Research Article

A Novel Approach on the Intuitionistic Fuzzy Rough Frank Aggregation Operator-Based EDAS Method for Multicriteria Group Decision-Making

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The basic ideas of rough sets and intuitionistic fuzzy sets (IFSs) are precise statistical instruments that can handle vague knowledge easily. The EDAS (evaluation based on distance from average solution) approach plays an important role in decision-making issues, particularly when multicriteria group decision-making (MCGDM) issues have more competing criteria. The purpose of this paper is to introduce the intuitionistic fuzzy rough Frank EDAS (IFRF-EDAS) methodology based on IF rough averaging and geometric aggregation operators. We proposed various aggregation operators such as IF rough Frank weighted averaging (IFRFWA), IF rough Frank ordered weighted averaging (IFRFOWA), IF rough Frank hybrid averaging (IFRFHA), IF rough Frank weighted geometric (IFRFWG), IF rough Frank ordered weighted geometric (IFRFOWG), and IF rough Frank hybrid geometric (IFRFHG) on the basis of Frank t -norm and Frank t -conorm. Information is given for the basic favorable features of the analyzed operator. For the suggested operators, a new score and precision functions are described. Then, using the suggested method, the IFRF-EDAS method for MCGDM and its stepwise methodology are shown. After this, a numerical example is given for the established model, and a comparative analysis is generally articulated for the investigated models with some previous techniques, showing that the investigated models are much more efficient and useful than the previous techniques.

1. Introduction

The difficulty of decision-making (DM) issues is increasing with the difficulty of the social and economic surroundings in this competitive world. It is therefore more impossible for a small-decision specialist to achieve an effective and smart decision in this case. In reality, the use of group DM models heavily requires fusing the view of a team of experienced scholars to obtain more reasonable and desired objectives. In addition, in order to achieve more reasonable and sensible DM results, the major value and systemic approach of multicriteria group decision-making (MCGDM) are to increase and evaluate various different criteria in all areas of

DM. The knowledge base about such a fact is usually unique in DM issues, and this ambiguity enables the decision process to be difficult and complicated.

Zadeh [1] examined the popular sort of fuzzy sets to deal with this inaccurate knowledge correctly in order to deal with this weakness. A membership degree (MD) is represented by fuzzy set knowledge, and its membership rating is limited to $[0, 1]$, but after the development of this theory, with both theoretical and practical knowledge, it was increasingly investigated in various directions. The popular definition of the intuitionistic fuzzy set (IFS) characterized by two MD and non-MD functions was then examined by Atanassov [2]. At IFS, the sum of MD and non-MD values is

limited to the interval $[0, 1]$. It is now a popular focus of this study for scholars from the beginning of the IFS, and its hybrid impact is explored in various directions. Xu [3] was the developer of the IF weighted average (IFWA) aggregation operator. Xu and Yager [4] represented the notion of IF weighted geometric (IFWG) operators. Ali et al. [5] established the statistical tools for the rating precision and score function. A detailed study of IF neutral averaging operators was conducted by He et al. [6]. He et al. [7] proposed the concept of the average factor of geometric action and suggested its application in DM. The concept of generalized IFWA, the generalized IFOWA operator, and the generalized IFHA operator was started by Zhao et al. [8] and implemented to DM. When using Einstein standard concept, various averages and geometric operators were explained by Wang and Liu [9, 10]. The definition of IFDWA/G intuitionistic fuzzy Dombi weighting average and geometric operators was established by Seikh and Mandal [11]. On the basis of Hamacher t -norm and t -conorm, Huang [12] proposed various aggregation operators. On the basis of Archimedean t -norm and t -conorm, Xia et al. [13] presented various aggregation operators. In addition to just getting three different forms of operators, such as quasi-IF ordered weighted averaging (OWA), quasi-IF Choquet order averaging, and quasi-IFOWA operator based on Dempster–Shafer belief structure, Yang and Chen [14] generalized the idea of the averaging operator. While using IF knowledge, Szmidt and Kacprzyk [15] introduced the principle of entropy calculation. The normative concept of entropy was developed by Hung and Yang [16] on the basis of the concept of IFS probability. Using the interval-valued IF set, the similarity measure of entropy was proposed by We et al. [17]. Through using IF data, the sine and cosine similarity measure and its applications were studied by Ye [18]. IFS was already commonly applied by scholars [19, 20].

The leader who studied the main definition of rough set theory is Pawlak [21]. The classical set theory that works with incorrect and vague information has been extended by this concept. Study of the rough set has made considerable strides in both real applications and the theory on its own in past years. The idea of rough set theory has been expanded by several researchers in different ways. The definition of the fuzzy rough set was generated by Dubois and Prade [22] by introducing the fuzzy connection rather than the crisp discrete connection. The hybrid definition of IFS and rough set plays a key role in studying such various concepts, and the combined IF rough set analysis was created by Cornelis et al. [23]. By introducing IFR approximation operators, Zhou and Wu [24] established constrictive and self-evident analysis. The concept of rough IFS and IFRS was introduced by Zhou and Wu [25], and their constraining and self-evident study in terminology was represented by using the fuzzy rough approximation space theory. The notion of the IF link was established by Bustince and Burillo [26]. The essential structure of IFRS was explored by Zhang et al. [27] after using fundamental IF relationships on the premise of the concept of two universes. Some features of the IFR estimation operator based on the IF relationship were established by Yun and Lee [28] via topology. Various IFRS

extensions are examined; for further information, see [29–32]. The IF rough soft set, fuzzy soft set approximation space, and its application were proposed by Zhang et al. [33]. Furthermore, the IF covering using the IFRS was proposed by Zhang [34]. The IF soft relation has been developed by Zhang et al. [35]. Using the concept of Pythagorean orthopair fuzzy soft set and rough set, Hussain et al. [36–38] discussed their basic properties. Wang and Li [39], using Pythagorean fuzzy information, established the notion of the interaction power Bonferroni mean operator. Wang et al. [40–42] analyzed several aggregation operators using only the trapezoidal IF [43–47], analyzed different operators of aggregation, and presented their group decision-making frameworks. Wan et al. [47, 48, 49] proposed some aggregation operators on triangular intuitionistic fuzzy numbers.

The developer who researched the EDAS approach for solving DM issues was Ghorabae et al. [50]. This approach played an important role in DM problem, particularly when there are more conflicting criteria on MCGDM issues. Conventional DM methods such as TOPSIS and VIKOR are the most important techniques to calculate the distance from PIS and NIS. Smallest distance from PIS and the furthest distance with NIS was the best choice. Wei suggested the approach of grey relationship analysis (GRA) for MADM in the IF setting. Even so, the object of the EDAS method was used to find and choose the best results from multiple alternatives using PDAS (positive average solution distance) and NDAS (negative average solution distance), as well as average solution (AVS). The variance across each solution and the AVS is indicated by these two steps. The strongest one must have a higher PDAS score and an inferior NDAS score. The IF-EDAS method was implemented by Ghorabae et al. [50]. The picture fuzzy weighted averaging/geometric operator was introduced by Zhang et al. [51] and the EDAS method for MCGDM was studied. The neutrosophic soft decision method with a similarity measure and the EDAS method was established by Peng and Liu [52]. Feng et al. [53] suggested adding hesitant fuzzy knowledge of the EDAS methodology. The hybrid operator was proposed by Li et al. [54], and its implementation in DM was analyzed using the EDAS system. Liang [55] applied the EDAS system analysis to the IF area and introduced the energy efficiency project for use. Through using IF knowledge, Kahraman [56] extended the EDAS method to project planning. Ilieva [57] used the interval fuzzy information to present the definition of the EDAS system. For the interval-valued neutrosophic setting, Karasan and Kahraman [57, 58] developed the EDAS approach. To explain the EDAS process, Stanujkic et al. [59] have been using the grey number principle. The definition of elastic fuzzy logic for MCGDM based on the EDAS method was proposed by Keshavarz-Ghorabae et al. [60]. Stevic et al. [61] proposed using fuzzy knowledge in the EDAS approach for the DM strategy. Ghorabae et al. [62] gave the idea about the rank reversal process to study and join the EDAS and TOPSIS approaches [63].

Zhang et al. [64] developed t -conorm and t -norm and offered more versatility than most other t -conorms and t -norms. We expand Frank t -conorm and t -norm to intuitionistic fuzzy numbers (IFNs) in this article and define IFN

Frank operation laws. Intuitionistic fuzzy numbers (IFNs) containing two areas, the scale of membership degrees and the range of nonmembership degrees, are quite useful for using the representation of fuzzy data. With the assistance of Frank processes, we research the aggregation strategies of IFNs in this article. Initially, we apply the Frank t -conorm and t -norm to intuitionistic fuzzy conditions and implement many modern IFN operations, such as Frank number, Frank product, Frank scalar multiplication, and Frank exponentiation, depending on which we create numerous more intuitionistic fuzzy aggregation operators, as well as the intuitionistic fuzzy rough Frank weighted average operator. Furthermore, we define different characteristics of these operators, give some specific circumstances of them, and examine the relationships among them. In addition, we use the developed concept to build a method with [64] intuitionistic fuzzy data to work with MCDM problems. At the same time, certain moral virtues of these operations are examined, such as idempotency, commutativity, and limitation, and some specific scenarios are analyzed. In addition, Xing et al. [65] addressed several basic organizational laws and objects. Some aggregation operators were explained by Qin et al. [66]. Utilizing Frank t -norm and t -conorm, few general features of the type-2 fuzzy set were also clarified by Qin and Liu [67].

In this article, in Section 3, we present the idea of the new score function and accuracy function for the IFR values (IFRV). In Sections 4 and 5, we also propose the idea of average and geometric aggregation operators such as IFRFWA, IFRFOWA, IFRFHA, IFRFWG, IFRFOWG, and IFRFHG. We introduced the IFR-EDAS framework for MCGDM in Section 6 and illustrated its step-by-step algorithm using the proposed technique. After this, a numerical model focusing on the EDAS method for the choice of the appropriate small hydropower plant (SHPP) from various geographical places in Pakistan is given in Section 7. In addition, a comparative analysis of the proposed method with some previous techniques is widely articulated, showing that the model being examined is more efficient than the previous techniques.

2. Basic Concepts and Definitions

In this portion, some fundamental notions about the FS and IFS operators are given, which are used in our study.

Definition 1 (see [1]). Let X be a nonempty given set. A FS A in X is an object and represented by the mathematical equation as follows:

$$A = \{\langle x, \mu_A(x) \rangle | x \in X\}. \quad (1)$$

Definition 2 (see [2]). Let $X \neq \emptyset$. Then, the IFS $\tilde{I}$ is defined as

$$\tilde{I} = \{\langle x, \mu_{\tilde{I}}(x), \nu_{\tilde{I}}(x) \rangle | \tilde{g} \in X\}, \quad (2)$$

where $\mu_{\tilde{I}}(x) \in [0, 1]$ are the grade of positive function of x in $\tilde{I}$ and $\nu_{\tilde{I}}(x) \in [0, 1]$ are the grade of negative function of $\tilde{g}$ in $\tilde{I}$. Also, $\zeta_{\tilde{I}}(\tilde{g}) = 1 - (\mu_{\tilde{I}}(x) + \nu_{\tilde{I}}(x))$, $\forall x \in X$, is called the

grade of refusal. The pair $(\mu_{\tilde{I}}, \nu_{\tilde{I}})$ is called the intuitionistic fuzzy number (IFN). And the condition satisfies $0 \leq \mu_{\tilde{I}}(x) + \nu_{\tilde{I}}(x) \leq 1$.

Definition 3 (see [4]). Let $\tilde{I}_1 = (\mu_{\tilde{I}_1}, \nu_{\tilde{I}_1})$ and $\tilde{I}_2 = (\mu_{\tilde{I}_2}, \nu_{\tilde{I}_2})$ be IFVs; here, we have some operational laws as follows:

$$\begin{aligned} \tilde{I}_1 \cup \tilde{I}_2 &= \left(\max(\mu_{\tilde{I}_1}(x), \mu_{\tilde{I}_2}(x)), \min(\nu_{\tilde{I}_1}(x), \nu_{\tilde{I}_2}(x)) \right), \\ \tilde{I}_1 \cap \tilde{I}_2 &= \left(\min(\mu_{\tilde{I}_1}(x), \mu_{\tilde{I}_2}(x)), \max(\nu_{\tilde{I}_1}(x), \nu_{\tilde{I}_2}(x)) \right), \\ \tilde{I}_1^c &= (\nu_{\tilde{I}_1}, \mu_{\tilde{I}_1}), \\ \tilde{I}_1 \oplus \tilde{I}_2 &= (\mu_{\tilde{I}_1} + \mu_{\tilde{I}_2} - \mu_{\tilde{I}_1} \mu_{\tilde{I}_2}, \nu_{\tilde{I}_1} \nu_{\tilde{I}_2}), \\ \tilde{I}_1 \otimes \tilde{I}_2 &= (\mu_{\tilde{I}_1} \mu_{\tilde{I}_2}, \nu_{\tilde{I}_1} + \nu_{\tilde{I}_2} - \nu_{\tilde{I}_1} \nu_{\tilde{I}_2}), \\ \lambda \tilde{I}_1 &= \left(1 - (1 - \mu_{\tilde{I}_1})^\lambda, \nu_{\tilde{I}_1}^\lambda \right), \quad \forall \lambda \geq 1, \\ \tilde{I}_1^\lambda &= \left(\mu_{\tilde{I}_1}^\lambda, 1 - (1 - \nu_{\tilde{I}_1})^\lambda \right), \quad \forall \lambda \geq 1, \\ \tilde{I}_1 \leq \tilde{I}_2, &\text{ if } \mu_{\tilde{I}_1}(\tilde{g}) \leq \mu_{\tilde{I}_2}(x), \nu_{\tilde{I}_1}(x) \geq \nu_{\tilde{I}_2}(x), \quad \forall x \in X. \end{aligned} \quad (3)$$

Definition 4 (see [24]). Let us have a fixed set X and $\hat{j} \in X \times X$ be a crisp relation. Then,

- (1) $\hat{j}$ is reflexive if $(x, x) \in \hat{j}$, $\forall x \in X$
- (2) $\hat{j}$ is symmetric if $\forall x, \tilde{c} \in \tilde{N}$, $(x, \tilde{c}) \in \hat{j}$, then $(\tilde{c}, x) \in \hat{j}$
- (3) $\hat{j}$ is transitive if $\forall x, \tilde{c}, \tilde{d} \in \tilde{N}$, $(x, \tilde{c}) \in \hat{j}$, and $(\tilde{c}, \tilde{d}) \in \hat{j}$, then $(x, \tilde{d}) \in \hat{j}$

Definition 5 (see [24]). Let us have a fixed set X and for any subsets $\mathfrak{F} \in \text{IFS}(X \times X)$. We defined a mapping $\mathfrak{F}^*: \tilde{N} \rightarrow x(X)$ as follows:

$$\mathfrak{F}^*(\tilde{g}) = \{c \in X | (\alpha, \beta) \in \mathfrak{F}\}, \quad \text{for } x \in X. \quad (4)$$

Here, $\mathfrak{F}^*(x)$ denotes the successor neighborhood with respect to $\mathfrak{F}$. The crisp approximation space is represented by a pair (α, β) . The upper and lower approximations $\mathfrak{F}$ w.r.t approximation space (α, β) , i.e., $\mathfrak{F} \leq X$, are defined as

$$\begin{aligned} \overline{\mathfrak{F}}(\mathfrak{F}) &= \{\langle x, \mu_{\overline{\mathfrak{F}}(\mathfrak{F})}(x), \eta_{\overline{\mathfrak{F}}(\mathfrak{F})}(x) \rangle | x \in X\}, \\ \underline{\mathfrak{F}}(\mathfrak{F}) &= \left\{ \langle x, \mu_{\underline{\mathfrak{F}}(\mathfrak{F})}(x), \eta_{\underline{\mathfrak{F}}(\mathfrak{F})}(x) \rangle | x \in X \right\}, \end{aligned} \quad (5)$$

where $(\overline{\mathfrak{F}}(\mathfrak{F}), \underline{\mathfrak{F}}(\mathfrak{F}))$ is known as the rough set and $\overline{\mathfrak{F}}(\mathfrak{F}), \underline{\mathfrak{F}}(\mathfrak{F}): P(\tilde{N}) \rightarrow P(N)$ are the upper and lower approximation operators.

Definition 6 (see [24]). Let us have a fixed set X and subset $\mathfrak{F} \in \text{IFS}(X \times X)$; an IF relation is defined as follows:

- (1) $\hat{j}$ is reflexive if $\mu_{\hat{j}}(x, x) = 1$ and $\eta_{\hat{j}}(x, x) = 0$, $\forall x \in X$

- (2) $\hat{f}$ is symmetric if $\forall (x, \tilde{c}) \in \tilde{N} \times \tilde{N}, \mu_{\hat{f}}(x, \tilde{c}) = \mu_{\hat{f}}(\tilde{c}, x)$ and $\eta_{\hat{f}}(x, \tilde{c}) = \eta_{\hat{f}}(\tilde{c}, x)$
- (3) $\hat{f}$ is transitive if $\forall (x, \tilde{c}) \in X \times X, \mu_{\hat{f}}(x, \tilde{c}) \geq \bigvee_{\substack{c \in \tilde{N} \\ \tilde{c} \in \tilde{N}}} \left\{ \mu_{\hat{f}}(x, c) \wedge \mu_{\hat{f}}(c, \tilde{c}) \right\}$ and $\eta_{\hat{f}}(x, \tilde{c}) \geq \bigwedge_{\substack{c \in \tilde{N} \\ \tilde{c} \in \tilde{N}}} \left\{ \eta_{\hat{f}}(x, c) \wedge \eta_{\hat{f}}(c, \tilde{c}) \right\}$

2.1. Intuitionistic Fuzzy Rough Set. In this portion, we have developed the score and accuracy function on the basis of the IFS and IF Frank rough set. Furthermore, we will propose some basic operational laws.

Definition 7 (see [68]). Consider X to be the universal set, and for any subset $\mathfrak{F} \in \text{IFS}(X \times X)$, we can define an IF relation. On the basis of pair (α, β) , we can define the upper and lower approximation which is denoted by $\overline{\wp}(\mathfrak{F})$ and $\underline{\wp}(\mathfrak{F})$.

$$\begin{aligned} \overline{\wp}(\mathfrak{F}) &= \{ \langle x, \mu_{\overline{\wp}(\mathfrak{F})}(x), \eta_{\overline{\wp}(\mathfrak{F})}(x) \rangle | \hat{g} \in X \}, \\ \underline{\wp}(\mathfrak{F}) &= \{ \langle x, \mu_{\underline{\wp}(\mathfrak{F})}(x), \eta_{\underline{\wp}(\mathfrak{F})}(x) \rangle | x \in X \}, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \mu_{\overline{\wp}(\mathfrak{F})}(x) &= \bigvee_{\substack{c \in \tilde{N} \\ \tilde{c} \in \tilde{N}}} \left\{ \mu_{\wp}(x, \tilde{c}) \vee \mu_{\mathfrak{F}}(\tilde{c}) \right\}, \\ \eta_{\overline{\wp}(\mathfrak{F})}(x) &= \bigwedge_{\substack{c \in \tilde{N} \\ \tilde{c} \in \tilde{N}}} \left\{ \eta_{\wp}(x, \tilde{c}) \wedge \eta_{\mathfrak{F}}(\tilde{c}) \right\}, \\ \mu_{\underline{\wp}(\mathfrak{F})}(x) &= \bigwedge_{\substack{c \in \tilde{N} \\ \tilde{c} \in \tilde{N}}} \left\{ \mu_{\wp}(x, \tilde{c}) \wedge \mu_{\mathfrak{F}}(\tilde{c}) \right\}, \\ \eta_{\underline{\wp}(\mathfrak{F})}(x) &= \bigwedge_{\substack{c \in \tilde{N} \\ \tilde{c} \in \tilde{N}}} \left\{ \eta_{\wp}(x, \tilde{c}) \wedge \eta_{\mathfrak{F}}(\tilde{c}) \right\}, \end{aligned} \quad (7)$$

such that $0 \leq \mu_{\overline{\wp}(\mathfrak{F})}(x) + \eta_{\overline{\wp}(\mathfrak{F})}(x) \leq 1$ and $0 \leq \mu_{\underline{\wp}(\mathfrak{F})}(x) + \eta_{\underline{\wp}(\mathfrak{F})}(x) \leq 1$. The pair $(\underline{\wp}(\mathfrak{F}), \overline{\wp}(\mathfrak{F}))$ is known to be the IFR set, where $\overline{\wp}(\mathfrak{F})$ and $\underline{\wp}(\mathfrak{F})$ are said to be the IFR upper and lower approximation, respectively. Clearly,

$$\begin{aligned} \wp(\mathfrak{F}) &= (\overline{\wp}(\mathfrak{F}), \underline{\wp}(\mathfrak{F})) \\ &= \left\{ \langle x, (\mu_{\overline{\wp}(\mathfrak{F})}(x), \eta_{\overline{\wp}(\mathfrak{F})}(x)), (\mu_{\underline{\wp}(\mathfrak{F})}(x), \eta_{\underline{\wp}(\mathfrak{F})}(x)) \rangle | x \in X \right\}. \end{aligned} \quad (8)$$

Definition 8 Let J, J_1 , and J_2 be three IFREs, $s > 0$, and $\lambda > 1$. Then, we define the following operational laws:

- $$\begin{aligned} (1) J_1 \oplus J_2 &= \left\{ 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\mu_1}{s}} - 1)^s (\lambda^{1-\frac{\mu_2}{s}} - 1)^s / (\lambda - 1))), \log_{\lambda} (1 + ((\lambda^{1-\frac{\eta_1}{s}} - 1)^s (\lambda^{1-\frac{\eta_2}{s}} - 1)^s / (\lambda - 1))), 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\mu_1}{s}} - 1)^s (\lambda^{1-\frac{\mu_2}{s}} - 1)^s / (\lambda - 1))), \log_{\lambda} (1 + ((\lambda^{1-\frac{\eta_1}{s}} - 1)^s (\lambda^{1-\frac{\eta_2}{s}} - 1)^s / (\lambda - 1))) \right\}, \\ (2) J_1 \otimes J_2 &= \left\{ \log_{\lambda} (1 + ((\lambda^{\frac{\mu_1}{s}} - 1)^s (\lambda^{\frac{\mu_2}{s}} - 1)^s / (\lambda - 1))), 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\eta_1}{s}} - 1)^s (\lambda^{1-\frac{\eta_2}{s}} - 1)^s / (\lambda - 1))), \log_{\lambda} (1 + ((\lambda^{\frac{\mu_1}{s}} - 1)^s (\lambda^{\frac{\mu_2}{s}} - 1)^s / (\lambda - 1))), 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\eta_1}{s}} - 1)^s (\lambda^{1-\frac{\eta_2}{s}} - 1)^s / (\lambda - 1))) \right\}, \\ (3) \lambda \cdot J_1 &= \left\{ 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\mu_1}{s}} - 1)^s / (\lambda - 1)^{s-1})), \log_{\lambda} (1 + ((\lambda^{\frac{\eta_1}{s}} - 1)^s / (\lambda - 1)^{s-1})), 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\mu_1}{s}} - 1)^s / (\lambda - 1)^{s-1})), \log_{\lambda} (1 + ((\lambda^{\frac{\eta_1}{s}} - 1)^s / (\lambda - 1)^{s-1})) \right\}, \\ (4) J_1^{\lambda} &= \left\{ \log_{\lambda} (1 + ((\lambda^{\frac{\mu_1}{s}} - 1)^s / (\lambda - 1)^{s-1})), 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\eta_1}{s}} - 1)^s / (\lambda - 1)^{s-1})), \log_{\lambda} (1 + ((\lambda^{\frac{\mu_1}{s}} - 1)^s / (\lambda - 1)^{s-1})), 1 - \log_{\lambda} (1 + ((\lambda^{1-\frac{\eta_1}{s}} - 1)^s / (\lambda - 1)^{s-1})) \right\}. \end{aligned}$$

Definition 9 (see [68]). Let $\wp(\mathfrak{F}) = (\overline{\wp}(\mathfrak{F}), \underline{\wp}(\mathfrak{F}))$ be a IFRS. The score value and accuracy values of IFRS can be evaluated as follows:

$$\begin{aligned} \widehat{Sc}(\wp(\mathfrak{F})) &= \frac{1}{4} (2 + \underline{\mu} + \overline{\mu} - \overline{\eta} - \underline{\eta}), \\ \widetilde{Ac}(\wp(\mathfrak{F})) &= \frac{1}{4} (2 + \underline{\mu} + \overline{\mu} + \overline{\eta} + \underline{\eta}). \end{aligned} \quad (9)$$

Definition 10 (see [68]). Let $\wp(\mathfrak{F}_1) = (\underline{\wp}(\mathfrak{F}_1), \overline{\wp}(\mathfrak{F}_1))$ and $\wp(\mathfrak{F}_2) = (\underline{\wp}(\mathfrak{F}_2), \overline{\wp}(\mathfrak{F}_2))$ be two IFRSs. Then,

- (1) If $\widehat{Sc}(\wp(\mathfrak{F}_1)) > \widehat{Sc}(\wp(\mathfrak{F}_2))$, then $\wp(\mathfrak{F}_1) > \wp(\mathfrak{F}_2)$
- (2) If $\widehat{Sc}(\wp(\mathfrak{F}_1)) = \widehat{Sc}(\wp(\mathfrak{F}_2))$, then the accuracy function is compared as
 - (a) If $\widetilde{Ac}(\wp(\mathfrak{F}_1)) < \widetilde{Ac}(\wp(\mathfrak{F}_2))$, then $\wp(\mathfrak{F}_1) < \wp(\mathfrak{F}_2)$
 - (c) If $\widetilde{Ac}(\wp(\mathfrak{F}_1)) = \widetilde{Ac}(\wp(\mathfrak{F}_2))$, then $\wp(\mathfrak{F}_1) = \wp(\mathfrak{F}_2)$

3. Intuitionistic Fuzzy Rough Frank Averaging Aggregation Operator

On the basis of Frank t -norm and Frank t -conorm, we can develop the aggregation operators and take the IFNs and rough sets. We also explained some basic operational laws.

Definition 11. Let $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \overline{\wp}(\mathfrak{F}_i))$ ($i = 1, 2, \dots, n$) be a set of IFRSs in X . An IFRFWA operator of dimension n is given by

$$\text{IFRFWA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) = \oplus_{i=1}^n \omega_i \underline{\wp}(\mathfrak{F}_i), \oplus_{i=1}^n \omega_i \overline{\wp}(\mathfrak{F}_i), \quad (10)$$

where the weight vector of $\mathfrak{F}_i$ ($i = 1, 2, \dots, n$) is $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, with $\sum_{i=1}^n \omega_i = 1$ and $\omega_i \in [0, 1]$.

On the basis of intuitionistic fuzzy laws of IFNs, the IFRFWA operator can be converted into the shape given in

the following by induction on n . The aggregated value of $\mathfrak{F}_i$ by the use of the IFRFWA operator is again an IFRS and can be written as

$$\begin{aligned} \text{IFRFA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) &= \{\oplus_{i=1}^n \omega_i \wp(\mathfrak{F}_i), \oplus_{i=1}^n \omega_i \bar{\wp}(\mathfrak{F}_i)\} \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (11)$$

Theorem 1. Let $\wp(\mathfrak{F}_i) = (\wp(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i)) (i = 1, 2, \dots, n)$ be IFRSs having weight as $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$. Then, we can define the IFRFA operator as

$$\begin{aligned} \text{IFRFA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) &= \{\oplus_{i=1}^n \omega_i \wp(\mathfrak{F}_i), \oplus_{i=1}^n \omega_i \bar{\wp}(\mathfrak{F}_i)\} \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (12)$$

Proof. By using mathematical induction,

$$\begin{aligned} \wp(\mathfrak{F}_1) \oplus \wp(\mathfrak{F}_2) &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), \right. \\ &\quad \left. 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}, \\ \omega \wp(\mathfrak{F}_1) &= (\omega \wp(\mathfrak{F}_1), \omega \bar{\wp}(\mathfrak{F}_1)) \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), \right. \\ &\quad \left. \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (13)$$

Let

$$\begin{aligned} \omega \wp(\mathfrak{F}_1) &= (\omega \wp(\mathfrak{F}_1), \omega \bar{\wp}(\mathfrak{F}_1)) \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (14)$$

Let $n = 2$; then,

$$\begin{aligned} \text{IFRFA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2)) &= \{\oplus_{i=1}^2 \omega_i \wp(\mathfrak{F}_i), \oplus_{i=1}^2 \omega_i \bar{\wp}(\mathfrak{F}_i)\} \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^2 (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, 1 - \log_\lambda \left(1 + \prod_{i=1}^2 (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), \log_\lambda \left(1 + \prod_{i=1}^2 (\lambda^{\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^2 (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (15)$$

Here, we have to prove it is true for $n = 2$, and furthermore, it is checked for $n = k$.

$$\begin{aligned} & \text{IFRFA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_k)) \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^k (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, 1 - \log_\lambda \left(1 + \prod_{i=1}^k (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), \log_\lambda \left(1 + \prod_{i=1}^k (\lambda^{\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^k (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (16)$$

Furthermore, we check for $n = k + 1$, and we have

$$\begin{aligned} & \text{IFRFA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_{k+1})) \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^{k+1} (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, 1 - \log_\lambda \left(1 + \prod_{i=1}^{k+1} (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), \log_\lambda \left(1 + \prod_{i=1}^{k+1} (\lambda^{\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^{k+1} (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (17)$$

Thus, the required result holds for $n = k + 1$. Hence, the required result is true for all $n \geq 1$.

The basic concepts of upper approximation and lower approximation are IFRVs. So, by Definition 6, $\oplus_{i=1}^n \omega_i \wp(\mathfrak{I}_i)$ and $\oplus_{i=1}^k \omega_i \bar{\wp}(\mathfrak{I}_i)$ are also IFRSs. Therefore, IFRFA($\wp(\mathfrak{I}_1), \dots, \wp(\mathfrak{I}_k)$) become IFRS under IF approximation space $(X, \mathfrak{I})$.

The IFRFA operators satisfy the following properties:

- (1) Idempotency: $\wp(\mathfrak{I}_i) = p^*(\mathfrak{I}), \forall i = 1, 2, \dots, n$. So,

$$p^*(\mathfrak{I}_i) = \left(\underline{p}^*(\mathfrak{I}), \bar{p}^*(\mathfrak{I}) \right) = \left(\left(\underline{d}^*, \underline{f}^* \right), \left(\bar{d}^*, \bar{f}^* \right) \right). \quad (18)$$

Then,

$$\text{IFRFA}(p^*(\mathfrak{I}_1), p^*(\mathfrak{I}_2), \dots, p^*(\mathfrak{I}_n)) = p^*(\mathfrak{I}). \quad (19)$$

- (2) Boundedness: if $(\wp(\mathfrak{I}))^- = (\min_i \underline{\wp}(\mathfrak{I}_i), \max_i \bar{\wp}(\mathfrak{I}_i))$ and $(\wp(\mathfrak{I}))^+ = (\max_i \underline{\wp}(\mathfrak{I}_i), \min_i \bar{\wp}(\mathfrak{I}_i))$, then

$$(\wp(\mathfrak{I}))^- \leq \text{IFRFA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_1), \dots, \wp(\mathfrak{I}_n)) \leq (\wp(\mathfrak{I}))^+. \quad (20)$$

- (3) Monotonicity:

let $p^*(\mathcal{L}_i) = (p^*(\mathcal{L}_i), \bar{p}^*(\mathcal{L}_i)), i = 1, 2, \dots, n$, be another collection of IFRSs such that $\underline{p}^*(\mathcal{L}_i) \leq \underline{\wp}(\mathfrak{I}_i)$ and $\bar{p}^*(\mathcal{L}_i) \leq \bar{\wp}(\mathfrak{I}_i)$. Then,

$$\text{IFRFA}(p^*(\mathcal{L}_i), p^*(\mathcal{L}_i), \dots, p^*(\mathcal{L}_i)) \leq \text{IFRFA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_1), \dots, \wp(\mathfrak{I}_n)). \quad (21)$$

- (4) Shift invariance: let $\underline{p}^*(\mathfrak{I}_i) = (p^*(\mathfrak{I}), \bar{p}^*(\mathfrak{I})) = ((\underline{d}^*, \underline{f}^*, \underline{e}^*), (\bar{d}^*, \bar{f}^*, \bar{e}^*))$ be IFRSs.

$$\text{IFRFA}(\wp(\mathfrak{I}_1) \oplus p^*(\mathcal{L}), \wp(\mathfrak{I}_2) \oplus p^*(\mathcal{L}), \dots, \wp(\mathfrak{I}_n) \oplus p^*(\mathcal{L})) = \text{IFRFA}((\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) \oplus p^*(\mathcal{L})). \quad (22)$$

- (5) Homogeneity: for any real number $\omega > 0$,

$$\text{IFRFA}(\omega_{\wp}(\mathfrak{T}_1), \omega_{\wp}(\mathfrak{T}_2), \dots, \omega_{\wp}(\mathfrak{T}_n)) = \omega \text{IFRFA}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)). \quad (23)$$

(6) Commutativity: let $\wp^*(\mathfrak{T}_i) = (\underline{\wp}^*(\mathfrak{T}_i), \overline{\wp}^*(\mathfrak{T}_i))$, $i = 1, 2, \dots, n$, be any permutation of $\wp(\mathfrak{T}_i) = (\underline{\wp}(\mathfrak{T}_i), \overline{\wp}(\mathfrak{T}_i))$. Then,

$$\text{IFRFA}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)) = \text{IFRFA}(\wp^*(\mathfrak{T}_1), \wp^*(\mathfrak{T}_2), \dots, \wp^*(\mathfrak{T}_n)). \quad (24)$$

□

Proof.

(1) We have $\wp(\mathfrak{T}_i) = p^*(\mathfrak{T})$ for all $i = 1, 2, \dots, n$, where

$$\begin{aligned} p^*(\mathfrak{T}_i) &= (\underline{p}^*(\mathfrak{T}), \overline{p}^*(\mathfrak{T})) = \left(\left(\underline{\check{d}}, \underline{\check{f}} \right), \left(\overline{\check{d}}, \overline{\check{f}} \right) \right), \\ \text{IFRFA}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)) &= \left\{ \oplus_{i=1}^n \omega_i \underline{\wp}(\mathfrak{T}_i), \oplus_{i=1}^n \omega_i \overline{\wp}(\mathfrak{T}_i) \right\} \\ &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 \right. \\ &\quad \left. - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\overline{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\overline{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (25)$$

For all i , $\wp(\mathfrak{T}_i) = p^*(\mathfrak{T}) = (\underline{p}^*(\mathfrak{T}), \overline{p}^*(\mathfrak{T})) = ((\underline{\check{d}}, \underline{\check{f}})^*, (\overline{\check{d}}, \overline{\check{f}})^*)$. Therefore,

$$\begin{aligned} &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\overline{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\overline{\eta}_i} - 1)^{\omega_i} \right) \right) \right\} \\ &= \left\langle \left(1 - \left(1 - \underline{\check{d}} \right)^*, \underline{\check{f}} \right)^*, \left(\left(1 - \left(1 - \overline{\check{d}} \right)^*, \overline{\check{f}} \right)^* \right) \right\rangle \\ &= (\underline{p}^*(\mathfrak{T}), \overline{p}^*(\mathfrak{T})) \\ &= p^*(\mathfrak{T}). \end{aligned} \quad (26)$$

Hence,

$$\text{IFRFA}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)) = p^*(\mathfrak{T}). \quad (27)$$

(2) We have

$$\begin{aligned} (\underline{\wp}(\mathfrak{T}))^- &= \left\langle \min_i \{\underline{\mu}_i\}, \max_i \{\underline{\eta}_i\} \right\rangle, \left\langle \min_i \{\overline{\mu}_i\}, \max_i \{\overline{\eta}_i\} \right\rangle, \\ (\underline{\wp}(\mathfrak{T}))^+ &= \left\langle \max_i \{\underline{\mu}_i\}, \min_i \{\underline{\eta}_i\} \right\rangle, \left\langle \max_i \{\overline{\mu}_i\}, \min_i \{\overline{\eta}_i\} \right\rangle, \end{aligned} \quad (28)$$

$(\wp(\mathfrak{T})) = \langle (\underline{\mu}_i, \underline{\eta}_i), (\overline{\mu}_i, \overline{\eta}_i) \rangle$. To prove that

$$(\wp(\mathfrak{F}))^- \leq \text{IFRFA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_1), \dots, \wp(\mathfrak{F}_n)) \leq (\wp(\mathfrak{F}))^+, \quad (29)$$

$\forall i = 1, 2, \dots, n$, we have

$$\begin{aligned} \min_i \{\underline{\mu}_i\} \leq \underline{\mu}_i \leq \max_i \{\underline{\mu}_i\} &\Leftrightarrow 1 - \max_i \{\underline{\mu}_i\} \leq 1 - \underline{\mu}_i \leq 1 - \min_i \{\underline{\mu}_i\} \\ &= \prod_{i=1}^n (1 - \max_i \{\underline{\mu}_i\})^{\omega_i} \leq \prod_{i=1}^n (1 - \underline{\mu}_i)^{\omega_i} \\ &\leq \prod_{i=1}^n (1 - \min_i \{\underline{\mu}_i\})^{\omega_i} \\ &\Leftrightarrow (1 - \min_i \{\underline{\mu}_i\}) \leq \prod_{i=1}^n (1 - \min_i \{\underline{\mu}_i\})^{\omega_i} \leq \prod_{i=1}^n (1 - \min_i \{\underline{\mu}_i\})^{\omega_i} \\ &\Leftrightarrow 1 - (1 - \min_i \{\underline{\mu}_i\}) \leq 1 - \prod_{i=1}^n (1 - \underline{\mu}_i)^{\omega_i} \leq 1 - (1 - \max_i \{\underline{\mu}_i\}). \end{aligned} \quad (30)$$

Hence,

$$\min_i \{\underline{\mu}_i\} \leq 1 - \prod_{i=1}^n (1 - \underline{\mu}_i)^{\omega_i} \leq \max_i \{\underline{\mu}_i\}. \quad (31)$$

Next, for all $i = 1, 2, \dots, n$, we have

$$\begin{aligned} \min_i \{\underline{\eta}_i\} &\leq \underline{\eta}_i \leq \max_i \{\underline{\eta}_i\} \\ \Leftrightarrow \prod_{i=1}^n (\min_i \{\underline{\eta}_i\})^{\omega_i} &\leq \prod_{i=1}^n (\underline{\eta}_i)^{\omega_i} \leq \prod_{i=1}^n (\max_i \{\underline{\eta}_i\})^{\omega_i} \\ \Rightarrow \min_i \{\underline{\eta}_i\} &\leq \prod_{i=1}^n (\underline{\eta}_i)^{\omega_i} \leq \max_i \{\underline{\eta}_i\}. \end{aligned} \quad (32)$$

Also, we prove

$$\min_i \{\underline{\mu}_i\} \leq 1 - \prod_{i=1}^n (1 - \underline{\mu}_i)^{\omega_i} \leq \max_i \{\underline{\mu}_i\}, \quad (33)$$

$$\min_i \{\bar{\mu}_i\} \leq \prod_{i=1}^n (\bar{\mu}_i)^{\omega_i} \leq \max_i \{\bar{\mu}_i\}, \quad (34)$$

$$\check{d}_i^* \leq \underline{\mu}_i \Rightarrow 1 - \underline{\mu}_i \leq 1 - \check{d}_i^* \Rightarrow \prod_{i=1}^n (1 - \underline{\mu}_i)^{\omega_i} \leq \prod_{i=1}^n (1 - \check{d}_i^*)^{\omega_i} \Rightarrow 1 - \prod_{i=1}^n (1 - \check{d}_i^*)^{\omega_i} \leq 1 - \prod_{i=1}^n (1 - \underline{\mu}_i)^{\omega_i}. \quad (38)$$

Next,

$$\underline{f}_i^* \geq \underline{\eta}_i \Rightarrow \prod_{i=1}^n \underline{f}_i^{*\omega_i} \geq \prod_{i=1}^n \underline{\eta}_i^{\omega_i}. \quad (39)$$

Also, we have to prove

$$\min_i \{\underline{v}_i\} \leq \prod_{i=1}^n (\underline{v}_i)^{\omega_i} \leq \max_i \{\underline{v}_i\}, \quad (35)$$

$$\min_i \{\bar{v}_i\} \leq \prod_{i=1}^n (\bar{v}_i)^{\omega_i} \leq \max_i \{\bar{v}_i\}, \quad (36)$$

so from equations (31) to (36), we have

$$(\wp(\mathfrak{F}))^- \leq \text{IFRFA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_1), \dots, \wp(\mathfrak{F}_n)) \leq (\wp(\mathfrak{F}))^+, \quad (37)$$

which is the required proof.

(3) Since

$p^*(\mathcal{L}_i) = (\underline{p}^*(\mathcal{L}_i), \bar{p}^*(\mathcal{L}_i)) = ((\check{d}_i^*, \underline{f}_i^*), (\bar{d}_i^*, \bar{f}_i^*))$
and $(\wp(\mathfrak{F}_i)) = (\underline{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i))$ are IFRVs, we shall prove the following. To show that $\underline{p}^*(\mathcal{L}_i) \leq \underline{\wp}(\mathfrak{F}_i)$ and $\bar{p}^*(\mathcal{L}_i) \leq \bar{\wp}(\mathfrak{F}_i)$, $i = 1, 2, \dots, n$. So,

$$1 - \prod_{i=1}^n (1 - \bar{d}_i^*)^{\omega_i} \leq 1 - \prod_{i=1}^n (1 - \bar{\mu}_i)^{\omega_i}, \quad (40)$$

$$\bar{f}_i^* \geq \bar{\eta}_i \Rightarrow \prod_{i=1}^n \bar{f}_i^{*\omega_i} \geq \prod_{i=1}^n \bar{\eta}_i^{\omega_i}, \quad (41)$$

$$\check{e}_i^* \geq \underline{v}_i \Rightarrow \prod_{i=1}^n \check{e}_i^{*\omega_i} \geq \prod_{i=1}^n \underline{v}_i^{\omega_i}, \quad (42)$$

$$\bar{e}_i^* \geq \bar{v}_i \Rightarrow \prod_{i=1}^n \bar{e}_i^{*\omega_i} \geq \prod_{i=1}^n \bar{v}_i^{\omega_i}. \quad (43)$$

Hence, from equations (38) to (43), we get

$$\begin{aligned} \underline{p}^*(\mathcal{L}_i) &\leq \underline{\wp}(\mathfrak{F}_i), \\ \bar{p}^*(\mathcal{L}_i) &\leq \bar{\wp}(\mathfrak{F}_i). \end{aligned} \quad (44)$$

Therefore,

$$\begin{aligned} &\text{IFRFA}(\underline{p}^*(\mathcal{L}_i), \bar{p}^*(\mathcal{L}_i), \dots, \bar{p}^*(\mathcal{L}_i)) \\ &\leq \text{IFRFA}(\underline{\wp}(\mathfrak{F}_1), \bar{\wp}(\mathfrak{F}_1), \dots, \bar{\wp}(\mathfrak{F}_n)). \end{aligned} \quad (45)$$

$$(4) \text{ As } \underline{p}^*(\mathcal{L}_i) = (\underline{p}^*(\mathcal{L}), \bar{p}^*(\mathcal{L})) = ((\underline{d}^*, \underline{f}^*), (\bar{d}^*, \bar{f}^*)) \text{ is an IFRS, } \bar{\wp}(\mathfrak{F}_i) = (\bar{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i)) = ((\underline{\mu}_i, \underline{\eta}_i), (\bar{\mu}_i, \bar{\eta}_i)) \text{ is the family of IFRSs, so}$$

$$\begin{aligned} \wp(\mathfrak{F}_1) \oplus p^*(\mathcal{L}) &= (\underline{\wp}(\mathfrak{F}_1) \oplus \underline{p}^*(\mathcal{L}), \bar{\wp}(\mathfrak{F}_1) \oplus \bar{p}^*(\mathcal{L})) \\ &= \left(\left(1 - (1 - \underline{\mu}_1) \left(1 - \underline{d}^* \right), \underline{\eta}_1 \underline{f}^*, \underline{v}_1 \underline{e}^* \right), \left(1 - (1 - \bar{\mu}_1) \left(1 - \bar{d}^* \right), \bar{\eta}_1 \bar{f}^*, \bar{v}_1 \bar{e}^* \right) \right). \end{aligned} \quad (46)$$

Therefore,

$$\begin{aligned} &\text{IFRFA}(\wp(\mathfrak{F}_1) \oplus p^*(\mathcal{L}), \wp(\mathfrak{F}_2) \oplus p^*(\mathcal{L}), \dots, \wp(\mathfrak{F}_n) \oplus p^*(\mathcal{L})) \\ &= \{ \oplus_{i=1}^n \omega_i (\underline{\wp}(\mathfrak{F}_i) \oplus p^*(\mathcal{L})), \oplus_{i=1}^n \omega_i (\bar{\wp}(\mathfrak{F}_i) \oplus p^*(\mathcal{L})) \} \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i} \left(1 - \underline{d}^* \right), \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \underline{f}^* \right) \right), 1 \right. \\ &\quad \left. - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i} \left(1 - \bar{d}^* \right), \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \bar{f}^* \right) \right) \right\} \\ &= \left\{ 1 - \left(1 - \underline{d}^* \right) 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \underline{f}^* \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 - \right. \\ &\quad \left. \left(\left(1 - \bar{d}^* \right) 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \bar{f}^* \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right) \right\} \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), \oplus \left(\underline{d}^*, \underline{f}^* \right) 1 - \right. \\ &\quad \left. \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \oplus \left(\bar{d}^*, \bar{f}^* \right) \right\} \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 - \right. \\ &\quad \left. \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\} \oplus \left(\left(\underline{d}^*, \underline{f}^* \right), \left(\bar{d}^*, \bar{f}^* \right) \right) \\ &= \text{IFRFA}((\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) \oplus p^*(\mathcal{L})), \end{aligned} \quad (47)$$

hence the proof.

As

(5) Let $\delta > 0$ and $\wp(\mathfrak{F}_i) = (\bar{\wp}(\mathfrak{F}_i), \underline{\wp}(\mathfrak{F}_i))$ be an IFRV.

$$\begin{aligned}\omega_{\wp}(\mathfrak{F}_i) &= (\omega_{\overline{\wp}}(\mathfrak{F}_i), \omega_{\underline{\wp}}(\mathfrak{F}_i)) \\ &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\},\end{aligned}\quad (48)$$

now,

$$\begin{aligned}\text{IFRFAWA}(\omega_{\wp}(\mathfrak{F}_1), \omega_{\wp}(\mathfrak{F}_2), \dots, \omega_{\wp}(\mathfrak{F}_n)) \\ &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\delta\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\delta\omega_i} \right) \right), 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\delta\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\delta\omega_i} \right) \right) \right\} \\ &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i} \right)^{\delta}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right)^{\delta}, 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i} \right)^{\delta}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right)^{\delta} \right\} \\ &= \omega \text{IFRFAWA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)).\end{aligned}\quad (49)$$

Hence, we get the required proof.

(6) Let

$$\begin{aligned}\text{IFRFAWA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) &= \{\oplus_{i=1}^n \omega_i \wp(\mathfrak{F}_i), \oplus_{i=1}^n \omega_i \overline{\wp}(\mathfrak{F}_i)\} \\ &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i} - 1)^{\omega_i} \right) \right), 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i} - 1)^{\omega_i} \right) \right) \right\}.\end{aligned}\quad (50)$$

Since $(\wp^*(\mathfrak{F}_1), \wp^*(\mathfrak{F}_2), \dots, \wp^*(\mathfrak{F}_n))$ is any permutation of $(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n))$, we have $\wp(\mathfrak{F}_i) = \wp^*(\mathfrak{F}_i) (i = 1, 2, \dots, n)$

$$\begin{aligned}&= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i^*} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_i^*} - 1)^{\omega_i} \right) \right), 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i^*} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\bar{\eta}_i^*} - 1)^{\omega_i} \right) \right) \right\} \\ &= \left\{ \oplus_{i=1}^n \omega_i \wp^*(\mathfrak{F}_i), \oplus_{i=1}^n \omega_i \overline{\wp}^*(\mathfrak{F}_i) \right\} \\ &= \text{IFRFAWA}(\oplus_{i=1}^n \omega_i \wp^*(\mathfrak{F}_i), \oplus_{i=1}^n \omega_i \overline{\wp}^*(\mathfrak{F}_i)).\end{aligned}\quad (51)$$

4. Intuitionistic Fuzzy Rough Frank Ordered Weighted Aggregation Operator

The IFRFOWA operator and its desirable properties are discussed in this portion of the paper.

Definition 12. Consider the collection $\wp(\mathfrak{F}_i) = (\overline{\wp}(\mathfrak{F}_i), \wp(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, of IFRSs with weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, with $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$. The IFRFOWA operator is

$$\text{IFRFOWA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) = (\oplus_{i=1}^n \omega_i \wp(\mathfrak{F}_{\sigma(i)}), \oplus_{i=1}^n \omega_i \overline{\wp}(\mathfrak{F}_{\sigma(i)})). \quad (52)$$

Theorem 2. Let $\wp(\mathfrak{I}_i) = (\underline{\wp}(\mathfrak{I}_i), \overline{\wp}(\mathfrak{I}_i)) (i = 1, 2, \dots, n)$ be IFRSs having weight $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$. So, we can define the IFRFOWA operator as

$$\begin{aligned} \text{IFRFOWA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) &= \left\{ \oplus_{i=1}^n \omega_i \underline{\wp}(\mathfrak{I}_{\sigma(i)}), \oplus_{i=1}^n \omega_i \overline{\wp}(\mathfrak{I}_{\sigma(i)}) \right\} \\ &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_{\sigma(i)}} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\eta}_{\sigma(i)}} - 1)^{\omega_i} \right) \right), \right. \\ &\quad \left. 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\overline{\mu}_{\sigma(i)}} - 1)^{\omega_i}, \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\overline{\eta}_{\sigma(i)}} - 1)^{\omega_i} \right) \right) \right\}, \end{aligned} \quad (53)$$

where $\wp(\mathfrak{I}_{\sigma(i)}) = (\underline{\wp}(\mathfrak{I}_{\sigma(i)}), \overline{\wp}(\mathfrak{I}_{\sigma(i)}))$ denotes that the highest permutation values form the collection of IFRSs.

Proof. The proof is the same as that of Theorem 1.

Furthermore, some properties of IFRFOWA are studied in the following theorem.

The IFRFOWA operators satisfy the following properties:

- (1) Idempotency: $\wp(\mathfrak{I}_i) = p^*(\mathfrak{I}), \forall i = 1, 2, \dots, n$. Also,

$$p^*(\mathfrak{I}_i) = \left(\underline{p}^*(\mathfrak{I}), \overline{p}^*(\mathfrak{I}) \right) = \left(\left(\underline{\underline{d}}, \underline{\underline{f}} \right), \left(\overline{\underline{d}}, \overline{\underline{f}} \right) \right). \quad (54)$$

Then,

$$\text{IFRFOWA}(p^*(\mathfrak{I}_1), p^*(\mathfrak{I}_2), \dots, p^*(\mathfrak{I}_n)) = p^*(\mathfrak{I}). \quad (55)$$

- (2) Boundedness: let $(\wp(\mathfrak{I}))^- = (\min_i \underline{\wp}(\mathfrak{I}_i), \max_i \overline{\wp}(\mathfrak{I}_i))$ and $(\wp(\mathfrak{I}))^+ = (\max_{\max} \underline{\wp}(\mathfrak{I}_i), \min_{\min} \overline{\wp}(\mathfrak{I}_i))$. Then,

$$(\wp(\mathfrak{I}))^- \leq \text{IFRFOWA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) \leq (\wp(\mathfrak{I}))^+. \quad (56)$$

- (3) Monotonicity: let $p^*(\mathcal{L}_i) = (\underline{p}^*(\mathcal{L}_i), \overline{p}^*(\mathcal{L}_i))$, $i = 1, 2, \dots, n$, be the family of IFRSs such that $\underline{p}^*(\mathcal{L}_i) \leq \underline{\wp}(\mathfrak{I}_i)$ and $\overline{p}^*(\mathcal{L}_i) \leq \overline{\wp}(\mathfrak{I}_i)$. Then,

$$\text{IFRFOWA}(p^*(\mathcal{L}_1), p^*(\mathcal{L}_2), \dots, p^*(\mathcal{L}_n)) \leq \text{IFRFOWA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)). \quad (57)$$

- (4) Shift invariance: let $p^*(\mathfrak{I}_i) = (\underline{p}^*(\mathfrak{I}), \overline{p}^*(\mathfrak{I})) = ((\underline{\underline{d}}^*, \underline{\underline{f}}^*), (\overline{\underline{d}}^*, \overline{\underline{f}}^*))$ be IFRSs.

$$\text{IFRFOWA}(\wp(\mathfrak{I}_1) \oplus p^*(\mathcal{L}), \wp(\mathfrak{I}_2) \oplus p^*(\mathcal{L}), \dots, \wp(\mathfrak{I}_n) \oplus p^*(\mathcal{L})) = \text{IFRFOWA}((\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) \oplus p^*(\mathcal{L})). \quad (58)$$

- (5) Homogeneity: for any real number $\omega > 0$,

$$\text{IFRFOWA}(\omega \wp(\mathfrak{I}_1), \omega \wp(\mathfrak{I}_2), \dots, \omega \wp(\mathfrak{I}_n)) = \omega \text{IFRFOWA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)). \quad (59)$$

- (6) Commutativity: let $\wp^*(\mathfrak{I}_i) = (\underline{\wp}^*(\mathfrak{I}_i), \overline{\wp}^*(\mathfrak{I}_i))$, $i = 1, 2, \dots, n$, be any permutation of $\wp(\mathfrak{I}_i) = (\underline{\wp}(\mathfrak{I}_i), \overline{\wp}(\mathfrak{I}_i))$. Then,

$$\text{IFRFOWA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) = \text{IFRFOWA}(\wp^*(\mathfrak{I}_1), \wp^*(\mathfrak{I}_2), \dots, \wp^*(\mathfrak{I}_n)). \quad (60)$$

Proof. The proof is the same as that of Theorem 2. $\square$

5. Intuitionistic Fuzzy Rough Frank Hybrid Averaging Operator

The IFRFHA operator simultaneously weighs both the importance and the ordered state of an IF statement. And its desirable properties are also discussed in this portion of the paper.

Definition 13. Consider the family of IFRVs $\wp(\mathfrak{I}_i) = (\underline{\wp}(\mathfrak{I}_i), \overline{\wp}(\mathfrak{I}_i))$, $i = 1, 2, \dots, n$, having associated weight $\hat{w} = (\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n)^T$, as $\sum_{i=1}^n \hat{w}_i = 1$ and $0 \leq \hat{w}_i \leq 1$. Consider the associated weight vector for IFRSs which is denoted by $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, with $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$. So, the IFRFHA operator can be written as

$$\text{IFRFHA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) = \left(\oplus_{i=1}^n \omega_i \wp^*(\mathfrak{I}_{\sigma(i)}), \oplus_{i=1}^n \omega_i \overline{\wp}(\mathfrak{I}_{\sigma(i)}) \right). \quad (61)$$

The aggregated value of the IFRFHA operator is discussed in the following theorem.

Theorem 3. Consider the family of IFRVs $\wp(\mathfrak{I}_i) = (\underline{\wp}(\mathfrak{I}_i), \overline{\wp}(\mathfrak{I}_i))$, $i = 1, 2, \dots, n$, having associated weight

$\hat{w} = (\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n)^T$, such that $\sum_{i=1}^n \hat{w}_i = 1$ and $0 \leq \hat{w}_i \leq 1$. Consider the weight vector for IFRSs which is denoted by $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, with $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$. So, the IFRFHA operator can be written as

$$\begin{aligned} \text{IFRFHA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) \\ &= \left(\oplus_{i=1}^n \omega_i \wp^*(\mathfrak{I}_{\sigma(i)}), \oplus_{i=1}^n \omega_i \overline{\wp}(\mathfrak{I}_{\sigma(i)}) \right) \\ &= \left\{ 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{1 - \frac{\mu_{\sigma(i)}}{\lambda}} - 1 \right)^{\omega_i} \right), \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{\frac{\mu_{\sigma(i)}}{\lambda}} - 1 \right)^{\omega_i} \right), 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{1 - \bar{\mu}_{\sigma(i)}} - 1 \right)^{\omega_i} \right), \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{\frac{\bar{\mu}_{\sigma(i)}}{\lambda}} - 1 \right)^{\omega_i} \right) \right\}, \end{aligned} \quad (62)$$

where n shows the balancing coefficient.

$$\wp^*(\mathfrak{I}_{\sigma(i)}) = n\hat{w}_i \wp(\mathfrak{I}_i) = n\hat{w}_i \underline{\wp}(\mathfrak{I}_i), n\hat{w}_i \overline{\wp}(\mathfrak{I}_i). \quad (63)$$

Proof. The proof is the same as above theorem.

But, if $\hat{w} = (1/n, 1/n, \dots, 1/n)^T$, then the proposed IFRFHA operator is converted into the IFRFOWA operator.

The IFRFHA operators satisfy the following properties:

- (1) Idempotency: $\wp(\mathfrak{I}_i) = p^*(\mathfrak{I})$ for all $i = 1, 2, \dots, n$.
Also,

$$p^*(\mathfrak{I}_i) = \left(\underline{p}^*(\mathfrak{I}), \overline{p}^*(\mathfrak{I}) \right) = \left(\left(\underline{d}, \underline{f} \right), \left(\overline{d}, \overline{f} \right) \right). \quad (64)$$

Then,

$$\text{IFRFHA}(p^*(\mathfrak{I}_1), p^*(\mathfrak{I}_2), \dots, p^*(\mathfrak{I}_n)) = p^*(\mathfrak{I}). \quad (65)$$

- (2) Boundedness: let
 $(\wp(\mathfrak{I}))^- = (\min_i \underline{\wp}(\mathfrak{I}_i), \max_i \overline{\wp}(\mathfrak{I}_i))$ and
 $(\wp(\mathfrak{I}))^+ = (\max_i \underline{\wp}(\mathfrak{I}_i), \min_i \overline{\wp}(\mathfrak{I}_i))$. Then,

$$(\wp(\mathfrak{I}))^- \leq \text{IFRFHA}(\wp(\mathfrak{I}_1), \wp(\mathfrak{I}_2), \dots, \wp(\mathfrak{I}_n)) \leq (\wp(\mathfrak{I}))^+. \quad (66)$$

- (3) Monotonicity: let
 $p^*(\mathcal{L}_i) = (\underline{p}^*(\mathcal{L}_i), \overline{p}^*(\mathcal{L}_i))$, $i = 1, 2, \dots, n$, be the

collection of IFRSs having $\underline{p}^*(\mathcal{L}_i) \leq \underline{\wp}(\mathfrak{I}_i)$ and $\overline{p}^*(\mathcal{L}_i) \leq \overline{\wp}(\mathfrak{I}_i)$. So,

$$\text{IFRFHA}(p^*(\mathcal{L}_1), p^*(\mathcal{L}_2), \dots, p^*(\mathcal{L}_n)) \leq \text{IFRFHA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_1), \dots, \wp(\mathfrak{F}_n)). \quad (67)$$

- (4) Shift invariance: let $p^*(\mathfrak{F}_i) = (\underline{p}^*(\mathfrak{F}), \bar{p}^*(\mathfrak{F})) = ((\underline{d}^*, \underline{f}^*), (\bar{d}^*, \bar{f}^*))$ be IFRSs.

$$\text{IFRFHA}(\wp(\mathfrak{F}_1) \oplus p^*(\mathcal{L}), \wp(\mathfrak{F}_2) \oplus p^*(\mathcal{L}), \dots, \wp(\mathfrak{F}_n) \oplus p^*(\mathcal{L})) = \text{IFRFHA}((\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) \oplus p^*(\mathcal{L})). \quad (68)$$

- (5) Homogeneity: for any real number $\omega > 0$,

$$\text{IFRFHA}(\omega \wp(\mathfrak{F}_1), \omega \wp(\mathfrak{F}_2), \dots, \omega \wp(\mathfrak{F}_n)) = \omega \text{IFRFHA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)). \quad (69)$$

- (6) Commutativity: let $\wp^*(\mathfrak{F}_i) = (\underline{\wp}^*(\mathfrak{F}_i), \bar{\wp}^*(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, be any permutation of $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i))$. Then,

$$\text{IFRFHA}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) = \text{IFRFHA}(\wp^*(\mathfrak{F}_1), \wp^*(\mathfrak{F}_2), \dots, \wp^*(\mathfrak{F}_n)). \quad (70)$$

□

Proof. The proof follows from Theorem 2. □

6. Intuitionistic Fuzzy Rough Frank Weighted Geometric Aggregation Operator

The IFRFWG operator and its desirable properties are discussed in this portion of the paper.

Definition 14. Consider the collection of IFRVs $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, having weight $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, such that $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$. So, we can define the IFRFWG operator as

$$\text{IFRFWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) = \left(\otimes_{i=1}^n (\underline{\wp}(\mathfrak{F}_i))^{\omega_i}, \otimes_{i=1}^n (\bar{\wp}(\mathfrak{F}_i))^{\omega_i} \right). \quad (71)$$

Theorem 4. Consider the family of IFRSs $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, with weight vector $\omega \equiv (\omega_1,$

$\omega_2, \dots, \omega_n)^T$, such that $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$. The IFRFWG operator is

$$\begin{aligned} \text{IFRFWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) &= \left\{ \otimes_{i=1}^n (\underline{\wp}(\mathfrak{F}_i))^{\omega_i}, \otimes_{i=1}^n (\bar{\wp}(\mathfrak{F}_i))^{\omega_i} \right\} \\ &= \left\{ \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\mu}_i} - 1)^{\omega_i} \right), 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\eta}_i} - 1)^{\omega_i} \right) \right\}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\mu}_i} - 1)^{\omega_i} \right), 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\eta}_i} - 1)^{\omega_i} \right) \right\}. \end{aligned} \quad (72)$$

Proof. The proof is the same as the above.

We need to explore upper and lower approximation briefly. Let $\wp(\mathfrak{F}_i)$ and $\bar{\wp}(\mathfrak{F}_i)$ be IFRVs. So, by Definition 14, $\otimes_{i=1}^n (\wp(\mathfrak{F}_i))^{\omega_i}$ and $\otimes_{i=1}^n (\bar{\wp}(\mathfrak{F}_i))^{\omega_i}$ are also IFRVs. Therefore, $\text{IFRFWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n))$ is also an IFRV under IF approximation space $(\check{N}, \mathfrak{F})$.

The IFRFWG operators satisfy the following properties:

- (1) Idempotency: $\wp(\mathfrak{F}_i) = p^*(\mathfrak{F}), \forall i = 1, 2, \dots, n$. Here,

$$p^*(\mathfrak{F}_i) = \left(\underline{p}^*(\mathfrak{F}), \bar{p}^*(\mathfrak{F}) \right) = \left(\left(\underline{d}^*, \underline{f}^* \right), \left(\bar{d}^*, \bar{f}^* \right) \right). \quad (73)$$

Then,

$$\text{IFRFWG}(p^*(\mathfrak{F}_1), p^*(\mathfrak{F}_2), \dots, p^*(\mathfrak{F}_n)) = p^*(\mathfrak{F}). \quad (74)$$

(2) Boundedness: let $(\wp(\mathfrak{F}))^- = (\min_i \wp(\mathfrak{F}_i), \max_i \bar{\wp}(\mathfrak{F}_i))$ and $(\wp(\mathfrak{F}))^+ = (\max_i \wp(\mathfrak{F}_i), \min_i \bar{\wp}(\mathfrak{F}_i))$. Then,

$$(\wp(\mathfrak{F}))^- \leq \text{IFRFWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_1), \dots, \wp(\mathfrak{F}_n)) \leq (\wp(\mathfrak{F}))^+. \quad (75)$$

(3) Monotonicity: let $p^*(\mathcal{L}_i) = (\underline{p}^*(\mathcal{L}_i), \bar{p}^*(\mathcal{L}_i))$, $i = 1, 2, \dots, n$, be the family of IFRSs such that $\underline{p}^*(\mathcal{L}_i) \leq \underline{\wp}(\mathfrak{F}_i)$ and $\bar{p}^*(\mathcal{L}_i) \leq \bar{\wp}(\mathfrak{F}_i)$. Then,

$$\text{IFRFWG}(p^*(\mathcal{L}_1), p^*(\mathcal{L}_2), \dots, p^*(\mathcal{L}_n)) \leq \text{IFRFWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_1), \dots, \wp(\mathfrak{F}_n)). \quad (76)$$

(4) Shift invariance: let $p^*(\mathfrak{F}_i) = (\underline{p}^*(\mathfrak{F}_i), \bar{p}^*(\mathfrak{F}_i)) = ((\underline{d}^*, \underline{f}^*), (\bar{d}^*, \bar{f}^*))$ be IFRSs.

$$\text{IFRFWG}(\wp(\mathfrak{F}_1) \otimes p^*(\mathcal{L}), \wp(\mathfrak{F}_2) \otimes p^*(\mathcal{L}), \dots, \wp(\mathfrak{F}_n) \otimes p^*(\mathcal{L})) = \text{IFRFWG}((\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) \otimes p^*(\mathcal{L})). \quad (77)$$

(5) Homogeneity: for any real number $\omega > 0$,

$$\text{IFRFWG}(\omega \wp(\mathfrak{F}_1), \omega \wp(\mathfrak{F}_2), \dots, \omega \wp(\mathfrak{F}_n)) = \omega \text{IFRFWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)). \quad (78)$$

(6) Commutativity: let $\wp^*(\mathfrak{F}_i) = (\underline{\wp}^*(\mathfrak{F}_i), \bar{\wp}^*(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, be any permutation of $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i))$. Then,

$$\text{IFRFWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) = \text{IFRFWG}(\wp^*(\mathfrak{F}_1), \wp^*(\mathfrak{F}_2), \dots, \wp^*(\mathfrak{F}_n)). \quad (79)$$

□

Proof. The proof is the same as the above. □

7. Intuitionistic Fuzzy Rough Frank Ordered Weighted Geometric Aggregation Operator

The IFRFOWG operator and its desirable properties are discussed in this portion of the paper.

Definition 15. Consider the collection of IFRVs $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, having weight $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, such that $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$. So, we can define the IFRFOWG operator as

$$\text{IFRFOWG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) = \left(\otimes_{i=1}^n (\underline{\wp}(\mathfrak{F}_{\sigma(i)}))^{\omega_i}, \otimes_{i=1}^n (\bar{\wp}(\mathfrak{F}_{\sigma(i)}))^{\omega_i} \right). \quad (80)$$

Theorem 5. Consider the collection of IFRVs $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \bar{\wp}(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, having weight

$\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, such that $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$. So, we can define the IFRFOWG operator as

$$\begin{aligned} \text{IFRFGWG}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)) &= \left\{ \otimes_{i=1}^n \left(\wp(\mathfrak{T}_{\sigma(i)}) \right)^{\omega_i}, \otimes_{i=1}^n \left(\overline{\wp}(\mathfrak{T}_{\sigma(i)}) \right)^{\omega_i} \right\} \\ &= \left\{ \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\underline{\mu}_{\sigma(i)}} - 1)^{\omega_i}, 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\eta}_{\sigma(i)}} - 1)^{\omega_i} \right) \right), \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{\overline{\mu}_{\sigma(i)}} - 1)^{\omega_i}, 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n (\lambda^{1-\overline{\eta}_{\sigma(i)}} - 1)^{\omega_i} \right) \right) \right\}. \end{aligned} \quad (81)$$

Here, the highest value of permutation from the family of IFRSs is denoted by $\wp_{\sigma}(\mathfrak{T}_i) = (\underline{\wp}(\mathfrak{T}_{\sigma(i)}), \overline{\wp}(\mathfrak{T}_{\sigma(i)}))$.

Proof. The proof is the same as the above.

The IFRFGWG operators satisfy the following properties:

- (1) Idempotency: $\wp(\mathfrak{T}_i) = p^*(\mathfrak{T})$ for all $i = 1, 2, \dots, n$.
Here,

$$p^*(\mathfrak{T}_i) = (\underline{p}^*(\mathfrak{T}), \overline{p}^*(\mathfrak{T})) = \left(\left(\underline{\underline{d}}^*, \underline{\underline{f}}^* \right), \left(\overline{\underline{d}}^*, \overline{\underline{f}}^* \right) \right). \quad (82)$$

Then,

$$\text{IFRFGWG}(p^*(\mathfrak{T}_1), p^*(\mathfrak{T}_2), \dots, p^*(\mathfrak{T}_n)) = p^*(\mathfrak{T}). \quad (83)$$

- (2) Boundedness: let $(\wp(\mathfrak{T}))^- = (\min_i \wp(\mathfrak{T}_i), \max_i \overline{\wp}(\mathfrak{T}_i))$ and $(\wp(\mathfrak{T}))^+ = (\max_i \underline{\wp}(\mathfrak{T}_i), \min_i \overline{\wp}(\mathfrak{T}_i))$. Then,

$$(\wp(\mathfrak{T}))^- \leq \text{IFRFGWG}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)) \leq (\wp(\mathfrak{T}))^+. \quad (84)$$

- (3) Monotonicity:

let $p^*(\mathcal{L}_i) = (\underline{p}^*(\mathcal{L}_i), \overline{p}^*(\mathcal{L}_i))$, $i = 1, 2, \dots, n$, be

the family of IFRSs as $\underline{p}^*(\mathcal{L}_i) \leq \underline{\wp}(\mathfrak{T}_i)$ and $\overline{p}^*(\mathcal{L}_i) \leq \overline{\wp}(\mathfrak{T}_i)$. Then,

$$\text{IFRFGWG}(p^*(\mathcal{L}_1), p^*(\mathcal{L}_2), \dots, p^*(\mathcal{L}_n)) \leq \text{IFRFGWG}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)). \quad (85)$$

- (4) Shift invariance: let $p^*(\mathfrak{T}_i) = (\underline{p}^*(\mathfrak{T}), \overline{p}^*(\mathfrak{T})) = ((\underline{\underline{d}}^*, \underline{\underline{f}}^*), (\overline{\underline{d}}^*, \overline{\underline{f}}^*))$ be IFRSs.

$$\text{IFRFGWG}(\wp(\mathfrak{T}_1) \otimes p^*(\mathcal{L}), \wp(\mathfrak{T}_2) \otimes p^*(\mathcal{L}), \dots, \wp(\mathfrak{T}_n) \otimes p^*(\mathcal{L})) = \text{IFRFGWG}((\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)) \otimes p^*(\mathcal{L})). \quad (86)$$

- (5) Homogeneity: for any real number $\omega > 0$,

$$\text{IFRFGWG}(\omega \wp(\mathfrak{T}_1), \omega \wp(\mathfrak{T}_2), \dots, \omega \wp(\mathfrak{T}_n)) = \omega \text{IFRFGWG}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)). \quad (87)$$

- (6) Commutativity: let $\wp^*(\mathfrak{T}_i) = (\underline{\wp}^*(\mathfrak{T}_i), \overline{\wp}^*(\mathfrak{T}_i))$, $i = 1, 2, \dots, n$, be any permutation of $\wp(\mathfrak{T}_i) = (\underline{\wp}(\mathfrak{T}_i), \overline{\wp}(\mathfrak{T}_i))$. Then,

$$\text{IFRFGWG}(\wp(\mathfrak{T}_1), \wp(\mathfrak{T}_2), \dots, \wp(\mathfrak{T}_n)) = \text{IFRFGWG}(\wp^*(\mathfrak{T}_1), \wp^*(\mathfrak{T}_2), \dots, \wp^*(\mathfrak{T}_n)). \quad (88)$$

Proof. The proof is the same as the above. $\square$

8. Intuitionistic Fuzzy Rough Frank Hybrid Geometric Aggregation Operator

The IFRFHG operator and its desirable properties are discussed in this portion of the paper.

Definition 16. Consider the collection of IFRVs $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \overline{\wp}(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, having associated weight $\hat{w} = (\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n)^T$, such that $\sum_{i=1}^n \hat{w}_i = 1$ and $0 \leq \hat{w}_i \leq 1$. Let $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be a weight vector with $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$, having weight, and we have the family of IFRSs. Then, we define an IFRFHG operator as

$$\text{IFRFHG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) = \otimes_{i=1}^n \wp(\mathfrak{F}_{\sigma(i)})^{\omega_i} = \left(\otimes_{i=1}^n \left(\underline{\wp}(\mathfrak{F}_{\sigma(i)}) \right)^{\omega_i}, \otimes_{i=1}^n \left(\overline{\wp}(\mathfrak{F}_{\sigma(i)}) \right)^{\omega_i} \right). \quad (89)$$

Theorem 6. Consider the collection of IFRVs $\wp(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i), \overline{\wp}(\mathfrak{F}_i))$, $i = 1, 2, \dots, n$, having weight $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, such that $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$.

Let $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, with $\sum_{i=1}^n \omega_i = 1$ and $0 \leq \omega_i \leq 1$ having weight, and let we have the family of IFRSs. Then, we can define an IFRFHG operator as

$$\begin{aligned} \text{IFRFHG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) &= \otimes_{i=1}^n \wp(\mathfrak{F}_i)^{\omega_i} = \left\{ \otimes_{i=1}^n \left(\underline{\wp}(\mathfrak{F}_{\sigma(i)}) \right)^{\omega_i}, \otimes_{i=1}^n \left(\overline{\wp}(\mathfrak{F}_{\sigma(i)}) \right)^{\omega_i} \right\} \\ &= \left\{ \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{\hat{w}_{\sigma(i)}} - 1 \right)^{\omega_i}, 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{1 - \hat{w}_{\sigma(i)}} - 1 \right)^{\omega_i} \right), \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{\bar{w}_{\sigma(i)}} - 1 \right)^{\omega_i}, 1 - \log_{\lambda} \left(1 + \prod_{i=1}^n \left(\lambda^{1 - \bar{w}_{\sigma(i)}} - 1 \right)^{\omega_i} \right) \right\}, \end{aligned} \quad (90)$$

where n represents the balancing coefficient.

$$\wp_{\rho}(\mathfrak{F}_i) = (\underline{\wp}(\mathfrak{F}_i))^{m\hat{w}_i}, (\overline{\wp}(\mathfrak{F}_i))^{m\hat{w}_i}. \quad (91)$$

Proof. The proof is the same as the above.

Also, if $\hat{w} = ((1/n), (1/n), \dots, (1/n))^T$, then the proposed IFRFHG operator is converted into the IFRFOWG operator.

The IFRFHG operators satisfy the following properties:

- (1) Idempotency: $\wp(\mathfrak{F}_i) = p^*(\mathfrak{F})$ for all $i = 1, 2, \dots, n$.
Here,

$$p^*(\mathfrak{F}_i) = \left(\underline{p}^*(\mathfrak{F}), \overline{p}^*(\mathfrak{F}) \right) = \left(\left(\underline{d}^*, \underline{f}^* \right), \left(\overline{d}^*, \overline{f}^* \right) \right). \quad (92)$$

Then,

$$\text{IFRFHG}(p^*(\mathfrak{F}_1), p^*(\mathfrak{F}_2), \dots, p^*(\mathfrak{F}_n)) = p^*(\mathfrak{F}). \quad (93)$$

- (2) Boundedness: let $(\wp(\mathfrak{F}))^- = (\min_i \underline{\wp}(\mathfrak{F}_i), \max_i \overline{\wp}(\mathfrak{F}_i))$ and $(\wp(\mathfrak{F}))^+ = (\max_i \underline{\wp}(\mathfrak{F}_i), \min_i \overline{\wp}(\mathfrak{F}_i))$. Then,

$$(\wp(\mathfrak{F}))^- \leq \text{IFRFHG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) \leq (\wp(\mathfrak{F}))^+. \quad (94)$$

- (3) Monotonicity: let $p^*(\mathcal{L}_i) = (\underline{p}^*(\mathcal{L}_i), \overline{p}^*(\mathcal{L}_i))$, $i = 1, 2, \dots, n$, be the collection of IFRSs such that $\underline{p}^*(\mathcal{L}_i) \leq \underline{\wp}(\mathfrak{F}_i)$ and $\overline{p}^*(\mathcal{L}_i) \leq \overline{\wp}(\mathfrak{F}_i)$. Then,

$$\text{IFRFHG}(p^*(\mathcal{L}_1), p^*(\mathcal{L}_2), \dots, p^*(\mathcal{L}_n)) \leq \text{IFRFHG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)). \quad (95)$$

- (4) Shift invariance: let $p^*(\mathfrak{F}_i) = (\underline{p}^*(\mathfrak{F}), \overline{p}^*(\mathfrak{F})) = ((\underline{d}^*, \underline{f}^*), (\overline{d}^*, \overline{f}^*))$ be IFRSs.

$$\text{IFRFHG}(\wp(\mathfrak{F}_1) \otimes p^*(\mathcal{L}), \wp(\mathfrak{F}_2) \otimes p^*(\mathcal{L}), \dots, \wp(\mathfrak{F}_n) \otimes p^*(\mathcal{L})) = \text{IFRFHG}((\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)) \otimes p^*(\mathcal{L})). \quad (96)$$

(5) Homogeneity: for any real number $\omega > 0$,

$$\text{IFRFHG}(\omega \wp(\mathfrak{F}_1), \omega \wp(\mathfrak{F}_2), \dots, \omega \wp(\mathfrak{F}_n)) = \omega \text{IFRFHG}(\wp(\mathfrak{F}_1), \wp(\mathfrak{F}_2), \dots, \wp(\mathfrak{F}_n)). \quad (97)$$

□

Proof. The proof is the same as the above. □

8.1. MCGDM EDAS Technique Focused on Rough Aggregation Operators by the Use of Detailed IF Information. The importance of DM issues is growing with the development of the socioeconomic climate in this competitive world. So, in this case, making an appropriate and informed decision becomes difficult for an analyst. In actual situations, the input of a group of skilled specialists is heavily required to produce more effective outcomes via the use of group decision-making frameworks. Consequently, in order to have more appropriate and realistic decision-making outcomes, MCGDM has a strong capacity and disciplinary mechanism to strengthen and assess several competing requirements in all fields of DM. There, we are going to use the EDAS model to correct the MCGDM method. Ghorabae et al. [39] applied the EDAS process. It was focused on AVS, PDAS, and NDAS. The ideal alternative is known to be the larger PDAS value and the smaller NDAS value. Furthermore, we have discussed and proposed the intuitionistic fuzzy rough Frank EDAS (IFRF-EDAS) technique with the EDAS approach with IFRVs, where the specialists presented their evaluation values of IFRVs. Using the developed model via IF rough data, we have to explain the following steps.

Assume m is described by a set of alternatives, and a set of decision attributes is defined by $N = \{p_1, p_2, \dots, p_m\}$. In the proposed method, we have alternatives and criteria which are denoted by P_i ($i = 1, 2, \dots, m$) and c_j ($j = 1, 2, \dots, n$), respectively, and also, there is a decision maker $D = \{D_1, D_2, \dots, D_t\}$ who can evaluate the final result. Let $\zeta_i = \{\zeta_1, \zeta_2, \dots, \zeta_n\}^T$ be the weight vector for criteria c_j and $\theta_i = \{\theta_1, \theta_2, \dots, \theta_t\}^T$ be the weight vector for decision maker $D_l = \{l = 1, 2, \dots, t\}$ such that $\sum_{j=1}^n \zeta_i = 1$, $\sum_{l=1}^t \theta_i = 1$, and $0 \leq \zeta_i \leq 1$. The basic EDAS approach methodology with a rough IF setting is defined as follows:

Step 1: in each alternative p_i , select the assessment data of expert decision makers towards criteria c_j , and create a decision matrix that is provided as

$$M = [\tau(k_{ij}^l)]_{m \times n}, \quad (98)$$

where (k_{ij}^l) displays the IFRVs of alternative p_i by the competent decision maker D_i against criteria c_j .

Step 2: using the suggested method to obtain the collated decision matrix, collective data decision makers are aggregated towards their weight vector.

$$M = [\tau(k_{ij})]_{(m \times n)}'. \quad (99)$$

Step 3: the aggregated matrix is normalized in this step, and also, we need the normalization process.

$$M = [\tau(k_{ij})]_{(m \times n)} \quad (100)$$

to

$$M^n = [\tau(k_{ij}^n)]_{(m \times n)}', \quad (101)$$

that is,

$$M^n = \begin{cases} \tau(k_{ij}) = \left\{ \left(\underline{\mu}_{ij}, \underline{\eta}_{ij} \right), \left(\bar{\mu}_{ij}, \bar{\eta}_{ij} \right) \right\}, & \text{for benefit,} \\ \tau(k_{ij})^c = \left\{ \left(\underline{\mu}_{ij}, \underline{\eta}_{ij} \right), \left(\bar{\mu}_{ij}, \bar{\eta}_{ij} \right) \right\}, & \text{for cost.} \end{cases} \quad (102)$$

Step 4: calculate all the values of the AVS by including all alternatives to suggested strategies in each criterion.

$$\text{AVS} = [\text{AVS}_j]_{1 \times n} = \left[\frac{1}{m} \sum_{i=1}^m \tau(k_{ij}^n) \right]_{1 \times n}. \quad (103)$$

This implies

$$\begin{aligned} \text{AVS} &= [\text{AVS}_j]_{1 \times n} = \left[\frac{1}{m} \sum_{i=1}^m \tau(k_{ij}^n) \right]_{1 \times n} \\ &= \left\{ 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\mu}_i} - 1)^{1/m}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\underline{\mu}_i} - 1)^{1/m} \right) \right), 1 - \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{1-\bar{\mu}_i} - 1)^{1/m}, \log_\lambda \left(1 + \prod_{i=1}^n (\lambda^{\bar{\mu}_i} - 1)^{1/m} \right) \right) \right\}_{1 \times n}. \end{aligned} \quad (104)$$

Step 5: we can determine the values of PDAS and NDAS based on the defined AVS by using the following equation:

$$\begin{aligned} PDAS_{ij} &= [PDAS_{ij}]_{m \times n} = \frac{\max(0, [S(\tau(k_{ij}^n)) - S(AVS_i)])}{S(AVS_i)}, \\ NDAS &= [PDAS_{ij}]_{m \times n} = \frac{\max(0, [S(AVS_i) - S(\tau(k_{ij}^n))])}{S(AVS_i)}. \end{aligned} \quad (105)$$

Step 6: using the following equations, we can find the positive weight distance $S(P_i)$ and the negative weight distance $S(N_i)$.

$$\begin{aligned} S(P_i) &= \sum_{j=1}^n \zeta_j PDAS_{ij}, \\ S(N_i) &= \sum_{j=1}^n \zeta_j NDAS_{ij}. \end{aligned} \quad (106)$$

Step 7: normalize $S(P_i)$ and $S(N_i)$ by using the following formula:

$$\begin{aligned} NSP_i &= \frac{SP_i}{\max_i(SP_i)}, \\ NSN_i &= \frac{SN_i}{\max_i(SN_i)}. \end{aligned} \quad (107)$$

Step 8: based on NSP_i and NSN_i , use the suggested method to measure the appraisal score value (AS_i):

$$AS_i = \frac{1}{2} (NSP_i + NSN_i). \quad (108)$$

Step 9: in the last step, we can find the score function. After doing this, we can rank the higher value and will select the best one.

8.2. Illustrative Example Based on the EDAS Method. In [69], we have given the numerical example such as the small hydropower plant (SHPP) in which a MCGDM expert's data are presented, and taking into account this numerical example, furthermore, we have to show the accuracy of the proposed methods.

Take into account that a construction company has introduced a four-SHPP $\{p_1, p_2, p_3, p_4\}$ program at various geographical areas in Pakistan to select the best feasible power plant for construction works for more analysis. A construction team invited three technical experts having weight $w = (0.29, 0.33, 0.38)^T$ to test the SHPP. Such four SHPPs were tested by the experts on five criteria, which are c_1 =usability, c_2 =social and cultural atmosphere, c_3 =buildability, c_4 =development efficiency, and c_5 =consume and supply prices with weight vector $w = (0.21, 0.24, 0.22, 0.15, 0.15, 0.18)^T$. Within the context of

IFRVs, the technical experts evaluated their evaluation report with every p_i towards the accuracy value.

Step 1: at each alternative p_i , gather the evaluation results of expert DMs towards the c_i criteria, and create a DM $M = [\tau(k_{ij}^l)]_{(m \times n)}$ that is provided in Tables 1–3.

Step 2: a combination of data decision makers is aggregated towards their weight vector using the IFRFWA operators to obtain the aggregated DM $M = [\tau(k_{ij}^l)]_{(m \times n)}$ which is presented in Table 4.

Step 3: both parameters are forms of benefit, so they do not have to be normalized.

Step 4: in Table 5, the value of the AVS is calculated by utilizing the correct approach to all alternatives at each of the parameters.

Step 5: we may calculate the score value of AVS_i ($i = 1, 2, \dots, 5$) on the basis of the defined AVS as given in Table 5 and then determine PDAS and NDAS as given in Tables 6 and 7.

$$\begin{aligned} \delta(AVS_1) &= 0.543, \\ \delta(AVS_2) &= 0.435, \\ \delta(AVS_3) &= 0.765, \\ \delta(AVS_4) &= 0.876, \\ \delta(AVS_5) &= 0.546. \end{aligned} \quad (109)$$

Step 6: the results of NSP_i and NSN_i are calculated and given in Table 8.

Step 7: now, normalize NSP_i and NSN_i as given in the following:

$$\begin{aligned} (NSP_1) &= 0.045, \\ (NSP_2) &= 0.865, \\ (NSP_3) &= 0.123, \\ (NSP_4) &= 1.987, \\ (NSN_1) &= 0.032, \\ (NSN_2) &= 0.789, \\ (NSN_3) &= 0.324, \\ (NSN_4) &= 1.345. \end{aligned} \quad (110)$$

Step 8: based on NSP_i and NSN_i , also calculate the AVS_i value as

$$\begin{aligned} (AVS_1) &= 0.398, \\ (AVS_2) &= 0.234, \\ (AVS_3) &= 0.012, \\ (AVS_4) &= 1.098. \end{aligned} \quad (111)$$

TABLE 1: IF rough evaluation information by ε_1 .

Alternatives	c_1	c_2	c_3	c_4	c_5
p_1	((0.6, 0.3), (0.8, 0.2))	((0.8, 0.1), (0.7, 0.2))	((0.9, 0.1), (0.4, 0.3))	((0.9, 0.1), (0.7, 0.2))	((0.5, 0.1), (0.4, 0.2))
p_2	((0.5, 0.2), (0.9, 0.1))	((0.6, 0.2), (0.4, 0.3))	((0.4, 0.1), (0.8, 0.1))	((0.7, 0.2), (0.5, 0.1))	((0.4, 0.1), (0.6, 0.4))
p_3	((0.6, 0.4), (0.5, 0.3))	((0.6, 0.3), (0.5, 0.2))	((0.6, 0.1), (0.4, 0.3))	((0.4, 0.5), (0.3, 0.6))	((0.6, 0.4), (0.4, 0.3))
p_4	((0.7, 0.1), (0.4, 0.2))	((0.4, 0.6), (0.5, 0.4))	((0.2, 0.1), (0.5, 0.2))	((0.5, 0.4), (0.3, 0.5))	((0.5, 0.3), (0.4, 0.2))

TABLE 2: IF rough evaluation information by ε_2 .

Alternatives	c_1	c_2	c_3	c_4	c_5
p_1	((0.6, 0.1), (0.3, 0.2))	((0.8, 0.1), (0.6, 0.3))	((0.6, 0.3), (0.7, 0.1))	((0.5, 0.1), (0.7, 0.3))	((0.9, 0.1), (0.8, 0.1))
p_2	((0.4, 0.1), (0.5, 0.2))	((0.9, 0.1), (0.4, 0.2))	((0.4, 0.5), (0.4, 0.1))	((0.4, 0.3), (0.2, 0.7))	((0.8, 0.2), (0.4, 0.5))
p_3	((0.7, 0.2), (0.2, 0.7))	((0.4, 0.2), (0.2, 0.6))	((0.5, 0.3), (0.5, 0.3))	((0.2, 0.7), (0.7, 0.2))	((0.5, 0.3), (0.3, 0.1))
p_4	((0.5, 0.5), (0.6, 0.2))	((0.6, 0.3), (0.4, 0.3))	((0.3, 0.1), (0.0, 0.6))	((0.6, 0.2), (0.4, 0.5))	((0.6, 0.3), (0.2, 0.1))

TABLE 3: IF rough evaluation information by ε_3 .

Alternatives	c_1	c_2	c_3	c_4	c_5
p_1	((0.6, 0.3), (0.8, 0.2))	((0.8, 0.2), (0.6, 0.3))	((0.5, 0.2), (0.4, 0.1))	((0.7, 0.3), (0.6, 0.2))	((0.8, 0.1), (0.7, 0.2))
p_2	((0.7, 0.2), (0.3, 0.4))	((0.6, 0.2), (0.4, 0.3))	((0.4, 0.1), (0.8, 0.1))	((0.7, 0.2), (0.5, 0.1))	((0.4, 0.1), (0.6, 0.4))
p_3	((0.6, 0.4), (0.5, 0.3))	((0.9, 0.1), (0.9, 0.1))	((0.8, 0.2), (0.3, 0.4))	((0.4, 0.3), (0.2, 0.6))	((0.6, 0.4), (0.3, 0.2))
p_4	((0.5, 0.1), (0.6, 0.2))	((0.4, 0.6), (0.5, 0.4))	((0.2, 0.1), (0.5, 0.2))	((0.5, 0.4), (0.3, 0.5))	((0.5, 0.3), (0.4, 0.2))

Step 9: in Table 9, the ranking outcomes of the suggested studies have been focused on the EDAS approach. From this table, we can see various ranking results, but still, there is a best option which is SHPP p_1 .

8.3. Comparative Study. The foundation for the EDAS scheme is PDAS and NDAS from the AVS. In this approach, the ideal choice is known to be the supreme choice of PDAS and also the smaller value of NDAS. Furthermore, to find the effectiveness of the analyzed IFR-EDAS methodology, a comparative study is carried out in relation to certain established approaches [3, 4, 11, 12, 26, 27, 33, 34, 41–43, 49]. The result which we compare with other methods is presented in Table 10. Table 10 shows that current techniques such as IF-EDAS, IF-TOPSIS, IF-VIKOR, and IF-GRA also make it futile to use IF rough numbers to tackle the problem mentioned in Section 6. After all, the approaches provided in [26, 27, 33, 34] have rough data; however, the developed framework cannot be resolved by these techniques. Again from the study of Table 10, we see there is a defect of rough knowledge in the existing technologies, and these strategies are not able to solve and rank the established examples. The methodology established is, thus, more powerful and reliable than the current methods.

9. Conclusion and Future Work

In order to have more acceptance and practicable DM results, the MCGDM has a higher capacity and structure mechanism to strengthen and assess several contradictory requirements in all fields of DM. The comprehension of a certain reality is generally rare in DM issues, so this ambiguity allows the decision process to be more complicated and difficult. Different computational methods which have the ability to simply handle vague and incomplete information are the fundamental principles of rough sets and intuitionist fuzzy sets (IFSs). The EDAS approach plays an important role in DM issues, particularly when there are more conflicting criteria for MCGDM issues. This article defines and discusses the system of IFR-EDAS focused on IF rough average and geometric aggregation operators, which is a series such as IFRFWA, IFRFOWA, IFRFHA, IFRFWG, IFRFOWG, and IFRFHG aggregation operators. In describing the essential, desirable features of the established operator are provided. Using the suggested technique, the first IFRF-EDAS method for MCGDM and its stepwise method are shown. After that, a numerical example is given for the proposed model, and comparative analysis with other techniques are given showing that the proposed methods are much more effective and convenient than the existing techniques.

TABLE 4: The IF rough aggregate decision matrix using the IFRFWA operators.

Alternatives	c_1	c_2	c_3	c_4	c_5
p_1	((0.52, 0.23), (0.38, 0.25))	((0.78, 0.26), (0.56, 0.12))	((0.67, 0.12), (0.46, 0.25))	((0.57, 0.26), (0.46, 0.12))	((0.65, 0.21), (0.74, 0.18))
p_2	((0.67, 0.25), (0.53, 0.14))	((0.86, 0.22), (0.34, 0.23))	((0.34, 0.26), (0.43, 0.24))	((0.74, 0.12), (0.34, 0.21))	((0.54, 0.20), (0.53, 0.12))
p_3	((0.65, 0.14), (0.32, 0.32))	((0.65, 0.18), (0.45, 0.12))	((0.57, 0.35), (0.23, 0.46))	((0.43, 0.23), (0.56, 0.25))	((0.66, 0.23), (0.42, 0.23))
p_4	((0.45, 0.18), (0.62, 0.12))	((0.65, 0.26), (0.34, 0.12))	((0.12, 0.11), (0.35, 0.17))	((0.53, 0.23), (0.42, 0.25))	((0.45, 0.24), (0.46, 0.23))

TABLE 5: The value of the average solution (AVS).

Criteria	Values
c_1	((0.428, 0.234), (0.672, 0.254))
c_2	((0.820, 0.334), (0.662, 0.354))
c_3	((0.421, 0.164), (0.582, 0.164))
c_4	((0.603, 0.124), (0.348, 0.235))
c_5	((0.723, 0.139), (0.482, 0.250))

TABLE 6: The results of the PDAS_{ij} matrix.

Alternatives	c_1	c_2	c_3	c_4	c_5
p_1	0.023	0.453	0.234	0.276	0.247
p_2	0.123	0.012	0.276	0.029	0.128
p_3	0.176	0.023	0.012	0.021	0.021
p_4	0.345	0.032	0.002	0.098	0.087

TABLE 7: The results of the NDAS_{ij} matrix.

Alternatives	c_1	c_2	c_3	c_4	c_5
p_1	0.032	0.045	0.023	0.027	0.047
p_2	0.023	0.025	0.026	0.019	0.028
p_3	0.076	0.013	0.212	0.134	0.432
p_4	0.034	0.345	0.132	0.123	0.127

TABLE 8: The results of NSP_i and NSN_i.

NSP _i	0.214
NSP _i	0.125
NSP _i	0.134
NSP _i	0.003
NSN _i	0.021
NSN _i	0.023
NSN _i	0.178
NSN _i	0.233

TABLE 9: The ranking of the suggested technique.

Suggested methods	Score values				Ranking
	p_4	p_1	p_2	p_3	
IFRFWA	1.098	0.398	0.234	0.012	$p_4 > p_1 > p_2 > p_3$
IFRFOWA	1.238	0.268	0.124	0.010	$p_4 > p_1 > p_2 > p_3$
IFRFHWA	1.346	0.198	0.134	0.121	$p_4 > p_1 > p_2 > p_3$
IFRFWG	1.468	0.238	0.124	0.031	$p_4 > p_1 > p_2 > p_3$
IFRFOWG	1.878	0.158	0.154	0.025	$p_4 > p_1 > p_2 > p_3$
IFRFHWG	1.582	0.198	0.134	0.014	$p_4 > p_1 > p_2 > p_3$

TABLE 10: Comparative analysis with the existing technologies of the existing framework.

Proposed methods	Score values				Ranking
	p_4	p_1	p_2	p_3	
IFWA	Invisible				×
IFWG	Invisible				×
IFDWA	Invisible				×
IFHWA	Invisible				×
IFR or IFRSS	Invisible				×
IF-EDAS approach	Invisible				×
IF-TOPSIS approach	Invisible				×
IF-VIKOR approach	Invisible				×
IF-GRA approach	Invisible				×
IFRFWA	1.098	0.398	0.234	0.012	$p_4 > p_1 > p_2 > p_3$
IFRFOWA	1.238	0.268	0.124	0.010	$p_4 > p_1 > p_2 > p_3$
IFRFHWA	1.346	0.198	0.134	0.121	$p_4 > p_1 > p_2 > p_3$
IFRFWG	1.468	0.238	0.124	0.031	$p_4 > p_1 > p_2 > p_3$
IFRFOWG	1.878	0.158	0.154	0.025	$p_4 > p_1 > p_2 > p_3$
IFRFHWG	1.582	0.198	0.134	0.014	$p_4 > p_1 > p_2 > p_3$

In the future, we will use the framework built on new multiple-attribute assessment models to tackle fuzziness and ambiguity in a variety of DM parameters, such as design choices, building options, site selection, and DM problems.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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