Research Article

MHD Mixed Convection Nanofluid Flow over Convectively Heated Nonlinear due to an Extending Surface with Soret Effect

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The aim of this paper is to investigate the flow of MHD mixed convection nanofluid flow under nonlinear heated due to an extending surface. The governing partial differential equations (PDEs) of the boundary layer are reduced to ordinary differential equations (ODEs) considering a technique of the transformation of similarity. The transformed equations are solved numerically considering the technique of an efficient numerical shooting applying the Runge–Kutta technique scheme from the fourth-fifth order. The results corresponding to the dimensionless speed, temperature, concentration profiles, and the Nusselt number reduced, and the Sherwood numbers are presented by figures to display the physical meaning of the phenomena. A comparison has been made between the obtained results with the previous results obtained by others and agrees with them if the new parameters vanish. The results obtained indicate the impacts of the nondimensional governing parameters, namely, magnetic field parameter $M$, Soret number $Sr$, heat source $\lambda$, thermal buoyancy parameter $\lambda_T$, and solutal buoyancy parameter $\lambda_C$, on the flow, temperature, and concentration profiles being discussed and presented graphically.

1. Introduction

Nanofluids are suspended particles of the fluid. They have particles with a nanometer size, and they have a less uniform dispersion in the rigid particles. Nanofluids have crucial usages in science and technology, marine engineering, and applications in the field of industry such as plastic, polymer industries, cancer home therapy, and building sciences. They flow through transferring vertical plane plate also having enormous applications in the field of aerosols engineering, aerodynamics, and civil engineering and because of this reason, the researchers are likely to investigate this field. Nonlinear warm air radiation and chemical reaction effects on MHD 3D Casson fluid movement in the porous medium were considered by Sulochana et al. [1]. Wahiduzzaman et al. [2] investigated MHD Casson fluid movement going through a nonisothermal porous linearly stretching sheet. Ramreddy et al. [3] analyzed the Soret effect on mixed convection flow in a nanofluid under convective boundary condition radiation and the Soret effects of MHD nanofluid moving freely from a moving vertical moving plate in the porous medium were discussed by Raju et al. [4]. The mixed convective flow of Maxwell nanofluid goes beyond an absorbent vertical stretched surface. An optimal solution is considered by Ramzan et al. [5]. Stagnation electrical MHD nanofluid varied convection with slip boundary on a stretching sheet was discussed by Hsiao [6]. Rafique et al. [7] have obtained the numerical solution of casson nanofluid streams over a nonlinear sloping surface through Soret and Dufour effects by the Keller–Box method. Mkhatshwa et al. [8] investigated the MHD mixed convective nanofluid flow about a vertical thin cylinder using the interfering
multidomain spectral collocation approach. Huang [9] analyzed the effect of non-Darcy and MHD on free convection of non-Newtonian fluids over a vertical plate which can be infiltrated in a pored medium with Soret/Dufour effects and thermal radiation.


Chemi et al. [31] researched the critical impendence load of double-wall carbon nanotubes using nonlocal theoretical elasticity. Timesli [32] studied the twisting analysis of double-wall carbon nanotubes embedded in a flexible Kerr medium under axial pressure using the nonlocal Donnell shell theory. Mirjavadi et al. [33] studied the analysis of fine nonlinear forced vibrations of two-phase magnetic flexible nanoparticles under elliptical type force. Some related investigations can be found in the article [34].

In this investigation, the researchers aim at presenting the influence of the prominent Soret impact on mixed convection heat and mass transfer in the border layer region of a semi-infinite vertical flat plate in a nanofluid, under the circulatory motion of the boundary conditions. The consequences of nondimensional governing parameters, namely, the volume fraction of nanoparticles, magnetic field parameter, radiation parameter, Soret number, buoyancy parameter, and porosity parameter on the flow, temperature, and concentration profiles are discussed and presented graphically. Also, the friction aspect and Nusselt and Sherwood numbers are deliberated and demonstrated in tabular form for two nanofluids separately. Some new contributions near to the present article are discussed in [36]–[44].

2. Construction of the Problem

Let us consider two-dimensional constant MHD nanofluid flows in the region \( y > 0 \) along nonlinearly stretched sheet assuming Cartesian coordinates \((x, y)\) chosen in which \( x \)-axis is measured along with the sheet whereas is \( y \)-axis normal to it under the influence of heat absorption/generation, magnetic field, suction, and injection parameters. The stretched sheet is assumed to have general power-law surface velocity distribution \( u_w(x) = ax^n \) where \( a > 0, n \geq 0 \) are constants. We consider that the flow is concerned with a variable magnetic field of strength \( B(x) = B_0 x^{(\alpha-1)/2} \). The induced magnetic field is neglected whereas the electric field is absent by assuming the low Magnetic Reynolds number. The temperature of the sheet is defined by a function \( T_w(x) = T_{w0} + Ax^n \) where \( A > 0 \) is constant, \( T_{w0} \) is the ambient fluid temperature, and \( C_{w0} \) is the ambient nanoparticles concentration. The coordinate system and physical flow are shown in Figure 1.

The equation of rheological state for the anisotropic flow of a Casson fluid [1, 2]:

\[
\tau_{ij} = \begin{cases} 
2 \left( \mu_B + \frac{P}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c, \\
2 \left( \mu_B + \frac{P}{\sqrt{2\pi}} \right) e_{ij}, & \pi < \pi_c,
\end{cases}
\]

where \( \pi = e_{ij} e_{ij} \).
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - u \left(1 + \frac{1}{\beta} \right) \left( \frac{\partial^2 u}{\partial y^2} \right) + g \left\{ \alpha_c (T - T_c) + \alpha_c (C - C_c) \right\} - \frac{\sigma B^2(x)}{\rho_f} u,
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q_0}{\rho c_p} (T - T_c) + \frac{Q_0}{T_c} \left[ \frac{D_B}{(\partial T/\partial y)} + \frac{D_T}{T_c} \left( \frac{\partial^2 T}{\partial y^2} \right) \right] + \frac{\nu}{c_p} \left(1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2,
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{K_T D_T}{T_c} \left( \frac{\partial^2 T}{\partial y^2} \right),
\]

where \( u \) and \( v \) are the velocity components along the \( x \)- and \( y \)-directions respectively, \( \nu \) is the kinematic viscosity, \( \beta = \mu_B \sqrt{2 \pi \lambda / P} \) is Casson fluid parameter, \( \sigma \) is the electrical conductivity, \( \rho \) is the fluid density, the thermal diffusivity is denoted by \( \alpha \), \( D_B \) and \( D_T \) are the coefficients of the thermophoresis and Brownian diffusion, \( Q_0 \) is the dimensional heat generation or absorption coefficient, \( c_p \) is the specific heat,

\[\tau = \rho_c \frac{f_c}{(\rho c)_f} \]

indicates the ratio of effective heat capacity of the nanoparticle material to the effective heat capacity of the fluid, \( g \) is the acceleration due to gravity, \( \alpha_T \) is the thermal expansion coefficient, \( \alpha_c \) is the concentration expansion coefficient, and \( K_T \) is the thermal diffusion ratio. The boundary conditions in the present problem are

\[
\begin{align*}
\{ & u = u_w(x) = ax^n, v = v_w, T = T_w(x) = T_c + Ax^n, D_B \left( \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_c} \frac{\partial T}{\partial y} \text{ at } y = 0, \\
u & \rightarrow 0, T \rightarrow T_c, C \rightarrow C_c \text{ as } y \rightarrow \infty.
\end{align*}
\]

The nondimensional transformations are taken as follows:

\[
\begin{align*}
\{ & u = ax^n f' \left( \eta \right), v = -ax^{(n-1)/2} \sqrt{\frac{n + 1}{2}} f \left( \eta \right) + \frac{n - 1}{2} \eta f' \left( \eta \right), \ \eta = \sqrt{\frac{n}{\nu}} \left( x - \frac{n + 1}{2} \right) y, \\
\theta(\eta) = & \frac{T - T_c}{T_w - T_c}, \ \phi(\eta) = \frac{C - C_c}{C_w - C_c}, \\
\left(1 + \frac{1}{\beta} \right) f'' \left( \eta \right) - n f' \left( \eta \right) \theta + \frac{n + 1}{2} \left[ f \left( \eta \right) f'' \left( \eta \right) - M f' \left( \eta \right) \right] + \lambda_T \theta + \lambda_c \phi = 0, \\
\frac{1}{Pr} \theta'' + \frac{n + 1}{2} f \left( \theta - n f' \left( \eta \right) \theta + \lambda \theta + Nb \phi' + Nt \theta^2 + \left(1 + \frac{1}{\beta} \right) Ec f'' \left( \eta \right) \right) = 0, \\
\phi'' + \frac{n + 1}{2} Le f \phi' + LeStr \theta'' = 0.
\end{align*}
\]
The boundary conditions are

\[ f(0) = f_w, f'(0) = 1, \theta(0) = 1, N_t\phi'(0) + N_t\theta'(0) = 0, \quad \text{at} \quad \eta = 0, \]
\[ f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0, \quad \text{as} \quad \eta \to \infty. \]  

(11)

where Ec is Eckert number, \( f_w \) is absorption/intromission variable (\( f_w > 0 \)) for absorption and (\( f_w < 0 \)) for introduction, \( M \) is the attractive field variable, \( N_b \) is the Brownian motion variable, \( N_t \) is the thermophoresis parameter, \( P_r \) is the Prandtl number, \( L_e \) is the Lewis number, \( \beta \) is thermal buoyancy parameter, \( N_t \) is the thermophoresis parameter, \( C_g \) is the local Reynolds number, \( \lambda \) is the attractive field variable, \( N_b \) is the Brownian motion variable, and \( \lambda_c \) is solutal buoyancy variable. These are specified as follows:

\[
M = \frac{\sigma B^2}{\alpha x^{n-1} \rho_f},
\]

\[
K = \frac{v}{\alpha x^{n-1} \mu},
\]

\[
\lambda_T = \frac{G_{x}}{R_{e_x}} = \frac{\alpha g}{\alpha x^{n-1}} (T_w - T_{\infty}),
\]

\[
\lambda_C = \frac{G_{C_x}}{R_{e_x}} = \frac{\alpha g}{\alpha x^{n-1} \mu} (C_w - C_{\infty}),
\]

\[
G_{x} = \frac{\alpha g (T_w - T_{\infty}) x^3}{\mu^2},
\]

\[
G_{C_x} = \frac{\alpha g (C_w - C_{\infty}) x^3}{\mu^2},
\]

\[
R_{e_x} = \frac{u_w(x)x}{\mu} = \frac{ax^{n+1}}{\mu},
\]

(12)

\[
P_r = \frac{v}{\alpha},
\]

\[
\lambda = \frac{Q_0}{\rho c_p},
\]

\[
N_b = \frac{\tau D_{b}}{(C_w - C_{\infty})},
\]

\[
N_t = \frac{\tau D_{C}}{(T_w - T_{\infty})},
\]

\[
E_{c} = \frac{u_w^2}{(T_w - T_{\infty})},
\]

\[
L_e = \frac{v}{D_{b}},
\]

\[
S_r = \left( \frac{K_{f} D_{f}}{v T_{\infty} (C_w - C_{\infty})} \right),
\]

The amounts of practical interest include the coefficient of skin abrasion \( C_{f_s} \), regional Nusselt volume \( N_{u_x} \), and local Sherwood volume \( S_{h_w} \) as follows:

\[
C_{f_s} = \frac{\tau_x}{\rho u_w},
\]

\[
N_{u_x} = \frac{x q_w}{\alpha (T_w - T_{\infty})},
\]

\[
S_{h_w} = \frac{x q_m}{D_{b} (C_w - C_{\infty})},
\]

where \( \tau_w \) is the stress of wall clipping; \( \alpha_f \) is the thermal conductivity of the nanofluid; and \( q_w, q_m \) are the wall heat and mass influx. These are given by

\[
\tau_w = \mu_B \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)_{y=0},
\]

\[
q_w = -\alpha_f \left( \frac{\partial T}{\partial y} \right)_{y=0},
\]

\[
q_m = -D_{b} \left( \frac{\partial C}{\partial y} \right)_{y=0}.
\]

Using equations (7) and (14) in equation (13), one obtains

\[
R_{e_x}^{1/2} C_{f_s} = \left( 1 + \frac{1}{\beta} \right) f'(0),
\]

(15)

\[
R_{e_x}^{-1/2} N_{u_x} = -\theta'(0),
\]

\[
R_{e_x}^{-1/2} S_{h_w} = -\phi'(0),
\]

where \( R_{e_x} = u_w x / \nu \) is the local Reynolds number.

3. Numerical Results and Discussion

The orders of not linear normal distinctive equation (3) for (5) with the limit situation (6) are detached in terms of numbers by the Runge–Kutta method with MATLAB package. The outcome displays the influences of the nondimensional controlling variable, which is magnetic area variable \( M \), Soret number \( S_r \), birthplace of heat \( \lambda \), thermal buoyancy parameter \( \lambda_T \), and solutal buoyancy parameter \( \lambda_C \) on the flow, heat, and concentricity profiles are shown and illustrated diagrammatically. The agent of rubbing and Nusselt and Sherwood numbers is discussed and given in tabular form for two nanofluids individually. The numeral outcomes are as follows:
\[
\begin{align*}
\text{Pr} &= 0.7, \\
\text{Le} &= 0.4, \\
\text{Nb} &= \text{Nt} = 0.3, \\
\text{Ec} &= 0.5, \\
\lambda &= 0.1, \\
\beta &= 0.5, \\
Fw &= 0.2, \\
n &= 2.
\end{align*}
\]

(16)

Figures 2–4 show that the difference of speed \(f'(\eta)\), heat \(\theta(\eta)\), and concentricity \(\phi(\eta)\) with respect to \(\eta\)-axis for various amounts of \(\lambda_C\). There is an observation of the velocity decrease along with the increase of \(\lambda_C\), whereas there is an increase of both the heat and concentricity profiles along with the increase of \(\lambda_C\) for magnetic field parameter \(M = 1, 2\). This causes development in both the thermic stretched sheet thickness and velocity and concentration sheet thickness. From these figures, it is observed that the increase of magnetic area parameter leads to the increase of velocity. It is also interesting to note from Figure 3 that temperature rises as \(M\) increases; Figure 4 also shows the fall of concentricity while the magnetic area parameter increases.

Figures 5–7 display variance of speed \(f'(\eta)\), heat \(\theta(\eta)\), and concentration \(\phi(\eta)\) with respect to \(\eta\)-axis for various amounts of thermic buoyancy parameter \(\lambda_T\). The increase of velocity along with the increase of thermic buoyancy parameter \(\lambda_T\) is noticed, whereas the heat and concentricity profiles reduce with the growing flux of magnetic area parameter. This causes development in both the thermic stretched sheet thickness and velocity and concentration sheet thickness.

Figures 8–10 show the alteration velocity \(f'(\eta)\), temperature \(\theta(\eta)\), and concentricity \(\phi(\eta)\) with respect to \(\eta\)-axis for various numbers of thermal buoyancy parameter \(\lambda_T\). It is noticed that the velocity value increases along with thermal buoyancy parameter \(\lambda_T\), whereas the temperature and the concentricity profiles fall with the increase of thermal buoyancy parameter \(\lambda_T\) of the flux for the Solutal buoyancy parameter \(\lambda_C = 0.5, 1\). This causes to make the thickness of both the thermic stretched sheet and velocity and concentricity sheet better.

Figures 11–13 plot the velocity variance \(f'(\eta)\), temperature \(\theta(\eta)\), and concentration \(\phi(\eta)\) with respect to \(\eta\)-axis for various numbers of magnetic area parameter \(M\). It is watched that, by the growing of magnetic area parameter, the velocity value grows too. However, the increase of the magnetic field area causes a reduction in the profiles of temperature and concentricity. This leads to developing the thermic stretched sheet thickness along with velocity and concentration sheet thickness.

Figures 14–16 clear the different values of velocity \(f'(\eta)\), temperature \(\theta(\eta)\), and concentration \(\phi(\eta)\) with respect to \(\eta\)-axis for changeable values of magnetic field parameter. By observation, it is clear that the increasing positive values of the magnetic area cause the reduction of velocity which also grows along with the reduction of negative values of a magnetic area, while it increases with decreasing of negative values of the magnetic field, though the temperature increases with decreasing the negative values of the magnetic field. Also, there is no effect of the positive values of magnetic field on the temperature, and the concentration outlines increase with increasing the negative values of the magnetic field parameter, while as well it is not affected by positive values of magnetic field on the concentration. This causes thermal stretched sheet thickness to get better along with velocity and concentration sheet thickness.

Figures 17–19 illustrate the difference of velocity \(f'(\eta)\), temperature \(\theta(\eta)\), and concentricity \(\phi(\eta)\) with respect to \(\eta\)-axis for different values of Soret number \(Sr\). The growth of profiles of both the velocity and concentricity is noted along with the increase of Soret number for \(n = 0.5, 2\), while the temperature profiles go down with the increase of Soret number of the flow for \(n = 0.5, 2\). This leads to making the thermal stretched sheet thickness better along with velocity and concentration sheet thickness.

Figures 20–22 show the variance of velocity \(f'(\eta)\), temperature \(\theta(\eta)\), and concentricity \(\phi(\eta)\) with respect to \(\eta\)-axis for different values of Soret number \(Sr\). It is watched that the increasing Soret number causes the increase of velocity and the decrease of temperature; however, both of them are not affected by Lewis number \(Le\) on the velocity and temperature, while the concentration profiles increase with increasing Soret number of the flow, but they are not affected by Lewis number on the concentration. These causes developing the thermal stretched sheet thickness along with velocity and concentration sheet thickness.

Figure 23 plots the diversity of Nusselt number \(\text{Nu}_x\Re_e^{1/2}\) with respect to Solutal buoyancy parameter \(\lambda_C\)-axis for various numbers of thermal buoyancy parameter \(\lambda_T\) and magnetic field \(M\). Through observation, it is clear that the Nusselt number grows up along with the increasing thermic buoyancy parameter but falls with the increasing magnetic area.

Figure 24 displays the difference of the Nusselt number \(\text{Nu}_x\Re_e^{1/2}\) with respect to Lewis number \(Le\)-axis for different values of Soret number \(Sr\) and thermal buoyancy parameter \(\lambda_T\). It is noticed that the increase of both the Soret number and thermic buoyancy parameter causes the Nusselt number to increase as well.

Figure 25 clears the variance of the Nusselt number \(\text{Nu}_x\Re_e^{1/2}\) with respect to \(\lambda\)-axis for different values of the Soret number \(Sr\) and thermal buoyancy parameter \(\lambda_T\). There is a notice that the Nusselt number grows along with the increase of both the Soret number and thermic buoyancy parameter.

Figure 26 shows the variation of the Sherwood number \(\text{Sh}_x\Re_e^{1/2}\) with respect to solutal buoyancy parameter \(\lambda_C\)-axis for various numbers of thermal buoyancy parameter \(\lambda_T\) and magnetic field \(M\). It is clear that the Sherwood number goes up with the progress of the thermic buoyancy parameter but it goes down with the progress of the magnetic field.
Table 1 displays a comparison for the heat transfer rate at
the sheet for different values of $n, \beta$, and $M$, when
$Ec = f_w = \lambda = 0, Nt = Nb = 0.5, Pr = 7$. It is strongly
obvious that the present study has original values near the results
obtained by the others [9].

Table 2 shows the heat transfer rate at the sheet for
numerous values of $Sr, \lambda_T, \lambda_C, \beta, f_w$, and $M$ when
$Nt = Nb = 0.5, Pr = 7, n = 1, Ec = 0.5$. It is clear that the
external parameters impact the heat transfer rate comparing
with the consequences obtained in the previous works.

Figure 2: Variation of velocity $f'$ concerning $\eta$ with various values of $\lambda_c$ ($M = 1$, $M = 2$).

Figure 3: Variation of temperature $\theta$ concerning $\eta$ with various values of $\lambda_c$ ($M = 1$, $M = 2$).
Figure 4: Variation concentration $\phi$ concerning $\eta$ with various values of $\lambda_c$ ($M = 1$, $M = 2$).

Figure 5: Variation of velocity $f'$ concerning $\eta$ with various values of $\lambda_T$ ($M = 1$, $M = 2$).
Table 1: Heat transfer rate at the sheet for numerous values of $n, \beta$, and $M$, when $Ec = f_w = \lambda = 0, Nt = Nb = 0.5, Pr = 7$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\beta$</th>
<th>$M$</th>
<th>Ref. [9] $-\theta^\prime (0)$</th>
<th>Present work $-\theta^\prime (0)$</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

Table 2: Heat transfer rate at the sheet for numerous values of $Sr, \lambda_T, \lambda_C, \beta, f_w$ and $M$ when $Nt = Nb = 0.5, Pr = 7, \eta = 1, Ec = 0.5$.

<table>
<thead>
<tr>
<th>$\lambda_T$</th>
<th>$\lambda_C$</th>
<th>$Sr$</th>
<th>$M$</th>
<th>$\beta$</th>
<th>$f_w$</th>
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Figure 6: Variation of temperature $\theta$ concerning $\eta$ with various values of $\lambda_T$ ($M = 1$, $M = 2$).
Figure 7: Distinction of concentration $\phi$ concerning $\eta$ with various values of $\lambda_T$ ($M = 1$, $M = 2$).

Figure 8: Diversity of velocity $f'$ concerning $\eta$ with various values of $\lambda_T$ ($\lambda_c = 0.5$, $\lambda_c = 1$).
Figure 9: Variation of temperature $\theta$ concerning $\eta$ with various values of $\lambda_T$ ($\lambda_c = 0.5$, $\lambda_c = 1$).

$\lambda_c = 0.5$
$\lambda_c = 1$

Figure 10: Variation of concentration $\phi$ concerning $\eta$ with various values of $\lambda_T$ ($\lambda_c = 0.5$, $\lambda_c = 1$).

$\lambda_c = 0.5$
$\lambda_c = 1$
\[ f'(\eta) \]

**Figure 11:** Variation of velocity \( f' \) concerning \( \eta \) with various values of \( M \).

\[ \theta(\eta) \]

**Figure 12:** Variation of temperature \( \theta \) concerning \( \eta \) with various values of \( M \).
Figure 13: Variation of concentration $\phi$ concerning $\eta$ with various values of $M$.

Figure 14: Variation of velocity $f'$ concerning $\eta$ with various values of $M$. 
\[ \begin{align*}
\text{Pr} &= 0.7, \text{Le} = 0.4, \text{Nb} = \text{Nt} = 0.3, \text{Ec} = 0.5, \lambda = 0.1, \\
\beta &= 0.5, F_w = 0.2, n = 2, \lambda_T = \lambda_C = \text{Sr} = 0
\end{align*} \]

\[ M = -0.7, -0.8, -1 \]

\[ M = -0.6 \]

**Figure 15:** Variation of temperature \( \theta \) concerning \( \eta \) with various values of \( M \).

\[ \begin{align*}
\text{Pr} &= 0.7, \text{Le} = 0.4, \text{Nb} = \text{Nt} = 0.3, \text{Ec} = 0.5, \lambda = 0.1, \\
\beta &= 0.5, F_w = 0.2, n = 2, \lambda_T = \lambda_C = \text{Sr} = 0
\end{align*} \]

\[ M = -0.6, -0.7, -0.8, -1 \]

\[ M = 0.2, 0.4 \]

\[ M = 0 \]

\[ M = -0.2, -0.4 \]

**Figure 16:** Variation of concentration \( \phi \) concerning \( \eta \) with various values of \( M \).
Figure 17: Variation of velocity \( f' \) concerning \( \eta \) with various values of Sr (\( n = 2 \), \( n = 0.5 \)).

Figure 18: Variation of temperature \( \theta \) concerning \( \eta \) with various values of Sr (\( n = 2 \), \( n = 0.5 \)).
Figure 19: Variation of concentration $\phi$ concerning $\eta$ with various values of $Sr$ ($n = 2$, $n = 0.5$).

Figure 20: Variation of velocity $f'$ concerning $\eta$ with various values of $Sr$ and $Le$. 
\[Pr = 0.7, \, Nb = Nt = 0.3, \, Ec = 0.5, \, \lambda = 0.1, \, n = 0.5, \, \beta = 0.5, \, M = 1, \, F_w = 0.2, \, \lambda_C = 0.1, \, \lambda_T = 0.1\]

\[Sr = 0.5, 1, 2, 3, 4, 5\]

**Figure 21:** Variation of temperature $\theta$ concerning $\eta$ with various values of $Sr$ and $Le$.

\[\phi(\eta)\]

\[Pr = 0.7, \, Nb = Nt = 0.3, \, Ec = 0.5, \, \lambda = 0.1, \, n = 0.5, \, \beta = 0.5, \, M = 1, \, F_w = 0.2, \, \lambda_C = 0.1, \, \lambda_T = 0.1\]

\[Sr = 0.5, 1, 2, 3\]

**Figure 22:** Variation of concentration $\phi$ concerning $\eta$ with various values of $Sr$ and $Le$. 
Figure 23: Variation of $N_u \cdot R_x^{-1/2}$ concerning $\lambda_c$ with various values of $\lambda_T$ and $M$.

Figure 24: Variation of $N_u \cdot R_x^{-1/2}$ concerning Le with various values of $\lambda_T$ and Sr.
4. Conclusion

In this research, the authors analyzed the impact of magnetic field $M$, Soret number $Sr$, heat source $\lambda$, thermal buoyancy parameter $\lambda_T$, and solutal buoyancy parameter $\lambda_C$ of a nanofluid flow over convectively nonlinear heated due to an extending surface. The partial differential equations that dominate flow are transformed into normal changed-in-variables equations using the similarity transform and then solved numerically. The impacts of nondimensional ruling parameters specifically the Soret number of nanoparticles, magnetic field parameter, thermal buoyancy parameter $\lambda_T$, Lewith number, and $\lambda_C$ on the flow, speed, temperature, and concentration profiles are discussed and presented graphically. The conclusions can be drawn as follows:

(1) The distribution of nanoparticle concentration is an increasing and decreasing function of the magnetic field and Soret number.
(2) Speed distribution decreases by increasing the values of the Soret number with growing values of the magnetic field.

(3) Temperature and concentration decrease with the increase of the values of thermal buoyancy parameter $\lambda_T$ and solutal buoyancy parameter $\lambda_C$.

(4) Nusselt and Sherwood local numbers are greater for the incremental values of Soret number $S_r$ and solutal buoyancy parameter $\lambda_C$.

For future work, we can use other influences of a nanofluid flow over convectively heated nonlinear due to an extending surface.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors state that there are no conflicts of interest to report concerning the present study.

**Authors’ Contributions**

All the authors contributed equally to this work.

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**References**


