Research Article

Complex Dynamics of a Ratio-Dependent Predator-Prey Model Induced by Spatial Motion

Caiyun Wang¹, Jing Li², and Ruiqiang He¹

¹Department of Mathematics, Xinzhou Teachers University, Xinzhou, Shanxi 034000, China

²Shanxi University of Finance and Economics, Taiyuan, Shanxi 034006, China

Correspondence should be addressed to Caiyun Wang; 494423138@qq.com

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One of the most efficient predator-prey models with spatial effects is the one with ratio-dependent functional response. However, there is a need to further explore the effects of spatial motion on the dynamic behavior of population. In this work, we study a ratio-dependent predator-prey model with diffusion terms. The aim of this work is to investigate the changes in predator's distribution in space as the prey populations change their mobility. We observe that the frequency diffusion of the prey gives rise to the sparse density of the predator. Moreover, we also observe that the increasing rate of the conversion into predator biomass induces pattern transitions of the predator. Specifically speaking, Turing pattern of the predator populations goes gradually from a spotted pattern to a black-eye pattern, with the intermediate state being the mixture of spot and stripe pattern. The simulation results and analysis of this work illustrate that the diffusion rate and the real intrinsic factor influence the persistence of the predator-prey system mutually.

1. Introduction

The dynamical behavior of the prey-predator model has been investigated by researchers for many years after the pioneering work was accomplished by Lotka and Volterra. The authors present the well-known Lotka-Volterra predator-prey model. It is established that the response function is the key element in the predator-prey models. The ratio-dependent functional response was first proposed by Leslie and Gower in 1973 [1, 2]. The authors discuss that the predator's growth is described by prey's density, as well as by the ratio of density of the predator to prey. Afterwards, the Holling-Tanner rate in the predator-prey model is presented in the literature. This model is also known as the Holling-Tanner predator-prey model [3]. The dynamics such as local and global stability, limit cycles, and pattern solutions of Holling-Tanner predator-prey model have already been investigated [4–7]. The factors such as delay and random are also considered in the recent literature [8–13]. The fuzzy approach is also considered to construct the predator-prey model [14, 15]. However, the

Holling-Tanner-type predator-prey models are important because of their appropriateness for modeling the real ecological interactions between predator and prey species. The Holling-Tanner predator-prey model is

$$\begin{cases} \frac{dN}{dt} = RN \left(1 - \frac{N}{K} \right) - \frac{MNP}{N + D}, \\ \frac{dP}{dt} = P \Theta \left(1 - \frac{HP}{N} \right), \end{cases} \quad (1)$$

where $N(T)$ and $P(T)$ denotes the prey and predator density, respectively, at time T . R, K, Θ, M, D , and H are all positive, and their definitions are given in Table 1.

The term $MNP/N + D$ in model (1) is known as Holling type II response function [16] which was proposed by Holling in 1965. In addition, it is assumed that the functional response is expressed by Holling type III response function [17–19], which is more suitable for the population of vertebrates. The ratio-dependent predator-prey model with Holling type III functional response is

TABLE 1: The definition of parameters used in model (1).

Parameter	Meaning
R	The prey's intrinsic growth rate
K	The carrying capacity of the prey
M	The largest amount of prey that predator can capture per unit time
Θ	Predator intrinsic growth rate
H	Conversion rate into the biomass of predator by prey
D	A saturation parameter

$$\begin{cases} \frac{dN}{dT} = RN \left(1 - \frac{N}{K}\right) - \frac{MN^2P}{AN^2 + 1}, \\ \frac{dP}{dT} = P\Theta \left(1 - \frac{HP}{N}\right), \end{cases} \quad (2)$$

where A is the half-saturation constant.

On the other hand, there are a large number of organisms with spatial pattern distributions [20–22]. The distribution of steady spatial pattern is called Turing pattern. This model was put forward when Turing presented the theory of mathematical explanation for the pattern formation of the animal skin [23]. Following Turing's famous

work, the researchers have already explained self-organized pattern formation in various fields such as geology, physics, chemistry, and biology [20–28]. Especially, for the diffusive predator-prey model in ecology, pattern formation helps us in a better way to understand the influence of predator's individual mobilities, the dynamics of stability, and oscillations of the prey species. There are various works that present the spatial predator-prey models [7, 29–35]. Wu and Song discussed the joint impact of nonlocal term and delay on the spatial predator-prey model [36]. Cao and Jiang indicate that the nonlocal reaction term can stabilize the spatially inhomogeneous periodic solutions [37]. Incorporating the diffusive terms into model (2), we have

$$\begin{cases} \frac{\partial N}{\partial T} = RN \left(1 - \frac{N}{K}\right) - \frac{MN^2P}{AN^2 + 1} + D_1 \nabla^2 N, & (x, T) \in \Omega \times (0, \infty), \\ \frac{\partial P}{\partial T} = \Theta P \left(1 - \frac{HP}{N}\right) + D_2 \nabla^2 P, & (x, T) \in \Omega \times (0, \infty), \\ \frac{\partial N(x, T)}{\partial n} = \frac{\partial P(x, T)}{\partial n} = 0, & (x, T) \in \partial\Omega \times (0, \infty), \\ N(x, 0) = N_0 > 0, & x \in \Omega, \\ P(x, 0) = P_0 > 0, & x \in \Omega, \end{cases} \quad (3)$$

where D_{11} and D_{22} represent the diffusion coefficients of the prey and predator, respectively, which are all positive.

The mathematical analysis of the spatial predator-prey model used to be based on linear stability analysis. However, linear stability analysis is only meaningful in the small domain of the equilibrium, where the linear system is approximated to nonlinear systems. The multiple scale method [38–40] is employed to study spatial dynamic behaviors of the model in our present paper. This method is effective when the system under consideration is a partial differential equation. Specifically, the parameters in system are decomposed into different scales. The amplitude equation is an important tool to understand the pattern selection near the Turing bifurcation boundary. By solving the amplitude equation, we obtain the domain of the controlled parameter, where the steady state pattern arises. The rest of our paper is organized as follows.

In Section 2, we present an example to show the conditions for the occurrence of Turing instability by bifurcation analysis. In Section 3, we derive the amplitude equations of our model on the basis of multiple scale analysis. In Section 4, we present the simulations performed to illustrate the different patterns in the corresponding Turing domain. Moreover, the predator's pattern transitions driven by prey's diffusion rate and conversion rate of biomass into predator are discussed in detail. Finally, in Section 5, we conclude our work.

2. Bifurcation Analysis

In this section, we present the conditions for the occurrence of Turing pattern. We choose the scale $\tilde{u} = N/K$, $\tilde{v} = MKP/R$, and $\tilde{t} = RT$, and we obtain

$$\begin{cases} \frac{\partial \tilde{u}}{\partial t} = \tilde{u}(1 - \tilde{u}) - \frac{\tilde{u}^2 \tilde{v}}{e\tilde{u}^2 + \tilde{v}} + d_1 \nabla^2 \tilde{u}, & (x, \tilde{t}) \in \Omega \times (0, \infty), \\ \frac{\partial \tilde{v}}{\partial t} = \tilde{v} \eta \left(1 - \frac{\gamma \tilde{v}}{\tilde{u}}\right) + d_2 \nabla^2 \tilde{v}, & (x, \tilde{t}) \in \Omega \times (0, \infty), \\ \frac{\partial \tilde{u}(x, \tilde{t})}{\partial n} = \frac{\partial \tilde{v}(x, \tilde{t})}{\partial n} = 0, & (x, t) \in \partial\Omega \times (0, \infty), \\ \tilde{u}(x, 0) = u_0 > 0, & x \in \Omega, \\ \tilde{v}(x, 0) = v_0 > 0, & x \in \Omega, \end{cases} \quad (4)$$

where $\eta = \Theta/R$, $e = AK^2$, $\gamma = HR/MK^2$, $d_1 = KD_1/R$, and $d_2 = RD_2/M^2K^3$.

The analysis of the existence and local stability of the positive equilibrium is presented in [41]. Here, we present an example to illustrate the existence and local stability of the positive equilibrium. Let $E^*(\tilde{u}^*, \tilde{v}^*)$ be an interior steady equilibrium of model (4). The Jacobin matrix of model (4) is

$$J = \begin{pmatrix} f_{\tilde{u}} & f_{\tilde{v}} \\ g_{\tilde{u}} & g_{\tilde{v}} \end{pmatrix}, \quad (5)$$

$$\tilde{u}(1 - \tilde{u}) - \frac{\tilde{u}^2 \tilde{v}}{e\tilde{u}^2 + 1} \triangleq f(\tilde{u}, \tilde{v}), \quad \tilde{v} \eta \left(1 - \frac{\gamma \tilde{v}}{\tilde{u}}\right) \triangleq g(\tilde{u}, \tilde{v}),$$

and the trace and determinant of matrix J are

$$\text{tr}(J) = \frac{-1 + e(\tilde{u}^*)^2 - 2e(\tilde{u}^*)^3}{1 + e(\tilde{u}^*)^2} - \eta, \quad (6)$$

$$\det(J) = \frac{\eta \tilde{u}^*}{1 + e(\tilde{u}^*)^2} F'(\tilde{u}^*).$$

On the basis of parameters presented in Table 2, we calculate the equilibrium and get the stability of the $E^*(\tilde{u}^*, \tilde{v}^*)$.

The characteristic polynomial at $E^*(\tilde{u}^*, \tilde{v}^*)$ is

$$|\lambda E - J_k| = 0, \quad (7)$$

where

$$J_k = \begin{pmatrix} f_{\tilde{u}} - d_1 k^2 & f_{\tilde{v}} \\ g_{\tilde{u}} & g_{\tilde{v}} - d_2 k^2 \end{pmatrix} \quad (8)$$

and k is the wave number. The eigenvalue is

$$\lambda_k^2 - \text{tr}_k \lambda_k + \Delta_k = 0, \quad (9)$$

where

$$\begin{aligned} \text{tr}_k &= (f_{\tilde{u}} + g_{\tilde{v}}) - k^2(d_1 + d_2), \\ \Delta_k &= f_{\tilde{u}} g_{\tilde{v}} - f_{\tilde{v}} g_{\tilde{u}} - k^2(d_2 f_{\tilde{u}} + d_1 g_{\tilde{v}}) + d_1 d_2 k^4 \\ &= \det(J) - k^2(d_2 f_{\tilde{u}} + d_1 g_{\tilde{v}}) + d_1 d_2 k^4. \end{aligned} \quad (10)$$

Therefore, the eigenvalue of (7) is

$$\lambda_k = \frac{\text{tr}_k \pm \sqrt{\text{tr}_k^2 - 4\Delta_k}}{2}. \quad (11)$$

The Hopf bifurcation occurs when the conditions $\text{Re}(\lambda_k) = 0$ and $\text{Im}(\lambda_k) \neq 0$ hold at $k = 0$. However, Turing bifurcation occurs when conditions $\text{Re}(\lambda_k) = 0$ and $\text{Im}(\lambda_k) = 0$ hold at $k = k_T > 0$. And, k_T satisfies

$$k_T^2 = \sqrt{\frac{f_{\tilde{u}} g_{\tilde{v}} - f_{\tilde{v}} g_{\tilde{u}}}{d_1 d_2}} = \sqrt{\frac{\det J}{d_1 d_2}}. \quad (12)$$

Substituting (12) into $\Delta_k = 0$, we get the expression of d_1^T . The value of this expression represents the critical value of Turing bifurcation parameter. Turing domain and the real part of eigenvalues of model 4 are shown in (Figures 1 and 2).

$$d_1^T = \frac{d_2 f_{\tilde{u}}}{2 \left(f_{\tilde{u}} g_{\tilde{v}} - f_{\tilde{v}} g_{\tilde{u}} + \sqrt{f_{\tilde{v}}^2 g_{\tilde{u}}^2 - f_{\tilde{u}} f_{\tilde{v}} g_{\tilde{u}} g_{\tilde{v}}} \right) / f_{\tilde{u}} - g_{\tilde{v}}}. \quad (13)$$

3. Multiple Scale Analysis

We choose d_1 and d_1^T as a control parameter and the bifurcation threshold, respectively. This is done in order to deduce the amplitude equations via multiple scale analysis [42, 43] and judge the type of the Turing pattern.

Near $d_1 = d_1^T$, system (4) is written at $E^*(\tilde{u}^*, \tilde{v}^*)$ as follows:

$$\frac{\partial U}{\partial t} = LU + N, \quad (14)$$

where

$$U = \begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix}, \quad L = \begin{pmatrix} f_{\tilde{u}} + d_1 \nabla^2 & f_{\tilde{v}} \\ g_{\tilde{u}} & g_{\tilde{v}} + d_2 \nabla^2 \end{pmatrix}, \quad (15)$$

$$\tilde{h}_3 = \begin{pmatrix} \frac{1}{6}p_{111}\tilde{u}_1^3 + \frac{1}{6}p_{222}\tilde{v}_1^3 + \frac{1}{6}(p_{112} + p_{121} + p_{211})\tilde{u}_1^2\tilde{v}_1 + \frac{1}{6}(p_{122} + p_{212} + p_{221})\tilde{u}_1\tilde{v}_1^2 \\ + p_{11}\tilde{u}_2\tilde{u}_1 + p_{22}\tilde{v}_2\tilde{v}_1 + \frac{1}{2}(p_{12} + p_{21})(\tilde{u}_2\tilde{v}_1 + \tilde{u}_1\tilde{v}_2) \\ \frac{1}{6}q_{111}\tilde{u}_1^3 + \frac{1}{6}q_{222}\tilde{v}_1^3 + \frac{1}{6}(q_{112} + q_{121} + q_{211})\tilde{u}_1^2\tilde{v}_1 + \frac{1}{6}(q_{122} + q_{212} + q_{221})\tilde{u}_1\tilde{v}_1^2 \\ + q_{11}\tilde{u}_2\tilde{u}_1 + q_{22}\tilde{v}_2\tilde{v}_1 + \frac{1}{2}(q_{12} + q_{21})(\tilde{u}_2\tilde{v}_1 + \tilde{u}_1\tilde{v}_2) \end{pmatrix}, \quad (16)$$

where p_{ijk} and q_{ijk} denote the derivative of i th, j th, and k th variable successively to f and g at $E^*(\tilde{u}^*, \tilde{v}^*)$. U , t , and N are expanded with a small parameter ε .

where

$$\begin{aligned} d_1^T - d_1 &= \varepsilon d_{11} + \varepsilon^2 d_{22} + \dots, \\ \tilde{t} &= T_0 + \varepsilon T_1 + \varepsilon^2 T_2, \\ U &= \begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix} \\ N &= \varepsilon^2 \tilde{h}_2 + \varepsilon^3 \tilde{h}_3 + o(\varepsilon^4), \end{aligned} \quad (17)$$

$$\begin{aligned} \tilde{h}_2 &= \begin{pmatrix} \frac{1}{2}p_{11}\tilde{u}_1^2 + \frac{1}{2}(p_{12} + p_{21})\tilde{u}_1\tilde{v}_1 + \frac{1}{2}p_{22}\tilde{v}_1^2 \\ \frac{1}{2}q_{11}\tilde{u}_1^2 + \frac{1}{2}(q_{12} + q_{21})\tilde{u}_1\tilde{v}_1 + \frac{1}{2}q_{22}\tilde{v}_1^2 \end{pmatrix}, \\ \tilde{h}_3 &= \begin{pmatrix} \frac{1}{6}p_{111}\tilde{u}_1^3 + \frac{1}{6}p_{222}\tilde{v}_1^3 + \frac{1}{6}(p_{112} + p_{121} + p_{211})\tilde{u}_1^2\tilde{v}_1 + \frac{1}{6}(p_{122} + p_{212} + p_{221})\tilde{u}_1\tilde{v}_1^2 \\ + p_{11}\tilde{u}_2\tilde{u}_1 + p_{22}\tilde{v}_2\tilde{v}_1 + \frac{1}{2}(p_{12} + p_{21})(\tilde{u}_2\tilde{v}_1 + \tilde{u}_1\tilde{v}_2) \\ \frac{1}{6}q_{111}\tilde{u}_1^3 + \frac{1}{6}q_{222}\tilde{v}_1^3 + \frac{1}{6}(q_{112} + q_{121} + q_{211})\tilde{u}_1^2\tilde{v}_1 + \frac{1}{6}(q_{122} + q_{212} + q_{221})\tilde{u}_1\tilde{v}_1^2 \\ + q_{11}\tilde{u}_2\tilde{u}_1 + q_{22}\tilde{v}_2\tilde{v}_1 + \frac{1}{2}(q_{12} + q_{21})(\tilde{u}_2\tilde{v}_1 + \tilde{u}_1\tilde{v}_2) \end{pmatrix}. \end{aligned} \quad (18)$$

Meanwhile, the linear operator L is dissembled by Taylor expansion and (14) transforms to

$$L = L_T + (d_1^T - d_1)M, \quad (19)$$

where

$$L_T = \begin{pmatrix} f_u + d_1^T \nabla^2 & f_v \\ g_u & g_v + d_2 \nabla^2 \end{pmatrix}, M = \begin{pmatrix} -\nabla^2 & 0 \\ 0 & 0 \end{pmatrix}. \quad (20)$$

The derivation of the amplitude is

$$\frac{\partial A}{\partial t} = \varepsilon \frac{\partial A}{\partial T_1} + \varepsilon^2 \frac{\partial A}{\partial T_2} + \dots \quad (21)$$

We substitute (17)–(21) into (14) and collect the ε , ε^2 , and ε^3 , represented by the following set of linear equations:

$$\varepsilon: L_T \begin{pmatrix} \tilde{u}_1 \\ \tilde{v}_1 \end{pmatrix} = 0, \quad (22)$$

$$\varepsilon^2: L_T \begin{pmatrix} \tilde{u}_2 \\ \tilde{v}_2 \end{pmatrix} = \frac{\partial}{\partial T_1} \begin{pmatrix} \tilde{u}_1 \\ \tilde{v}_1 \end{pmatrix} - d_{11}M \begin{pmatrix} \tilde{u}_1 \\ \tilde{v}_1 \end{pmatrix} - \tilde{h}_2 \triangleq \begin{pmatrix} F_u \\ F_v \end{pmatrix}, \quad (23)$$

TABLE 2: An example to illustrate the existence and stability of the equilibrium of model (4).

η	γ	e	$\tilde{u}^*$	$\tilde{v}^*$	$\text{tr}(J)$	$\det(J)$
0.7	0.01040281036	140	0.351	33.74086284	-0.3731305371	0.2532783223

FIGURE 1: Turing domain of model (4).

$$\varepsilon^3: L_T \begin{pmatrix} \tilde{u}_3 \\ \tilde{v}_3 \end{pmatrix} = \frac{\partial}{\partial T_1} \begin{pmatrix} \tilde{u}_2 \\ \tilde{v}_2 \end{pmatrix} + \frac{\partial}{\partial T_2} \begin{pmatrix} \tilde{u}_1 \\ \tilde{v}_1 \end{pmatrix} - d_{11} M \begin{pmatrix} \tilde{u}_2 \\ \tilde{v}_2 \end{pmatrix} - d_{22} M \begin{pmatrix} \tilde{u}_1 \\ \tilde{v}_1 \end{pmatrix} - \tilde{h}_3 \triangleq \begin{pmatrix} H_u \\ H_v \end{pmatrix}, \quad (24)$$

where

$$\begin{pmatrix} F_u \\ F_v \end{pmatrix} = \sum_{j=1}^3 \begin{pmatrix} F_u^j \\ F_v^j \end{pmatrix} e^{i\mathbf{k}_j \cdot \mathbf{r}} + c.c. \quad (25)$$

and

$$\begin{pmatrix} H_u \\ H_v \end{pmatrix} = \sum_{j=1}^3 \begin{pmatrix} H_u^j \\ H_v^j \end{pmatrix} e^{i\mathbf{k}_j \cdot \mathbf{r}} + c.c.. \quad (26)$$

According to [43], solving (22), the solution is expressed as

FIGURE 2: Dispersion relation of system (4), which shows the real part of eigenvalues of model (4).

$$\begin{pmatrix} \tilde{u}_1 \\ \tilde{v}_1 \end{pmatrix} = \begin{pmatrix} l \\ 1 \end{pmatrix} (W_1 e^{i\mathbf{k}_1 \cdot \mathbf{r}} + W_2 e^{i\mathbf{k}_2 \cdot \mathbf{r}} + W_3 e^{i\mathbf{k}_3 \cdot \mathbf{r}}) + c.c., \quad (27)$$

where $l = f_u d_2 - g_v d_1^T / 2g_u d_1^T$ and $|\mathbf{k}_j| = k_T$. $c.c.$ represents the complex conjugate and the pattern of the mode $\exp(i\mathbf{k}_j \cdot \mathbf{r})$ is W_j ($j = 1, 2, 3$) under the first-order perturbation.

Substituting (27) into right-hand side of (23), the eigenvector of the operator L_T^+ is as follows:

$$\begin{pmatrix} 1 \\ l_2 \end{pmatrix} e^{-i\mathbf{k}_j \cdot \mathbf{r}} + c.c., \quad (j = 1, 2, 3), \quad (28)$$

where L_T^+ represents the eigenvector of the zero eigenvalue of the adjoint operator of L_T and

$$l_2 = \frac{2f_v d_1^T}{f_u d_2 - g_v d_1^T}. \quad (29)$$

On the basis of Fredholm solvability condition, the orthogonality condition is given as

$$(1, l_2) \begin{pmatrix} F_u^j \\ F_v^j \end{pmatrix} = 0, \quad (j = 1, 2, 3). \quad (30)$$

(30) yields

$$\begin{cases} (l + l_2) \frac{\partial W_1}{\partial T_1} = d_{11} k_T^2 l W_1 + [p_{11} l^2 + (p_{12} + p_{21})l + p_{22} + l_2(q_{11} l^2 + (q_{12} + q_{12})l + q_{22})] \overline{W}_2 \overline{W}_3, \\ (l + l_2) \frac{\partial W_2}{\partial T_1} = d_{11} k_T^2 l W_2 + [p_{11} l^2 + (p_{12} + p_{21})l + p_{22} + l_2(q_{11} l^2 + (q_{12} + q_{12})l + q_{22})] \overline{W}_1 \overline{W}_3, \\ (l + l_2) \frac{\partial W_3}{\partial T_1} = d_{11} k_T^2 l W_3 + [p_{11} l^2 + (p_{12} + p_{21})l + p_{22} + l_2(q_{11} l^2 + (q_{12} + q_{12})l + q_{22})] \overline{W}_1 \overline{W}_2. \end{cases} \quad (31)$$

Let the solution of (23) have the following form:

$$\begin{aligned} \begin{pmatrix} \tilde{u}_2 \\ \tilde{v}_2 \end{pmatrix} &= \begin{pmatrix} \tilde{U}_0 \\ \tilde{V}_0 \end{pmatrix} + \sum_{j=1}^3 \begin{pmatrix} \tilde{U}_j \\ \tilde{V}_j \end{pmatrix} e^{i\mathbf{k}_j \cdot \mathbf{r}} + \sum_{j=1}^3 \begin{pmatrix} \tilde{U}_{jj} \\ \tilde{V}_{jj} \end{pmatrix} e^{i2\mathbf{k}_j \cdot \mathbf{r}} \\ &+ \begin{pmatrix} \tilde{U}_{12} \\ \tilde{V}_{12} \end{pmatrix} e^{i(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}} \\ &+ \begin{pmatrix} \tilde{U}_{23} \\ \tilde{V}_{23} \end{pmatrix} e^{i(\mathbf{k}_2 - \mathbf{k}_3) \cdot \mathbf{r}} + \begin{pmatrix} \tilde{U}_{31} \\ \tilde{V}_{31} \end{pmatrix} e^{i(\mathbf{k}_3 - \mathbf{k}_1) \cdot \mathbf{r}} + c.c.. \end{aligned} \quad (32)$$

$$\begin{aligned} \begin{pmatrix} \tilde{U}_0 \\ \tilde{V}_0 \end{pmatrix} &= \begin{pmatrix} \tilde{u}_0 \\ \tilde{v}_0 \end{pmatrix} (|W_1|^2 + |W_2|^2 + |W_3|^2), \\ \tilde{U}_j &= l \tilde{V}_j, \\ \begin{pmatrix} \tilde{U}_{ij} \\ \tilde{V}_{ij} \end{pmatrix} &= \begin{pmatrix} \tilde{u}_{11} \\ \tilde{v}_{11} \end{pmatrix} W_j^2, \\ \begin{pmatrix} \tilde{U}_{mn} \\ \tilde{V}_{mn} \end{pmatrix} &= \begin{pmatrix} \tilde{u}_{mn} \\ \tilde{v}_{mn} \end{pmatrix} W_m \overline{W}_n, \end{aligned} \quad (33)$$

where

Substituting (32) into (23) and merging the coefficients of $\exp(0)$, $\exp(i\mathbf{k}_j \cdot \mathbf{r})$, $\exp(i2\mathbf{k}_j \cdot \mathbf{r})$, and $\exp(i(\mathbf{k}_j - \mathbf{k}_i) \cdot \mathbf{r})$, we obtain the following resultant relation:

$$\begin{aligned} \begin{pmatrix} \tilde{u}_0 \\ \tilde{v}_0 \end{pmatrix} &= \frac{1}{f_u \tilde{g}_v - f_v \tilde{g}_u} \begin{pmatrix} -\tilde{g}_v [p_{11} l^2 + (p_{12} + p_{21})l + p_{22}] + p_v [q_{11} l^2 + (q_{12} + q_{12})l + q_{22}] \\ -f_u [q_{11} l^2 + (q_{12} + q_{12})l + q_{22}] + q_u [p_{11} l^2 + (p_{12} + p_{21})l + p_{22}] \end{pmatrix}, \\ \begin{pmatrix} \tilde{u}_{11} \\ \tilde{v}_{11} \end{pmatrix} &= \frac{1}{2} \frac{1}{(f_u - 4K_T^2 d_1^T)(\tilde{g}_v - 4K_T^2 d_2) - f_v \tilde{g}_u} \begin{pmatrix} -(\tilde{g}_v - 4K_T^2 d_2) [p_{11} l^2 + (p_{12} + p_{21})l + p_{22}] + p_v [q_{11} l^2 + (q_{12} + q_{12})l + q_{22}] \\ -(f_u - 4K_T^2 d_1^T) [q_{11} l^2 + (q_{12} + q_{12})l + q_{22}] + \tilde{g}_u [p_{11} l^2 + (p_{12} + p_{21})l + p_{22}] \end{pmatrix}, \\ \begin{pmatrix} \tilde{u}_{mn} \\ \tilde{v}_{mn} \end{pmatrix} &= \frac{1}{(f_u - 3K_T^2 d_1^T)(\tilde{g}_v - 3K_T^2 d_{22}) - f_v \tilde{g}_u} \begin{pmatrix} -(\tilde{g}_v - 3K_T^2 d_2) [p_{11} l^2 + (p_{12} + p_{21})l + p_{22}] + f_v [q_{11} l^2 + (q_{12} + q_{12})l + q_{22}] \\ -(f_u - 3K_T^2 d_1^T) [q_{11} l^2 + (q_{12} + q_{12})l + q_{22}] + q_u [p_{11} l^2 + (p_{12} + p_{21})l + p_{22}] \end{pmatrix}. \end{aligned} \quad (34)$$

We make use of the Fredholm solvability condition in (24). Now, we have

$$(1, l_2) \begin{pmatrix} H_u^j \\ H_v^j \end{pmatrix} = 0 \quad (j = 1, 2, 3). \quad (35)$$

From (35), we get

$$\begin{cases} (l + l_2) \left(\frac{\partial V_1}{\partial T_1} + \frac{\partial W_1}{\partial T_2} \right) = lk_T^2 (d_{11}V_1 + d_{22}W_1) + D(\bar{V}_2\bar{W}_3 + \bar{V}_3\bar{W}_2) + (E_1|W_1|^2 + E_2(|W_2|^2 + |W_3|^2))W_1, \\ (l + l_2) \left(\frac{\partial V_2}{\partial T_1} + \frac{\partial W_2}{\partial T_2} \right) = lk_T^2 (d_{11}V_2 + d_{22}W_2) + D(\bar{V}_1\bar{W}_3 + \bar{V}_3\bar{W}_1) + (E_1|W_2|^2 + E_2(|W_1|^2 + |W_3|^2))W_2, \\ (l + l_2) \left(\frac{\partial V_3}{\partial T_1} + \frac{\partial W_3}{\partial T_2} \right) = lk_T^2 (d_{11}V_3 + d_{22}W_3) + D(\bar{V}_1\bar{W}_2 + \bar{V}_2\bar{W}_1) + (E_1|W_3|^2 + E_2(|W_1|^2 + |W_2|^2))W_3, \end{cases} \quad (36)$$

where

$$\begin{aligned} D &= (lr_1 + s_1) + l_2(lr_2 + n_2), \\ E_1 &= \left(br_1 + a_1s_1 + \frac{P_1}{2} \right) + l_2 \left(br_2 + b_1s_2 + \frac{P_2}{2} \right), \\ E_2 &= (cr_1 + c_1s_1 + P_1) + l_2(cr_2 + c_1s_2 + P_2), \end{aligned} \quad (37)$$

and

$$\begin{aligned} b &= \tilde{u}_0 + \tilde{u}_{11}, \quad c = \tilde{u}_0 + \tilde{u}_{mn}, \\ b_1 &= \tilde{v}_0 + \tilde{v}_{11}, \quad c_1 = \tilde{v}_0 + \tilde{v}_{mn}, \\ r_1 &= p_{11}l + \frac{1}{2}(p_{12} + p_{21}), \quad s_1 = p_{22} + \frac{1}{2}(p_{12} + p_{21})l, \\ r_2 &= q_{11}l + \frac{1}{2}(q_{12} + q_{21}), \quad s_2 = q_{22} + \frac{1}{2}(q_{12} + q_{21})l, \\ P_1 &= p_{111}l^3 + (p_{112} + p_{121} + p_{211})l^2 \\ &\quad + (p_{122} + p_{212} + p_{221})l + p_{222}, \\ P_2 &= q_{111}l^3 + (q_{112} + q_{121} + q_{211})l^2 \\ &\quad + (q_{122} + q_{212} + q_{221})l + q_{222}. \end{aligned} \quad (38)$$

Multiplying (31) and (36) by ε and ε^2 , together with (21) and

$$D_i = \tilde{D}_i^u = l\tilde{D}_i^v, \quad (i = 1, 2, 3), \quad (39)$$

we have the following amplitude equations:

$$\begin{cases} \tau_0 \frac{\partial A_1}{\partial t} = \mu D_1 + h\bar{D}_2\bar{D}_3 - (g_1|D_1|^2 + g_2(|D_2|^2 + |D_3|^2))D_1, \\ \tau_0 \frac{\partial A_2}{\partial t} = \mu D_2 + h\bar{D}_1\bar{D}_3 - (g_1|D_2|^2 + g_2(|D_1|^2 + |D_3|^2))D_2, \\ \tau_0 \frac{\partial A_3}{\partial t} = \mu D_3 + h\bar{D}_1\bar{D}_2 - (g_1|D_3|^2 + g_2(|D_1|^2 + |D_2|^2))D_3, \end{cases} \quad (40)$$

where

$$\begin{aligned} \tau_0 &= \frac{l + l_2}{d_1^T l k_T^2}, \quad \mu = \frac{d_1^T - d_1}{d_1^T}, \quad h = \frac{D}{d_1^T l^2 k_T^2}, \quad g_1 \\ &= -\frac{E_1}{d_1^T l^3 k_T^2}, \quad g_2 = -\frac{E_2}{d_1^T l^3 k_T^2}. \end{aligned} \quad (41)$$

On the basis of aforementioned analysis, we obtain the following results about the pattern selection of system (4) shown in (Table 3).

4. Numerical Simulations of Patterns

4.1. Pattern Selection. In this section, we select the suitable value of d_1 and perform numerical simulations to verify the theoretical results, i.e., to see whether the patterns arise in the corresponding Turing domain. We make use of the zero-flux boundary condition with 200×200 lattice sites. The space step is $\Delta h = 1$, and the time step is $\Delta t = 0.01$. For the sake of the similar type of patterns of the predator and prey, we choose the predator to exhibit our investigations. We start with initial spatial state as the heterogeneity space. The simulations are performed until the patterns do not change their characteristic anymore. The value of parameters is presented in Tables 4 and 5.

Figure 3 shows the evolutions of spatial pattern of the predator at final instants when $d_1 = 0.95$, $d_1 = 0.5$, $d_1 = 0.47$, and $d_1 = 0.2$, respectively. As presented in Figure 3, we obtain three different patterns in the corresponding Turing domains, which are hot spot patterns, i.e., Figure 3(a), mixture of spots and stripes, i.e., Figures 3(b) and 3(c), and stripe patterns, i.e., Figure 3(d). Thus, the simulation results shown are aligned with the theoretical analysis. However, it is noticeable that there is a little difference between the two mixtures in Figure 3(c) and in Figure 3(d), because they have different wavelengths.

4.2. Effect of the Prey's Diffusion on Patterns of Predator. In this section, we discuss the pattern formation driven by the diffusion rate of the prey by varying the value of the prey's diffusion, while the values of the other parameters are kept the same as in Section 4.1.

TABLE 3: The pattern selection of model (4).

μ	Pattern	Stability
$\mu_2 < \mu = 0.1115957587 < \mu_3$	Spot pattern	Stable
$\mu_3 < \mu = 0.5064420882 < \mu_4$	Coexistence of the spots and stripe patterns	Bistable
$\mu = 0.8025768353 > \mu_4$	Tripe patterns	Stable

TABLE 4: The parameters set in system (4).

η	γ	e	d_2
0.6	0.002056779432	525	4

TABLE 5: The parameters calculated in system (4).

d_1^T	g_1	g_2	h	μ_1	μ_2	μ_3	μ_4
1.013052345	76.6130004	171.1217394	4.675460751	-0.01304738873	0	0.1875026305	0.7938085466

FIGURE 3: The spatial patterns of the predator with different values of 31: (a) 31 = 0.95; (b) 31 = 0.5; (c) 31 = 0.47; (d) 31 = 0.2.

FIGURE 4: The spatial patterns of the predator with different value of 31: (a) $31 = 1.025$; (b) $31 = 1.009$; (c) $31 = 0.9$; (d) $31 = 0.83$.

First, we fix $d_1 = 1.025$ which is larger than d_1^T and transcends the bifurcation domain. Please note that there are no patterns emerging in Figure 4(a). This is in line with our theoretical results. As d_1 becomes smaller than d_1^T decreasing from 1.009 to 0.83 in Figures 4(b)–4(d), the spot pattern gets more and more dense. In order to analyze the variation in density, we compare Figure 4(b) with Figure 4(d) at the same iterations in detail. On the one hand, from Figures 5 and 6, we find that Figure 6 transforms faster than Figure 5. For example, Figure 6(b) starts to become the spot pattern while Figure 5(b) is still irregular in shape after 3400 iterations. On the contrary, we observe that the spots in Figure 5(d) are sparser than in Figure 6(d). Thus, the density of the predator will decrease as there is an increase in the mobility of the prey. According to the Reichenbach's viewpoint [44], we can illustrate our results effectively, i.e., when the prey species move frequently, the prey will encounter the predator easily. This leads to the decrease of the prey and aggravate the competition of the predator. As a result, the predators confront the danger of extinction. Moreover, we conclude that the pattern formation is induced by the diffusion rate of the prey by comparing Figure 4(a) with Figures 4(b)–4(d).

4.3. Pattern Transitions of the Predator Driven by Parameter γ . We fit the values of parameters η , e , d_1 , and d_2 presented in Table 6, while the value of γ gradually increases. We investigate the changes in the spatial distribution of the predator as the conversion rate into predator biomass varies. Specifically, when γ is fixed $\gamma = 0.007041633659$, hot spot pattern dominates the spatial region in Figure 7(a). In Figure 7(b), when we have $\gamma = 0.008509501103$, then the mixture of hot spots and stripes appears. As γ continues to grow to 0.009332994621, there is just the stripe pattern that is presented in Figure 5(c). Similarly, when γ increases to $\gamma = 0.01032204568$, the mixture of cold spots and stripes arises, which is shown in Figure 7(d). Finally, when γ becomes larger, we simulate the cold spot pattern presented in Figure 7(e). Figure 7 shows the orderly changes of the pattern transition, as well as the density of the predator increases from 85 to 90 and then decreases to 85 again with an increase in the value of γ , because γ denotes the number of the prey species which are captured by a single predator. The higher the value of parameter γ , the more food the predator species eat. However, please note that there is an effect of the environmental capacity, so the number of the predators cannot

FIGURE 5: The iterations of stripe patterns of the predator with $31 = 1.009$. The number of iterations is (a) 0, (b) 3600, (c) 130000, and (d) 150000.

FIGURE 6: Continued.

FIGURE 6: The iterations of stripe patterns of the predator with $\beta_1 = 0.83$. The number of iterations is (a) 0, (b) 3600, (c) 130000, and (d) 150000.

TABLE 6: The parameters selected in system (4).

η	e	d_1	d_2
0.6	350	0.2	4

FIGURE 7: Continued.

FIGURE 7: The transitions patterns of the predator induced by the conversion rate of the prey into predator biomass and the other parameters are set in Table 6. Iterations: (a) 0.007041633659; (b) 0.008509501103; (c) 0.009332994621; (d) 0.01032204568; (e) 0.01092704597.

increase forever. So, the population of the predator decreases again when it exceeds the threshold value. The simulation results of this work present a clear biological meaning.

5. Conclusions

In this work, we study a ratio-dependent prey-predator model. The pattern selection of our model has been investigated via multiple scale analysis near the Turing bifurcation point. First, we obtain the condition of the Turing instability of our model. Then, by choosing d_1 as a bifurcation parameter, we obtained the amplitude equations and presented the Turing pattern selection of model (4). The simulation results presented in this work are aligned with the theoretical analysis, i.e., H_0 spot patterns arise when μ lies in domain (μ_2, μ_3) with $d_1 = 0.95$; the stable mixture of spots and stripes appears when $\mu_3 < \mu < \mu_4$ with $d_1 = 0.5$ and $d_1 = 0.47$, while only stripe pattern is simulated when $\mu > \mu_4$ and $d_1 = 0.2$. On the contrary, we study the effect of the spatial motion and conversion rate of predator biomass on density of the predator. The decreasing value of the former agent d_1 causes the sparse density in the predator population, while the increasing value of the latter factor induces the variation of the density of predator species. The simulation results presented in this work reveal the inherent mechanism of our model. Moreover, on the basis of our analysis, the slight value changes in d_1 lead to density changes of the predator. This observation shows that the diffusive rate of the prey has a great effect on prey and the whole system. Above all, the intrinsic agents, i.e., diffusion and predation, control the persistence or extinction of the spatial predator-prey model [45].

However, it is noticeable that there is a little defect in the analysis of pattern selection via amplitude equations. That is, pattern selection is only valid when the value of the controlled parameter d_1 is set close to the threshold d_1^T . Thus, pattern selection is only valid when the value is very close to the controlled parameter d_1^T . This inspires us to think about the best distance of the parameter away from the controlled parameter. Besides, the method of the amplitude equations is also applicable in the epidemic model and the vegetation

model to study the pattern formation of the epidemic and vegetation in the future [46–49].

Data Availability

The data in the manuscript are not the public data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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