solutions can be determined. Positive ideal solution is given by $z^+ = \{z_1^+, z_2^+, \ldots, z_n^+\}$, where, for each specific s, $z_s^+ = \max\{z_{1s}, z_{2s}, \ldots, z_{ns}\}$. Negative ideal solution is given by $z^- = \{z_1^-, z_2^-, \ldots, z_n^-\}$, and, for each specific s, $z_s^- = \min\{z_{1s}, z_{2s}, \ldots, z_{ns}\}$. According to the obtained positive and negative ideal solutions, the positive and negative ideal solution distances can be determined: $d^+ = \sqrt{\sum_{j=1}^{m} (z_{is} - z_s^+)^2}$and $d^- = \sqrt{\sum_{j=1}^{m} (z_{is} - z_s^-)^2}$. Finally, the relative closeness degree can be calculated: $D = (d^-/d^- + d^+)$. The larger $D$is, the better the scheme is and the higher the ranking is.

Through the foregoing method, the problem of multiattribute group decision-making is realized.

3.4. Algorithm. This section proposes a group decision-making method that combines extended outranking relation based on probabilistic linguistic term sets, clustering method, consensus mechanism, feedback mechanism, and improved TOPSIS ranking method based on the net credibility and weight adjustment. First, according to the probabilistic linguistic term set matrix, the credibility index based on probabilistic linguistic term sets is obtained through the extended outranking method based on probabilistic linguistic term sets. Using the clustering algorithm to cluster, calculating the overall consensus degree through the consensus mechanism, and using the feedback mechanism to improve the overall consensus degree, and finally substituting the weight vector to obtain the corresponding plan ranking result are the steps in the improved TOPSIS method.

Large-scale multiattribute group decision-making methods can effectively solve complex decision problems.

For this type of question, this article takes the following symbols:

There are $n$schemes, denoted as $A = \{A_1, A_2, \ldots, A_n\}$

There are $m$attributes, denoted as $G = \{G_1, G_2, \ldots, G_m\}$

There are $q$experts, recorded as $X = \{x_1, x_2, \ldots, x_q\}$

The initial expert weight vector is $\omega_x = \{\omega_{x_1}, \omega_{x_2}, \ldots, \omega_{x_q}\}$

A probabilistic linguistic term set $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$

The degree to which $A_i$is better than $A_k$on attribute G_j, denoted as $r_j(A_i, A_k)$

The degree of rejection scheme $A_i$is better than that of $A_k$on attribute G_j, denoted as $t_j(A_i, A_k)$

The credibility index of schemes $A_i$and $A_k$is denoted as $U(A_i, A_k)$

To effectively rank the schemes, the following steps are adopted:

Step 1: a probabilistic linguistic term set matrix (PLTSM) is given, denoted as $\mathrm{PLTSM}_{x_q} = [L(p)_{x_q}]_{n\times m}$

The score function is used to calculate the scoring values of the expert scores one by one, and the score function matrix $\mathrm{FM}_{x_q} = [F_j(L_{P_i})_{x_q}]_{n\times m}$is listed

Step 2: for each attribute, the indifference threshold q, preference threshold p, and veto threshold $v$are given by expert group discussion and related calculations, respectively.

In general, the threshold of each attribute is given by the function of the relevant historical research or the experience of the expert or related regulations and is empirical. There is generally a correlation between q, p, and v.

Step 3: the expert's score function matrix is given as $[F_j(L_{P_i})_{x_q}]_{n\times m}$. According to the score function, the degree of merits and demerits between the two schemes under each attribute is obtained by the extended outranking method based on probabilistic linguistic term sets, and the credibility index between the two schemes under each attribute is calculated according to the degree of the advantages and disadvantages.

Step 4: using the clustering method such as K-means, cluster the expert groups to obtain the final clustering $\theta = \{\theta_1, \theta_2, \ldots, \theta_K\}$.

Step 5: use the consensus mechanism to obtain the overall consensus degree ocd. If the obtained ocd > $\overline{\mathrm{ocd}}$, then the overall consensus degree meets the expected requirements, and go directly to the improved TOPSIS method of Step 7 for group decision analysis; otherwise, go to Step 6.

Step 6: for the case of ocd < $\overline{\mathrm{ocd}}$, as a situation where the expected requirements are not met, a feedback mechanism is required. The expert weights are adjusted according to the feedback mechanism described in Section 3.2.

Step 7: the ranking results are obtained using the improved TOPSIS ranking method based on the net credibility and weight adjustment in Section 3.3.

4. An Illustrative Example

PPP is a partnership between the government and private capital owners. As a way of building a public foundation project, it effectively reduces government financial pressures and allows private capital to participate in projects that were previously impossible or difficult to initiate. The public and private sectors share the risks and can also effectively reduce the risks faced by a single party.

PPP projects often have a pool of experts. Experts conduct a comprehensive evaluation of a project to effectively avoid project risks. However, experts often give highly subjective suggestions. By combining with the multiattribute group decision-making method based on probabilistic linguistic term sets proposed in this work, it can effectively avoid the subjective problem of the original expert evaluation.

There are five PPP projects ($A1, A2, A3, A4, A5$)that can be selected by a city investment company, depending on the company's funds and other factors. Under the existing priority given to the projects with a higher ranking, this requires a comprehensive ranking of the five projects. The following factors need to be considered comprehensively: the government support dimension ($G1$), the project risk dimension ($G2$), the project sustainability dimension ($G3$), the project benefit dimension ($G4$), and the macroeconomic dimension ($G5$). For convenience, the weights of the attributes are equal. The existing 20 PPP project experts $X = \{x_1, x_2, \ldots, x_{20}\}$have equal initial weights, and the weight vector is $\omega_x = ((1/20), (1/20), \ldots, (1/20))$. Twenty experts scored five attributes of the five projects, which were divided into VL, L, M, H, and VH(five-scale linguistic evaluation sets: VL—S0, L—S1, M—S2, H—S3, VH—S4), and can give a continuous score, such as (VL, L).

Step 1: expert evaluation matrices are given, where the elements of the matrix row represent the scores of the attributes of the scheme, and the elements of the matrix column represent the scores of the schemes of the attribute. Due to the large number of elements, please refer to Appendix for details.

Due to space limitations, this article only shows the process of obtaining the credibility matrix of expert 1 and so on for the rest of the experts:

The score function matrix for all scores is obtained from equation (2):

$$\mathrm{FM}_{x_1} = \begin{pmatrix} 1.8835 & 1.0484 & 0.8084 & 0.8340 & 1.6200 \\ 0.5340 & 1.8430 & 1.3869 & 0.9148 & 0.6456 \\ 0.7085 & 1.3684 & 1.5978 & 1.0490 & 1.0407 \\ 1.1680 & 1.4758 & 0.6851 & 0.5967 & 1.4681 \\ 1.1897 & 1.2006 & 1.2705 & 0.7717 & 1.2264 \end{pmatrix}. \tag{34}$$

Step 2: the expert group gives the indifference threshold, the preference threshold, and the veto threshold for each attribute, which is common to all experts (see Table 1).

Step 3: from equations (4) and (5), the matrix of the pros and cons of the experts under different attributes is calculated according to the given threshold:

$(r_1(A_i, A_k))_{n\times n}$and $(t_1(A_i, A_k))_{n\times n}$of expert 1 under the $G1$attribute:

$$\begin{aligned} (r_1(A_i, A_k))_{n\times n} &= \begin{pmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 1.0000 & 1.0000 & 0 & 0 \\ 0 & 1.0000 & 1.0000 & 0.1621 & 0.0753 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 0 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{pmatrix}, \\ (t_1(A_i, A_k))_{n\times n} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0.2680 & 0.3114 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 0.4311 & 0 & 0 & 0 & 1.0000 \\ 0.3877 & 0 & 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{35}$$

$(r_2(A_i, A_k))_{n\times n}$and $(t_2(A_i, A_k))_{n\times n}$of expert 1 under the $G2$attribute:

$$\begin{aligned} (r_2(A_i, A_k))_{n\times n} &= \begin{pmatrix} 1.0000 & 0 & 0.4002 & 0 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.1641 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0 & 1.0000 & 0.6240 & 1.0000 \end{pmatrix}, \\ (t_2(A_i, A_k))_{n\times n} &= \begin{pmatrix} 0 & 0.9865 & 0 & 0.0685 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1866 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6060 & 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{36}$$

$(r_3(A_i, A_k))_{n\times n}$and $(t_3(A_i, A_k))_{n\times n}$of expert 1 under the $G3$attribute:

Table 1: The indifference threshold, preference threshold, and veto threshold of the five attributes.

Threshold	$G1$	$G2$	$G3$	$G4$	$G5$
Indifference threshold	0.25	0.2	0.3	0.25	0.25
Preference threshold	0.5	0.4	0.6	0.5	0.5
Veto threshold	1	0.8	1.2	1	1

$$(r_3(A_i, A_k))_{n\times n} = \begin{pmatrix} 1.0000 & 0.0716 & 0 & 1.0000 & 0.4598 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0 & 0 & 1.0000 & 0.0487 \\ 1.0000 & 1.0000 & 0.9087 & 1.0000 & 1.0000 \end{pmatrix},$$
$$(t_3(A_i, A_k))_{n\times n} = \begin{pmatrix} 0 & 0 & 0.3157 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1697 & 0.5213 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{37}$$

$(r_4(A_i, A_k))_{n\times n}$and $(t_4(A_i, A_k))_{n\times n}$of expert 1 under the $G4$attribute:

$$(r_4(A_i, A_k))_{n\times n} = \begin{pmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.7273 & 0.1906 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 0.8908 & 1.0000 & 1.0000 \end{pmatrix},$$
$$(t_4(A_i, A_k))_{n\times n} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{38}$$

$(r_5(A_i, A_k))_{n\times n}$and $(t_5(A_i, A_k))_{n\times n}$of expert 1 under the $G5$attribute:

$$\left(r_5\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 1.0000 & 0.4195 & 0 & 0 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.7273 & 0.1906 & 1.0000 & 1.0000 \\ 0.4257 & 1.0000 & 0.8908 & 1.0000 & 1.0000 \end{pmatrix},$$
$$\left(t_5\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.9488 & 0 & 0 & 0.6451 & 0.1616 \\ 0.1586 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (39)$$

From equation (3), $\left(R_1\left(A_i, A_k\right)\right)_{n\times n}$of expert 1 can be obtained:

$$\left(R_1\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 1.0000 & 0.6143 & 0.6800 & 0.8000 & 0.8920 \\ 0.6000 & 1.0000 & 0.8839 & 0.6000 & 0.6000 \\ 0.6000 & 0.8000 & 1.0000 & 0.6905 & 0.8151 \\ 0.8000 & 0.5783 & 0.6381 & 1.0000 & 0.8097 \\ 0.6851 & 0.8000 & 0.9599 & 0.9248 & 1.0000 \end{pmatrix}. \quad (40)$$

From equation (6), $\left(U_1\left(A_i, A_k\right)\right)_{n\times n}$of expert 1 can be obtained:

$$U_1 = \left(U_1\left(A_i, A_k\right)\right)_{n\times n} = \begin{pmatrix} 1.0000 & 0.0215 & 0.6800 & 0.8000 & 0.8920 \\ 0 & 1.0000 & 0.8839 & 0.5323 & 0.6000 \\ 0 & 0.8000 & 1.0000 & 0.6905 & 0.8151 \\ 0.8000 & 0.5783 & 0.6381 & 1.0000 & 0.8097 \\ 0.6851 & 0.8000 & 0.9599 & 0.9248 & 1.0000 \end{pmatrix}. \quad (41)$$

The same method is used to obtain the credibility matrices given by other experts under the five attributes.

Step 4: after obtaining the credibility matrix of the 20 experts, remove the main diagonal as described in Step 1 of the classification mechanism, retain other elements and group them into credibility vectors one by one, and cluster. For example, for $U_1 = \left(U_1\left(A_i, A_k\right)\right)_{n\times n}$, $U_1 =$ {0.0215,0.6800,0.8000,0.8920,0.0000,0.8839,0.5323,0.6000,0.0000,0.8000,0.6905,0.8151,0.8000,0.5783,0.6381, 0.8907,0.6851,0.8000,0.9599,0.9248} (see Table 2).

Clustering is performed by equations (7)–(11), and Kin the K-means cluster is set to 5, and the result is clustered:

$$\begin{aligned} \theta_1 &= \{U_1, U_2, U_4, U_6, U_7, U_{11}, U_{16}, U_{17}\}, \\ \theta_2 &= \{U_3, U_{13}, U_{14}\}, \\ \theta_3 &= \{U_9, U_{10}, U_{15}, U_{19}\}, \\ \theta_4 &= \{U_5, U_{12}, U_{20}\}, \\ \theta_5 &= \{U_8, U_{18}\}. \end{aligned} \quad (42)$$

From equations (12)–(14) for clusters 1 and 2, the intercluster similarity matrix is as follows:

Table 2: The element value of the credibility vector.

ξ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
U_1	0.0215	0.6800	0.8000	0.8920	0.0000	0.8839	0.5323	0.6000	0.0000	0.8000	0.6905	0.8151	0.8000	0.5783	0.6381	0.8097	0.6851	0.8000	0.9599	0.9248
U_2	0.4962	0.9278	0.9293	0.8708	1.0000	1.0000	1.0000	1.0000	0.9137	0.5484	0.9613	1.0000	0.8000	0.2264	1.0000	1.0000	0.9451	0.5911	1.0000	0.9175
U_3	0.8000	0.8679	0.8163	0.6000	0.6000	0.7446	0.4000	0.0000	0.9834	0.8862	0.7715	0.3371	1.0000	0.8000	0.9035	0.6000	0.8989	0.8000	0.8000	0.6857
U_4	0.8000	1.0000	1.0000	1.0000	0.8919	1.0000	0.8545	0.8201	0.8178	0.7608	0.6451	0.7963	1.0000	0.8000	1.0000	1.0000	0.7462	0.4591	0.8799	0.6957
U_5	0.0000	0.8407	0.8128	0.4101	0.6135	1.0000	0.0000	1.0000	0.7365	0.0000	0.4244	0.8004	0.9815	0.0000	0.8360	0.3636	0.4467	0.0000	1.0000	0.0000
U_6	1.0000	1.0000	0.8000	1.0000	0.5485	0.6580	0.5377	0.9054	0.2878	0.8000	0.7390	0.8167	0.8000	0.8000	0.9739	0.8000	0.6966	1.0000	0.8292	0.7156
U_7	0.8000	0.8595	0.1608	0.8000	0.7925	0.7856	0.7959	0.8000	0.8000	0.8000	0.8000	0.7449	0.8256	0.8272	1.0000	0.7901	1.0000	0.7924	0.8000	0.0000
U_8	0.6000	0.6622	0.8000	0.9334	0.0000	0.6396	0.0000	0.7734	0.7893	0.9073	0.7299	0.8251	0.8605	0.7747	0.7900	0.8000	0.0000	0.3499	0.1959	0.0000
U_9	0.6333	0.0000	0.6569	0.8000	0.8000	0.0000	0.8000	0.8000	1.0000	0.8000	0.7723	1.0000	0.7233	0.8000	0.0000	0.7475	1.0000	0.7000	0.0000	0.4184
U_{10}	1.0000	0.0000	0.8000	0.8429	0.7528	0.0650	0.0000	0.8000	0.7654	0.8405	0.1960	1.0000	1.0000	1.0000	0.1398	0.9160	0.8156	1.0000	0.1981	0.2982
U_{11}	0.9901	1.0000	0.7315	0.8000	0.8707	1.0000	0.8344	0.8000	0.7047	0.7201	0.8208	0.6092	0.5782	0.7457	0.9998	0.6564	0.9663	1.0000	1.0000	0.8000
U_{12}	0.6688	1.0000	0.8206	0.7371	0.0000	0.9040	0.7341	0.6067	0.7002	0.5157	0.6000	0.7509	0.6873	0.0000	0.8735	0.2061	0.0000	0.1697	0.8006	0.7585
U_{13}	0.8590	0.9498	0.6000	0.0000	0.7461	0.8550	0.4226	0.0000	0.8000	0.9882	0.8000	0.0000	0.7732	0.6122	0.9544	0.0040	1.0000	1.0000	1.0000	1.0000
U_{14}	0.0534	0.8081	0.0000	0.0000	0.8000	1.0000	0.9210	0.6000	0.8000	0.8000	0.8000	0.6000	0.7321	0.9902	0.8036	0.7674	0.8526	0.5417	0.7013	0.7479
U_{15}	0.9816	0.8000	0.8040	1.0000	0.7505	0.0000	0.5869	1.0000	0.8000	0.9995	0.6815	1.0000	0.8000	0.8656	0.7520	0.9179	0.2947	0.8205	0.0000	0.0000
U_{16}	0.8000	1.0000	0.7418	0.8000	0.6718	0.9448	0.2423	0.8000	0.6192	0.8000	0.4629	0.6467	0.8000	0.7083	1.0000	0.8000	0.7598	1.0000	1.0000	0.5300
U_{17}	1.0000	0.7797	1.0000	0.8276	0.7962	0.3290	0.9078	0.4158	0.5937	0.6000	0.8000	0.6000	0.4711	0.8000	0.5707	0.6363	0.8313	0.8000	0.8801	1.0000
U_{18}	0.9452	0.3841	0.0000	0.0000	0.0000	0.5165	0.0000	0.4139	0.8000	1.0000	0.4980	0.8000	0.8000	0.9623	0.6000	0.0000	0.0000	1.0000	0.5316	0.0000
U_{19}	0.8000	0.0000	0.6000	0.6819	0.6634	0.2854	0.9656	0.9498	1.0000	0.8622	0.8000	0.8000	0.0000	0.9389	0.3175	0.8000	0.8748	0.9285	0.5471	1.0000
U_{20}	0.7232	0.7341	1.0000	0.9381	1.0000	1.0000	1.0000	1.0000	0.9822	0.7387	1.0000	1.0000	0.3060	0.0387	0.1813	0.9740	0.0000	0.0380	0.5338	1.0000

$$sm_{12} = \begin{pmatrix} - & 0.8323 & 0.9694 & 0.7017 & 0.3262 \\ 0.9811 & - & 0.9586 & 0.8681 & 0.4323 \\ 0.7310 & 0.8372 & - & 0.9494 & 0.5587 \\ 0.9243 & 0.8849 & 0.9893 & - & 0.6456 \\ 0.9116 & 0.9752 & 0.9151 & 0.8868 & - \end{pmatrix}. \quad (43)$$

By analogy, other intercluster similarity matrices are obtained. The weight of each cluster is as follows:

$$\begin{aligned} \omega_{\theta_1}^0 &= 0.4, \\ \omega_{\theta_2}^0 &= 0.15, \\ \omega_{\theta_3}^0 &= 0.20, \\ \omega_{\theta_4}^0 &= 0.15, \\ \omega_{\theta_5}^0 &= 0.10. \end{aligned} \quad (44)$$

The following are available from equations (15)–(18): $ca_1 = 0.7304$, $ca_2 = 0.7102$, $ca_3 = 0.8092$, $ca_4 = 0.7417$, $ca_5 = 0.6427$, and ocd = 0.7268.

Let $\overline{\text{ocd}} = 0.75$; then ocd < 0.75, triggering the feedback mechanism.

The following are available from equations (19)–(23), the degrees of consensus for all clusters and groups: $g^{\theta_1} = 0.8920$, $g^{\theta_2} = 0.8020$, $g^{\theta_3} = 0.7931$, $g^{\theta_4} = 0.8088$, and $g^{\theta_5} = 0.7631$, where $g^{\theta_1} > g^{\theta_4} > g^{\theta_2} > g^{\theta_3} > g^{\theta_5}$.

The weights of the lowest cluster $\theta_5$are adjusted by formulas (24)–(25). After consultation with the expert group, $\mu_{\max} = 0.8$is set. After the adjusted $\theta_5$experts negotiate, it is decided to give θ_1 50%$\omega_{\theta_5}^0$weight and give θ_4 30%$\omega_{\theta_5}^0$weight; then the weight of each cluster is as follows:

$$\begin{aligned} \omega_{\theta_1}^0 &= 0.45, \\ \omega_{\theta_2}^0 &= 0.15, \\ \omega_{\theta_3}^0 &= 0.2, \\ \omega_{\theta_4}^0 &= 0.18, \\ \omega_{\theta_5}^0 &= 0.02. \end{aligned} \quad (45)$$

Reaggregate the calculations to arrive at a new consensus: $ca_1 = 0.7403$, $ca_2 = 0.7612$, $ca_3 = 0.7993$, $ca_4 = 0.7291$, $ca_5 = 0.6717$, and ocd = 0.7403.

Now, ocd < 0.75, and it triggers the feedback mechanism again.

Consensus of each cluster and group is as follows: $g^{\theta_1} = 0.8889$, $g^{\theta_2} = 0.7886$, $g^{\theta_3} = 0.7877$, $g^{\theta_4} = 0.8169$, and $g^{\theta_5} = 0.7449$, where $g^{\theta_1} > g^{\theta_4} > g^{\theta_2} > g^{\theta_3} > g^{\theta_5}$.

As $\theta_5$has been adjusted, adjust the weight of the second smallest cluster θ_4. After the adjusted $\theta_4$experts had negotiated, it was decided to give θ_1 40%$\omega_{\theta_4}^1$weight and give θ_3 40%$\omega_{\theta_4}^1$weight, at which time the cluster weights are as follows:

$$\begin{aligned} \omega_{\theta_1}^0 &= 0.53, \\ \omega_{\theta_2}^0 &= 0.15, \\ \omega_{\theta_3}^0 &= 0.04, \\ \omega_{\theta_4}^0 &= 0.26, \\ \omega_{\theta_5}^0 &= 0.02. \end{aligned} \quad (46)$$

Reaggregate calculations to arrive at a new consensus: $ca_1 = 0.7731$, $ca_2 = 0.8004$, $ca_3 = 0.7963$, $ca_4 = 0.7285$, $ca_5 = 0.6666$, and ocd = 0.7530.

Now, ocd > 0.75does not trigger the feedback mechanism. The weights of the experts obtained by formulas (26) and (27) are as follows:

$$\begin{aligned} \omega_{x_1} &= 0.06625, \\ \omega_{x_2} &= 0.6625, \\ \omega_{x_3} &= 0.05, \\ \omega_{x_4} &= 0.06625, \\ \omega_{x_5} &= 0.08667, \\ \omega_{x_6} &= 0.06625, \\ \omega_{x_7} &= 0.01, \\ \omega_{x_8} &= 0.01, \\ \omega_{x_9} &= 0.01, \\ \omega_{x_{10}} &= 0.06625, \\ \omega_{x_{11}} &= 0.08667, \\ \omega_{x_{12}} &= 0.05, \\ \omega_{x_{13}} &= 0.05, \\ \omega_{x_{14}} &= 0.05, \\ \omega_{x_{15}} &= 0.01, \\ \omega_{x_{16}} &= 0.06625, \\ \omega_{x_{17}} &= 0.06625, \\ \omega_{x_{18}} &= 0.01, \\ \omega_{x_{19}} &= 0.01, \\ \omega_{x_{20}} &= 0.08667. \end{aligned} \quad (47)$$

The net credibility of each scheme is calculated from equations (28)–(30). Finally, the constructed relative closeness degree matrix is used to find the distance and relative closeness degree from equations (31)–(33) as summarized in Table 3.

Table 3: The distance from each scheme to the ideal solution.

Distance	d^+	d^-	$D = (d^- / d^- + d^+)$
$A1$	0.0681	0.1152	0.6284
$A2$	0.0878	0.0873	0.4987
$A3$	0.1349	0.0335	0.1989
$A4$	0.0874	0.0910	0.5103
$A5$	0.0887	0.0945	0.5157

In summary, the program ranks $A1 > A5 > A4 > A2 > A3$.

5. Comparison and Discussion

The probabilistic linguistic term set effectively explains the degree of hesitation and probability distribution of expert opinions in the nonideal case, which can make the linguistic information flexibly expressed. Through combination with TOPSIS, VIKOR [50], and other methods, the method proposed in this article can effectively solve the problem of multiattribute group decision-making based on the probabilistic linguistic term sets.

In group decision-making, there may be situations where the opinions of experts are extremely conflicted, which requires an increase in consensus and consistency. There are currently several ways to increase consensus, including adjusting expert weights or adjusting expert preferences. How to adjust the relationship of expert preferences and the distribution of expert weights is a question worth exploring. In [44], the authors used the clustering algorithm to divide the expert group into three clusters. After clustering, the preference of the adjustment expert was selected. After the expert preference was adjusted cluster by cluster, clustering was rerun. The consensus degree increased from 0.6925 to 0.7861 and then increased to 0.7970; thus, the consensus degree basically met the requirements. In [44], choosing to adjust the preferences of experts and improving the consensus degree through consultation and exchange methods may have problems in group decision-making operations that require multiple consultations. Repeated consultations will inevitably take time and effort, and it is not as convenient to adjust the weight as the method given in this article.

In [45], after the authors used K-means clustering, the expert group was divided into six clusters. In contrast to the method proposed in this article, the method given in [45] directly withdraws the clusters with poor consensus and calculates the weights of these experts to other expert clusters, thus improving the group consensus. If the cluster of experts is directly withdrawn, the opinions of these experts are actually meaningless and have problems. In practice, there may be some opinions of this group of experts which cannot be completely ignored. However, the weight adjustment and improvement made in this paper have a maximum adjustment threshold, which effectively avoids the problem of total loss of the opinions of some experts as seen in [45].

6. Conclusions

This work proposes integrating a new extended outranking-TOPSIS method with probabilistic linguistic term sets for multiattribute group decision-making. First, for the application of probabilistic linguistic term sets in multiattribute group decision-making problems, a new extended outranking relation based on probabilistic linguistic term sets is proposed to determine the superior and inferior relationships between the schemes. Second, according to the expert opinions obtained, the expert consensus improvement mechanism based on clustering improvement is used to determine and improve the consensus degree. Finally, an improved TOPSIS ranking method based on the net credibility and weight adjustment is proposed to rank the schemes. This article also provides an application case of PPP to illustrate the method proposed in this article.

The theory and calculation of the extended outranking relation based on probabilistic linguistic term sets proposed in this work are not complicated, which is convenient for practical application. Furthermore, it solves the problem of ignoring the differences in the degree of hesitation which exists in similar outranking methods. The proposed expert consensus improvement mechanism based on clustering improvement can also effectively respect the opinions of marginal expert groups, respect the concept of group decision-making, and facilitate the development of group decision-making, and it can be effectively applied to various group decision-making application problems. By improving the TOPSIS method, it can be effectively applied to the ranking of schemes based on probabilistic linguistic term sets.

This study requires some improvements. Keeping expert opinions from too much influence of the model is a point that needs to be paid attention to in the consensus study of multiattribute group decision-making models. The K-means clustering method is used in this article, and K in K-means clustering method is set manually as a hyperparameter. Due to the relative subjectivity of manual settings, some expert opinions that should be maintained may be affected. In the future, methods such as grid search will be used to traverse each K to find the situation with the largest initial group decision consensus degree, so as to minimize the extent of expert weight adjustment transfer. It helps to maintain the opinions of the expert group, making the analysis results closer to the initial opinion of the expert group.

At present, there are few studies regarding the application of PLTS in large-scale multiattribute group decision-making problems. In future work, for the combination of PLTS and large-scale multiattribute group decision-making, the following research can be carried out. First, we can study the use of different operators and distance measurement methods to reduce information loss in the large-scale multiattribute group decision-making process. For the study of consensus mechanisms, in addition to clustering, there are other algorithms that can be utilized to improve expert weights and expert preference relationships. In addition, more optimization algorithms or a combination of multiple optimization algorithms, such as other machine learning methods, can be introduced to make large-scale multiattribute group decision-making more comprehensive and more accurate.

Appendix

All the probabilistic linguistic term sets in the illustrative example in this article are listed as follows:

PLTSM_{x_1}

$$\mathrm{PLTSM}_{x_2}=\begin{pmatrix}
\{S_0(0.05),S_1(0.35),S_2(0.15),S_3(0.05),S_4(0.15)\} & \{S_0(0.25),S_1(0.1),S_2(0.05),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.35),S_3(0.2),S_4(0.2)\} & \{S_0(0.2),S_1(0.2),S_2(0.05),S_3(0.05),S_4(0.2)\} & \{S_0(0.05),S_1(0.3),S_2(0.3),S_3(0.15),S_4(0.2)\} \\
\{S_0(0.05),S_1(0.1),S_2(0.25),S_3(0.35),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.1),S_1(0.1),S_2(0.45),S_3(0.15),S_4(0.1)\} & \{S_0(0.05),S_1(0.15),S_2(0.25),S_3(0.2),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.2),S_4(0.35)\} \\
\{S_0(0.1),S_1(0.15),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.2),S_1(0.35),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.3),S_4(0.05)\} & \{S_0(0.05),S_1(0.3),S_2(0.2),S_3(0.35),S_4(0.1)\} & \{S_0(0.25),S_1(0.05),S_2(0.2),S_3(0.25),S_4(0.25)\} \\
\{S_0(0.1),S_1(0.1),S_2(0.1),S_3(0.2),S_4(0.15)\} & \{S_0(0.25),S_1(0.05),S_2(0.15),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.3),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.15),S_1(0.1),S_2(0.3),S_3(0.1),S_4(0.05)\} & \{S_0(0.3),S_1(0.15),S_2(0.05),S_3(0.2),S_4(0.2)\} \\
\{S_0(0.1),S_1(0.3),S_2(0.35),S_3(0.1),S_4(0.1)\} & \{S_0(0.3),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0)\} & \{S_0(0.1),S_1(0.25),S_2(0.1),S_3(0.25),S_4(0.2)\} & \{S_0(0.15),S_1(0.05),S_2(0.15),S_3(0.3),S_4(0.15)\} & \{S_0(0.25),S_1(0.15),S_2(0.05),S_3(0.15),S_4(0.05)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_3}=\begin{pmatrix}
\{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.25),S_4(0.15)\} & \{S_0(0.3),S_1(0.05),S_2(0.1),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.15),S_2(0.2),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.05),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.05),S_2(0.2),S_3(0.25),S_4(0.15)\} \\
\{S_0(0.3),S_1(0.1),S_2(0.25),S_3(0.25),S_4(0.05)\} & \{S_0(0.1),S_1(0.35),S_2(0.1),S_3(0.2),S_4(0.2)\} & \{S_0(0.35),S_1(0.35),S_2(0.1),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.1),S_2(0.2),S_3(0.35),S_4(0.2)\} & \{S_0(0.2),S_1(0.3),S_2(0.05),S_3(0.1),S_4(0.35)\} \\
\{S_0(0.35),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.15),S_4(0.15)\} & \{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.25)\} & \{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.25),S_4(0)\} & \{S_0(0.05),S_1(0.25),S_2(0.2),S_3(0.25),S_4(0.05)\} \\
\{S_0(0.1),S_1(0.15),S_2(0.1),S_3(0.15),S_4(0.05)\} & \{S_0(0.25),S_1(0.3),S_2(0.05),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.05),S_2(0.1),S_3(0.05),S_4(0.05)\} & \{S_0(0.25),S_1(0.2),S_2(0.15),S_3(0.25),S_4(0)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.3)\} \\
\{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.1)\} & \{S_0(0),S_1(0.35),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.35)\} & \{S_0(0.35),S_1(0.3),S_2(0.05),S_3(0),S_4(0)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.1),S_4(0.2)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_4}=\begin{pmatrix}
\{S_0(0.2),S_1(0),S_2(0),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.25),S_2(0.2),S_3(0.1),S_4(0.1)\} & \{S_0(0.15),S_1(0.3),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.1),S_1(0.15),S_2(0.35),S_3(0.35),S_4(0.05)\} & \{S_0(0.25),S_1(0),S_2(0),S_3(0.2),S_4(0.35)\} \\
\{S_0(0.15),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.2)\} & \{S_0(0.15),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.35)\} & \{S_0(0.05),S_1(0.45),S_2(0.15),S_3(0.1),S_4(0.15)\} & \{S_0(0.05),S_1(0.35),S_2(0.3),S_3(0.1),S_4(0.1)\} & \{S_0(0.35),S_1(0.25),S_2(0.15),S_3(0.05),S_4(0.35)\} \\
\{S_0(0.15),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.2)\} & \{S_0(0.3),S_1(0.15),S_2(0.1),S_3(0.05),S_4(0.3)\} & \{S_0(0.3),S_1(0.25),S_2(0.05),S_3(0.1),S_4(0)\} & \{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0.35)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.1)\} \\
\{S_0(0.05),S_1(0.35),S_2(0.1),S_3(0.3),S_4(0.1)\} & \{S_0(0.15),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.05),S_4(0.2)\} & \{S_0(0.05),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.25)\} & \{S_0(0.2),S_1(0.15),S_2(0.05),S_3(0.05),S_4(0.25)\} \\
\{S_0(0.15),S_1(0.05),S_2(0.35),S_3(0.3),S_4(0.05)\} & \{S_0(0.25),S_1(0.25),S_2(0.1),S_3(0.15),S_4(0.1)\} & \{S_0(0.2),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.15)\} & \{S_0(0.3),S_1(0.15),S_2(0.25),S_3(0),S_4(0.1)\} & \{S_0(0.25),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.25)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_5}=\begin{pmatrix}
\{S_0(0.35),S_1(0.1),S_2(0.3),S_3(0.05),S_4(0.15)\} & \{S_0(0.15),S_1(0.3),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0),S_1(0.35),S_2(0.3),S_3(0.25),S_4(0)\} & \{S_0(0.1),S_1(0.15),S_2(0.05),S_3(0.15),S_4(0.1)\} & \{S_0(0.05),S_1(0.2),S_2(0.3),S_3(0.2),S_4(0.1)\} \\
\{S_0(0),S_1(0.15),S_2(0.3),S_3(0.15),S_4(0.2)\} & \{S_0(0.05),S_1(0.05),S_2(0.35),S_3(0.2),S_4(0.05)\} & \{S_0(0.3),S_1(0.05),S_2(0.2),S_3(0.25),S_4(0.2)\} & \{S_0(0.05),S_1(0.35),S_2(0.1),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.25),S_2(0.4),S_3(0.05),S_4(0.15)\} \\
\{S_0(0.2),S_1(0),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.1)\} & \{S_0(0.15),S_1(0.2),S_2(0.1),S_3(0.15),S_4(0.3)\} & \{S_0(0.2),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.25)\} & \{S_0(0.1),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.25)\} \\
\{S_0(0.3),S_1(0.15),S_2(0.25),S_3(0),S_4(0.25)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.35),S_4(0.25)\} & \{S_0(0.05),S_1(0.05),S_2(0.1),S_3(0.35),S_4(0.2)\} & \{S_0(0.15),S_1(0.5),S_2(0.15),S_3(0),S_4(0)\} & \{S_0(0.1),S_1(0.2),S_2(0.3),S_3(0.1),S_4(0.1)\} \\
\{S_0(0),S_1(0.2),S_2(0.2),S_3(0.3),S_4(0.2)\} & \{S_0(0.35),S_1(0.2),S_2(0.2),S_3(0.15),S_4(0.1)\} & \{S_0(0.3),S_1(0.1),S_2(0.1),S_3(0.25),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.1),S_4(0.2)\} & \{S_0(0.2),S_1(0.15),S_2(0.05),S_3(0.25),S_4(0.1)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_6}=\begin{pmatrix}
\{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.35),S_4(0)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.15),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0),S_1(0.05),S_2(0.35),S_3(0.3),S_4(0.2)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.35),S_4(0.25)\} \\
\{S_0(0.35),S_1(0.1),S_2(0.05),S_3(0.25),S_4(0.15)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.1),S_1(0.35),S_2(0.05),S_3(0.25),S_4(0.15)\} & \{S_0(0.25),S_1(0.2),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.15),S_1(0.25),S_2(0.3),S_3(0.1),S_4(0)\} \\
\{S_0(0.15),S_1(0.2),S_2(0.35),S_3(0.3),S_4(0.3)\} & \{S_0(0.25),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.35),S_4(0.1)\} & \{S_0(0.15),S_1(0.15),S_2(0.3),S_3(0.15),S_4(0.15)\} \\
\{S_0(0.25),S_1(0.2),S_2(0.25),S_3(0.25),S_4(0.05)\} & \{S_0(0.3),S_1(0.2),S_2(0.3),S_3(0.05),S_4(0.15)\} & \{S_0(0.05),S_1(0.1),S_2(0.3),S_3(0.35),S_4(0)\} & \{S_0(0.05),S_1(0.1),S_2(0.3),S_3(0.3),S_4(0.1)\} & \{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.3),S_4(0)\} \\
\{S_0(0.35),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.35),S_4(0.15)\} & \{S_0(0.2),S_1(0.05),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.2),S_2(0.05),S_3(0.25),S_4(0.2)\} & \{S_0(0.15),S_1(0.1),S_2(0.05),S_3(0.2),S_4(0.3)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_7}=\begin{pmatrix}
\{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.35),S_4(0.1)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.15),S_4(0.1)\} & \{S_0(0.25),S_1(0.15),S_2(0.05),S_3(0.5),S_4(0.05)\} & \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.25),S_4(0.05)\} & \{S_0(0.3),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.1)\} \\
\{S_0(0.1),S_1(0.05),S_2(0.05),S_3(0.35),S_4(0.3)\} & \{S_0(0.25),S_1(0.2),S_2(0.2),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.2),S_3(0.2),S_4(0.15)\} & \{S_0(0.25),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.2)\} & \{S_0(0.2),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0.2)\} \\
\{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.05),S_1(0.35),S_2(0.45),S_3(0.05),S_4(0.05)\} & \{S_0(0.35),S_1(0.25),S_2(0.2),S_3(0.15),S_4(0)\} & \{S_0(0.35),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.15)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.05),S_4(0.1)\} \\
\{S_0(0.05),S_1(0.35),S_2(0.1),S_3(0.2),S_4(0.1)\} & \{S_0(0.2),S_1(0.1),S_2(0.2),S_3(0.05),S_4(0.15)\} & \{S_0(0.35),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0.1)\} & \{S_0(0.25),S_1(0.15),S_2(0.1),S_3(0.2),S_4(0.05)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.3),S_4(0.05)\} \\
\{S_0(0.2),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.25)\} & \{S_0(0.05),S_1(0.15),S_2(0.2),S_3(0.05),S_4(0.2)\} & \{S_0(0.2),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.2)\} & \{S_0(0.1),S_1(0.2),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.35),S_1(0.1),S_2(0.1),S_3(0.1),S_4(0.15)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_8}=\begin{pmatrix}
\{S_0(0.15),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.3),S_4(0.35)\} & \{S_0(0.05),S_1(0.3),S_2(0.05),S_3(0.15),S_4(0.05)\} & \{S_0(0.3),S_1(0.35),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.35),S_1(0.1),S_2(0.15),S_3(0.15),S_4(0.15)\} \\
\{S_0(0.3),S_1(0.4),S_2(0),S_3(0),S_4(0.2)\} & \{S_0(0.1),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.3)\} & \{S_0(0.25),S_1(0.05),S_2(0.3),S_3(0.15),S_4(0.15)\} & \{S_0(0.1),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.25)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.05),S_4(0.25)\} \\
\{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.05),S_2(0.35),S_3(0.25),S_4(0.1)\} & \{S_0(0.3),S_1(0.05),S_2(0.2),S_3(0.15),S_4(0.25)\} & \{S_0(0.1),S_1(0.2),S_2(0.05),S_3(0.25),S_4(0.15)\} & \{S_0(0.15),S_1(0.2),S_2(0.45),S_3(0.05),S_4(0.15)\} \\
\{S_0(0.25),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.35)\} & \{S_0(0.05),S_1(0.1),S_2(0.05),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.15),S_2(0.1),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.35),S_2(0.05),S_3(0.35),S_4(0.05)\} \\
\{S_0(0.1),S_1(0.15),S_2(0.05),S_3(0.15),S_4(0.05)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.15),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.2)\} & \{S_0(0.3),S_1(0.2),S_2(0.35),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.25)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_9}=\begin{pmatrix}
\{S_0(0.15),S_1(0.1),S_2(0.25),S_3(0.1),S_4(0.2)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.2),S_4(0.35)\} & \{S_0(0.25),S_1(0.05),S_2(0.2),S_3(0.2),S_4(0.2)\} & \{S_0(0.2),S_1(0.15),S_2(0.3),S_3(0.35),S_4(0)\} & \{S_0(0.25),S_1(0.1),S_2(0.05),S_3(0.25),S_4(0.35)\} \\
\{S_0(0.3),S_1(0.05),S_2(0.15),S_3(0.25),S_4(0.25)\} & \{S_0(0.1),S_1(0.15),S_2(0.15),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.25),S_2(0.05),S_3(0.3),S_4(0.05)\} & \{S_0(0),S_1(0.3),S_2(0.3),S_3(0.15),S_4(0.2)\} & \{S_0(0.35),S_1(0.2),S_2(0.15),S_3(0.2),S_4(0.05)\} \\
\{S_0(0.05),S_1(0.2),S_2(0.3),S_3(0.2),S_4(0.1)\} & \{S_0(0),S_1(0),S_2(0.05),S_3(0.5),S_4(0.25)\} & \{S_0(0.3),S_1(0.2),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.15),S_1(0.15),S_2(0.25),S_3(0.25),S_4(0.15)\} & \{S_0(0.2),S_1(0.05),S_2(0.35),S_3(0.1),S_4(0.2)\} \\
\{S_0(0.3),S_1(0.1),S_2(0.2),S_3(0.15),S_4(0.25)\} & \{S_0(0.15),S_1(0.05),S_2(0.15),S_3(0.35),S_4(0.15)\} & \{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.1),S_4(0.25)\} & \{S_0(0.25),S_1(0.35),S_2(0.05),S_3(0.2),S_4(0.15)\} & \{S_0(0.05),S_1(0.2),S_2(0.1),S_3(0.35),S_4(0.1)\} \\
\{S_0(0),S_1(0.35),S_2(0.35),S_3(0.1),S_4(0)\} & \{S_0(0.05),S_1(0.15),S_2(0.2),S_3(0.25),S_4(0.15)\} & \{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.05),S_4(0.3)\} & \{S_0(0.15),S_1(0.3),S_2(0.2),S_3(0.05),S_4(0.1)\} & \{S_0(0.35),S_1(0.1),S_2(0.15),S_3(0.15),S_4(0.25)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{10}}=\begin{pmatrix}
\{S_0(0.1),S_1(0.3),S_2(0.2),S_3(0.25),S_4(0.15)\} & \{S_0(0.05),S_1(0.35),S_2(0.15),S_3(0.2),S_4(0.2)\} & \{S_0(0.05),S_1(0.15),S_2(0.25),S_3(0.1),S_4(0.2)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.1),S_4(0,05)\} & \{S_0(0.35),S_1(0.25),S_2(0.05),S_3(0.3),S_4(0)\} \\
\{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.05),S_4(0.25)\} & \{S_0(0.15),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.35)\} & \{S_0(0.05),S_1(0.25),S_2(0.05),S_3(0.35),S_4(0.05)\} & \{S_0(0.3),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0)\} & \{S_0(0.3),S_1(0.1),S_2(0.2),S_3(0.35),S_4(0.05)\} \\
\{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.15),S_4(0.2)\} & \{S_0(0.25),S_1(0.2),S_2(0.1),S_3(0.35),S_4(0.1)\} & \{S_0(0),S_1(0.05),S_2(0.2),S_3(0.35),S_4(0.15)\} & \{S_0(0.1),S_1(0.1),S_2(0.15),S_3(0.05),S_4(0.25)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.3),S_4(0.1)\} \\
\{S_0(0.2),S_1(0.1),S_2(0.2),S_3(0.1),S_4(0.3)\} & \{S_0(0.05),S_1(0.15),S_2(0.1),S_3(0.15),S_4(0.3)\} & \{S_0(0.1),S_1(0.1),S_2(0.15),S_3(0.4),S_4(0.1)\} & \{S_0(0.15),S_1(0.2),S_2(0.05),S_3(0.5),S_4(0.05)\} & \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.15),S_4(0.2)\} \\
\{S_0(0.2),S_1(0.2),S_2(0.15),S_3(0.1),S_4(0.15)\} & \{S_0(0.25),S_1(0.2),S_2(0),S_3(0.15),S_4(0.15)\} & \{S_0(0.15),S_1(0.2),S_2(0.3),S_3(0.15),S_4(0.1)\} & \{S_0(0.1),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.35),S_2(0.15),S_3(0.15),S_4(0.15)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{11}}=\begin{pmatrix}
\{S_0(0.1),S_1(0.05),S_2(0.25),S_3(0.3),S_4(0.25)\} & \{S_0(0.15),S_1(0.15),S_2(0.05),S_3(0.25),S_4(0.3)\} & \{S_0(0.2),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.15)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.25),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.2),S_3(0.25),S_4(0.15)\} \\
\{S_0(0.1),S_1(0.2),S_2(0.3),S_3(0.35),S_4(0.05)\} & \{S_0(0.3),S_1(0.1),S_2(0.05),S_3(0.05),S_4(0.25)\} & \{S_0(0.15),S_1(0.35),S_2(0.1),S_3(0.05),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.1)\} & \{S_0(0.1),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.1)\} \\
\{S_0(0.3),S_1(0.05),S_2(0.35),S_3(0.15),S_4(0.1)\} & \{S_0(0.3),S_1(0.1),S_2(0.35),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.25),S_2(0.3),S_3(0.1),S_4(0.25)\} & \{S_0(0.3),S_1(0.05),S_2(0.3),S_3(0.2),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.1),S_4(0.05)\} \\
\{S_0(0.2),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.1)\} & \{S_0(0.3),S_1(0.45),S_2(0.05),S_3(0),S_4(0)\} & \{S_0(0.1),S_1(0.1),S_2(0.1),S_3(0.35),S_4(0.05)\} & \{S_0(0.05),S_1(0.3),S_2(0.35),S_3(0.1),S_4(0.2)\} & \{S_0(0.15),S_1(0.15),S_2(0.2),S_3(0.15),S_4(0.35)\} \\
\{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.2),S_4(0.1)\} & \{S_0(0.25),S_1(0.25),S_2(0.3),S_3(0.1),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0.3),S_3(0.1),S_4(0)\} & \{S_0(0.1),S_1(0.05),S_2(0.05),S_3(0.2),S_4(0.1)\} & \{S_0(0.05),S_1(0.3),S_2(0.1),S_3(0.25),S_4(0.25)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{12}}=\begin{pmatrix}
\{S_0(0),S_1(0.05),S_2(0.35),S_3(0.35),S_4(0)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.2),S_1(0.1),S_2(0.3),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.1),S_2(0.35),S_3(0.2),S_4(0.1)\} & \{S_0(0.15),S_1(0.15),S_2(0.3),S_3(0.15),S_4(0.2)\} \\
\{S_0(0.25),S_1(0.25),S_2(0.3),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.15),S_2(0.15),S_3(0.1),S_4(0.1)\} & \{S_0(0.2),S_1(0.35),S_2(0.15),S_3(0.2),S_4(0)\} & \{S_0(0.3),S_1(0),S_2(0.05),S_3(0.35),S_4(0.15)\} & \{S_0(0),S_1(0.3),S_2(0.3),S_3(0.3),S_4(0)\} \\
\{S_0(0.1),S_1(0.2),S_2(0.15),S_3(0.35),S_4(0.15)\} & \{S_0(0.2),S_1(0.3),S_2(0.35),S_3(0),S_4(0.1)\} & \{S_0(0.1),S_1(0.3),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.35),S_1(0.35),S_2(0.15),S_3(0.15),S_4(0)\} & \{S_0(0.05),S_1(0.35),S_2(0.2),S_3(0.1),S_4(0.3)\} \\
\{S_0(0.1),S_1(0.2),S_2(0.05),S_3(0.35),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0.05),S_3(0.3),S_4(0.15)\} & \{S_0(0.35),S_1(0.2),S_2(0.1),S_3(0.1),S_4(0.15)\} & \{S_0(0.1),S_1(0.2),S_2(0.3),S_3(0.15),S_4(0.15)\} & \{S_0(0),S_1(0.25),S_2(0.35),S_3(0.35),S_4(0)\} \\
\{S_0(0.35),S_1(0.2),S_2(0),S_3(0.3),S_4(0)\} & \{S_0(0.25),S_1(0.25),S_2(0.05),S_3(0.25),S_4(0.1)\} & \{S_0(0.05),S_1(0.15),S_2(0.15),S_3(0.35),S_4(0.3)\} & \{S_0(0.15),S_1(0.05),S_2(0.35),S_3(0.15),S_4(0)\} & \{S_0(0.05),S_1(0.3),S_2(0.35),S_3(0.05),S_4(0.05)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{13}}=\begin{pmatrix}
\{S_0(0.1),S_1(0.05),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.15),S_1(0.35),S_2(0.2),S_3(0),S_4(0.15)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.3),S_1(0.5),S_2(0.15),S_3(0.2),S_4(0.1)\} & \{S_0(0.1),S_1(0.35),S_2(0.25),S_3(0.05),S_4(0.1)\} \\
\{S_0(0.05),S_1(0.35),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.25),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.2)\} & \{S_0(0.25),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.05)\} & \{S_0(0.1),S_1(0.25),S_2(0.5),S_3(0),S_4(0.5)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.2),S_4(0.35)\} \\
\{S_0(0.15),S_1(0.45),S_2(0.2),S_3(0.15),S_4(0)\} & \{S_0(0.15),S_1(0.15),S_2(0.2),S_3(0.1),S_4(0.35)\} & \{S_0(0.3),S_1(0.05),S_2(0.35),S_3(0.05),S_4(0.1)\} & \{S_0(0.25),S_1(0.05),S_2(0),S_3(0.3),S_4(0.1)\} & \{S_0(0.2),S_1(0.2),S_2(0.15),S_3(0.05),S_4(0.25)\} \\
\{S_0(0.3),S_1(0.2),S_2(0.1),S_3(0.35),S_4(0.05)\} & \{S_0(0.05),S_1(0.1),S_2(0.25),S_3(0.2),S_4(0.3)\} & \{S_0(0.1),S_1(0.35),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.35),S_1(0.1),S_2(0.2),S_3(0.1),S_4(0.15)\} & \{S_0(0.3),S_1(0.15),S_2(0.2),S_3(0.25),S_4(0.05)\} \\
\{S_0(0.1),S_1(0.2),S_2(0.35),S_3(0.2),S_4(0.1)\} & \{S_0(0),S_1(0),S_2(0.35),S_3(0.5),S_4(0.05)\} & \{S_0(0.05),S_1(0.15),S_2(0.35),S_3(3),S_4(0.05)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.35)\}
\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{14}}=\begin{pmatrix}\{S_0(0.05),S_1(0.25),S_2(0.2),S_3(0.1),S_4(0.3)\} & \{S_0(0.25),S_1(0.05),S_2(0.15),S_3(0.25),S_4(0.3)\} & \{S_0(0.15),S_1(0),S_2(0),S_3(0.35),S_4(0.25)\} & \{S_0(0.15),S_1(0),S_2(0),S_3(0.35),S_4(0.2)\} & \{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.1),S_4(0.25)\}\\ \{S_0(0.1),S_1(0.3),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.3),S_4(0.15)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.1),S_1(0.3),S_2(0.15),S_3(0.1),S_4(0.3)\}\\ \{S_0(0.05),S_1(0.25),S_2(0),S_3(0.45),S_4(0.2)\} & \{S_0(0.15),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.05),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.25)\}\\ \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.05),S_1(0.05),S_2(0.45),S_3(0.15),S_4(0.1)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.25),S_2(0.05),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.05),S_2(0.2),S_3(0.05),S_4(0.2)\}\\ \{S_0(0.3),S_1(0.1),S_2(0.05),S_3(0.25),S_4(0.05)\} & \{S_0(0.2),S_1(0.05),S_2(0.25),S_3(0),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.1),S_1(0),S_2(0.2),S_3(0.35),S_4(0)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{15}}=\begin{pmatrix}\{S_0(0.15),S_1(0.35),S_2(0.05),S_3(0.15),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.25),S_1(0.1),S_2(0.1),S_3(0.35),S_4(0)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.15),S_4(0.2)\} & \{S_0(0.05),S_1(0.2),S_2(0.15),S_3(0.05),S_4(0.05)\}\\ \{S_0(0.2),S_1(0.1),S_2(0.25),S_3(0.15),S_4(0.15)\} & \{S_0(0.15),S_1(0.1),S_2(0.25),S_3(0.15),S_4(0.1)\} & \{S_0(0.05),S_1(0.25),S_2(0.3),S_3(0.15),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0),S_3(0.15),S_4(0.35)\} & \{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.15),S_4(0.2)\}\\ \{S_0(0.25),S_1(0.25),S_2(0.1),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.05),S_1(0.2),S_2(0.25),S_3(0.1),S_4(0.35)\} & \{S_0(0.3),S_1(0.05),S_2(0.15),S_3(0.2),S_4(0.3)\} & \{S_0(0.1),S_1(0.2),S_2(0.2),S_3(0.05),S_4(0.25)\}\\ \{S_0(0.05),S_1(0.25),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.15),S_4(0.2)\} & \{S_0(0.15),S_1(0.05),S_2(0.5),S_3(0.1),S_4(0)\} & \{S_0(0.15),S_1(0.25),S_2(0.45),S_3(0.05),S_4(0.1)\} & \{S_0(0.15),S_1(0.15),S_2(0.1),S_3(0.05),S_4(0.15)\}\\ \{S_0(0.2),S_1(0.2),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.25),S_1(0.2),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0.15),S_1(0.1),S_2(0.3),S_3(0.35),S_4(0.1)\} & \{S_0(0.25),S_1(0.35),S_2(0.25),S_3(0.05),S_4(0.1)\} & \{S_0(0.2),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0.15)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{16}}=\begin{pmatrix}\{S_0(0.1),S_1(0.1),S_2(0.35),S_3(0.25),S_4(0.1)\} & \{S_0(0.15),S_1(0.3),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0.3),S_1(0.1),S_2(0.1),S_3(0.2),S_4(0.1)\} & \{S_0(0),S_1(0.35),S_2(0.3),S_3(0.1),S_4(0.25)\} & \{S_0(0.3),S_1(0.05),S_2(0.05),S_3(0.35),S_4(0.15)\}\\ \{S_0(0.15),S_1(0.25),S_2(0.25),S_3(0.05),S_4(0.2)\} & \{S_0(0.25),S_1(0.15),S_2(0.1),S_3(0.1),S_4(0.2)\} & \{S_0(0.35),S_1(0.2),S_2(0.05),S_3(0.35),S_4(0)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.2),S_4(0.35)\} & \{S_0(0.1),S_1(0.05),S_2(0.1),S_3(0.25),S_4(0.05)\}\\ \{S_0(0.3),S_1(0.05),S_2(0.05),S_3(0.35),S_4(0.1)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.25),S_4(0.3)\} & \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.2),S_4(0.25)\} & \{S_0(0.3),S_1(0.35),S_2(0.2),S_3(0.15),S_4(0)\} & \{S_0(0.25),S_1(0.15),S_2(0.2),S_3(0.2),S_4(0.15)\}\\ \{S_0(0.3),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.25),S_2(0.2),S_3(0.25),S_4(0.2)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.05),S_4(0.1)\} & \{S_0(0.1),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.25)\} & \{S_0(0.2),S_1(0.25),S_2(0.1),S_3(0.05),S_4(0.35)\}\\ \{S_0(0.25),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.2)\} & \{S_0(0.3),S_1(0.15),S_2(0.35),S_3(0.1),S_4(0.1)\} & \{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.1),S_4(0.1)\} & \{S_0(0.1),S_1(0.25),S_2(0.35),S_3(0.15),S_4(0.05)\} & \{S_0(0.05),S_1(0.25),S_2(0.3),S_3(0.3),S_4(0.05)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{17}}=\begin{pmatrix}\{S_0(0),S_1(0.25),S_2(0.15),S_3(0.35),S_4(0.1)\} & \{S_0(0.1),S_1(0.25),S_2(0.15),S_3(0.2),S_4(0.15)\} & \{S_0(0.25),S_1(0.15),S_2(0.05),S_3(0.05),S_4(0.2)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.1),S_4(0.35)\} & \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.2),S_4(0)\}\\ \{S_0(0.05),S_1(0.1),S_2(0.35),S_3(0.2),S_4(0)\} & \{S_0(0.2),S_1(0.05),S_2(0.15),S_3(0.15),S_4(0.35)\} & \{S_0(0.3),S_1(0.05),S_2(0.2),S_3(0.1),S_4(0.25)\} & \{S_0(0.35),S_1(0.1),S_2(0.15),S_3(0.15),S_4(0)\} & \{S_0(0.15),S_1(0.1),S_2(0.15),S_3(0.1),S_4(0)\}\\ \{S_0(0.15),S_1(0.1),S_2(0.35),S_3(0.05),S_4(0.25)\} & \{S_0(0.05),S_1(0.2),S_2(0.25),S_3(0.3),S_4(0.15)\} & \{S_0(0.05),S_1(0.05),S_2(0.25),S_3(0.3),S_4(0.35)\} & \{S_0(0.1),S_1(0.25),S_2(0.2),S_3(0.25),S_4(0.1)\} & \{S_0(0.35),S_1(0.3),S_2(0.05),S_3(0.1),S_4(0.2)\}\\ \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.05),S_4(0)\} & \{S_0(0.25),S_1(0.15),S_2(0.25),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.3),S_2(0.05),S_3(0.15),S_4(0.35)\} & \{S_0(0.15),S_1(0.2),S_2(0.25),S_3(0.25),S_4(0.1)\} & \{S_0(0.2),S_1(0.15),S_2(0.3),S_3(0.25),S_4(0.05)\}\\ \{S_0(0.15),S_1(0.2),S_2(0.05),S_3(0.3),S_4(0.05)\} & \{S_0(0.1),S_1(0.3),S_2(0.3),S_3(0.25),S_4(0)\} & \{S_0(0.15),S_1(0.05),S_2(0.2),S_3(0.15),S_4(0.15)\} & \{S_0(0),S_1(0.1),S_2(0.35),S_3(0.35),S_4(0.1)\} & \{S_0(0.15),S_1(0.05),S_2(0.1),S_3(0.15),S_4(0.3)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{18}}=\begin{pmatrix}\{S_0(0.05),S_1(0.05),S_2(0.1),S_3(0.35),S_4(0.3)\} & \{S_0(0.35),S_1(0.05),S_2(0.35),S_3(0.05),S_4(0)\} & \{S_0(0.2),S_1(0.35),S_2(0.3),S_3(0),S_4(0)\} & \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.05),S_4(0.1)\} & \{S_0(0.35),S_1(0.05),S_2(0.25),S_3(0.1),S_4(0.2)\}\\ \{S_0(0.25),S_1(0.25),S_2(0.1),S_3(0.3),S_4(0)\} & \{S_0(0.2),S_1(0.1),S_2(0.05),S_3(0.3),S_4(0.2)\} & \{S_0(0.15),S_1(0.35),S_2(0.15),S_3(0.3),S_4(0.05)\} & \{S_0(0.2),S_1(0.05),S_2(0.1),S_3(0.3),S_4(0.1)\} & \{S_0(0.25),S_1(0.05),S_2(0.1),S_3(0.05),S_4(0.3)\}\\ \{S_0(0.1),S_1(0.2),S_2(0.25),S_3(0.2),S_4(0.15)\} & \{S_0(0.1),S_1(0.2),S_2(0.1),S_3(0.1),S_4(0.05)\} & \{S_0(0.05),S_1(0.2),S_2(0.05),S_3(0.3),S_4(0.05)\} & \{S_0(0.25),S_1(0.05),S_2(0.15),S_3(0.1),S_4(0.2)\} & \{S_0(0.05),S_1(0.25),S_2(0.3),S_3(0.15),S_4(0.1)\}\\ \{S_0(0.1),S_1(0.1),S_2(0.25),S_3(0.2),S_4(0.1)\} & \{S_0(0.2),S_1(0.35),S_2(0.15),S_3(0.05),S_4(0.2)\} & \{S_0(0.3),S_1(0.15),S_2(0.2),S_3(0.1),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.3),S_3(0.05),S_4(0.35)\} & \{S_0(0),S_1(0.1),S_2(0.1),S_3(0.35),S_4(0.3)\}\\ \{S_0(0.25),S_1(0.3),S_2(0.2),S_3(0.25),S_4(0)\} & \{S_0(0.05),S_1(0.1),S_2(0.1),S_3(0.15),S_4(0.25)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.1),S_4(0.1)\} & \{S_0(0.1),S_1(0.35),S_2(0.2),S_3(0.2),S_4(0.05)\} & \{S_0(0.2),S_1(0.3),S_2(0.45),S_3(0.05),S_4(0)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{19}}=\begin{pmatrix}\{S_0(0.35),S_1(0.05),S_2(0.05),S_3(0.1),S_4(0.3)\} & \{S_0(0.35),S_1(0.35),S_2(0.05),S_3(0.15),S_4(0.05)\} & \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.25),S_4(0.15)\} & \{S_0(0.15),S_1(0.3),S_2(0.15),S_3(0.3),S_4(0.2)\} & \{S_0(0.1),S_1(0),S_2(0.5),S_3(0.3),S_4(0.05)\}\\ \{S_0(0.15),S_1(0.15),S_2(0.3),S_3(0.3),S_4(0)\} & \{S_0(0.2),S_1(0.3),S_2(0.25),S_3(0.1),S_4(0.15)\} & \{S_0(0.1),S_1(0.1),S_2(0.05),S_3(0.2),S_4(0.3)\} & \{S_0(0.3),S_1(0.25),S_2(0.05),S_3(0.15),S_4(0.2)\} & \{S_0(0.25),S_1(0.15),S_2(0.15),S_3(0.15),S_4(0.05)\}\\ \{S_0(0.3),S_1(0.1),S_2(0.1),S_3(0.1),S_4(0.15)\} & \{S_0(0),S_1(0.3),S_2(0.35),S_3(0.25),S_4(0)\} & \{S_0(0.05),S_1(0.2),S_2(0.35),S_3(0.3),S_4(0.05)\} & \{S_0(0.25),S_1(0.2),S_2(0.3),S_3(0.05),S_4(0.25)\} & \{S_0(0.05),S_1(0.2),S_2(0.3),S_3(0.25),S_4(0.2)\}\\ \{S_0(0.15),S_1(0.05),S_2(0.3),S_3(0.25),S_4(0.1)\} & \{S_0(0.2),S_1(0.05),S_2(0.25),S_3(0.2),S_4(0.2)\} & \{S_0(0.15),S_1(0.05),S_2(0.25),S_3(0.05),S_4(0.3)\} & \{S_0(0.05),S_1(0.2),S_2(0.15),S_3(0.15),S_4(0.1)\} & \{S_0(0.35),S_1(0.35),S_2(0.1),S_3(0),S_4(0)\}\\ \{S_0(0.1),S_1(0.1),S_2(0.3),S_3(0.05),S_4(0.05)\} & \{S_0(0.2),S_1(0.15),S_2(0.15),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.35),S_2(0.25),S_3(0.1),S_4(0.1)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.1),S_4(0.25)\} & \{S_0(0.1),S_1(0.25),S_2(0.2),S_3(0.2),S_4(0.05)\}\end{pmatrix},$$

$$\mathrm{PLTSM}_{x_{20}}=\begin{pmatrix}\{S_0(0.1),S_1(0.1),S_2(0.3),S_3(0.05),S_4(0.25)\} & \{S_0(0.1),S_1(0.05),S_2(0.3),S_3(0.05),S_4(0.35)\} & \{S_0(0.1),S_1(0.3),S_2(0.15),S_3(0.25),S_4(0.15)\} & \{S_0(0.25),S_1(0.25),S_2(0.25),S_3(0.15),S_4(0)\} & \{S_0(0.2),S_1(0.1),S_2(0.15),S_3(0.25),S_4(0)\}\\ \{S_0(0),S_1(0.2),S_2(0.5),S_3(0.2),S_4(0)\} & \{S_0(0.05),S_1(0.25),S_2(0.15),S_3(0.05),S_4(0.35)\} & \{S_0(0.15),S_1(0.25),S_2(0.25),S_3(0.05),S_4(0.1)\} & \{S_0(0.05),S_1(0.05),S_2(0.15),S_3(0.2),S_4(0.05)\} & \{S_0(0.15),S_1(0.05),S_2(0.15),S_3(0.1),S_4(0.15)\}\\ \{S_0(0.1),S_1(0.35),S_2(0.15),S_3(0.1),S_4(0.05)\} & \{S_0(0.1),S_1(0.2),S_2(0.2),S_3(0.2),S_4(0.25)\} & \{S_0(0.1),S_1(0.35),S_2(0.05),S_3(0.1),S_4(0.35)\} & \{S_0(0.1),S_1(0.05),S_2(0.15),S_3(0.3),S_4(0.3)\} & \{S_0(0.1),S_1(0.1),S_2(0.1),S_3(0.2),S_4(0.25)\}\\ \{S_0(0.3),S_1(0.1),S_2(0.25),S_3(0.25),S_4(0.05)\} & \{S_0(0.25),S_1(0.35),S_2(0.15),S_3(0.05),S_4(0.15)\} & \{S_0(0.1),S_1(0.25),S_2(0.2),S_3(0.15),S_4(0.25)\} & \{S_0(0.15),S_1(0.25),S_2(0.1),S_3(0.15),S_4(0.2)\} & \{S_0(0.2),S_1(0.35),S_2(0.05),S_3(0.05),S_4(0.35)\}\\ \{S_0(0.25),S_1(0.25),S_2(0.35),S_3(0.1),S_4(0)\} & \{S_0(0.05),S_1(0.1),S_2(0.15),S_3(0.05),S_4(0.3)\} & \{S_0(0.15),S_1(0.15),S_2(0.15),S_3(0.25),S_4(0.25)\} & \{S_0(0.15),S_1(0.15),S_2(0.2),S_3(0.25),S_4(0.05)\} & \{S_0(0.2),S_1(0.3),S_2(0.1),S_3(0.35),S_4(0.05)\}\end{pmatrix}. \tag{A.1}$$

Data Availability

All of the data used to support the study have been included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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