Research Article

Adaptive Fixed-Time Trajectory Tracking Control for Underactuated Hovercraft with Prescribed Performance in the Presence of Model Uncertainties

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This paper develops an adaptive fixed-time trajectory tracking controller of an underactuated hovercraft with a prescribed performance in the presence of model uncertainties and unknown time-varying environment disturbances. It is the first time that the proposed method is applied to the motion control of the hovercraft. To begin with, based on the hovercraft's four degrees of freedom (DOF) model, the virtual control laws are designed using an error transforming function and the fixed-time stability theory to guarantee that the position tracking errors are constrained within the prescribed convergence rates and minimum overshoot. In addition, by combining the Lyapunov direct method and the adaptive radial basis function neural network (ARBFNN), the actual control laws are designed to ensure that the velocity tracking errors converge to a small region containing zero while handling model uncertainties and external disturbances effectively. Finally, all tracking errors of the closed-loop system are uniformly ultimately bounded and fixed-time convergent. Results from a comparative simulation study verify the effectiveness and advantage of the proposed method.

1. Introduction

The underactuated hovercraft as shown in Figure 1 possesses a flexible skirt around the bottom of it to seal the cushion air [1]. It can be lifted to the sail by a large enough air cushion force to obtain a unique amphibious performance. Therefore, the hovercraft has a higher speed than normal surface vessels. In recent years, because of its special amphibious performance, the hovercraft has been getting increasing attention in oceanic research work, for instance, oceanic resources exploration, scientific investigation, ocean expeditions, military missions, transportation, rescue, and other fields [2].

What is particularly noteworthy is that the hovercraft in navigation possesses very little contact with the sailing surface; therefore, it has smaller righting moments than normal surface vessels. It is easy to generate a large roll angle and large drift angle when turning the rudder so that the hovercraft runs in a dangerous situation. The air cushion

force's lateral component under a large roll angle results in the hovercraft drifting sideways and losing course stability [3–5]. The roll motion occurs always when the hovercraft is turning and is more strenuous with the increase in the turn rate and drift angle. Regarding that the above dangerous sailing mechanism is closely relevant to the roll motion of the hovercraft; a four-DOF motion mathematical model of a hovercraft by taking the roll degree of freedom into account is established in this paper, which is closer to the sailing characteristics of the actual hovercraft than the three-DOF model.

Usually, the hovercraft has the characteristics of the typical underactuated vessel. The major difficulty for motion control of the underactuated surface vessels is that the lateral motion is not controlled directly, namely, the number of DOF is more than the number of independent actuators in the underactuated system. The challenge to control underactuated vessels is how to apply two independent actuators to regulate three or more DOFs' motions under

FIGURE 1: Reference frame of the hovercraft.

unknown dynamic environments. Over the past two decades, for overcoming the above control problems, various control strategies have been presented by the endeavours of many research workers, and significant achievements have been achieved. In [6], an adaptive fuzzy stabilization control method was proposed to carry out trajectory tracking control of the underactuated surface vessel in the presence of unknown time-varying environment disturbances. Considering the robustness of the underactuated vessel's control, the literature [7] adopted the biologically inspired method to perform motion control of an underactuated surface vessel and used the single-layer neural network to handle the unknown dynamics. An adaptive dynamic sliding mode control method for trajectory tracking control of an underactuated underwater unmanned vehicle was proposed in [8], but the proposed adaptive sliding mode control method requires some assumptions, such as the existence of the first derivative of external disturbance and control input. In [9], the combination of neural network and the terminal sliding mode was used to design a finite-time trajectory tracking controller for an underactuated hovercraft. A H_∞ observer-based fuzzy controller design for a vessel dynamic positioning system was investigated in [10], and the simulation results show that this method is effective for both known/unknown premise variable cases.

Unfortunately, one important problem, the transient and steady error performance constraint playing a significant role in practice, is not addressed in the above papers. The transient performance constraints can guarantee that high-speed hovercraft avoids collision hazards and provides stability simultaneously. Recently, a performance constraint technique was proposed in [11–13]. This technique can guarantee that the transient errors converge at a designed exponential rate with a predesigned maximum overshoot. Now, the prescribed performance control strategy has been applied to many practical system controls, such as servo system [14], air vehicles [15, 16], and robots [17, 18]. With the further research of scientific researchers, a series of

improved prescribed performance control technology has been proposed in [19–23] to constrain the transient and steady-state performance of the control system. An envelope-constraint-based tracking control method for air-breathing hypersonic vehicles with unknown nonaffine formulations is proposed in [19], and the desired transient performance is guaranteed for velocity and attitude tracking errors using envelope constraint. In [21], a novel performance function without the information of the initial error is proposed for limiting the tracking error of hypersonic flight vehicles into a prescribed range. A new prescribed performance control approach utilizing transformed errors instead of initial tracking errors is proposed for uncertain nonlinear dynamic systems in [22]. For the velocity dynamics of air-breathing hypersonic vehicles, an adaptive neural controller containing only one neural network is addressed using the prescribed performance control method in [23]. The highlights of the method in [23] are that the presented control method has a concise control architecture and low computational cost. Subsequently, the prescribed performance technique has been applied practically to the motion control of the underactuated vessels. For example, a robust adaptive controller was presented to perform the motion control of the underactuated ship with prescribed performance in the presence of model uncertainties and external disturbances [24]. In [25], the transient and steady-state performance was also featured for trajectory tracking control of underactuated underwater vessels.

On the other hand, the controller designed in the practical engineering applications of the ship only guarantees that the stability of the system is not to make much sense. The system convergence speed is always an important index to evaluate the designed controller's performance. In the last few years, the control technique for specifying the system convergence time was proposed; for example, one type of delayed memristor-based fractional-order neural networks based on the finite-time stability problem was studied in [26]. A novel adaptive fixed-time controller was

designed to perform output tracking problems of a class of multiinput multioutput nonlinear systems in the presence of system uncertainties [27]. A fixed-time sliding mode manifold was proposed [28], and the convergence time of the sliding mode manifold is shown to be independent of the initial conditions of the control system. In [29], a non-singular fixed-time fast terminal sliding mode manifold with fixed-time convergence was presented, and the preestablished convergence time is also developed; unfortunately, the control law designed is discontinuous.

Inspired by the abovementioned observations, a nonlinear trajectory tracking control method for an underactuated hovercraft is presented that ensures a prescribed performance, specifies the system convergence time, and handles the system uncertainties. Incorporating the prescribed performance and specified system convergence time simultaneously in the trajectory tracking control design of the underactuated hovercraft has been unprecedented. Accordingly, the main contributions of this paper are summarized as follows:

- (1) Unlike the existing trajectory tracking control methods of the underactuated hovercraft in [9], the presented control strategy avoids the dangerous navigation situation when the hovercraft sails in a narrow channel. Compared with the barrier Lyapunov function method [30], one of the advantages of the prescribed performance method is that it can increase the space of the initial feasible solution of the system and ensure that the tracking error of the system can converge to any small region set in advance.
- (2) To solve the unknown dynamic model uncertainties of the controlled system and external disturbances, an ARBFNN is applied to the trajectory tracking control scheme that is simple and easy to implement in practice.
- (3) By selecting the control parameters appropriately, the convergence time that is independent of the initial values of the controlled system can be pre-specified to guarantee a fast convergence performance of the nonlinear trajectory tracking system.
- (4) It is proven that, by applying the presented controller, all tracking errors of the closed-loop system are uniformly ultimately bounded and uniformly fixed-time convergent despite the presence of system uncertainties.

This paper is arranged as follows. The preliminaries and problem formulation are given in Section 2. Section 3 is devoted to the design of the adaptive neural-based fixed-time trajectory tracking controller for the hovercraft with prescribed performance. Numerical simulation results are shown in Section 4. Section 5 concludes the work of this paper.

2. Preliminaries and Problem Formulation

2.1. Preliminaries

Notation. Throughout this paper, $(\cdot)^T$ denotes the transpose of a matrix $(\cdot)$, $\|\cdot\|$ represents the Euclidean norm of a

vector, $|\cdot|$ signifies the absolute value of a scalar, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ mean the minimum and maximum eigenvalues of a given matrix $B \in R^{n \times n}$, respectively, $U_1 \cup U_2$ denotes the union of interval U_1 and interval U_2 , and $\text{sign}(\cdot)$ denotes the signum function.

Lemma 1 (see [31]). *For any real numbers $x_i \in R$, $i = 1, 2, \dots, n$, we can obtain the following inequalities:*

$$\sum_{i=1}^n |x_i|^p \geq \begin{cases} \left(\sum_{i=1}^n |x_i| \right)^p, & 0 < p \leq 1, \\ n^{1-p} \left(\sum_{i=1}^n |x_i| \right)^p, & 1 < p < +\infty. \end{cases} \quad (1)$$

Lemma 2 (see [32]). *For any continuous unknown real function $f(x): R^m \rightarrow R$, the radius basis function neural network (NN) can be employed to approximate it over the compact $\Omega \subseteq R^m$ as follows:*

$$f(x) = w^{*T} h(x) + \delta(x), \quad (2)$$

where $x \in R^m$ is the input vector of NN, $w^* = [w_1^*, \dots, w_n^*]^T \in R^n$ denotes the NN ideal bounded weight vector, n denotes the hidden node number of NN, and $\delta(x)$ represents the ideal approximation error with satisfying $|\delta(x)| \leq \bar{\delta}$, where $\bar{\delta}$ is a constant. The ideal weight value can be determined by the following equation:

$$w^* = \arg \min_{\hat{w}} \left\{ \sup_{x \in \Omega} |f(x) - \hat{w}^T h(x)| \right\}, \quad (3)$$

where $\hat{w}$ is the estimation of the ideal weight value w^* , which is usually calculated by the adaptive updating law based on the Lyapunov stability theorem. The radius basis function vector is $h(x) = [h_1(x), \dots, h_n(x)]^T: \Omega \rightarrow R^n$, and $h_i(x)$ are selected as the Gaussian function in the following form:

$$h_i(x) = \exp\left(-\frac{\|x - \phi_i\|^2}{\varepsilon_i^2}\right), \quad (i = 1, 2, \dots, n), \quad (4)$$

where $\phi_i \in R^m$ and $\varepsilon_i \in R$ denote the centre and width of the radius basis function, respectively.

Definition 1. Considering the following first-order system (5), if the system is finite-time stable and the upper bound of convergence time is independent of the initial values of the control system, the system is fixed-time stable:

$$\dot{x} = f(x) + \tau + d, \quad (5)$$

where $x \in R$ and $\tau \in R$ are the state and input of system (5), respectively, $f(x)$ is a known continuous function, and d is an unknown continuous bounded function which signifies model uncertainties or/and external disturbances.

Lemma 3 (see [33]). *Considering system (5), if there exists a Lyapunov function $V(x)$ which satisfies*

$$\dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x), \quad (6)$$

where α , p , β , and q are positive constants and satisfy $0 < p < 1 < q$, system (5) is fixed-time stable and the convergence time can be estimated as follows:

$$T \leq \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)}. \quad (7)$$

Lemma 4 (see [34]). In the light of Lemma 3, if there is a positive bounded function Δ and a Lyapunov function $V(x)$ which satisfy

$$\dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x) + \Delta, \quad (8)$$

where α , p , β , and q are positive constants and satisfy $0 < p < 1 < q$, the system state can converge to $D = \{x: V(x) \leq \min\{(\Delta/\alpha\theta_p)^{1/p}, (\Delta/\beta\theta_q)^{1/q}\}\}$ within time T :

$$T \leq \frac{1}{\alpha(1-\theta_p)(1-p)} + \frac{1}{\beta(1-\theta_q)(q-1)}, \quad (9)$$

where $\theta_p, \theta_q \in (0, 1)$ are positive constants.

2.2. Hovercraft Model. According to [2], the following kinematic model and dynamic model are employed to characterize the four-DOF motions of the hovercraft in Figure 1:

$$\begin{cases} \dot{x} = u \cos \psi - v \sin \psi \cos \phi, \\ \dot{y} = u \sin \psi + v \cos \psi \cos \phi, \\ \dot{\phi} = p, \\ \dot{\psi} = r \cos \phi, \\ \dot{u} = vr + \frac{F_{xD0}}{m_0} + f_u + \frac{\tau_u}{m_0}, \\ \dot{v} = -ur + \frac{F_{yD0}}{m_0} + f_v, \\ \dot{p} = \frac{M_{xD0}}{J_{x0}} + f_p, \\ \dot{r} = \frac{M_{zD0}}{J_{z0}} + f_r + \frac{\tau_r}{J_{z0}}, \end{cases} \quad (10)$$

with

$$\begin{aligned} f_u &= \frac{1}{m_0} (-\Delta m \dot{u} + \Delta m v r + \Delta F_{xD}) + \frac{d_u(t)}{m_0}, \\ f_v &= -\frac{1}{m_0} (\Delta m (\dot{v} + ur) + \Delta F_{yD}) + \frac{d_v(t)}{m_0}, \\ f_p &= \frac{1}{J_{x0}} (-\Delta J_x \dot{p} + \Delta M_{xD}) + \frac{d_p(t)}{J_{x0}}, \\ f_r &= \frac{1}{J_{z0}} (-\Delta J_z \dot{r} + \Delta M_{zD}) + \frac{d_r(t)}{J_{z0}}, \end{aligned} \quad (11)$$

where u and v denote velocities, p and r denote the angular velocities in the body-fixed frame, x and y represent positions, and ϕ and ψ represent attitudes with respect to the earth-fixed frame. The control inputs are represented by τ_u and τ_r . The known hull design mass and moment of inertia are signified by m_0 , J_{x0} , and J_{z0} . Δm , ΔJ_x , and ΔJ_z are viewed as the uncertainty of the ship's mass and moment of inertia. The known part of the current resistance models of the hovercraft, for example, air momentum force, skirt resistance, air resistance, and wave-making resistance, is denoted by F_{xD0} , F_{yD0} , M_{xD0} , and M_{zD0} , respectively, that can be determined through wind tunnel test and tank test. ΔF_{xD} , ΔF_{yD} , ΔM_{xD} , and ΔM_{zD} are viewed as the uncertainties of the model in hydrodynamic and aerodynamic drag caused by modelling errors. $d_u(t)$, $d_v(t)$, $d_p(t)$, and $d_r(t)$ signify external environment resistance. f_u , f_v , f_p , and f_r contain the total model uncertainties and external environment resistances. F_{xD0} , F_{yD0} , M_{xD0} , and M_{zD0} can be calculated by the following equations:

$$\begin{aligned} F_{xD0} &= F_{xa} + F_m \cos \beta + F_{wm} \cos \beta + F_{sk} \cos \beta, \\ F_{yD0} &= F_{ya} + F_m \sin \beta + F_{wm} \sin \beta + F_{sk} \sin \beta + F_c, \\ M_{xD0} &= M_{xa} + F_m z_m \sin \beta + F_{wm} z_{wm} \sin \beta + G h_m \tan \phi + F_{sk} z_{sk} \sin \beta + F_c z_c, \\ M_{zD0} &= M_{za} + F_m x_m \sin \beta + F_m y_m \cos \beta + F_{wm} x_{wm} \sin \beta + F_{wm} y_{wm} \cos \beta \\ &\quad + F_{sk} x_{sk} \sin \beta + F_{sk} y_{sk} \cos \beta + F_c x_c, \end{aligned} \quad (12)$$

where

$$\begin{aligned}
F_{xa} &= -0.5\rho_a V_a^2 C_{xa} S_{PP}, \\
F_{ya} &= -0.5\rho_a V_a^2 C_{ya} S_{LP}, \\
M_{xa} &= -0.5\rho_a V_a^2 C_{mxa} S_{PP} l_c + F_{ya} z_a, \\
M_{za} &= -0.5\rho_a V_a^2 C_{mza} S_{HP} H_{hov} + F_{ya} x_a + F_{xa} y_a, \\
F_m &= \rho_a V_a Q, \\
F_{wm} &= C_{wm} p_c^2 B_c / \rho_w g, \\
F_{sk} &= 0.5 V_a^2 C_{sk} (h/l_{sk})^{-0.34} l_{sk} S_c^{0.5} \\
&\quad + (2.8167 (p_c/l_c)^{-0.259} - 1) F_{wm}, \\
F_c &= 2l_c \phi p_c (0.5 B_c \tan \phi + h_0), \\
M_G &= G h_m \tan \phi,
\end{aligned} \tag{13}$$

where F_{xa} , F_{ya} , M_{xa} , and M_{za} denote air resistance, F_{wm} , F_m , F_c , and F_{sk} denote wave-making resistance, air momentum force, cushion resistance, and skirt resistance, respectively. C_{xa} , C_{ya} , C_{mxa} , C_{mza} , C_{wm} , and C_{sk} represent the relevant resistance parameters. M_G means roll restoring moment. The length and width of the cushion are represented by l_c and B_c , respectively. S_{LP} , S_{PP} , and S_{HP} denote lateral, positive, and horizontal projection areas, respectively. S_c and p_c are the area and pressure of the cushion, the average clearance for air leakage in static hovering mode is signified by h , h_m denotes the metacentric height, h_0 denotes the initial lifting height, the total length of the skirt is represented by l_{sk} , the flow coefficient is signified by ϕ , H_{hov} means the hovercraft's height, the air density and water density are signified by ρ_a and ρ_w , z_a , z_m , z_{wm} , z_{sk} , and z_c denote heights of each force's acting point with respect to the mass centre of the hovercraft, and (x_a, y_a) , (x_m, y_m) , (x_{wm}, y_{wm}) , (x_{sk}, y_{sk}) , and (x_c, y_c) signify the coordinates of the force's acting points. β signifies slip angle, and V_a denotes the relative wind speed which can be calculated by the following equations:

$$\begin{aligned}
\beta &= \arctan\left(\frac{v}{u}\right), \\
V_a &= \sqrt{(u + V_\omega \cos(\beta_\omega - \psi))^2 + (v + V_\omega \sin(\beta_\omega - \psi))^2}, \\
\beta_a &= \arctan\left[\frac{v + V_\omega \sin(\beta_\omega - \psi)}{u + V_\omega \cos(\beta_\omega - \psi)}\right],
\end{aligned} \tag{14}$$

where β_a signifies the relative wind direction, V_ω represents the absolute wind speed, and β_ω denotes the absolute wind direction.

Assumption 1. In the control process, we ignore the motion of the hovercraft's pitch and heave and set the pressure of each air chamber and the flow of the cushion fan to be constants.

Assumption 2. The same two air propellers and the same two air rudders are symmetrically mounted at the tail of the hovercraft. In addition, the air rudder provides the moment of steering and the propeller only provides the forward thrust.

Assumption 3. The model uncertainties f_u , f_v , f_p , and f_r are continuously bounded, namely, $|f_i| \leq T_{i\max}$ ($i = u, v, p, r$) with $T_{i\max}$ being positive constants.

Assumption 4. The hovercraft's surge, sway velocities, and yaw angular velocity are bounded.

Remark 1. In practice, the velocity and yaw angular velocity of the hovercraft cannot be infinite because of the restraints of aerodynamic resistance, hydrodynamic damping term, and the ability of the actuator [35].

2.3. Problem Formulation. To expediently describe the control problem, we firstly define the reference trajectory in this paper. The hovercraft tracks that the desired path is generated by the virtual surface vessel:

$$\begin{bmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{\psi}_d \end{bmatrix} = \begin{bmatrix} \cos \psi_d & -\sin \psi_d & 0 \\ \sin \psi_d & \cos \psi_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{dset} \\ v_{dset} \\ r_{dset} \end{bmatrix}. \tag{15}$$

Assumption 5. The virtual vessel states x_d , y_d , and ψ_d and their first derivatives with respect to time $\dot{x}_d$, $\dot{y}_d$, and $\dot{\psi}_d$ are bounded.

The position tracking errors can be defined as follows:

$$\begin{aligned}
x_e &= x - x_d, \\
y_e &= y - y_d.
\end{aligned} \tag{16}$$

By utilizing (10) and (15), the derivatives of the position error with respect to time can be calculated as follows:

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi \cos \phi \\ \sin \psi & \cos \psi \cos \phi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix}. \tag{17}$$

Subsequently, we define the velocity and yaw angular velocity tracking errors as follows:

$$\begin{aligned}
u_e &= u - \alpha_u, \\
v_e &= v - \alpha_v, \\
r_e &= r - \alpha_r,
\end{aligned} \tag{18}$$

where α_u , α_v , and α_r are the virtual control laws to be designed later on, which are viewed as the desired velocities and the desired yaw angular velocity of the u, v , and r , respectively.

In terms of the equation of the hovercraft dynamics (10), the derivatives of formula (18) with respect to time can be calculated as follows:

$$\begin{aligned}
\dot{u}_e &= vr + \frac{F_{xD0}}{m_0} + f_u + \frac{\tau_u}{m_0} - \dot{\alpha}_u, \\
\dot{v}_e &= -ur + \frac{F_{yD0}}{m_0} + f_v - \dot{\alpha}_v, \\
\dot{r}_e &= \frac{M_{zD0}}{J_{z0}} + f_r + \frac{\tau_r}{J_{z0}} - \dot{\alpha}_r.
\end{aligned} \tag{19}$$

To avoid the dangerous navigation situation when the hovercraft sails in a narrow channel, we consider the transient performance during the position tracking error convergence. The errors can be defined to satisfy the following prescribed performance [36]:

$$\begin{aligned}
-\alpha_1\eta(t) &< x_e < \beta_1\eta(t), \\
-\alpha_2\eta(t) &< y_e < \beta_2\eta(t),
\end{aligned} \tag{20}$$

where $\eta(t) = (\eta_0 - \eta_\infty)e^{-at} + \eta_\infty$ is a smooth performance constraint function with parameters satisfying $\eta_0 > \eta_\infty > 0$, $\alpha_i > 0$, $\beta_i > 0$, $i = 1, 2$.

Assumption 6. Initial position tracking errors satisfy the strict conditions $-\alpha_1\eta(0) < x_e(0) < \beta_1\eta(0)$ and $-\alpha_2\eta(0) < y_e(0) < \beta_2\eta(0)$.

The control objective in this paper can be formulated as follows.

Considering the hovercraft model (10) subject to model uncertainties and external disturbances, an adaptive neural-based fixed-time trajectory tracking controller is designed to generate surge force τ_u , virtual control law α_r , and yaw moment τ_r to guarantee velocity tracking errors u_e and v_e , and yaw angular velocity tracking error r_e converges to the small region containing zero within the fixed time. Then, the virtual control laws α_u and α_v are reasonably designed to ensure the position tracking errors remain within the given prescribed performance bounds while the desired trajectory is being tracked by the hovercraft.

3. Main Results

3.1. Error Transformation. To guarantee the position tracking errors always satisfy the prescribed performance (20), the following transformations are applied:

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \eta(t) \begin{bmatrix} S_x(\varepsilon_x) \\ S_y(\varepsilon_y) \end{bmatrix}, \tag{21}$$

where $S_i(\varepsilon_i)$, $i = x, y$ are strictly monotonically increasing functions of ε_i . Select $S_i(\varepsilon_i)$ as follows:

$$\begin{bmatrix} S_x(\varepsilon_x) \\ S_y(\varepsilon_y) \end{bmatrix} = \begin{bmatrix} \frac{\beta_1 e^{(\varepsilon_x + (1/2)\ln(\alpha_1/\beta_1))} - \alpha_1 e^{-(\varepsilon_x + (1/2)\ln(\alpha_1/\beta_1))}}{e^{(\varepsilon_x + (1/2)\ln(\alpha_1/\beta_1))} + e^{-(\varepsilon_x + (1/2)\ln(\alpha_1/\beta_1))}} \\ \frac{\beta_2 e^{(\varepsilon_y + (1/2)\ln(\alpha_2/\beta_2))} - \alpha_2 e^{-(\varepsilon_y + (1/2)\ln(\alpha_2/\beta_2))}}{e^{(\varepsilon_y + (1/2)\ln(\alpha_2/\beta_2))} + e^{-(\varepsilon_y + (1/2)\ln(\alpha_2/\beta_2))}} \end{bmatrix}. \tag{22}$$

Since $\eta(t) \neq 0$, the transformed errors ε_x and ε_y can be altered as follows:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\ln\left(\frac{x_e}{\eta}\beta_1 + \alpha_1\beta_1\right) - \frac{1}{2}\ln\left(\alpha_1\beta_1 - \frac{x_e}{\eta}\alpha_1\right) \\ \frac{1}{2}\ln\left(\frac{y_e}{\eta}\beta_2 + \alpha_2\beta_2\right) - \frac{1}{2}\ln\left(\alpha_2\beta_2 - \frac{y_e}{\eta}\alpha_2\right) \end{bmatrix}. \tag{23}$$

Theorem 1. Regarding the position tracking errors x_e and y_e and the transformed errors ε_x and ε_y , if ε_x and ε_y are bounded and the initial tracking errors satisfy $-\alpha_1\eta(0) < x_e(0) < \beta_1\eta(0)$, $-\alpha_2\eta(0) < y_e(0) < \beta_2\eta(0)$, then x_e and y_e will be ensured to stay within the prescribed performance (20) for all $t \geq 0$.

Proof. We can infer that (22) and (23) have the following properties:

- (1) $-\alpha_1 < S_x(\varepsilon_x) < \beta_1$, $-\alpha_2 < S_y(\varepsilon_y) < \beta_2$
- (2) $\lim_{\varepsilon_x \rightarrow -\infty} S_x(\varepsilon_x) = -\alpha_1$, $\lim_{\varepsilon_x \rightarrow \infty} S_x(\varepsilon_x) = \beta_1$, $\lim_{\varepsilon_y \rightarrow -\infty} S_y(\varepsilon_y) = -\alpha_2$, $\lim_{\varepsilon_y \rightarrow \infty} S_y(\varepsilon_y) = \beta_2$
- (3) $S_x(0) = 0$, $S_y(0) = 0$
- (4) $x_e \rightarrow 0$ when $\varepsilon_x \rightarrow 0$, $y_e \rightarrow 0$ when $\varepsilon_y \rightarrow 0$

Accordingly, if ε_x and ε_y are not infinite and the initial tracking errors x_e and y_e that are within the performance limits, we can deduce that the above properties (1), (2), (3), and (4) will guarantee always that x_e and y_e satisfy the prespecified performance (20). And when ε_x and ε_y converge to a small neighbourhood around zero, x_e and y_e can converge to a small neighbourhood around zero as well. $\square$

3.2. Design of the Virtual Control Laws. In this subsection, we apply the Lyapunov direct method to design the desired velocities as the virtual control law of position error. Firstly, differentiate the transformed errors (23) as follows:

$$\begin{bmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \end{bmatrix} = \begin{bmatrix} \xi_x & 0 \\ 0 & \xi_y \end{bmatrix} \left\{ \begin{bmatrix} \cos \psi & -\sin \psi \cos \phi \\ \sin \psi & \cos \psi \cos \phi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} \dot{x}_d + x_e \frac{\dot{\eta}}{\eta} \\ \dot{y}_d + y_e \frac{\dot{\eta}}{\eta} \end{bmatrix} \right\}, \tag{24}$$

where

$$\begin{aligned}
\xi_x &= \frac{1}{2\eta} \left(\frac{1}{x_e/\eta + \alpha_1} - \frac{1}{x_e/\eta - \beta_1} \right), \\
\xi_y &= \frac{1}{2\eta} \left(\frac{1}{y_e/\eta + \alpha_2} - \frac{1}{y_e/\eta - \beta_2} \right).
\end{aligned} \tag{25}$$

We can easily obtain that $\xi_x \neq 0$ and $\xi_y \neq 0$.

According to the fixed-time stability lemma and Lyapunov direct method, the desired velocities are designed as follows:

$$\begin{bmatrix} \alpha_u \\ \alpha_v \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \\ \cos \phi & \cos \phi \end{bmatrix} \times \begin{bmatrix} \dot{x}_d + x_e \frac{\dot{\eta}}{\eta} - \frac{1}{\xi_x} \left(k_{p11} \text{sign}(\varepsilon_x) |\varepsilon_x|^{p_1} + k_{p12} \text{sign}(\varepsilon_x) |\varepsilon_x|^{q_1} + k_x \varepsilon_x \right) \\ \dot{y}_d + y_e \frac{\dot{\eta}}{\eta} - \frac{1}{\xi_y} \left(k_{p21} \text{sign}(\varepsilon_y) |\varepsilon_y|^{p_1} + k_{p22} \text{sign}(\varepsilon_y) |\varepsilon_y|^{q_1} + k_y \varepsilon_y \right) \end{bmatrix}, \quad (26)$$

where $k_x, k_y, k_{p11}, k_{p12}, k_{p21}, k_{p22}, p_1$, and q_1 are positive constants with satisfying the inequality $0 < p_1 < 1 < q_1$.

Assumption 7. The hovercraft's roll angle ϕ satisfies $|\phi| < 90^\circ$.

Remark 2. In the motion process of the hovercraft, the roll angle is impossible to reach $\pm 90^\circ$ because of the effect of roll restoring moment. The control task of this paper will lose its significance when the environmental forces break this balanced relationship.

By substituting (26) into (24), we have the following:

$$\begin{aligned} \begin{bmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \end{bmatrix} &= \begin{bmatrix} \xi_x & 0 \\ 0 & \xi_y \end{bmatrix} \left(\begin{bmatrix} \cos \psi & -\sin \psi \cos \phi \\ \sin \psi & \cos \psi \cos \phi \end{bmatrix} \begin{bmatrix} u_e + \alpha_u \\ v_e + \alpha_v \end{bmatrix} - \begin{bmatrix} \dot{x}_d + x_e \frac{\dot{\eta}}{\eta} \\ \dot{y}_d + y_e \frac{\dot{\eta}}{\eta} \end{bmatrix} \right) \\ &= - \begin{bmatrix} k_{p11} \text{sign}(\varepsilon_x) |\varepsilon_x|^{p_1} + k_{p12} \text{sign}(\varepsilon_x) |\varepsilon_x|^{q_1} + k_x \varepsilon_x \\ k_{p21} \text{sign}(\varepsilon_y) |\varepsilon_y|^{p_1} + k_{p22} \text{sign}(\varepsilon_y) |\varepsilon_y|^{q_1} + k_y \varepsilon_y \end{bmatrix} \\ &\quad + \begin{bmatrix} u_e \xi_x \cos \psi - v_e \xi_x \sin \psi \cos \phi \\ u_e \xi_y \sin \psi + v_e \xi_y \cos \psi \cos \phi \end{bmatrix}. \end{aligned} \quad (27)$$

Consider the following candidate Lyapunov function,

$$V_p = \frac{1}{2} \varepsilon_x^2 + \frac{1}{2} \varepsilon_y^2. \quad (28)$$

The time derivative of the Lyapunov function defined by (28) along (27) is as follows:

$$\begin{aligned} \dot{V}_p &= \varepsilon_x \dot{\varepsilon}_x + \varepsilon_y \dot{\varepsilon}_y \\ &= -k_{p11} |\varepsilon_x|^{p_1+1} - k_{p12} |\varepsilon_x|^{q_1+1} - k_x \varepsilon_x^2 \\ &\quad - k_{p21} |\varepsilon_y|^{p_1+1} - k_{p22} |\varepsilon_y|^{q_1+1} - k_y \varepsilon_y^2 \\ &\quad + \varepsilon_x u_e \xi_x \cos \psi - \varepsilon_x v_e \xi_x \sin \psi \cos \phi \\ &\quad + \varepsilon_y u_e \xi_y \sin \psi + \varepsilon_y v_e \xi_y \cos \psi \cos \phi. \end{aligned} \quad (29)$$

The result in (29) will be applied to the stability analysis of the designed controller in Subsection 3.6.

3.3. Design of the Surge Control Law. The surge control law is designed to guarantee that the surge velocity tracking error converges to a small neighbourhood around zero within a fixed time. Consider the following candidate Lyapunov function:

$$V_u = \frac{1}{2} u_e^2 + \frac{1}{2} \tilde{w}_u^T \Gamma_u^{-1} \tilde{w}_u, \quad (30)$$

where $\Gamma_u \in R^{l_u \times l_u}$ is the positive definite diagonal matrix, l_u is the hidden node number, and $\tilde{w}_u = w_u^* - \hat{w}_u$ is the estimation error of the weight value with $\hat{w}_u$ representing the estimation value of the ideal weight value w_u^* .

The time derivative of the Lyapunov function is defined by (30) along (19) yields the following:

$$\begin{aligned}\dot{V}_u &= u_e \dot{u}_e + \tilde{w}_u^T \Gamma_u^{-1} (-\dot{\tilde{w}}_u) \\ &= u_e \left(vr + \frac{F_{xD0}}{m_0} + f_u + \frac{\tau_u}{m_0} - \dot{\alpha}_u \right) + \tilde{w}_u^T \Gamma_u^{-1} (-\dot{\tilde{w}}_u).\end{aligned}\quad (31)$$

The ARBFNN is employed to deal with the model uncertainty f_u of (31). According to Lemma 2, we have the following:

$$\begin{aligned}f_u &= w_u^{*T} h_u(z_u) + \delta_u(z_u), \\ \hat{f}_u &= \hat{w}_u^T h_u(z_u),\end{aligned}\quad (32)$$

where $\hat{f}_u$ is the estimation value of the f_u , $\delta_u(z_u)$ signifies the minimum approximation error that satisfies $|\delta_u(z_u)| \leq \bar{\delta}_u$ with $\bar{\delta}_u$ being the designed positive constant, $z_u = [u, \dot{u}]^T \in \Omega_u$ denotes the input of the NN, and $h_u(z_u) \in R^{l_u}$ is the Gaussian basis function.

According to (31), we design the surge control law as follows:

$$\begin{aligned}\tau_u &= m_0 \left(-vr - \frac{F_{xD0}}{m_0} - \hat{f}_u + \dot{\alpha}_u - k_u u_e - k_{u1} \text{sign}(u_e) |u_e|^{p_1} \right. \\ &\quad \left. - k_{u2} \text{sign}(u_e) |u_e|^{q_1} - \varepsilon_x \xi_x \cos \psi - \varepsilon_y \xi_y \sin \psi \right),\end{aligned}\quad (33)$$

where $k_u > (1/2)$, k_{u1} , and k_{u2} are positive constants and the updated law of the NN weight is designed as follows:

$$-\dot{\tilde{w}}_u = -u_e \Gamma_u h_u(z_u) + \sigma_u \Gamma_u \tilde{w}_u. \quad (34)$$

By substituting (32) and (33) into (31), we can obtain the derivative of the Lyapunov function V_u as follows:

$$\begin{aligned}\dot{V}_u &= u_e \left(vr + \frac{F_{xD0}}{m_0} + f_u + \frac{\tau_u}{m_0} - \dot{\alpha}_u \right) + \tilde{w}_u^T \Gamma_u^{-1} (-\dot{\tilde{w}}_u) \\ &= u_e (f_u - \hat{f}_u) - k_u u_e^2 - k_{u1} |u_e|^{p_1+1} \\ &\quad - k_{u2} |u_e|^{q_1+1} - u_e \varepsilon_x \xi_x \cos \psi \\ &\quad - u_e \varepsilon_y \xi_y \sin \psi + \tilde{w}_u^T \Gamma_u^{-1} (-u_e \Gamma_u h_u(z) + \sigma_u \tilde{w}_u) \\ &= -k_u u_e^2 - k_{u1} |u_e|^{p_1+1} - k_{u2} |u_e|^{q_1+1} - u_e \varepsilon_x \xi_x \cos \psi \\ &\quad - u_e \varepsilon_y \xi_y \sin \psi + \tilde{w}_u^T \sigma_u \tilde{w}_u + u_e \delta_u(z_u) \\ &\leq -\left(k_u - \frac{1}{2}\right) u_e^2 - k_{u1} |u_e|^{p_1+1} - k_{u2} |u_e|^{q_1+1} - u_e \varepsilon_x \xi_x \cos \psi \\ &\quad - u_e \varepsilon_y \xi_y \sin \psi + \tilde{w}_u^T \sigma_u \tilde{w}_u + \frac{1-\bar{\delta}_u}{2}.\end{aligned}\quad (35)$$

Inequality (35) will be used to analyse the stability of the controlled system in Subsection 3.6.

3.4. Design of the Desired Angular Velocity in Yaw. The hovercraft is a kind of typical underactuated surface vessel because its lateral axis is not directly actuated. Thus, the virtual control law is designed to stabilize the sway velocity tracking error. Consider the following candidate Lyapunov function:

$$V_v = \frac{1}{2} v_e^2 + \frac{1}{2} \tilde{w}_v^T \Gamma_v^{-1} \tilde{w}_v, \quad (36)$$

where $\Gamma_v \in R^{l_v \times l_v}$ signifies the design positive definite diagonal matrix with l_v being the hidden node number, $\tilde{w}_v = w_v^* - \hat{w}_v$ denotes the estimation error of the weight value, and the ideal weight value w_v^* is estimated by the $\hat{w}_v$.

The derivative of the Lyapunov function V_v with respect to time is given by the following:

$$\begin{aligned}\dot{V}_v &= v_e \dot{v}_e + \tilde{w}_v^T \Gamma_v^{-1} (-\dot{\tilde{w}}_v) \\ &= v_e \left(-ur + \frac{F_{yD0}}{m_0} + f_v - \dot{\alpha}_v \right) + \tilde{w}_v^T \Gamma_v^{-1} (-\dot{\tilde{w}}_v).\end{aligned}\quad (37)$$

The model uncertainty f_v of (37) is handled by applying the ARBFNN. According to Lemma 2, we can obtain the following:

$$\begin{aligned}f_v &= w_v^{*T} h_v(z_v) + \delta_v(z_v), \\ \hat{f}_v &= \hat{w}_v^T h_v(z_v),\end{aligned}\quad (38)$$

where $\hat{f}_v$ is the estimation value of f_v , the minimum approximation error is represented by $\delta_v(z_v)$ and satisfies $|\delta_v(z_v)| \leq \bar{\delta}_v$ with $\bar{\delta}_v$ being the design positive constant, $z_v = [v, \dot{v}]^T \in \Omega_v$ denotes the input of the NN, and $h_v(z_v) \in R^{l_v}$ is the Gaussian basis function.

According to (37), the design desired yaw angular velocity as a stability function:

$$\begin{aligned}\alpha_r &= \frac{1}{u} \left(\frac{F_{yD0}}{m_0} - \dot{\alpha}_v + \hat{f}_v + k_v v_e + k_{v1} \text{sign}(v_e) |v_e|^{p_1} + k_{v2} \text{sign}(v_e) |v_e|^{q_1} \right. \\ &\quad \left. - \varepsilon_x \xi_x \sin \psi \cos \phi + \varepsilon_y \xi_y \cos \psi \cos \phi \right),\end{aligned}\quad (39)$$

where $k_v > (1/2)$, k_{v1} , and k_{v2} are positive constants and the updated law of the NN weight is designed as follows:

$$-\dot{\tilde{w}}_v = -v_e \Gamma_v h_v(z) + \sigma_v \Gamma_v \tilde{w}_v. \quad (40)$$

Remark 3. In the practical motion control of the hovercraft, the pitch angle is always set to the positive value. Only in special cases, the negative pitch angle can be set, such as when the hovercraft enters and leaves the mother ship. Accordingly, the surge velocity is always set to $u > 0$ in the motion control process of the hovercraft.

By substituting (38) and (39) into (40), we can obtain the derivative of the Lyapunov function V_v as follows:

$$\begin{aligned}
\dot{V}_v &= v_e \left(-ur + \frac{F_{yD0}}{m_0} + f_v - \dot{\alpha}_v \right) + \tilde{w}_v^T \Gamma_v^{-1} (-\dot{\hat{w}}_v) \\
&= v_e \left(-k_v v_e - k_{v1} \text{sign}(v_e) |v_e|^{p_1} - k_{v2} \text{sign}(v_e) |v_e|^{q_1} + f_v - \hat{f}_v \right) \\
&\quad + \tilde{w}_v^T \Gamma_v^{-1} (-\dot{\hat{w}}_v) + \varepsilon_x v_e \xi_x \sin \psi \cos \phi - \varepsilon_y v_e \xi_y \cos \psi \cos \phi \\
&= -uv_e r_e + v_e \delta_v(z_v) - k_v v_e^2 - k_{v1} |v_e|^{p_1+1} - k_{v2} |v_e|^{q_1+1} + \sigma_v \tilde{w}_v^T \hat{w}_v \\
&\leq -\left(k_v - \frac{1}{2}\right) v_e^2 - k_{v1} |v_e|^{p_1+1} - k_{v2} |v_e|^{q_1+1} + \sigma_v \tilde{w}_v^T \hat{w}_v \\
&\quad + \frac{1}{2} \delta_v^2 - uv_e r_e + \varepsilon_x v_e \xi_x \sin \psi \cos \phi - \varepsilon_y v_e \xi_y \cos \psi \cos \phi.
\end{aligned} \tag{41}$$

Inequality (41) will be used to perform the stability analysis of the nonlinear system in Subsection 3.6.

3.5. Design of the Yaw Control Law. Subsequently, we design the yaw control law τ_r to guarantee that the tracking error of angular velocity in the yaw converges to a small region around zero within a fixed time. Select the Lyapunov candidate function as follows:

$$V_r = \frac{1}{2} r_e^2 + \frac{1}{2} \tilde{w}_r^T \Gamma_r^{-1} \tilde{w}_r, \tag{42}$$

where $k_r > (1/2)$, k_{r1} and k_{r2} are positive constants, and the updated law of the NN weight is selected as follows:

$$-\dot{\hat{w}}_r = -r_e \Gamma_r h_r(z_r) + \sigma_r \Gamma_r \hat{w}_r. \tag{46}$$

Then, substituting (44) and (45) into (43), the derivative of the Lyapunov function is expressed as follows:

$$\begin{aligned}
\dot{V}_r &= r_e \left(f_r - \hat{f}_r - k_r r_e - k_{r1} \text{sign}(r_e) |r_e|^{p_1} \right. \\
&\quad \left. - k_{r2} \text{sign}(r_e) |r_e|^{q_1} + uv_e \right) \\
&\quad + \tilde{w}_r^T \Gamma_r^{-1} (-r_e \Gamma_r h_r(z_r) + \sigma_r \Gamma_r \hat{w}_r) \\
&= r_e \left(\tilde{w}_r^T h_r(z_r) + \delta_r(z_r) \right) - k_{r1} |r_e|^{p_1+1} - k_{r2} |r_e|^{q_1+1} + uv_e r_e \\
&\quad + \tilde{w}_r^T \Gamma_r^{-1} (-r_e \Gamma_r h_r(z_r) + \sigma_r \Gamma_r \hat{w}_r) - k_r r_e^2 \\
&= r_e \delta_r(z_r) - k_{r1} |r_e|^{p_1+1} - k_{r2} |r_e|^{q_1+1} \\
&\quad + uv_e r_e + \sigma_r \tilde{w}_r^T \hat{w}_r - k_r r_e^2 \\
&\leq -\left(k_r - \frac{1}{2}\right) r_e^2 - k_{r1} |r_e|^{p_1+1} - k_{r2} |r_e|^{q_1+1} \\
&\quad + uv_e r_e + \sigma_r \tilde{w}_r^T \hat{w}_r + \frac{1}{2} \delta_r^2.
\end{aligned} \tag{47}$$

where $\Gamma_r \in R^{l_r \times l_r}$ is the designed positive definite diagonal matrix, l_r is the hidden node number, and the estimation error of the weight value is represented by $\tilde{w}_r = w_r^* - \hat{w}_r$ with $\hat{w}_r$ denoting the estimation value of the ideal weight value w_r^* .

The time derivative of the Lyapunov function defined by (42) along (19) is calculated as follows:

$$\begin{aligned}
\dot{V}_r &= r_e \dot{r}_e + \tilde{w}_r^T \Gamma_r^{-1} (-\dot{\hat{w}}_r) \\
&= r_e \left(\frac{M_{zD0}}{J_{z0}} + f_r + \frac{\tau_r}{J_{z0}} - \dot{\alpha}_r \right) + \tilde{w}_r^T \Gamma_r^{-1} (-\dot{\hat{w}}_r).
\end{aligned} \tag{43}$$

The model uncertainty f_r of (43) is tackled by applying the ARBFNN. Accordingly, in terms of Lemma 2, we have the following:

$$\begin{aligned}
f_r &= w_r^{*T} h_r(z_r) + \delta_r(z_r), \\
\hat{f}_r &= \tilde{w}_r^T h_r(z_r),
\end{aligned} \tag{44}$$

where $\hat{f}_r$ is the estimation value of the f_r , the minimum approximation error is signified by $\delta_r(z_r)$ and satisfies $|\delta_r(z_r)| \leq \bar{\delta}_r$ with $\bar{\delta}_r$ being the designed positive constant, $z_r = [r, \dot{r}]^T \in \Omega_r$ denotes the input of the NN, and $h_r(z_r) \in R^{l_r}$ is the Gaussian basis function.

According to (43), design the yaw control law as follows:

$$\tau_r = J_{z0} \left(\dot{\alpha}_r - \frac{M_{zD0}}{J_{z0}} - \hat{f}_r - k_r r_e - k_{r1} \text{sign}(r_e) |r_e|^{p_1} - k_{r2} \text{sign}(r_e) |r_e|^{q_1} + uv_e \right), \tag{45}$$

The result in (47) will be applied to the stability analysis of the control system in the next subsection.

3.6. Stability Analysis

Theorem 2. Consider the hovercraft trajectory tracking nonlinear system (10) under model uncertainty and external environment disturbance, and suppose that Assumptions 1–7 are satisfied. If the desired velocities α_u and α_v and desired angular velocity α_r are calculated by (26) and (39), the adaptive fixed-time controllers τ_u and τ_r are obtained by (33) and (45), the model uncertainties and external disturbances are approximated by the ARBFNNs (32), (38), and (44), and the initial condition satisfies $-\alpha_1 \eta(0) < x_e(0) < \beta_1 \eta(0)$ and $-\alpha_2 \eta(0) < y_e(0) < \beta_2 \eta(0)$, then the position tracking errors can be guaranteed to remain within the given prescribed performance bounds (20); all tracking errors can converge to a small region containing zero within the fixed time and are uniformly ultimately bounded.

Proof. The proof process is divided into two steps. Firstly, we indicate that all error signals are uniformly ultimately bounded. Secondly, verify that all tracking errors will converge uniformly to a small neighbourhood around zero within a fixed time. $\square$

Step 1. Consider the Lyapunov candidate function of the system:

With the help of (29), (35), (41), and (47), the time derivative of (48) satisfies the following:

$$\begin{aligned}
 V &= V_p + V_u + V_v + V_r \\
 &= \frac{1}{2}\varepsilon_x^2 + \frac{1}{2}\varepsilon_y^2 + \frac{1}{2}u_e^2 + \frac{1}{2}\tilde{w}_u^T \Gamma_u^{-1} \tilde{w}_u \\
 &\quad + \frac{1}{2}v_e^2 + \frac{1}{2}\tilde{w}_v^T \Gamma_v^{-1} \tilde{w}_v + \frac{1}{2}r_e^2 + \frac{1}{2}\tilde{w}_r^T \Gamma_r^{-1} \tilde{w}_r.
 \end{aligned} \tag{48}$$

$$\begin{aligned}
 \dot{V} &\leq -k_{p11}|\varepsilon_x|^{p_1+1} - k_{p12}|\varepsilon_x|^{q_1+1} - k_x\varepsilon_x^2 - k_{p21}|\varepsilon_y|^{p_1+1} - k_{p22}|\varepsilon_y|^{q_1+1} - k_y\varepsilon_y^2 \\
 &\quad + \varepsilon_x u_e \xi_x \cos \psi - \varepsilon_x v_e \xi_x \sin \psi \cos \phi + \varepsilon_y u_e \xi_y \sin \psi + \varepsilon_y v_e \xi_y \cos \psi \cos \phi \\
 &\quad - \left(k_u - \frac{1}{2}\right)u_e^2 - k_{u1}|u_e|^{p_1+1} - k_{u2}|u_e|^{q_1+1} - u_e \varepsilon_x \xi_x \cos \psi - u_e \varepsilon_y \xi_y \sin \psi \\
 &\quad + \tilde{w}_u^T \sigma_u \hat{w}_u + \frac{1}{2}\bar{\delta}_u^2 - \left(k_v - \frac{1}{2}\right)v_e^2 - k_{v1}|v_e|^{p_1+1} - k_{v2}|v_e|^{q_1+1} + \sigma_v \tilde{w}_v^T \hat{w}_v \\
 &\quad + \frac{1}{2}\bar{\delta}_v^2 - uv_e r_e + \varepsilon_x v_e \xi_x \sin \psi \cos \phi - \varepsilon_y v_e \xi_y \cos \psi \cos \phi \\
 &\quad - \left(k_r - \frac{1}{2}\right)r_e^2 - k_{r1}|r_e|^{p_1+1} - k_{r2}|r_e|^{q_1+1} + uv_e r_e + \sigma_r \tilde{w}_r^T \hat{w} + \frac{1}{2}\bar{\delta}_r^2 \\
 &= -k_{p11}|\varepsilon_x|^{p_1+1} - k_{p12}|\varepsilon_x|^{q_1+1} - k_x\varepsilon_x^2 - k_{p21}|\varepsilon_y|^{p_1+1} - k_{p22}|\varepsilon_y|^{q_1+1} - k_y\varepsilon_y^2 \\
 &\quad - \left(k_u - \frac{1}{2}\right)u_e^2 - k_{u1}|u_e|^{p_1+1} - k_{u2}|u_e|^{q_1+1} - \left(k_v - \frac{1}{2}\right)v_e^2 - k_{v1}|v_e|^{p_1+1} \\
 &\quad - k_{v2}|v_e|^{q_1+1} - \left(k_r - \frac{1}{2}\right)r_e^2 - k_{r1}|r_e|^{p_1+1} - k_{r2}|r_e|^{q_1+1} - \sigma_u \tilde{w}_u^T \hat{w}_u + \frac{1}{2}\bar{\delta}_u^2 \\
 &\quad - \sigma_v \tilde{w}_v^T \hat{w}_v - \sigma_r \tilde{w}_r^T \hat{w}_r + \sigma_u \tilde{w}_u^T w_u^* + \sigma_v \tilde{w}_v^T w_v^* + \sigma_r \tilde{w}_r^T w_r^* + \frac{1}{2}\bar{\delta}_v^2 + \frac{1}{2}\bar{\delta}_r^2.
 \end{aligned} \tag{49}$$

In the light of Young's inequality, the following inequalities can be obtained:

$$\begin{aligned}\sigma_u \tilde{w}_u^T w_u^* &\leq \frac{\sigma_u}{2} \tilde{w}_u^T \tilde{w}_u + \frac{\sigma_u}{2} w_u^{*T} w_u^*, \\ \sigma_v \tilde{w}_v^T w_v^* &\leq \frac{\sigma_v}{2} \tilde{w}_v^T \tilde{w}_v + \frac{\sigma_v}{2} w_v^{*T} w_v^*, \\ \sigma_r \tilde{w}_r^T w_r^* &\leq \frac{\sigma_r}{2} \tilde{w}_r^T \tilde{w}_r + \frac{\sigma_r}{2} w_r^{*T} w_r^*.\end{aligned}\quad (50)$$

Thus, we can rewrite (49) as follows:

$$\begin{aligned}\dot{V} &\leq -k_{p11}|\varepsilon_x|^{p_1+1} - k_{p12}|\varepsilon_x|^{q_1+1} - k_x \varepsilon_x^2 - k_{p21}|\varepsilon_y|^{p_1+1} - k_{p22}|\varepsilon_y|^{q_1+1} - k_y \varepsilon_y^2 - \left(k_u - \frac{1}{2}\right) u_e^2 \\ &\quad - k_{u1}|u_e|^{p_1+1} - k_{u2}|u_e|^{q_1+1} - \frac{\sigma_u}{2} \tilde{w}_u^T \tilde{w}_u - \left(k_v - \frac{1}{2}\right) v_e^2 - k_{v1}|v_e|^{p_1+1} - k_{v2}|v_e|^{q_1+1} - \frac{\sigma_v}{2} \tilde{w}_v^T \tilde{w}_v \\ &\quad - \left(k_r - \frac{1}{2}\right) r_e^2 - k_{r1}|r_e|^{p_1+1} - k_{r2}|r_e|^{q_1+1} - \frac{\sigma_r}{2} \tilde{w}_r^T \tilde{w}_r + \frac{\sigma_u}{2} w_u^{*T} w_u^* + \frac{\sigma_v}{2} w_v^{*T} w_v^* + \frac{\sigma_r}{2} w_r^{*T} w_r^* \\ &\quad + \frac{1}{2} \bar{\delta}_v^2 + \frac{1}{2} \bar{\delta}_u^2 + \frac{1}{2} \bar{\delta}_r^2 \\ &\leq -k_x \varepsilon_x^2 - k_y \varepsilon_y^2 - \left(k_u - \frac{1}{2}\right) u_e^2 - \frac{\sigma_u}{2} \tilde{w}_u^T \tilde{w}_u \\ &\quad - \left(k_v - \frac{1}{2}\right) v_e^2 - \frac{\sigma_v}{2} \tilde{w}_v^T \tilde{w}_v - \left(k_r - \frac{1}{2}\right) r_e^2 - \frac{\sigma_r}{2} \tilde{w}_r^T \tilde{w}_r \\ &\quad + \frac{\sigma_u}{2} w_u^{*T} w_u^* + \frac{\sigma_v}{2} w_v^{*T} w_v^* + \frac{\sigma_r}{2} w_r^{*T} w_r^* + \frac{1}{2} \bar{\delta}_v^2 + \frac{1}{2} \bar{\delta}_u^2 + \frac{1}{2} \bar{\delta}_r^2 \\ &\leq -2\rho V + C_1,\end{aligned}\quad (51)$$

where

$$\begin{aligned}\rho &= \min\left\{k_x, k_y, \left(k_u - \frac{1}{2}\right), \left(k_v - \frac{1}{2}\right), \left(k_r - \frac{1}{2}\right), \frac{\sigma_u}{2} \lambda_{\min}(\Gamma_u), \frac{\sigma_v}{2} \lambda_{\min}(\Gamma_v), \frac{\sigma_r}{2} \lambda_{\min}(\Gamma_r)\right\}, \\ C_1 &= \frac{\sigma_u}{2} w_u^{*T} w_u^* + \frac{\sigma_v}{2} w_v^{*T} w_v^* + \frac{\sigma_r}{2} w_r^{*T} w_r^* + \frac{1}{2} \bar{\delta}_v^2 + \frac{1}{2} \bar{\delta}_u^2 + \frac{1}{2} \bar{\delta}_r^2.\end{aligned}\quad (52)$$

By integrating both sides of (51), the following inequality is obtained:

$$0 \leq V \leq \left(V(0) - \frac{C_1}{2\rho}\right) e^{-2\rho t} + \frac{C_1}{2\rho}. \quad (53)$$

Therefore, it is clear that V is bounded, which implies that the tracking errors are uniformly ultimately bounded. Thus, in terms of Theorem 1, we can deduce that the position tracking errors x_e and y_e always satisfy their prescribed performance limits (20). Furthermore, we can reasonably

suppose that there is always a positive constant Δ_M such that $\|\tilde{w}_i\| \leq \Delta_M, i = u, v, r$.

Step 2. For indicating that the tracking errors of the controlled system are uniformly fixed-time stable, we alter inequality (51) as follows:

$$\begin{aligned}
\dot{V} \leq & -k_{p11}2^{(p_1+1/2)}\left(\frac{1}{2}\varepsilon_x^2\right)^{(p_1+1/2)} - k_{p21}2^{(p_1+1/2)}\left(\frac{1}{2}\varepsilon_y^2\right)^{(p_1+1/2)} - k_{u1}2^{(p_1+1/2)}\left(\frac{1}{2}u_e^2\right)^{(p_1+1/2)} \\
& - k_{v1}2^{(p_1+1/2)}\left(\frac{1}{2}v_e^2\right)^{(p_1+1/2)} - k_{r1}2^{(p_1+1/2)}\left(\frac{1}{2}r_e^2\right)^{(p_1+1/2)} - k_{p12}2^{(q_1+1/2)}\left(\frac{1}{2}\varepsilon_x^2\right)^{(q_1+1/2)} \\
& - k_{p22}2^{(q_1+1/2)}\left(\frac{1}{2}\varepsilon_y^2\right)^{(q_1+1/2)} - k_{u2}2^{(q_1+1/2)}\left(\frac{1}{2}u_e^2\right)^{(q_1+1/2)} - k_{v2}2^{(q_1+1/2)}\left(\frac{1}{2}v_e^2\right)^{(q_1+1/2)} \\
& - k_{r2}2^{(q_1+1/2)}\left(\frac{1}{2}r_e^2\right)^{(q_1+1/2)} + \frac{\sigma_u}{2}w_u^{*T}w_u^* + \frac{\sigma_v}{2}w_v^{*T}w_v^* + \frac{\sigma_r}{2}w_r^{*T}w_r^* \\
& - (\sigma_u\lambda_{\min}(\Gamma_u))^{(p_1+1/2)}\left(\frac{1}{2}\tilde{w}_u^T\Gamma_u^{-1}\tilde{w}_u\right)^{(p_1+1/2)} + \left(\frac{\sigma_u}{2}\lambda_{\min}(\Gamma_u)\tilde{w}_u^T\Gamma_u^{-1}\tilde{w}_u\right)^{(p_1+1/2)} \\
& - (\sigma_v\lambda_{\min}(\Gamma_v))^{(p_1+1/2)}\left(\frac{1}{2}\tilde{w}_v^T\Gamma_v^{-1}\tilde{w}_v\right)^{(p_1+1/2)} + \left(\frac{\sigma_v}{2}\lambda_{\min}(\Gamma_v)\tilde{w}_v^T\Gamma_v^{-1}\tilde{w}_v\right)^{(p_1+1/2)} \\
& - (\sigma_r\lambda_{\min}(\Gamma_r))^{(p_1+1/2)}\left(\frac{1}{2}\tilde{w}_r^T\Gamma_r^{-1}\tilde{w}_r\right)^{(p_1+1/2)} + \left(\frac{\sigma_r}{2}\lambda_{\min}(\Gamma_r)\tilde{w}_r^T\Gamma_r^{-1}\tilde{w}_r\right)^{(p_1+1/2)} \\
& - \frac{\sigma_u}{2}\lambda_{\min}(\Gamma_u)\tilde{w}_u^T\Gamma_u^{-1}\tilde{w}_u - \frac{\sigma_v}{2}\lambda_{\min}(\Gamma_v)\tilde{w}_v^T\Gamma_v^{-1}\tilde{w}_v - \frac{\sigma_r}{2}\lambda_{\min}(\Gamma_r)\tilde{w}_r^T\Gamma_r^{-1}\tilde{w}_r \\
& - (\sigma_u\lambda_{\min}(\Gamma_u))^{(q_1+1/2)}\left(\frac{1}{2}\tilde{w}_u^T\Gamma_u^{-1}\tilde{w}_u\right)^{(q_1+1/2)} + \left(\frac{\sigma_u}{2}\lambda_{\min}(\Gamma_u)\tilde{w}_u^T\Gamma_u^{-1}\tilde{w}_u\right)^{(q_1+1/2)} \\
& - (\sigma_v\lambda_{\min}(\Gamma_v))^{(q_1+1/2)}\left(\frac{1}{2}\tilde{w}_v^T\Gamma_v^{-1}\tilde{w}_v\right)^{(q_1+1/2)} + \left(\frac{\sigma_v}{2}\lambda_{\min}(\Gamma_v)\tilde{w}_v^T\Gamma_v^{-1}\tilde{w}_v\right)^{(q_1+1/2)} \\
& - (\sigma_r\lambda_{\min}(\Gamma_r))^{(q_1+1/2)}\left(\frac{1}{2}\tilde{w}_r^T\Gamma_r^{-1}\tilde{w}_r\right)^{(q_1+1/2)} + \left(\frac{\sigma_r}{2}\lambda_{\min}(\Gamma_r)\tilde{w}_r^T\Gamma_r^{-1}\tilde{w}_r\right)^{(q_1+1/2)} \\
& + \frac{1}{2}\delta_u^2 + \frac{1}{2}\delta_v^2 + \frac{1}{2}\delta_r^2.
\end{aligned} \tag{54}$$

With the fact that for all real numbers $x \geq 0$, we can obtain the following inequality:

$$x^{(p_1+1/2)} - x \leq \left(\frac{p_1+1}{2}\right)^{(p_1+1/2-p_1)} - \left(\frac{p_1+1}{2}\right)^{(2/1-p_1)} = \vartheta_0. \tag{55}$$

According to Lemma 1 and (55), inequality (54) can be expressed as follows:

$$\dot{V} \leq -\rho_1 V^{(p_1+1/2)} - 8^{(1-q_1/2)}\rho_2 V^{(q_1+1/2)} + C_2, \tag{56}$$

where

$$\begin{aligned}
\rho_1 &= \min \begin{cases} k_{p11} 2^{(p_1+1/2)}, k_{p21} 2^{(p_1+1/2)}, k_{u1} 2^{(p_1+1/2)}, (\sigma_r \lambda_{\min}(\Gamma_r))^{(p_1+1/2)}, \\ k_{v1} 2^{(p_1+1/2)}, k_{r1} 2^{(p_1+1/2)}, (\sigma_u \lambda_{\min}(\Gamma_u))^{(p_1+1/2)}, (\sigma_v \lambda_{\min}(\Gamma_v))^{(p_1+1/2)}, \end{cases} \\
\rho_2 &= \min \begin{cases} k_{p12} 2^{(q_1+1/2)}, k_{p22} 2^{(q_1+1/2)}, (\sigma_r \lambda_{\min}(\Gamma_r))^{(q_1+1/2)}, \\ k_{u2} 2^{(q_1+1/2)}, k_{v2} 2^{(q_1+1/2)}, (\sigma_u \lambda_{\min}(\Gamma_u))^{(q_1+1/2)}, \\ k_{r2} 2^{(q_1+1/2)}, (\sigma_v \lambda_{\min}(\Gamma_v))^{(q_1+1/2)}, \end{cases} \\
\vartheta_1 &= \left(\frac{\sigma_u}{2} \lambda_{\min}(\Gamma_u) \lambda_{\max}(\Gamma_u^{-1}) \right)^{(q_1+1/2)} + \left(\frac{\sigma_v}{2} \lambda_{\min}(\Gamma_v) \lambda_{\max}(\Gamma_v^{-1}) \right)^{(q_1+1/2)} + \left(\frac{\sigma_r}{2} \lambda_{\min}(\Gamma_r) \lambda_{\max}(\Gamma_r^{-1}) \right)^{(q_1+1/2)}, \\
C_2 &= \frac{1}{2} \delta_v^2 + \frac{1}{2} \delta_u^2 + \frac{1}{2} \delta_r^2 + 3\vartheta_0 + \vartheta_1 \Delta_M^{q_1+1} + \frac{\sigma_u}{2} w_u^{*T} w_u^* + \frac{\sigma_v}{2} w_v^{*T} w_v^* + \frac{\sigma_r}{2} w_r^{*T} w_r^*.
\end{aligned} \tag{57}$$

In terms of Lemma 4, all tracking errors can converge to the residual set $D_{\text{total}} = D_{\text{total}_1} \cup D_{\text{total}_2}$ within a fixed time T_{total} . Then, D_{total} and T_{total} can be formulated as follows:

$$\begin{aligned}
D_{\text{total}_1} &= \{x: |x| \leq \sqrt{2}\Xi, x = \varepsilon_x, \varepsilon_y, u_e, v_e, r_e\}, \\
D_{\text{total}_2} &= \{\|\tilde{w}_i\|: \|\tilde{w}_i\| \leq \sqrt{2\lambda_{\max}(\Gamma_i)\Xi}, i = u, v, r\}, \\
\Xi &= \min \left\{ \left(\frac{C_2}{\rho_1 \theta_{\rho_1}} \right)^{2/p_1+1}, \left(\frac{C_2}{8^{(1-q_1)/2} \rho_2 \theta_{\rho_2}} \right)^{2/q_1+1} \right\}, \\
T_{\text{total}} &\leq \frac{2}{\rho_1(1-\theta_{\rho_1})(1-p_1)} + \frac{2}{8^{(1-q_1)/2} \rho_2(1-\theta_{\rho_2})(q_1-1)},
\end{aligned} \tag{58}$$

where $\theta_{\rho_1}, \theta_{\rho_2} \in (0, 1)$ are positive constants.

In the light of Theorem 1 and the analyses of Steps 1 and 2, the boundedness of ε_x and ε_y ensures that the position tracking errors x_e and y_e always satisfy their prescribed performance restraints. Furthermore, in terms of (21), we can conclude that the position tracking errors x_e and y_e will converge to a small region $D_p = D_{p1} \cup D_{p2}$ within a fixed time T_{total} , given by the following:

$$\begin{aligned}
D_{p1} &= \{x_e: \eta(T_{\text{total}})S_x(-\sqrt{2}\Xi) \leq x_e \leq \eta(T_{\text{total}})S_x(\sqrt{2}\Xi)\}, \\
D_{p2} &= \{y_e: \eta(T_{\text{total}})S_y(-\sqrt{2}\Xi) \leq y_e \leq \eta(T_{\text{total}})S_y(\sqrt{2}\Xi)\},
\end{aligned} \tag{59}$$

where $\eta(T_{\text{total}}) = (\eta_0 - \eta_{\infty})e^{-aT_{\text{total}}} + \eta_{\infty}$.

Remark 4. In the light of (58), we know that the coefficients ρ_1, p_1, ρ_2 , and q_1 determine the precision of the tracking errors, namely, the small enough tracking errors are obtained by selecting the suitable coefficients. If ε_x and ε_y approach zero within a fixed time T_{total} by choosing suitable coefficients, then the position tracking errors x_e and y_e approach zero within a fixed time based on (21) as well.

Remark 5. It is worth noting that the small tracking errors will bring about the large control energy so that the input saturation will occur in the control process. Accordingly, in practical applications, the coefficients ρ_1, p_1, ρ_2 , and q_1 should be adjusted discreetly for obtaining superior control performance.

Remark 6. For dealing with uncertainty in the control system, many successful methods have been proposed by researchers. For example, the multiobjective optimal control method [37, 38], fuzzy approximation strategy [39], and neural approximation strategy [40] can effectively solve uncertainty in control problems and may yield low computational cost. However, an ARBFNN is used to approximate the model uncertainty in this paper to better combine with the fixed-time theory. In future work, we will combine the multiobjective optimal control method and fuzzy/neural approximation strategy to effectively deal with the uncertainty in the hovercraft's control system for obtaining the optimal control performance and lower computational cost in the real-time control process.

Remark 7. Through the efforts of researchers, the general prescribed performance constraint control method is improved positively such as [41, 42]. The improved performance function was designed to overcome the existing prescribed performance constraint control method's shortcoming that the initial errors have to be known in advance for control designs. In this paper, it is the first time that fixed-time stability theory is introduced into the general prescribed performance control method. Therefore, the proposed method improved the convergence speed of the transformation errors, so that the convergence speed of the position tracking errors is also accelerated. Accordingly, this paper only uses a general prescribed performance control method that is better introduced into fixed-time stability theory, and the improved prescribed performance control method [41, 42] will be introduced into future work.

TABLE 1: Main particulars of the hovercraft.

m_0 (kg)	4000	J_{z0} (kg · m ²)	1.8×10^6
J_{x0} (kg · m ²)	2.5×10^5	S_{pp} (m ²)	45
S_{LP} (m ²)	93	S_{HP} (m ²)	260
g (m/s ²)	9.8	S_c (m ²)	212
l_{sk} (m)	65	B_c (m)	8.9
β_w (deg)	45	h_m (m)	2.4
l_c (m)	23.6	h (m)	1
H_{hov} (m)	5.9	V_w (knot)	5

FIGURE 2: The actual and desired trajectory of the hovercraft.

4. Simulation Results

In this section, we complete the computer simulation experiments by comparison with the finite-time terminal sliding mode control method in [9] to verify the effectiveness and robustness of the proposed control method. In simulations, the hovercraft's main particulars are shown in Table 1 [2, 9].

The control parameters are set as $k_{p11} = 0.01$, $k_{p12} = 0.02$, $k_{p21} = 0.001$, $k_{p22} = 0.001$, $k_x = 0.0005$, $k_y = 0.005$, $p_1 = 0.7$, $q_1 = 1.2$, $k_u = 1$, $k_v = 1.5$, $k_r = 2$, $k_{u1} = 0.1$, $k_{u2} = 0.4$, $k_{v1} = 1.2$, $k_{v2} = 0.15$, $k_{r2} = 3.5$, $k_{r1} = 0.02$, $\alpha_1 = \alpha_2 = 1$, $\beta_1 = \beta_2 = 1$, $\eta_0 = 200$, $\eta_\infty = 2.2$, $a = 0.02$, and $\delta_u = \delta_v = \delta_r = 0.1$. Since ARBFNN is utilized to approximate the unknown terms f_u , f_v , and f_r , the hidden node numbers for $\hat{w}_u^T h_u(z_u)$, $\hat{w}_v^T h_v(z_v)$, and $\hat{w}_r^T h_r(z_r)$ are set to $n = 13$ and the centres ϕ_{l_1} ($l_1 = 1, \dots, n$), ϕ_{l_2} ($l_2 = 1, \dots, n$), and ϕ_{l_3} ($l_3 = 1, \dots, n$) are evenly spaced in $[30, 40]$, $[-2, 2]$, and $[-1, 1]$, respectively. The widths are chosen as $\varepsilon_{l_1} = 2$, $\varepsilon_{l_2} = 1$, and $\varepsilon_{l_3} = 1$, respectively.

The hovercraft's initial values are set as $x(0) = -100$ m, $y(0) = 100$ m, $\phi(0) = 0$, $\psi(0) = 30^\circ$, $u(0) = 30$ knots, $v(0) = 0$, $p(0) = 0$, and $r(0) = 0$.

The virtual surface vessel's parameters are set as $x_d(0) = 0$, $y_d(0) = 0$, $\psi_d(0) = 45^\circ$, $u_{dset}(t) = 35$ knots, $v_{dset}(t) = 0$, and $r_{dset}(t) = 0.35^\circ/s$.

The total uncertainties suffered by the controlled system are characterized by the following formula:

$$\begin{bmatrix} f_u \\ f_v \\ f_r \end{bmatrix} = \begin{bmatrix} \frac{2u \sin(0.05t)}{(10m_0)} \\ \frac{v \cos(0.03t)}{m_0} \\ \frac{10^2 r \cos(0.02t)}{J_{z0}} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \quad (60)$$

where $b = [b_1 \ b_2 \ b_3]^T \in R^3$ and $\dot{b} = -T^{-1}b + Aw_n$ signifies the first-order Markov process with $w_n \in R^3$ representing the zero-mean Gaussian white noises' vector, and the rest coefficients of the first-order Markov process can be expressed as follows:

$$\begin{aligned} b(0) &= [2 \times 10^4, 2 \times 10^4, 2 \times 10^4]^T, \\ T &= \text{diag}(10^3, 10^3, 10^3), \\ A &= \text{diag}(1 \times 10^4, 1 \times 10^4, 1 \times 10^4). \end{aligned} \quad (61)$$

The simulation results of the two different methods with and without prescribed performance constraints are shown

FIGURE 3: Position tracking errors.

FIGURE 4: Input of two methods.

FIGURE 5: Velocity and yaw angular velocity tracking errors.

FIGURE 6: Roll angular and yaw angular states.

FIGURE 7: Velocity and yaw angular velocity states.

in Figures 2–7 and labelled as proposed controller and sliding mode controller, respectively. Figure 2 shows the hovercraft can follow the virtual surface vessel despite the presence of the system uncertainties. The desired trajectory is constructed by a circle, which can represent practical significance in the problem of path following or trajectory tracking. Furthermore, from Figure 3, it is obvious that the proposed method can ensure the prespecified transient and steady performance of the position tracking errors. However, the constraints on the position tracking errors are violated under the sliding mode controller.

Subsequently, from Figures 3 and 5, it is observed that the controllers designed in this paper are effective and can ensure that the position tracking error, velocity tracking errors, and the yaw angular velocity tracking error converge to a very small region around zero. Further observation shows that the proposed controller has a faster convergence speed and higher tracking precision than the sliding mode controller. The curves of the roll angle, yaw angle, and velocity states evolved with respect to time are shown in

Figures 6 and 7. These two controllers' control inputs are shown in Figure 4. All simulation results illustrate the superiority and robustness of the proposed method.

5. Conclusion

Driven by the practical requirements for the underactuated hovercraft to sail in confined channels, the Lyapunov direct method and fixed-time stability theory-based trajectory tracking control algorithm with a prescribed performance is proposed for the hovercraft's motion control while being subjected to model uncertainties and external disturbances. The prescribed performance bound technique and fixed-time stability theory, which are integrated into Lyapunov stability synthesis, successfully guarantee that the position tracking errors of the hovercraft remain within the pre-specified performance limits. It is shown that with the proposed control strategy, the requirement of the pre-specified performance is always not violated and all tracking errors are not only uniformly ultimately bounded but also

fixed-time convergent to an arbitrarily small region containing zero. In the future, we will consider that the velocity states are constrained to guarantee the safe sailing of the high-speed hovercraft.

Data Availability

The data in this paper are derived from the International Cooperation Projects. Hence, the data are private.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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