Research Article

On Computation of Recently Defined Degree-Based Topological Indices of Some Families of Convex Polytopes via $M$-Polynomial

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Topological indices are of incredible significance in the field of graph theory. Convex polytopes play a significant role both in various branches of mathematics and also in applied areas, most notably in linear programming. We have calculated some topological indices such as atom-bond connectivity index, geometric arithmetic index, $K$-Banhatti indices, and $K$-hyper-Banhatti indices and modified $K$-Banhatti indices from some families of convex polytopes through $M$-polynomials. The $M$-polynomials of the graphs provide us with a great help to calculate the topological indices of different structures.

1. Introduction

Graph theory is a powerful and definable field of mathematics that in every field of science has countless adaptations. In graph theory, different characteristics of graphs are defined [1]. The topological indices display the graphical structure and many other characteristics in graphs. They are typically based on the distances between the vertices, on vertex degrees, or on the graph depicted by the matrix.

Using the general polynomial is the general method by which we can generate the unique form of topological indices. Hosoya polynomial is an example of this form [2]. Similarly $M$-polynomial [3] that is an algebraic polynomial can also explain the behavior of the molecular structure. It is also graph representative mathematical object which is the most general polynomial developed till now and gives us the formulas that are very close to the degree-based topological indices.

The $M$-polynomial introduced in 2015 for a graph $G$ by Emeric Deutsch and Sandi Klavžar [4] is defined as

$$M(G,x,y) = \sum_{\delta \leq i \leq j \leq \Delta} m_{ij}(G)x^i y^j.$$  \hspace{1cm} (1)

Here, $m_{ij}(G)$ is the number of edges $uv \in E(G)$, such that $\{d_u, d_v\} = \{i, j\}$, $\delta = \min\{d_v \mid v \in V(G)\}$, and $\Delta = \max\{d_v \mid v \in V(G)\}$.

We generally find topological indices using the definitions, but we can also find them using the chosen graph’s derivatives and integrals on $M$-polynomials. So if we have an $M$-polynomial of the graph, we can measure its various topological indices. Several $M$-polynomials of several graphs have been added over the last few years [3, 5, 6].

In 1947, first index, the wiener index suggested by Wiener. A well-known degree-based topological invariant known as atom-bond connectivity index [7] was purposed in 1998, by Estrada et al. This index is denoted by ABC. Damir Vukičević and Boris Furtula suggested a new index in 2009, known as geometric arithmetic index [8] which is denoted by GA. $K$-Banhatti and $K$-hyper-Banhatti indices were introduced by Kulli in 2016 [9, 10]. Similarly, in 2017, Kulli introduced some more important topological indices [11] named as modified $K$-Banhatti indices. Here we will use the operators which are defined as
2. Some Families of Convex Polytopes and Their Computational Aspects

If we add new edges in the graph of convex polytopes \( Q_n \), then we get a graph named as convex polytopes \( T_n \). It will consist of three-sided face, five-sided face, and \( n \)-sided faces. \( a_{i_1}b_{i_1}, \) i.e., \( V(T_n) = V(Q_n) \) and \( V(A_n) = (R_n) \cup \{ b_{i_2}c_{i_2} \mid 1 \leq i_2 \leq n \} \) (for example, see Figure 1).

Similarly, the graph of convex polytope \( A_n \), i.e., double antiprism, can be acquired from the graph of convex polytopes \( R_n \), by adding new edges \( b_{i_1}c_{i_1} \), i.e., \( V(A_n) = V(R_n) \) and \( V(S_n) = V(Q_n) \cup \{ c_{i_2}c_{i_2} \mid 1 \leq i_2 \leq n \} \) (for example, see Figure 2).

Convex polytopes (double antiprism) \( S_n \) graph can be obtained from the graph of convex polytope \( Q_n \) by adding new edges \( c_{i_1}c_{i_1} \), i.e., \( V(S_n) = V(Q_n) \cup \{ c_{i_1}c_{i_1} \mid 1 \leq i_1 \leq n \} \) (for example, see Figure 3).

3. M-Polynomial and Topological Indices of Convex Polytopes \( T_n \)

Now we will find the topological indices of \( T_n \) using its \( M \)-polynomial.

**Theorem 1.** The \( M \)-polynomial of \( T_n \) is \( M[T_n; x; y] = 2nx^3y^3 + 2nx^3y^6 + nx^4y^4 + 2nx^4y^6 + nx^6y^6 \). Then

\begin{align*}
1 & \quad ABC[T_n] = (n/12)(16 + 4\sqrt{14} + 3\sqrt{6} + \sqrt{48} + \sqrt{10}) \\
2 & \quad GA[T_n] = (n/15)(30 + 10\sqrt{2} + 6\sqrt{6}) \\
3 & \quad B_1[T_n] = 178n \\
4 & \quad B_2[T_n] = 502n \\
5 & \quad HB_1[T_n] = 2126n \\
6 & \quad HB_2[T_n] = 2518n \\
7 & \quad mB_1[T_n] = (17033/10920)n \\
8 & \quad mB_2[T_n] = (1171/1680)n \\
9 & \quad H_b[T_n] = (17033/5460)n.
\end{align*}

**Proof**

\[
\begin{align*}
D_x g(x, y) &= 6nx^3y^3 + 6nx^3y^6 + 4nx^4y^4 + 8nx^4y^6 + 6nx^6y^6; \\
D_y g(x, y) &= 6nx^3y^3 + 12nx^3y^6 + 4nx^4y^4 + 12nx^4y^6 + 6nx^6y^6; \\
\left(D_x + D_y\right) [g(x, y)] &= 12nx^3y^3 + 18nx^3y^6 + 8nx^4y^4 + 20nx^4y^6 + 12nx^6y^6; \\
S_y^{1/2} g(x, y) &= \frac{2}{\sqrt{3}} nx^3y^3 + \frac{2}{\sqrt{6}} nx^3y^6 + \frac{n}{2} x^4y^4 + \frac{2}{\sqrt{6}} nx^4y^6 + \frac{n}{\sqrt{2}} x^6y^6, \\
S_x^{1/2} S_y^{1/2} g(x, y) &= \frac{2}{3} nx^3y^3 + \frac{2}{3\sqrt{2}} nx^3y^6 + \frac{n}{4} x^4y^4 + \frac{1}{\sqrt{6}} nx^4y^6 + \frac{n}{6} x^6y^6.
\end{align*}
\]
\[
J_S^{1/2} J_x^{1/2} g(x, y) = \frac{2}{3} nx^6 + \frac{2}{3\sqrt{2}} nx^9 + \frac{n}{4} x^8 + \frac{1}{\sqrt{6}} nx^{10} + \frac{n}{6} x^{12},
\]

\[
Q_x J_S^{1/2} J_x^{1/2} g(x, y) = \frac{2}{3} nx^4 + \frac{2}{3\sqrt{2}} nx^7 + \frac{n}{4} x^6 + \frac{1}{\sqrt{6}} nx^8 + \frac{n}{6} x^{10},
\]

\[
D_x^{1/2} Q_x J_S^{1/2} J_x^{1/2} g(x, y) = \frac{4}{3} nx^4 + \frac{\sqrt{14}}{3} nx^7 + \frac{\sqrt{6}}{4} nx^6 + \frac{\sqrt{8}}{\sqrt{6}} nx^8 + \frac{\sqrt{10}}{6} nx^{10},
\]

\[
D_y^{1/2} [g(x, y)] = 2 \sqrt{3} nx^3 y^3 + 2 \sqrt{6} nx^3 y^6 + 2 nx^4 y^4 + 2 \sqrt{6} nx^4 y^6 + n \sqrt{6} nx^6 y^6,
\]

\[
D_x^{1/2} Q_y J_x^{1/2} g(x, y) = 6 nx^3 y^3 + 2 \sqrt{6} \sqrt{3} nx^3 y^6 + 4 nx^4 y^4 + 2 \sqrt{3} \sqrt{6} nx^4 y^6 + 6 nx^6 y^6,
\]

\[
J D_x^{1/2} D_y^{1/2} [g(x, y)] = 6 nx^6 + 2 \sqrt{6} \sqrt{3} nx^9 + 4 nx^8 + 2 \sqrt{6} \sqrt{6} nx^{10} + 6 nx^{12},
\]

\[
S_x J D_x^{1/2} D_y^{1/2} [g(x, y)] = nx^6 + \frac{2}{3} \sqrt{2} nx^9 + \frac{1}{2} nx^8 + \frac{2}{5} \sqrt{6} nx^{10} + \frac{1}{2} nx^{12},
\]

\[
2 S_x J D_x^{1/2} D_y^{1/2} [g(x, y)] = 2 nx^6 + \frac{4}{3} \sqrt{2} nx^9 + nx^8 + \frac{4}{5} \sqrt{6} nx^{10} + nx^{12},
\]

\[
J g(x, y) = 2 nx^6 + nx^8 + 2 nx^9 + nx^{10} + nx^{12};
\]

\[
Q_x J g(x, y) = 2 nx^4 + nx^6 + 2 nx^7 + 2 nx^8 + nx^{10};
\]

\[
D_x Q_x J g(x, y) = 8 nx^4 + 6 nx^6 + 14 nx^7 + 16 nx^8 + 10 nx^{10};
\]

\[
2 D_x Q_x J g(x, y) = 16 nx^4 + 12 nx^6 + 28 nx^7 + 32 nx^8 + 20 nx^{10};
\]

\[
J(D_x + D_y) g(x, y) = 12 nx^6 + 18 nx^7 + 8 nx^8 + 20 nx^{10} + 12 nx^{12};
\]

\[
Q_x J(D_x + D_y) g(x, y) = 12 nx^4 + 18 nx^7 + 8 nx^6 + 20 nx^8 + 12 nx^{10};
\]

\[
D_x Q_x J(D_x + D_y) g(x, y) = 48 nx^4 + 126 nx^7 + 48 nx^6 + 160 nx^8 + 120 nx^{10};
\]

\[
D_x g(x, y) = 18 nx^3 y^3 + 18 nx^3 y^6 + 16 nx^4 y^4 + 32 nx^4 y^6 + 36 nx^6 y^6;
\]

\[
D_y g(x, y) = 18 nx^3 y^3 + 72 nx^3 y^6 + 16 nx^4 y^4 + 72 nx^4 y^6 + 36 nx^6 y^6;
\]

\[
D_x Q_x J g(x, y) = 32 nx^4 + 98 nx^7 + 36 nx^6 + 128 nx^8 + 100 nx^{10};
\]

\[
2 D_x Q_x J g(x, y) = 64 nx^4 + 196 nx^7 + 72 nx^6 + 256 nx^8 + 200 nx^{10};
\]

\[
2 D_x Q_x J(D_x + D_y) g(x, y) = 96 nx^4 + 252 nx^7 + 96 nx^6 + 320 nx^8 + 240 nx^{10};
\]

\[
(D_x^2 + D_y^2) g(x, y) = 36 nx^3 y^3 + 90 nx^3 y^6 + 32 nx^4 y^4 + 104 nx^4 y^6 + 72 nx^6 y^6;
\]

\[
J(D_x^2 + D_y^2) g(x, y) = 36 nx^6 + 90 nx^9 + 32 nx^8 + 104 nx^{10} + 72 nx^{12};
\]

\[
Q_x J(D_x^2 + D_y^2) g(x, y) = 36 nx^4 + 90 nx^7 + 32 nx^6 + 104 nx^8 + 72 nx^{10};
\]

\[
D_x Q_x J(D_x^2 + D_y^2) g(x, y) = 576 nx^4 + 4410 nx^7 + 1152 nx^6 + 6656 nx^8 + 7200 nx^{10};
\]

\[
L_x g(x, y) = 2 nx^6 y^3 + 2 nx^6 y^6 + nx^8 y^4 + 2 nx^8 y^6 + nx^{12} y^6;
\]

\[
L_y g(x, y) = 2 nx^3 y^6 + 2 nx^3 y^{12} + nx^4 y^8 + 2 nx^4 y^{12} + nx^6 y^{12};
\]

\[
J(L_x + L_y) g(x, y) = 4 nx^9 + 4 nx^{12} + 2 nx^{14} + 2 nx^{15} + 2 nx^{16} + 2 nx^{18};
\]

\[
Q_x J(L_x + L_y) g(x, y) = 4 nx^7 + 4 nx^{10} + 2 nx^{12} + 2 nx^{13} + 2 nx^{14} + 2 nx^{16};
\]

\[
S_x Q_x J(L_x + L_y) g(x, y) = \frac{4}{7} nx^7 + \frac{4}{10} nx^{10} + \frac{1}{6} nx^{12} + \frac{2}{13} nx^{13} + \frac{1}{7} nx^{14} + \frac{1}{8} nx^{16};
\]

\[
S_y g(x, y) = \frac{2}{3} nx^3 y^3 + \frac{1}{3} nx^3 y^6 + \frac{1}{4} nx^4 y^4 + \frac{1}{3} nx^4 y^6 + \frac{1}{6} nx^6 y^6;
\]

\[
S_x g(x, y) = \frac{2}{3} nx^3 y^3 + \frac{2}{3} nx^3 y^6 + \frac{1}{4} nx^4 y^4 + \frac{1}{2} nx^4 y^6 + \frac{1}{6} nx^6 y^6;
\]

\[
(S_x + S_y) g(x, y) = \frac{4}{3} nx^3 y^3 + \frac{1}{2} nx^4 y^4 + \frac{5}{6} nx^4 y^6 + \frac{1}{3} nx^6 y^6;
\]
Figure 1: Graph of convex polytope $T_6$.

Figure 2: Graph of double antiprism $A_6$.

Figure 3: Graph of double antiprism $S_6$. 

4 Complexity
Here, we will find the topological indices of \( A_n \).

**Theorem 2.** The \( M \)-polynomial of \( A_n \) is

\[
M[(A_n); x; y] = 2nx^4y^4 + 4nx^4y^6 + nx^6y^6. \tag{4}
\]

Then,

(1) \( ABC[A_n] = (n/36)(18\sqrt{6} + 48\sqrt{3} + 6\sqrt{10}) \)

(2) \( GA[A_n] = (n/5)(15 + 8\sqrt{6}) \)

(3) \( B_1[A_n] = 176n \)

(4) \( B_2[A_n] = 536n \)

(5) \( HB_1[A_n] = 2272n \)

(6) \( HB_2[A_n] = 2768n \)

(7) \( mB_1[A_n] = (961/840)n \)

(8) \( mB_2[A_n] = (49/120)n \)

(9) \( H_b[A_n] = (961/420)n. \)

**Proof**

Here, \( D_x \) and \( D_y \) are the first-order differentiation operators with respect to \( x \) and \( y \), respectively. We will present the following identities:

\[
D_x 1/(S_x + S_y) g(x, y) = \frac{4}{3} nx^6 + nx^9 + \frac{1}{2} nx^8 + \frac{5}{6} nx^{10} + \frac{1}{3} nx^{12},
\]

\[
Q^{-1} D_x 1/(S_x + S_y) g(x, y) = \frac{4}{3} nx^4 + nx^7 + \frac{1}{2} nx^6 + \frac{5}{6} nx^8 + \frac{1}{3} nx^{10},
\]

\[
S_x Q^{-1} D_x 1/(S_x + S_y) g(x, y) = \frac{1}{3} nx^4 + \frac{2}{3} nx^7 + \frac{1}{12} nx^6 + \frac{5}{48} nx^8 + \frac{1}{30} nx^{10},
\]

\[
2 S_x Q^{-1} D_x (L_x + L_y) g(x, y) = \frac{8}{7} nx^2 + \frac{4}{5} nx^{10} + \frac{1}{3} nx^{12} + \frac{4}{13} nx^{13} + \frac{2}{7} nx^{14} + \frac{1}{4} nx^{16}. \tag{3}
\]

4. \( M \)-Polynomial and Topological Indices of Convex Polytopes \( A_n \)

Here, we will find the topological indices of \( A_n \) using its \( M \)-polynomial.
\[ D_{y}^{1/2} [g(x, y)] = 4nx^4 y^4 + 4\sqrt{6}nx^4 y^6 + \sqrt{6}nx^6 y^6, \]
\[ D_{x}^{1/2} D_{y}^{1/2} [g(x, y)] = 8nx^4 y^4 + 8\sqrt{6}nx^4 y^6 + 6nx^6 y^6, \]
\[ JD_{y}^{1/2} D_{y}^{1/2} [g(x, y)] = 8nx^8 + 8\sqrt{6}nx^{10} + 6nx^{12}, \]
\[ S_x JD_{x}^{1/2} D_{y}^{1/2} [g(x, y)] = nx^8 + 4\sqrt{6}nx^{10} + \frac{1}{2}nx^{12}, \]
\[ 2S_x J D_{x}^{1/2} D_{y}^{1/2} [g(x, y)] = 2nx^8 + 8\sqrt{6}nx^{10} + nx^{12}, \]
\[ J_x g(x; y) = 2nx^8 + 4nx^{10} + nx^{12}, \]
\[ Q_x J_x g(x; y) = 2nx^6 + 4nx^8 + nx^{10}, \]
\[ D_x Q_x J_x g(x; y) = 12nx^6 + 32nx^8 + 10nx^{10}, \]
\[ 2D_x Q_x J_x g(x; y) = 24nx^6 + 64nx^8 + 20nx^{10}, \]
\[ J(D_x + D_y) g(x; y) = 16nx^8 + 40nx^{10} + 12nx^{12}, \]
\[ Q_x J(D_x + D_y) g(x; y) = 16nx^6 + 40nx^8 + 12nx^{10}, \]
\[ D_x Q_x J(D_x + D_y) g(x; y) = 96nx^6 + 320nx^8 + 120nx^{10}, \]
\[ D_{x}^2 g(x; y) = 32nx^8 y^4 + 64nx^8 y^6 + 36nx^6 y^6; \]
\[ D_{y}^2 g(x; y) = 32nx^4 y^4 + 144nx^4 y^6 + 36nx^6 y^6; \]
\[ (D_{x}^2 + D_{y}^2) g(x; y) = 64nx^4 y^4 + 208nx^4 y^6 + 72nx^6 y^6; \]
\[ D_{x}^2 Q_x J g(x; y) = 72nx^6 + 256nx^8 + 100nx^{10}; \]
\[ 2D_{x}^2 Q_x J g(x; y) = 144nx^6 + 512nx^8 + 200nx^{10}, \]
\[ 2D_x Q_x J(D_x + D_y) g(x; y) = 192nx^6 + 640nx^8 + 240nx^{10}; \]
\[ J(D_{x}^2 + D_{y}^2) g(x; y) = 64nx^8 + 208nx^{10} + 72nx^{12}; \]
\[ Q_x J(D_{x}^2 + D_{y}^2) g(x; y) = 64nx^6 + 208nx^8 + 72nx^{10}; \]
\[ D_{x}^2 Q_x J(D_{x}^2 + D_{y}^2) g(x; y) = 384nx^6 + 1664nx^8 + 720nx^{10}; \]
\[ L_x g(x; y) = 2nx^4 y^4 + 4nx^6 y^6 + nx^{12} y^6; \]
\[ L_y g(x; y) = 2nx^4 y^4 + 4nx^8 y^6 + nx^{12} y^6; \]
\[ J(L_x + L_y) g(x; y) = 4nx^{12} + 4nx^{16} + 4nx^{14} + 2nx^{18}; \]
\[ Q_x J(L_x + L_y) g(x; y) = 4nx^{10} + 4nx^{14} + 4nx^{12} + 2nx^{16}; \]
\[ S_x Q_x J(L_x + L_y) g(x; y) = \frac{2}{5}nx^{10} + \frac{2}{7}nx^{14} + \frac{1}{3}nx^{12} + \frac{1}{8}nx^{16}, \]
\[ S_y g(x; y) = \frac{1}{2}nx^4 y^4 + \frac{2}{3}nx^4 y^6 + \frac{n}{6}nx^6 y^6, \]
\[ S_x g(x; y) = \frac{1}{2}nx^4 y^4 + nx^4 y^6 + \frac{n}{6}nx^6 y^6; \]
\[ (S_x + S_y) g(x; y) = nx^4 y^4 + \frac{5}{3}nx^4 y^6 + \frac{n}{3}nx^6 y^6, \]
\[ J(S_x + S_y) g(x; y) = nx^8 + \frac{5}{3}nx^{10} + \frac{n}{3}nx^{12}, \]
\[ Q_x J(S_x + S_y) g(x; y) = nx^6 + \frac{5}{3}nx^8 + \frac{n}{3}nx^{10}, \]
\[ S_x Q_x J(S_x + S_y) g(x; y) = \frac{n}{6} nx^6 + \frac{7}{24}nx^8 + \frac{n}{30}nx^{10}, \]
\[ 2S_x Q_x J(L_x + L_y) g(x; y) = \frac{4}{5}nx^{10} + \frac{4}{7}nx^{14} + \frac{2}{3}nx^{12} + \frac{1}{4}nx^{16}. \]
Hence, we conclude the following results:

(1) $ABC[A_n] = D_x^{1/2} Q_{-2} J x^{1/2} [g(x, y)]_{x=1} = (n/36)(18\sqrt{6}) + 48\sqrt{3} + 6\sqrt{10}$

(2) $GA[A_n] = 2S_x D^{1/2} y^{1/2} [g(x, y)]_{x=1} = (n/5)(15 + 8\sqrt{6})$

(3) $B_1[A_n] = (D_x + D_y + 2D_x Q_{-2} J)[g(x, y)]_{x=1} = 176n$

(4) $B_2[A_n] = D_x Q_{-2} J (D_x + D_y)[g(x, y)]_{x=1} = 536n$

(5) $HB_1[A_n] = (D_x^2 + D_y^2 + 2D_x^2 Q_{-2} J + 2D_y Q_{-2} J ) (D_x + D_y)[g(x, y)]_{x=1} = 2272n$

(6) $HB_2[A_n] = D_x^2 Q_{-2} J (D_x^2 + D_y^2)[g(x, y)]_{x=1} = 2768n$

(7) $mB_1[A_n] = S_x Q_{-2} J (L_x + L_y) [g(x, y)]_{x=1} = (961/840)n$

(8) $mB_2[A_n] = S_x Q_{-2} J (S_x + S_y) [g(x, y)]_{x=1} = (49/120)n$

(9) $H_b[A_n] = 2S_x Q_{-2} J (L_x + L_y) [g(x, y)]_{x=1} = (961/420)n$. □

5. $M$-Polynomial and Topological Indices of Convex Polytopes $S_n$

Now we will find the topological indices of $S_n$ using its $M$-polynomial.

\[\begin{align*}
D_x g(x, y) &= 6nx^3 y^3 + 6nx^3 y^5 + 20nx^5 y^5, \\
D_y g(x, y) &= 6nx^3 y^3 + 10nx^3 y^5 + 20nx^5 y^5, \\
(D_x + D_y) [g(x, y)] &= 12nx^3 y^3 + 16nx^3 y^5 + 40nx^5 y^5, \\
S_x^{1/2} g(x, y) &= \frac{2}{\sqrt{3}}nx^3 y^3 + \frac{2}{\sqrt{5}}nx^3 y^5 + \frac{4}{\sqrt{5}}nx^5 y^5, \\
S_y^{1/2} g(x, y) &= \frac{2}{3}nx^3 y^3 + \frac{2}{\sqrt{15}}nx^3 y^5 + \frac{4}{5}nx^5 y^5, \\
JS_x^{1/2} g(x, y) &= \frac{2}{3}nx^6 + \frac{2}{\sqrt{15}}nx^8 + \frac{4}{5}nx^{10}, \\
Q_{-2}JS_x^{1/2} g(x, y) &= \frac{2}{3}nx^4 + \frac{2}{\sqrt{15}}nx^6 + \frac{4}{5}nx^8, \\
D_x^{1/2} Q_{-2} JS_x^{1/2} g(x, y) &= \frac{4}{3}nx^4 + \frac{2\sqrt{6}}{\sqrt{15}}nx^6 + \frac{8\sqrt{5}}{5}nx^8, \\
D_y^{1/2} [g(x, y)] &= 2\sqrt{3}nx^3 y^3 + 2\sqrt{5}nx^3 y^5 + 4\sqrt{5}nx^5 y^5, \\
D_x^{1/2} D_y^{1/2} [g(x, y)] &= 6nx^3 y^3 + 2\sqrt{15}nx^3 y^5 + 20nx^5 y^5, \\
J D_x^{1/2} D_y^{1/2} [g(x, y)] &= 6nx^6 + 2\sqrt{15}nx^8 + 20nx^{10}, \\
S_x J D_x^{1/2} D_y^{1/2} [g(x, y)] &= nx^6 + \frac{\sqrt{15}}{4}nx^8 + 2nx^{10}, \\
2S_x J D_x^{1/2} D_y^{1/2} [g(x, y)] &= 2nx^6 + \frac{\sqrt{15}}{2}nx^8 + 4nx^{10},
\end{align*}\]

Theorem 3. The $M$-polynomial of $S_n$ is

\[M[S_n; x; y] = 2nx^3 y^3 + 2nx^3 y^5 + 4nx^5 y^5.\]  

Then,

(1) $ABC[S_n] = (n/15)(20 + 6\sqrt{10} + 24\sqrt{2})$

(2) $GA[S_n] = (n/2)(12 + \sqrt{15})$

(3) $B_1[S_n] = 172n$

(4) $B_2[S_n] = 464n$

(5) $HB_1[S_n] = 15824n$

(6) $HB_2[S_n] = 1952$

(7) $mB_1[S_n] = (14332/9009)n$

(8) $mB_2[S_n] = (32/45)n$

(9) $H_b[S_n] = (28664/9009)n$. 

Proof
Hence, we conclude the following results:

(1) ABC \([S_n]=D_{x}^{1/2}Q_{x}J_{x}^{1/2}J_{y}^{1/2}[g(x,y)]\) \(x=1\) = \((n/15)
\)
\(2) G A \([S_n]=2S_{x}ID_{x}^{1/2}D_{x}^{1/2}J_{y}^{1/2}[g(x,y)]\) \(x=1\) = \((n/2)(12+\sqrt{15})\)
\(3) B_{x} \[S_n]=[D_{x}+D_{y}+2D_{x}Q_{x}J_{y}][g(x,y)]\) \(x=1\) = 172n
\(4) B_{y} \[S_n]=[D_{x}Q_{x}J_{y}][g(x,y)]\) \(x=1\) = 464n
\(5) HB_{x} \[S_n] = (D_{x}^{2} + D_{x}^{2} + 2D_{x}Q_{x}J_{y} + 2D_{x}Q_{x}J_{y})[g(x,y)]\) \(x=1\) = 15824n
\(6) HB_{y} \[S_n] = D_{y}^{2}Q_{x}J_{y} + D_{y}^{2}J_{y} g(x,y)]\) \(x=1\) = 1952n
\(7) m_{B_{x}} \[S_n] = S_{x}Q_{x}J_{y} \left\{L_{x} + L_{y}\right\} \; [g(x,y)]\) \(x=1\) = (14332/9009)n
\(8) m_{B_{y}} \[S_n] = S_{x}Q_{x}J_{y} \left\{S_{x} + S_{y}\right\} \; [g(x,y)]\) \(x=1\) = (32/45)n
\(9) H_{x} \[S_n] = 2S_{x}Q_{x}J_{y} \left\{L_{x} + L_{y}\right\} \; [g(x,y)]\) \(x=1\) = (28664/9009)n.
Figure 4: The plot of topological indices of convex polytope $T_n$. (a) Atom-bond connectivity index, (b) geometric arithmetic index, (c) first $K$-Banhatti index, (d) second $K$-Banhatti index, (e) first $K$-hyper-Banhatti index, (f) second $K$-hyper-Banhatti index, (g) modified first $K$-Banhatti index, (h) modified second $K$-Banhatti index, and (i) harmonic $K$-Banhatti index.

Figure 5: The plot of topological indices of convex polytope $A_n$. (a) Atom-bond connectivity index, (b) geometric arithmetic index, (c) first $K$-Banhatti index, (d) second $K$-Banhatti index, (e) first $K$-hyper-Banhatti index, (f) second $K$-hyper-Banhatti index, (g) modified first $K$-Banhatti index, (h) modified second $K$-Banhatti index, and (i) harmonic $K$-Banhatti index.
6. Graphical Analysis

Graphs are helpful as they elucidate the facts in visual form. Moreover, we demonstrate the behavior of $M$-polynomial and topological indices by drawing graphics. A graph stores information of the chemical structures that is equal to several words. The information that could not be represented in terms of words can be clarified in a better way by means of a graph. This graphical description provides us a great way to understand results against parameters (see Figures 4–6).

### 7. Conclusion

Topological indices are a numerical quantity computed from the graph of any compound and typically remain the same under graph isomorphism. These are used to predict and model certain properties of different compounds. We can easily understand the properties, for example, the chemical reactivity, biological activities, and physical features of associated molecules. In this work, we have calculated very important degree-based topological indices such as atom-bond connectivity index, geometric arithmetic index, $K$-Banhatti indices, $K$-hyper-Banhatti indices and modified $K$-Banhatti indices of some families of cycle graphs which are given in Table 1 with the help of their Polynomials. These findings can play a huge role in industry and pharmacy because these indices are absolutely functions of chemical graphs and explain many chemical properties as viscosity, strain energy, and heat of formation etc.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare no conflicts of interest.
References


