Research Article

New Rough Approximations Based on E-Neighborhoods

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This paper puts forward some rough approximations which are motivated from topology. Given a subset \( R \subseteq U \times U \), we can use 8 types of \( E \)-neighborhoods to construct approximations of an arbitrary \( X \subseteq U \) on the one hand. On the other hand, we can also construct approximations relying on a topology which is induced by an \( E \)-neighborhood. Properties of these approximations and relationships between them are studied. For convenience of use, we also give some useful and easy-to-understand examples and make a comparison between our approximations and those in the published literature.

1. Introduction and Preliminaries

The problem of imperfect knowledge became a crucial issue for computer scientists, especially in the area of information system and artificial intelligence [1, 2]. There are various approaches to manipulate and understand imperfect knowledge, among which is rough set theory. Rough set was proposed by Pawlak [3, 4] in 1982 which has been generalized in many ways [5–13]. What we are concerned about are those methods whose ideas are motivated from topology, for example, methods constructing the lower and upper approximations by using different kinds of neighborhoods, such as right and left neighborhoods [10, 14], minimal right neighborhoods [15], and intersection and union neighborhoods (see [16, 17]). In fact, a combination of rough set theory and topological theory became the main goal of many studies (see [18–25]).

Now, we recall some basic properties and results of rough set theory, particularly those related to some types of neighborhood systems.

**Definition 1** (see [3, 5, 16])

1. A subset \( R \subseteq U \times U \) (also called a binary relation on \( U \)) is said to be an equivalence relation if it is reflexive (i.e., \((v, v) \in R \) for each \( v \in U \)), symmetric (i.e., \((u, v) \in R \) if \((v, u) \in R \) ), and transitive (i.e., \((u, w) \in R \) whenever \((u, v) \in R \) and \((v, w) \in R \)). It is said to be a preorder (or quasi-order) if it is reflexive and transitive. It is said to be a partial order if it is an antisymmetric (i.e., \( u = v \) whenever \((u, v) \in R \) and...
Proposition 1 (see [3, 4] for a special case). The lower approximations and the upper approximations have the following properties: \((E, F) \cup \bigcup_{i \in I} E_i \subseteq 2^U \), the power set of \(U \).

1. \( R(\emptyset) = R(\emptyset) = \emptyset \) and \( R(U) = R(U) = U \).
2. \( R(E) \subseteq E \subseteq R(E) \) and \( R(E) = [R(E')]^T \).
3. \( R(\cup_{i \in I} E_i) = \cup_{i \in I} R(E_i) \) and \( R(\bigcap_{i \in I} E_i) = \bigcap_{i \in I} R(E_i) \). Particularly, \( R(E) \subseteq R(F) \) and \( R(E) \subseteq R(F) \) if \( E \subseteq F \).
4. \( R[R(E)] = R(E) \) and \( R[R(E)] = R(E) \).

Definition 2. A subset \( \mathcal{J} \subseteq 2^U \) is called a topology on \( U \) (and \( (U, \mathcal{J}) \) is called a topological space) if it is closed under union and finite intersection. A topology satisfying that every open set is also closed is called a clopen topology.

In this section, we introduce the notions of \( E \)-neighborhoods using \( j \)-neighborhoods and study their properties.

Definition 5. Let \( R \subseteq U^2 \). The \( E \)-neighborhoods are defined as follows:

1. \( E_r(x) = \{ y \in U : N_r(x) \cap N_r(y) \neq \emptyset \} \)
2. \( E_j(x) = \{ y \in U : N_j(x) \cap N_j(y) \neq \emptyset \} \)
3. \( E_J(x) = \{ y \in U : N_J(x) \cap N_J(y) \neq \emptyset \} \)
4. \( E_{\omega}(x) = \{ y \in U : N_{\omega}(x) \cap N_{\omega}(y) \neq \emptyset \} \)

We give the following example to illustrate how we calculate different types of neighborhoods. Also, we will benefit from this example to clarify some obtained results.

Example 1. Let \( U = \{ v, w, x, y \} \) and \( R = \{ (v, v), (y, y), (v, x), (v, y), (y, w), (w, y) \} \). Then, the \( j \)-neighborhoods and \( E \)-neighborhoods of a point are as in Table 1.

Theorem 1. \( E \)-neighborhoods have the following properties:

1. \( E_r(v) \subseteq E_i(v) \) for each \( i \in I \)
2. \( E_{\omega}(v) \subseteq E_r(v) \) for each \( r \in R \)
3. \( v \in E_j(v)(j \in J) \)
4. \( E_r(v) \subseteq E_j(v)(j \in J) \)
5. \( E_j(v) \subseteq E_{\omega}(v) \) for each \( j \in J \)
6. \( v \in E_{\omega}(v)(j \in J) \)
7. \( v \in E_{\omega}(v)(j \in J) \)
8. \( v \in E_{\omega}(v)(j \in J) \)
9. \( v \in E_{\omega}(v)(j \in J) \)
10. \( v \in E_{\omega}(v)(j \in J) \)
Proof. (3) Obviously, \( v \in E_i(x) \Leftrightarrow N_j(v) \cap N_j(x) \neq \emptyset \Leftrightarrow x \in E_i(x) \) for each \( j \in \{ l, i, \langle r, \rangle, \langle j \rangle \} \). Then, \( v \in E_i(x) \Leftrightarrow x \in E_i(x) \) for each \( j \in \{ l, u, \langle i, i \rangle, \langle u \rangle \} \).

(4) Step 1: since \( R \) is reflexive, \( \cap_{x \in N_j(x)} N_j(x) \subseteq N_j(v) \) and \( \cap_{x \in N_j(x)} N_j(x) \subseteq N_j(v) \). This implies that \( E_i(v) \subseteq E_i(v) \) and \( E_i(v) \subseteq E_i(v) \). Consequently, \( E_i(v) \subseteq E_i(v) \) and \( E_i(v) \subseteq E_i(v) \).

Step 2: let \( x \in N_j(v) \). By reflexivity of \( R \), we have \( \{ x \} \subseteq N_j(x) \cap N_j(x) \). Therefore, \( x \in E_i(x) \); thus, \( N_j(v) \subseteq E_i(x) \). Also, let \( x \in P_j(v) \). Then, \( N_j(x) \subseteq N_j(v) \) since \( R \) is reflexive, \( N_j(x) \cap N_j(x) \neq \emptyset \). Therefore, \( x \in E_i(x) \); thus, \( P_j(v) \subseteq E_i(x) \). Hence, we obtain the desired result.

(5) Since \( R \) is symmetric, \( N_j(v) = N_j(v) \). Therefore, \( N_j(v) \cap N_j(z) \neq \emptyset \) \( \Leftrightarrow \cap_{x \in N_j(x)} N_j(x) \neq N_j(v) \). Thus, \( E_i(v) = E_i(v) \). Consequently, \( E_i(v) = E_i(v) = E_i(v) = E_i(v) \). Similarly, \( E_i(v) = E_i(v) = E_i(v) = E_i(v) = E_i(v) \).

(6) Let \( a \in N_j(v) \). Then, \( vRz \). For each \( N_j(x) \) containing \( v \), we have \( xRz \). Since \( R \) is transitive, \( xRz \). Therefore, \( a \in N_j(x) \); thus, \( a \in \cap_{x \in N_j(x)} N_j(x) = N_j(v) \). Hence, \( N_j(v) \subseteq N_j(v) \); consequently, \( E_i(v) \subseteq E_i(v) \) and \( E_i(v) \subseteq E_i(v) \). This implies that \( E_i(v) \subseteq E_i(v) \) and \( E_i(v) \subseteq E_i(v) \).

(7) It follows from (4) and the fact \( v \in E_i(x) \Leftrightarrow N_j(v) \cap N_j(x) \neq \emptyset \Rightarrow x \in E_i(v) \) (j \( \in \{ l, i, \langle r, \rangle, \langle j \rangle \} \}).

(8) Step 1: we prove the case \( j = r \). Let \( x \in E_i(v) \). Then, \( N_r(x) \cap N_r(v) \neq \emptyset \), i.e., there exists \( z \in N_r(x) \cap N_r(v) \), \( xRz \), and \( vRz \). Since \( R \) is symmetric and transitive, \( vRz \). Therefore, \( x \in N_r(v) \); thus, \( E_i(v) \subseteq E_i(v) \).

Step 2: since \( R \) is symmetric, \( E_i(v) = E_i(v) = E_i(v) \) by (5). We only prove the case \( j = l \). Let \( v \in E_i(w) \). Then, \( N_j(v) \cap N_j(w) \neq \emptyset \); therefore, there is \( a \in U \) such that \( aRv \) and \( bRw \). Now, let \( v \in E_i(v) \). Then, \( N_j(x) \cap N_j(w) \neq \emptyset \), i.e., there exists a \( b \in U \) such that \( bRx \) and \( bRw \). Note that \( aRb \); this means that \( aRx \); consequently, \( a \in N_j(x) \); also, \( a \in N_j(w) \); thus, \( N_j(x) \cap N_j(w) \neq \emptyset \). Hence, \( x \in E_i(w) \), as required.

(10) Step 1: by (4) and (8), \( E_i(v) = N_j(v) \) and \( P_j(v) \subseteq E_j(v) \). It remains to prove \( E_i(v) \subseteq P_j(v) \). Since \( R \) is symmetric, \( E_i(v) = E_i(v) = E_i(v) \) by (5). Let \( x \in E_i(v) \). Then, \( N_j(x) \cap N_j(x) \neq \emptyset \). Since \( R \) is equivalence, \( N_j(x) = N_j(v) \). Therefore, \( x \in P_j(v) \); consequently, \( E_i(v) \subseteq P_j(v) \). Thus, \( P_j(v) = E_i(v) \) is proved.

Step 2: we only prove the case \( j = l \). Let \( v \in E_i(w) \). Then, \( N_j(v) \cap N_j(w) \neq \emptyset \). Since \( R \) is equivalence, \( N_j(v) = N_j(w) \). Therefore, \( E_i(v) = E_i(w) \). Conversely, \( E_i(v) = E_i(w) \). Since \( R \) is reflexive, \( v \in N_j(v) \). Hence, \( v \in E_i(w) \).

3. Rough Approximations Using \( E \)-Neighborhoods Directly

Let \((U, R, \lambda_j)\) be \( j \)-NS and \( \mathcal{R}_j = \{ E_j(v) : v \in U \} (j \in J) \). We devote this section to formulate the following concepts using \( E \)-neighborhoods directly: \( R \)-lower approximation, \( R \)-upper approximation, \( R \)-boundary region, \( R \)-positive region, \( R \)-negative region, and \( R \)-accuracy measure of a subset \( X \). We will also illustrate the relationships between them and reveal the main properties with the help of examples.

Definition 6. Let \( X \subseteq U \) and \( j \in J \). Then, \( R_j - \text{accuracy measure of} \ X \) is \( R_j - \text{accuracy measure of} \ X \) (if \( 0 < R_j - \text{accuracy measure of} \ X \) and \( 0 < R_j - \text{accuracy measure of} \ X \)).

Theorem 2. \( R \)-approximations have the following properties (\( R \subseteq U, \{ E_i, F_j \} \cup \{ E_i \}_{i \in \{ 1 \}} \subseteq 2^U, j \in J \)):

1. \( R_j - \text{upper approximation} \) is \( R_j - \text{upper approximation} \).
2. \( R_j - \text{lower approximation} \) is \( R_j - \text{lower approximation} \).
 Complexity

4 Complexity

Rough Approximations Induced byhoods indirectly in this section. Let particular, \( R_j(E) \subseteq R_j(F) \) and \( R_j(E) \subseteq R_j(F) \) if \( E \subseteq F \).

(4) \( R_j(E) = [R_j^-(E)^+] \) and \( R_j(E) = [R_j^+(E)^-] \).

(5) In general, \( R_j[R_j^-(E)] \neq R_j(E) \) and \( R_j[R_j^+(E)] \neq R_j(E) \).

Proof. We only prove (3) and (4).

(3) Step 1: obviously, \( R_j(\cap_i E_i) \subseteq (\cap_i R_j^-(E_i)) \). Conversely, for each \( x \in R_j^-(\cap_i E_i) \) (i.e., \( x \in R_j^-(E_i) \) for each \( i \in I \)), we have \( E_i(x) \subseteq E_i \), and thus, \( E_i(x) \subseteq \cap_i E_i \). Hence, \( x \in R_j(\cap_i E_i) \), as required.

Step 2: obviously, \( \cup_i R_j^+(E_i) \subseteq R_j^+(\cup_i E_i) \). Conversely, for each \( x \in R_j^+(\cup_i E_i) \), we have \( E_i(x) \cap (\cup_i E_i) \neq \emptyset \), i.e., \( E_i(x) \cap E_i \neq \emptyset \) for some \( i_0 \in I \). Thus, \( x \in R_j^+(\cup_i E_i) \), as required.

(4) \( x \in R_j^+(E) \) or \( E(x) \subseteq E(x) \). From Table 2 (where \( Y = \{v, w, y\} \) and \( Z = \{v, x, y\} \)). From Table 2, we can see the approximations in this paper and those in [6] (resp., [10]) are incomparable.

4. Rough Approximations Induced by E-Topologies

We will construct rough approximations using E-neighborhoods indirectly in this section. Let \( (U, R, \lambda) \) be \( j \)-NS \((j \in J)\).

We first employ \( E_i \)-neighborhoods to generate a topology \( F_{R,j} \) (called an \( E_i \)-topology) and then call the interior \( F_{R,j}(X) \) and the closure \( F_{R,j}^+(X) \) of a subset \( X \subseteq U \) the \( F_{R,j} \)-lower approximation and \( F_{R,j}^+ \)-upper approximation of \( X \), respectively. These kinds of approximations are compared with those in Section 3.

Theorem 3

(1) \( F_{R,j} = \{A \in 2^U; E_j(x) \subseteq A (\forall x \in E_j)\} \) is a topology on \( U \) satisfying \( U - A \subseteq F_{R,j} \) whenever \( A \in F_{R,j} \) \((j \in J)\).

(2) Both \( F_{R,j}(X) \subseteq F_{R,j}(X) \) and \( F_{R,j}(X) \subseteq F_{R,j}(X) \) are topologies on \( U \) \((j \in J)\).

(3) \( F_{R,j} \subseteq F_{R,j} \subseteq F_{R,j} \subseteq F_{R,j} \subseteq F_{R,j} \).

(4) \( F_{R,j}(X) \subseteq F_{R,j}(X) \subseteq F_{R,j}(X) \subseteq F_{R,j}(X) \subseteq F_{R,j}(X) \).

(5) If \( R \) is reflexive, then \( F_{R,j}(X) \subseteq F_{R,j} \) \((j \in J)\).

(6) If \( R \) is serial, then \( F_{R,j}(X) \subseteq F_{R,j} \) \((j \in \{1, 2, 3, 4\})\).

(7) If \( R \) is a equivalence relation, then \( F_{R,j} \) is constant for all \( j \in J \).

Proof. We only prove (1). Obviously, \( F_{R,j} \) is a topology on \( U \). For each \( A \in F_{R,j} \), and each \( x \in U - A \), we need to prove \( E_j(x) \subseteq U - A \). Without loss of generality, we assume \( E_j(x) \neq \emptyset \). Suppose \( v \in E_j(x) \). Then, \( x \in E_j(v) \) (by Theorem (3) and \( E_j(v) \subseteq A \) (as \( A \in F_{R,j} \)), and thus, \( x \in A \). This is a contradiction. Therefore, \( E_j(x) \cap A = \emptyset \), i.e., \( E_j(x) \subseteq U - A \).

Definition 7. Let \( X \subseteq U \) and \( j \in J \). Then, \( F_{R,j}(X) = X^0 \) (the interior of \( X \) in \( (U, F_{R,j}) \)) and \( F_{R,j}^+(X) = X^\ominus \) (the closure of \( X \) in \( (U, F_{R,j}) \)) are called the \( F_{R,j} \)-lower approximation and \( F_{R,j}^+ \)-upper approximation, respectively; \( F_{R,j}(X) - F_{R,j}^+(X) \), \( F_{R,j}^-(X) \), and \( F_{R,j}^- \) are called \( F_{R,j} \)-boundary, \( F_{R,j}^+ \)-positive, and \( F_{R,j}^- \)-negative regions of \( X \), respectively; \( M_{F,j}(X) = \|F_{R,j}(X)\|/\|F_{R,j}^+(X)\| \) is called the \( F_{R,j} \)-accuracy measure of \( X \) (if \( 0 < \|F_{R,j}(X)\| < \infty \)).

The relation between \( M_{F,j}(X) \) and \( M_{F,j}(X) \) \((j \in J)\) is given by the following.

Theorem 4. \( M_{F,j}(X) \leq M_{F,j}(X) \) for each \( j \in J \) (if \( 0 < \|F_{R,j}(X)\| < \infty \)).

Proof

(i) Step 1: for each \( z \in F_{R,j}^+(X) \), we have \( z \in F_{R,j}^+(X) \) (because \( F_{R,j}^+(X) = X^0 \) and \( E_j(z) \subseteq X^0 \subseteq X \), and thus, \( F_{R,j}^+(X) \subseteq F_{R,j}^-(X) \cap X \), which implies
\[
\|F_{R,j}(X)\| \leq \|F_{R,j}^+(X)\| \cap X \.
\] (1)

(ii) Step 2: let \( z \in F_{R,j}^+(X) \cup X \). If \( z \in X \), then \( z \in F_{R,j}^+(X) \). If \( z \notin X \), then \( z \in F_{R,j}^+(X) \), and thus, \( E_j(z) \cap X \neq \emptyset \). This means there exists \( v \in U - z \) such that \( v \in E_j(z) \) and \( v \in X \). Consequently, for any \( v \in F_{R,j} \), containing \( z \), we have \( v \in V \). Therefore, \( V \cap X \neq \emptyset \), and thus, \( z \in F_{R,j}(X) \) (because \( F_{R,j}(X) = X^\ominus \)). It follows that \( F_{R,j}(X) \subseteq X \subseteq F_{R,j}(X) \), and thus,
\[
\frac{1}{\|F_{R,j}(X)\|} \leq \frac{1}{\|F_{R,j}^+(X)\|} \cup X \quad (2)
\]

From (1) and (2), we can see that \( M_{F,j}(X) = (\|F_{R,j}(X)\|)^1/\|F_{R,j}^+(X)\| \leq (\|F_{R,j}(X)\|/\|F_{R,j}^+(X)\|) \cap X \cap X \equiv M_{F,j}(X) \).

Example 3

(1) Now, we exemplify an application of rough approximations introduced in this paper. Let \( X = \{x_1, x_2, \ldots, x_{100}\} \) be a group of people who have just reached Xian Yang Airport (but are not allowed to outbound station) from two countries (by two planes involving 200 people denoted by a set
Table 2: $R_j$-lower approximations and $R_j$-upper approximations ($j \in \{r, l, i, u\}$).

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Table 3: $J_{R,j}$-lower approximations and $J_{R,j}$-upper approximations ($j \in \{r, l, i, u\}$).

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<th>$J_{R,j}^+(X)$</th>
<th>$J_{R,j}^-(X)$</th>
<th>$J_{R,j}^+(X)$</th>
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The set $U = \{x_1, x_2, \ldots, x_{500}\}$ and will attend a meeting holding in Xi’an. Assume that $x_1$ and $x_2$ are actually infected of asymptomatic infection of new coronavirus, $E_j(x) \in 2^U$ consists of $x$ and all $y \in U$ who contacted $x$ after $x$ had contacted one of $x_1$ and $x_2$. To insulate the safety of this meeting, $R_j^-(X) = \{x \in U \exists (x \in X) \cap (x \notin \emptyset)\}$ and $R_j^+(X) = \{x \in U \exists (x \in X) \cap (x \notin \emptyset)\}$ each person contacting $x$ after $x$ contacts one of $x_1$ and $x_2$ is in X can be looked to be the set of all people who must run a nucleic acid test, and $R_j^+(X) = X \cup \{x \in U \exists (x \in X) \cap (x \notin \emptyset)\}$ some person contacting $x$ after $x$ contacts one of $x_1$ and $x_2$ is in X can be looked to be the set of all people who should run a nucleic acid test.

(2) For $j$-NS $(U, R, J_j)$ in Example 1, the $J_{R,j}$-lower approximations and $J_{R,j}$-upper approximations $(j \in \{r, l, i, u\})$ and the $M_{j,j}$-accuracy measure are given in Table 3.

5. Concluding Remarks

Motivated by topology, this article has initiated two new rough approximations by introducing a new class of neighborhood systems (called $E_j$-neighborhoods) using $j$-neighborhoods. We have probed the main features and formulated the concepts of $E_j$-lower and $E_j$-upper approximations and $E_j$-accuracy measure which are induced from different types of $j$ and compared them. We complete this work by studying these concepts from a topological view and comparing them. In all comparisons, we obtain higher accurate approximations in the case of $j = i$.

In the upcoming works, we will study new types of neighborhoods in rough set theory and use them to define a
topological structure. Also, we will investigate the $E_j$-neighborhoods and approximations on the fuzzy rough set motivated from fuzzy control problems and fractional-order nonlinear systems.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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**References**


