Hindawi
Complexity
Volume 2023, Article ID 9767809, 1 page
https://doi.org/10.1155/2023/9767809

Retraction

Retracted: Emergency Optimization Decision-Making with Incomplete Probabilistic Information under the Background of COVID-19

Complexity

Received 17 October 2023; Accepted 17 October 2023; Published 18 October 2023

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] M. Fu, L. Wang, J. Zhu, and B. Zheng, "Emergency Optimization Decision-Making with Incomplete Probabilistic Information under the Background of COVID-19," *Complexity*, vol. 2021, Article ID 6658006, 16 pages, 2021.

Hindawi
Complexity
Volume 2021, Article ID 6658006, 16 pages
https://doi.org/10.1155/2021/6658006

Research Article

Emergency Optimization Decision-Making with Incomplete Probabilistic Information under the Background of COVID-19

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Received 1 December 2020; Revised 8 January 2021; Accepted 1 February 2021; Published 23 July 2021

Academic Editor: M. Irfan Uddin

At present, the whole world is facing the serious challenge of COVID-19, and it has reached a consensus that taking appropriate measures timely is the key to prevent and control infectious diseases. This paper proposes an algorithm to solve the problem of how to choose the most appropriate alternative from numerous alternatives in the limited time from the perspective of management. First of all, we have compared various data structures for keeping the comparison results of alternatives. After comparisons, we adopt the hesitant fuzzy incomplete probabilistic linguistic preference relation matrix to save the information which can keep the first-hand valuable collected data to the maximum extent; then, we can obtain the missing values with the help of the fault tree analysis method, which can consider both subjective evaluation data and objective historical data simultaneously. Meanwhile, the fault tree analysis method can find development laws with the help of similar infectious diseases that have occurred in the past. The definition of consistency index is also introduced which can measure whether there are contradictions and the degree of contradiction in the decision results. Only those data that meet the consistency requirements can be used for decision-making and then a method is proposed to effectively reduce the degree of inconsistency. The information aggregation method will be adopted subsequently, and we can obtain the ranking of alternatives. An instance with specific execution steps is also introduced to illustrate the feasibility and efficiency of the algorithm proposed in this paper; in the end, several types of comparisons with typical algorithms proposed by other scholars are carried out, and all the experimental results show that the algorithm proposed in this paper is effective and innovative in some aspects.

1. Introduction

COVID-19 is presenting society with unprecedented challenges, risks, and disruptions [1], and the losses caused will be immeasurable. At present, the primary goal for the whole world is to control the spread of the virus as soon as possible. From a macroperspective, every country must implement a series of measures immediately. In addition, different countries should take different measures nationally or subnationally based on their specific contexts, management systems, and characteristics [2]. They should collect and summarize data regularly as the infectious disease evolves [3]. From a microperspective, communities, schools, companies, and even families should also adopt appropriate measures as needed.

Scientific decision-making is particularly important in this public health emergency; furthermore, the time for decision-makers to make critical decisions is very limited and the information used for decision-making is often incomplete and uncertain. We have always believed that information is the basis of decision-making; however, in most cases, we can only get probability information about the situation of the disease and it is therefore hard to obtain definite information because of the hesitation among different values in the decision-makers' mind. How to effectively describe uncertain information and supplement incomplete data in a limited time is the first critical problem in front of us.

National and local authorities must balance the prevention work of infectious diseases and the effect on economic development [4]. They should continually adjust measures and policies according to the actual pandemic situation. Prevention and control measures that are too strict will have a serious impact on economic development; however, the measures that are too loose will be difficult to effectively control the spread of the disease [5]. How to aggregate the limited information and then find the appropriate measure is the second critical problem in front of us.

Fortunately, the fuzzy theory can not only accurately describe uncertain information but also aggregate probability information efficiently. The fuzzy theory, which was first proposed by professor Zadeh in 1965 [6], introduced the important concept of "membership" to describe the uncertainties and vagueness of the research object. Subsequently, Marinos published some research achievements on the fuzzy logic field [7] in 1969 based on the fuzzy theory of Zadeh. Since then, more and more attention has been paid to the research and application of the fuzzy theory. Group decision-making is one of the most common applications of the fuzzy theory, the main research object of group decision-making is to find the most appropriate alternative from multiple alternatives, and one of the critical steps is that a group of experts present their preferences over a series of possible alternatives. In this step, the form of preference relations is particularly important, which directly affects the integrity of information. Recently, research on the subject of preference relations has developed quickly, such as the fuzzy preference relations [8], the linguistic preference relations [9], the hesitant fuzzy linguistic preference relations [10], the probabilistic linguistic preference relations [11], and the hesitant fuzzy incomplete probabilistic linguistic preference relations.

There are at least four reasons why this paper adopts the fuzzy theory to deal with emergency optimization decision-making problems under the background of COVID-19:

(i) The fuzzy theory is widely used in the decision-making field, and the problem discussed in this paper is a decision-making problem essentially

(ii) The evaluation data can be effectively and accurately indicated by the incomplete probabilistic linguistic term sets of the fuzzy theory

(iii) The algorithms of filling missing values are relatively mature in the fuzzy theory and the missing values in the collected data need to be supplemented

(iv) Several efficient information aggregation algorithms are available, and they can make full use of information and can provide support for decision-making in the limited time

The main contribution of this paper is that we consider the problem of epidemic prevention and control as an optimization decision-making problem under multiple constraints.

2. The Fundamental Theory

The preference relation is one of the representation forms in the MCDM problem, and the theory of the preference relation has developed quickly in recent years, such as fuzzy preference relations [8], linguistic preference relations [9], probabilistic linguistic preference relations [11], double hierarchy hesitant fuzzy linguistic preference relations [12], and hesitant fuzzy linguistic preference relations [13].

The linguistic preference relation is one of the closest forms to human opinion compared with other preference relations. As a cognitive linguistic information representation tool, Gou et al. [14] proposed the definition of the double hierarchy linguistic preference relation (DHLPR) based on the double hierarchy hesitant fuzzy linguistic preference relations. The DHLPR can reflect the cognitive information and the relationships of any two alternatives more clearly.

2.1. The Multicriteria Decision-Making Problem. Most problems in real life need decision-making support and they belong to the research field of multicriteria decision-making (MCDM) problems. The MCDM problems try to find the most appropriate solution from many alternatives according to several conflicting criteria. The solutions to MCDM problems can assist decision-makers with making critical decisions. The emergency optimization decision-making problem with incomplete probabilistic information under the background of COVID-19 is a typical MCDM problem.

The definition of the MCDM can be simply described as follows; suppose there are m alternatives in total, which can be denoted as $A = \{A_1, A_2, \ldots, A_m\}$, and there are n indicators in total, which can be denoted as $I = \{I_1, I_2, \ldots, I_n\}$. Additionally, the selection of indicators is very important, as we need to select the indicators which can describe the main characteristics of the problem. Too many indicators will greatly increase the workload of the data acquisition and are also not conducive to the role of key indicators; conversely, too few indicators are difficult to describe the details of the problem. First of all, we need to collect data for each indicator, and then we should normalize the original data according to certain algorithms. Different MCDM problems may adopt different data structures for indicators and data normalized processing algorithms. In the end, we can obtain the decision matrix D which can be denoted as

$$D = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{22} & \cdots & D_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ D_{m1} & D_{m2} & \cdots & D_{mn} \end{bmatrix}. \quad (1)$$

The ultimate goal is to obtain the most appropriate alternative through the calculation for the decision matrix by information aggregation algorithms. There are various information aggregation algorithms, and part of them will be introduced in the following sections.

2.2. The Hesitant Fuzzy Linguistic Term Set. Some indicators cannot be described quantitatively but only evaluated qualitatively in the actual decision-making process, such as the disease severity, the anxiety level, and the severity of the disease spread. It may make decision-makers feel more comfortable and intuitive if we can directly record information by linguistic terms. Zadeh proposed the fuzzy linguistic method to simulate and manage uncertain linguistic information. The evaluation values of experts can be regarded as linguistic variables in the fuzzy linguistic method, and the linguistic variable is not a numerical value but a word or a phrase [15]. Certainly, the linguistic information has to be transformed into a specific form which can be processed easily by computer algorithms after all the comments are given by experts. Xu proposed the definition of the subscript symmetric additive linguistic term set which can be denoted as

$$S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}. \tag{2}$$

The intermediate scale s_0 indicates the moderate value and other scales are evenly distributed on both sides [16]. $s_{-\tau}$ and s_τ indicate the upper and lower bounds, respectively, and τ is a positive integer. The subscript symmetric additive linguistic term set S meets the following conditions: (i) the inequality $s_\alpha > s_\beta$ will hold if the condition $\alpha > \beta$ holds and (ii) the inverse operation $\text{neg}(s_\alpha) = s_{-\alpha}$ holds; however, the equation $\text{neg}(s_0) = s_0$ is a special case.

After that, other scholars have proposed the definition of subscript asymmetry additive linguistic term set which can be denoted as follows: $S' = \{s_t | t = 0, \ldots, \tau, \ldots, 2\tau\}$.

The intermediate scale s_τ indicates the moderate value and s_0 and $s_{2\tau}$ indicate the upper and lower bounds, respectively. The inverse operation $\text{neg}(s_\alpha) = s_{2\tau-\alpha}$ holds; however, the equation $\text{neg}(s_\tau) = s_\tau$ is a special case. An example is given to illustrate the specific usage of the subscript asymmetry additive linguistic term set.

$$\begin{aligned} S' = \{&s_0 = \text{"terrible"}, s_1 = \text{"bad"}, s_2 = \text{"identity"}, \\ &s_3 = \text{"good"}, s_4 = \text{"good"}\}. \end{aligned} \tag{3}$$

Usually, many critical decisions must be made within a short period of time according to the incomplete probabilistic information, especially in the process of the prevention and control of COVID-19. Decision-makers (DMs) often find that the information obtained may be uncertain and vague to a certain extent [17]. For example, suppose there are five emergency levels in total, they are S_0: very low, S_1: low, S_2: medium, S_3: high, and S_4: very high, respectively, and we can construct a subscript asymmetry additive linguistic term set. When a DM evaluates the risk level of the infectious disease spread, he/she may be hesitant among multiple values because of the limited information and time allowed for consideration, for example, he/she may consider that the emergency levels of low, medium, and high are all possible, not just one. Different from traditional algorithms, hesitant fuzzy linguistic elements (HFLEs) can record all the data details and avoid any information loss according to the fuzzy theory. So the risk level of the infectious disease spread can be recorded as a hesitant fuzzy linguistic element which can be denoted as $h = \{S_1\text{: low}; S_2\text{: medium}; S_3\text{: high}\}$ Mathematically, it can be also recorded as $h = \{S_1, S_2, S_3\}$ for short. If all indicators of each alternative are indicated by the form of HFLE [18], they will constitute a hesitant fuzzy linguistic term set (HFLTS) which is shown in equation (3), whereby the symbol n indicates the total number of indicators. Each indicator corresponds to an HFLE, and each alternative corresponds to an HFLTS. All the HFLTSs construct the hesitant fuzzy linguistic decision matrix which is shown as follows:

$$\begin{aligned} H &= \{h_1, h_2, \ldots, h_n\}, \\ \text{HM} &= \begin{pmatrix} H_1 \\ H_2 \\ \vdots \\ H_m \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{pmatrix}. \end{aligned} \tag{4}$$

2.3. The Hesitant Fuzzy Probabilistic Linguistic Term Set. The definition of the hesitant fuzzy probabilistic linguistic term set (HFPLTS) is proposed based on the definition of the hesitant fuzzy linguistic term set [19]. We have realized that every evaluation value may have multiple values; however, the probability of each value is not necessarily equal. As mentioned in the above example, the DM may consider that the emergency levels of low, medium, and high are all possible; besides that, he/she may consider that the occurrence probability of the low level is the highest, and the occurrence probability of the high level is the lowest [20]. Furthermore, he/she may be able to give the probability values for different evaluation values; for example, the probability values are 0.4, 0.35, and 0.25 for the emergency levels of low, medium, and high, respectively. The probabilities are also very important to describe the information details. So we can record the indicator by using the hesitant fuzzy probabilistic linguistic element (HFPLE) [21], the indicator can be indicated as $\text{ph} = \{S_1(0.4), S_2(0.35), S_3(0.25)\}$ mathematically, and then, the HFPLEs of all the indicators will constitute a hesitant fuzzy probabilistic linguistic term set PH. All the alternatives will construct the hesitant fuzzy probabilistic linguistic term set matrix PHM.

$$\text{PH} = \{\text{ph}_1, \text{ph}_2, \ldots, \text{ph}_n\}, \tag{5}$$

$$\text{PHM} = \begin{pmatrix} \text{PH}_1 \\ \text{PH}_2 \\ \vdots \\ \text{PH}_m \end{pmatrix} = \begin{pmatrix} \text{ph}_{11} & \text{ph}_{12} & \cdots & \text{ph}_{1n} \\ \text{ph}_{21} & \text{ph}_{22} & \cdots & \text{ph}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{ph}_{m1} & \text{ph}_{m2} & \cdots & \text{ph}_{mn} \end{pmatrix}. \tag{6}$$

2.4. The Hesitant Fuzzy Incomplete Probabilistic Linguistic Term Set. The definition of the hesitant fuzzy incomplete probabilistic linguistic term set (HFIPLTS) is proposed based on the definition of the hesitant fuzzy probabilistic linguistic term set (HFPLTS). Each evaluation value may have multiple values, and the probability of each value is not necessarily equal as mentioned above; however, we find that sometimes it is difficult to determine part of the probability values. Therefore, we have to advance the definition of the hesitant fuzzy incomplete probabilistic linguistic term set [22]. It can be divided into two categories according to whether all or part of the probability values are unknown. As mentioned in the above example, the hesitant fuzzy incomplete probabilistic linguistic element (HFIPLE) may consider that all the emergency levels of low, medium, and high are possible, and he/she also realizes that the probabilities of the three cases may be different; the first category is that he/she cannot decide all the specific probability values at present. So we can record the indicator using the hesitant fuzzy incomplete probabilistic linguistic element, $\text{iph}_1 = \{S_1(x_1), S_2(x_2), S_3(x_3)\}$. The second category is that he/she cannot decide part of probability values at present [23], as he/she can only decide that the probability value of the high level is 0.4, so we can also record the indicator using the hesitant fuzzy incomplete probabilistic linguistic element, $\text{iph}_2 = \{S_1(x_1), S_2(x_2), S_3(0.4)\}$. The HFIPLEs of all the indicators will constitute a hesitant fuzzy incomplete probabilistic linguistic term set, and all the alternatives will constitute the hesitant fuzzy incomplete probabilistic linguistic term set matrix.

$$\text{IPH} = \{\text{iph}_1, \text{iph}_2, \ldots, \text{iph}_n\}, \tag{7}$$

$$\text{IPHM} = \begin{pmatrix} \text{IPH}_1 \\ \text{IPH}_2 \\ \vdots \\ \text{IPH}_m \end{pmatrix} = \begin{pmatrix} \text{iph}_{11} & \text{iph}_{12} & \cdots & \text{iph}_{1n} \\ \text{iph}_{21} & \text{iph}_{22} & \cdots & \text{iph}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{iph}_{m1} & \text{iph}_{m2} & \cdots & \text{iph}_{mn} \end{pmatrix}. \tag{8}$$

If all the probability values are known, the hesitant fuzzy incomplete probabilistic linguistic term set will turn into the hesitant fuzzy probabilistic linguistic term set immediately, and similarly, if all the probability values are equal, the hesitant fuzzy probabilistic linguistic term set will turn into the hesitant fuzzy linguistic term set immediately. In other words, the hesitant fuzzy incomplete probabilistic linguistic term set is a special case of the hesitant fuzzy probabilistic linguistic term set; meanwhile, the hesitant fuzzy probabilistic linguistic term set is also a special case of the hesitant fuzzy linguistic term set [24].

2.5. The Hesitant Fuzzy Incomplete Probabilistic Linguistic Preference Relation. Usually, people are used to making pairwise comparisons for alternatives and then constructing a preference relation matrix in the decision-making process [25]. When people compare one alternative with another, he/she may give a series of possible preferences because of the complexity and the lack of information. Therefore, the best data structure for indicating preference values is the hesitant fuzzy incomplete probabilistic linguistic element and it can retain the original information to the maximum extent.

Suppose there are m alternatives in total; we must find the most appropriate measure from the m alternatives in the limited time. In general, the decision-making process can be divided into the following steps: firstly, we will obtain a $m \times m$ matrix through pairwise comparisons of the m alternatives, and each element of the matrix is indicated by the form of hesitant fuzzy incomplete probabilistic linguistic element [26]. Then, we must complete the missing values in the matrix IPLM which we will introduce in the following sections; the consistency of the matrix will be also verified and adjusted after the previous step. Subsequently, the information will be aggregated comprehensively through aggregation algorithms for each alternative [27]. Finally, the most appropriate alternative will be obtained and the analysis results will be provided to decision-makers to make critical decisions for epidemic prevention and control.

$$\text{IPLM} = \begin{pmatrix} \text{IPH}_{11} & \text{IPH}_{12} & \cdots & \text{IPH}_{1m} \\ \text{IPH}_{21} & \text{IPH}_{22} & \cdots & \text{IPH}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \text{IPH}_{m1} & \text{IPH}_{m2} & \cdots & \text{PH}_{mm} \end{pmatrix}. \tag{9}$$

2.6. The Consistency of the HFIPLPR. It is often difficult to precisely rank alternatives only by simple observation; however, people are better at pairwise comparison in the decision-making process. Therefore, the preference matrix will be obtained through many pairwise comparisons of the alternatives. Furthermore, there may exist as contradictory phenomena in the preference matrix because all the alternatives cannot be compared simultaneously and each pairwise comparison is done independently only between two different alternatives [28]. So, some concepts of preference relation consistency will be introduced.

Definition 1. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives; the symbol R denotes the preference relation of alternatives A. The symbol R represents a $m \times m$ matrix which can be denoted as $R = (\gamma_{ij})_{m\times m}$, $\forall i, j = 1, 2, \ldots, m$, and all the values of the matrix are real numbers, and they satisfy the following conditions, $\gamma_{ij} \geq 0$, $\gamma_{ii} = 0.5$, $\gamma_{ij} + \gamma_{ji} = 1$, and $\gamma_{ij} \in [0, 1]$. The value of γ_{ij} indicates the preference degree of alternative A_i over alternative A_j. Alternative A_i will be better than alternative A_j if the inequality $\gamma_{ij} > 0.5$ holds; similarly, alternative A_i will be worse than alternative A_j if the inequality $\gamma_{ij} < 0.5$ holds. The matrix will be satisfied with the strict additive consistent fuzzy preference relation if the following equation holds:

$$\gamma_{ij} = \gamma_{ik} + \gamma_{jk} + 0.5, \quad i, j, k = 1, 2, \ldots, m. \tag{10}$$

The values of the preference relation matrix discussed in this paper are hesitant fuzzy incomplete probabilistic linguistic elements, not real numbers; therefore, the definition of hesitant fuzzy incomplete probabilistic linguistic preference relation has both similarities and differences with Definition 1 [29]. We introduce Definition 2 for the hesitant fuzzy incomplete probabilistic linguistic preference relation with the help of Definition 1.

Definition 2. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives; the symbol HFIPLPR denotes the hesitant fuzzy incomplete probabilistic linguistic preference relation based on alternatives A. The HFIPLPR is also a $m \times m$ matrix which can be denoted as HFIPLPR $= (L(x)_{ij})_{m\times m}$, $\forall i, j = 1, 2, \ldots, m$; the biggest difference from Definition 1 is that the values of the matrix are hesitant fuzzy probabilistic linguistic elements, not real numbers, and it can be denoted as $L(x)_{ij} = \{L_{ij,l}(x_{ij,l}) | l = 1, 2, \ldots, \#l(x)_{ij}\}$. It also indicates the preference degree of alternative A_i over alternative A_j. The HFPLPR will be satisfied with the strict additive consistent fuzzy preference relation if the following equation holds:

$$E\left(L(x)_{ij}\right) = E\left(L(x)_{ik}\right) + E\left(L(x)_{jk}\right) + 0.5, \quad i, j, k = 1, 2, \ldots, m. \tag{11}$$

The symbol $E(L(x)_{ij})$ denotes the expected mean value of the HFIPLE $L(x)_{ij}$, the symbol $r_{ij,l}$ denotes one of the evaluation values, and the symbol $x_{ij,l}$ denotes its corresponding probability value.

$$E\left(L(x)_{ij}\right) = \sum_{l=1}^{\#L(x)_{ij}} r_{ij,l} \cdot x_{ij,l}. \tag{12}$$

2.7. The Fault Tree Analysis (FTA). The fault tree analysis method is introduced [30] to find out the laws of the development trend, internal logic relationships, and root causes of the infectious disease. The fault tree analysis method is a special logic causality diagram of an inverted tree, which uses event symbols, logic gate symbols, and transition symbols to describe the causal relationships among various events in the complex system. The input events of the logic gates are the "cause" of the output events, and the output events of the logic gates are the "result" of the input events. The events in the FTA can be roughly classified into three categories: basic events, intermediate events, and undesirable events. Generally, the fault tree analysis method can be roughly divided into the following five steps:

(i) Define several undesirable events according to the research target, especially the most undesirable event which is the main target to avoid. In this paper, the large-scale spread of the infectious disease COVID-19 definitely is the most undesirable event.

(ii) Get as much information as possible about the research target and possible causes of the undesirable events, and their occurrence probabilities should also be obtained and analyzed. It is necessary to carry out quantitative statistics and classification according to the previous monitoring data.

(iii) Build the fault tree analysis model. Select the main causes and make clear the logical relationships and tiers among them, and then build the fault tree analysis model graphically.

(iv) Calculate the probabilities of the undesirable events by the fault tree analysis.

(v) Find out the main factors that threaten the security of the system, and also find out the method to make the system safer through the fault tree analysis. The purpose of this paper is to find out the most effective way to prevent and control the spread of the infectious COVID-19.

The hesitant fuzzy method can retain the subjective judgment information to the maximum extent; besides that, the fault tree analysis method can make full use of all types of past objective data. The subjective judgment and past objective data can be organically combined in the algorithm proposed in this paper and they can play their respective advantages to find out the most appropriate alternative.

3. The Emergency Optimization Decision-Making Algorithm with Incomplete Probabilistic Information

When infectious diseases break out, we need to make appropriate decisions timely based on the limited information, as the quality of the decisions often directly affects the working effect of the infectious disease prevention and control. Generally, the decision-making process can be roughly divided into three steps [31]. Firstly, relevant information should be collected and the causes of infectious diseases should be found out, and then the experts will propose a series of alternatives. Secondly, we usually try to find out the most appropriate alternative only by simple comparisons, although unfortunately it is often difficult to find out the alternative which is significantly better than others, meaning we therefore need to compare alternatives quantitatively by certain algorithms [32]. Finally, we can obtain the most appropriate alternative according to the calculation results of the algorithm, and then we will closely monitor the practical implementation effect of the selected alternative. So how to scientifically compare alternatives is the main focus of this paper after the above analysis and the specific steps are listed as follows:

Step 1: construct the decision matrix. Suppose there are m alternatives in total, we can obtain a $m \times m$ decision matrix through pairwise comparisons. We only compare two alternatives at a time instead of multiple alternatives because the pairwise comparison is more conducive to make a scientific and reasonable ranking result especially in the face of a complex epidemic situation. Therefore, the data structure of the decision matrix M can be denoted as equation (13). Additionally, the elements of the matrix are not integers or real numbers, but hesitant fuzzy incomplete probabilistic linguistic sets, giving mainly three advantages. Firstly, people are better at using linguistic terms to express the comparison results. Secondly, it is often difficult to accurately express the pairwise comparison results only by one integer or real numbers, and there may be multiple values because of the hesitation in the decision-maker's mind. Thirdly, the probability of each evaluation value given by the decision-maker may be different or unknown. We find the facts that the values of h_{11}, h_{22}, ..., h_{mm} are equal to 0.5 and h_{ij} and h_{ji} are complementary to each other which can be denoted as $h_{ji} = h_{ij}^c$. Equation (14) is a simple example to illustrate the specific calculation method of h_{ij}^c. Therefore, we just need to determine the values of the upper triangular in the decision matrix, and all the values will be obtained through simple calculations.

$$M = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & \cdots & h_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mm} \end{pmatrix}, \tag{13}$$

$$h_{ji} = h_{ij}^c = \left\{S_{2\tau-\chi}\left(x_\chi\right), S_{2\tau-\beta}\left(x_\beta\right), S_{2\tau-\alpha}\left(x_\alpha\right)\right\}, \quad \text{when } h_{ij} = \left\{S_\alpha\left(x_\alpha\right), S_\beta\left(x_\beta\right), S_\chi\left(x_\chi\right)\right\}. \tag{14}$$

Step 2: complete missing values of the decision matrix with the help of the fault tree analysis. The elements of the decision matrix are hesitant fuzzy incomplete probabilistic linguistic sets. Let us take $h_{ij} = \left\{S_\alpha(x_\alpha), S_\beta(x_\beta), S_\chi(x_\chi)\right\}$ as an example, where the symbol h_{ij} indicates the comparison result of alternative A_i with A_j, the subscripts α, β, and χ indicate the evaluation values, and these values will be given by experts; meanwhile, x_α, x_β, and x_χ are their corresponding probability values which may be partially or wholly unknown. Therefore, firstly we need to obtain these unknown values and complete the decision matrix. The complement algorithm can be roughly divided into the following steps:

(i) Define a series of undesirable events that can be denoted as $U = \{U_l | l = 1, 2, \ldots, t\}$.

(ii) Define basic and intermediate events according to the historical data or similar cases. Furthermore, find out the internal logical relationships among basic events, intermediate events, and undesirable events. The logical relationships mainly include logical "AND" which can be indicated by the logical operator "$\otimes$" and logical "OR" which can be indicated by the logical operator "$\oplus$".

(iii) Establish the fault tree analysis model graphically according to the logical relationships.

This paper illustrates the specific usages and skills of the fault tree analysis method in detail with an example shown in Figure 1. The symbols can be divided into three categories; the first category consists of U_1, U_2, and U_3 which are called undesirable events. It should be also noted that the severity degree will increase with the increase of the subscript value; therefore, the most undesirable event in the example is U_3. The second category consists of M_1, M_2, and M_3, which are called intermediate events, and indicates the intermediate states of the system. The third category consists of X_1, X_2, X_3, X_4, X_5, X_6, and X_7, which are called basic events, and they are often the triggers of the epidemic

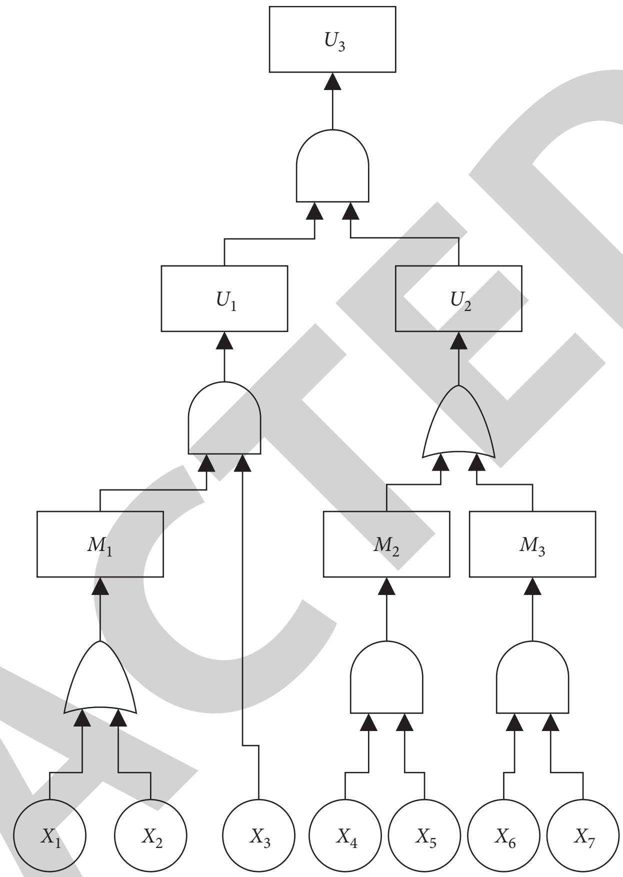

Figure 1: The graphical fault tree analysis model.

development. Generally, basic events trigger the development of intermediate events, which subsequently trigger the development of undesirable events according to certain logical rules. The occurrence probabilities of the undesirable events can be obtained by the logical rules of the fault tree which are shown as follows:

$$F_{U_l} = F_l\left(X_1, X_2, \ldots, X_k\right), \quad l = 1, 2, \ldots, t. \tag{15}$$

We can obtain the occurrence probability matrix of the basic events when implementing different alternatives according to the historical data and comprehensive judgment, and the form of the probability matrix is shown as follows:

$$\Theta = [\rho_{ij}]_{m\times k} = \begin{matrix} & X_1 & X_2 & \cdots & X_k \\ A_1 & \rho_{11} & \rho_{12} & \cdots & \rho_{1k} \\ A_2 & \rho_{21} & \rho_{22} & \cdots & \rho_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \rho_{m1} & \rho_{m2} & \cdots & \rho_{mk} \end{matrix}. \tag{16}$$

Therefore, the probabilities of the undesirable events when implementing different alternatives can be obtained according to equations (15) and (16). The undesirable events probability matrix can be denoted as follows:

$$\Lambda = [\zeta_{ij}]_{m\times t} = \begin{matrix} & U_1 & U_2 & \cdots & U_t \\ A_1 & \zeta_{11} & \zeta_{12} & \dots & \zeta_{1t} \\ A_2 & \zeta_{21} & \zeta_{22} & \dots & \zeta_{2t} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \zeta_{m1} & \zeta_{m2} & \cdots & \zeta_{mt} \end{matrix}, \quad (17)$$

$\zeta_{ij} = f_{ij}(\rho_{i1}, \rho_{i2}, \dots, \rho_{ik})$ $i = 1, 2, \dots, m; j = 1, 2, \dots, t,$

$$\zeta_{ij} = f_{ij}(\rho_{i1}, \rho_{i2}, \dots, \rho_{ik}), \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, t. \quad (18)$$

The symbol f_{ij} indicates one type of rule which can be obtained according to the logical relationships of the fault tree analysis model. The calculation formulas of the most important logical relationships are listed as follows:

$$\rho_{ij} \oplus \rho_{uv} = \rho_{ij} + \rho_{uv} - \rho_{ij}\rho_{uv}, \quad (19)$$

$$\rho_{ij} \otimes \rho_{uv} = \rho_{ij}\rho_{uv}. \quad (20)$$

Then the undesirable events probability matrix $\Lambda = [\zeta_{ij}]_{m\times t}$ will be further processed and we can obtain the normalized undesirable event probability matrix $\overline{\Lambda} = [\overline{\zeta}_{ij}]_{m\times t}$.

$$\overline{\zeta_{ij}} = \begin{cases} \dfrac{\zeta_{ij}}{\sum_{j=1}^{t} \zeta_{ij}}, & j > 1; i = 1, 2, \dots, m, \\[2ex] \dfrac{\zeta_{i1}}{\sum_{i=1}^{m} \zeta_{i1}}, & j = 1; i = 1, 2, \dots, m. \end{cases} \quad (21)$$

We will calculate the missing probability values of the decision matrix according to the normalized undesirable event probability matrix $\overline{\Lambda} = [\overline{\zeta}_{ij}]_{m\times t}$. The algorithm will be divided into two categories according to the actual situation.

(i) If the total number of undesirable events equals one, then the missing probability values can be calculated as follows:

$$p^*_{ij,l} = \frac{\left(1/\left(1 + \lambda_{ij,l}\right)\right), \quad \left(\overline{\zeta_i} + \lambda_{ij,l} \cdot x_{ij,l}\right)}{\sum_{l=1}^{\eta}\left(1/\left(1 + \lambda_{ij,l}\right)\right), \quad \left(\overline{\zeta_i} + \lambda_{ij,l} \cdot x_{ij,l}\right)}, \quad i, j = 1, 2, \dots, m. \quad (22)$$

The symbol η denotes the total number of items of the hesitant fuzzy incomplete probabilistic linguistic preference relation sets.

(ii) If the number of undesirable events is greater than one, the algorithm will be more complex. Firstly, we must increase or decrease the number of the undesirable events to ensure that they have the same number of items as the hesitant fuzzy incomplete probabilistic linguistic preference relation sets. Then, the missing probability values can be obtained according to the undesirable events probability matrix. The method of increasing or decreasing the number of undesirable events will be introduced, respectively.

Firstly, we introduce the reduction algorithm, the total number of undesirable events will be reduced step by step, and the reduction will be one at a time. The number of undesirable events will be reduced from t to $t-1$ and the method is shown as follows:

$$\overline{\Lambda}' = \begin{matrix} & U_{1.5} & U_{2.5} & \cdots & U_{\frac{2t-1}{2}} \\ A_1 & \dfrac{\overline{\zeta}_{11} + \overline{\zeta}_{12}}{2} & \dfrac{\overline{\zeta}_{12} + \overline{\zeta}_{13}}{2} & \cdots & \dfrac{\overline{\zeta}_{1(t-1)} + \overline{\zeta}_{1t}}{2} \\ A_2 & \dfrac{\overline{\zeta}_{21} + \overline{\zeta}_{22}}{2} & \dfrac{\overline{\zeta}_{22} + \overline{\zeta}_{23}}{2} & \cdots & \dfrac{\overline{\zeta}_{2(t-1)} + \overline{\zeta}_{2t}}{2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \dfrac{\overline{\zeta}_{m1} + \overline{\zeta}_{m2}}{2} & \dfrac{\overline{\zeta}_{m2} + \overline{\zeta}_{m3}}{2} & \cdots & \dfrac{\overline{\zeta}_{m(t-1)} + \overline{\zeta}_{mt}}{2} \end{matrix}. \quad (23)$$

Besides that, we must normalize the matrix $\overline{\Lambda}'$ again according to equation (21) and obtain the normalized matrix $\Lambda'' = [\zeta''_{ij}]_{m\times(t-1)}$.

Similarly, the total number of undesirable events will be increased step by step, and the increment will also be one at a time. It can be divided into two categories according to the total number of undesirable events which can be denoted as symbol t. The first category is that if t is an even number, then insert the average values of the column $t/2$ and the column $(t/2)+1$, and the specific formula is shown as equation (24); the second category is that if t is an odd number, then insert the average values of the column $(t-1)/2$ and the column $(t+1)/2$, and the specific formula is shown as equation (25). Certainly, the new matrix must also be normalized according to equation (21) and we can obtain the normalized matrix $\Lambda'' = [\zeta'''_{ij}]_{m\times(t+1)}$.

$$\Lambda'' = \begin{matrix} & U_1 & \dots & U_{t/2} & U' & U_{(t+1)/2} & \dots & U_t \\ A_1 & \overline{\zeta}_{11} & \dots & \overline{\zeta}_{1(t/2)} & \dfrac{\overline{\zeta}_{1(t/2)} + \overline{\zeta}_{1((t+1)/2)}}{2} & \overline{\zeta}_{1((t+1)/2)} & \dots & \overline{\zeta}_{1t} \\ A_2 & \overline{\zeta}_{21} & \dots & \overline{\zeta}_{2(t/2)} & \dfrac{\overline{\zeta}_{2(t/2)} + \overline{\zeta}_{2((t+1)/2)}}{2} & \overline{\zeta}_{2((t+1)/2)} & \dots & \overline{\zeta}_{2t} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \overline{\zeta}_{m1} & \dots & \overline{\zeta}_{m(t/2)} & \dfrac{\overline{\zeta}_{m(t/2)} + \overline{\zeta}_{m((t+1)/2)}}{2} & \overline{\zeta}_{m((t+1)/2)} & \dots & \overline{\zeta}_{mt} \end{matrix}, \quad (24)$$

$$\Lambda''' = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccccc} U_1 & \cdots & U_{(t-1)/2} & U' & U_{(t+1)/2} & \cdots & U_t \end{array} \\ \begin{pmatrix} \bar{\zeta}_{11} & \cdots & \bar{\zeta}_{1((t-1)/2)} & \dfrac{\bar{\zeta}_{1((t-1)/2)} + \bar{\zeta}_{1((t+1)/2)}}{2} & \bar{\zeta}_{1((t+1)/2)} & \cdots & \bar{\zeta}_{1t} \\ \bar{\zeta}_{21} & \cdots & \bar{\zeta}_{2((t-1)/2)} & \dfrac{\bar{\zeta}_{2((t-1)/2)} + \bar{\zeta}_{2((t+1)/2)}}{2} & \bar{\zeta}_{2((t+1)/2)} & \cdots & \bar{\zeta}_{2t} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{\zeta}_{m1} & \cdots & \bar{\zeta}_{m((t-1)/2)} & \dfrac{\bar{\zeta}_{m((t-1)/2)} + \bar{\zeta}_{m((t+1)/2)}}{2} & \bar{\zeta}_{m((t+1)/2)} & \cdots & \bar{\zeta}_{mt} \end{pmatrix} \end{array}, \tag{25}$$

$$p_{ij,l}' = \frac{\left(1/\left(1+\lambda_{ij,l}\right)\right), \quad \left(\overline{\zeta_{il}} + \lambda_{ij,l} \cdot x_{ij,l}\right)}{\sum_{l=1}^{\eta}\left(1/\left(1+\lambda_{ij,l}\right)\right), \quad \left(\overline{\zeta_{il}} + \lambda_{ij,l} \cdot x_{ij,l}\right)}, \quad i, j = 1, 2, \ldots, m. \tag{26}$$

The missing probability values will be calculated according to equation (26). The value of $\lambda_{ij,l}$ will be zero if the corresponding value of $x_{ij,l}$ is equal to zero; in contrast, the value of $\lambda_{ij,l}$ will be one if the corresponding value of $x_{ij,l}$ is not equal to zero.

Step 3: the emergency decision-making algorithm. We have already obtained the completed hesitant fuzzy probabilistic linguistic preference relation decision matrix after the above steps, and then we try to get the alternative ranking results according to the matrix. Let $W = (W_1, W_2, \ldots, W_m)$ be the priority values of the m alternatives, respectively, the results will rank the alternatives from large to small according to the value of W, and then we can obtain the most appropriate alternative.

We can construct the following model according to the perfect consistency of the decision matrix to obtain the solution of W. Equation (27) is the calculation method of the expected value for each element of the decision matrix, in which the symbol $r_{ij,l}$ denotes the evaluation value and the symbol $x_{ij,l}$ indicates the corresponding probability value. If every expected value satisfies equation (28), then the whole decision matrix will satisfy the perfect consistency. However, we find that the perfect consistency is hard to achieve, and we can only approach as close as possible to the perfect consistency which is also the main solution idea of the model.

$$E\left(L(x)_{ij}\right) = \sum_{l=1}^{\#L(x)_{ij}} r_{ij,l} \cdot x_{ij,l}, \tag{27}$$

$$E\left(L(x)_{ij}\right) = \log_{\theta} W_i - \log_{\theta} W_j + \tau, \tag{28}$$

$$\text{Model 1} \quad \begin{aligned} Z = & \min \sum_{i=1}^{m-1} \sum_{j=2, j>i}^{m} \left(d_{ij}^{+} + d_{ij}^{-}\right) \\ \text{s.t.} & \begin{cases} \log_{\theta} W_i - \log_{\theta} W_j + \tau - \sum_{l=1}^{\#L(x)_{ij}} r_{ij,l} \cdot x_{ij,l} - d_{ij}^{+} + d_{ij}^{-} = 0, \quad i, j = 1, 2, \ldots, m; i < j \\ \sum_{l=1}^{\#L(x)_{ij}} x_{ij,l} = 1, \quad x_{ij,l} \geq 0 \\ \sum_{i=1}^{m} W_i = 1, \quad W_i \geq 0. \end{cases} \end{aligned} \tag{29}$$

The symbol d_{ij}^{+} denotes the positive deviation, and similarly, the symbol d_{ij}^{-} denotes the negative deviation. The model is a linear optimization model and can be solved easily by the Lingo software which is a professional tool to solve such problems. We can quickly obtain the minimum sum of the deviations, all the deviation values, and the priority values W_i with the help of the Lingo software. The perfect consistency is almost impossible to achieve which is mentioned above; therefore, we further propose the definition of the acceptable consistency index which can be calculated as follows:

$$\text{CI} = \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=2, i<j}^{m} \left(d_{ij}^{+} + d_{ij}^{-}\right). \tag{30}$$

We set a threshold ε; if the inequality $\text{CI} \leq \varepsilon$ holds, then the decision matrix satisfies the acceptable consistency, and we can obtain the ranking result according to the values of W_i. However, if the inequality $\text{CI} \leq \varepsilon$ does not hold, meaning that the decision matrix does not satisfy the acceptable consistency requirement, it denotes that there may be some contradictions which cannot be ignored in the decision matrix, and we must therefore adjust part of data in the decision matrix. Definitely, the equation $\text{CI} = 0$ will hold for all the decision matrices when they satisfy the perfect consistency.

Step 4: the consistency repairing method of the decision matrix. Suppose the element of the decision matrix can be denoted as $h_{ij} = \{r_{ij,1}(x_{ij,1}), r_{ij,2}(x_{ij,2}), \ldots, r_{ij,l}(x_{ij,l})\}$, and the symbol $r_{ij,k}$ indicates the evaluation value and the evaluation will be better with the increase of k value. The symbol $x_{ij,k}$ indicates the corresponding probability value, and the equation $\sum_{k=1}^{l} x_{ij,k} = 1$ holds. We must adjust some values of the decision matrix until it satisfies the acceptable consistency requirement, which is because only the decision matrix which satisfies the consistency is reliable. Every element of the decision matrix is composed of the evaluation value and its corresponding probability value. Compared with the evaluation value, the corresponding probability value is more subjective, and therefore, we choose to adjust probability values until the decision matrix satisfies the acceptable consistency. The main flowchart of the algorithm is shown in Figure 2 and the specific algorithm is roughly described as follows.

Firstly, the maximum deviation value will be obtained according to the following equation .:

$$d^* = \max\left\{d_{ij}^+, d_{ij}^- | i = 1, 2, \ldots, m-1; j = 2, 3, \ldots, m; i < j\right\}. \quad (31)$$

Secondly, the algorithm will be divided into two categories according to the type of the maximum deviation value. The first category is that if the equation $d^* = d_{ij}^+$ holds, then the corresponding probability value will be modified by the equations $x_{ij,l}^* = x_{ij,l} + d^*$ and $x_{ij,1}^* = x_{ij,1} - d^*$ (in particular, when the inequality $x_{ij,l} + d^* \geq 1$ holds, then $x_{ij,l}^* = 1$; similarly, when the inequality $x_{ij,1} - d^* \leq 0$ holds, then $x_{ij,1}^* = 0$); the second category is that if the equation $d^* = d_{ij}^-$ holds, then the corresponding probability value will be modified by the equations $x_{ij,l}^* = x_{ij,l} - d^*$ and $x_{ij,1}^* = x_{ij,1} + d^*$ (in particular, when the inequality $x_{ij,l} - d^* \leq 0$ holds, then $x_{ij,l}^* = 0$; similarly, when the inequality $x_{ij,1} + d^* \geq 1$ holds, then $x_{ij,1}^* = 1$). Therefore, the repaired decision matrix will be obtained after the above steps.

Thirdly, we must verify the consistency of the repaired decision matrix again. If the acceptable consistency cannot be achieved, then we must repeat Step 4 until it satisfies the acceptable consistency requirement. When the acceptable consistency is achieved, then the most appropriate alternative will be obtained according to the solutions of model 1 subsequently.

4. An Emergency Optimization Decision-Making Instance

In this section, we will illustrate the specific execution steps through an emergency optimization decision-making instance under the background of COVID-19:

Step 1: construct the decision matrix for all the alternatives. Suppose there are four alternatives at present to prevent and control the development of COVID-19. We must compare these alternatives and find out the most appropriate alternative within the limited time, and then we should implement it immediately and observe the implementation effect continuously.

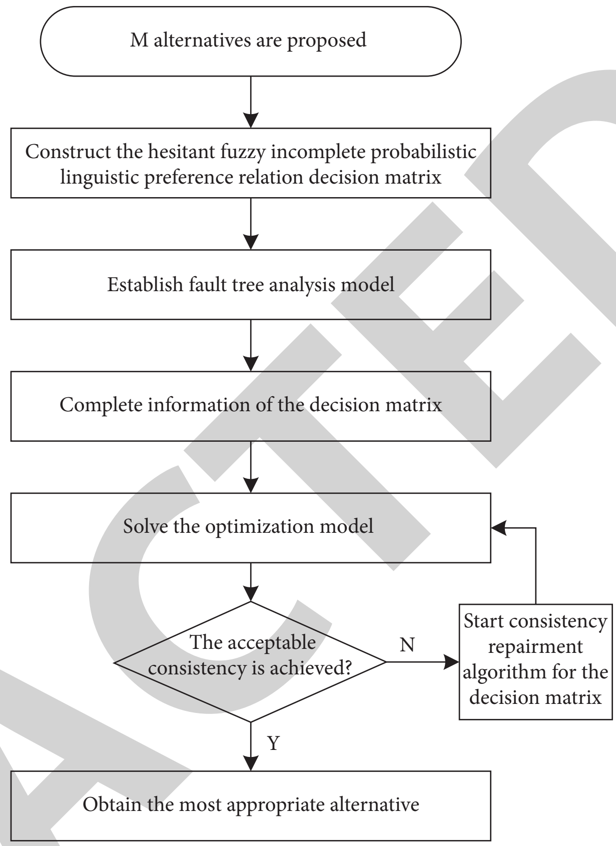

Figure 2: The main flowchart of the decision-making algorithm.

Suppose there are six evaluation levels in total which can be denoted as $\{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$, respectively, in which the symbol s_3 indicates that there is no difference between the two alternatives, the symbols $\{s_0, s_1, s_2\}$ indicate the first alternative is worse than the second one, and the symbol s_0 indicates the worst evaluation. In contrast, the symbols $\{s_4, s_5, s_6\}$ indicate the first alternative is better than the second one and the symbol s_6 indicates the best evaluation. The evaluation value of alternative A_i to A_j is the complement of the corresponding evaluation value of alternative A_j to A_i, which can be denoted as $h_{ij} = h_{ji}^c$ mathematically; all the evaluation values will be s_3 when the alternatives compare with themselves. Therefore, only the evaluation values in the upper triangle of the decision matrix must be evaluated by experts. The hesitant fuzzy incomplete probabilistic linguistic preference relations are shown in Table 1.

We find that there are often multiple values in a single evaluation value, such as $h_{12} = \{s_1(0.4), s_2(x), s_3(x)\}$, which is because of the hesitation in the expert's mind or the evaluation values are given, respectively, by several experts and they could not convince each other, so we keep all the evaluation values given by experts. The values in the brackets indicate the corresponding probability values of the evaluation values. Sometimes the values can be given definitely by experts; however, sometimes the experts fail to give these values within

Table 1: The hesitant fuzzy incomplete probabilistic linguistic preference relations.

	A_1	A_2	A_3	A_4
A_1	$\{s_3(1)\}$	$\{s_2(0.4), s_3(x), s_4(x)\}$	$\{s_2(x), s_4(x)\}$	$\{s_2(x), s_3(0.4), s_4(x)\}$
A_2	$\{s_2(x), s_3(x), s_4(0.4)\}$	$\{s_3(1)\}$	$\{s_2(x), s_4(x)\}$	$\{s_2(x), s_3(0.26), s_4(x), s_5(x)\}$
A_3	$\{s_2(x), s_4(x)\}$	$\{s_2(x), s_4(x)\}$	$\{s_3(1)\}$	$\{s_2(x), s_3(x), s_4(0.45)\}$
A_4	$\{s_2(x), s_3(0.4), s_4(x)\}$	$\{s_1(x), s_2(x), s_3(0.26), s_4(x)\}$	$\{s_2(0.45), s_3(x), s_4(x)\}$	$\{s_3(1)\}$

the limited time, and we can denote all of them as the symbol x. We find that it is difficult to obtain the ranking list of the alternatives just by simple observation of Table 1; therefore, the work must be processed by complex algorithms.

Step 2: obtain the missing probability values with the help of fault tree analysis. The development of COVID-19 often has potential logical rules, and the occurrences of different basic events will lead to different trends of the infectious diseases spread. For example, insufficient investigation of the infected person, the control of personnel circulation not being strict, and the infected person not being identified in time are all basic events. The fault tree analysis model can reveal the logical rules to a certain extent. Suppose the fault tree analysis model used in the instance is Figure 1 mentioned above. The formulas of the undesirable events shown as follows are obtained according to the fault tree analysis model:

$$F_{U_1} = (X_1 \oplus X_2) \otimes X_3, \tag{32}$$

$$F_{U_2} = (X_4 \otimes X_5) \oplus (X_6 \otimes X_7), \tag{33}$$

$$F_{U_3} = ((X_1 \oplus X_2) \otimes X_3) \otimes ((X_4 \otimes X_5) \oplus (X_6 \otimes X_7)). \tag{34}$$

The occurrence probability matrix of the basic events when implementing different alternatives can be obtained according to historical records and practice experiences. The matrix is shown as follows:

$$\Theta = [\rho_{ij}]_{4\times 7} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccccccc} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 \end{array} \\ \begin{pmatrix} 0.3 & 0.5 & 0.6 & 0.4 & 0.7 & 0.6 & 0.8 \\ 0.7 & 0.3 & 0.6 & 0.3 & 0.6 & 0.4 & 0.5 \\ 0.3 & 0.6 & 0.5 & 0.4 & 0.6 & 0.8 & 0.3 \\ 0.4 & 0.3 & 0.6 & 0.5 & 0.4 & 0.7 & 0.5 \end{pmatrix} \end{array}. \tag{35}$$

The probability matrix of the undesirable events when implementing different alternatives can be obtained according to formulas (32)–(35) and the matrix is shown as follows:

$$\Lambda = [\zeta_{ij}]_{4\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} U_1 & U_2 & U_3 \end{array} \\ \begin{pmatrix} 0.39 & 0.63 & 0.24 \\ 0.47 & 0.34 & 0.16 \\ 0.36 & 0.42 & 0.15 \\ 0.35 & 0.48 & 0.17 \end{pmatrix} \end{array}. \tag{36}$$

We normalize the matrix $\Lambda = [\zeta_{ij}]_{4\times 3}$ according to formula (21) and get the normalized probability matrix of the undesirable events $\overline{\Lambda} = [\overline{\zeta}_{ij}]_{4\times 3}$.

$$\overline{\Lambda} = [\overline{\zeta}_{ij}]_{4\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} U_1 & U_2 & U_3 \end{array} \\ \begin{pmatrix} 0.31 & 0.50 & 0.19 \\ 0.48 & 0.35 & 0.17 \\ 0.39 & 0.45 & 0.16 \\ 0.35 & 0.48 & 0.17 \end{pmatrix} \end{array}. \tag{37}$$

We can obtain the missing probability values according to the algorithm mentioned in Step 2 of the previous section, and the completed hesitant fuzzy probabilistic linguistic preference relations are shown in Table 2 after the above complement step.

It is a complex process to supplement the missing values; it can be divided into three categories according to the total number of items.

The first category likes the element $h_{12} = \{s_2(0.4), s_3(x), s_4(x)\}$, where the number of items is equal to the number of undesirable events, and so we can obtain the missing probability values directly according to formula (26).

The second category likes the element $h_{13} = \{s_2(x), s_4(x)\}$, where the number of items is less than the number of undesirable events, and so we must decrease the number of undesirable events gradually according to equation (23) and obtain the decreased matrix shown as equation (38). Subsequently, the normalized matrix shown as equation (39) will be obtained based on the decreased matrix.

$$\overline{\Lambda}' = [\overline{\zeta}'_{ij}]_{4\times 2} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cc} U_{1.5} & U_{2.5} \end{array} \\ \begin{pmatrix} 0.405 & 0.345 \\ 0.415 & 0.260 \\ 0.420 & 0.305 \\ 0.415 & 0.325 \end{pmatrix} \end{array}, \tag{38}$$

$$\overline{\Lambda}'' = [\overline{\zeta}''_{ij}]_{4\times 2} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cc} U_1 & U_2 \end{array} \\ \begin{pmatrix} 0.54 & 0.46 \\ 0.61 & 0.39 \\ 0.58 & 0.42 \\ 0.56 & 0.44 \end{pmatrix} \end{array}. \tag{39}$$

Table 2: The completed hesitant fuzzy probabilistic linguistic preference relations.

	A_1	A_2	A_3	A_4
A_1	$\{s_3(1)\}$	$\{s_2(0.34), s_3(0.48), s_4(0.18)\}$	$\{s_2(0.54), s_4(0.46)\}$	$\{s_2(0.33), s_3(0.47), s_4(0.20)\}$
A_2	$\{s_2(0.18), s_3(0.48), s_4(0.34)\}$	$\{s_3(1)\}$	$\{s_2(0.61), s_4(0.39)\}$	$\{s_2(0.35), s_3(0.28), s_4(0.25), s_5(0.12)\}$
A_3	$\{s_2(0.46), s_4(0.54)\}$	$\{s_2(0.39), s_4(0.61)\}$	$\{s_3(1)\}$	$\{s_2(0.34), s_3(0.39), s_4(0.27)\}$
A_4	$\{s_2(0.20), s_3(0.47), s_4(0.33)\}$	$\{s_1(0.12), s_2(0.25), s_3(0.28), s_4(0.35)\}$	$\{s_2(0.27), s_3(0.39), s_4(0.34)\}$	$\{s_3(1)\}$

The third category likes the element $h_{24} = \{s_2(x), s_3(0.26), s_4(x), s_5(x)\}$, where the number of items is more than the number of undesirable events, and so we must increase the number of undesirable events gradually according to formula (24) and obtain the increased matrix shown as equation (40). Subsequently, the normalized matrix shown as equation (41) will be obtained based on the increased matrix.

$$\overline{\Lambda}''' = [\overline{\zeta}'''_{ij}]_{4\times 4} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{cccc} U_1 & U_{1.5} & U_2 & U_3 \end{array} \\ \begin{pmatrix} 0.31 & 0.405 & 0.50 & 0.19 \\ 0.48 & 0.415 & 0.35 & 0.17 \\ 0.39 & 0.420 & 0.45 & 0.16 \\ 0.35 & 0.415 & 0.48 & 0.17 \end{pmatrix} \end{array}, \quad (40)$$

$$\widetilde{\Lambda}''' = [\widetilde{\zeta}'''_{ij}]_{4\times 4} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{cccc} U_1 & U_2 & U_3 & U_4 \end{array} \\ \begin{pmatrix} 0.22 & 0.29 & 0.36 & 0.13 \\ 0.34 & 0.29 & 0.25 & 0.12 \\ 0.27 & 0.30 & 0.32 & 0.11 \\ 0.25 & 0.29 & 0.34 & 0.12 \end{pmatrix} \end{array}. \quad (41)$$

Step 3: rank the alternatives by the solutions of model 1. First of all, we can construct the following model according to model 1 and the completed decision matrix. The value of the symbol τ should be three because there are seven evaluation levels in total which are from 0 to 6 and the third level is the moderate evaluation level. We set the acceptable consistency index at CI = 0.05.

Model 2

$$\min Z = d_{12}^+ + d_{12}^- + d_{13}^+ + d_{13}^- + d_{14}^+ + d_{14}^- + d_{23}^+ + d_{23}^- + d_{24}^+ + d_{24}^- + d_{34}^+ + d_{34}^- \begin{cases} \text{In}W_1 - \text{In}W_2 + 3 - 2.84 - d_{12}^+ + d_{12}^- = 0, \\ \text{In}W_1 - \text{In}W_3 + 3 - 2.92 - d_{13}^+ + d_{13}^- = 0, \\ \text{In}W_1 - \text{In}W_4 + 3 - 2.87 - d_{14}^+ + d_{14}^- = 0, \\ \text{In}W_2 - \text{In}W_3 + 3 - 2.78 - d_{23}^+ + d_{23}^- = 0, \\ \text{In}W_2 - \text{In}W_4 + 3 - 3.14 - d_{24}^+ + d_{24}^- = 0, \\ \text{In}W_3 - \text{In}W_4 + 3 - 2.93 - d_{34}^+ + d_{34}^- = 0. \end{cases} \quad (42)$$

We solve the model by the Lingo software and the solutions are shown as follows:

$$\begin{aligned} &Z = 0.43, \\ &d_{23}^+ = 0.3, \\ &d_{24}^- = 0.11, \\ &d_{34}^+ = 0.02, \\ &d_{12}^+ = d_{12}^- = d_{13}^+ = d_{13}^- = d_{14}^+ = d_{14}^- = d_{23}^- = d_{24}^+ = d_{34}^- = 0, \\ &W = \{W_1, W_2, W_3, W_4\} = \{0.228, 0.267, 0.246, 0.259\}. \end{aligned} \quad (43)$$

We can obtain the acceptable consistency index CI = 0.072 according to formula (30). We must start the consistency repairing algorithm immediately because the value of CI is greater than the threshold of 0.05, which means there are some contradictions that cannot be ignored in the decision matrix.

We will calculate the maximum deviation according to the consistency adjustment algorithm mentioned above:

$$d_{ij}^* = \max\{d_{23}^+, d_{24}^-, d_{34}^+\} = \{0.3, 0.11, 0.02\} = d_{23}^+ = 0.3. \quad (44)$$

We find that the maximum deviation is d_{23}^+; therefore, we must adjust the probabilities of the element $h_{23} = \{s_2(0.61), s_4(0.39)\}$ in the decision matrix. The new probability of s_4 will be 0.39 + 0.3 = 0.69; meanwhile, the new probability of s_2 will be 0.61 − 0.3 = 0.31. So the element can be denoted as $h_{23}' = \{s_2(0.31), s_4(0.69)\}$. Certainly, the element $h_{32}' = h_{23}^{c'}$ will be also adjusted accordingly. The revised preference relations are shown in Table 3.

Table 3: The revised hesitant fuzzy probabilistic linguistic preference relations.

	A_1	A_2	A_3	A_4
A_1	$\{s_3(1)\}$	$\{s_2(0.34), s_3(0.48), s_4(0.18)\}$	$\{s_2(0.54), s_4(0.46)\}$	$\{s_2(0.33), s_3(0.47), s_4(0.20)\}$
A_2	$\{s_2(0.18), s_3(0.48), s_4(0.34)\}$	$\{s_3(1)\}$	$\{s_2(0.31), s_4(0.69)\}$	$\{s_2(0.35), s_3(0.28), s_4(0.25), s_5(0.12)\}$
A_3	$\{s_2(0.46), s_4(0.54)\}$	$\{s_2(0.69), s_4(0.31)\}$	$\{s_3(1)\}$	$\{s_2(0.34), s_3(0.39), s_4(0.27)\}$
A_4	$\{s_2(0.20), s_3(0.47), s_4(0.33)\}$	$\{s_1(0.12), s_2(0.25), s_3(0.28), s_4(0.35)\}$	$\{s_2(0.27), s_3(0.39), s_4(0.34)\}$	$\{s_3(1)\}$

We can rebuild the model which is shown as follows according to the data in Table 3. The model which consists of linear equations can be solved easily with the help of the Lingo software.

Model 3

$$\min Z = d_{12}^{+} + d_{12}^{-} + d_{13}^{+} + d_{13}^{-} + d_{14}^{+} + d_{14}^{-} + d_{23}^{+} + d_{23}^{-} + d_{24}^{+} + d_{24}^{-} + d_{34}^{+} + d_{34}^{-} \begin{cases} \text{In}W_1 - \text{In}W_2 + 3 - 2.84 - d_{12}^{+} + d_{12}^{-} = 0 \\ \text{In}W_1 - \text{In}W_3 + 3 - 2.92 - d_{13}^{+} + d_{13}^{-} = 0 \\ \text{In}W_1 - \text{In}W_4 + 3 - 2.87 - d_{14}^{+} + d_{14}^{-} = 0 \\ \text{In}W_2 - \text{In}W_3 + 3 - 3.38 - d_{23}^{+} + d_{23}^{-} = 0 \\ \text{In}W_2 - \text{In}W_4 + 3 - 3.14 - d_{24}^{+} + d_{24}^{-} = 0 \\ \text{In}W_3 - \text{In}W_4 + 3 - 2.93 - d_{34}^{+} + d_{34}^{-} = 0 \end{cases}, \tag{45}$$

$$\begin{aligned} Z &= 0.3, \\ \text{CI} &= 0.05, \\ d_{12}^{-} &= 0.11, \\ d_{13}^{+} &= 0.02, \\ d_{23}^{-} &= 0.17, \\ d_{12}^{+} &= d_{13}^{-} = d_{14}^{+} = d_{14}^{-} = d_{23}^{+} = d_{24}^{+} = d_{24}^{-} = d_{34}^{+} = d_{34}^{-} = 0, \\ W &= \{W_1, W_2, W_3, W_4\} = \{0.222, 0.290, 0.235, 0.253\}. \end{aligned} \tag{46}$$

The solutions can be obtained which are shown above, and the acceptable consistency CI = 0.05 meets the consistency threshold requirement. Therefore, the solutions are valid and the ranking of the alternatives is $W_2 > W_4 > W_3 > W_1$. Alternative A_2 is the most appropriate one compared with other alternatives and we should implement the alternative immediately.

5. Comparisons with Other Typical Algorithms

We will compare the algorithm proposed in this paper with other typical algorithms proposed by other scholars in this section.

5.1. Comparison with the Hesitant Fuzzy Information Aggregation Algorithm. The hesitant fuzzy preference relation decision matrix shown in Table 4 will be obtained immediately by transforming from Table 1. It only considers the information of the evaluation values, and the information of the corresponding probability values will be ignored.

We can construct model 4 and it can be solved by the Lingo software, and we consider the probabilities of various evaluation values are equal in such case. The model and solutions are shown as follows:

Table 4: The hesitant fuzzy preference relations.

	A_1	A_2	A_3	A_4
A_1	$\{s_3\}$	$\{s_2, s_3, s_4\}$	$\{s_2, s_4\}$	$\{s_2, s_3, s_4\}$
A_2	$\{s_2, s_3, s_4\}$	$\{s_3\}$	$\{s_2, s_4\}$	$\{s_2, s_3, s_4, s_5\}$
A_3	$\{s_2, s_4\}$	$\{s_2, s_4\}$	$\{s_3\}$	$\{s_2, s_3, s_4\}$
A_4	$\{s_2, s_3, s_4\}$	$\{s_1, s_2, s_3, s_4\}$	$\{s_2, s_3, s_4\}$	$\{s_3\}$

Model 4

$$\min Z = d_{12}^{+} + d_{12}^{-} + d_{13}^{+} + d_{13}^{-} + d_{14}^{+} + d_{14}^{-} + d_{23}^{+} + d_{23}^{-} + d_{24}^{+} + d_{24}^{-} + d_{34}^{+} + d_{34}^{-} \begin{cases} \mathrm{In}W_1 - \mathrm{In}W_2 + 3 - 3 - d_{12}^{+} + d_{12}^{-} = 0 \\ \mathrm{In}W_1 - \mathrm{In}W_3 + 3 - 3 - d_{13}^{+} + d_{13}^{-} = 0 \\ \mathrm{In}W_1 - \mathrm{In}W_4 + 3 - 3 - d_{14}^{+} + d_{14}^{-} = 0 \\ \mathrm{In}W_2 - \mathrm{In}W_3 + 3 - 3 - d_{23}^{+} + d_{23}^{-} = 0 \\ \mathrm{In}W_2 - \mathrm{In}W_4 + 3 - 3.5 - d_{24}^{+} + d_{24}^{-} = 0 \\ \mathrm{In}W_3 - \mathrm{In}W_4 + 3 - 3 - d_{34}^{+} + d_{34}^{-} = 0 \end{cases}, \tag{47}$$

$$\begin{aligned} &Z = 0.5, \\ &\mathrm{CI} = 0.083, \\ &d_{24}^{-} = 0.5, \\ &d_{12}^{+} = d_{12}^{-} = d_{13}^{+} = d_{13}^{-} = d_{14}^{+} = d_{14}^{-} = d_{23}^{+} = d_{23}^{-} = d_{24}^{+} = d_{34}^{+} = d_{34}^{-} = 0, \\ &W = \{W_1, W_2, W_3, W_4\} = \{0.25, 0.25, 0.25, 0.25\}. \end{aligned} \tag{48}$$

The algorithm has several shortcomings identified through the analysis of the results. Firstly, it cannot rank the alternatives because all the alternatives are equal and can be denoted as $\{A_1 \sim A_2 \sim A_3 \sim A_4\}$; secondly, the consistency does not meet the requirements; thirdly, the algorithm cannot adjust the consistency; fourthly, treating the probabilities of various evaluation values equally leads to the loss of information.

5.2. Comparison with the Additive Consistency Traditional Algorithm. The additive consistency traditional approach is one of the best algorithms to solve the decision-making problem with incomplete information. The algorithm idea can be mathematically described by model 5, and the specific model of the instance discussed in this paper can be constructed which is shown as model 6. The execution result of the traditional algorithm is shown in Figure 3.

Model 5

$$\begin{aligned} &\min \quad Y = \sum_{i=1}^{m-1} \sum_{j=2, j>i}^{m} \left(d_{ij}^{+} + d_{ij}^{-}\right) \\ &\text{s.t.} \quad \begin{aligned} &\sum_{l=1}^{\#L(x)_{ij}} r_{ij,l} \cdot x_{ij,l} - \left(W_i - W_j\right) - d_{ij}^{+} + d_{ij}^{-} = 0, \quad i, j = 1, 2, \ldots, m; i < j \\ &\sum_{l=1}^{\#L(x)_{ij}} x_{ij,l} = 1, \quad x_{ij,l} \geq 0 \sum_{i=1}^{m} W_i = 1, \quad W_i \geq 0, \end{aligned} \end{aligned} \tag{49}$$

Model 6

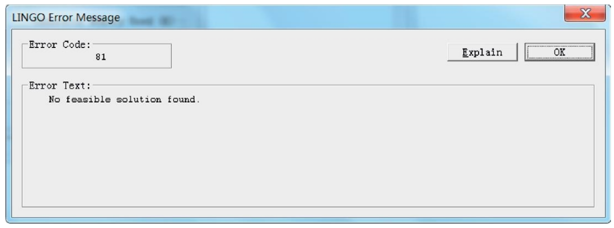

Figure 3: The execution result of the traditional algorithm.

$$\min Y = d_{12}^{+} + d_{12}^{-} + d_{13}^{+} + d_{13}^{-} + d_{14}^{+} + d_{14}^{-} + d_{23}^{+} + d_{23}^{-} + d_{24}^{+} + d_{24}^{-} + d_{34}^{+} + d_{34}^{-} \begin{cases} 2*0.4 + 3*x_{12,2} + 4*\left(1 - 0.4 - x_{12,2}\right) + W_1 - W_2 - d_{12}^{+} + d_{12}^{-} = 0 \\ 2*x_{13,1} + 4*\left(1 - x_{13,1}\right) + W_1 - W_3 - d_{13}^{+} + d_{13}^{-} = 0 \\ 2*x_{14,1} + 3*0.4 + 4*\left(1 - 0.4 - x_{14,1}\right) + W_1 - W_4 - d_{14}^{+} + d_{14}^{-} = 0 \\ 2*x_{23,1} + 4*\left(1 - x_{23,1}\right) + W_2 - W_3 - d_{23}^{+} + d_{23}^{-} = 0 \\ 2*x_{24,1} + 3*0.26 + 4*x_{24,3} + 5*\left(1 - 0.26 - x_{24,1} - x_{24,3}\right) + W_2 - W_4 - d_{24}^{+} + d_{24}^{-} = 0 \\ 2*x_{34,1} + 3*\left(1 - 0.45 - x_{34,1}\right) + 4*0.45 + W_3 - W_4 - d_{34}^{+} + d_{34}^{-} = 0 \end{cases} \quad (50)$$

We find that the linear optimization model has no feasible solution. Therefore, the additive consistency traditional approach cannot be applied to the case discussed in this paper. The main reason is that there are too many unknown probabilities in the instance, and the additive consistency traditional approach is only suitable for solving the model with a few unknown parameters. This further verifies the advanced nature of the algorithm proposed in this paper.

5.3. Comparisons with the Existed Outstanding Research Achievements. Several scholars have done research in this field and have made some progress. One of the typical representatives is Gao et al. [33], who have performed research on the problem and proposed a complement algorithm for the missing information. We find that the algorithm proposed in this paper is superior mainly in two aspects by carefully comparing the two algorithms. Firstly, there is a constraint which is that the item number of each evaluation value must be equal to the number of undesirable events in their algorithm; otherwise, the complement algorithm will fail. However, the algorithm proposed in our paper overcomes the problem. The instance discussed in this paper cannot be solved by the algorithm proposed by Gao et al. The second aspect is that the algorithm makes the consistency meet requirement by adjusting the evaluation values in the algorithm proposed by Gao et al.; however, we adjust the probability values instead of the evaluation values, meaning the results of the experiments show that our method is more efficient and objective. This is because the evaluation values are given definitely by experts and they cannot be changed arbitrarily; however, some of the corresponding probability values are unknown and can be given new values.

6. Conclusions

Measures must be taken in time to prevent and control infectious diseases, and the core of the work is to take the appropriate measures timely. In essence, this work belongs to the category of optimization decision-making problem. There are two main constraints in the problem which are limited time and incomplete information.

We adopt the data recording method of the hesitant fuzzy incomplete probabilistic linguistic preference relation to keep all the possible values of the pairwise comparisons among alternatives. This method has the following advantages; the method can keep multiple values for each element which can simulate the hesitations in the decision process and each evaluation value corresponds to a probability value which can retain information details as much as possible. Furthermore, it is worth noting that this method allows the existence of unknown probability values.

The fault tree analysis method which can deduce the development process of the infectious disease is introduced subsequently and it can complete the missing values. The method considers not only the objective historical data but also the subjective evaluation data.

The definition of the consistency index is also introduced, which can measure whether there are contradictions in the decision-making process. A consistency algorithm is proposed subsequently which can effectively improve consistency, and this is also one of the innovations of this paper. In the end, we can obtain the ranking result of alternatives by using the information aggregation algorithm.

This paper has also compared the algorithm proposed in this paper with typical research achievements in this field from three aspects. We find that the algorithm proposed in this paper has certain innovations through the system simulation and theoretical analyses. Although we find that some typical algorithms can not solve the problem proposed in this paper, however, we have to admit that the algorithm proposed in this paper is inspired by their algorithms.

As the development of infectious diseases is dynamic, group dynamic decision algorithms will be the focus and difficulty of future work for our research group.

Data Availability

The goal of this paper is more inclined to propose an intelligent optimization algorithm, and the data used in this paper are assumed according to the actual decision-making situations.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by "the six outstanding and one top-notch" outstanding talent training innovation project of Anhui University of Finance and Economics, Anhui Province (aclzy2020010 and 2020zyrc005).

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