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Research Article

Disturbance Observer-Based Robust Formation-Containment of Discrete-Time Multiagent Systems with Exogenous Disturbances

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This paper investigates robust formation-containment control of discrete-time multiagent systems (MASs) with exogenous disturbances. Based on the discrete-time disturbance observer method, both state feedback and output feedback control protocols are proposed. Formation-containment conditions are obtained and convergency analysis is given according to Lyapunov stability theory. And, the corresponding control gains are obtained by solving some discrete-time algebraic Riccati equations. Numerical simulations are presented to illustrate the theoretical findings.

1. Introduction

Recently, the distributed cooperative control of the MASs has drawn much attention from various disciplines. In distributed cooperative control issues, consensus plays a significant role, which means that the final states of all agents can reach a common value. Due to its widespread applications, many works about consensus have been reported in the past decades [1–6]. Relative research studies refer to synchronization [7, 8], flocking [9], formation [10, 11], and so on.

In recent years, as an important extension of consensus, containment of MASs has been intensively studied for its wide applications in real world. Containment means that there exist multiple leaders in a network, and all followers can asymptotically enter into the convex hull spanned by the leaders. Li et al. [12, 13] addressed the distributed containment control of MASs with general linear dynamics. Ma and Miao [14] proposed the distributed dynamic output feedback control law by using relative output information of neighboring agents. Containment of second-order MASs was studied by using sampled-data position under continuous communication and intermittent communication topology, respectively [15, 16]. Containment was investigated for discrete-time linear MASs with input saturation and intermittent communication [17].

All the works mentioned above do not take the formation of the leaders into account. However, in some real networks, there may be information exchange between the leaders and the leaders will form a special formation, such as robot football. Therefore, formation-containment as a new research topic has attracted much attention, which combines the property of containment control and formation control. Han et al. [18] addressed formation-containment control of second-order dynamics MASs with time-varying. Formation-containment control protocol of high-order linear systems with time-delayed and time-invariant was analysed and designed [19]. In [20], formation-containment control of continuous-time nonlinear Euler–Lagrange MASs with input saturation was studied. Zuo et al. and Wang et al. [21–23] investigated the out formation-containment control, that is, the outputs of all followers converge to the convex hull spanned by the outputs of all leaders, and the outputs of all leaders can maintain a formation structure. Zuo et al. [21] proposed the distributed static and dynamic output feedback control protocols for homogeneous and heterogeneous MASs with time-varying. Wang et al. [22, 23] used the intermittent control and impulsive control to achieve the output formation-containment control of heterogeneous MASs, respectively.

However, the above papers mainly investigated formation-containment control problem for continuous-time MASs without disturbances. Disturbances often exist and destroy the performance of the controlled systems. Therefore, it is very important and meaningful to research the coordination of MASs with exogenous disturbances. In [24], the finite-time leaderless consensus of double-integrator MASs under the fixed network topology with external bounded disturbances was investigated. The leader-following consensus of first-order and second-order nonlinear MASs with unknown bounded external disturbances were discussed [25]. In [26], identical and nonidentical external disturbances were investigated for the leader-following output consensus of discrete-time linear MASs with input saturation. Containment of continuous-time MASs with exogenous disturbances was investigated in virtue of disturbance observer (DO) technique [27]. Du and Li [28] employed the event-triggered control approach to deal with the robust stabilization problem of delayed systems with parameter uncertainties and exogenous disturbances.

Inspired by the above literatures, this paper concentrates on the formation-containment control of high-order discrete-time MASs with exogenous disturbances. The main contributions are as follows:

(i) Discrete-time MASs is discussed in this paper, which can be used to model more plants under computer control technology. And, the stability analysis is more challenging than the continuous-time MASs.

(ii) Formation-containment of MASs is investigated in this paper. The formation of the leader is considered, which has more applications than ordinary containment problems.

(iii) Exogenous disturbances are considered, and a discrete-time disturbance observer method is proposed for attenuating the disturbances.

The rest of the paper is organized as follows. Section 2 states the model considered in the paper and gives some basic definitions, lemmas, and assumptions. In Section 3, discrete-time DO-based state feedback formation-containment protocol is proposed. In Section 4, discrete-time DO-based output feedback formation-containment protocol is offered. Numerical examples are included to demonstrate the proposed protocols in Section 5. Finally, Section 6 gives a conclusion for this paper.

Notation: R denotes the set of real numbers. $R^{N\times M}$, I_N, and 1_N represent the set of $N\times M$ real matrices, $N\times N$ identity matrix, and the N-dimension column vector that all the elements are 1, respectively. A^T (or x^T) represents the transpose of the matrix A (or the vector x). $\|*\|$ represents the Euclid norm of $*$. $\otimes$ denotes the Kronecker product.

2. Preliminaries

A graph $\mathscr{G}=(V,E,A)$ represents a network topology, which includes a set of nodes $\mathscr{V}=\{1,2,\ldots,N+M\}$, a set of edges $\mathscr{E}\subseteq V\times V$, and an adjacent matrix $\mathscr{A}=[a_{ij}]$. For a directed graph, $a_{ij}>0\Leftrightarrow(j,i)\in\mathscr{E}$, that is, j sends information to i. G is an undirected graph if $a_{ij}=a_{ji}$. $\mathscr{N}_i=\{j\mid(j,i)\in\mathscr{E}\}$ is a neighbor set of the ith node. $L=D-\mathscr{A}=[l_{ij}]$ is the Laplacian matrix, where $D=\operatorname{diag}(\sum_{j=1,j\neq i}^{N+M}a_{ij})$. Therefore, $l_{ii}=\sum_{j=1,j\neq i}^{N+M}a_{ij}$ and $l_{ij}=-a_{ij}, i\neq j$.

$\mathscr{F}=\{1,2,\ldots,N\}$ and $\mathscr{L}=\{N+1,N+2,\ldots,N+M\}$ represent a set of followers and leaders, respectively. Suppose that the edges among followers and among leaders are undirected, and edges between leaders and followers are directed. Thus, L can be rewritten as

$$\begin{bmatrix} L_1 & L_2 \\ 0 & L_3 \end{bmatrix}. \tag{1}$$

The dynamics of the ith agent are described by

$$\begin{aligned} x_i(k+1) &= Ax_i(k)+B\left(u_i(k)+d_i(k)\right), \\ y_i(k) &= Cx_i(k), \quad i\in\mathscr{V}, \end{aligned} \tag{2}$$

where $x_i\in R^n$, $u_i\in R^m$, $d_i\in R^m$, and $y_i\in R^p$ denote the state, the control input, the exogenous disturbance, and the measurement output of the ith agent, respectively. $A\in R^{n\times n}$, $B\in R^{n\times m}$, and $C\in R^{p\times n}$ are the constant matrices.

It is assumed that the disturbance $\omega_i(k)$, $i=1,\ldots,N+M$ are generated by the following exogenous system:

$$\begin{aligned} \omega_i(k+1) &= S\omega_i(k), \\ d_i(k) &= F\omega_i(k), \quad i\in\mathscr{V}, \end{aligned} \tag{3}$$

where $\omega_i\in R^l$ is the disturbance state of the ith agent of the exogenous system and $S\in R^{l\times l}$ and $F\in R^{m\times l}$ are the constant matrices of the exogenous system.

Suppose there is a virtual leader in the network, whose dynamics are described as

$$\begin{aligned} x_0(k+1) &= Ax_0(k), \\ y_0(k) &= Cx_0(k), \end{aligned} \tag{4}$$

where $x_0\in R^n$ and $y_0\in R^p$ are the state and the measured output of the virtual leader, respectively. Then, one has $h_i(k+1)=Ah_i(k)$, $i\in\mathscr{L}$, where $h_i\in R^n$ is the desired relative position between the leader i and the virtual leader. Let $h_l(k)=(h_{N+1}^T(k),h_{N+2}^T(k),\ldots,h_{N+M}^T(k)^T)$; then, it follows $h_l(k+1)=(I_M\otimes A)h_l(k)$.

The following definitions, assumptions, and lemmas are necessary for the main results of this paper.

Definition 1 (see [29]). Let $\mathscr{C}$ be a subset of R^n, the set $\mathscr{C}$ is said to be convex if any x and y in $\mathscr{C}$ and any $\alpha\in[0,1]$, and the point $(1-\alpha)x+\alpha y\in\mathscr{C}$. The convex hull of a set of points $X=\{x_1,x_2,\ldots,x_n\}$ is the minimal convex set containing all points in X. We denote the convex hull of X as $\operatorname{Co}(X)$.

Definition 2 (see [23]). The linear MASs (2) is said to achieve the formation-containment if, for any given initial states $x_i(0)$, $i \in \mathscr{V}$, there exist $\lim_{k\longrightarrow\infty}\|x_i(k)- h_i(k)- x_0(k)\| = 0$, $i \in \mathscr{L}$ and $\lim_{k\longrightarrow\infty} \inf_{x_r \in Co(X)} \|x_i(k) - x_r\| = 0$, $i \in \mathscr{F}$.

Assumption 1. Suppose that the edges among the followers are undirected, i.e., all the followers can access each other's information. Moreover, for each follower, there exists at least one leader that has a directed path to that follower.

Assumption 2. The matrix pair (A, B) is stabilizable.

Assumption 3. The matrix pair (A, C) is detectable.

Lemma 1 (see [30]). *Under Assumption 2, there exists a unique positive defined matrix P, satisfying the modified algebraic Riccati equation:*

$$A^T \mathrm{PA} - P - A^T \mathrm{PB}\left(B^T \mathrm{PB} + I\right)^{-1} B^T \mathrm{PA} + I = 0. \tag{5}$$

Lemma 2 (see [12]). *Under Assumption 1, L_1 is positive definite, each entry of $-L_1^{-1}L_2$ is nonnegative, and each row of $-L_1^{-1}L_2$ has a sum equal to one.*

3. DO-Based State Feedback of Formation-Containment

In this section, DO-based distributed state feedback protocol of the formation-containment is proposed and the formation-containment criteria are derived. The dynamics of discrete-time DO based on the state are proposed as follows:

$$\begin{aligned}
v_i(k+1) &= (S + \mathrm{HBF})\left(v_i(k) - Hx_i(k)\right) + H\left(Ax_i(k) + Bu_i(k)\right),\\
\widehat{\omega}_i(k) &= v_i(k) - Hx_i(k),\\
\widehat{d}_i(k) &= F\widehat{\omega}_i(k), \quad i \in \mathscr{V},
\end{aligned} \tag{6}$$

where $v_i \in R^l$ is the internal state variable of the observer, $\widehat{d}_i$ and $\widehat{\omega}_i$ are the estimated value of d_i and ω_i, respectively, and $H \in R^{l\times n}$ is the gain matrix of the observer.

Remark 1. The disturbance is assumed to be generated by a exogenous system, which leads to that the agents in the network cannot get the information of the disturbances. For attenuating the disturbance, all agents have to estimate the value of the exogenous disturbances only using the relative state or relative output information.

According to (2) and (6), one has

$$\widehat{\omega}_i(k+1) = (S + \mathrm{HBF})\widehat{\omega}_i(k) - \mathrm{HBF}\omega_i(k). \tag{7}$$

Then, denote $\delta_i(k) = \omega_i(k) - \widehat{\omega}_i(k)$, and from (3) and (7), one has

$$\delta_i(k+1) = (S + \mathrm{HBF})\delta_i(k). \tag{8}$$

Let $\delta_f(k) = \delta_1^T(k), \delta_2^T(k), \ldots, \delta_N^T(k)^T$ and $\delta_l(k) = \delta_{N+1}^T(k), \delta_{N+2}^T(k), \ldots, \delta_{N+M}^T(k)^T$; then, (8) can be rewritten as the following form:

$$\begin{aligned}
\delta_f(k+1) &= \left(I_N \otimes (S + \mathrm{HBF})\right)\delta_f(k),\\
\delta_l(k+1) &= \left(I_M \otimes (S + \mathrm{HBF})\right)\delta_l(k).
\end{aligned} \tag{9}$$

The distributed DO-based state feedback formation-containment control protocol is proposed as

$$\begin{aligned}
u_i(k) &= K_1\left(\sum_{j\in N_i} a_{ij}\left(x_i(k) - x_j(k)\right)\right) - \widehat{d}_i(k), \quad i \in \mathscr{F},\\
u_i(k) &= K_2\left\{\sum_{j\in N_i}\left(a_{ij}\left(x_i(k) - h_i(k) - \left(x_j(k) - h_j(k)\right)\right) + g_i\left(x_i(k) - h_i(k) - x_0(k)\right)\right) - \widehat{d}_i(k), \quad i \in \mathscr{L},\right.
\end{aligned} \tag{10}$$

where K_1 and K_2 are the gain matrices to be designed and g_i represents the information interaction between the leader i and the virtual leader, where $g_i = 1$ if the virtual leader can send information to the leader i, otherwise $g_i = 0$.

Remark 2. The formation-containment protocol depends on the estimated value of d_i and the local relative information, which consists of two parts, the local relative information and the estimated value $\widehat{d}_i$.

Substituting control protocol (10) into system (2), one has

$$
\begin{aligned}
x_i(k+1) &= Ax_i + BK_1\left(\sum_{j\in N_i} a_{ij}\left(x_i(k) - x_j(k)\right)\right) + \mathrm{BF}\delta_i(k), \quad i \in \mathscr{F},\\
x_i(k+1) &= Ax_i(k) + BK_2\left\{\sum_{j\in N_i}\left(a_{ij}\left(x_i(k) - h_i(k) - x_j(k) - h_j(k)\right) + g_i\left(x_i(k) - h_i(k) - x_0(k)\right)\right)\right\} + \mathrm{BF}\delta_i(k), \quad i \in \mathscr{L}.
\end{aligned}
\tag{11}
$$

Let $x_f(k) = x_1^T(k), x_2^T(k), \ldots, x_N^T(k)^T$ and $x_l(k) = x_{N+1}^T(k), x_{N+2}^T(k), \ldots, x_{N+M}^T(k)^T$; then, (11) can be rewritten as follows:

$$
\begin{aligned}
x_f(k+1) &= \left(I_N \otimes A + L_1 \otimes \mathrm{BK}_1\right)x_f(k) + \left(L_2 \otimes \mathrm{BK}_1\right)x_l(k) + \left(I_N \otimes \mathrm{BF}\right)\delta_f(k),\\
x_l(k+1) &= \left(I_M \otimes A\right)x_l(k) + \left(J \otimes \mathrm{BK}_2\right)\left(x_l(k) - h_l(k) - \overline{x}_0(k)\right) + \left(I_M \otimes \mathrm{BF}\right)\delta_l(k),
\end{aligned}
\tag{12}
$$

where $J = L_3 + G$ is a positive definite matrix, $G = \mathrm{diag}\{g_{N+1}, g_{N+2}, \ldots, g_{N+M}\}$, and $\overline{x}_0(k) = 1_M \otimes x_0(k)$. On the basis of equation (4), one has $\overline{x}_0(k+1) = (I_M \otimes A)\overline{x}_0(k)$.

The following theorem gives a sufficient and necessary condition to achieve the formation-containment via state feedback protocol.

Theorem 1. *Suppose Assumptions 1 and 2 hold. Under DO-based state feedback protocol (10) with* $K_1 = -1/\lambda_1(B^TPB + I)^{-1}B^TPA$ *and* $K_2 = -1/\mu_1(B^TPB + I)^{-1}B^TPA$, *the formation-containment of system (2) can be realized if and only if* $S + HBF$ *is Schur stable, where* $P > 0$ *is the unique solution of the algebraic Riccati equation (5) and* λ_1 *and* μ_1 *are the minimum eigenvalue of* L_1 *and* J, *respectively.*

Proof. Denote the error of the system as $e_i(k) = \sum_{j\in N_i} a_{ij}(x_i(k) - x_j(k))$, $i \in \mathscr{F}$, and $\psi_i(k) = \sum_{j\in N_i}(a_{ij}((x_i(k) - h_i(k) - x_j(k) - h_j(k))) + g_i(x_i(k) - h_i(k) - x_0(k)))$, $i \in \mathscr{L}$. Let $e(k) = e_1^T(k), e_2^T(k), \ldots, e_N^T(k)^T$ and $\psi(k) = \psi_{N+1}^T(k), \psi_{N+2}^T(k), \ldots, \psi_{N+M}^T(k)^T$; then, it follows

$$
\begin{aligned}
e(k) &= \left(L_1 \otimes I_n\right)x_f(k) + \left(L_2 \otimes I_n\right)x_l(k),\\
\psi(k) &= \left(J \otimes I_n\right)\left(x_l(k) - h_l(k) - \overline{x}_0(k)\right).
\end{aligned}
\tag{13}
$$

Substituting (12) into $e(k+1)$ and $\psi(k+1)$ of (13), one derives

$$
\begin{aligned}
e(k+1) &= \left(I_N \otimes A + L_1 \otimes \mathrm{BK}_1\right)e(k) + \left(L_2 \otimes \mathrm{BK}_2\right)\psi(k) + \left(L_1 \otimes \mathrm{BF}\right)\delta_f(k) + \left(L_2 \otimes \mathrm{BF}\right)\delta_l(k),\\
\psi(k+1) &= \left(I_M \otimes A + J \otimes \mathrm{BK}_2\right)\psi(k) + \left(J \otimes \mathrm{BF}\right)\delta_l(k).
\end{aligned}
\tag{14}
$$

Then, the error system can be rewritten as the following form by (9) and (14):

$$
\begin{pmatrix} e(k+1) \\ \psi(k+1) \\ \delta_f(k+1) \\ \delta_l(k+1) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & A_{22} & 0 & A_{24} \\ 0 & 0 & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \end{pmatrix}\begin{pmatrix} e(k) \\ \psi(k) \\ \delta_f(k) \\ \delta_l(k) \end{pmatrix},
\tag{15}
$$

where $A_{11} = I_N \otimes A + L_1 \otimes \mathrm{BK}_1$, $A_{12} = L_2 \otimes \mathrm{BK}_2$, $A_{13} = L_1 \otimes \mathrm{BF}$, $A_{14} = L_2 \otimes \mathrm{BF}$, $A_{22} = I_M \otimes A + J \otimes \mathrm{BK}_2$, $A_{24} = J \otimes \mathrm{BF}$, $A_{33} = I_N \otimes (S + \mathrm{HBF})$, and $A_{44} = I_M \otimes (S + \mathrm{HBF})$.

(15) is Schur stable if and only if A_{11}, A_{22}, A_{33}, and A_{44} are Schur stable. On the one side, $A_{11} = I_N \otimes A + L_1 \otimes \mathrm{BK}$ and $A_{22} = I_M \otimes A + J \otimes \mathrm{BK}_2$ are Schur stable if and only if $A + \lambda_i\mathrm{BK}_1$ and $A + \mu_j\mathrm{BK}_2$ are Schur stable, where λ_i, $i = 1, \ldots, N$, and μ_j, $j = 1, \ldots, M$, are the eigenvalues of L_1 and J, respectively. According to Lemma 2, under Assumption 1, L_1 is positive definite and of course is nonsingular. Then, one has $\lambda_i > 0$, $i = 1, \ldots, N$. Consider the following discrete-time system as $x(k+1) = (A + \lambda_i\mathrm{BK}_1)x(k)$ with $K_1 = -1/\lambda_1(B^T\mathrm{PB} + I)^{-1}B^T\mathrm{PA}$, where λ_1 is the minimum eigenvalue of L_1. Choosing the discrete-time Lyapunov function as $V(k) = x^T(k)Px(k)$, where P is the unique solution of the modified discrete-time algebra Riccati equation (5), then one has

$$\begin{aligned}V(k+1)-V(k) &= x^T(k+1)Px(k+1)-x^T(k)Px(k)\\&= x^T(k)\Big(\big(A+\lambda_i\mathrm{BK}_1\big)^T P\big(A+\lambda_i\mathrm{BK}_1\big)-P\Big)x(k)\\&= x^T(k)\Big(A^T\mathrm{PA}+2\lambda_i A^T\mathrm{PBK}_1+\lambda_i^2K_1^TB^T\mathrm{PBK}_1-P\Big)x(k)\\&\le x^T(k)\Big(A^T\mathrm{PA}-P-2A^T\mathrm{PB}\big(B^T\mathrm{PB}+I\big)^{-1}B^T\mathrm{PA}\Big)\\&\quad\Big(+A^TPB\big(B^T\mathrm{PB}+I\big)^{-1}B^T\mathrm{PB}\big(B^T\mathrm{PB}+I\big)^{-1}B^T\mathrm{PA}\Big)x(k)\\&\le x^T(k)\Big(A^TPA-P-A^T\mathrm{PB}\big(B^T\mathrm{PB}+I\big)^{-1}B^T\mathrm{PA}\Big)x(k)\\&= -x^T(k)x(k).\\&< 0.\end{aligned} \tag{16}$$

Hence, $V(k)\longrightarrow 0$ as $k\longrightarrow\infty$, and then, one can conclude that $A+\lambda_i\mathrm{BK}$ is Schur stable for $i=1,\ldots,N$. This leads to that $A_{11}=I_N\otimes A+L_1\otimes\mathrm{BK}$ is Schur stable. Similarly, one has $A_{22}=I_M\otimes A+J\otimes\mathrm{BK}_2$ is Schur stable with $K_2=-1/\mu_1(B^T\mathrm{PB}+I)^{-1}B^T\mathrm{PA}$, where μ_1 is the minimum eigenvalue of J.

On the other side, $S+\mathrm{HBF}$ is Schur stable if and only if $A_{33}=I_N\otimes(S+\mathrm{HBF})$ and $A_{44}=I_M\otimes(S+\mathrm{HBF})$ are Schur stable. Because A_{11}, A_{22}, A_{33}, and A_{44} are Schur stable, the error system (15) is Schur stable. Therefore, $\lim_{k\longrightarrow\infty}\|e(k)\|=0$, $\lim_{k\longrightarrow\infty}\|\psi(k)\|=0$, $\lim_{k\longrightarrow\infty}\|\delta_f(k)\|=0$, and $\lim_{k\longrightarrow\infty}\|\delta_l(k)\|=0$, that is, $\lim_{k\longrightarrow\infty}\|x_f(k)+(L_1^{-1}L_2\otimes I_n)x_l(k)\|=0$ and $\lim_{k\longrightarrow\infty}\|x_l(k)-h_l(k)-\overline{x}_0(k)\|=0$; thus, $\lim_{k\longrightarrow\infty}\inf_{x_r\in Co(X)}\|x_i(k)-x_r\|=0$, $i\in\mathcal{F}$, and $\lim_{k\longrightarrow\infty}\|x_i(k)-h_i(k)-x_0(k)\|=0$, $i\in\mathcal{L}$, on the basis of Definition 1, which means Theorem 1 holds. □

4. DO-Based Output Feedback of Formation-Containment

When the state of all agents cannot be obtained, the state observer can be used to estimate the state. Therefore, in this section, the discrete-time state observer and disturbance observer are given. Meanwhile, DO-based output feedback formation-containment protocol is proposed.

The state observer is designed as

$$\begin{aligned}\widehat{x}_i(k+1)&=A\widehat{x}_i(k)+B\big(u_i(k)+d_i(k)\big)+D\big(C\widehat{x}_i(k)-y_i(k)\big),\\\widehat{y}_i(k)&=C\widehat{x}_i(k),\quad i\in\mathcal{V},\end{aligned} \tag{17}$$

where $\widehat{x}_i\in R^n$ and $\widehat{y}_i\in R^p$ are the estimated value of x_i and y_i, respectively, $D\in R^{n\times p}$ is the constant matrix, and the rest variable are same as the previous part.

The discrete-time disturbance observer based on output information is proposed as

$$\begin{aligned}v_i(k+1)&=(S+\mathrm{HBF})\big(v_i(k)-H\widehat{x}_i(k)\big)+H\big(A\widehat{x}_i(k)+Bu_i(k)+D\big(C\widehat{x}_i(k)-y_i(k)\big)\big),\\\widehat{\omega}_i(k)&=v_i(k)-H\widehat{x}_i(k),\\\widehat{d}_i(k)&=F\widehat{\omega}_i(k),\end{aligned} \tag{18}$$

where $v_i\in R^l$ is the internal state variable of the observer, $\widehat{\omega}_i$ and $\widehat{d}_i$ are the estimated value of ω_i and d_i, respectively, and $H\in R^{l\times n}$ is the gain matrix of the observer.

Remark 3. For the case that the state of each agent cannot be obtained, the state observer can be used to estimate the state. Moreover, the disturbances exist in the subsystems. One has to design corresponding controller to attenuate the disturbances. Discrete-time output-based disturbance observer (18) is proposed.

Denoting $\delta_i(k)=\omega_i(k)-\widehat{\omega}i(k)$, by (3) and (18), one has

$$\delta_i(k+1)=(S+\mathrm{HBF})\delta_i(k). \tag{19}$$

Let $\delta_f(k)=\delta_1^T(k),\delta_2^T(k),\ldots,\delta_N^T(k)^T$ and $\delta_l(k)=\delta_{N+1}^T(k),\delta_{N+2}^T(k),\ldots,\delta_{N+M}^T(k)^T$; then, $\delta_i(k+1)$ of (19) can be rewritten as follows:

$$\begin{aligned}\delta_f(k+1)&=\big(I_N\otimes(S+\mathrm{HBF})\big)\delta_f(k),\\\delta_l(k+1)&=\big(I_M\otimes(S+\mathrm{HBF})\big)\delta_l(k).\end{aligned} \tag{20}$$

And, the formation-containment control protocol depends on output information and can be designed as

$$\begin{aligned} u_i(k) &= K_3\left(\sum_{j\in N_i} a_{ij}\left(\hat{x}_i(k)-\hat{x}_j(k)\right)\right)-\hat{d}_i(k), \quad i\in\mathcal{F},\\ u_i(k) &= K_4\left\{\sum_{j\in N_i}\left(a_{ij}\left(\hat{x}_i(k)-h_i(k)-\hat{x}_j(k)-h_j(k)\right)+g_i\left(\hat{x}_i(k)-h_i(k)-x_0(k)\right)\right)\right\}-\hat{d}_i(k), \quad i\in\mathcal{L}, \end{aligned} \tag{21}$$

where K_3 and K_4 are the gain matrices to be designed. g_i is mentioned in the previous section.

Substituting output feedback law (21) into the system of state observer (17) and denoting $\tilde{x}_i(k)=x_i(k)-\hat{x}_i(k)$, $i\in\mathcal{V}$, as the error of the real value and the estimated value of the state of the ith agent, it follows

$$\begin{aligned} \hat{x}_i(k+1) &= A\hat{x}_i(k)+\mathrm{BK}_3\left(\sum_{j\in N_i} a_{ij}\left(\hat{x}_i(k)-\hat{x}_j(k)\right)\right)+\mathrm{BF}\delta_i(k)-\mathrm{DC}\tilde{x}_i(k), \quad i\in\mathcal{F},\\ \hat{x}_i(k+1) &= A\hat{x}_i(k)+\mathrm{BK}_4\left\{\sum_{j\in N_i}\left(a_{ij}\left(\hat{x}_i(k)-h_i(k)-\hat{x}_j(k)-h_j(k)\right)+g_i\left(\hat{x}_i(k)-h_i(k)-x_0(k)\right)\right)\right\}+\mathrm{BF}\delta_i(k)-\mathrm{DC}\tilde{x}_i(k), \quad i\in\mathcal{L},\\ \tilde{x}_i(k+1) &= (A+\mathrm{DC})\tilde{x}_i(k), \quad i\in\mathcal{V}. \end{aligned} \tag{22}$$

Let $\hat{x}_f(k)=\hat{x}_1^T(k),\hat{x}_2^T(k),\ldots,\hat{x}_N^T(k)^T$, $\hat{x}_l(k)=\hat{x}_{N+1}^T(k),\hat{x}_{N+2}^T(k),\ldots,\hat{x}_{N+M}^T(k)^T$, $\tilde{x}_f(k)=\tilde{x}_1^T(k),\tilde{x}_2^T(k),\ldots,\tilde{x}_N^T(k)^T$, and $\tilde{x}_l(k)=\tilde{x}_{N+1}^T(k),\tilde{x}_{N+2}^T(k),\ldots,\tilde{x}_{N+M}^T(k)^T$; then, (22) can be rewritten as follows:

$$\begin{aligned} \hat{x}_f(k+1) &= \left(I_N\otimes A+L_1\otimes \mathrm{BK}_3\right)\hat{x}_f(k)+\left(L_2\otimes \mathrm{BK}_3\right)\hat{x}_l(k)+\left(I_N\otimes \mathrm{BF}\right)\delta_f(k)-\left(I_N\otimes \mathrm{DC}\right)\tilde{x}_f(k),\\ \hat{x}_l(k+1) &= \left(I_M\otimes A\right)\hat{x}_l(k)+\left(J\otimes \mathrm{BK}_4\right)\left(\hat{x}_l(k)-\hat{h}_l(k)-\overline{x}_0(k)\right)+\left(I_M\otimes \mathrm{BF}\right)\delta_l(k)-\left(I_M\otimes \mathrm{DC}\right)\tilde{x}_l(k),\\ \tilde{x}_f(k+1) &= \left(I_N\otimes (A+\mathrm{DC})\right)\tilde{x}_f(k),\\ \tilde{x}_l(k+1) &= \left(I_M\otimes (A+\mathrm{DC})\right)\tilde{x}_l(k), \end{aligned} \tag{23}$$

where related concept of J and $\overline{x}_0(k)$ are the same as Section 3.

A sufficient and necessary condition to achieve the formation-containment via output feedback law is given by the following theorem.

Theorem 2. *Suppose Assumptions 1–3 hold. Under DO-based output feedback protocol (21) with $K_3=-1/\lambda_1\left(B^TPB+I\right)^{-1}B^TPA$ and $K_4=-1/\mu_1\left(B^TPB+I\right)^{-1}B^TPA$, the formation-containment of system (2) can be realized if and only if both $S+HBF$ and $A+DC$ are Schur stable, where $P>0$ is the unique solution of the algebraic Riccati equation (5) and λ_1 and μ_1 are the minimum eigenvalue of L_1 and J, respectively.*

Proof. Consider the error of the state observer system as $\hat{e}_i(k)=\sum_{j\in N_i}a_{ij}\left(\hat{x}_i(k)-\hat{x}_j(k)\right)$, $i\in\mathcal{F}$, and $\hat{\psi}_i(k)=\sum_{j\in N_i}\left(a_{ij}\left(\hat{x}_i(k)-h_i(k)-\hat{x}_j(k)-h_j(k)\right)+g_i\left(\hat{x}_i(k)-h_i(k)-x_0(k)\right)\right)$, $i\in\mathcal{L}$. Let $\hat{e}(k)=\hat{e}_1^T(k),\hat{e}_2^T(k),\ldots,\hat{e}_N^T(k)^T$ and $\hat{\psi}(k)=\hat{\psi}_{N+1}^T(k),\hat{\psi}_{N+2}^T(k),\ldots,\hat{\psi}_{N+M}^T(k)^T$; then, one has

$$\begin{aligned} \hat{e}(k) &= \left(L_1\otimes I_n\right)\hat{x}_f(k)+\left(L_2\otimes I_n\right)\hat{x}_l(k),\\ \hat{\psi}(k) &= \left(J\otimes I_n\right)\left(x_l(k)-h_l(k)-\overline{x}_0(k)\right). \end{aligned} \tag{24}$$

Substituting (23) into (24), then $\widehat{e}(k+1)$ and $\widehat{\psi}(k+1)$ have the following form:

$$\widehat{e}(k+1) = (I_N \otimes A + L_1 \otimes \mathrm{BK}_3)\widehat{e}(k) + (L_2 \otimes \mathrm{BK}_4)\widehat{\psi}(k) + (L_1 \otimes \mathrm{BF})\delta_f(k) + (L_2 \otimes \mathrm{BF})\delta_l(k) - (L_1 \otimes \mathrm{DC})\widetilde{x}_f(k) - (L_2 \otimes \mathrm{DC})\widetilde{x}_l(k),$$
$$\widehat{\psi}(k+1) = (I_M \otimes A + J \otimes \mathrm{BK}_4)\psi(k) + (J \otimes \mathrm{BF})\delta_l(k) - (J \otimes \mathrm{DC})\widetilde{x}_l(k). \tag{25}$$

According to (20), (23), and (25), the error system can be rewritten as the form of matrix multiplication:

$$\begin{pmatrix} \widehat{e}(k+1) \\ \widehat{\psi}(k+1) \\ \delta_f(k+1) \\ \delta_l(k+1) \\ \widetilde{x}_f(k+1) \\ \widetilde{x}_l(k+1) \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{15} & B_{16} \\ 0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\ 0 & 0 & B_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & B_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & B_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & B_{66} \end{pmatrix} \begin{pmatrix} \widehat{e}(k) \\ \widehat{\psi}(k) \\ \delta_f(k) \\ \delta_l(k) \\ \widetilde{x}_f(k) \\ \widetilde{x}_l(k) \end{pmatrix}, \tag{26}$$

where $B_{11} = I_N \otimes A + L_1 \otimes \mathrm{BK}_3$, $B_{12} = L_2 \otimes \mathrm{BK}_4$, $B_{13} = L_1 \otimes \mathrm{BF}$, $B_{14} = L_2 \otimes \mathrm{BF}$, $B_{15} = -L_1 \otimes \mathrm{DC}$, $B_{16} = -L_2 \otimes \mathrm{DC}$, $B_{22} = I_M \otimes A + J \otimes \mathrm{BK}_4$, $B_{24} = J \otimes \mathrm{BF}$, $B_{26} = -J \otimes \mathrm{DC}$, $B_{33} = I_N \otimes (S + \mathrm{HBF})$, $B_{44} = I_M \otimes (S + \mathrm{HBF})$, $B_{55} = I_N \otimes (A + \mathrm{DC})$, and $B_{66} = I_M \otimes (A + \mathrm{DC})$.

(26) is Schur stable if and only if B_{11}, B_{22}, B_{33}, B_{44}, B_{55}, and B_{66} are Schur stable. According to (16) of Theorem 1, we similarly have $B_{11} = I_N \otimes A + L_1 \otimes \mathrm{BK}_3$ with $K_3 = -1/\lambda_1 (B^T\mathrm{PB} + I)^{-1}B^T\mathrm{PA}$ and $B_{22} = I_M \otimes A + J \otimes \mathrm{BK}_4$ with $K_4 = -1/\mu_1 (B^T\mathrm{PB} + I)^{-1}B^T\mathrm{PA}$ are Schur stable. And, $S + \mathrm{HBF}$ and $A + \mathrm{DC}$ are Schur stable if and only if $B_{33} = I_N \otimes (S + \mathrm{HBF})$, $B_{44} = I_M \otimes (S + \mathrm{HBF})$, $B_{55} = I_N \otimes (A + \mathrm{DC})$, and $B_{66} = I_M \otimes (A + \mathrm{DC})$ are Schur stable.

By the above analysis, it is easy to say that (26) is Schur stable, which means that $\lim_{k\longrightarrow\infty}\|\widehat{e}(k)\| = 0$, $\lim_{k\longrightarrow\infty}\|\widehat{\psi}(k)\| = 0$, $lim_{k\longrightarrow\infty}\|\delta_f(k)\| = 0$, $\lim_{k\longrightarrow\infty}\|\delta_l(k)\| = 0$, $\lim_{k\longrightarrow\infty}\|\widetilde{x}_f(k)\| = 0$, and $\lim_{k\longrightarrow\infty}\|\widetilde{x}_l(k)\| = 0$; by the definitions of $\widehat{e}(k)$, $\widehat{\psi}(k)$, $\widetilde{x}_f(k)$, and $\widetilde{x}_l(k)$, one has $\lim_{k\longrightarrow\infty}\|\widehat{x}_f(k) + (L_1^{-1}L_2 \otimes I_n)\widehat{x}_l(k)\| = 0$, $\lim_{k\longrightarrow\infty}\|\widehat{x}_l(k) - h_l(k) - \overline{x}_0(k)\| = 0$, $\lim_{k\longrightarrow\infty}\|x_i(k) - \widehat{x}_i(k)\| = 0$, $i \in \mathscr{V}$, that is, $\lim_{k\longrightarrow\infty}\inf_{x_r \in Co(X)}\|x_i(k) - x_r\| = 0$, $i \in \mathscr{F}$, and $\lim_{k\longrightarrow\infty}\|x_i(k) - h_i(k) - x_0(k)\| = 0$, $i \in \mathscr{L}$. Therefore, the formation-containment via output feedback is achieved, which means Theorem 2 holds. □

5. Simulations

In this section, numerical simulations are given to illustrate the theoretical results. The topology of the network is shown as the graph in Figure 1, which consists of six followers labeled as $\mathscr{F} = \{1, 2, \ldots, 6\}$, three leaders labeled as $\mathscr{L} = \{7, 8, 9\}$, and one virtual leader labeled as 0.

Choose the system matrices A, B, C, D, F, and S as follows:

$$A = \begin{bmatrix} 1.0 & -2.0 \\ 0 & -1.0 \end{bmatrix}, B = \begin{bmatrix} 0.03 \\ -0.05 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & -1.0 \\ 1.0 & 2.0 \end{bmatrix}, D = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, \tag{27}$$
$$F = \begin{bmatrix} -0.02 & 1.8 \end{bmatrix}, S = \begin{bmatrix} -1.0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Then, one has the following dynamics: $i \in \mathscr{V} = \{1, 2, \ldots, 8, 9\}$, $x_{i1}(k+1) = x_{i1}(k) - 2x_{i2}(k) + 0.03(u_i(k) + d_i(k))$ and $x_{i2}(k+1) = -x_{i2}(k) - 0.05(u_i(k) + d_i(k))$. For virtual formation structure, $x_{01}(k+1) = x_{01}(k) - 2x_{02}(k)$, $x_{02}(k+1) = -x_{02}(k)$, and $i \in \mathscr{L}$, $h_{i1}(k+1) = h_{i1}(k) - 2h_{i2}(k)$ and $h_{i2}(k+1) = -h_{i2}(k)$. It is easy to verify that (A, B) is stabilizable, and (A, C) is detectable. The eigenvalues of A are 1.0 and −1.0, respectively. The eigenvalues of S are −1.0 and 0, respectively, which means that A and S are not Schur stable. Choose the gain matrix as

$$H = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.5 \end{bmatrix}. \tag{28}$$

And, the eigenvalues of $A + \mathrm{DC}$ are −0.7899 and 0.1899, respectively, and the eigenvalues of $S + \mathrm{HBF}$ are −0.9999 and 0.045, respectively, which means that both $A + \mathrm{DC}$ and $S + \mathrm{HBF}$ are Schur stable. Hence, the conditions of Theorem 1 and Theorem 2 are satisfied. According to algebraic Ricatti equation (5), one has a positive definite matrix as follows:

$$P = \begin{bmatrix} 13.3479 & -13.5550 \\ -13.5550 & 43.9266 \end{bmatrix}. \tag{29}$$

Then, one can obtain the corresponding controller gain matrices as $K_1 = [-0.8931, -0.3699]$, $K_2 = [-1.9822, -0.8210]$, $K_3 = [-0.8931, -0.3699]$, and $K_4 = [-1.9822, -0.8210]$. By state feedback, one can have Figures 2–4, and by output feedback, one has Figures 5–7. According to Figures 2 and 5, one can know all followers converge to the convex hull spanned by the leaders, which means $\lim_{k\longrightarrow\infty}\inf_{x_r \in Co(X)}\|x_i(k) - x_r\| = 0$, $i \in \mathscr{F}$. In the figures, red lines denote the leaders' trajectories. Figures 3 and 6 denote $\lim_{k\longrightarrow\infty}\|x_i(k) - h_i(k) - x_0(k)\| = 0$, $i \in \mathscr{L}$. By Figures 4 and 7, we can obtain three leaders which can maintain a triangular structure. Form which, the formation-containment control via state feedback and output feedback control protocols are achieved.

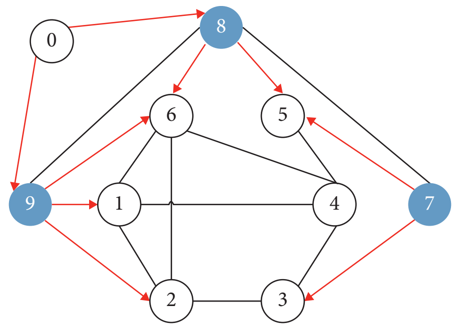

Figure 1: Topology graph.

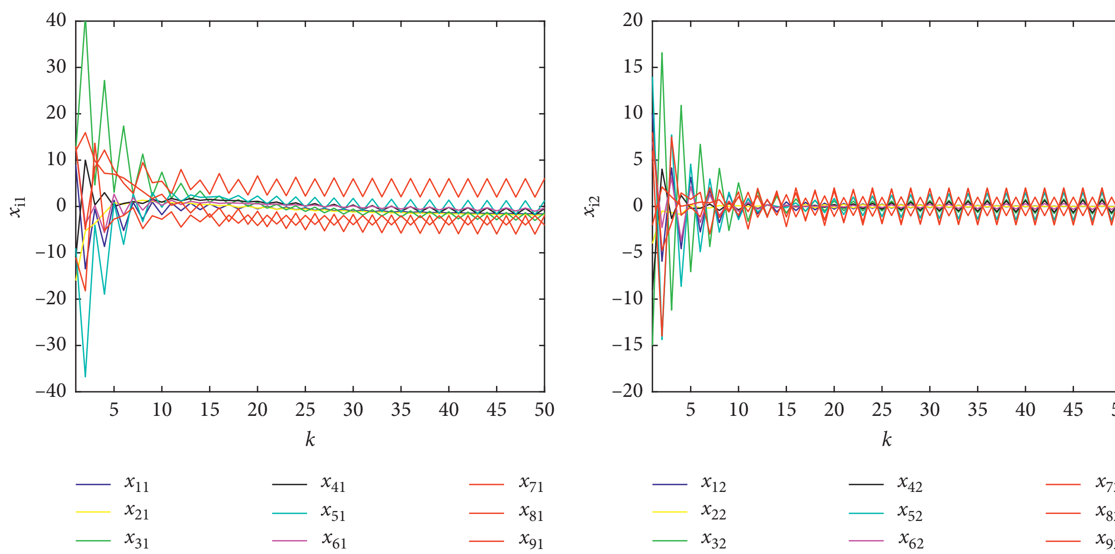

Figure 2: The trajectories of the state x_i, $i = \{1, 2, \ldots, 9\}$, via state feedback.

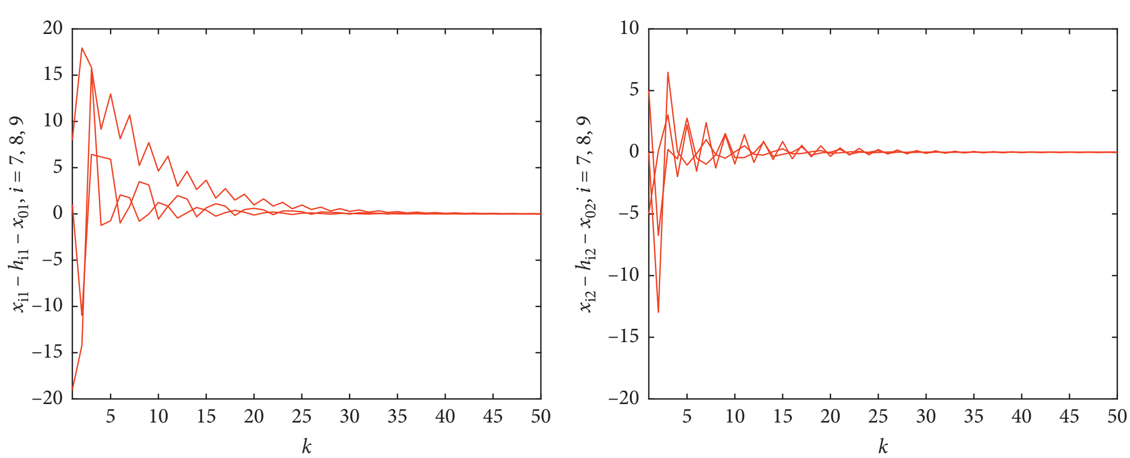

Figure 3: The trajectories of $x_i - h_i - x_0$, $i = \{7, 8, 9\}$, via state feedback.

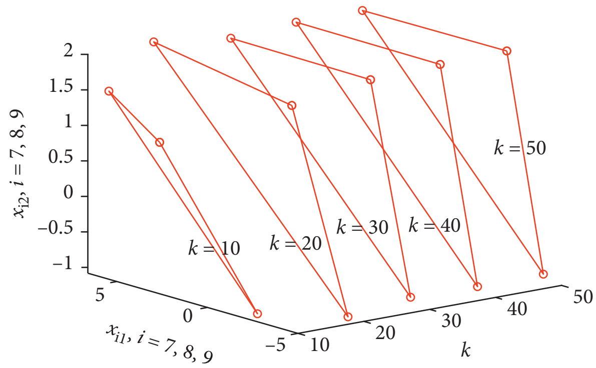

Figure 4: The formation structure of three leaders via state feedback.

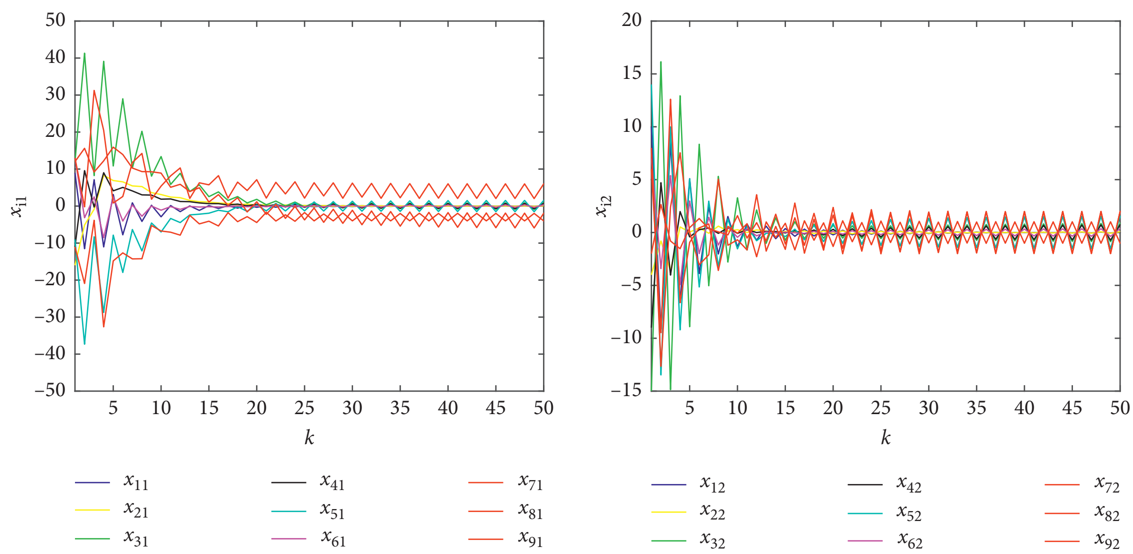

Figure 5: The trajectories of the state x_i, $i = \{1, 2, \ldots, 9\}$. via output feedback.

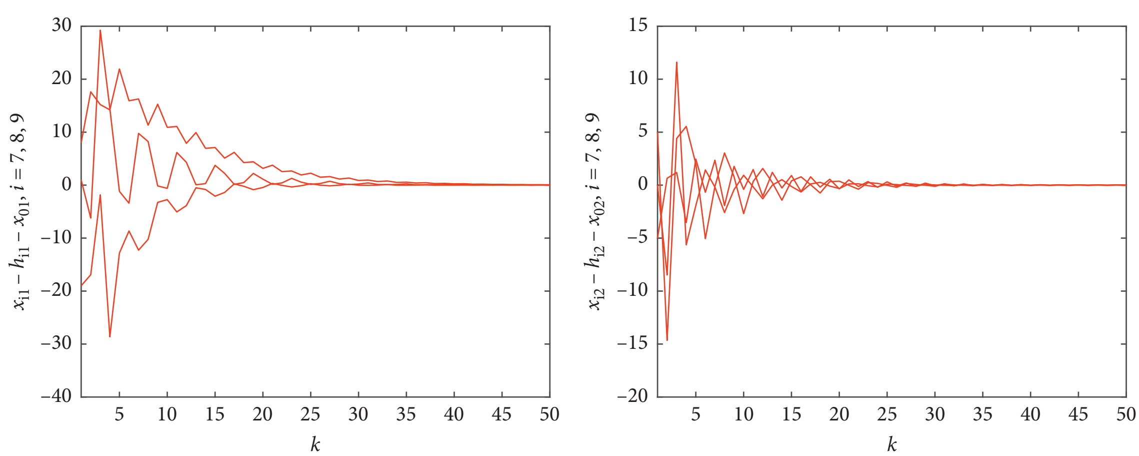

Figure 6: The trajectories of the $x_i - h_i - x_0$, $i = \{7, 8, 9\}$, via output feedback.

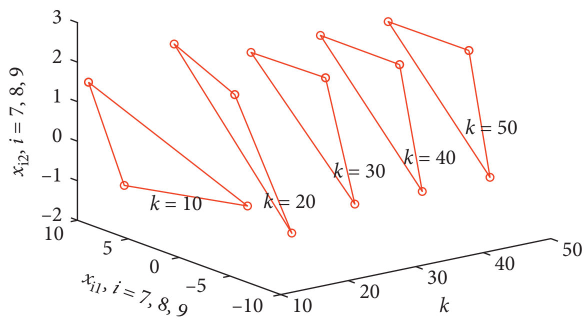

Figure 7: The formation structure of three leaders via output feedback.

6. Conclusions

In this paper, formation-containment control is investigated for high-order discrete-time MASs with exogenous disturbances. Two protocols are proposed via DO-based state feedback and output feedback, respectively. Formation-containment conditions are available, formation-containment analysis is given, and the gain matrices are obtained by solving the discrete-time algebraic Riccai equations. Future works will focus on the output formation-containment of heterogeneous MASs.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of the study.

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References

[1] X. Wang, X. Wang, H. Su, and L. James, "Coordination control for uncertain networked systems using interval observers," *Institute of Electrical and Electronics Engineers Transactions on Cybernetics*, vol. 50, no. 9, pp. 4008–4019, 2020.

[2] C. Xu, H. Xu, H. Su, and C. Liu, "Disturbance-observer based consensus of linear multi-agent systems with exogenous disturbance under intermittent communication," *Neurocomputing*, vol. 404, pp. 26–33, 2020.

[3] J. Qu, Z. Ji, C. Lin, and H. Yu, "Fast consensus seeking on networks with antagonistic interactions," *Complexity*, vol. 201815 pages, 2018.

[4] X. Wang and H. Su, "Completely model-free RL-based consensus of continuous-time multi-agent systems," *Applied Mathematics and Computation*, vol. 382, Article ID 125312, 2020.

[5] H. Su, Y. Liu, and Z. Zeng, "Second-order consensus for multiagent systems via intermittent sampled position data control," *Institute of Electrical and Electronics Engineers Transactions on Cybernetics*, vol. 50, no. 5, pp. 2063–2072, 2020.

[6] X. Wang, G.-P. Jiang, H. Su, and Z. Zeng, "Consensus-based distributed reduced-order observer design for LTI systems," *Institute of Electrical and Electronics Engineers Transactions on Cybernetics*, vol. 202011 pages, 2020.

[7] C. Xu, Y. Zhao, B. Qin, and H. Zhang, "Adaptive synchronization of coupled harmonic oscillators under switching topology," *Journal of the Franklin Institute*, vol. 356, no. 2, pp. 1067–1087, 2019.

[8] C. Xu, H. Su, C. Liu, and G. Zhang, "Robust adaptive synchronization of complex network with bounded disturbances," *Advances in Difference Equations*, vol. 2019, p. 483, 2019.

[9] S. Chen, H. Pei, Q. Lai, and H. Yan, "Multitarget tracking control for coupled heterogeneous inertial agents systems based on flocking behavior," *Institute of Electrical and Electronics Engineers Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 12, pp. 2605–2611, 2019.

[10] Z. Lin, L. Wang, Z. Han, and M. Fu, "Distributed formation control of multi-agent systems using complex Laplacian," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 59, no. 7, pp. 1765–1777, 2014.

[11] X. Ge and Q.-L. Han, "Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism," *Institute of Electrical and Electronics Engineers Transactions on Industrial Electronics*, vol. 64, no. 10, pp. 8118–8127, 2017.

[12] Z. Li, W. Ren, X. Liu, and M. Fu, "Distributed containment control of multi-agent systems with general linear dynamics in the presence of multiple leaders," *International Journal of Robust and Nonlinear Control*, vol. 23, no. 5, pp. 534–547, 2013.

[13] B. Li, H.-Y. Yang, Z.-Q. Chen, and Z.-X. Liu, "Distributed containment control of multi-agent systems with general linear dynamics and time-delays," *International Journal of Control, Automation and Systems*, vol. 16, no. 6, pp. 2718–2726, 2018.

[14] Q. Ma and G. Miao, "Distributed containment control of linear multi-agent systems," *Neurocomputing*, vol. 133, pp. 399–403, 2014.

[15] Y. Liu and H. Su, "Some necessary and sufficient conditions for containment of second-order multi-agent systems with sampled position data," *Neurocomputing*, vol. 378, pp. 228–237, 2020.

[16] Y. Liu and H. Su, "Containment control of second-order multi-agent systems via intermittent sampled position data communication," *Applied Mathematics and Computation*, vol. 362, Article ID 124522, 2019.

[17] C. Xu, B. Li, and L. Yang, "Semi-global containment of discrete-time high-order multi-agent systems with input saturation via intermittent control," *IET Control Theory & Applications*, vol. 14, no. 16, pp. 2303–2309, 2020.

[18] L. Han, X. Dong, Q. Li, and Z. Ren, "Formation-containment control for second-order multi-agent systems with time-varying delays," *Neurocomputing*, vol. 218, pp. 439–447, 2016.

[19] X. Dong, Z. Shi, G. Lu, and Y. Zhong, "Formation-containment analysis and design for high-order linear time-invariant

swarm systems," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 17, pp. 3439–3456, 2015.

[20] C. Li, L. Chen, Y. Guo, and G. Ma, "Formation-containment control for networked Euler-Lagrange systems with input saturation," *Nonlinear Dynamics*, vol. 91, no. 2, pp. 1307–1320, 2018.

[21] S. Zuo, Y. Song, F. L. Lewis, and A. Davoudi, "Time-varying output formation containment of general linear homogeneous and heterogeneous multiagent systems," *Institute of Electrical and Electronics Engineers Transactions on Control of Network Systems*, vol. 6, no. 2, pp. 537–548, 2018.

[22] Y.-W. Wang, X.-K. Liu, J.-W. Xiao, and X. Lin, "Output formation-containment of coupled heterogeneous linear systems under intermittent communication," *Journal of the Franklin Institute*, vol. 354, no. 1, pp. 392–414, 2017.

[23] Y.-W. Wang, X.-K. Liu, J.-W. Xiao, and Y. Shen, "Output formation-containment of interacted heterogeneous linear systems by distributed hybrid active control," *Automatica*, vol. 93, pp. 26–32, 2018.

[24] X. He and Q. Wang, "Distributed finite-time leaderless consensus control for double-integrator multi-agent systems with external disturbances," *Applied Mathematics and Computation*, vol. 295, pp. 65–76, 2017.

[25] M.-C. Fan and Y. Wu, "Global leader-following consensus of nonlinear multi-agent systems with unknown control directions and unknown external disturbances," *Applied Mathematics and Computation*, vol. 331, pp. 274–286, 2018.

[26] H. Su, Y. Qiu, and L. Wang, "Semi-global output consensus of discrete-time multi-agent systems with input saturation and external disturbances," *ISA Transactions*, vol. 67, pp. 131–139, 2017.

[27] C. Xu, Y. Zheng, H. Su, and H.-B. Zeng, "Containment for linear multi-agent systems with exogenous disturbances," *Neurocomputing*, vol. 160, pp. 206–212, 2015.

[28] Y. Du and B. Li, "Event-based robust stabilization for delayed systems with parameter uncertainties and exogenous disturbances," *Advances in Difference Equations*, vol. 2020, no. 1, 16 pages, 2020.

[29] Z. Zhao, W. Yang, and H. Shi, "Semi-global containment control of discrete-time linear systems with actuator position and rate saturation," *Neurocomputing*, vol. 349, no. 15, pp. 173–182, 2019.

[30] Z. Lin, *Low Gain Feedback, Lecture Notes in Control and Information Sciences*, Vol. 240, Springer, London, UK, 1998.