Research Article

On ISRC Rumor Spreading Model for Scale-Free Networks with Self-Purification Mechanism

Zijun Wang\textsuperscript{1,2} and An Chen\textsuperscript{1,2}

\textsuperscript{1}Institutes of Science and Development, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{2}University of Chinese Academy of Sciences, Beijing 100190, China

Correspondence should be addressed to An Chen; change1970@163.com

Received 9 October 2020; Revised 26 December 2020; Accepted 16 January 2021; Published 3 February 2021

Abstract

At present, the feasibility of using self-purification mechanism to inhibit rumor spreading has been confirmed by studies from different perspectives. This paper improves the classical rumor spreading models with self-purification mechanism, analyzes the correlation between spreading threshold in the model and its self-purification level theoretically, and conducts numerical simulations to study the impact of the changes of model parameters on key indicators in the process of rumor spreading. The simulation results show that changes of model parameters, including self-purification level and forgetting rate, exert significant influences on rumor spreading exactly.

1. Introduction

Rumors generally refer to unconfirmed information. Although they contain uncertain elements, most of them are false and wrong\cite{1,2}. For example, after the nuclear leakage caused by the Japanese earthquake, the rumor that eating iodized salt can prevent radiation led to a large number of people buying salt in many places, which not only caused public panic but also seriously disturbed the normal social order.

The systematic study of rumors began in World War II. Knapp\cite{3}, whose research laid a foundation for the study, sorted out the rumors generated during the war and classified them. In 1947, Allport and Postman\cite{4} believed that rumors were affected by personal values and other psychological factors and proposed the formula for the influence of rumors: \(R = I \times A\) (ambiguity of event). The research study on the dynamics of rumor spreading arose during the 1960s. Because the process of rumor spreading is similar to the process of disease infection, the nodes in the model of rumor spreading is frequently divided into three categories by many studies using the dynamic model of disease infection for reference. These are the ignorant, spreaders, and stiflers, corresponding to the susceptible, the infected, and the recovered, respectively, in the model of disease infection. The existing dynamical models of rumor spreading are based on two classical models: DK model\cite{5} and MT model\cite{6}. However, these models failed to accurately describe the rumor spreading process in large-scale social networks with increasing complexity of the objects studied. Therefore, scholars paid more attention to the phenomenon of rumor spreading in complex networks. Moreno et al.\cite{7,8} examined the rumor spreading thresholds in homogeneous and heterogeneous networks, respectively. Mo and Guo\cite{9} proposed a new control protocol to force the multiagent systems to achieve robust consensus. Zanette\cite{10,11} applied the rumor spreading model to static and dynamic small-world networks and verified the existence of critical threshold of the rumor spreading. Pan et al.\cite{12} proved that high clustering of the network could effectively resist the spread of rumors. Zhao et al.\cite{13,14} investigated the dynamic characteristics of the rumor spreading in the BA scale-free network and BBV network and held a viewpoint that the topology of the network has a great influence on the rumor spreading. Scholars also consider the influence of
suppress the spread of rumors and actively spreading the truth of information are two main methods to cope with rumor spreading in practice. However, some problems such as high coping costs, difficulty in confirming the truth in time, and insufficient effectiveness of external intervention keep emerging during this process [35]. Thus, scholars discuss how to apply the self-purification mechanism to solve relevant troubles [36], that is, to encourage users to release the complementation and correction of information, as well as refute false information. At present, the feasibility of using self-purification mechanism to inhibit rumor spreading has been confirmed by studies from different perspectives. Ge et al. [37] analyzed the inhibition effect of collective intelligence on the spread of false information in social media. Xia et al. [35] developed a new rumor spreading model for social media with self-purification mechanism and tested simulations, which unfolded the ability of social media to self-purify rumors. Tanaka et al. [38] proved that public questions and criticisms of rumors can affect individual judgments so that the spread of rumors can be suppressed.

Most of the current research studies on self-purification mechanism of networks are qualitative or case studies, and some other rumor spreading models are built with self-purification and skepticism mechanism. Zan et al. [19] who considered counterattack and self-resistance mechanism based on SIR model proposed the SICR rumor spreading model. Wang and Zhao [39] studied the SIQR rumor spreading model with skepticism mechanism and found that rumor truth disseminating rate plays an important role in rumor spreading process. Zhao et al. [40] added the group of doubters and constructed an SIHR rumor spreading model with self-protection awareness and skepticism mechanism, which suggests that many countermeasures can effectively prevent the dissemination of rumor, such as reducing the real contact rate between the ignorant and spreaders, improving the attention-degree of media to skeptics, and cutting down the depletion rate of mass media.

It is worth noting that the above models with self-purification mechanism are mostly based on homogeneous networks, while the theoretical analysis and related simulation studies of the complex networks, such as online social networks, are less involved. Moreover, once the status of doubters in most models is determined, even if they contact with other groups, the identity of doubters will not change. That is not completely consistent with the actual situation that rumor spreaders are mostly skeptical before spreading rumors [41]. In order to improve these shortcomings, the new rumor spreading model is introduced, traditional interaction rules are innovated, and correlation between spreading threshold in the model and its self-purification level is analyzed in this paper. In addition, simulations are designed and conducted to prove that the changes of model parameters, including self-purification level and forgetting rate, exert significant influences on rumor spreading exactly.

In this paper, the novelties are as follows. (1) adding the rule that “the criticizer may become a rumor spreader after hearing rumors for many times” as a complement to classical rules; (2) theoretical analysis of correlation between spreading threshold and self-purification level of the network is conducted; and (3) the influence of combinations of varying rumor spreading rate, forgetting rate, and self-purification capacity on the key indicators during rumor spreading process is explored. In the next section, the rumor spreading model for scale-free networks is built and relevant interaction rules, transformation relationships, and mean-field equations are introduced. Theoretical analysis about the correlation between spreading threshold and self-purification level is performed in Section 3. In Section 4, relevant numerical simulations are tested. Finally, conclusions and discussions are shown in Section 5.

2. Rumor Spreading Model Building

When building the rumor spreading model with self-purification mechanism, “whether an individual believes in rumor, whether an individual spreads rumor, and whether an individual criticizes rumor” are taken as a basis for grouping, and the situation of criticizing is a sufficient but non-necessary condition for the situation of disbelieving.

The rumor spreading model in this paper is called the ISRC model, where ‘I’ stands for the ignorant, ‘S’ means spreaders, ‘R’ denotes the recovered, and ‘C’ indicates criticizers. $I(t)$ refers to the density of the ignorant who do not hear the rumor at time $t$, and for the same reason, $S(t)$, $R(t)$, and $C(t)$ show the density of spreaders who believe in and spread the rumor at time $t$, the density of the recovered who can determine falsity of the rumor and do not spread or criticize it at time $t$, and the density of criticizers who criticize the rumor at time $t$, respectively. It is a remarkable fact that the criticizers consist of the individuals who can accurately identify the rumor and the others who criticize the rumor without understanding the truth of rumor information. The latter is likely to be assimilated by rumor spreaders due to the herd effect after getting access to the rumor for many times. That corresponds to the conclusion in the previous study that “the criticizer may become a rumor spreader after hearing rumors for many times [41].”

Correspondingly, $I_k(t)$ represents the density of the ignorant with connectivity $k$ at time $t$. The meanings of $S_k(t)$, $R_k(t)$, and $C_k(t)$ are similar to that of $I_k(t)$ and will not be repeated here. We have that $I(t) = \sum_k I_k(t) p(k)$ with $p(k)$ the degree distribution and so do $S(t)$, $R(t)$, and $C(t)$. In addition, $I(t) + S(t) + R(t) + C(t) = 1$ and $I_k(t) + S_k(t) + R_k(t) + C_k(t) = 1$.

Here, we assume that (1) the rumor spreading model targets a single rumor and does not consider the interaction among multiple rumors; (2) the method without nodes joining or leaving is used to keep the total number of nodes
in the network constant in the selected period; (3) the transformation of these four groups must be based on the information exchange between groups, and only spreaders and critics can spread the information, the others are receivers of the information; (4) considering the herd effect, some critics who do not understand the truth may be assimilated by rumor spreaders after repeated exposures to the rumor; similarly, rumor spreaders have chance to be the recovered after hearing criticizing information for many times; and (5) rumor exchange within the group of spreaders has no effect on the change of status of both sides.

For scale-free networks, the transformation relationships and interaction rules among $I$, $S$, $R$, and $C$ are illustrated in Figure 1.

The following formulas hold $\alpha + a_2 + a_3 = 1$ and $b_1 + b_2 = 1$. According to the relevant dynamical method, the mean-field equations can be described as follows:

\[
\begin{align*}
\frac{dI_k(t)}{dt} &= -kI_k(t) \sum_{k'} C_{k'}(t)p\left(\frac{k'}{k}\right) + \sum_{k'} S_{k'}(t)p\left(\frac{k'}{k}\right), \\
\frac{dS_k(t)}{dt} &= a_kI_k(t) \sum_{k'} S_{k'}(t)p\left(\frac{k'}{k}\right) + a_3kC_k(t) \sum_{k'} S_{k'}(t)p\left(\frac{k'}{k}\right) - \beta S_k(t) - \theta kS_k(t) \sum_{k'} C_{k'}(t)p\left(\frac{k'}{k}\right), \\
\frac{dR_k(t)}{dt} &= \beta S_k(t) + \beta C_k(t) + \theta kS_k(t) \sum_{k'} C_{k'}(t)p\left(\frac{k'}{k}\right) + a_1kI_k(t) \sum_{k'} S_{k'}(t)p\left(\frac{k'}{k}\right) + b_1kI_k(t) \sum_{k'} C_{k'}(t)p\left(\frac{k'}{k}\right), \\
\frac{dC_k(t)}{dt} &= a_2kI_k(t) \sum_{k'} S_{k'}(t)p\left(\frac{k'}{k}\right) + b_2kI_k(t) \sum_{k'} C_{k'}(t)p\left(\frac{k'}{k}\right) - \beta C_k(t) - a_3kC_k(t) \sum_{k'} S_{k'}(t)p\left(\frac{k'}{k}\right).
\end{align*}
\]

where $p(k'/k)$ denotes the conditional probability that a node with $k$ links is connected to a node with degree $k'$. It can be written as $p(k'/k) = k' \cdot p(k')/\langle k \rangle$, where $\langle k \rangle$ means the average degree and $p(k')$ denotes the degree distribution $[15]$.

### 3. Analysis of Correlation

In this section, we analyze the correlation between spreading threshold in the ISRC model for scale-free networks and self-purification level. Considering the model shown in Figure 1, only nodes of type I and type R exist in the final network.

Referring to the idea on the SIR-like model for complex networks in reference [42], spreaders $S$ and critics $C$ are grouped into one category in this section and recorded as Information Spreaders, denoted by $X$. This model is called IXR model, and its transformation relationships and interaction rules among $I$, $X$, and $R$ are shown in Figure 2.

Mean-field equations are depicted by the following equations:

\[
\begin{align*}
\frac{dI_k(t)}{dt} &= -kI_k(t) \sum_{k'} X_{k'}(t)p\left(\frac{k'}{k}\right), \\
\frac{dX_k(t)}{dt} &= p_1kI_k(t) \sum_{k'} X_{k'}(t)p\left(\frac{k'}{k}\right) - p_3kX_k(t) \sum_{k'} X_{k'}(t)p\left(\frac{k'}{k}\right) - p_XX_k(t), \\
\frac{dR_k(t)}{dt} &= p_2kI_k(t) \sum_{k'} X_{k'}(t)p\left(\frac{k'}{k}\right) + p_3kX_k(t) \sum_{k'} X_{k'}(t)p\left(\frac{k'}{k}\right) + p_XX_k(t).
\end{align*}
\]

where $p_1 + p_2 = 1$ and $p(k'/k)$ stand for the conditional probability, represented as $p(k'/k) = k' \cdot p(k')/\langle k \rangle$, where $\langle k \rangle$ means average degree and $p(k')$ denotes degree distribution.

We assume a homogeneous initial distribution of the ignorant $I_k(0) = I(0)$, and set $I_k(0) = 1$ without loss of generality. In this case, equation (2) can be integrated directly yielding

\[
I_k(t) = e^{-kp(t)},
\]

where an auxiliary function is defined as follows:
integrating terms in the equation, we obtain

\[
\Phi(t) = \int \sum_k \langle XX_k(t') \rangle \, dt',
\]

with the shortened form \( \langle MM_k(t') \rangle = \sum_k M_k(t')q(k) \), and \( q(k') \) is the simple mark for \( p(k'/k) \) for convenience.

Multiplying equation (3) with \( q(k) \), summing over, and integrating terms in the equation, we obtain

\[
\frac{d\Phi(t)}{dt} = p_1 \sum_k kI_k(t)q(k)\langle XX_k(t') \rangle \, dt' - p_3 \sum_k kX_k(t')q(k)\langle XX_k(t') \rangle \, dt' + p_4 \Phi(t)
\]

\[
= p_1 [1 - \langle e^{-k\Phi(t)} \rangle] - p_3 \int_0^t \langle XX_k(t') \rangle \langle XX_k(t') \rangle \, dt' - p_4 \Phi(t).
\]

Because \( \Phi(t) \) and \( \Phi(\infty) = \lim_{t \to \infty} \Phi(t) \) are very small when close to the critical threshold, we assert \( \Phi(t) = f(t)\Phi(\infty) \), where \( f(t) \) denotes a finite function. We solve equation (3) using the method to ODE, solve equation

\[
(7) \text{using Taylor series expansion and retaining leading terms and eventually derive the expression of } X_k(t) \text{ as follows:}
\]

\[
X_k(t) = p_1 \int_0^t kI_k(t') \sum_k X_k(t')q(k') \, dt' - p_3 \int_0^t kX_k(t') \sum_k X_k(t')q(k') \, dt'
\]

\[
- p_4 \int_0^t X_k(t') \, dt'
\]

\[
= p_1 k\Phi(\infty) \left[ f(t) - p_4 \int_0^t f(t')e^{p_3(t'-t)} \, dt' \right] + O(\Phi^2(\infty)) + O(p_3).
\]

When \( t \to \infty \), \( (d\Phi(t)/dt) \to 0 \). Therefore, according to equation (7), we have

\[
(9) \quad \Phi(t) \to \Phi(\infty) = 0.
\]

Insert equations (8) into (9) and expand the exponential to the relevant order in \( \Phi(\infty) \) yielding

\[
(10) \quad L = \int_0^\infty \left[ p_1 (f(t) - p_4 \int_0^t f(t')e^{p_3(t'-t)} \, dt')^2 \right] \, dt
\]

is a positive-defined integral.

Consequently,
where $\Phi(\infty) = 0$ is always a solution. We can find the nonzero solution as follows:

$$\Phi(\infty) = \frac{2(p_1\langle k \rangle - p_s)}{\langle k^2 \rangle [p_1 + 2p_s\langle k \rangle L]}$$

(12)

From expression of $\Phi(\infty)$ in equation (12), we learn $\langle k^2 \rangle [p_1 + 2p_s\langle k \rangle L]$ is always positive. $2(p_1\langle k \rangle - p_s) > 0$ needs to be true to ensure $\Phi(\infty)$ is positive, i.e., $p_1 > p_s\langle k \rangle = p_s\langle k \rangle / \langle k^2 \rangle$. Therefore, we obtain the spreading threshold in the IXR model:

$$\lambda = \frac{p_s\langle k \rangle}{\langle k^2 \rangle}$$

(13)

This result shows that the spreading threshold in the IXR model is not only related to the degree of nodes in the network $k$ but also depends on the forgetting rate $p_s$. That actually indicates that spreading threshold in the IXR model for scale-free networks is uncorrelated with the self-purification level $p_s$. It can be deduced that there is no correlation between spreading threshold in the ISRC model for scale-free networks and self-purification level of the network.

4. Numerical Simulations

In this section, the Runge–Kutta method is used to solve the system of differential equation (1), and the numerical simulations are conducted by using NetLogo to analyze the influence of changes of rumor spreading rate, forgetting rate, and self-purification level on process and results of rumor spreading in scale-free networks. Maximum value of the sum of the density of group $S$ in the process of rumor spreading is regarded as peak value of rumor influence (abbreviated as PVI), and the moment when this situation is reached is called arrival time of peak value of rumor influence (abbreviated as TPVI). Since the types of remaining nodes in the final network will only be part or all of $I$ and $R$, the situation in which the density of $I$ and the density of $R$ is no longer changing is set a sign of the end of the rumor spreading process. The duration of rumor spreading is abbreviated as DRS. These three indicators (PVI, TPVI, and DRS) reflect the pros and cons of the effect to inhibit rumor spreading. For example, if with smaller PVI, earlier TPVI, and shorter DRS, there will be better effect to inhibit rumor spreading.

In this section, the rumor is set to spread in a scale-free network with $N = 10^3$ nodes, power exponent $\gamma = 2.8$, and average degree $\langle k \rangle = 5$. In simulations, there are 10 spreaders in the initial network, i.e., $S(0) = 10^3/10^3$, $I(0) = (10^3 - 10^3/10^3)$, $R(0) = 0$, and $C(0) = 0$. 30 simulations under each condition is performed, and average value of all results under each condition as final result is taken (the values of PVI are account to two decimal places, and the values of TPVI and DRS are accurate to one decimal place).

Figure 3 shows the influence of changes of rumor spreading rate $\alpha$ and self-purification level $\theta$ on three indicators (PVI, TPVI, and DRS) during the process of rumor spreading. Figure 3(a) displays how PVI changes for four values of $\alpha$. It can be seen from Figure 3(a) that PVI decreases as $\theta$ increases, and for the same level of self-purification, the higher the rumor spreading rate $\alpha$ is, the greater the PVI is. Figure 3(a) reflects that the improving self-purification level and reducing rumor spreading rate are conducive to reducing the peak value of rumor influence in process of rumor spreading. Correspondingly, Figures 3(b) and 3(c) reveal how TPVI and DRS change. From Figure 3(b), we can obtain that with small $\alpha$ ($\alpha = 0.2, 0.4, 0.6$), TPVI is negatively correlated with $\theta$, but TPVI has little changes in the situation of $\alpha = 0.8$. When $\theta < 0.3$, TPVI decreases as $\alpha$ increases, while $\theta > 0.3$, TPVI decreases in $\alpha$. Therefore, Figure 3(b) implies that if rumor spreading rate remains at a high level, the improvement of self-purification level of networks may not affect the arrival time of peak value of rumor influence, and Figure 3(b) also indicates that $\theta = 0.3$ is a critical value; when $\theta < 0.3$, the higher the rumor spreading rate is, the earlier the arrival time of peak value of rumor influence is; when $\theta > 0.3$, result is the opposite. Figure 3(c) manifests that $\alpha$ and $\theta$ have minor effects on DRS, which also means duration of rumor spreading is not clearly related to rumor spreading rate or self-purification level. Therefore, as a whole, at a low rumor spreading rate and high self-purification level, there is a better effect to inhibit rumor spreading in the network, while in the case of high rumor spreading rate, as the level of self-purification increases, the effect to inhibit rumor spreading may become worse.

Figure 4 displays the influence of changes of forgetting rate $\beta$ and self-purification level $\theta$ on three indicators (PVI, TPVI, and DRS) during the process of rumor spreading. Figure 4(a) shows how PVI changes for four values of $\beta$. We can learn from Figure 4(a) that PVI is negatively correlated with $\theta$, and for the same level of self-purification, the higher the forgetting rate $\beta$ is, the smaller the PVI is. Figure 4(a) reveals that the improvement of self-purification level and forgetting rate is conducive to reducing the peak value of rumor influence in process of rumor spreading. Accordingly, Figure 4(b) demonstrates how TPVI changes. For same $\beta$, as $\theta$ increases, the values of TPVI tend to decrease. However, as forgetting rate $\beta$ increases, the changes of TPVI become more and more irregular and erratic. It is conveyed by Figure 4(c) that DRS is negatively correlated with $\beta$, and for same forgetting rate, the change of $\theta$ has a minor effect on DRS. Figure 4(c) also means that duration of rumor spreading is considerably related to forgetting rate. Thus, whether the network is at a high or low forgetting rate, on the whole, the increase in self-purification level will lead to a tendency to suppress rumor spreading. However, compared with the case of high forgetting rate, when the forgetting rate
Figure 3: Influence of changes of rumor spreading rate $\alpha$ and self-purification level $\theta$ on three indicators PVI, TPVI, and DRS. Simulations are conducted under the condition $a_3 = 0.33, b_1 = b_2 = 0.5, \beta = 0.1$, and $a_1 = a_2$. (a) PVI change for four values of $\alpha$ ($\alpha = 0.2, 0.4, 0.6, 0.8$), (b) TPVI change, and (c) DRS change.
is low, the improvement of self-purification level has a more significant effect to inhibit rumor spreading.

5. Conclusions and Discussion

This paper improves classical rumor spreading models with self-purification mechanism, analyzes the correlation between spreading threshold in the model and its self-purification level theoretically, and conducts numerical simulations to prove that the changes of model parameters, including self-purification level and forgetting rate, exert significant influences on rumor spreading. Novel features and significant results are summarized into three respects:

(1) When building the model, take “the criticizer may become a rumor spreader after hearing rumors for many times” into account and add variable \(a_3\) to convey the probability of a criticizer transforming to a rumor spreader under the influence of rumor spreaders.

(2) Through theoretical analysis about the correlation between spreading threshold in the model and its self-purification level, we find the spreading
threshold in the ISRC rumor spreading model for scale-free networks has no correlation with self-purification mechanism. More precisely, the spreading threshold has no correlation with the probability of a criticizer transforming to a rumor spreader under the influence of rumor spreaders.

(3) Numerical simulations are conducted to study the impact of changes of model parameters on key indicators in process of rumor spreading. The results manifest that changes of model parameters exert significant influences on rumor spreading exactly. On the one hand, at a low rumor spreading rate and high self-purification level, there is a better effect to inhibit rumor spreading in the network, while in the case of high rumor spreading rate, as self-purification level increases, the effect may become worse; on the other hand, whether the network is at a high or low forgetting rate, on the whole, the increase in self-purification level will lead to a tendency to suppress rumor spreading. However, compared with the case of high forgetting rate, when the forgetting rate is low, the improvement of self-purification level has a more significant effect to inhibit rumor spreading.

It should be noted that the authoritative effect of individuals is not considered in this paper, and the forgetting rate is regarded as a fixed value. In real social networks, there are “opinion leaders,” which mirror that the more fans a person has, the greater his or her assimilation influence on other individuals is. Considering the forgetting rate tends to change over time in real networks, the authoritative effect of individuals and the dynamic forgetting rate can be combined in the follow-up research to further explore the influence of changes of related parameters on the process of rumor spreading.

Data Availability

The data in this paper are obtained through simulations by NetLogo 6.1.0. The specific code and data are available from the author (fightingzj@163.com) on reasonable request.

Conflicts of Interest

The authors declare that they have no potential conflicts of interest.

Supplementary Materials

The code within Code.txt is run using Netlogo 6.1.0 and adjusted the parameters to get simulation results (results of authors are shown in Data.txt). Figures are plotted according to the data within Data.txt and eventually Figures 3(a)–3(c) and Figures 4(a)–4(c) are obtained in this paper. (Supplementary Materials)

References


