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Research Article

Synchronization of a Hyperchaotic Finance System

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In this article, we propose a series of control strategies to synchronize two chaotic financial systems. Due to the characteristics of chaotic systems, the system is very sensitive to its initial values. Thus, the behaviour of two systems with different initial values will be completely different. In order to realize the synchronization of two financial chaotic systems, we designed a series of controls including controllers to realize global asymptotic synchronization and controllers to realize global exponential synchronization to make the two systems fully synchronized. We provide mathematical proofs to show that the designed controls are effective. Numerical methods are used to verify the effectiveness of the controls.

1. Introduction

In recent years, it has been found that there exists chaos in many engineering systems and scientific systems such as chemistry models and ecology models. For example, Lorentz chaotic system family [1–3], Rössler chaotic system [4], and Chua's chaotic circuit [5] have great applications in the field of natural science and engineering. More complex multidimensional, multiscroll, and multiwing chaotic systems have also been studied [6–8]. The dynamic behaviours of chaotic systems with time-delay have also been studied extensively [9–11]. Chaos in many financial systems has also been considered in the literature [12–15]. These models show that the interactions of financial factors can lead to complex phenomena that are difficult to estimate. An important characteristic of chaotic systems is their extreme sensitivity to initial conditions. In chaotic systems, very small initial state differences will lead to great differences in systematic trajectories. Another characteristic of chaotic systems is their global boundedness. The global boundedness of a variety of chaotic systems has been studied in the literature [16–19].

In recent years, the work of Ott, Grebogi, and Yorke (OGY method) [20] has pioneered the academic research of chaos control. Chaos leads to irregular and unpredictable situation. Therefore, eliminating or completely suppressing chaos in nonlinear dynamic systems has practical application value. A variety of control strategies were used to suppress chaos in the literature [21, 22]. Pecora and Carrol [23, 24] have carried out pioneering work on chaos synchronization. In recent years, chaos synchronization has been extensively studied [7, 25–27]. Synchronization of chaotic systems has been studied extensively in recent years. Mu et al. investigated route to broadband optical chaos generation and synchronization using dual-path optically injected semiconductor lasers [28]. Lü et al. studied a new synchronization tracking technique for uncertain discrete network with spatiotemporal chaos behaviours [29]. Lai et al. analysed the synchronization control of an unusual chaotic system with exponential term and coexisting attractors [30]. Yadav et al. studied difference synchronization among three chaotic systems [31]. Zheng et al. studied a hybrid model for construction of digital chaos and local synchronization [32].

In this paper, we consider the chaotic system proposed in [33]. The system presented in [33] is based on a system of differential equations which describes the interactions between elements in the financial system. The interactions between these elements trigged chaos. Chaos phenomenon will bring unpredictable risks to the financial system. Thus, it is very important to control the system. In this paper, we propose a series of control laws to achieve the synchronization of the system. Our study provides insight into hedging risks in the financial system. The purpose of this article is to apply feedback to controls to this financial model to realize the synchronization of the chaotic system. Furthermore, we adopt the Lyapunov direct method to prove our results. We also use numerical simulations to verify the effectiveness of the designed controls.

The rest of this paper is organized as follows. In Section 2, we present the preliminary of this paper, where the idea of chaos synchronization is outlined. Then, in Section 3, we design control laws for the hyperchaotic financial model. Under such control laws, two systems of the model with different initial conditions are synchronized. We use mathematical analysis to prove the effectiveness of the control laws and use numerical simulation to show the applications of the designed control laws. Conclusions are drawn in Section 4.

2. Preliminary

Tong et al. [33] proposed a hyperchaotic financial system, which is described by the following system of differential equations:

$$\begin{aligned} \dot{x} &= z + (y - a)x + w, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz, \\ \dot{w} &= -0.05xz + rw, \end{aligned} \tag{1}$$

where the dot denotes differentiation with respect to time t. System (1) displays complicated dynamical behaviours. As shown in Figure 1, for $a = 0.9$, $b = 0.1$, $c = 1$, and $r = -0.6$, the trajectory of system is a chaotic attractor. By calculating the Lyapunov exponent of system (1), we find that the system has Lyapunov exponents 0.828, 0.046, −0.582, and −0.976. Since the system has two positive Lyapunov exponents, the system is hyperchaotic.

Next, we assume that there are two hyperchaotic financial systems including a drive system with subscript d and a receiving system with subscript r. For system (1), the drive system is given by

$$\begin{aligned} \dot{x}_d &= z_d + (y_d - a)x_d + w_d, \\ \dot{y}_d &= 1 - by_d - x_d^2, \\ \dot{z}_d &= -x_d - cz_d, \\ \dot{w}_d &= -0.05x_d z_d + rw_d. \end{aligned} \tag{2}$$

The corresponding receiving system is obtained as

$$\begin{aligned} \dot{x}_r &= z_r + (y_r - a)x_r + w_r + u_1, \\ \dot{y}_r &= 1 - by_r - x_r^2 + u_2, \\ \dot{z}_r &= -x_r - cz_r + u_3, \\ \dot{w}_r &= -0.05x_r z_r + rw_r + u_4, \end{aligned} \tag{3}$$

where u_1, u_2, u_3, and u_4 are nonlinear controllers to be designed such that drive system (2) and driven system (3) can be synchronized. For this purpose, we obtain the error dynamical system by subtracting equation (2) from equation (3). Thus, the error system is obtained as

$$\begin{aligned} \dot{e}_x &= z_r + (y_r - a)x_r + w_r - (z_d + (y_d - a)x_d + w_d) + u_1, \\ \dot{e}_y &= 1 - by_r - x_r^2 - (1 - by_d - x_d^2) + u_2, \\ \dot{e}_z &= -x_r - cz_r - (-x_d - cz_d) + u_3, \\ \dot{e}_w &= -0.05x_r z_r + rw_r - (-0.05x_d z_d + rw_d) + u_4, \end{aligned} \tag{4}$$

where $e_x = x_r - x_d$, $e_y = y_r - y_d$, $e_z = z_r - z_d$, and $e_w = w_r - w_d$.

Equation (4) can be simplified as

$$\begin{aligned} \dot{e}_x &= e_z + e_w - ae_x + y_r x_r - y_d x_d + u_1, \\ \dot{e}_y &= -be_y - x_r^2 + x_d^2 + u_2, \\ \dot{e}_z &= -e_x - ce_z + u_3, \\ \dot{e}_w &= re_w - 0.05x_r z_r + 0.05x_d z_d + u_4. \end{aligned} \tag{5}$$

When there is no control, i.e., $u_1 = 0$, $u_2 = 0, u_3 = 0$, and $u_4 = 0$ if the two hyperchaotic finance systems have different initial conditions, i.e.,

$$[x_d(t_0), \; y_d(t_0), z_d(t_0), w_d(t_0)] \neq t[x_r(t_0), \; y_r(t_0), z_r(t_0), w_r(t_0)]. \tag{6}$$

The trajectories of the two hyperchaotic systems are quite different.

In this work, our aim is to establish appropriate control laws u_1, u_2, u_3, and u_4 such that the drive hyperchaotic financial system (2) and the receiving hyperchaotic system (3) are synchronized. That is, to say, we need

$$\lim_{t \longrightarrow \infty} \mathbf{e}(t) = 0, \tag{7}$$

where $\mathbf{e}(t) = [e_1, e_2, e_3, e_4]^T$.

3. Synchronization Scheme

In this section, we will design a series of control strategies to synchronization two hyperechaotic systems with different initial conditions.

Definition 1. ∀ initial condition of drive system (2)$x_d(t_0)$, $y_d(t_0), z_d(t_0)$, and $w_d(t_0) \in R^4$ and the corresponding initial condition of receiving system (3) $x_r(t_0)$, $y_r(t_0), z_r(t_0)$, and $w_r(t_0) \in R^4$, if the zero solution of error system (4) is locally asymptotically stable, then we say

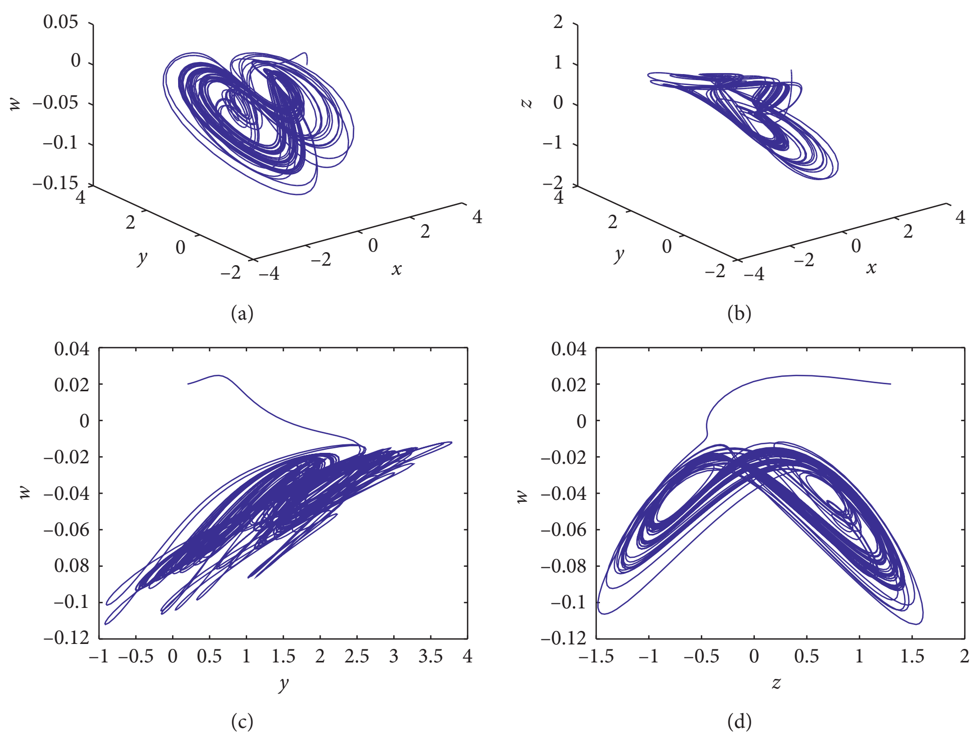


FIGURE 1: Simulation results of system (1) when $a = 0.9$, $b = 0.1$, $c = 1$, and $r = -0.6$ (a) in the x-y-w space, (b) in the x-y-z space, (c) projected on the y-w plane, and (d) projected on the z-w plane.

that drive system (2) and receiving system (18) are locally asymptotically synchronized.

Theorem 1. *For any arbitrary initial conditions, drive system (2) and receiving system (3) are globally asymptotically synchronized by the following control law:*

$$\begin{aligned} u_1 &= -y_r x_r + y_d x_d, \\ u_2 &= x_r^2 - x_d^2, \\ u_3 &= 0, \\ u_4 &= 0.05 x_r z_r - 0.05 x_d z_d. \end{aligned} \tag{8}$$

Proof. Under control law (6), we linearize system (4) at the equilibrium (0,0,0,0) to obtain

$$\begin{bmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & r \end{bmatrix}. \tag{9}$$

Matrix (7) has the eigenvalues $\lambda_1 = r$, $\lambda_2 = -b$, $\lambda_3 = -(a/2) - (c/2) + (\sqrt{a^2 - 2ac + c^2 - 4}/2)$, $\lambda_4 = -(a/2) - (c/2) - (\sqrt{a^2 - 2ac + c^2 - 4}/2)$. It can be seen that all these eigenvalues are negative. Therefore, the zero solution of system (4) is locally asymptotically stable. Hence, drive system (2) and driven system (3) are locally asymptotically synchronized with the control law (6). The proof is complete.

In the following, we use numerical simulations to verify that the designed control law (6) is effective. In Figure 2, we plot the time history of error system (4). As shown in the simulation results, the zero solution of system (4) is stable.

Next, we investigate the global exponential synchronization of drive system (2) and the corresponding receiving system (3). □

Definition 2. ∀ initial condition of drive system (2)$x_d(t_0)$, $y_d(t_0)$, $z_d(t_0)$ and $w_d(t_0) \in R^4$, and the corresponding initial condition of receiving system (3) $x_r(t_0)$, $y_r(t_0)$, $z_r(t_0)$ and $w_r(t_0) \in R^4$, if the zero solution of error system (4) satisfies the inequality

$$e_x^2(t) + e_y^2(t) + e_z^2(t) + e_w^2(t) \le \beta(e(t_0)) e^{-\mu t}, \tag{10}$$

where $\beta(e(t_0))$ is a constant depending on $e(t_0)$ and $\mu > 0$, then the zero solution of (4) is globally, exponentially stable, that is, system (2) and system (3) are globally, exponentially synchronized.

Theorem 2. *For any arbitrary initial conditions, the drive system (2) and the receiving system (3) are globally exponentially synchronized under the following control law:*

$$\begin{aligned} u_1 &= -y_r x_r + y_d x_d - e_w, \\ u_2 &= x_r^2 - x_d^2, \\ u_3 &= 0, \\ u_4 &= 0.05 x_r z_r - 0.05 x_d z_d. \end{aligned} \tag{11}$$

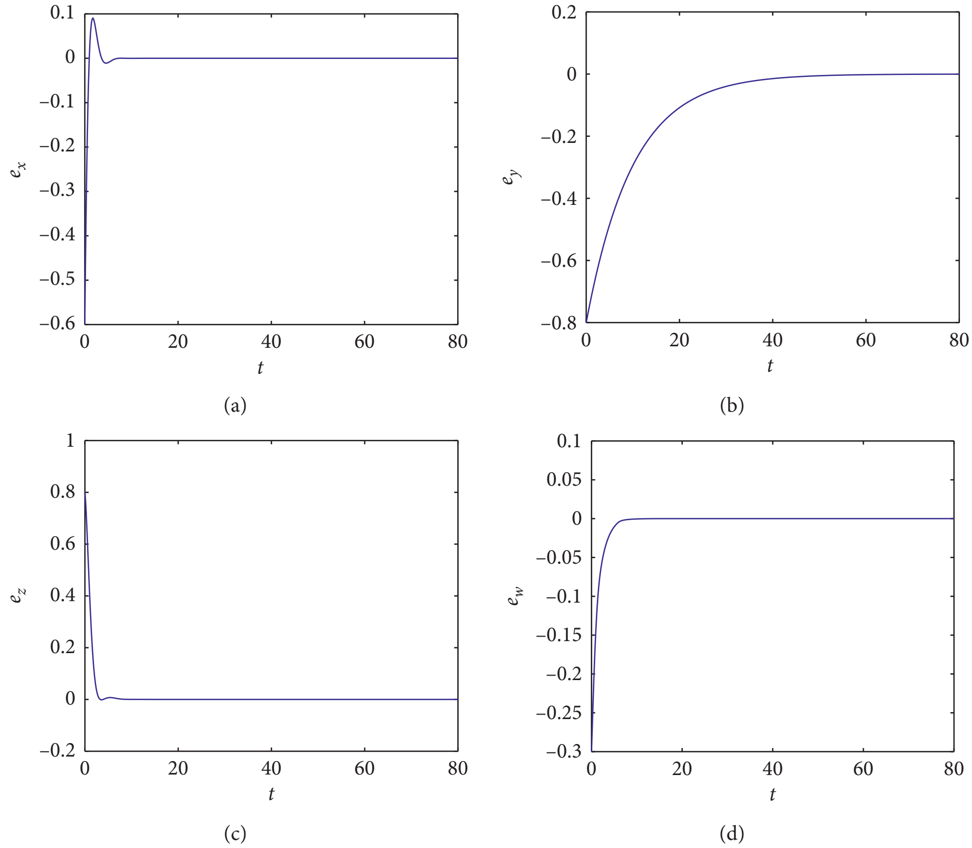


Figure 2: Time history of error system (4) under the control law given in Theorem 1. The values of parameters are the same as those in Figure 1.

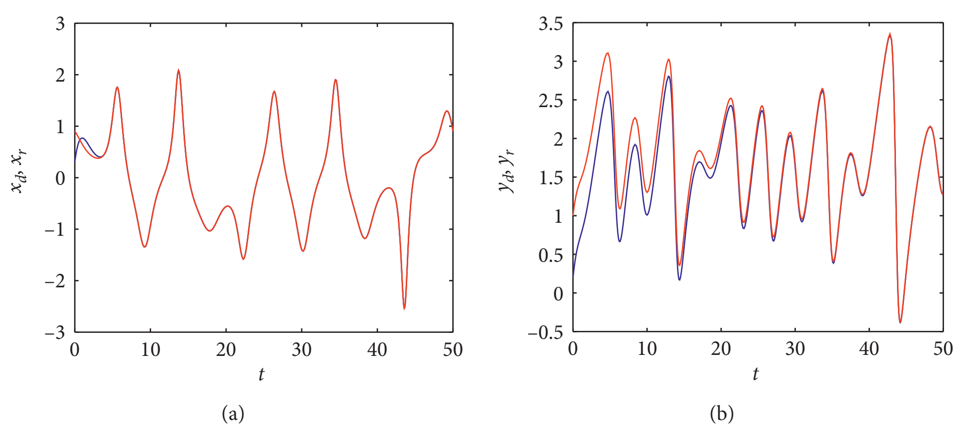
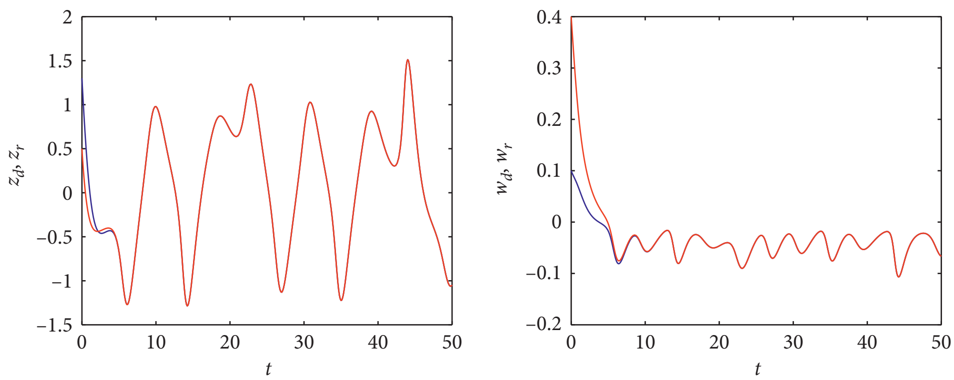


Figure 3: Continued.



(c) (d)

Figure 3: Time histories of drive system (2) (blue curve) and driven system (3) (red curve) under the control law given in Theorem 2.

Proof. For system (4), we design positive definite, radially unbounded Lyapunov function

$$V = e_x^2 + e_y^2 + e_z^2 + e_z^2. \tag{12}$$

We then calculate the derivative of V along the trajectory of system (4) with control law (8). Then, we have

$$\begin{aligned}\frac{dV}{dt} &= 2e_x\dot{e}_x + 2e_y\dot{e}_y + 2e_z\dot{e}_z + 2e_w\dot{e}_w = 2e_x\left(e_z - ae_x\right) + 2e_y\left(-be_y\right) + 2e_z\left(-e_x - ce_z\right) + 2e_w\left(re_w\right)\\ &= 2e_xe_z - 2ae_x^2 - 2be_y^2 - 2e_xe_z - 2ce_z^2 + 2re_w^2\\ &= -2ae_x^2 - 2be_y^2 - 2ce_z^2 + 2re_w^2\\ &= \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_w \end{bmatrix}^T \begin{bmatrix} -2a & 0 & 0 & 0 \\ 0 & -2b & 0 & 0 \\ 0 & 0 & -2c & 0 \\ 0 & 0 & 0 & 2r \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_w \end{bmatrix}.\end{aligned} \tag{13}$$

Therefore, the zero solution of (4) is globally, exponentially stable, and thus systems (2) and (3) are globally, exponentially synchronized.

Now, we use numerical method to verify that the control law proposed in Theorem 2 is correct. In Figure 3, we plot the time histories of both drive system (2) and receiving system (3) in the same figure. Here, the time history of system (2) is plotted using blue curve, and the time history of the receiving system is plotted using red curve. As shown in Figure 3, the control strategy proposed in Theorem 2 is effective.

The chaos in the financial system makes it difficult to predict, which poses a challenge to the analysis of the system. By designing chaos synchronization schemes, two chaotic financial systems can behave synchronously and are beneficial to the control and early warning of financial risks. Specifically, on the one hand, synchronized financial systems are controllable. On the other hand, since the behaviours of the drive system and the driven systems are synchronized, we can use the drive system to predict the behaviours of the driven systems to forecast potential financial risks. □

4. Conclusions

In this work, we investigate the synchronization of a financial hyperchaotic system. Through studying the properties of the model, we design appropriate control strategies to synchronize the two such financial systems with different initial conditions. We provide mathematical proofs showing the effectiveness of the control law. Moreover, we conduct numerical simulations to confirm that the designed control strategies are effective and applicable. Therefore, the controls proposed in this paper provide methods for achieving synchronous chaotic systems. This is a major contribution of this work because the chaotic behaviour of a financial system creates uncertainty and unpredictable risk. In contrast, a synchronized financial system can be predicted, and thus the risk is hedged. In short, this study provides insights into the risk management of chaotic financial systems.

The method to realize chaotic synchronization is not unique. In this work, we only design a few synchronization methods for the system. In our future work, we will design more strategies to achieve synchronizations for a variety of chaotic systems. Surely, this method could also be implemented in other fields such as supply chain management, especially the application mentioned in studies [34–36].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] E. N. Lorenz, "Deterministic nonperiodic flow," *Journal of the Atmospheric Sciences*, vol. 20, no. 2, pp. 130–141, 1963.

[2] G. Chen and T. Ueta, "Yet another chaotic attractor," *International Journal of Bifurcation and Chaos*, vol. 09, no. 7, pp. 1465-1466, 1999.

[3] J. Lü and G. Chen, "A new chaotic attractor coined," *International Journal of Bifurcation and Chaos*, vol. 12, no. 3, pp. 659–661, 2002.

[4] O. E. Rössler, "An equation for continuous chaos," *Physics Letters A*, vol. 57, no. 5, pp. 397-398, 1976.

[5] L. Chua, M. Komuro, and T. Matsumoto, "The double scroll family," *IEEE Transactions on Circuits and Systems*, vol. 33, no. 11, pp. 1072–1118, 1986.

[6] F. Xu, P. Yu, and X. Liao, "Global analysis on n-scroll chaotic attractors of modified Chua's circuit," *International Journal of Bifurcation and Chaos*, vol. 19, no. 1, pp. 135–157, 2009.

[7] F. Xu, P. Yu, and X. Liao, "Synchronization and stabilization of multi-scroll integer and fractional order chaotic attractors generated using trigonometric functions," *International Journal of Bifurcation and Chaos*, vol. 23, no. 8, p. 1350145, 2013.

[8] F. Xu, "A class of integer order and fractional order hyperchaotic systems via the Chen system," *International Journal of Bifurcation and Chaos*, vol. 26, no. 6, p. 1650109, 2016.

[9] P. Yu and F. Xu, "A common phenomenon in chaotic systems linked by time delay," *International Journal of Bifurcation and Chaos*, vol. 16, no. 12, pp. 3727–3736, 2006.

[10] M. Jiang, Y. Shen, J. Jian, and X. Liao, "Stability, bifurcation and a new chaos in the logistic differential equation with delay," *Physics Letters A*, vol. 350, no. 3-4, pp. 221–227, 2006.

[11] S. Wang, S. He, A. Yousefpour, H. Jahanshahi, R. Repnik, and M. Perc, "Chaos and complexity in a fractional-order financial system with time delays," *Chaos, Solitons and Fractals*, vol. 131, p. 109521, 2020.

[12] J.-H. Ma and Y.-S. Chen, "Study for the bifurcation topological structure and the global complicated character of a kind of nonlinear finance system (i)," *Applied Mathematics and Mechanics*, vol. 22, no. 11, pp. 1240–1251, 2001.

[13] J.-H. Ma and Y.-S. Chen, "Study for the bifurcation topological structure and the global complicated character of a kind of nonlinear finance system (ii)," *Applied Mathematics and Mechanics*, vol. 22, no. 12, pp. 1375–1382, 2001.

[14] F. Xu, Y. Lai, and X.-B. Shu, "Chaos in integer order and fractional order financial systems and their synchronization," *Chaos, Solitons & Fractals*, vol. 117, pp. 125–136, 2018.

[15] F. Zhang, G. Yang, Y. Zhang, X. Liao, and G. Zhang, "Qualitative Study of a 4D Chaos Financial System," *Complexity*, vol. 2018, Article ID 3789873, , 2018.

[16] G. A. Leonov, "Bounds for attractors and the existence of homoclinic orbits in the Lorenz system," *Journal of Applied Mathematics and Mechanics*, vol. 65, no. 1, pp. 19–32, 2001.

[17] F. Zhang, X. Liao, and G. Zhang, "On the global boundedness of the Lü system," *Applied Mathematics and Computation*, vol. 284, pp. 332–339, 2016.

[18] F. Zhang and G. Zhang, "Dynamics of a low-order atmospheric circulation chaotic model," *Optik*, vol. 127, no. 8, pp. 4105–4108, 2016.

[19] F. Zhang, R. Chen, and X. Chen, "Analysis of a Generalized Lorenz–Stenflo Equation," *Complexity*, vol. 2017, Article ID 7520590, , 2017.

[20] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Physical Review Letters*, vol. 64, no. 11, pp. 1196–1199, 1990.

[21] L. Chen and G. Chen, "Controlling chaos in an economic model," *Physica A: Statistical Mechanics and Its Applications*, vol. 374, no. 1, pp. 349–358, 2007.

[22] K. B. Kim, J. B. Park, Y. H. Choi, and G. Chen, "Control of chaotic dynamical systems using radial basis function network approximators," *Information Sciences*, vol. 130, no. 1-4, pp. 165–183, 2000.

[23] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Physical Review Letters*, vol. 64, no. 8, pp. 821–824, 1990.

[24] T. L. Carroll and L. M. Pecora, "Synchronizing chaotic circuits," *IEEE Transactions on Circuits and Systems*, vol. 38, no. 4, pp. 453–456, 1991.

[25] X. Wu, G. Chen, and J. Cai, "Chaos synchronization of the master-slave generalized Lorenz systems via linear state error feedback control," *Physica D: Nonlinear Phenomena*, vol. 229, no. 1, pp. 52–80, 2007.

[26] F. Xu and P. Yu, "Global stabilization and synchronization of n-scroll chaotic attractors in a modified Chua's circuit with hyperbolic tangent function," *International Journal of Bifurcation and Chaos*, vol. 19, no. 8, pp. 2563–2572, 2009.

[27] C. Liu, Z. Yang, D. Sun, X. Liu, and W. Liu, "Synchronization of chaotic systems with time delays via periodically intermittent control," *Journal of Circuits, Systems and Computers*, vol. 26, no. 9, p. 1750139, 2017.

[28] P. Mu, W. Pan, L. Yan, B. Luo, and X. Zou, "Route to broadband optical chaos generation and synchronization using dual-path optically injected semiconductor lasers," *Optik*, vol. 124, no. 21, pp. 4867–4872, 2013.

[29] L. Lü, L. Chen, S. Bai, and G. Li, "A new synchronization tracking technique for uncertain discrete network with spatiotemporal chaos behaviors," *Physica A Statistical Mechanics and its Applications*, vol. 460, pp. 314–325, 2016.

[30] Q. Lai, A. Akgul, M. Varan, J. Kengne, and A. Turan Erguzel, "Dynamic analysis and synchronization control of an unusual chaotic system with exponential term and coexisting attractors," *Chinese Journal of Physics*, vol. 56, no. 6, pp. 2837–2851, 2018.

[31] V. K. Yadav, V. K. Shukla, and S. Das, "Difference synchronization among three chaotic systems with exponential term and its chaos control," *Chaos, Solitons and Fractals*, vol. 124, pp. 36–51, 2019.

[32] J. Zheng, H. Hu, H. Ming, and Y. Zhang, “Design of a hybrid model for construction of digital chaos and local synchronization,” *Applied Mathematics and Computation*, vol. 392, p. 125673, 2021.
[33] X.-J. Tong, M. Zhang, Z. Wang, Y. Liu, and J. Ma, “An image encryption scheme based on a new hyperchaotic finance system,” *Optik*, vol. 126, no. 20, pp. 2445–2452, 2015.
[34] J. Li, L. Yi, V. Shi, and X. Chen, “Supplier encroachment strategy in the presence of retail strategic inventory: centralization or decentralization?” *Omega*, vol. 98, p. 102213, 2021.
[35] P. He, Z. Wang, V. Shi, and Y. Liao, “The direct and cross effects in a supply chain with consumers sensitive to both carbon emissions and delivery time,” *European Journal of Operational Research*, vol. 292, no. 1, pp. 172–183, 2021.
[36] F. Wang, A. Diabat, and L. Wu, “Supply chain coordination with competing suppliers under price-sensitive stochastic demand,” *International Journal of Production Economics*, vol. 234, p. 108020, 2021.