Based on the current water crisis scenario, effective water resources management can play an essential role. Reservoir operation optimization is part of water resources management. Reservoir operation optimization is difficult as it involves a large number of variables and constraints to achieve this goal. The present study aims at exploring the performance of recently developed heuristic algorithms—Rao algorithms as applied to the reservoir operation studies for the first time. Rao algorithms are metaphor-less algorithms that require only basic parameters—population size and function evaluations. In the present study, Rao algorithms have been applied to two case studies: discrete four-reservoir operation system problem and continuous four-reservoir operation system problem (benchmark problems) for the assessment of their performance vis-à-vis other algorithms from the literature. The results showed that the Rao-1 algorithm provided the optimal solution with the least function evaluations when compared to Rao-2, Rao-3, and other algorithms applied in the past to the same benchmark problem. Consequently, the Rao-1 model is found to be superior to these approaches by taking less computational time. Hence, the Rao-1 algorithm can be considered suitable for application to reservoir operation optimization problems.

1. Introduction

The ever-rising population and change in regime towards the accelerated demand of water have a prerequisite for the complex optimization problems towards the global sustainability of the available water resources of the earth [1]. The sustenance for the ubiquitous natured water is very important for the attainment of ecological balance, and also to satisfy the rising need for water, it is important to utilize the available water optimally for its sustenance [2–5]. Thus, reservoir operation optimization is of prime significance in the current scenario, which overwhelms a huge number of variables and constraints. In general, water demands from the reservoir fulfilled are based on reservoir operating rules with available input variables and present water storage level along with the hydrological conditions [6–8]. Researchers are developing various optimization methods and applying newly developed approaches to achieve the best optimal solutions [9–11]. However, in recent decades, the field of populace-centered metaheuristic processes is engulfed with several ‘new’ algorithms based on the comparison of some natural phenomena or behavior of animals, fishes, insects, societies, cultures, planets, musical instruments, etc. [12, 13]. Optimization techniques have been evolved from traditional to evolutionary techniques. Sreenivasan and Vedula [14] applied chance-constrained Linear Programming (LP) to a multipurpose reservoir based on the reliability level the optimal solution was obtained. To optimize the nonlinear hydropower function using the LP method, the function was linearized and the solution was obtained within the tolerance limits. Kumar and Prakash [15] developed the Nonlinear Programming (NLP) model to analyze the operation of the multipurpose Koyna dam, India. It was analyzed for different dependable inflows and was found that after relaxing
the release, it can generate more hydropower as it was the vital importance. To incorporate the uncertainties due to inflow, Dynamic Programming was combined with fuzzy rule, and it resulted in the satisfying target performances for Dez and Karoon reservoirs in Iran [16].

One of the earliest review research conducted on the implementation of traditional models such as Discrete Differential Dynamic Programming (DDDPP), LP, NLP, and stochastic models for the reservoir operation optimization and management, was carried out [17]. The study described the pros and cons of these techniques. The review findings evidenced the difficulty of obtaining a generalized model that can be applied for all real-world optimization problems. The traditional models have certain shortcomings like convergence depends upon the initial solution, inefficient in handling discrete search space, and sometimes get stuck in local optima. To overcome the shortcomings of the traditional approaches, evolutionary approaches were developed. Jalali et al. [18] proposed one of the well-established nature-inspired optimization algorithms called Ant Colony Optimization (ACO) to be statistically implemented at the Dez reservoir, Iran. The authors concluded that with proper tuning of parameters global optimal solution is achieved. Labadie 2015 [17] developed and implemented Bat Algorithm to Karoun-4 reservoirs and hypothetical systems and presented its excellence over the traditional approaches. Biogeography Based Optimization Algorithm was validated using a mathematical function and was further applied to the single and multireservoir system [19]. Asadieh and Afshar [20] presented a comparative analysis of the Charged System Search Algorithm (CSSA) with Particle Swarm Optimization (PSO), Genetic Algorithm (GA), gradient-based NLP, and ACO for benchmark problem and Dez reservoir in Iran. CSSA is found to be superior in comparison to other methods. Crow algorithm outperformed other techniques when applied to the multireservoir system in China [21]. The optimal cropping pattern for the Bilaspur project, Rajasthan, India, was developed using the Differential Evolution technique [22]. Garousi-Nejad et al. [23] tested the Firefly Algorithm on mathematical benchmark functions and operation optimization of the reservoir with irrigation and hydropower as the purposes. Firefly was found to be superior to GA in terms of convergence rate and variance. Rule curves for the Pechiparai reservoir, Tamil Nadu, India, were derived [24] using a reliability-based GA model. The harmony search method was found to have potential when tested on the benchmark problem and effectively solved the flood management problem of the Narmab reservoir in Iran [25]. Afshar et al. [26] concluded that HBMO results comparable to LP and other well-developed optimization techniques. Hybrid Algorithm (HA) of Particle Swarm Optimization (PSO) Algorithm and Artificial Fish Swarm Algorithm (AFSA) was developed and implemented by Yaseen [27] for the analysis of the Karun-4 hydropower system. Hybridization overcomes the drawback of AFSA and PSO when assessed based on reliability, resilience, and vulnerability indices. Janga Reddy and Nagesh Kumar [28] applied Particle Swarm Optimization (PSO), Elitist Mutated PSO (EMPSO), and GA to the Bhadra reservoir system, India. EMPSO outperformed standard PSO and GA. Shark Algorithm (SA) was found to produce good results when applied to complex reservoir problems [29].

Another hybrid AI-based model was proposed in [30], which predicted water level prediction and uncertainty analysis at Urmia lake in Iran. This model was based on hybridization of improved adaptive neurofuzzy inference system (ANFIS) and multilayer perceptron (MLP) models are hybridized with a sunflower optimization (SO) algorithm and shown a significant improvement in improving lake’s water level.

The applications of recently explored evolutionary algorithms, such as the Water Cycle Algorithm (WCA) [31], Weed Optimization Algorithm (WOA) [32], and Wolf Search Algorithm (WSA) [33], have been remarkably established over the past decade. The recent review research was conducted on the feasibility of evolutionary computing algorithms for reservoir operation modeling [34]. The review research confirmed the capacity of the evolutionary algorithms as advanced computer aid models owing to their capability to improve the stochastic complexity and for a better understanding of simulated reservoir operation. These approaches are adopted mostly from nature like Particle Swarm Optimization, Crow Algorithm, Weed Optimization, Shark Algorithm, and many more mimicking the behavior of particular species from nature; hence, are called Metaphor algorithms. These optimization algorithms need regulation of system explicit parameters. Subsequently, these are descriptions that proliferate the exercises in tweaking as well as the phase. The algorithm-specific parameter fewer optimization applications can be embarked upon by metaphor-less algorithm as introduced by Rao. The metaphor-less algorithm has an advantage over metaheuristic techniques in that it does not require algorithm-specific parameters to tune the algorithm. The metaphor-less algorithms applied so far in reservoir operation studies are Teaching Learning Based Optimization (TLBO) and Jaya Algorithm (JA). Kumar and Yadav [35] reported the satisfactory performances of TLBO and JA when applied to the benchmark studies. Palwal et al. [36] tested JA on the benchmark problem and found it to result better than other approaches in the past and was also applied to a real case of Mula reservoir, Maharashtra, India. Chong et al. [37] applied JA for hydropower operation optimization to a reservoir system in Malaysia. In this study, the uncertainty of inflows is handled using the Thomas–Fiering model. Results obtained are compared with the results obtained from various metaheuristic approaches, and performance indices are calculated, which indicated JA is efficient in handling reservoir operation optimization problems. Motevali Bashi Naeini and Soltaninia [38] combined branch and bound (BB) with a hybrid of PSO-LP and applied it to benchmark problems to obtain a computationally efficient operation optimization algorithm at the dam design stage. Recently, Rao algorithms have been developed [39] and were tested on 23 benchmark functions along with 25 unconstrained and 2 constrained optimization problems. Wang et al. [40] applied the Rao-1 algorithm to parameter estimation of the photovoltaic cell model and found it to be suitable for such problems. Rao and Pawar [41] applied the Rao algorithm to mechanical system problems which are constrained in nature and found Rao algorithms to be superior to other algorithms.
The need for water resources management can be achieved by optimizing the existing reservoir operation [42, 43]. Various approaches have been adapted to achieve this goal in the past and still, researchers are going on in view of achieving a better strategy [44, 45]. From the literature review, it is found that Rao algorithms have never been applied to the reservoir operation optimization problems, albeit having shown enough promise in the other areas of engineering optimization problems. This led to the thought of the application of these algorithms to the complex reservoir operation optimization problem to assess its applicability in such problems.

In this paper, this novel approach for reservoir operation optimization using the three Rao algorithms is presented. Rao algorithms are recently developed metaphor-less heuristic algorithms which just need mathematical operators in their equations and do not depend upon algorithm-specific parameters. Hence, it is considered highly suitable for problems such as reservoir operation that involve a large number of variables and constraints. To assess the potential of the proposed algorithms, it has been tested on two benchmark problems (discrete-four reservoir operation (DFRO) problem and continuous four-reservoir operation (CFRO) problem) from the literature. The performance comparison of proposed algorithms with the other existing optimization algorithms for reservoir operation optimization is also presented, referring to the past studies.

2. Materials and Methods

2.1. Description of the Rao Algorithms. Rao algorithms are metaphor-less algorithms and only need common control parameters like that in TLBO and JA. The update equation in Rao algorithms is inherited from JA. Similar to JA, they also require only mathematical operators to upgrade the solution based on the best and the worst solution. In JA, interactions were made between the candidate solution to be updated with the best and with the worst solution. In the Rao-1 algorithm, interaction is between the best and the worst values. In the other two algorithms, along with the interaction between the worst and the best value, there is random interaction between the candidate solutions based on their performances. The reservoir operation optimization process for the four-reservoir system problems using Rao algorithms has been demonstrated in Figure 1.

2.2. Methodology of the Proposed Algorithms: Rao Algorithms. The independent variables are initialized using minimum, maximum bounds of the particular variable and random number, as shown in the following equation:

\[ X_{b,c} = (X_b) + r \ast ((X_b) - \min(X_b)), \]

where \((X_b)\) is the minimum bound for the \(b^{th}\) variable, \(r\) is a random number \((0, 1)\), \((X_b)\) is the maximum bound for the \(b^{th}\) variable, \(X_{b,c} = \text{Value of } b^{th} \text{ variable for } c^{th} \text{ candidate solution. Dependable variable values are generated using the values of independent variables. Then, the objective function is computed further, considering constraint violation. The function value is obtained considering penalties for the violation. Penalties are added to the objective function value for a minimization problem to obtain the function value and vice versa.}

The best and the worst solutions are selected amongst the candidate solutions for the \(a^{th}\) iteration. Let \(c\) be the candidate solution for \(a^{th}\) iteration, then the updated value of \(b^{th}\) variable is obtained using equation (2a) for Rao-1, (2b) for Rao-2, and (2c) for Rao-3, respectively:

\[ X_{b,c,a} = X_{b,c,a} + r_{b,a} (X_{b,best,a} - X_{b,worst,a}) + r_{b,a} \left( (X_{b,c,a} or X_{b,d,a}) - \left| X_{b,d,a} or X_{b,c,a} \right| \right), \]

\[ X_{b,c,a} = X_{b,c,a} + r_{b,a} (X_{b,best,a} - X_{b,worst,a}) + r_{b,a} \left( (X_{b,c,a} or X_{b,d,a}) - \left| X_{b,d,a} or X_{b,c,a} \right| \right), \]

where, \(X_{b,c,a} = \text{value of } b^{th} \text{ variable for } c^{th} \text{ candidate solution for } a^{th} \text{ iteration, } X_{b,best,a} = \text{value of } b^{th} \text{ variable for the best candidate solution for } a^{th} \text{ iteration, } X_{b,worst,a} = \text{value of } b^{th} \text{ variable for the worst candidate solution for } a^{th} \text{ iteration, } X_{b,c,a}' = \text{updated value of } b^{th} \text{ variable for } c^{th} \text{ candidate solution for } a^{th} \text{ iteration, and } r_{b,a} \text{ and } r_{b,a} \text{ are random numbers for } b^{th} \text{ variable during } a^{th} \text{ iteration. In equations (2b) and (2c), the terms } X_{b,c,a} \text{ and } X_{b,d,a} \text{ represents the variable values corresponding to } c^{th} \text{ and } d^{th} \text{ candidate solution and random interaction between them. If the value corresponding to } c^{th} \text{ is better than } d^{th} \text{ then the term } "X_{b,c,a} or X_{b,d,a}" \text{ becomes } X_{b,c,a} \text{ and } "X_{b,c,a} or X_{b,d,a}" \text{ becomes } X_{b,d,a} \text{ and vice versa in the opposite case.}

New function values that are computed using updated variable values are compared to the respective function values. The adopted function is the best one, and the worst was rejected. The best values for the respective candidate solution are now the preliminary set for the following iteration. The same process continues until the termination criterion is reached.

2.3. Case Study 1: Discrete Four-Reservoir Operation (DFRO) System Problem. A hypothetical discrete four-reservoir system introduced by Larson [46] has been used as the case study: a benchmark problem to test the potential of Rao algorithms. The schematic view of this case study is shown in Figure 2. This system was also used as a benchmark in past studies to test other optimization techniques in the field of reservoir operation studies. The system is a series and parallel combination of four reservoirs. Reservoirs 1–3 produce hydropower, while reservoir 4 is a multipurpose reservoir serving irrigation as well as hydropower production. For maximization of the profits from this system, a twelve-hour operating period is considered in the objective function.

Data for the benchmark problem are shown in Table 1. The objective function \((F)\) is the maximization of net profit obtained from all four reservoirs. Mathematically, it can be expressed as follows:
Max $F = \sum_{i=1}^{4} \sum_{j=1}^{12} b_1(t) \cdot R_i(t) + \sum_{i=1}^{4} b_5(t) \cdot R_4(t)$, \hspace{1cm} (3)

where $b_1(t) = 4 \times 12$ matrices of benefit from hydropower production from all the four reservoirs. The benefit function matrix is as follows:

$$b_1(t) = \begin{bmatrix}
1.1 & 1 & 1 & 1.2 & 1.8 & 2.5 & 2.2 & 2 & 1.8 & 2.2 & 1.8 & 1.4 \\
1.4 & 1.1 & 1 & 1.2 & 1.8 & 2.5 & 2.2 & 2 & 1.8 & 2.2 & 1.8 \\
1 & 1 & 1.2 & 1.8 & 2.5 & 2.2 & 2 & 1.8 & 2.2 & 1.8 & 1.4 & 1.1 \\
1 & 1.2 & 1.8 & 2.5 & 2.2 & 2 & 1.8 & 2.2 & 1.8 & 1.4 & 1.1 & 1
\end{bmatrix}.$$ \hspace{1cm} (4)

$b_5(t)$ is the benefit from irrigation for reservoir 4, and $b_5(t) = [1.61, 71.81, 92221, 91.81, 71.61, 5].$

The objective function is subjected to the following constraints.

2.3.1. Continuity Constraint. The continuity constraints for each reservoir over each operating period “$t$” are as follows:

$$S_i(t + 1) = S_i(t) + I_i(t) + M \cdot R_i(t),$$ \hspace{1cm} (5)

where $S_i(t + 1)$ denotes the reservoir storage at period “$t$” and for reservoirs $i = 1$ to 4. $S_i(t)$ presents the reservoir storage at the beginning of period “$t$” and for reservoirs $i = 1$ to 4. $I_i(t)$ indicates the reservoir inflows during the period “$t$” and for the reservoirs $i = 1$ to 4. $R_i(t)$ denotes the reservoirs releases during the period “$t$” and for the reservoirs $i = 1$ to 4. $M = 4 \times 4$ matrix of indices of reservoir connections, $M = \begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 \\
1 & 0 & 1 & -1
\end{bmatrix}.$

2.3.2. End Storage Constraint. End storage ($S_i(13)$) should be at least 5, 5, 5, and 7 units for reservoirs 1, 2, 3, and 4, respectively, to maintain the continuity of the operation. The
penalty function applied for this constraint violation is the same as that applied by the other researches and is as follows:

\[
g_i[S_i(13, d_i)] = 40 \left| S_i(13) - d_i \right|^2, \quad \text{for } S_i(13) \leq d_i, \\
g_i[S_i(13, d_i)] = 0, \quad \text{for } S_i(13) > d_i. \tag{6}
\]

Hence, the modified function is represented as follows:

\[
\text{Max } F = \sum_{i=1}^{4} \sum_{t=1}^{12} b_i(t) \cdot R_i(t) + \sum_{t=1}^{12} b_3(t) \cdot R_4(t) - g_i[S_i(13, d_i)]. \tag{7}
\]

2.4. Case Study 2: Continuous Four-Reservoir Operation (CFRO) System Problem. CFRO was introduced by Te Chow and Cortes Rivera [47] in 1974. The schematic sketch of the CFRO is shown in Figure 2. The CFRO problem has similarities with the DFRO problem. The difference is in the inflows and bounds. The connections between the reservoirs in the system and benefits functions are the same as DFRO. Hence, the continuity equation and objective function are the same.

The minimum releases for time period \( t = 1 \) to 12 for reservoir 1, 2, 3, and 4 are 0.005, 0.0005, 0.0005, and 0.005, respectively. The maximum releases for periods \( t = 1 \) to 12 for reservoirs 1, 2, 3, and 4 are 4, 0.5, 0.5, and 4, respectively. The initial and final storages for reservoirs 1, 2, 3, and 4 are 4, 6, 6, and 8, respectively. The minimum storages for \( t > 1 \) to \( t = 12 \) are 1 for all 4 reservoirs. The inflows for \( t = 1 \) to 12 are shown in Table 2. The maximum storages for \( t = 2 \) to 12 are given in Table 3. The penalty factor was 40 for DFRO and 13 for CFRO based on past studies.

3. Results and Discussion

3.1. DFRO. Rao algorithms were applied to the DFRO system optimization: the first considered benchmark problem. The population size is set to 50, 40, and 40 for Rao-1, Rao-2, and Rao-3 algorithms, respectively, after conducting the sensitivity analysis. The population size was initially set as 50 based on past studies (Kumar and Yadav [35]). Rao-1 resulted in the optimal solution for a population size of 50, while Rao-2 and Rao-3 did not. Rao-2 and Rao-3 yielded the optimal solutions for a population size of 40 in both cases. The global optimal solution with the objective function value of 401.3 has been achieved for the DFRO problem using the Rao-1 algorithm with a population size of 50 and maximum Function Evaluations (FEs) of 1,50,000. For Rao-2, the optimal value of 401.23 is achieved at a population size of 40 and Max FEs of 11,00,000 and that for Rao-3, the optimal value achieved is 401.4 at a population size of 40 and Maximum FEs of 12,51,000. Rao-3 led to a higher value of an objective function with a slight violation of constraints. According to the past studies also 401.3 is the global optimal solution for the provided benchmark problem without constraint violation, which is achieved in the case of the Rao-1 algorithm model. These algorithms result in three values— the best, the mean, and the worst for a particular solution. Runs represent the number of times the same model is operated for the same given set of conditions. Generally, it is preferred to be selected as 10. It can have a higher value also like 15 or 20, depending upon the variation observed in the results for the same set of conditions. In the present study, it has been adopted for runs as 10. For 10 runs, the best, the mean, and the worst values for these algorithms corresponding to their particular solution along with the standard deviation are shown in Table 4. Rao-1 showed the least standard deviation of 0.45, and the higher standard deviation is in the case of Rao-3. From Table 4, it can also be observed that there is less variation in the best and the worst values of the function. Hence, it can be said that Rao-1 has less standard deviation as it has confined the exploration of the updated value between the worst and the best. In the updated equation of Rao-1, it shows the random interaction between the best and the worst values rather than with the

![Figure 2: Four reservoir systems. 1, 2, 3, and 4: number of reservoirs; I1, I2, I3, and I4: inflows to reservoirs 1, 2, 3, and 4, respectively; R1, R2, R3, and R4: releases from reservoirs 1, 2, 3, and 4, respectively.](image-url)
random variable value or specific value, thereby reducing the range. Hence, fewer changes of deviation in the solution, which may further lead to faster convergence in the case of the Rao-1 algorithm. On the other hand, Rao-2 and Rao-3 algorithms involve the interaction between the best and the worst along with random interactions between the variable of different candidate solutions, which may again need some more time (FEs) but involves a more random nature. This results in a higher standard deviation and the requirement of more number function evaluations. Figure 3 demonstrates the resulting release pattern for the four reservoirs using Rao-1, Rao-2, Rao-3, JA, and LP algorithms. The release pattern for reservoir 1 is found to be the same for Rao algorithms and JA but is different for LP at time step 1. Rao-1 and JA have 2 units' higher releases than LP, while at Step 4 of the period, LP has 2 units' higher releases for nearly the same benefit function in both cases, which shows the balance of net benefit for reservoir 1. For reservoir 2, Rao-2 and Rao-3 algorithm models have produced the same releases till 10 time steps and for the last 2 time steps showed different trends, thereby balancing the releases in this case. While Rao-1 synchronized with JA except for 3 time steps but both the models have led to an equal quantity of releases for this reservoir. Similarly, LP showed a different trend with these algorithms but has led to some amount of release for reservoir 2. In the case of reservoir 3, the releases obtained from the Rao-1 algorithm varied from those obtained from the other approaches; however, the net benefits achieved are the same, leading to the same optimal solution. Rao-1 and Rao-2 resulted in the optimal solution with releases varying from those of the other approaches at 1 and 2 time steps, respectively. Rao-2 released nearly the same quantity as that by other approaches while Rao-1 released a bit higher than the others. Since it is a multipurpose multireservoir operation, the releases for different reservoirs obtained from different approaches can be different, leading to the same optimal solution as the single objective function has been framed by using the benefit function associated with the releases from each reservoir.

The performance comparison of these algorithms with other approaches developed and applied to this system in the past has been shown in Table 5. From Table 5, it can be observed that Rao-1 results in the optimal value of 401.3 for 150,000 FEs while Rao-2 and Rao-3 resulted in the optimal value of 401.23 and 401.4 for the Max FEs of 1,100,000 and 1,251,000, respectively. From Table 5, it can be seen that JA has obtained the optimal value of 401.4 with FEs = 350,000 (Kumar and Yadav [35]) and 325,000 [36], while Rao-1 resulted in the optimal solution with FEs = 150,000, which is nearly 50% of the FEs in JA and a way better than Rao-2 and Rao-3 algorithms in terms of the number of function evaluations. Hence, it can be said that the Rao-1 algorithm model achieved the optimal solution in less computational time, indicating the faster convergence of the Rao-1 algorithm model. Thus, the Rao-1 algorithm model is found to be superior to other optimization algorithms and as well as to its parent algorithm for this case study.

3.2. CFRO. Rao algorithms have also been applied to the second case study, i.e., CFRO system optimization: another benchmark problem. The population size was initially set as 50, based on past studies (Kumar and Yadav [35]). The optimal solution with the objective function value of 308.8 has been achieved for the CFRO system problem using the Rao-1 algorithm with a population size of 50 and maximum Function Evaluations (FEs) of 155,000. For Rao-2, the optimal value of 304.64 is achieved at a population size of 50 and Max FEs of 725,000 and that for Rao-3, the optimal value achieved is 307 at a population size of 50 and Maximum FEs of 700,000. The LP model has obtained the value of 308.3 as the optimal objective function value. The objective function is the maximization of benefits which has better been achieved by the Rao-1 algorithm. Rao algorithms result in three values—the best, the mean, and the worst for a particular solution. Runs represent the number of times the same model is operated for the same given set of conditions.

### Table 2: Inflows for the CFRO system.

<table>
<thead>
<tr>
<th>Time</th>
<th>Reservoir 1</th>
<th>Reservoir 2</th>
<th>Reservoir 3</th>
<th>Reservoir 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>2.5</td>
<td>3.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>1.25</td>
<td>2.5</td>
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<td>0</td>
</tr>
<tr>
<td>9</td>
<td>1.25</td>
<td>1.3</td>
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<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.75</td>
<td>1.2</td>
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<td>11</td>
<td>1.75</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>12</td>
<td>1</td>
<td>0.7</td>
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### Table 3: Maximum storages for the CFRO system.

<table>
<thead>
<tr>
<th>Time</th>
<th>Reservoir 1</th>
<th>Reservoir 2</th>
<th>Reservoir 3</th>
<th>Reservoir 4</th>
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<td>18</td>
<td>8</td>
<td>15</td>
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</table>

### Table 4: Best, mean, and worst objective function values obtained corresponding to the optimal solution using Rao algorithms for the DFRO system problem.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Runs</th>
<th>Best</th>
<th>Mean</th>
<th>Worst</th>
<th>Standard deviation</th>
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<tbody>
<tr>
<td>Rao-1</td>
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<td>401.3</td>
<td>401.01</td>
<td>400.69</td>
<td>0.45</td>
</tr>
<tr>
<td>Rao-2</td>
<td>10</td>
<td>401.23</td>
<td>397.2</td>
<td>395.07</td>
<td>1.83</td>
</tr>
<tr>
<td>Rao-3</td>
<td>10</td>
<td>401.4</td>
<td>398.34</td>
<td>395.74</td>
<td>1.97</td>
</tr>
</tbody>
</table>
In the present study, it has been adopted for runs as 15. For 15 runs, the best, the mean, and the worst values for these algorithms corresponding to their particular solution along with the standard deviation are shown in Table 6. The highest standard deviation is observed for the Rao-3 algorithm.

Figure 4 demonstrates the resulting release pattern for the CFRO using Rao-1, Rao-2, Rao-3, and LP algorithms. Releases obtained from Rao-1, LP, Rao-3 are the same for 12 time steps, while Rao-2 shows different releases at 2 time steps for reservoir 1. While, for reservoir 2, release patterns are different in the case of all the optimization algorithms. Variation in releases obtained from Rao algorithms and LP at few time steps is observed in the case of reservoirs 3 and 4.

The performance comparison of these algorithms with other approaches developed and applied to this system in the past has been shown in Table 7. From Table 5, it can be seen that...
observed that Rao-1 results in the optimal value of 308.8 for 155,000 FEs while Rao-2 and Rao-3 resulted in the optimal value of 304.64 and 307 for the Max FEs of 725,000 and 700,000, respectively. From Table 7, it can be seen that JA has obtained the optimal value of 308.4 with FEs = 350,000 (Kumar and Yadav [35]), which is the highest objective function value in the past, while Rao-1 resulted in the optimal solution of 308.8 with FEs = 155,000, which is nearly 50% of the FEs in JA and a way better than Rao-2 and Rao-3 algorithms in terms of the number of functions evaluations. The objective function is maximization which is better achieved by Rao-1 with slight constraint violation leading to the highest objective function value with a considerable range of violations and lesser FEs. Hence, it can be said that the Rao-1 algorithm model achieved the optimal solution in less computational time, indicating the faster convergence of the Rao-1 algorithm model. Thus, the Rao-1 algorithm model is found to be suitable for its application the complex reservoir operation problems.

4. Conclusions

The Rao Algorithms are similar to the Jaya Algorithm (JA) as these are also metaphor-less algorithms and do not need any algorithm-specific parameter. These algorithms just need mathematical operators in the update equation along with the best and the worst values. The complexity of JA has been examined empirically in terms of big-O notations using the GuessCompx tool [53, 54]. The complexity is compared with the metaheuristic technique (GA) and both methods showed linear complexity (Paliwal et al. [36]). Hence, these algorithms reduce the computational complexity. They are also found to be easy in application and efficient too. The parameters in the update equation of JA are shuffled to prepare

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Runs</th>
<th>Best</th>
<th>Mean</th>
<th>Worst</th>
<th>Standard deviation</th>
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<tbody>
<tr>
<td>Rao-1</td>
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<td>308.8</td>
<td>304.04</td>
<td>295.07</td>
<td>4.43</td>
</tr>
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<td>295.6</td>
<td>288.19</td>
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<tr>
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<td>307</td>
<td>299.74</td>
<td>279.32</td>
<td>7.76</td>
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</tbody>
</table>

Figure 4: Release pattern for the CFRO system obtained through Rao algorithms. (a) Reservoir 1. (b) Reservoir 2. (c) Reservoir 3. (d) Reservoir 4.
the update equations for Rao algorithms. In this study, outcomes from Rao Algorithms have been compared to other approaches applied in the past and also to its parental algorithm (JA). Rao-1 Algorithm is found to be superior to other algorithms for these case studies. Rao-1 Algorithm leads to the global optimal solution with the least FEs as compared to all other optimization models found in the literature for DFRO and higher objective function value with fewer FEs with considerable constraint violation for CFRO system problem. Rao-1 Algorithm model outperformed every other model in terms of computational time. Hence, it can be concluded that the Rao-1 Algorithm model can be utilized in the field of reservoir operation optimization as it can lead to near global optimal solution with much fewer function evaluations. Future research studies can be adopted on the investigation of the proposed optimization algorithm for other water resources management and operation.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions


References


V. Te Chow and G. Cortes-Rivera, “Application of DDDP in water resources planning,” Final report, University of Illinois at Urbana-Champaign, Champaign, IL, USA, 1974.
