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Research Article

Flow of Brinkman Fluid with Heat Generation and Chemical Reaction

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Unsteady magnetohydrodynamics (MHD) flow of fractionalized Brinkman-type fluid over a vertical plate is discussed. In the model of problem, additional effects such as heat generation/absorption and chemical reaction are also considered. The model is solved by using the Caputo fractional derivative. The governing dimensionless equations for velocity, concentration, and temperature profiles are solved using the Laplace transform method and compared graphically. The effects of different parameters like fractional parameter, heat generation/absorption Q, chemical reaction R, and magnetic parameter M are discussed through numerous graphs. Furthermore, comparison among ordinary and fractionalized velocity fields are also drawn. From the figures, it is observed that chemical reaction and magnetic field have decreasing effect on velocity profile, whereas thermal radiation and mass Grashof numbers have increasing effect on the velocity of the fluid.

1. Introduction

The important significance of non-Newtonian fluids can be seen in applied mathematics, engineering, and physics. It has various significances in many areas, such as uses of lubricants, biological fluid food processing, or plastic manufacturing. Some commonly examples of non-Newtonian fluids are custard, colloids, melted butter, paint, ketchup, starch suspensions, blood, toothpaste, gels, shampoo, and corn starch.

Mass transfer and heat transfer occurs mostly in nature due to temperature and concentration differences, respectively. Today, research work in magnetohydrodynamics (MHD) has substantial significance as these flows are absolutely prevailing in nature.

Convection flow with porous media has numerous applications such as flows in soils, solar power collectors, heat transfer correlated with geothermal systems, heat source in the field of agricultural storage system, heat transfer in nuclear reactors, heat transfer in aerobic and anaerobic reactions, heat evacuation from nuclear fuel detritus, and heat exchangers for porous material.

MHD fluid has many implementations in meteorology, distillation of gasoline, energy generators, geophysics, accelerators, petroleum industry, astrophysics, polymer technology, aerodynamics, and boundary layer control and in material processes such as glass fiber drawing, extrusion, and casting wire. The flow of viscous fluid through a perpendicular plate is analyzed by Swamy et al. [1]. The effect of mass diffusion on MHD fluid with porosity has been observed by Chaudhary et al. [2]. Exact solution for magnetohydrodynamics flow through a perpendicular plate in the existence of porosity is obtained by Sivaiah et al. [3]. The solution for unsteady flow of viscous fluid with porosity is obtained by Das and Jana [4].

Furthermore, convection flow in the existence of porosity has wide applications such as ground water hydrology, oil extraction, geothermal systems, cooling systems, storage of nuclear waste materials, energy-efficient drying processes, solid matrix heat exchangers, and wall-cooled catalytic reactors. Kataria and Patel [5] analyzed the impact of magnetic field with heat transfer over a plate. The authors of [6] discussed the solution of viscous fluid flow with thermal

radiation. Chamkha [7] discussed the effect of heat source on MHD fluid through a moving plate.

The authors of [8] studied the flow of polar fluid through a plate. Rahman and Sattar [9] studied the flow of fluid with a heat source. Rajesh and Varma [10] studied the influence of mass diffusion on magnetohydrodynamic fluid flow. The authors of [11] analyzed the solution of convection flow through a vertical plate. They also discussed the solution for time-dependent concentration and temperature. Convection flow immersed in a porous media through a surface is discussed in [12–14].

The impact of conjugate flow of MHD fluid is discussed by Khan et al. [15]. Rajesh et al. [16] discussed the MHD flow through a moving plate. MHD flow through an accelerated surface in the existence of porous media is discussed by Chaudhary et al. [17]. The authors also analyzed the solution of velocity field graphically. Das [18] analyzed the solution of magnetohydrodynamics of convection flow through a plate. Pal et al. [19] examined the solution of viscous fluid with thermal radiation on magnetohydrodynamics flow, whereas the solution for convection flow with nonuniform temperature through a moving plate is obtained by Seth et al. [20]. The solution of nanofluid with ramped temperature is studied by Khalid et al. [21].

The discussion of mass diffusion has empirical use in numerous areas of engineering and applied sciences. These phenomena play a vital role in cooling of a nuclear reactor and tabular reactor, chemical industry, mixture of terracotta material, petroleum industry, and decomposition of rigid materials. Seddeek et al. [22] examined the MHD fluid flow with thermal radiation. An intensive study of chemical reaction with heat source/sink is studied by Shah et al. [23]. Seth et al. [24] obtained the solution of unsteady magnetohydrodynamic flow of the fluid over a plate with ramped condition. The solution of convection flow of MHD fluid over a plate with heat generation/absorption is obtained by Shateyi and Motsa[25]. MHD fluid flow with Ohmic heating and heat generation is analyzed by Kasim et al. [26]. The exact solution of MHD fluid with mass transfer immersed in a porous media is discussed by Ali et al. [27]. The exact solution of magnetohydrodynamic flow of a Brinkman fluid perpendicular to the plate is analyzed by Khan et al. [28]. The analytical investigation of Brinkman fluid flow with variable concentration, temperature, and velocity is obtained by Ali et al. [29]. The flow of nanofluid with thermal radiation is studied in [30–35]. Patel et al. [36] studied the effect of Joule's heating on ferrofluid. The influence of Brownian motion and thermophoresis is studied by Mittal and Kataria [37]. Kataria [38] studied the effect of radiation and magnetic field on Casson fluid. Hashemi et al. [39] analyzed the solution of a circular rod. Some flows of fluids with numerical and computational methods are discussed in [40–42].

In this problem, the model of unsteady magnetohydrodynamic free convection flow of Brinkman fluid through a plate is considered. The impact of chemical reaction and heat absorption/generation is added into account. Firstly,

the governing equations have been made nondimensional and then solved semianalytically. The results for velocity profile, temperature profile, and concentration profile are obtained and then analyzed graphically. Various graphs are plotted and discussed for different parameters, which are used in the flow model. The comparison between ordinary and fractionalized fluid is drawn graphically and shows that Caputo fractional derivative is the best choice for controlled fluid velocity.

2. Mathematical Description of the Model

The magnetohydrodynamic flow of Brinkman fluid through a plate with mass and heat transfer is considered. The fluid is flowing along the $x^{.}$ axis. The motion of fluid depends on $y^{.}$-axis and time $t^{.}_1$. The plate and fluid have concentration $C^{.}_\infty$ and temperature $T^{.}_\infty$ at constant $t^{.}_1 = 0$ with zero velocity. But, for $t^{.}_1 > 0$, the plate starts to move in the plane with uniform velocity $U_1 e^{at^{.}_1}$. The concentration and temperature of the plate increased linearly to $C^{.}_w$ and $T^{.}_w$ with time $t^{.}$. A constant strength β_0 of magnetic field is applied normally. In view of the above assumption and using Boussinesq's approximation, the convection flow of Brinkman fluid with chemical reaction, and magnetic field through a plate, the linear momentum equation is

$$\rho \frac{\partial u_1 \left(y^{.}, t^{.}_1 \right)}{\partial t^{.}_1} + B_1 u_1 \left(y^{.}, t^{.}_1 \right) = \frac{\partial \tau \left(y^{.}, t^{.}_1 \right)}{\partial y^{.}} + g\beta_{T^{.}} \left(T^{.} - T^{.}_\infty \right)$$

$$- \sigma \beta_0^2 u_1 \left(y^{.}, t^{.}_1 \right) + g\beta_{C^{.}} \left(C^{.} - C^{.}_\infty \right). \tag{1}$$

Shear stress τ is

$$\tau = \mu \frac{\partial u_1 \left(y^{.}, t^{.}_1 \right)}{\partial y^{.}}. \tag{2}$$

Thermal equation is

$$\rho C_p \frac{\partial T^{.} \left(y^{.}, t^{.}_1 \right)}{\partial t^{.}_1} = -\frac{\partial q_1 \left(y^{.}, t^{.}_1 \right)}{\partial y^{.}} + Q_1 \left(T^{.} - T^{.}_\infty \right). \tag{3}$$

According to Fourier's Law, $q_1 \left(y^{.}, t^{.}_1 \right)$ is given by

$$q_1 \left(y^{.}, t^{.}_1 \right) = -\alpha_0 \frac{\partial T^{.} \left(y^{.}, t^{.}_1 \right)}{\partial y^{.}}. \tag{4}$$

Diffusion equation is

$$\frac{\partial C^{.} \left(y^{.}, t^{.}_1 \right)}{\partial t^{.}_1} = -\frac{\partial J_1 \left(y^{.}, t^{.}_1 \right)}{\partial y^{.}} - R_1 \left(C^{.} - C^{.}_\infty \right). \tag{5}$$

According to Fick's Law, $J_1 \left(y^{.}, t^{.}_1 \right)$ is given by

$$J_1 \left(y^{.}, t^{.}_1 \right) = -D_m \frac{\partial C^{.} \left(y^{.}, t^{.}_1 \right)}{\partial y^{.}}. \tag{6}$$

The boundary conditions for the flow model are

$$u_1\left(y^{'},t_1^{'}\right) = 0,$$
$$T^{'}\left(y^{'},t_1^{'}\right) = T_{\infty}^{'},$$
$$C^{'}\left(y^{'},t_1^{'}\right) = C_{\infty}^{'},$$
$$y^{'} > 0, t_1^{'} = 0. \tag{7}$$

$$u_1\left(0,t_1^{'}\right) = U_1 f\left(t_1^{'}\right) = U_1 e^{at_1^{'}},$$
$$T^{'}\left(0,t_1^{'}\right) = T_w^{'},$$
$$C^{'}\left(0,t_1^{'}\right) = C_w^{'},$$
$$t_1^{'} > 0, \tag{8}$$

$$u_1\left(y^{'},t_1^{'}\right) \longrightarrow 0,$$
$$T^{'}\left(y^{'},t_1^{'}\right) \longrightarrow 0,$$
$$C^{'}\left(y^{'},t_1^{'}\right) \longrightarrow 0,$$
$$y^{'} \longrightarrow \infty, t_1^{'} > 0. \tag{9}$$

To write the flow model in dimensionless form, we used the following dimensionless variables:

$$y^* = \frac{Uy^{'}}{\nu},$$

$$t^* = \frac{U^2 t_1^{'}}{\nu},$$

$$Pr^* = \frac{\rho \nu C_P}{\alpha_0},$$

$$\nu^* = \frac{u_1}{U^{'}},$$

$$Gr^* = \frac{\rho \nu \beta_{T^{'}}\left(T_w^{'} - T_{\infty}^{'}\right)}{U^3},$$

$$M^* = \frac{\beta_0^2 \sigma}{\rho U^2}, \tag{10}$$

$$Q^* = \frac{Q_1 \nu}{U^2 \rho C_P},$$

$$R^* = \frac{R_1 \nu}{U^2},$$

$$C^* = \frac{C^{'} - C_{\infty}^{'}}{C_w^{'} - C_{\infty}^{'}},$$

$$Sc^* = \frac{\nu}{D_1} Gm^* = \frac{\rho \nu \beta_{C^{'}}\left(C_w^{'} - C_{\infty}^{'}\right)}{U^3}.$$

Using nondimensional variables from equations in (10) into the equations (1)–(9), we have

$$\left(\frac{\partial}{\partial t} + B\right)\nu(y,t) = K_1 \frac{\partial \tau}{\partial y} - M\nu(y,t) + GrT(y,t)$$
$$+ GmC(y,t), \tag{11}$$

$$\tau = L_1 \frac{\partial \nu(y,t)}{\partial y}, \tag{12}$$

$$\frac{\partial T(y,t)}{\partial t} + K_2 \frac{\partial q}{\partial y} - QT(y,t) = 0, \tag{13}$$

$$q = -m_1 \frac{\partial T(y,t)}{\partial y}, \tag{14}$$

$$\frac{\partial C(y,t)}{\partial t} + K_3 \frac{\partial J}{\partial y} + RC(y,t) = 0, \tag{15}$$

$$J = -n_1 \frac{\partial C(y,t)}{\partial y}, \tag{16}$$

with boundary conditions as

$$\nu(y,0) = T(y,0) = C(y,0) = 0, \quad y > 0, t = 0,$$
$$\nu(y,t) = e^{at}, T(y,t) = 1, C(y,t) = 1, \quad t > 0, \tag{17}$$
$$\nu(\infty,t) = T(\infty,t) = C(\infty,t) \longrightarrow 0, \quad t > 0,$$

where

$$K_1 = \frac{\tau}{U^2 \rho},$$

$$K_2 = \frac{q}{U\rho C_P\left(T_w - T_{\infty}\right)},$$

$$K_3 = \frac{J_1}{\left(C_w - C_{\infty}\right)U},$$

$$L_1 = \frac{U^2 \mu}{q_1 \nu}, \tag{18}$$

$$m_1 = \frac{\alpha_0\left(T_w - T_{\infty}\right)U}{q\nu},$$

$$n_1 = \frac{D_m\left(C_w - C_{\infty}\right)U}{J\nu},$$

where Gr, B, Sc, Q, M, Pr, Gm, and ν represent the Grashof number for heat transfer, Brinkman parameter, Schmidt number, nondimensional heat source, magnetic field, Prandtl number, mass Grashof number, and velocity of the fluid, respectively.

3. Generalized Model

Equation (12) is frictionally generalized by Blair and Caffyn [43]:

$$\tau = L_{1-\beta} D_t^{1-\beta} \frac{\partial u(y,t)}{\partial y}, \quad 1 \geq \beta > 0. \tag{19}$$

Equation (14) is generalized by using Fourier Law defined by Povstenko and Hristov [44, 45]:

$$q = -m_{1-\gamma} D_t^{1-\gamma} \frac{\partial T(y,t)}{\partial y}, \quad 1 \geq \gamma > 0. \tag{20}$$

By using Fick's Law, equation (16) is generalized as

$$J = -n_{1-\alpha}D_t^{1-\alpha}\frac{\partial C(y,t)}{\partial y}, \quad 1 \geq \alpha > 0. \tag{21}$$

Using equation (19) into equation (11), equation (20) into equation (13), and equation (21) into equation (15), we have

$$\left[\frac{\partial}{\partial t} + B\right]v(y,t) = K_1\frac{\partial}{\partial y}\left[L_{1-\beta}D_t^{1-\beta}\frac{\partial v(y,t)}{\partial y}\right] - Mv(y,t) + \mathrm{Gr}T(y,t) + \mathrm{Gm}C(y,t), \tag{22}$$

$$\frac{\partial T(y,t)}{\partial t} = K_2\frac{\partial}{\partial y}\left[m_{1-\gamma}D_t^{1-\gamma}\frac{\partial T(y,t)}{\partial y}\right] + QT(y,t), \tag{23}$$

$$\frac{\partial C(y,t)}{\partial t} = K_3\frac{\partial}{\partial y}\left[n_{1-\alpha}D_t^{1-\alpha}\frac{\partial C(y,t)}{\partial y}\right] - RC(y,t). \tag{24}$$

Taking inversion left operator in equations (22)–(24), we obtain

$$\left[I_t^{1-\beta}\frac{\partial}{\partial t} + BI_t^{1-\beta}\right]v(y,t) = \left[D_t^\beta + BI_t^{1-\beta}\right]v(y,t) = P_1\frac{\partial^2 v(y,t)}{\partial y^2} - $$
$$I_t^{1-\beta}Mv(y,t) + I_t^{1-\beta}\mathrm{Gr}T(y,t) + I_t^{1-\beta}\mathrm{Gm}C(y,t), \tag{25}$$

$$I_t^{1-\gamma}\frac{\partial T(y,t)}{\partial t} = D_t^\gamma T(y,t) = P_2\frac{\partial^2 T(y,t)}{\partial y^2} + QI_t^{1-\gamma}T(y,t), \tag{26}$$

$$I_t^{1-\alpha}\frac{\partial C(y,t)}{\partial t} = D_t^\alpha C(y,t) = P_3\frac{\partial^2 C(y,t)}{\partial y^2} - RI_t^{1-\alpha}C(y,t), \tag{27}$$

where $P_1 = K_1L_{1-\beta} = 1$ when $\beta \longrightarrow 1$, $P_2 = K_2m_{1-\gamma} = 1/\mathrm{Pr}$ when $\gamma \longrightarrow 1$, $P_3 = K_3n_{1-\alpha} = 1/\mathrm{Sc}$ when $\alpha \longrightarrow 1$, and $D_t^\alpha v(y,t)$ represents the Caputo fractional derivative of $v(y,t)$ as

$$D_t^\alpha v(y,t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)}\displaystyle\int_0^t \dfrac{1}{(t-q)^\alpha}\dfrac{\partial v(y,q)}{\partial q}\,\mathrm{d}q, & 0 \leq \alpha < 1, \\[12pt] \dfrac{\partial v(y,t)}{\partial t}, & \alpha = 1, \end{cases} \tag{28}$$

and the left inverse operator of the derivative operator is

$$I_t^\alpha g(y,t) = \frac{1}{\Gamma(\alpha)}\int_0^t g(y,s)(t-s)^{\alpha-1}\mathrm{d}s, \tag{29}$$

with

$$I_t^{1-\alpha}\frac{\partial g(y,t)}{\partial t} = D_t^\alpha g(y,t). \tag{30}$$

4. Solution of the Problem

Equations (25)–(27) with initial and boundary conditions are solved semianalytically.

4.1. Calculation of Concentration.

Solution of equation (27) is

$$s^\alpha\overline{C}(y,s) = P_3\frac{\partial^2\overline{C}(y,s)}{\partial y^2} - R\frac{\overline{C}(y,s)}{s^{1-\alpha}}. \tag{31}$$

Boundary conditions satisfying equation (31) are

$$\begin{aligned} \overline{C}(0,s) &= s^{-1}, \\ \overline{C}(y,s) &\longrightarrow 0, \\ y &\longrightarrow \infty. \end{aligned} \tag{32}$$

Equation (31) is solved by using conditions given in equation (32), and we have

$$\overline{C}(y,s) = s^{-1}e^{-y\sqrt{(1/P_3)((s+R)/s^{1-\alpha})}}, \tag{33}$$

which is complicated and cannot be solved analytically. The numerical result of equation (33) is obtained by using the algorithm in [46, 47].

4.2. Calculation of Temperature.

Solution of equation (26) is

$$s^\gamma\overline{T}(y,s) = P_2\frac{\partial^2\overline{T}(y,s)}{\partial y^2} + Q\frac{\overline{T}(y,s)}{s^{1-\gamma}}. \tag{34}$$

Boundary conditions satisfying equation (34) are

$$\overline{T}(0,s) = s^{-1},$$

$$\overline{T}(y,s) \longrightarrow 0, \qquad (35)$$

$$y \longrightarrow \infty.$$

Equation (34) is solved by using conditions given in equation (35), which results in

$$\overline{T}(y,s) = \frac{1}{s} e^{-y\sqrt{(1/P_2)((s-Q)/s^{1-\gamma})}}, \qquad (36)$$

which is complicated one and cannot be solved analytically. The numerical result of equation (36) is obtained by using the algorithm in [46, 47].

4.3. Calculation of Velocity. Solution of equation (25) is

$$\left[s^\beta + \frac{B}{s^{1-\beta}}\right]v(y,t) = P_1 \frac{\partial^2 v(y,t)}{\partial y^2} - \frac{1}{s^{1-\beta}}Mv(y,t)$$

$$+ \frac{1}{s^{1-\beta}}\mathrm{Gr}T(y,t) + \frac{1}{s^{1-\beta}}\mathrm{Gm}C(y,t). \qquad (37)$$

Boundary conditions satisfying equation (37) are

$$\overline{v}(0,s) = \frac{1}{s-a},$$

$$\overline{v}(y,s) \longrightarrow 0, \qquad (38)$$

$$y \longrightarrow \infty.$$

Equation (37) is solved by using conditions given in equation (38), and we obtain

$$\overline{v}(y,s) = \frac{1}{s-a}e^{-y\sqrt{(1/P_1)\left((s+H)/s^{1-\beta}\right)}} + \frac{\mathrm{Gr}}{s\left(s^{1-\beta}\right)\left[(P_1/P_2)\left((s-Q)/s^{1-\gamma}\right) - \left((s+H)/s^{1-\beta}\right)\right]}$$

$$\left[e^{-y\sqrt{(1/P_1)\left((s+H)/s^{1-\beta}\right)}} - e^{-y\sqrt{(1/P_2)((s-Q)/s^{1-\gamma})}}\right] + \frac{\mathrm{Gm}}{s\left(s^{1-\beta}\right)\left[(P_1/P_3)\left((s+R)/s^{1-\alpha}\right) - \left((s+H)/s^{1-\beta}\right)\right]} \qquad (39)$$

$$\left[e^{-y\sqrt{(1/P_1)\left((s+H)/s^{1-\beta}\right)}} - e^{-y\sqrt{(1/P_3)\left((s+R)/s^{1-\alpha}\right)}}\right],$$

which is much complicated, so it cannot be solved analytically. Numerical result of equation (39) will be obtained by using the algorithm in [46, 47].

5. Results and Discussion

Semianalytical solution for MHD flow of Brinkman fluid with a combined concentration and temperature gradient over a plate is obtained. The generalized model is solved with a Caputo fractional derivative. The graph of concentration profile, temperature profile, and velocity profile are plotted for different parameters.

Figure 1 represents the effect of B on $v(y,t)$. It is noted that the $v(y,t)$ decreases with increasing values of the Brinkman parameter. Physically, Brinkman is the relation between drag force and density; therefore, drag force increases with increasing values of the Brinkman parameter which decays down the fluid motion. The behavior of Gm is reported in Figure 2. From this graph, it is concluded that the magnitude of fluid velocity rises by raising the values of Gm. Gm is the relative strength of viscous force and concentration buoyancy force. As Gm increases, the motion of fluid is accelerated due to an increment of buoyancy force. Figure 3 represents the impact of different values of Gr on $v(y,t)$. From this graph, it is noted that velocity distribution is directly proportional with Gr. Physically, Gr is a relation between viscous force and buoyancy force. Therefore, with

an increment in the values of Gr, buoyancy force is increased which raises the magnitude of $v(y,t)$.

Figure 4 displays the behavior of M and Q on fluid motion. The speed of fluid layer is reduced for raising values of M as shown in the graph. Physically, the Lorentz force creates the low resistivity which increases the thickness or width of momentum boundary layer in the solution. The impact of Q on fluid velocity is displayed in Figure 4. The $v(y,t)$ is increased by increasing values of Q as depicted in the figure. Physically, when the values of heat generation parameter are increased, the thermal conductivity becomes dominant. Fluid particles attract each other weakly which improve the fluid motion.

Figure 5 represents the behavior of R and Pr on the $v(y,t)$. The graph shows that $v(y,t)$ falls down for larger values of R. The impact of various values of Pr on $v(y,t)$ is displayed in Figure 5. Pr represents the ratio of momentum (product of mass and velocity) diffusion to thermal diffusion. In the problems of heat transfer, Pr manages the thickness of boundary layer and momentum (velocity). For larger value of Pr, diffusion of heat becomes slow as compared to the fluid momentum (velocity) which decreases the thermal conductivity (thickness) and raises the boundary layer momentum. The influence of R on fluid motion is shown in Figure 5. From the figure, it is concluded that fluid motion decays by increasing the values of R. Physically, boundary layer thickness is increased by increasing values of
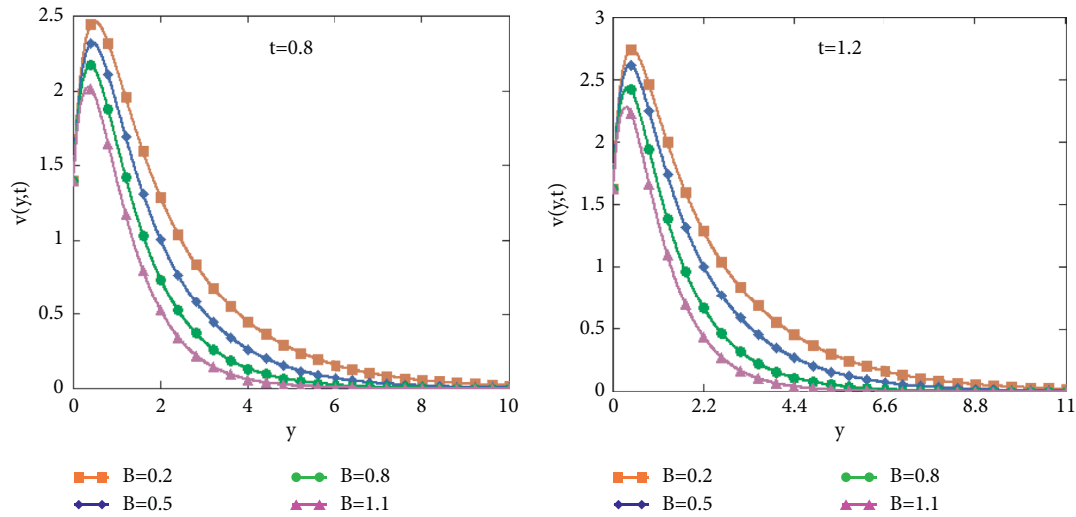
FIGURE 1: Velocity diagram $v(y,t)$ for various values of Brinkman parameter B at $R = 1.4, Q = 0.4, \text{Gr} = 9, \text{Gm} = 6, M = .85,$ $\alpha = \beta = \gamma = 0.5, \text{Sc} = 4.5,$ and $\text{Pr} = 4.0$.
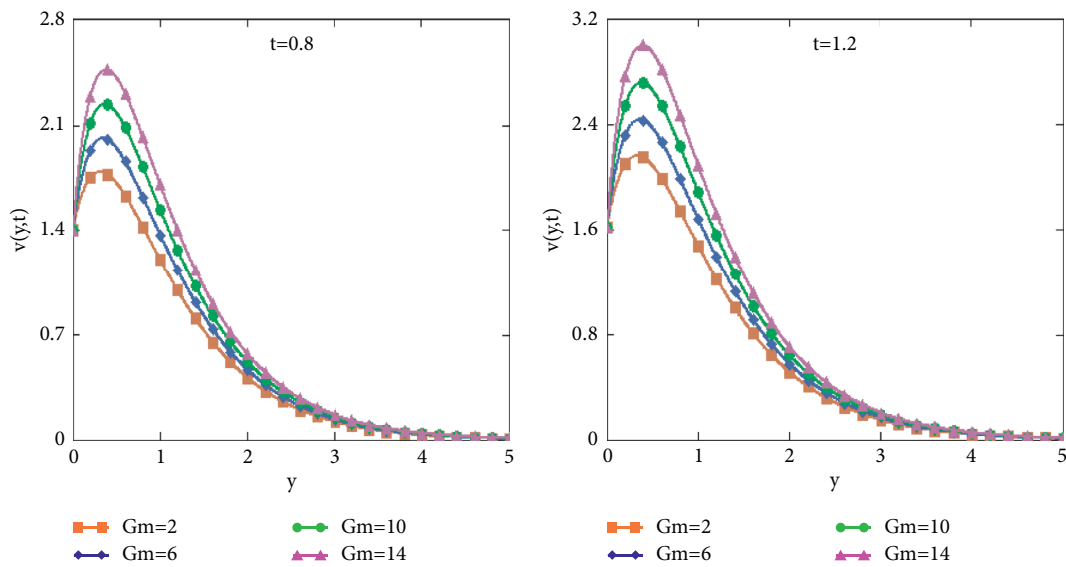

FIGURE 2: Profiles of velocity $v(y,t)$ for various values of the mass Grashof number Gm at $R = 1.4, Q = 0.4, \text{Gr} = 9, M = .85, \text{Sc} = 4.5,$ $\alpha = \beta = \gamma = 0.5, B = 0.2,$ and $\text{Pr} = 4.0$.

R which slows down the velocity distribution. Figure 6 shows the influence of Sc and α, β, and γ on $v(y,t)$. The graph shows that for increasing values of Sc, the diffusion of the molecule increases which reduces the fluid level. However, fluid velocity rises with increasing values of fractional parameters.

The behavior of heat generation Q and Pr on $T(y,t)$ is displayed in Figure 7. This figure shows that temperature increases with increment in the values of Q. Figure 7 indicates the influence of Pr on temperature $T(y,t)$. Temperature distribution is accelerated with decreasing values of Pr as shown in the graph. Physically, the increase in Pr minimizes viscosity which reduces the thermal boundary layer.

The behavior of chemical reaction R and Sc on $C(y,t)$ is shown in Figure 8. The concentration level is accelerated with decreasing R as depicted in the graph. Physically, boundary layer thickness is increased by increasing values of R which slows down the concentration distribution. Figure 8 shows the influence of Sc on $C(y,t)$. The concentration level increases with reducing values of Sc as highlighted in the figure. The graph shows that for increasing values of Sc, the diffusion of molecule increases which reduces the fluid level. Figure 9 shows the comparison of Brinkman-type fractional fluid with Olisa [48]. From the figure, it is concluded that fractional derivative is the best choice to enhance the fluid motion. Figure 9(b) represents that if we take fractional parameters $\beta = \gamma = \alpha \longrightarrow 1$, $\text{Gm} = F(s) = 0$, and $B = 0$, the
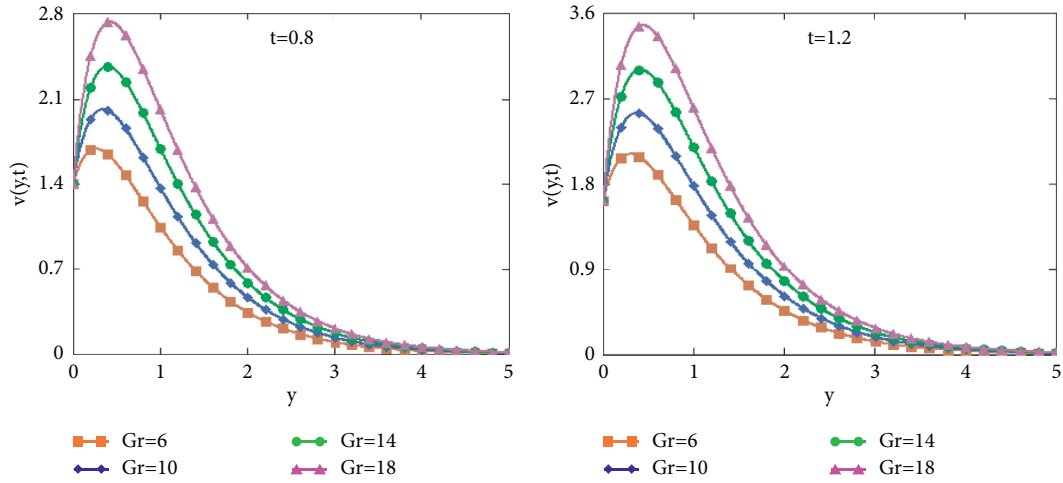
FIGURE 3: Velocity profile $v(y,t)$ for various values of Gr at $R = 1.4, Q = 0.4, Gm = 6, M = .85, Sc = 4.5, \alpha = \beta = \gamma = 0.5, B = 0.2,$ and Pr = 4.0.
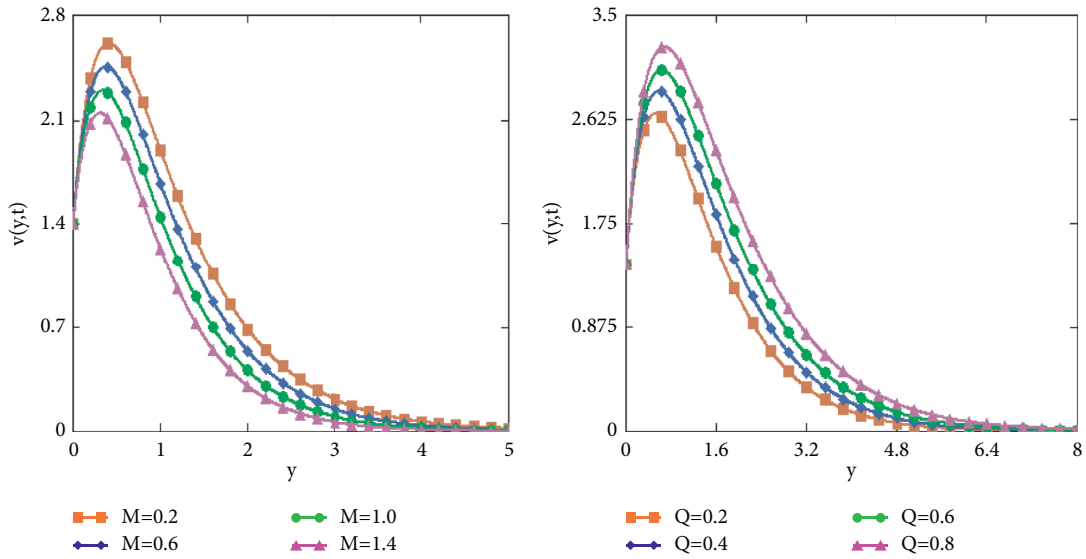

FIGURE 4: Velocity distribution $v(y,t)$ for distinct values of M and heat generation parameter Q at $R = 1.4, Gr = 9, Gm = 6, Sc = 4.5, \alpha = \beta = \gamma = 0.5, B = 0.2,$ and Pr = 4.0.
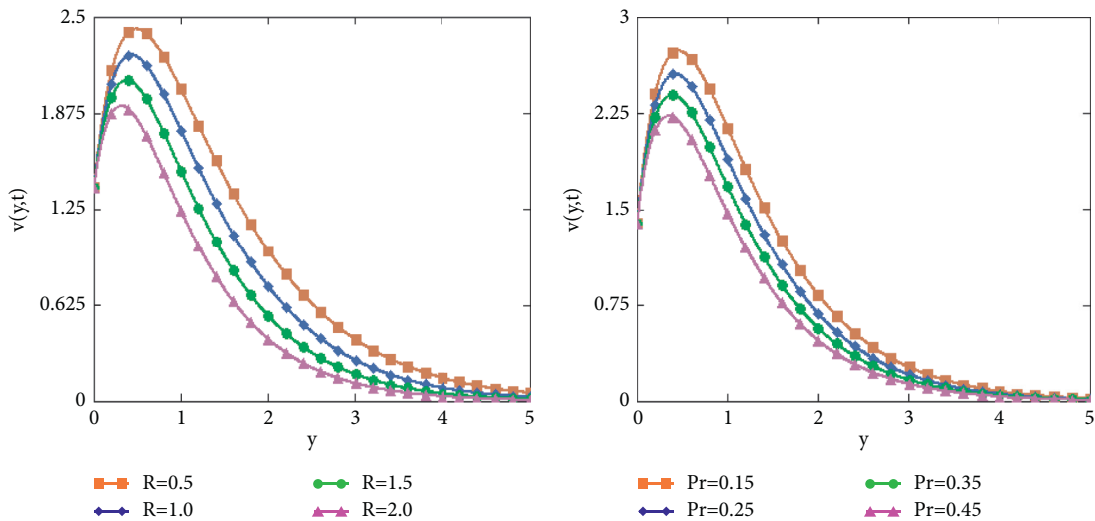

FIGURE 5: Velocity diagram $v(y,t)$ for distinct values of chemical reaction parameter R and Pr at $Q = 0.4, Gr = 9, Gm = 6, M = .85, Sc = 4.5, \alpha = \beta = \gamma = 0.5,$ and $B = 0.2$.
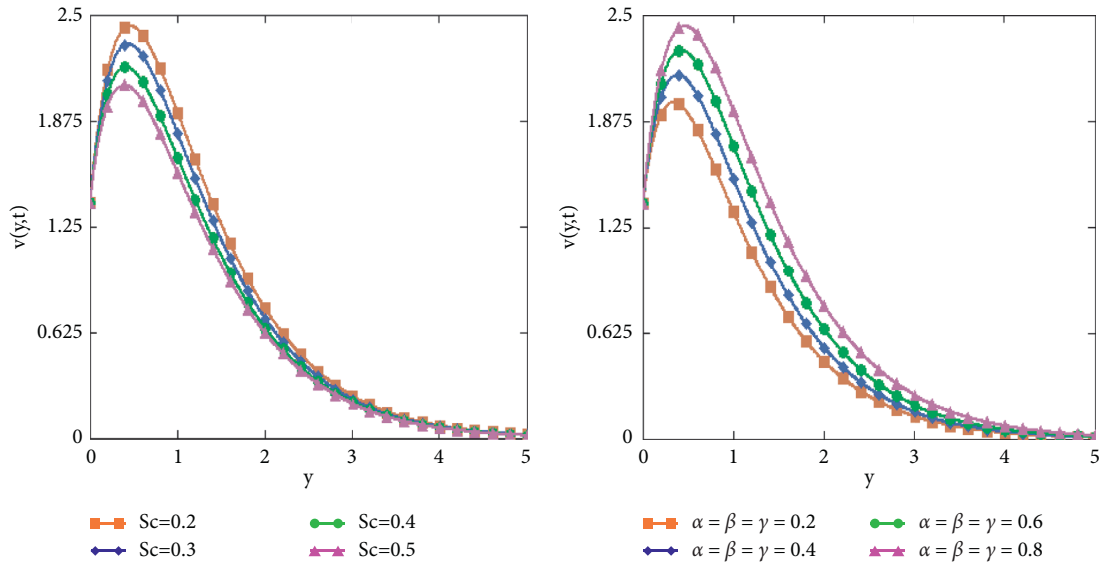
FIGURE 6: $v(y, t)$ for distinct values of Sc and fractional parameters at $R = 1.4, Q = 0.4, \mathrm{Gr} = 9, \mathrm{Gm} = 6, M = .85, B = 0.2,$ and $\mathrm{Pr} = 4.0$.
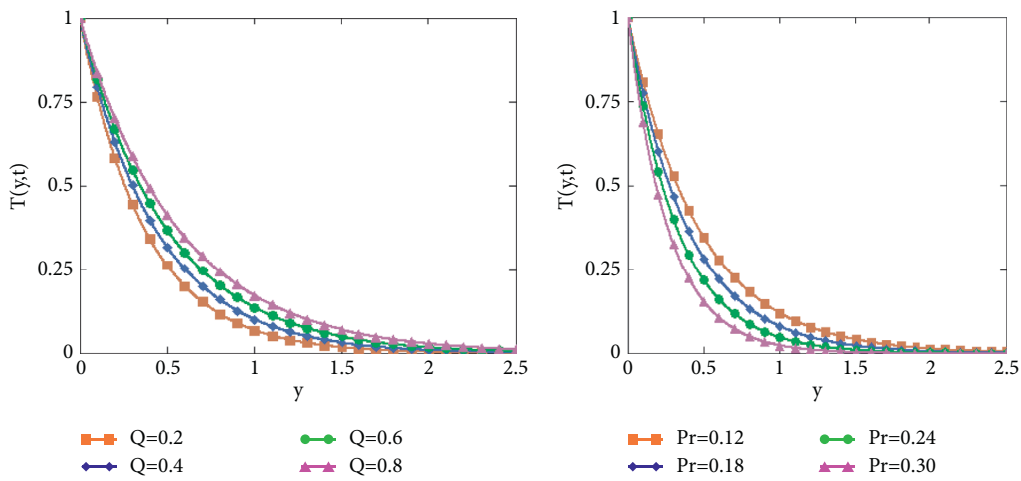

FIGURE 7: Variation in temperature profile $T(y, t)$ for various values of heat generation parameter Q and Pr at $\gamma = 0.5$.
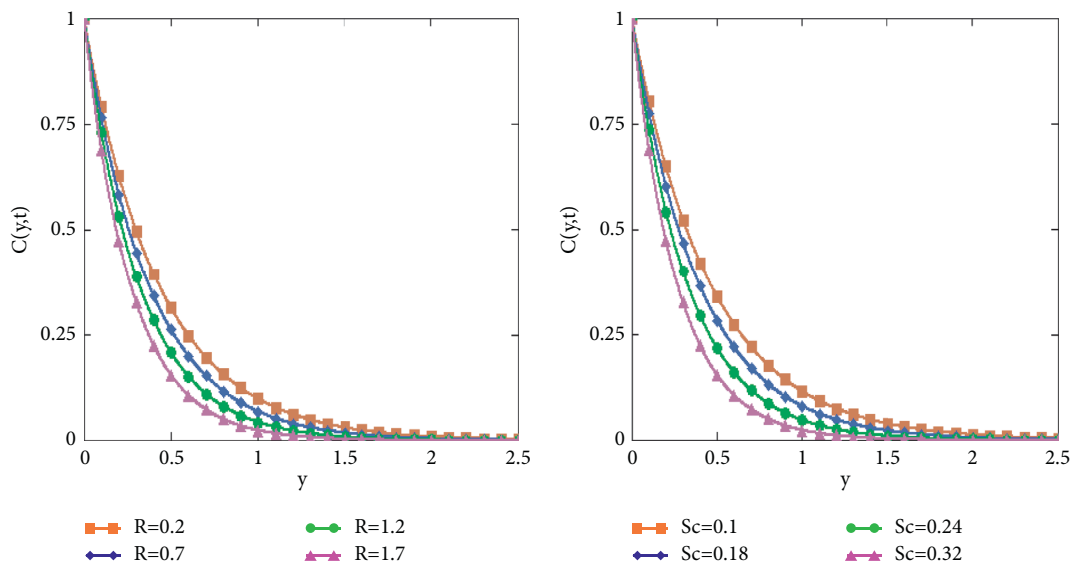

FIGURE 8: $C(y, t)$ for distinct values of chemical reaction parameter R and Sc at $\alpha = 0.5$.
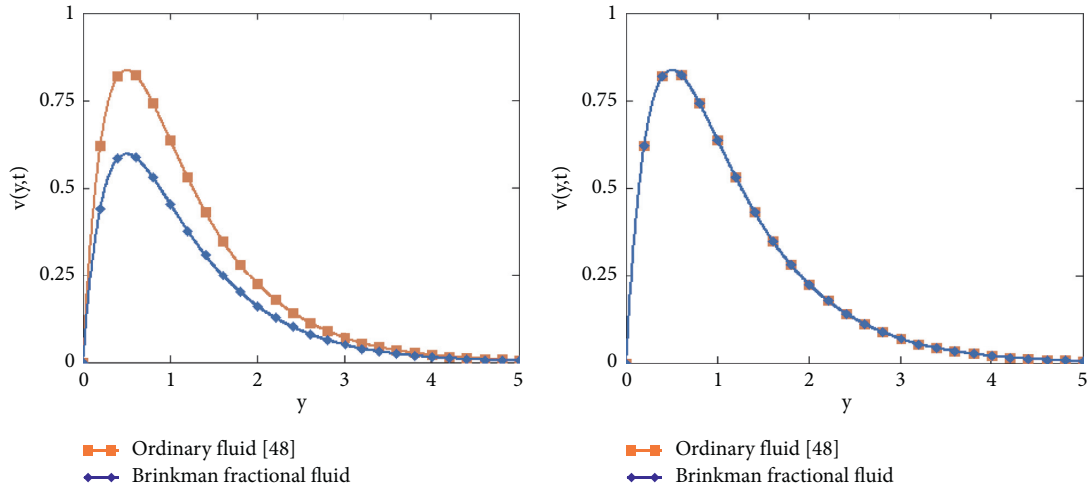
Figure 9: Velocity distribution for comparison of our work with Olisa [48].
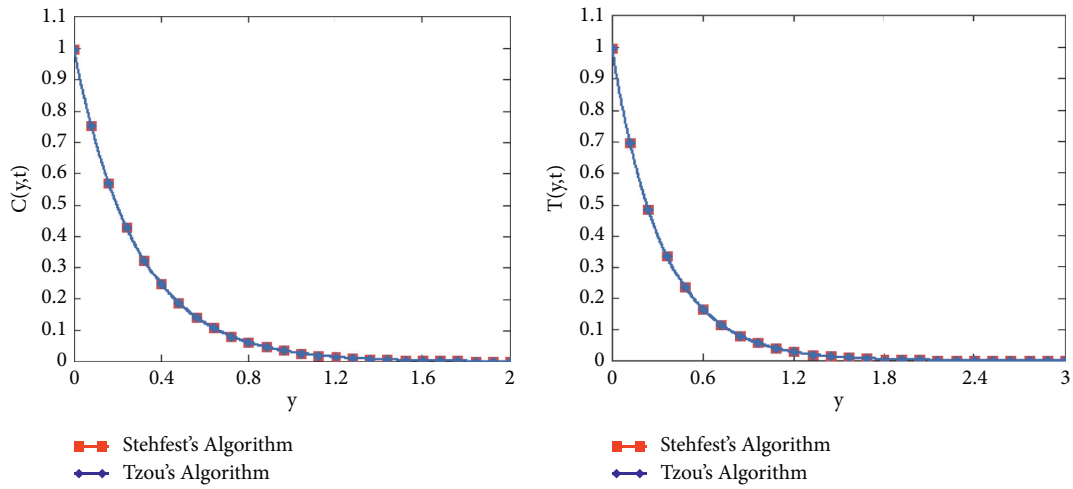

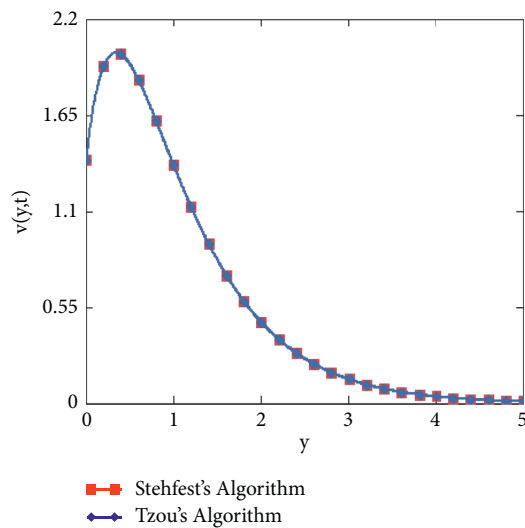Figure 10: Concentrations and temperatures obtained by Stehfest's and Tzou's algorithms.

Figure 11: Velocity profile obtained by Stehfest's and Tzou's algorithms.

fluid profiles are identical which shows the authenticity of the present work. Figure 10 represents the validity of inversion algorithms for concentration and temperature profiles. The overlapping velocity profiles show the validity of inversion algorithms as shown in Figure 11.

6. Conclusion

Solution of free convection magnetohydrodynamic flow of Brinkman-type fluid has been obtained via Laplace transform. Different parameters used in the model are plotted and discussed. The model is solved with a fractional derivative known as Caputo fractional derivative.

Here are the main points which have been summarized for this model:

(i) Velocity distribution retards with decreasing values of fractional parameter

(ii) Thermal buoyancy forces lead to accelerate the $v(y, t)$

(iii) The $v(y, t)$ decreases as magnetic parameter, chemical reaction parameter, Prandtl number, and Sc increases

(iv) The Brinkman parameter is a decreasing function of velocity field

(v) The larger values of Q increased the $T(y, t)$

(vi) The larger values of Pr reduced the $T(y, t)$

(vii) The concentration level is a decreasing function of Sc

(viii) The smaller values of R reduce the concentration profile

(ix) Caputo fractional derivative is the best choice to enhance the fluid motion as compared to ordinary fluid

Data Availability

All used data are included within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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