Research Article

Initial Stress and Gravity on P-Wave Reflection from Electromagneto-Thermo-Microstretch Medium in the Context of Three-Phase Lag Model

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The present paper studied the reflection of thermo-microstretch waves under the generalized thermoelasticity theory which is employed to study the reflection of plane harmonic waves from a semi-infinite elastic solid under the effect of the electromagnetic field, initial stress, and gravity. The formulation is applied under the thermoelasticity theory with three-phase lag, and the reflection coefficient ratio variations with the angle of incidence under different conditions are obtained. Numerical results obtained from the present study are presented graphically and discussed. It is observed that the initial stress, gravitation, and electromagnetic field exert some influence in the thermo-microstretch medium due to reflection of P-waves.

1. Introduction

In recent decades, the influences of electromagnetic field, initial stress, gravitation, and thermal field have been much pronounced in diverse fields, especially, engineering, geophysics, geology, acoustics, plasma, and physics because of its utilitarian aspects in these fields. The generalized theories of thermoelasticity, which admit the finite speed of a thermal signal, have been the center of interest of active research. Allam et al. [1] discussed the Green–Lindsay model of reflection of P and SV waves from the free surface of thermoelastic diffusion, solid under influence of the electromagnetic field, and initial stress. Abo-Dahab and Kilicman [2] illustrated the reflection and transmission of P- and SV-wave phenomena at the interface between solid and liquid media with magnetic field and two thermal relaxation times. Othman and Song [3] developed the reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation. Abd-Alla et al.

[4] studied SV wave's incidence at the interface between solid and liquid media under electromagnetic field and initial stress in the context of three thermoelastic theories. Aboueregail and Abo-Dahab [5] discussed the dual-phase model on magneto-thermoelasticity infinite nonhomogeneous solid having a spherical cavity. Zhou et al. [6] developed the reflection and transmission of plane waves at the interface of pyroelectric bimetals. Singh [7] illustrated the reflection of P and SV waves from the free surface of an elastic solid with generalized thermodiffusion. Sinha and Sinha [8–10] studied the reflection of thermoelastic waves at a solid half-space with thermal relaxation. Angel and Achenbach [11] discussed the reflection and transmission of elastic waves by a periodic array of crack. Kilany et al. [12] developed photothermal and void effect of a semiconductor rotational medium based on the Lord–Shulman theory. Quintanilla and Racke [13] studied a note on stability of three-phase lag heat conduction. Kothari et al. [14] developed the fundamental solutions of generalized

thermoelasticity with three-phase lags. Banik and Kanoria [15] illustrated generalized thermoelastic interaction in a functionally graded isotropic unbounded medium due to varying heat source with three-phase lag effect. Chou and Yang [16] developed two-dimensional dual-phase lag thermal behavior in single-/multilayer structures using the space-time conservation element and solution element (CESE) method. Liu [17] studied numerical analysis of dual-phase lag and heat transfer in a layered cylinder with nonlinear interface boundary conditions. Marzougui et al. [18] used a computational analysis of heat transport irreversibility phenomenon in a magnetized porous channel. Abo-Dahab et al. [19] developed MHD Casson nanofluid flow over nonlinearly heated porous medium in the presence of extending surface effect with suction/injection. Zaim et al. [20] used Galerkin finite element analysis of magneto-hydrodynamic natural convection of Cu-water-nanoliquid in a baffled U-shaped enclosure. Mebarek-Oudina et al. [21] illustrated magneto-thermal convection stability in an inclined cylindrical annulus filled with a molten metal. Ezzat and Othman [22] investigated the plane waves in an electromagneto-thermoelastic perfect conductivity medium with two relaxation times.

The objective of the present investigation is to determine the reflection of P waves from the thermo-electro-magneto-microstretch medium in the context of three-phase lag model under the effect of rotation, electromagnetic field, and gravity with initial stress. The reflection coefficient ratios of various reflected waves with the angle of incidence have been obtained from the 3PHL model and LS theory. Also, the effect of the electromagnetic field and gravity is discussed numerically and illustrated graphically. The physical quantities are obtained and tested by a numerical study using the parameters of Cu as a target and presented graphically. The distribution of these quantities is represented graphically in the presence and absence of electro-magnetic field, initial stress, and gravity field.

2. Formulation of the Problem

The equations of generalized thermo-microstretch in a rectangular coordinate system (x, y, z) with z -axis directed in the media are used. Constant magnetic field intensity $H = (0, H_0, 0)$ is taken as the direction of the y -axis. We begin our

consideration with linearized equations of electrodynamics of slowly moving media.

$$\begin{aligned} J &= \text{curl } h - \epsilon_0 \dot{E}, \\ \text{curl } E &= -\mu_0 \dot{h}, \\ E &= -\mu_0 (\dot{u} \times H), \\ \nabla \cdot h &= 0. \end{aligned} \quad (1)$$

The constitutive equation in the presence of gravitational and electromagnetic field is

$$\sigma_{ji,i} + F_i + G_i = \rho \ddot{u}_i, \quad (2)$$

$$\begin{aligned} F_i &= \mu_0 (J \times H)_i, \\ G_i &= \rho g \left(\frac{\partial w}{\partial x}, 0, -\frac{\partial u}{\partial x} \right). \end{aligned} \quad (3)$$

From the above equations, we can obtain

$$\begin{aligned} E &= \mu_0 H_0 (\dot{w}, 0, -\dot{u}), \\ h &= (0, -H_0 e, 0), \\ J &= (-h_z - \epsilon_0 \mu_0 H_0 \ddot{w}, 0, h_x + \epsilon_0 \mu_0 H_0 \ddot{u}). \end{aligned} \quad (4)$$

From equations (3) and (4), we obtain

$$F = (F_x, F_y, F_z) = (\mu_0 H_0^2 e_x - \epsilon_0 \mu_0^2 H_0^2 \ddot{u}, 0, \mu_0 H_0^2 e_z - \epsilon_0 \mu_0^2 H_0^2 \ddot{w}). \quad (5)$$

So, the displacement vector $\mathbf{u}$ has the components $u_x = u(x, z, t)$, $u_y = 0$, and $u_z = w(x, z, t)$.

$$\begin{aligned} \sigma_{ij} &= (\lambda_0 \varphi^* - P + \lambda u_{k,k}) \delta_{ij} + (\mu + k) u_{j,i} + \mu u_{i,j} - k \epsilon_{ijk} \varphi_k \\ &\quad - \hat{\gamma} T \delta_{ij} - P \omega_{ij}, \\ m_{ij} &= \alpha \varphi_{k,k} \delta_{ij} + \beta \varphi_{i,j} + \gamma \varphi_{j,i}, \\ \lambda_i &= \alpha_0 \varphi_i^*, \end{aligned} \quad (6)$$

where $\omega_{ij} = (1/2)(u_{j,i} - u_{i,j})$.

In component form, the basic governing equations become

$$\left(\mu + k - \frac{P}{2} \right) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + \left(\lambda + \mu + \frac{P}{2} \right) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) - k \frac{\partial \varphi_2}{\partial z} + \lambda_0 \frac{\partial \varphi^*}{\partial x} - \hat{\gamma} \frac{\partial T}{\partial x} + F_1 + \rho g \frac{\partial w}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (7)$$

$$\left(\mu + k - \frac{P}{2} \right) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + \left(\lambda + \mu + \frac{P}{2} \right) \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} \right) + k \frac{\partial \varphi_2}{\partial x} + \lambda_0 \frac{\partial \varphi^*}{\partial z} - \hat{\gamma} \frac{\partial T}{\partial z} + F_3 - \rho g \frac{\partial u}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}, \quad (8)$$

$$(\alpha + \beta + \gamma) \nabla (\nabla \cdot \varphi) - \gamma \nabla \times (\nabla \times \varphi) + k (\nabla \times u) - 2k \varphi = \rho j \frac{\partial^2 \varphi}{\partial t^2}, \quad (9)$$

$$\alpha_0 \nabla^2 \varphi^* - \frac{1}{3} \lambda_1 \varphi^* - \frac{1}{3} \lambda_0 (\nabla \cdot \mathbf{u}) + \frac{1}{3} \hat{\gamma}_1 T = \frac{3}{2} \rho j \frac{\partial^2 \varphi^*}{\partial t^2}, \quad (10)$$

$$K^* \nabla^2 T + \tau_v^* \nabla^2 \dot{T} + K \tau_T \nabla^2 \ddot{T} = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2} \frac{\partial^2}{\partial t^2} \right) [\rho C_e \ddot{T} + \hat{\gamma} T_0 \ddot{e} + \hat{\gamma}_1 T_0 \ddot{\varphi}^*], \quad (11)$$

such that $\tau_v^* = K + K^* \tau_v$, $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial z^2)$.

The constitutive relation can be written as

$$\sigma_{xx} = (\lambda + 2\mu + k) \frac{\partial u}{\partial x} + \lambda \frac{\partial w}{\partial z} + \lambda_0 \varphi^* - \hat{\gamma} T - P, \quad (12)$$

$$\sigma_{zz} = (\lambda + 2\mu + k) \frac{\partial w}{\partial z} + \lambda \frac{\partial u}{\partial x} + \lambda_0 \varphi^* - \hat{\gamma} T - P, \quad (13)$$

$$\begin{aligned} \sigma_{xz} &= \left(\mu + \frac{P}{2} \right) \frac{\partial u}{\partial z} + \left(\mu + k - \frac{P}{2} \right) \frac{\partial w}{\partial x} + k \varphi_2, \\ \sigma_{zx} &= \left(\mu + \frac{P}{2} \right) \frac{\partial w}{\partial x} + \left(\mu + k - \frac{P}{2} \right) \frac{\partial u}{\partial z} - k \varphi_2, \end{aligned} \quad (14)$$

$$\begin{aligned} m_{xy} &= \gamma \frac{\partial \varphi_2}{\partial x}, \\ m_{zy} &= \gamma \frac{\partial \varphi_2}{\partial z}, \end{aligned} \quad (15)$$

$$\begin{aligned} \lambda_x &= \alpha_0 \frac{\partial j^*}{\partial x}, \\ \lambda_z &= \alpha_0 \frac{\partial j^*}{\partial z}. \end{aligned} \quad (16)$$

To facilitate the solution, the following dimensionless quantities are introduced:

$$\begin{aligned} x'_i &= \frac{\omega^*}{c_0} x_i, \\ u'_i &= \frac{\rho c_0 \omega^*}{\hat{\gamma} T_0} u_i, \\ \Theta' &= \frac{\hat{\gamma}}{\rho c_0^2} (T - T_0), \end{aligned} \quad (17)$$

$$\{t', \tau'_T, \tau'_v, \tau'_q\} = \omega^* \{t, \tau_T, \tau_v, \tau_q\},$$

$$\begin{aligned} \sigma'_{ij} &= \frac{\sigma_{ij}}{\hat{\gamma} T_0}, \\ m'_{ij} &= \frac{\omega^*}{c_0 \hat{\gamma} T_0} m_{ij}, \\ \lambda'_i &= \frac{\omega^*}{c_0 \hat{\gamma} T_0} \lambda_i, \\ \varphi'^* &= \frac{\rho c_0^2}{\hat{\gamma} T_0} \varphi^*, \\ \varphi'_2 &= \frac{\rho c_0^2}{\hat{\gamma} T_0} \varphi_2, \\ g' &= \frac{g}{c_0 \omega^*}, \\ \omega^* &= \frac{\rho C_e c_0^2}{k}, \\ h' &= (h/H_0), \text{ and} \\ c_0^2 &= (\lambda + 2\mu + k/\rho), \\ i, j &= 1, 2, 3 \end{aligned} \quad (18)$$

Using equation (18), the governing equations (7)–(16) recast in the following form (after suppressing the primes):

$$\left(\frac{2\mu + 2k - P}{2\rho c_0^2} \right) \nabla^2 u + \left(\frac{2\lambda + 2\mu + P}{2\rho c_0^2} + R_H \right) \frac{\partial e}{\partial x} - \frac{k}{\rho c_0^2} \frac{\partial \varphi_2}{\partial z} + \frac{\lambda_0}{\rho c_0^2} \frac{\partial \varphi^*}{\partial x} - \frac{\rho c_0^2}{\gamma T_0} \frac{\partial \Theta}{\partial x} + g \frac{\partial w}{\partial x} = \beta^2 \frac{\partial^2 u}{\partial t^2}, \quad (19)$$

$$\left(\frac{2\mu + 2k - P}{2\rho c_0^2} \right) \nabla^2 w + \left(\frac{2\lambda + 2\mu + P}{2\rho c_0^2} + R_H \right) \frac{\partial e}{\partial z} + \frac{k}{\rho c_0^2} \frac{\partial \varphi_2}{\partial x} + \frac{\lambda_0}{\rho c_0^2} \frac{\partial \varphi^*}{\partial z} - \frac{\rho c_0^2}{\gamma T_0} \frac{\partial \Theta}{\partial z} - g \frac{\partial u}{\partial z} = \beta^2 \frac{\partial^2 w}{\partial t^2}, \quad (20)$$

$$\nabla^2 \varphi_2 - \frac{2kc_0^2}{\gamma \omega^{*2}} \varphi_2 + \frac{kc_0^2}{\gamma \omega^{*2}} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = \frac{\rho j c_0^2}{\gamma} \frac{\partial^2 \varphi_2}{\partial t^2}, \quad (21)$$

$$\left(\frac{C_1^2}{c_0^2} \nabla^2 - \frac{C_2^2}{\omega^{*2}} - \frac{\partial^2}{\partial t^2} \right) \varphi^* - \frac{C_3^2}{\omega^{*2}} e + a_0 \Theta = 0, \quad (22)$$

$$C_k \nabla^2 \Theta + C_v \nabla^2 \dot{\Theta} + C_T \nabla^2 \ddot{\Theta} = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2} \frac{\partial^2}{\partial t^2} \right) [\ddot{\Theta} + \varepsilon_1 \ddot{e} + \varepsilon_2 \ddot{\varphi}^*], \quad (23)$$

$$\sigma_{xx} = u_x + a_1 w_z + a_2 \varphi^* - a_3 \Theta - a_4, \quad (24)$$

$$\sigma_{zz} = w_z + a_1 u_x + a_2 \varphi^* - a_3 \Theta - a_4, \quad (25)$$

$$\sigma_{xz} = a_5 u_z + a_6 w_x + a_7 \varphi_2, \quad (26)$$

$$\sigma_{zx} = a_5 w_x + a_6 u_z - a_7 \varphi_2, \quad (27)$$

$$\begin{aligned} m_{xy} &= a_8 \frac{\partial \varphi_2}{\partial x}, \\ m_{zy} &= a_8 \frac{\partial \varphi_2}{\partial z}, \end{aligned} \quad (28)$$

$$\begin{aligned} \lambda_x &= a_9 \frac{\partial \varphi^*}{\partial x}, \\ \lambda_z &= a_9 \frac{\partial \varphi^*}{\partial z}, \end{aligned} \quad (29)$$

where

$$\begin{aligned} R_H &= \frac{\mu_0 H_0^2}{\rho c_0^2}, \\ \beta^2 &= \left(1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} \right), \\ (C_1^2, C_2^2, C_3^2) &= \frac{2}{9\rho j} (3\alpha_0, \lambda_0, \lambda_1), \\ a_0 &= \frac{2\rho c_0^4 \hat{\gamma}_1}{9j\hat{\gamma}^2 T_0 \omega^{*2}}, \\ (C_k, C_\nu, C_T) &= \frac{1}{\rho c_0^2 C_e} (K^*, \tau_\nu^*, K\tau_T \omega^*), \\ \varepsilon_1 &= \frac{\hat{\gamma}^3 T_0}{\rho^3 c_0^4 C_e}, \\ \varepsilon_2 &= \frac{\hat{\gamma}_1 \hat{\gamma}^2 T_0^2}{\rho^3 c_0^4 C_e}, \\ a_0 &= \frac{2\rho c_0^4}{9j\hat{\gamma} T_0 \omega^{*2}}, \\ a_4 &= \left(\frac{P}{\hat{\gamma} T_0} + 1 \right), \\ (a_1, a_2, a_5, a_6, a_7) &= \frac{1}{\rho c_0^2} \left(\lambda, \lambda_0, \mu + \frac{P}{2}, \mu + k - \frac{P}{2}, k \right), \\ (a_8, a_9) &= \frac{\omega^{*2}}{\rho c_0^4} (\gamma, \alpha_0). \end{aligned} \quad (30)$$

The displacement components $u(x, z, t)$ and $w(x, z, t)$ may be written in terms of potential functions $\Phi(x, z, t)$ and $\Psi(x, z, t)$ as

$$\begin{aligned} u &= \Phi_x + \Psi_z, \\ w &= \Phi_z - \Psi_x. \end{aligned} \quad (31)$$

Using equation (31) in equations (19)–(23), we obtain

$$\left[\nabla^2 - \zeta_0 \frac{\partial^2}{\partial t^2} \right] \Phi - \zeta_1 \frac{\partial \Psi}{\partial x} - \zeta_2 \Theta + \zeta_3 \varphi^* = 0, \quad (32)$$

$$\zeta_4 \frac{\partial \Phi}{\partial x} + \left[\nabla^2 - \zeta_5 \frac{\partial^2}{\partial t^2} \right] \Psi - \zeta_6 \varphi_2 = 0, \quad (33)$$

$$\frac{kc_0^2}{\gamma \omega^{*2}} \nabla^2 \Psi + \left[\nabla^2 - \frac{j\rho c_0^2}{\gamma} \frac{\partial^2}{\partial t^2} - \frac{2kc_0^2}{\gamma \omega^{*2}} \right] \varphi_2 = 0, \quad (34)$$

$$\left(\frac{C_1^2}{c_0^2} \nabla^2 - \frac{C_2^2}{\omega^{*2}} - \frac{\partial^2}{\partial t^2} \right) \varphi^* - \frac{C_3^2}{\omega^{*2}} \nabla^2 \Phi + a_0 \Theta = 0, \quad (35)$$

$$\begin{aligned} &C_k \nabla^2 \Theta + C_\nu \nabla^2 \dot{\Theta} + C_T \nabla^2 \ddot{\Theta} \\ &= \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2} \frac{\partial^2}{\partial t^2} \right) [\ddot{\Theta} + \varepsilon_1 \nabla^2 \ddot{\Phi} + \varepsilon_2 \ddot{\varphi}^*], \end{aligned} \quad (36)$$

where

$$\begin{aligned} (\zeta_0, \zeta_1, \zeta_2, \zeta_3) &= \frac{1}{1 + R_H} \left(\beta^2, g, \frac{\rho c_0^2}{\gamma T_0}, \frac{\lambda_0}{\rho c_0^2} \right), (\zeta_4, \zeta_5, \zeta_6) \\ &= \frac{2\rho c_0^2}{2\mu + 2k - P} (g, \beta^2, k). \end{aligned} \quad (37)$$

3. Solution of the problem

We assume now that the solution of equations (32)–(36) takes the following form:

$$\{\Phi, \Psi, \Theta, \varphi^*, \varphi_2\} = \{\overline{\Phi}, \overline{\Psi}, \overline{\Theta}, \overline{\varphi^*}, \overline{\varphi_2}\} \exp \cdot [i\xi(x \sin \theta + z \cos \theta) - i\omega t]. \quad (38)$$

Substituting equation (38) into equations (32)–(36), we get

$$[-\xi^2 + \zeta_0 \omega^2] \overline{\Phi} + i\zeta_1 \xi \sin \theta \overline{\Psi} - \zeta_2 \overline{\Theta} + \zeta_3 \overline{\varphi^*} = 0, \quad (39)$$

$$[\xi^2 - \omega^2 \zeta_5] \overline{\Psi} - \zeta_6 \overline{\varphi_2} = -i\zeta_4 \sin \theta \overline{\Phi}, \quad (40)$$

$$-p^* \xi^2 \overline{\Psi} + \left[\xi^2 - \frac{j\rho c_0^2 \omega^2}{\gamma} + \frac{2kc_0^2}{\gamma \omega^{*2}} \right] \overline{\varphi_2} = 0, \quad (41)$$

$$\frac{C_3^2}{\omega^{*2}} \xi^2 \overline{\Phi} + a_0 \overline{\Theta} - \left[\frac{C_1^2}{c_0^2} \xi^2 + \frac{C_2^2}{\omega^{*2}} - \omega^2 \right] \overline{\varphi^*} = 0, \quad (42)$$

$$-\varepsilon_1 \omega^2 \tau_q^* \xi^2 \overline{\Phi} + [(-C_k - i\omega C_v + \omega^2 C_T) \xi^2 + \omega^2 \tau_q^*] \overline{\Theta} + \varepsilon_2 \omega^2 \tau_q^* \overline{\varphi^*} = 0, \quad (43)$$

where $\tau_q^* = 1 - i\omega \tau_q - (\omega^2 \tau_q^2/2)$, $p^* = (kc_0^2/\gamma \omega^{*2})$.

From equations (39)–(43), we get

$$\begin{vmatrix} -\xi^2 + \zeta_0 \omega^2 & i\zeta_1 \xi \sin \theta & -\zeta_2 & \zeta_3 & 0 \\ i\zeta_4 \xi \sin \theta & \xi^2 - \omega^2 \zeta_5 & 0 & 0 & -\zeta_6 \\ 0 & -p^* \xi^2 & 0 & 0 & \xi^2 - \frac{j\rho c_0^2 \omega^2}{\gamma} + \frac{2kc_0^2}{\gamma \omega^{*2}} \\ \frac{C_3^2}{\omega^{*2}} \xi^2 & 0 & a_0 & -\frac{C_1^2}{C_0^2} \xi^2 - \frac{C_2^2}{\omega^{*2}} + \omega^2 & 0 \\ -\varepsilon_1 \omega^2 \tau_q^* \xi^2 & 0 & (-C_k - i\omega C_v + \omega^2 C_T) \xi^2 + \omega^2 \tau_q^* & \varepsilon_2 \omega^2 \tau_q^* & 0 \end{vmatrix} = 0. \quad (44)$$

which tends to

$$A\xi^{10} + B\xi^8 + C\xi^6 + D\xi^4 + E\xi^2 + F = 0, \quad (45)$$

where A, B, C, D, E , and F in equation (45) are given in Appendix A

From equation (45), there are five waves with five different velocities.

Then $\Phi, \Psi, \Theta, \varphi^*$, and φ_2 will take the following forms:

$$\begin{aligned} \{\Phi, \Psi, \Theta, \varphi^*, \varphi_2\} &= \{1, \eta_0, \kappa_0, \chi_0, \vartheta_0\} A_0 \exp[i\xi(x \sin \theta_0 + z \cos \theta_0) - i\omega t] \\ &+ \sum_{j=1}^5 \{1, \eta_j, \kappa_j, \chi_j, \vartheta_j\} A_j \exp[i\xi_j(x \sin \theta_j - z \cos \theta_j) - i\omega t]. \end{aligned} \quad (46)$$

From equation (40) and (41), we get η_j, κ_j, χ_j , and ϑ_j which are given in Appendix B.

3.1. The Boundary Conditions. The boundary conditions for the problem be taken as

$$\sigma_{zz} + \tau_{zz} = 0, \sigma_{xz} + \tau_{xz} = 0, \frac{\partial T}{\partial z} = 0, m_{yz} = 0, \lambda_x = 0, \text{ at } z = 0, \quad (47)$$

$$\begin{aligned} & \sum_{j=1}^5 \left[a_1 \vartheta_j - \mu_e H_0^2 \xi_j^2 - a_3 \xi_j^2 \left(\sin^2 \theta_j + \frac{\eta_j}{2} \sin 2\theta_j \right) - a_2 \xi_j^2 \left(\cos^2 \theta_j - \frac{\eta_j}{2} \sin 2\theta_j + \frac{k_j}{\xi_j^2} \right) \right] A_j \\ &= \left[a_1 \vartheta_1 - \mu_e H_0^2 \xi_1^2 - a_3 \xi_1^2 \left(\sin^2 \theta_0 + \frac{\eta_1}{2} \sin 2\theta_0 \right) - a_2 \xi_1^2 \left(\cos^2 \theta_0 - \frac{\eta_1}{2} \sin 2\theta_0 + \frac{k_1}{\xi_1^2} \right) \right] A_0, \end{aligned} \quad (48)$$

$$\begin{aligned} & \sum_{j=1}^5 \left[(a_4 + a_5) \frac{\xi_j^2}{2} \sin 2\theta_j + a_4 \xi_j^2 \eta_j \cos^2 \theta_j - a_5 \xi_j^2 \eta_j \sin^2 \theta_j + a_6 \xi_j \right] A_j \\ &= \left[-(a_4 + a_5) \frac{\xi_1^2}{2} \sin 2\theta_0 + a_4 \xi_1^2 \eta_1 \cos^2 \theta_0 - a_5 \xi_1^2 \eta_1 \sin^2 \theta_0 + ca_6 \xi_1 \right] A_0, \end{aligned} \quad (49)$$

$$\sum_{j=1}^5 k_j \xi_j \cos \theta_j A_j = k_1 \xi_1 \cos \theta_0 A_0, \quad (50)$$

$$\sum_{j=1}^5 \zeta_j \xi_j \cos \theta_j A_j = \zeta_1 \xi_1 \cos \theta_0 A_0, \quad (51)$$

$$\sum_{j=1}^5 \vartheta_j \xi_j \cos \theta_j A_j = -\vartheta_1 \xi_1 \cos \theta_0 A_0. \quad (52)$$

From equations (55)–(60), we get

$$a_{ij} Z_j = B_i, \quad i, j = (1, 2, \dots, 5), \quad (53)$$

$$Z_j = \frac{A_j}{A_0},$$

$$a_{1j} = a_1 \vartheta_j - \xi_j^2 \left[a_3 \left(\sin^2 \theta_j + \frac{\eta_j}{2} \sin 2\theta_j \right) - a_2 \left(\cos^2 \theta_j - \frac{\eta_j}{2} \sin 2\theta_j + \frac{k_j}{\xi_j^2} \right) + \mu_e H_0^2 \right], \quad (54)$$

$$a_{2j} = (a_4 + a_5) \frac{\xi_j^2}{2} \sin 2\theta_j + a_4 \xi_j^2 \eta_j \cos^2 \theta_j - a_5 \xi_j^2 \eta_j \sin^2 \theta_j + a_6 \xi_j, \quad (55)$$

$$\begin{aligned} a_{3j} &= k_j \xi_j \cos \theta_j, \\ a_{4j} &= \zeta_j \xi_j \cos \theta_j A_j, \\ a_{5j} &= \vartheta_j \xi_j \cos \theta_j, \end{aligned} \quad (56)$$

FIGURE 1: Geometry of the problem.

$$B_1 = -a_1 \vartheta_1 + \xi_j^2 \left[a_3 \left(\sin^2 \theta_0 + \frac{\eta_1}{2} \sin 2\theta_0 \right) - a_2 \left(\cos^2 \theta_0 - \frac{\eta_1}{2} \sin 2\theta_0 + \frac{k_j}{\xi_j^2} \right) + \mu_e H_0^2 \right], \quad (57)$$

$$B_2 = -(a_4 + a_5) \frac{\xi_1^2}{2} \sin 2\theta_0 - a_4 \xi_1^2 \eta_1 \cos^2 \theta_1 + a_5 \xi_1^2 \eta_1 \sin^2 \theta_1 - a_6 \xi_1, \quad (58)$$

$$\begin{aligned} B_3 &= k_1 \xi_1 \cos \theta_0, \\ B_4 &= \zeta_1 \xi_1 \cos \theta_0, \\ B_5 &= -\vartheta_1 \xi_1 \cos \theta_0. \end{aligned} \quad (59)$$

4. Numerical Results and Discussion

The physical constants used are [3]

$$\begin{aligned} i &= \sqrt{-1}, \alpha_0 = 0.779 \times 10^{-4}, \lambda_0 = 0.5 \times 10^{11}, \lambda_1 = 0.5 \times 10^{11}, j = 0.2 \times 10^{-15}, \rho = 8954, C_e = 383.1, \\ k &= 386, T_0 = 293, \lambda = 7.76 \times 10^{10}, \mu = 3.86 \times 10^{10}, \gamma = 0.779 \times 10^{-4}, K^* = 2.97 \times 10^{13}, \mu_0 = 0.1, \\ \varepsilon_0 &= 0.1, \omega = 0.24, \tau_T = 0.002, \tau_v = 0.001, \tau_q = 0.005. \end{aligned} \quad (60)$$

Figure 2 shows the variation of the amplitude of reflected P_1 wave $|Z_1|$, amplitude of reflected P_2 wave $|Z_2|$, amplitude of reflected P_3 wave $|Z_3|$, amplitude of reflected P_4 wave $|Z_4|$, and amplitude of reflected P_5 wave $|Z_5|$ with respect to the angle of incidence θ for different values of the initial stress. The amplitude of the reflected wave increases with increasing of the angle of incidence until it reaches a maximum value at θ about 90. The amplitude of reflected P_2 wave $|Z_2|$, the amplitude of reflected P_3 wave $|Z_3|$, the amplitude of reflected P_4 wave $|Z_4|$, and the amplitude of reflected P_5 wave $|Z_5|$ each of them decrease with increasing of the angle of incidence until it becomes a minimum value at $\theta = 90$. In addition, all lines begin to coincide when the horizontal

angle of incidence θ increases to the maximum. These results obey physical reality for the behavior of reflection of P waves [3].

Figure 3 shows the variation of the amplitude of reflected P_1 wave $|Z_1|$, amplitude of reflected P_2 wave $|Z_2|$, amplitude of reflected P_3 wave $|Z_3|$, amplitude of reflected P_4 wave $|Z_4|$, and amplitude of reflected P_5 wave $|Z_5|$ with respect to the angle of incidence θ for different values of the gravitation. The amplitude of reflected P_1 wave $|Z_1|$, the amplitude of the reflected wave $|Z_2|$, and the amplitude of reflected P_5 wave $|Z_5|$ increases with increasing angle of incidence until it reaches a maximum value at θ about 90. The amplitude of reflected P_3 wave $|Z_3|$ and the amplitude of reflected P_4 wave

FIGURE 2: Variation of the amplitudes $|Z_i|$ ($i = 1, 2, \dots, 5$) with the angle of incidence of P waves for variation of initial stress.

$|Z_4|$ decreases with increasing angle of incidence until it becomes a minimum value at $\theta = 90$. It is noticed that the reflection of P waves is strongly affected by the presence of gravity [23].

Figures 4 and 5: show the variation of the amplitude of reflected P_1 wave $|Z_1|$, amplitude of reflected P_2 wave $|Z_2|$, amplitude of reflected P_3 wave $|Z_3|$, amplitude of reflected P_4 wave $|Z_4|$, and amplitude of reflected P_5 wave $|Z_5|$ with respect to the angle of incidence θ for different values of the electric field and magnetic field, respectively. The amplitude of reflected P_1 wave $|Z_1|$, amplitude of reflected P_2 wave $|Z_2|$, amplitude of reflected P_3 wave $|Z_3|$, amplitude of reflected P_4 wave $|Z_4|$, and amplitude of reflected P_5 wave $|Z_5|$

increase with increasing electric field and magnetic field. The electric field and magnetic field increase the magnitude of reflection of P waves. This is mainly due to the fact that the electric field and magnetic field correspond to a term signifying a positive force that tends to accelerate the charge carries [2].

Figure 6 shows the variation of the amplitude of reflected P_1 wave $|Z_1|$, amplitude of reflected P_2 wave $|Z_2|$, amplitude of reflected P_3 wave $|Z_3|$, amplitude of reflected P_4 wave $|Z_4|$, and amplitude of reflected P_5 wave $|Z_5|$ with respect to the angle of incidence θ . The change of amplitude of reflected to wave appeared in this figure, where we notice $|Z_3| > |Z_4| > |Z_5| > |Z_2| > |Z_1|$ of the existence of gravity field

FIGURE 3: Variation of the amplitudes $|Z_i|$ ($i = 1, 2, \dots, 5$) with the angle of incidence of P waves for variation of gravity field.

FIGURE 4: Variation of the amplitudes $|Z_i|$ ($i = 1, 2, \dots, 5$) with the angle of incidence of P waves for variation of magnetic field.

FIGURE 5: Variation of the amplitudes $|Z_i|$ ($i = 1, 2, \dots, 5$) with the angle of incidence of P waves for variation of electric field.

gravitation, electric field, magnetic field, and initial stress. These figures are very important to study the dependence of these physical quantities on the vertical amplitude of reflected of

distance. The curves obtained are highly depending on the vertical distance from origin, and all the physical quantities satisfy boundary condition and are moving in P waves [24].

FIGURE 6: Variation of the amplitudes $|Z_i|$ ($i = 1, 2, \dots, 5$) with the angle of incidence of P waves: $P = 10^{10}$, $\varepsilon_0 = 0.1$, $g = 0.1$, and $H_0 = 10^3$.

5. Conclusion

We obtain the following conclusions based on the above analysis:

- (1) The reflected P waves and amplitude of the reflected P waves depend on the angle of incidence, rotation, initial stress, gravity field, and magnetic field, and the nature of this dependence is different for different reflected P waves.
- (2) The rotation, initial stress, gravity field, and electromagnetic field play a significant role, and the effects have the inverse trend for the reflected P waves and amplitude of the reflected P waves.
- (3) The rotation, initial stress, gravity field, and electromagnetic field have a strong effect on the reflected P waves and amplitude of the reflected P waves.
- (4) The result provides a motivation to investigate the reflection of P waves from thermo-magneto-microstretch materials as a new class of applications thermoelectric solids. The results presented in this paper should prove to be useful for researchers in material science, designers of new materials, microsystem technologies, physicists, as well as for those working on the development of thermo-electro-magneto-microstretch and in practical situations as in geophysics, optics, acoustics, and geomagnetic and oil prospecting.
- (5) The equations were solved by using the wave function assumption which is the basis of the normal mode method and the features of this system. We can find the amplitude of the primary wave under the influence of different variables, whether gravity, primary gain, and electric and magnetic fields, by varying relaxation time.

It is observed that the amplitudes of reflected P waves change in the presence of rotation, initial stress, gravity field, and magnetic field. Hence, the presence of rotation, initial stress, gravity field, and electromagnetic field significantly affected the reflection phenomena.

Nomenclature

$\vec{B}$:	Magnetic induction vector
C_e :	Specific heat per unit mass
$\vec{E}$:	Electric intensity vector
e_{ij} :	Strain tensor
$\vec{F}_i$:	Lorentz's body force vector
g :	Gravity field
$\vec{H}$:	Magnetic field vector
$\vec{h}$:	Perturbed magnetic field vector
$\vec{H}_0$:	Primary constant magnetic field vector
$\vec{i} = \sqrt{-1}, j$:	Microinertia moment
$\vec{J}$:	Electric current density vector
K, K^* :	Thermal conduction coefficients
m_{ij} :	Coupled stress tensor
P :	Initial stress
t :	Time
T_0 :	The reference temperature
T :	The temperature distribution
$\mathbf{u}$:	Displacement vector
$\alpha_0, \lambda_0, \lambda_1$:	Microstretch constants
α_t :	Coefficient of linear thermal expansion
δ_{ij} :	Kronecker delta
ε_0 :	Electric permeability
φ :	Rotation vector
φ^* :	Scalar microstretch
k, α, β, γ :	Micropolar constants
λ, μ :	Lame' constants
μ_0 :	Magnetic permeability
$v = (\omega/\xi)$:	Velocity of the coupled waves
ρ :	Mass density
σ_{ij} :	Stress tensor
τ_{ij} :	Maxwell's stress tensor
τ_q :	The phase-lag of the heat flux
τ_T :	The phase-lag of temperature gradient
τ_v :	Thermal displacement phase lag
ω :	Complex frequency
ξ :	Wave number
$\hat{\gamma}$:	$(3\lambda + 2\mu + k)\alpha_{t_1}$
$\hat{\gamma}_1$:	$(3\lambda + 2\mu + k)\alpha_{t_2}$

Appendix

The coefficients of Eq. (45)

$$A = -\frac{C_1^2}{c_0^2} L,$$

$$B = \frac{C_1^2}{c_0^2} \left(p^* L \zeta_6 - \zeta_2 \varepsilon_1 \omega^2 \tau_q^* + L \omega^2 \zeta_5 + \zeta_0 \omega^2 L + LS - \omega^2 \tau_q^* \right) - \frac{C_2^2}{\omega^{*2}} L + \omega^2 L + \zeta_3 L \frac{C_3^2}{\omega^{*2}},$$

$$\begin{aligned} C = & -a_0 \varepsilon_2 \omega^2 \tau_q^* - \frac{C_2^2}{\omega^{*2}} \omega^2 \tau_q^* + \omega^4 \tau_q^* + \frac{C_1^2}{c_0^2} \omega^4 \tau_q^* \zeta_5 + \frac{C_2^2}{\omega^{*2}} L \omega^2 \zeta_5 - \omega^4 L \zeta_6 + \frac{C_1^2}{c_0^2} \omega^4 \tau_q^* \zeta_0 + \frac{C_2^2}{\omega^{*2}} L \zeta_0 \omega^2 \\ & - \omega^4 L \zeta_0 - \frac{C_1^2}{c_0^2} L \zeta_0 \zeta_5 \omega^4 + \frac{C_1^2}{c_0^2} \omega^2 \tau_q^* S + \frac{C_2^2}{\omega^{*2}} LS - C_1^2 LS \omega^2 \zeta_5 - \omega^2 LS - \frac{C_1^2}{c_0^2} LS \zeta_0 \omega^2 - \zeta_1 \zeta_4 \sin^2 \vartheta \frac{C_1^2}{c_0^2} L \\ & + p^* \frac{C_1^2}{c_0^2} \omega^2 \tau_q^* \zeta_6 + p^* \frac{C_2^2}{\omega^{*2}} L \zeta_6 + p^* \frac{C_2^2}{\omega^{*2}} \omega^2 \tau_q^* \zeta_6 - p^* \omega^2 L \zeta_6 - \zeta_6 \zeta_0 \omega^2 \frac{C_2^2}{c_0^2} L p^* + \zeta_2 \frac{C_3^2}{\omega^{*2}} \varepsilon_2 \omega^2 \tau_q^* \\ & - \zeta_2 \varepsilon_1 \omega^2 \tau_q^* \frac{C_2^2}{\omega^{*2}} + \zeta_2 \varepsilon_1 \omega^4 \tau_q^* - \zeta_3 LS \frac{C_3^2}{\omega^{*2}} - \zeta_3 \zeta_5 L \omega^2 \frac{C_3^2}{\omega^{*2}} - \zeta_3 \zeta_6 p^* L \frac{C_3^2}{\omega^{*2}} + \zeta_2 \varepsilon_1 \omega^2 \tau_q^* S \frac{C_1^2}{c_0^2} \\ & + \zeta_2 \zeta_5 \varepsilon_1 \omega^4 \tau_q^* \frac{C_1^2}{c_0^2} - \zeta_2 \zeta_6 p^* \varepsilon_1 \omega^2 \tau_q^* \frac{C_1^2}{c_0^2} + \zeta_3 \omega^2 \tau_q^* \frac{C_3^2}{\omega^{*2}} + \zeta_3 a_0 \varepsilon_1 \omega^2 \tau_q^*, \end{aligned}$$

$$\begin{aligned} D = & \omega^4 \zeta_5 a_0 \varepsilon_2 \tau_q^* + \frac{C_2^2}{\omega^{*2}} \omega^4 \tau_q^* \zeta_5 - \omega^6 \tau_q^* \zeta_5 - a_0 \varepsilon_2 \omega^4 \zeta_0 \tau_q^* + \frac{C_2^2}{\omega^{*2}} \omega^4 \zeta_0 \tau_q^* - \omega^6 \zeta_0 \tau_q^* + \frac{C_2^2}{\omega^{*2}} \omega^2 \tau_q^* S \\ & - \frac{C_1^2}{c_0^2} \omega^6 \tau_q^* \zeta_0 \zeta_5 - \frac{C_2^2}{\omega^{*2}} L \zeta_0 \zeta_5 \omega^4 + \omega^6 L \zeta_0 \zeta_5 + a_0 \varepsilon_2 \omega^2 S \tau_q^* - \omega^4 \tau_q^* S - S \zeta_0 \omega^2 \frac{C_2^2}{\omega^{*2}} L + \omega^4 LS \zeta_0 \\ & - \frac{C_1^2}{c_0^2} \omega^4 \tau_q^* S \zeta_5 - \frac{C_2^2}{\omega^{*2}} LS \omega^2 \zeta_5 + \omega^4 LS \zeta_5 - \frac{C_1^2}{c_0^2} \omega^4 \tau_q^* S \zeta_0 - \zeta_1 \zeta_4 \sin^2 \vartheta \frac{C_1^2}{c_0^2} \omega^2 \tau_q^* - \zeta_1 \zeta_4 \sin^2 \vartheta \frac{C_2^2}{\omega^{*2}} \tau_q^* L \\ & + S \zeta_0 \zeta_5 \omega^4 \frac{C_1^2}{c_0^2} L + \zeta_1 \zeta_4 \sin^2 \vartheta \frac{C_1^2}{c_0^2} LS - \zeta_0 \zeta_6 p^* \omega^4 \tau_q^* \frac{C_1^2}{c_0^2} - \zeta_0 \zeta_6 p^* \omega^2 \frac{C_2^2}{\omega^{*2}} L - p^* \omega^4 \tau_q^* \zeta_6 \\ & + \zeta_1 \zeta_4 \sin^2 \vartheta \omega^2 L + p^* a_0 \varepsilon_2 \omega^2 \tau_q^* \zeta_6 + \zeta_2 S \frac{C_2^2}{\omega^{*2}} \varepsilon_1 \tau_q^* \omega^2 - \zeta_2 S \varepsilon_1 \tau_q^* \omega^4 - \zeta_2 \zeta_5 \frac{C_3^2}{\omega^{*2}} \varepsilon_2 \tau_q^* \omega^4 - \zeta_2 \zeta_5 \varepsilon_1 \tau_q^* \omega^6 \\ & + \zeta_0 \zeta_6 p^* \omega^4 L - \zeta_2 S \frac{C_3^2}{\omega^{*2}} \varepsilon_2 \tau_q^* \omega^2 - \zeta_3 \zeta_5 \omega^4 \tau_q^* a_0 \varepsilon_1 + \zeta_3 \zeta_5 \omega^2 LS \frac{C_3^2}{\omega^{*2}} - \zeta_3 \zeta_6 \omega^2 \tau_q^* a_0 \varepsilon_1 - \zeta_3 \zeta_5 \omega^4 \tau_q^* \frac{C_3^2}{\omega^{*2}} \\ & + \zeta_2 \zeta_5 \frac{C_2^2}{\omega^{*2}} \varepsilon_1 \tau_q^* \omega^4 - \zeta_2 \zeta_5 \varepsilon_1 \tau_q^* \omega^4 S \frac{C_1^2}{c_0^2} + \zeta_2 \zeta_6 p^* \omega^2 \tau_q^* \frac{C_3^2}{\omega^{*2}} - \zeta_3 S \omega^2 \tau_q^* \frac{C_3^2}{\omega^{*2}} - \zeta_3 S a_0 \varepsilon_1 \tau_q^* \omega^2 \end{aligned}$$

$$\begin{aligned}
& -\zeta_2 \zeta_6 P^* \omega^2 \tau_q^* \varepsilon_1 \frac{C_2^2}{\omega^{*2}} + \zeta_2 \zeta_6 P^* \omega^4 \tau_q^* \varepsilon_1, \\
E = & a_0 \varepsilon_2 \omega^6 \zeta_0 \zeta_5 \tau_q^* - \frac{C_2^2}{\omega^{*2}} \omega^6 \tau_q^* \zeta_0 \zeta_5 + \omega^8 \zeta_0 \zeta_5 \tau_q^* - S \omega^4 \zeta_5 a_0 \varepsilon_2 \tau_q^* - \frac{C_2^2}{\omega^{*2}} \omega^4 \tau_q^* \zeta_5 S - \zeta_3 \zeta_6 P^* \omega^2 \tau_q^* \frac{C_3^2}{\omega^{*2}} \\
& + \omega^6 \tau_q^* \zeta_5 S - \omega^4 \tau_q^* \zeta_0 S a_0 \varepsilon_2 - \frac{C_2^2}{\omega^{*2}} \omega^4 \tau_q^* S \zeta_0 + \omega^6 \tau_q^* \zeta_0 S + \frac{C_1^2}{C_0^2} \omega^6 \tau_q^* S \zeta_0 \zeta_5 - S \zeta_0 \zeta_5 \omega^6 L \\
& + \frac{C_2^2}{\omega^{*2}} L S \zeta_0 \zeta_5 \omega^4 + \sin^2 \vartheta \zeta_1 \zeta_4 S \frac{C_1^2}{C_0^2} \omega^2 \tau_q^* + \zeta_1 \zeta_4 \sin^2 \vartheta \frac{C_2^2}{\omega^{*2}} L S + \zeta_1 \zeta_4 \sin^2 \vartheta \omega^4 \tau_q^* + \zeta_2 \zeta_5 \omega^6 S \varepsilon_1 \tau_q^* \\
& - \zeta_1 \zeta_4 \sin^2 \vartheta \omega^2 L S - \zeta_1 \zeta_4 \sin^2 \vartheta \omega^2 a_0 \varepsilon_2 \tau_q^* - \zeta_1 \zeta_4 \sin^2 \vartheta \frac{C_2^2}{\omega^{*2}} \omega^2 \tau_q^* + \zeta_2 \zeta_5 \omega^4 S \frac{C_3^2}{\omega^{*2}} \varepsilon_2 \tau_q^* - \zeta_2 \zeta_5 \omega^4 S \frac{C_2^2}{\omega^{*2}} \varepsilon_1 \tau_q^* \\
& + \zeta_0 \zeta_6 \omega^6 P^* \tau_q^* - \zeta_0 \zeta_6 P^* a_0 \varepsilon_2 \omega^4 \tau_q^* - \zeta_0 \zeta_6 P^* \omega^4 \tau_q^* \frac{C_2^2}{\omega^{*2}} + \zeta_3 \zeta_5 \omega^4 S \varepsilon_1 \tau_q^* a_0 + \zeta_3 \zeta_5 \omega^4 S \frac{C_3^2}{\omega^{*2}} \tau_q^*, \\
F = & \tau_q^* S \omega^2 \left(\zeta_0 \zeta_5 \omega^4 a_0 \varepsilon_2 + \zeta_0 \zeta_5 \omega^4 \frac{C_2^2}{\omega^{*2}} + \zeta_1 \zeta_4 \sin^2 \vartheta a_0 \varepsilon_2 + \zeta_1 \zeta_4 \sin^2 \vartheta \frac{C_2^2}{\omega^{*2}} - \zeta_1 \zeta_4 \sin^2 \vartheta \omega^2 \right), \\
S = & \frac{-j \rho c_0^2 \omega^2}{\tilde{\gamma}} + \frac{2 k c_0^2}{\tilde{\gamma} \omega^{*2}}, \\
L = & -C_k - i \omega C_v + \omega^2 C_T.
\end{aligned} \tag{A.1}$$

The amplitudes of Eq. (46)

$$\begin{aligned}
\eta_j &= \frac{-i \xi_j \zeta_4 \sin \theta_j M_2}{(\xi_j^2 - \omega^2 \zeta_5) M_2 - \rho^* \xi_j^2 \zeta_6}, \\
\kappa_j &= \frac{i \rho^* \xi_j^3 \zeta_4 \sin \theta_j}{(\xi_j^2 - \omega^2 \zeta_5) M_2 - \rho^* \xi_j^2 \zeta_6}, \\
\chi_j &= \frac{-\varepsilon_2 \omega^2 \tau_q^* \xi_j^2 (C_3^2 / \omega^{*2}) - \varepsilon_1 \omega^2 \tau_q^* M_1 \xi_j^2}{a_0 \varepsilon_2 \omega^2 \tau_q^* - [L \xi_j^2 + \omega^2 \tau_q^*] M_1}, \\
\vartheta_j &= \frac{a_0 \varepsilon_1 \omega^2 \tau_q^* \xi_j^2 + (C_3^2 / \omega^{*2}) \xi_j^2 [L \xi_j^2 + \omega^2 \tau_q^*]}{a_0 \varepsilon_2 \omega^2 \tau_q^* - [L \xi_j^2 + \omega^2 \tau_q^*] M_1}, \\
M_1 &= \left[-\frac{C_1^2}{C_0^2} \xi_j^2 - \frac{C_2^2}{\omega^{*2}} + \omega^2 \right], \\
M_2 &= \xi_j^2 + S.
\end{aligned} \tag{A.2}$$

Data Availability

The data used to support the findings of this study are available from the authors upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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