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Research Article

Robust Iterative Learning Control for 2-D Singular Fornasini–Marchesini Systems with Iteration-Varying Boundary States

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This study first investigates robust iterative learning control (ILC) issue for a class of two-dimensional linear discrete singular Fornasini–Marchesini systems (2-D LDSFM) under iteration-varying boundary states. Initially, using the singular value decomposition theory, an equivalent dynamical decomposition form of 2-D LDSFM is derived. A simple P-type ILC law is proposed such that the ILC tracking error can be driven into a residual range, the bound of which is relevant to the bound parameters of boundary states. Specially, while the boundary states of 2-D LDSFM satisfy iteration-invariant boundary states, accurate tracking on 2-D desired surface trajectory can be accomplished by using 2-D linear inequality theory. In addition, extension to 2-D LDSFM without direct transmission from inputs to outputs is presented. A numerical example is used to illustrate the effectiveness and feasibility of the designed ILC law.

1. Introduction

Two-dimensional (2-D) singular dynamical systems derived from the discretization of spatiotemporal dynamical systems with singular matrices or singular distributed parameter systems have received much attention due to their extensive applications in physical phenomena and industrial processes, such as electrical circuits [1], nanoelectronics [2], transmission lines in signal propagation [3], and power systems [4]. In recent years, fruitful results on 2-D singular systems in the infinite coordinate domain have been reported, mainly including the detectability and observer design [5], H_∞ control [6], and stability analysis [7]. However, in practical industrial applications, 2-D singular systems are often required to execute some specific tracking control tasks repetitively over the finite coordinate domain. For example, in form-closure grasps, the immobilized manipulation of serial chains described by 2-D singular systems could be regarded as a repetitive control problem [8]. Also, in the field of mold processing and material surface manufacturing, it is usually required to obtain high-precision 2-D reference surface by repeated operations of the controlled processing units [9]. Obviously, for the repetitively tracking cases mentioned above, the traditional tracking control approaches for 2-D singular systems in the infinite coordinate domain is difficult to be applicable.

Iterative learning control (ILC), as an intelligent control method, is used to address the repetitive trajectory tracking problem for one-dimensional (1-D) dynamical systems with less precise model knowledge, which makes ILC be widely prevalent in practical applications. A large number of ILC research results for 1-D dynamical systems have been reported in the past decades [10–16]. However, only very few ILC results for 2-D dynamical systems is involved in [17–24]. For 2-D linear discrete nonsingular Fornasini–Marchesini systems (2-D LDNFM), a two-gain ILC algorithm is proposed in [20] to address the robust ILC issue under the iteration-varying boundary states, and the detailed proof process of convergence analysis is given. Also, to track a class of nonrepetitive reference surface trajectory represented by a high-order internal model (HOIM) operator, two HOIM-based ILC laws were, respectively, investigated in [21] for 2-D LDNFM by using HOIM-based linear inequality theory, but the ultimate ILC tracking error can only converge to a bounded range. To accomplish the objective of zero tracking error, an adaptive ILC technique is proposed in [24] to deal with the tracking problem of iteration-varying reference surface trajectory. Unfortunately, it requires that the gain matrix of 2-D LDNFM must be positive-definite (or negative-definite), such that the proposed adaptive ILC algorithm, in practical applications, are severely confined. It is worth emphasizing that the aforementioned ILC results have concerned on a nonsingular case. However, in practical life and industry, there exist some 2-D singular dynamical systems, such as electrical circuits, transmission lines in signal propagation, and power systems, which are often required to execute some specific tracking control tasks repetitively. Based on these practical applications, it is essential to exploit ILC techniques for 2-D singular dynamical systems.

The main aim of this paper is to investigate the robustness and convergence property of P-type ILC law for two classes of two-dimensional linear discrete singular Fornasini–Marchesini systems (2-D LDSFM) under iteration-varying boundary states. By using singular value decomposition theory, an equivalent dynamical decomposition form of 2-D LDSFM is derived. By using 2-D linear inequality theory, it can guarantee that the ultimate tracking error tends to a bounded range, the bound of which is relevant to the bound parameters of boundary states. The main contributions of this paper relative to the related works are summarized as follows:

(1) In the existing ILC results for 2-D linear discrete systems [17–24], they are concerned on a nonsingular case. To the best of our knowledge, this is the first time to investigate robust ILC algorithms for 2-D discrete singular systems in this paper.

(2) Compared with the adaptive ILC algorithm for 2-D LDNFM in [24], the ILC algorithm proposed in this paper has no restriction on the numbers of system inputs and outputs.

(3) In the study of ILC algorithms for 1-D singular systems [25–27], the λ-norm and identical boundary condition have been widely used to analysis the convergence of the proposed ILC laws. However, the λ-norm might not provide a satisfactory measurement of ILC tracking errors [22], and thus, it is not involved in analysing the proposed ILC law in the paper.

The remaining section of this paper is arranged as follows: The problem description is provided in Section 2. Sections 3 and 4, respectively, present robustness and convergence analysis of P-type ILC law for 2-D LDSFM (1) and extension to 2-D LDSFM with $D = 0$. In Section 5, a simulation example is introduced. Finally, Section 6 gives a conclusion.

2. Problem Description

Consider a ILC issue for the following two-dimensional linear discrete singular Fornasini–Marchesini systems (2-D LDSFM) [28] over finite region $n_1 \in \{0, 1, \ldots, N_1 - 1\}$ and $n_2 \in \{0, 1, \ldots, N_2 - 1\}$:

$$\begin{cases} E_g x_k(n_1+1, n_2+1) = A_1 x_k(n_1+1, n_2) + A_2 x_k(n_1, n_2) \\ + A_3(n_1, n_2+1) + B u_k(n_1, n_2), \\ y_k(n_1, n_2) = C x_k(n_1, n_2) + D u_k(n_1, n_2), \end{cases} \tag{1}$$

where $u_k(n_1, n_2) \in R^m$, $x_k(n_1, n_2) \in R^p$, and $y_k(n_1, n_2) \in R^s$ represent, respectively, control input, system state, and system output and E_g, A_1, A_2, A_3, B, C, and D are real matrices with appropriate dimensions. $k \in \{0, 1, 2, \ldots\}$ denotes the k-th iteration of controlled system (1); n_1 and n_2 are, respectively, horizontal dynamical index and vertical dynamical index. It is worth noting that as E_g is nonsingular (without loss of generality, let $E_g = I_p$, where I_p represents identity matrix with $(p \times p)$), (1) is called a regular 2-D LDFFM [28], which has been investigated in [21]. However, in this paper, a singular matrix E_g with $\operatorname{rank}(E_g) = r < p$ is considered.

For $n_1 \in \{0, 1, \ldots, N_1\}$ and $n_2 \in \{0, 1, \ldots, N_2\}$, let an achievable desired surface trajectory and the corresponding tracking error at k-th iteration be denoted as $y_d(n_1, n_2)$ and $e_k(n_1, n_2)$, respectively. The control objective of ILC for 2-D LDSFM (1) is to generate a control input sequence $u_k(n_1, n_2)$, where $n_1 \in \{0, 1, \ldots, N_1 - 1\}$ and $n_2 \in \{0, 1, \ldots, N_2 - 1\}$ with an iterative way, such that the actual tracking output $y_k(n_1, n_2)$ can accurately track $y_d(n_1, n_2)$, i.e.,

$$\lim_{k \longrightarrow +\infty} y_k(n_1, n_2) = y_d(n_1, n_2), \tag{2}$$

where $n_1 \in \{0, 1, \ldots, N_1\}$ and $n_2 \in \{0, 1, \ldots, N_2\}$.

According to the singular value matrix theory [29], there exist two nonsingular matrices $P \in R^{p\times p}$ and $Q \in R^{p\times p}$ such that

$$\begin{aligned} PE_gQ &= \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \\ PA_1Q &= \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{bmatrix}, \\ PA_2Q &= \begin{bmatrix} \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{21} & \widehat{A}_{22} \end{bmatrix}, \\ PA_3Q &= \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix}, \\ PB &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \\ CQ &= \begin{bmatrix} C_1 & C_2 \end{bmatrix}. \end{aligned} \tag{3}$$

2-D LDSFM (1) can be transformed into a decomposition form

$$\begin{aligned} z_k^1(n_1+1,n_2+1) = {} & \overline{A}_{11} z_k^1(n_1+1,n_2) + \widetilde{A}_{12} z_k^2(n_1+1,n_2) + \widehat{A}_{11} z_k^1(n_1,n_2) + \widehat{A}_{12} z_k^2(n_1,n_2) \\ & + \widetilde{A}_{11} z_k^1(n_1,n_2+1) + \widetilde{A}_{12} z_k^2(n_1,n_2+1) + B_1 u_k(n_1,n_2), \end{aligned} \tag{4}$$

$$\begin{aligned} 0 = {} & \overline{A}_{21} z_k^1(n_1+1,n_2) + \overline{A}_{22} z_k^2(n_1+1,n_2) + \widehat{A}_{21} z_k^1(n_1,n_2) + \widehat{A}_{22} z_k^2(n_1,n_2) \\ & + \widetilde{A}_{21} z_k^1(n_1,n_2+1) + \widetilde{A}_{22} z_k^2(n_1,n_2+1) + B_2 u_k(n_1,n_2), \end{aligned} \tag{5}$$

$$y_k(n_1,n_2) = C_1 z_k^1(n_1,n_2) + C_2 z_k^2(n_1,n_2) + D u_k(n_1,n_2), \tag{6}$$

where $z_k(n_1,n_2) = \begin{bmatrix} z_k^1(n_1,n_2) \\ z_k^2(n_1,n_2) \end{bmatrix} = Q^{-1} x_k(n_1,n_2)$, $z_k^1(n_1,n_2) \in R^r$, $z_k^2(n_1,n_2) \in R^{p-r}$, and $Q^{-1} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$. For some thermal processes in chemical reactors, heat exchangers and pipe furnaces, $z_k(n_1,n_2)$, usually represents temperature at space n_1 and time n_2 in [6, 30].

For the convenience of discussing the ILC problem, Definitions 1 and 2 on nonnegative matrix (vector), Assumptions 1–3, and Lemma 1 for 2-D LDSFM (1) are provided.

Definition 1. If every element of a matrix (or vector) is nonnegative, then the matrix (or vector) is said to be nonnegative, i.e., for $A = [a_{ij}] \in R^{L_1 \times L_2}$, if $a_{ij} \geq 0$, where $i \in \{1, 2, \ldots, L_1\}$ and $j \in \{1, 2, \ldots, L_2\}$, then it is denoted that $A \geq 0$.

Definition 2. For two matrices $P = [p_{ij}] \in R^{L_1 \times L_2}$ and $S = [s_{ij}] \in R^{L_1 \times L_2}$, $P \leq S$ is denoted if $p_{ij} \leq s_{ij}$ for $i \in \{1, 2, \ldots, L_1\}$ and $j \in \{1, 2, \ldots, L_2\}$.

Assumption 1. For 2-D LDSFM (1), let boundary states $x_k(0,n_2)$ and $x_k(n_1,0)$ be satisfied as $\|x_k(0,n_2) - x_0(0,n_2)\| \leq \Lambda_1$, with $n_2 \in \{1, 2, \cdots, N_2\}$ and $\|x_k(n_1,0) - x_0(n_1,0)\| \leq \Lambda_2$, with $n_1 \in \{0, 1, \cdots, N_1\}$, where $x_0(0,n_2)$ and $x_0(n_1,0)$ are time-varying functions with respective to n_2 and n_1, respectively; Λ_1 and Λ_2 are unknown constants. $\|\cdot\|$ represents the infinite norm of vector/matrix in this paper.

Assumption 2. $\widetilde{A}_{22}$ is an invertible matrix.

Assumption 3. 2-D LDSFM (1) is regular, if there exists two complex numbers z_1 and z_2 to make $\det(E_g z_1 z_2 - A_1 z_1 - A_2 - A_3 z_2) \neq 0$.

Lemma 1. *Consider the following 3-D linear discrete inequality system for $n_1 \in \{0, 1, \ldots, N_1\}$, $n_2 \in \{0, 1, \ldots, N_2\}$, and $k \in \{0, 1, 2, \ldots\}$:*

$$\theta_{k+1}(n_1,n_2) \leq \alpha \theta_k(n_1,n_2) + \delta_k(n_1,n_2), \tag{7}$$

where $\theta_k(n_1,n_2) \in R$ and $\delta_k(n_1,n_2) \in R$, respectively, are state and control input and α denotes real constant. Suppose the boundary state $\theta_0(n_1,n_2) = f(n_1,n_2)$, where $f(n_1,n_2)$ is a bounded vector function. When $\limsup_{k \longrightarrow +\infty} \delta_k(n_1,n_2) \leq b_\delta$, if $0 < \alpha < 1$ is satisfied, then

$$\limsup_{k \longrightarrow +\infty} \theta_k(n_1,n_2) \leq \frac{b_\delta}{1-\alpha}. \tag{8}$$

The proof process of Lemma 1 is similar with [22].

Remark 1. In the existing ILC results for 2-D LDNFM [20, 21, 23], it is usually required that boundary states are satisfied as identical boundary states. However, in practical ILC applications for 2-D systems, it is difficult to obtain in each repetitive operation. To this end, ILC for 2-D LDSFM (1) under iteration-varying boundary states is investigated in this paper. Assumptions 2 and 3 are basic and reasonable assumptions in control theory of the 2-D singular system [28, 31].

3. Robustness and Convergence Analysis of P-Type ILC Law for 2-D LDSFM (1)

According to the system characteristic of 2-D LDSFM (1), the following P-type ILC law is proposed for $n_1 \in \{0, 1, \ldots, N_1\}$ and $n_2 \in \{0, 1, \ldots, N_2\}$:

$$u_{k+1}(n_1,n_2) = u_k(n_1,n_2) + K e_k(n_1,n_2), \tag{9}$$

where the learning gain $K \in R^{m \times s}$ is to be designed.

Theorem 1. *For 2-D LDSFM (1) under Assumptions 1-3, use the P-type ILC law (9). If there exists the learning gain K satisfying $\|I_s - DK\| < 1$, then the tracking error $e_k(n_1,n_2)$, $n_1 \in \{0, 1, \ldots, N_1\}$, $n_2 \in \{0, 1, \ldots, N_2\}$ converges to a bounded range, i.e.,*

$$\limsup_{k \longrightarrow +\infty} \|e_k(n_1,n_2)\| \leq \beta_e, \tag{10}$$

where β_e is a certain bound related to the bound parameters Λ_1 and Λ_2 in Assumption 1.

Proof. For $n_1 \in \{0, 1, \ldots, N_1\}$ and $n_2 \in \{0, 1, \ldots, N_2\}$, let

$$\delta z_{k+1}^1(n_1,n_2) = z_{k+1}^1(n_1,n_2) - z_k^1(n_1,n_2), \tag{11}$$

$$\delta z_{k+1}^2(n_1,n_2) = z_{k+1}^2(n_1,n_2) - z_k^2(n_1,n_2). \tag{12}$$

Using (11) and (5), and considering (4) and (12), there is

$$
\begin{aligned}
\delta z_{k+1}^{1}(n_1+1,n_2+1) &= z_{k+1}^{1}(n_1+1,n_2+1) - z_{k}^{1}(n_1+1,n_2+1) \\
&= \overline{A}_{11}\delta z_{k+1}^{1}(n_1+1,n_2) + \overline{A}_{12}\delta z_{k+1}^{2}(n_1+1,n_2) + \widehat{A}_{11}\delta z_{k+1}^{1}(n_1,n_2) \\
&\quad + \widehat{A}_{12}\delta z_{k+1}^{2}(n_1,n_2) + \widetilde{A}_{11}\delta z_{k+1}^{1}(n_1,n_2+1) + \widetilde{A}_{12}\delta z_{k+1}^{2}(n_1,n_2+1) + B_1\left[u_{k+1}(n_1,n_2) - u_k(n_1,n_2)\right], \\
0 &= \overline{A}_{21}\delta z_{k+1}^{1}(n_1+1,n_2) + \overline{A}_{22}\delta z_{k+1}^{2}(n_1+1,n_2) + \widehat{A}_{21}\delta z_{k+1}^{1}(n_1,n_2) \\
&\quad + \widehat{A}_{22}\delta z_{k+1}^{2}(n_1,n_2+1) + \widetilde{A}_{21}\delta z_{k+1}^{1}(n_1,n_2+1) + \widehat{A}_{22}\delta z_{k+1}^{2}(n_1,n_2+1) + B_2\left[u_{k+1}(n_1,n_2) - u_k(n_1,n_2)\right]
\end{aligned}
\tag{13}
$$

where $n_1 \in \{0,1,\ldots,N_1-1\}$ and $n_2 \in \{0,1,\ldots,N_2-1\}$.

Then, using ILC law (8), we have

$$
\begin{aligned}
\delta z_{k+1}^{1}(n_1+1,n_2+1) &= \overline{A}_{11}\delta z_{k+1}^{1}(n_1+1,n_2) + \overline{A}_{12}\delta z_{k+1}^{2}(n_1+1,n_2) + \widehat{A}_{11}\delta z_{k+1}^{1}(n_1,n_2) + \widehat{A}_{12}\delta z_{k+1}^{2}(n_1,n_2) \\
&\quad + \widetilde{A}_{11}\delta z_{k+1}^{1}(n_1,n_2+1) + \widetilde{A}_{12}\delta z_{k+1}^{2}(n_1,n_2+1) + B_1 K e_k(n_1,n_2),
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
0 &= \overline{A}_{21}\delta z_{k+1}^{1}(n_1+1,n_2) + \overline{A}_{22}\delta z_{k+1}^{2}(n_1+1,n_2) + \widehat{A}_{21}\delta z_{k+1}^{1}(n_1,n_2) + \widehat{A}_{22}\delta z_{k+1}^{2}(n_1,n_2) \\
&\quad + \widetilde{A}_{21}\delta z_{k+1}^{1}(n_1,n_2+1) + \widetilde{A}_{22}\delta z_{k+1}^{2}(n_1,n_2+1) + B_2 K e_k(n_1,n_2).
\end{aligned}
\tag{15}
$$

According to Assumption 2, premultiplying by $\widetilde{A}_{22}^{-1}$ on both sides of (15), we obtain

$$
\begin{aligned}
\delta z_{k+1}^{2}(n_1,n_2+1) &= -\widetilde{A}_{22}^{-1}\overline{A}_{21}\delta z_{k+1}^{1}(n_1+1,n_2) - \widetilde{A}_{22}^{-1}\overline{A}_{22}\delta z_{k+1}^{2}(n_1+1,n_2) - \widetilde{A}_{22}^{-1}\widehat{A}_{21}\delta z_{k+1}^{1}(n_1,n_2) \\
&\quad - \widetilde{A}_{22}^{-1}\widehat{A}_{22}\delta z_{k+1}^{2}(n_1,n_2) - \widetilde{A}_{22}^{-1}\widetilde{A}_{21}\delta z_{k+1}^{1}(n_1,n_2+1) - \widetilde{A}_{22}^{-1}B_2 K e_k(n_1,n_2).
\end{aligned}
\tag{16}
$$

Substituting (16) into (12), it yields

$$
\begin{aligned}
\delta z_{k+1}^{1}(n_1+1,n_2+1) &= \left(\overline{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{21}\right)\delta z_{k+1}^{1}(n_1+1,n_2) + \left(\overline{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{22}\right)\delta z_{k+1}^{2}(n_1+1,n_2) \\
&\quad + \left(\widehat{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21}\right)\delta z_{k+1}^{1}(n_1,n_2) + \left(\widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21}\right)\delta z_{k+1}^{2}(n_1,n_2) \\
&\quad + \left(\widetilde{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{A}_{21}\right)\delta z_{k+1}^{1}(n_1,n_2+1) + \left(B_1 K - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}B_2 K\right)e_k(n_1,n_2).
\end{aligned}
\tag{17}
$$

On the other hand, according to $e_k(n_1,n_2) = y_d(n_1,n_2) - y_k(n_1,n_2)$ and (6), it generates

$$
\begin{aligned}
e_{k+1}(n_1,n_2) - e_k(n_1,n_2) &= y_d(n_1,n_2) - y_{k+1}(n_1,n_2) - y_d(n_1,n_2) + y_k(n_1,n_2) \\
&= -C_1\left[z_{k+1}^{1}(n_1,n_2) - z_k^{1}(n_1,n_2)\right] - C_2\left[z_{k+1}^{2}(n_1,n_2) - z_k^{2}(n_1,n_2)\right] - D\left[u_{k+1}(n_1,n_2) - u_k(n_1,n_2)\right],
\end{aligned}
\tag{18}
$$

where $n_1 \in \{0,1,\ldots,N_1\}$ and $n_2 \in \{0,1,\ldots,N_2\}$. From (11)-(12) and the ILC law (9), it yields

$$
e_{k+1}(n_1,n_2) = -C_1\delta z_{k+1}^{1}(n_1,n_2) - C_2\delta z_{k+1}^{2}(n_1,n_2) + (I_s - DK)e_k(n_1,n_2).
\tag{19}
$$

Let

$$\delta Z_k^1(n_2) = \left[\, \delta z_k^{1,T}(1,n_2) \;\; \delta z_k^{1,T}(2,n_2) \;\; \dots \;\; \delta z_k^{1,T}(N_1,n_2) \,\right]^T, \tag{20}$$

$$\delta Z_k^2(n_2) = \left[\, \delta z_k^{2,T}(1,n_2) \;\; \delta z_k^{2,T}(2,n_2) \;\; \dots \;\; \delta z_k^{2,T}(N_1,n_2) \,\right]^T, \tag{21}$$

$$E_k(n_2) = \left[\, e_k^T(0,n_2) \;\; e_k^T(1,n_2) \;\; \dots \;\; e_k^T(N_1,n_2) \,\right]^T. \tag{22}$$

From (20)–(22), (17), (16), and (19) can be rewritten as

$$\begin{aligned} \Phi_1 \delta Z_{k+1}^1(n_2+1) = {} & \Phi_2 \delta Z_{k+1}^1(n_2) + \Phi_3 \delta Z_{k+1}^2(n_2) + \Phi_4 E_k(n_2) + \Phi_5 \delta z_{k+1}^1(0,n_2+1) \\ & + \Phi_6 \delta z_{k+1}^1(0,n_2) + \Phi_7 \delta z_{k+1}^2(0,n_2), \end{aligned} \tag{23}$$

$$\begin{aligned} \Psi_0 \delta Z_{k+1}^2(n_2+1) = {} & \Psi_1 \delta Z_{k+1}^1(n_2+1) + \Psi_2 \delta Z_{k+1}^1(n_2) + \Psi_3 \delta Z_{k+1}^2(n_2) + \Psi_4 E_k(n_2) \\ & + \Psi_5 \delta z_{k+1}^1(0,n_2+1) + \Psi_6 \delta z_{k+1}^1(0,n_2) + \Psi_7 \delta z_{k+1}^2(0,n_2) + \Psi_8 \delta z_{k+1}^2(0,n_2+1), \end{aligned} \tag{24}$$

$$E_{k+1}(n_2) = \Theta_1 \delta Z_{k+1}^1(n_2) + \Theta_2 \delta Z_{k+1}^2(n_2) + \Theta_3 E_k(n_2) + \Theta_4 \delta z_{k+1}^1(0,n_2) + \Theta_5 \delta z_{k+1}^2(0,n_2), \tag{25}$$

where Φ_i, $i \in \{1,2,\dots,7\}$, Ψ_j, $j \in \{0,1,2,\dots,8\}$, and Θ_h, $h \in \{1,2,\dots,5\}$, are given in the next page. Since Φ_1 is a nonsingular block matrix, and premultiplying Φ_1^{-1} on both sides of (23), we get

$$\begin{aligned} \delta Z_{k+1}^1(n_2+1) = {} & \Phi_1^{-1}\Phi_2 \delta Z_{k+1}^1(n_2) + \Phi_1^{-1}\Phi_3 \delta Z_{k+1}^2(n_2) + \Phi_1^{-1}\Phi_4 E_k(n_2) + \Phi_1^{-1}\Phi_5 \delta z_{k+1}^1(0,n_2+1) \\ & + \Phi_1^{-1}\Phi_6 \delta z_{k+1}^1(0,n_2) + \Phi_1^{-1}\Phi_7 \delta z_{k+1}^2(0,n_2). \end{aligned} \tag{26}$$

Substituting (26) into (24), we have

$$\begin{aligned} \Psi_0 \delta Z_{k+1}^2(n_2+1) = {} & \left(\Psi_1\Phi_1^{-1}\Phi_2 + \Psi_2\right) \delta Z_{k+1}^1(n_2) + \left(\Psi_1\Phi_1^{-1}\Phi_3 + \Psi_3\right) \delta Z_{k+1}^2(n_2) \\ & + \left(\Psi_1\Phi_1^{-1}\Phi_4 + \Psi_4\right) E_k(n_2) + \left(\Psi_1\Phi_1^{-1}\Phi_5 + \Psi_5\right) \delta z_{k+1}^1(0,n_2+1) \\ & + \left(\Psi_1\Phi_1^{-1}\Phi_6 + \Psi_6\right) \delta z_{k+1}^1(0,n_2) + \left(\Psi_1\Phi_1^{-1}\Phi_7 + \Psi_7\right) \delta z_{k+1}^2(0,n_2) + \Psi_8 \delta z_{k+1}^2(0,n_2+1). \end{aligned} \tag{27}$$

Taking norms on both sides of (26), (27), and (25), respectively, and considering $\|\Psi_0\| = 1$, there is

$$\begin{aligned} \left\|\delta Z_{k+1}^1(n_2+1)\right\| \le {} & \left\|\Phi_1^{-1}\Phi_2\right\| \left\|\delta Z_{k+1}^1(n_2)\right\| + \left\|\Phi_1^{-1}\Phi_3\right\| \left\|\delta Z_{k+1}^2(n_2)\right\| + \left\|\Phi_1^{-1}\Phi_4\right\| \left\|E_k(n_2)\right\| \\ & + \left\|\Phi_1^{-1}\Phi_5\right\| \left\|\delta z_{k+1}^1(0,n_2+1)\right\| + \left\|\Phi_1^{-1}\Phi_6\right\| \left\|\delta z_{k+1}^1(0,n_2)\right\| + \left\|\Phi_1^{-1}\Phi_7\right\| \left\|\delta z_{k+1}^2(0,n_2)\right\|, \end{aligned} \tag{28}$$

$$\begin{aligned} \left\|\delta Z_{k+1}^2(n_2+1)\right\| \le {} & \left\|\Psi_1\Phi_1^{-1}\Phi_2 + \Psi_2\right\| \left\|\delta Z_{k+1}^1(n_2)\right\| + \left\|\Psi_1\Phi_1^{-1}\Phi_3 + \Psi_3\right\| \left\|\delta Z_{k+1}^2(n_2)\right\| \\ & + \left\|\Psi_1\Phi_1^{-1}\Phi_4 + \Psi_4\right\| \left\|E_k(n_2)\right\| + \left\|\Psi_1\Phi_1^{-1}\Phi_5 + \Psi_5\right\| \left\|\delta z_{k+1}^1(0,n_2+1)\right\| \\ & + \left\|\Psi_1\Phi_1^{-1}\Phi_6 + \Psi_6\right\| \left\|\delta z_{k+1}^1(0,n_2)\right\| + \left\|\Psi_1\Phi_1^{-1}\Phi_7 + \Psi_7\right\| \left\|\delta z_{k+1}^2(0,n_2)\right\| + \left\|\Psi_8\right\| \left\|\delta z_{k+1}^2(0,n_2+1)\right\|, \end{aligned} \tag{29}$$

$$\begin{aligned} \left\|E_{k+1}(n_2)\right\| \le {} & \left\|\Theta_1\right\| \left\|\delta Z_{k+1}^1(n_2)\right\| + \left\|\Theta_2\right\| \left\|\delta Z_{k+1}^2(n_2)\right\| + \left\|\Theta_3\right\| \left\|E_k(n_2)\right\| + \left\|\Theta_4\right\| \left\|\delta z_{k+1}^1(0,n_2)\right\| \\ & + \left\|\Theta_5\right\| \left\|\delta z_{k+1}^2(0,n_2)\right\|, \end{aligned} \tag{30}$$

where

$$\Phi_1 = \begin{bmatrix} I_r & 0 & 0 & \cdots & 0 \\ -\widetilde{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{A}_{21} & I_r & 0 & \cdots & 0 \\ 0 & -\widetilde{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{A}_{21} & I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -\widetilde{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{A}_{21} & I_r \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} \overline{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{21} & 0 & 0 & \cdots & 0 \\ \widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21} & \overline{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{21} & 0 & \ddots & 0 \\ 0 & \widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21} & \overline{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{21} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21} & \overline{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{21} \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} \overline{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{22} & 0 & 0 & \cdots & 0 \\ \widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{22} & \overline{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{22} & 0 & \ddots & 0 \\ 0 & \widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{22} & \overline{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{22} & \overline{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{22} \end{bmatrix},$$

$$\Phi_4 = \begin{bmatrix} B_1K - \widetilde{A}_{12}\widetilde{A}_{22}B_2K & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & B_1K - \widetilde{A}_{12}\widetilde{A}_{22}B_2K & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$\Phi_5 = \begin{bmatrix} \widetilde{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{A}_{21} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Phi_6 = \begin{bmatrix} \widehat{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Phi_7 = \begin{bmatrix} \widehat{A}_{12} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{22} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Psi_0 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ I_{p-r} & 0 & 0 & \cdots & 0 \\ 0 & I_{p-r} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & 0 \\ 0 & \cdots & 0 & I_{p-r} & 0 \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ -\widetilde{A}_{22}^{-1}\widetilde{A}_{21} & 0 & 0 & \dots & 0 \\ 0 & -\widetilde{A}_{22}^{-1}\widetilde{A}_{21} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -\widetilde{A}_{22}^{-1}\widetilde{A}_{21} & 0 \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} -\widetilde{A}_{22}^{-1}\overline{A}_{21} & 0 & 0 & \dots & 0 \\ -\widetilde{A}_{22}^{-1}\widehat{A}_{21} & -\widetilde{A}_{22}^{-1}\overline{A}_{21} & 0 & \dots & 0 \\ 0 & -\widetilde{A}_{22}^{-1}\widehat{A}_{21} & -\widetilde{A}_{22}^{-1}\overline{A}_{21} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -\widetilde{A}_{22}^{-1}\widehat{A}_{21} & -\widetilde{A}_{22}^{-1}\overline{A}_{21} \end{bmatrix},$$

$$\Psi_3 = \begin{bmatrix} -\widetilde{A}_{22}^{-1}\overline{A}_{22} & 0 & 0 & \dots & 0 \\ -\widetilde{A}_{22}^{-1}\widehat{A}_{22} & -\widetilde{A}_{22}^{-1}\overline{A}_{22} & 0 & \dots & 0 \\ 0 & -\widetilde{A}_{22}^{-1}\widehat{A}_{22} & -\widetilde{A}_{22}^{-1}\overline{A}_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -\widetilde{A}_{22}^{-1}\widehat{A}_{22} & -\widetilde{A}_{22}^{-1}\overline{A}_{22} \end{bmatrix},$$

$$\Psi_4 = \begin{bmatrix} -\widetilde{A}_{22}^{-1}B_2K & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & -\widetilde{A}_{22}^{-1}B_2K & 0 \\ 0 & \dots & 0 & 0 \end{bmatrix},$$

$$\Psi_5 = \begin{bmatrix} -\widetilde{A}_{22}^{-1}\widetilde{A}_{21} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Psi_6 = \begin{bmatrix} -\widetilde{A}_{22}^{-1}\widehat{A}_{21} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Psi_7 = \begin{bmatrix} -\widetilde{A}_{22}^{-1}\widehat{A}_{22} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\Psi_8 = \begin{bmatrix} I_{p-r} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Theta_1 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ -C_1 & 0 & 0 & \dots & 0 \\ 0 & -C_1 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -C_1 & 0 \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ -C_2 & 0 & 0 & \ddots & \vdots \\ 0 & -C_2 & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -C_2 & 0 \end{bmatrix},$$

$$\Theta_3 = \begin{bmatrix} I_s - DK & 0 & \dots & 0 \\ 0 & I_s - DK & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & I_s - DK \end{bmatrix},$$

$$\Theta_4 = \begin{bmatrix} -C_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Theta_5 = \begin{bmatrix} -C_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{31}$$

Letting $\delta Z_k(n_2) = \left[\ \|\delta Z_k^1(n_2)\| \ \ \|\delta Z_k^2(n_2)\|\ \right]^T$ and considering Definitions 1-2, (28)–(30) are reformulated as

$$\begin{aligned}\delta Z_{k+1}(n_2+1) \le \overline{\Phi}_1 \delta Z_{k+1}(n_2) + \overline{\Phi}_2 \left\|E_k(n_2)\right\| + \overline{\Phi}_3 \left\|\delta z_{k+1}^1(0,n_2+1)\right\| + \overline{\Phi}_4 \left\|\delta z_{k+1}^1(0,n_2)\right\| \\ + \overline{\Phi}_5 \left\|\delta z_{k+1}^2(0,n_2)\right\| + \overline{\Phi}_6 \left\|\delta z_{k+1}^2(0,n_2+1)\right\|,\end{aligned} \tag{32}$$

$$\left\|E_{k+1}(n_2)\right\| \le \overline{\Phi}_7 \delta Z_{k+1}(n_2) + \overline{\Phi}_8 \left\|E_k(n_2)\right\| + \overline{\Phi}_9 \left\|\delta z_{k+1}^1(0,n_2)\right\| + \overline{\Phi}_{10} \left\|\delta z_{k+1}^2(0,n_2)\right\|, \tag{33}$$

where

$$\begin{aligned}
\overline{\Phi}_1 &= \begin{bmatrix} \left\|\Phi_1^{-1}\Phi_2\right\| & \left\|\Phi_1^{-1}\Phi_3\right\| \\ \left\|\Psi_1\Phi_1^{-1}\Phi_2 + \Psi_2\right\| & \left\|\Psi_1\Phi_1^{-1}\Phi_3 + \Psi_3\right\| \end{bmatrix}, \\
\overline{\Phi}_2 &= \begin{bmatrix} \left\|\Phi_1^{-1}\Phi_4\right\| \\ \left\|\Psi_1\Phi_1^{-1}\Phi_4 + \Psi_4\right\| \end{bmatrix}, \\
\overline{\Phi}_3 &= \begin{bmatrix} \left\|\Phi_1^{-1}\Phi_5\right\| \\ \left\|\Psi_1\Phi_1^{-1}\Phi_5 + \Psi_5\right\| \end{bmatrix}, \\
\overline{\Phi}_4 &= \begin{bmatrix} \left\|\Phi_1^{-1}\Phi_6\right\| \\ \left\|\Psi_1\Phi_1^{-1}\Phi_6 + \Psi_6\right\| \end{bmatrix}, \\
\overline{\Phi}_5 &= \begin{bmatrix} \left\|\Phi_1^{-1}\Phi_7\right\| \\ \left\|\Psi_1\Phi_1^{-1}\Phi_7 + \Psi_7\right\| \end{bmatrix}, \\
\overline{\Phi}_6 &= \begin{bmatrix} 0 \\ \left\|\Psi_8\right\| \end{bmatrix}, \\
\overline{\Phi}_7 &= \left[\ \left\|\Theta_1\right\| \ \ \left\|\Theta_2\right\|\ \right], \\
\overline{\Phi}_8 &= \left\|\Theta_3\right\|, \\
\overline{\Phi}_9 &= \left\|\Theta_4\right\|, \\
\overline{\Phi}_{10} &= \left\|\Theta_5\right\|.
\end{aligned} \tag{34}$$

From Assumption 1, we derive

$$\begin{aligned}
\left\|\delta z_{k+1}^1(0,n_2)\right\| &= \left\|Q_{11}\left[x_{k+1}^1(0,n_2) - x_k^1(0,n_2)\right] + Q_{12}\left[x_{k+1}^2(0,n_2) - x_k^2(0,n_2)\right]\right\| \le \overline{\Lambda}_1, \\
\left\|\delta z_{k+1}^2(0,n_2)\right\| &= \left\|Q_{21}\left[x_{k+1}^1(0,n_2) - x_k^1(0,n_2)\right] + Q_{22}\left[x_{k+1}^2(0,n_2) - x_k^2(0,n_2)\right]\right\| \le \overline{\Lambda}_2, \\
\left\|\delta z_{k+1}^1(n_1,0)\right\| &= \left\|Q_{11}\left[x_{k+1}^1(n_1,0) - x_k^1(n_1,0)\right] + Q_{12}\left[x_{k+1}^2(0,n_2) - x_k^2(0,n_2)\right]\right\| \le \overline{\Lambda}_3, \\
\left\|\delta z_{k+1}^2(n_1,0)\right\| &= \left\|Q_{21}\left[x_{k+1}^1(n_1,0) - x_k^1(n_1,0)\right] + Q_{22}\left[x_{k+1}^2(n_1,0) - x_k^2(n_1,0)\right]\right\| \le \overline{\Lambda}_4, \\
\left\|\delta Z_{k+1}^1(0)\right\| &= \left\|\begin{bmatrix} \delta z_{k+1}^1(1,0) \\ \delta z_{k+1}^1(2,0) \\ \vdots \\ \delta z_{k+1}^1(N_1,0) \end{bmatrix}\right\| \le \overline{\Lambda}_5,
\end{aligned} \tag{35}$$

$$\left\|\delta Z_{k+1}^2(0)\right\| = \left\|\begin{bmatrix} \delta z_{k+1}^2(1,0) \\ \delta z_{k+1}^2(2,0) \\ \vdots \\ \delta z_{k+1}^2(N_1,0) \end{bmatrix}\right\| \le \overline{\Lambda}_6, \tag{36}$$

where $\overline{\Lambda}_1$, $\overline{\Lambda}_2$, $\overline{\Lambda}_3$, $\overline{\Lambda}_4$, $\overline{\Lambda}_5$, and $\overline{\Lambda}_6$ are dependent on bound parameters Λ_1 and Λ_2 in Assumption 1. From (35) and (36), we know that $\delta Z_{k+1}(0)$ is bounded. In addition, $\|E_0(n_2)\|$ is bounded for $n_2 \in \{0, 1, \ldots, N_2 - 1\}$ due to the boundedness property of $y_d(n_1, n_2)$ and $y_0(n_1, n_2)$. Using Lemma 2 in [22], if $\overline{\Phi}_8 < 1$ (equivalently, $\|I_s - DK\| < 1$), there is

$$\limsup_{k \longrightarrow +\infty} \delta Z_{k+1}(n_2) \leq \beta_{\delta Z}, \quad n_2 \in \{0, 1, \ldots, N_2\}, \tag{37}$$

$$\limsup_{k \longrightarrow +\infty} \|E_k(n_2)\| \leq \beta_E, \quad n_2 \in \{0, 1, \ldots, N_2 - 1\}. \tag{38}$$

In addition, taking $n_2 = N_2$ in (19), we have

$$e_{k+1}(n_1, N_2) = -C_1 \delta z^1_{k+1}(n_1, N_2) - C_2 \delta z^2_{k+1}(n_1, N_2) + (I_s - DK) e_k(n_1, N_2), \tag{39}$$

where $n_1 \in \{0, 1, \ldots, N_1\}$. Taking norm on both sides of (39), we have

$$\|e_{k+1}(n_1, N_2)\| \leq \|C_1\| \|\delta z^1_{k+1}(n_1, N_2)\| + \|C_2\| \|\delta z^2_{k+1}(n_1, N_2)\| + \|I_s - DK\| \|e_k(n_1, N_2)\|. \tag{40}$$

From (37), we know $\|\delta z^1_{k+1}(n_1, N_2)\|$ and $\|\delta z^2_{k+1}(n_1, N_2)\|$ are bounded. Using Lemma 1, if $\|I_s - DK\| < 1$ is satisfied, there is

$$\limsup_{k \longrightarrow +\infty} \|e_k(n_1, N_2)\| \leq b_e, \quad n_1 \in \{0, 1, \ldots, N_1\}. \tag{41}$$

From (38), (41), and the definition on $E_k(n_2)$ in (22), it yields

$$\limsup_{k \longrightarrow +\infty} \|e_k(n_1, n_2)\| \leq \beta_e, \tag{42}$$

where $n_1 \in \{0, 1, \ldots, N_1\}$ and $n_2 \in \{0, 1, \ldots, N_2\}$. Theorem 1 is completed.

As iteration-invariant boundary states are imposed on 2-D LDSFM (1), there is Corollary 1. □

Corollary 1. *For 2-D LDSFM (1) under Assumption 1 with $\Lambda_1 = 0$ and $\Lambda_2 = 0$ and Assumptions 2-3, use P-type ILC law (9). If there exists the learning gain K satisfying $\|I_s - DK\| < 1$, then, the tracking error $e_k(n_1, n_2)$ converges to zero, i.e.,*

$$\lim_{k \longrightarrow +\infty} e_k(n_1, n_2) = 0, \tag{43}$$

where $n_1 \in \{0, 1, \ldots, N_1\}$ and $n_2 \in \{0, 1, \ldots, N_2\}$.

4. Extension to 2-D LDSFM with $D = 0$

In this section, under Assumptions 1–3, we investigate the ILC issue for 2-D LDSFM with $D = 0$ [28], which is described in the following:

$$\begin{cases} E_g x_k(n_1 + 1, n_2 + 1) = A_1 x_k(n_1 + 1, n_2) + A_2 x_k(n_1, n_2) + A_3(n_1, n_2 + 1) + B u_k(n_1, n_2), \\ y_k(n_1, n_2) = C x_k(n_1, n_2), \end{cases} \tag{44}$$

where E_g, A_1, A_2, A_3, B, and C have been described in (1).

Based on Assumptions 1–3, we will discuss the robustness and convergence property of ILC law (45) for 2-D LDSFM (44). Theorem 2 is given as follows.

Theorem 2. *For 2-D LDSFM (44) under Assumptions 1–3, the following P-type ILC law for $n_1 \in \{0, 1, \ldots, N_1 - 1\}$ and $n_2 \in \{0, 1, \ldots, N_2 - 1\}$,*

$$u_{k+1}(n_1, n_2) = u_k(n_1, n_2) + K e_k(n_1 + 1, n_2 + 1), \tag{45}$$

is applied. If there exists the learning gain K satisfying

$$\overline{\Phi}_8 < 1, \tag{46}$$

where $\overline{\Phi}_8 = \|I_{sN_1} + \widehat{\Theta}_1 \Phi_1^{-1} \Phi_4\| + \|\widehat{\Theta}_2\| \|\Psi_1 \Phi_1^{-1} \Phi_4 + \Psi_4\|$, where $\widehat{\Theta}_1$, $\widehat{\Theta}_2$, Φ_1, Φ_4, Ψ_1, and Ψ_4 are given in (52)–(55); the tracking error $e_k(n_1, n_2)$, with $n_1 \in \{1, 2, \ldots, N_1\}$ and $n_2 \in \{1, 2, \ldots, N_2\}$ converges to a bounded range, i.e.,

$$\limsup_{k \longrightarrow +\infty} \|e_k(n_1, n_2)\| \leq \beta'_e, \tag{47}$$

where β'_e is a certain bound relevant with the bound parameters Λ_1 and Λ_2 in Assumption 1.

Proof. For $n_1 \in \{0, 1, \ldots, N_1\}$ and $n_2 \in \{0, 1, \ldots, N_2\}$, let $\delta z^1_{k+1}(n_1, n_2)$ and $\delta z^2_{k+1}(n_1, n_2)$ be defined as (11) and (12). Using singular value decomposition theory on (43) and following the same procedure as (17) and (16), there is

$$\begin{aligned}\delta z_{k+1}^{1}(n_1+1,n_2+1) &= \left(\overline{A}_{11}-\widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{21}\right)\delta z_{k+1}^{1}(n_1+1,n_2)+\left(\overline{A}_{12}-\widetilde{A}_{12}\widetilde{A}_{22}^{-1}\overline{A}_{22}\right)\delta z_{k+1}^{2}(n_1+1,n_2)\\ &\quad+\left(\widehat{A}_{11}-\widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21}\right)\delta z_{k+1}^{1}(n_1,n_2)+\left(\widehat{A}_{12}-\widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widehat{A}_{21}\right)\delta z_{k+1}^{2}(n_1,n_2)\\ &\quad+\left(\widetilde{A}_{11}-\widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{A}_{21}\right)\delta z_{k+1}^{1}(n_1,n_2+1)+\left(B_1K-\widetilde{A}_{12}\widetilde{A}_{22}^{-1}B_2K\right)e_k(n_1+1,n_2+1),\\ \delta z_{k+1}^{2}(n_1,n_2+1) &= -\widetilde{A}_{22}^{-1}\overline{A}_{21}\delta z_{k+1}^{1}(n_1+1,n_2)-\widetilde{A}_{22}^{-1}\overline{A}_{22}\delta z_{k+1}^{2}(n_1+1,n_2)-\widetilde{A}_{22}^{-1}\widehat{A}_{21}\delta z_{k+1}^{1}(n_1,n_2)\\ &\quad-\widetilde{A}_{22}^{-1}\widehat{A}_{22}\delta z_{k+1}^{2}(n_1,n_2)-\widetilde{A}_{22}^{-1}\widetilde{A}_{21}\delta z_{k+1}^{1}(n_1,n_2+1)-\widetilde{A}_{22}^{-1}B_2Ke_k(n_1+1,n_2+1).\end{aligned} \tag{48}$$

On the other hand, according to $e_k(n_1,n_2)=y_d(n_1,n_2)-y_k(n_1,n_2)$ and (44), it generates

$$\begin{aligned}&e_{k+1}(n_1+1,n_2+1)-e_k(n_1+1,n_2+1)=y_d(n_1+1,n_2+1)-y_{k+1}(n_1+1,n_2+1)-y_d(n_1+1,n_2+1)\\ &\quad+y_k(n_1+1,n_2+1)=-C_1\left[z_{k+1}^{1}(n_1+1,n_2+1)-z_k^{1}(n_1+1,n_2+1)\right]-C_2\left[z_{k+1}^{2}(n_1+1,n_2+1)-z_k^{2}(n_1+1,n_2+1)\right],\end{aligned} \tag{49}$$

where $n_1\in\{0,1,\ldots,N_1-1\}$ and $n_2\in\{0,1,\ldots,N_2-1\}$.

From (11)-(12), it becomes

$$e_{k+1}(n_1+1,n_2+1)-e_k(n_1+1,n_2+1)=-C_1\delta z_{k+1}^{1}(n_1+1,n_2+1)-C_2\delta z_{k+1}^{2}(n_1+1,n_2+1). \tag{50}$$

Let

$$\widehat{E}_k(n_2)=\left[\,e_k^T(1,n_2)\;\; e_k^T(2,n_2)\;\;\ldots\;\; e_k^T(N_1,n_2)\,\right]^T. \tag{51}$$

From (51), (50) can be reformulated as

$$\widehat{E}_{k+1}(n_2+1)=\widehat{\Theta}_1\delta Z_{k+1}^{1}(n_2+1)+\widehat{\Theta}_2\delta Z_{k+1}^{2}(n_2+1)+\widehat{E}_k(n_2+1), \tag{52}$$

where $\delta Z_{k+1}^{1}(n_2)$ and $\delta Z_{k+1}^{2}(n_2)$ are described in (20) and (21). $\widehat{\Theta}_1$ and $\widehat{\Theta}_2$ are given as

$$\begin{aligned}\widehat{\Theta}_1&=\begin{bmatrix}-C_1 & 0 & \ldots & 0\\ 0 & -C_1 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \ldots & 0 & -C_1\end{bmatrix},\\ \widehat{\Theta}_2&=\begin{bmatrix}-C_2 & 0 & \ldots & 0\\ 0 & -C_2 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \ldots & 0 & -C_2\end{bmatrix}.\end{aligned} \tag{53}$$

Following the similar deduction process as (26) and (27), it yields

$$\begin{aligned}\delta Z_{k+1}^{1}(n_2+1)&=\Phi_1^{-1}\Phi_2\delta Z_{k+1}^{1}(n_2)+\Phi_1^{-1}\Phi_3\delta Z_{k+1}^{2}(n_2)+\Phi_1^{-1}\Phi_4\widehat{E}_k(n_2+1)+\Phi_1^{-1}\Phi_5\delta z_{k+1}^{1}(0,n_2+1)\\ &\quad+\Phi_1^{-1}\Phi_6\delta z_{k+1}^{1}(0,n_2)+\Phi_1^{-1}\Phi_7\delta z_{k+1}^{2}(0,n_2),\end{aligned} \tag{54}$$

$$\begin{aligned}\Psi_0\delta Z_{k+1}^{2}(n_2+1)&=\left(\Psi_1\Phi_1^{-1}\Phi_2+\Psi_2\right)\delta Z_{k+1}^{1}(n_2)+\left(\Psi_1\Phi_1^{-1}\Phi_3+\Psi_3\right)\delta Z_{k+1}^{2}(n_2)\\ &\quad+\left(\Psi_1\Phi_1^{-1}\Phi_4+\Psi_4\right)\widehat{E}_k(n_2+1)+\left(\Psi_1\Phi_1^{-1}\Phi_5+\Psi_5\right)\delta z_{k+1}^{1}(0,n_2+1)\\ &\quad+\left(\Psi_1\Phi_1^{-1}\Phi_6+\Psi_6\right)\delta z_{k+1}^{1}(0,n_2)+\left(\Psi_1\Phi_1^{-1}\Phi_7+\Psi_7\right)\delta z_{k+1}^{2}(0,n_2)+\Psi_8\delta z_{k+1}^{2}(0,n_2+1),\end{aligned} \tag{55}$$

where Φ_i, $i \in \{1,2,3,5,6,7\}$, and Ψ_j, $j \in \{0,1,2,3,5,6,7,8\}$, have been described in (23) and (24); Φ_4 and Ψ_4 are given as

$$\Phi_4 = \begin{bmatrix} B_1K - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}B_2K & 0 & \cdots & 0 \\ 0 & B_1K - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}B_2K & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & B_1K - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}B_2K \end{bmatrix},$$
$$\Psi_4 = \begin{bmatrix} -\widetilde{A}_{22}^{-1}B_2K & 0 & \cdots & 0 \\ 0 & -\widetilde{A}_{22}^{-1}B_2K & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -\widetilde{A}_{22}^{-1}B_2K \end{bmatrix}. \tag{56}$$

Substituting (54) into (52), we have

$$\begin{aligned} \widehat{E}_{k+1}(n_2+1) = {} & \widehat{\Theta}_1\Phi_1^{-1}\Phi_2\delta Z_{k+1}^1(n_2) + \widehat{\Theta}_1\Phi_1^{-1}\Phi_3\delta Z_{k+1}^2(n_2) + \left(I_{sN_1} + \widehat{\Theta}_1\Phi_1^{-1}\Phi_4\right)\widehat{E}_k(n_2+1) \\ & + \widehat{\Theta}_2\delta Z_{k+1}^2(n_2+1) + \widehat{\Theta}_1\Phi_1^{-1}\Phi_5\delta z_{k+1}^1(0,n_2+1) + \widehat{\Theta}_1\Phi_1^{-1}\Phi_6\delta z_{k+1}^1(0,n_2) + \widehat{\Theta}_1\Phi_1^{-1}\Phi_7\delta z_{k+1}^2(0,n_2). \end{aligned} \tag{57}$$

Taking norms on both sides of (54)-(55), respectively, and considering $\|\Psi_0\| = 1$, there is

$$\begin{aligned} \left\|\delta Z_{k+1}^1(n_2+1)\right\| \le {} & \left\|\Phi_1^{-1}\Phi_2\right\|\left\|\delta Z_{k+1}^1(n_2)\right\| + \left\|\Phi_1^{-1}\Phi_3\right\|\left\|\delta Z_{k+1}^2(n_2)\right\| + \left\|\Phi_1^{-1}\Phi_4\right\|\left\|\widehat{E}_k(n_2+1)\right\| \\ & + \left\|\Phi_1^{-1}\Phi_5\right\|\left\|\delta z_{k+1}^1(0,n_2+1)\right\| + \left\|\Phi_1^{-1}\Phi_6\right\|\left\|\delta z_{k+1}^1(0,n_2)\right\| + \left\|\Phi_1^{-1}\Phi_7\right\|\left\|\delta z_{k+1}^2(0,n_2)\right\|, \end{aligned} \tag{58}$$

$$\begin{aligned} \left\|\delta Z_{k+1}^2(n_2+1)\right\| \le {} & \left\|\Psi_1\Phi_1^{-1}\Phi_2 + \Psi_2\right\|\left\|\delta Z_{k+1}^1(n_2)\right\| + \left\|\Psi_1\Phi_1^{-1}\Phi_3 + \Psi_3\right\|\left\|\delta Z_{k+1}^2(n_2)\right\| \\ & + \left\|\Psi_1\Phi_1^{-1}\Phi_4 + \Psi_4\right\|\left\|\widehat{E}_k(n_2+1)\right\| + \left\|\Psi_1\Phi_1^{-1}\Phi_5 + \Psi_5\right\|\left\|\delta z_{k+1}^1(0,n_2+1)\right\| \\ & + \left\|\Psi_1\Phi_1^{-1}\Phi_6 + \Psi_6\right\|\left\|\delta z_{k+1}^1(0,n_2)\right\| + \left\|\Psi_1\Phi_1^{-1}\Phi_7 + \Psi_7\right\|\left\|\delta z_{k+1}^2(0,n_2)\right\| + \left\|\Psi_8\right\|\left\|\delta z_{k+1}^2(0,n_2+1)\right\|. \end{aligned} \tag{59}$$

Taking norms on both sides of (57), and considering (59), it yields

$$\begin{aligned} \left\|\widehat{E}_{k+1}(n_2+1)\right\| \le {} & \left(\left\|\widehat{\Theta}_1\Phi_1^{-1}\Phi_2\right\| + \left\|\widehat{\Theta}_2\right\|\left\|\Psi_1\Phi_1^{-1}\Phi_2 + \Psi_2\right\|\right)\left\|\delta Z_{k+1}^1(n_2)\right\| \\ & + \left(\left\|\widehat{\Theta}_1\Phi_1^{-1}\Phi_3\right\| + \left\|\widehat{\Theta}_2\right\|\left\|\Psi_1\Phi_1^{-1}\Phi_2 + \Psi_3\right\|\right)\left\|\delta Z_{k+1}^2(n_2)\right\| + \left(\left\|I_{sN_1} + \widehat{\Theta}_1\Phi_1^{-1}\Phi_4\right\| + \left\|\widehat{\Theta}_2\right\|\left\|\Psi_1\Phi_1^{-1}\Phi_4 + \Psi_4\right\|\right)\left\|\widehat{E}_k(n_2+1)\right\| \\ & + \left(\left\|\widehat{\Theta}_1\Phi_1^{-1}\Phi_5\right\| + \left\|\widehat{\Theta}_2\right\|\left\|\Psi_1\Phi_1^{-1}\Phi_5 + \Psi_5\right\|\right)\left\|\delta z_{k+1}^1(0,n_2+1)\right\| + \left(\left\|\widehat{\Theta}_1\Phi_1^{-1}\Phi_6\right\| + \left\|\widehat{\Theta}_2\right\|\left\|\Psi_1\Phi_1^{-1}\Phi_6 + \Psi_6\right\|\right)\left\|\delta z_{k+1}^1(0,n_2)\right\| \\ & + \left(\left\|\widehat{\Theta}_1\Phi_1^{-1}\Phi_7\right\| + \left\|\widehat{\Theta}_2\right\|\left\|\Psi_1\Phi_1^{-1}\Phi_7 + \Psi_7\right\|\right)\left\|\delta z_{k+1}^2(0,n_2)\right\| + \left\|\widehat{\Theta}_2\right\|\left\|\Psi_8\right\|\left\|\delta z_{k+1}^2(0,n_2+1)\right\|. \end{aligned} \tag{60}$$

Define $\delta Z_k(n_2) = \left[\|\delta Z_k^1(n_2)\| \ \ \|\delta Z_k^2(n_2)\| \right]^T$, and (58)–(60) can be rewritten as

$$\begin{aligned}
\delta Z_{k+1}(n_2+1) \le{}& \overline{\Phi}_1 \delta Z_{k+1}(n_2) + \overline{\Phi}_2 \left\|E_k(n_2+1)\right\| + \overline{\Phi}_3 \left\|\delta z_{k+1}^1(0,n_2+1)\right\| + \overline{\Phi}_4 \left\|\delta z_{k+1}^1(0,n_2)\right\| \\
&+ \overline{\Phi}_5 \left\|\delta z_{k+1}^2(0,n_2)\right\| + \overline{\Phi}_6 \left\|\delta z_{k+1}^2(0,n_2+1)\right\|, \\
\left\|\widehat{E}_{k+1}(n_2+1)\right\| \le{}& \overline{\Phi}_7 \delta Z_{k+1}(n_2) + \overline{\Phi}_8 \left\|\widehat{E}_k(n_2+1)\right\| + \overline{\Phi}_9 \left\|\delta z_{k+1}^1(0,n_2+1)\right\| + \overline{\Phi}_{10} \left\|\delta z_{k+1}^1(0,n_2)\right\| \\
&+ \overline{\Phi}_{11} \left\|\delta z_{k+1}^2(0,n_2)\right\| + \overline{\Phi}_{12} \left\|\delta z_{k+1}^2(0,n_2+1)\right\|,
\end{aligned} \tag{61}$$

where $\overline{\Phi}_i$, $i \in \{1,2,\ldots,6\}$, are given in (33)-(34); $\overline{\Phi}_j$, $j \in \{7,8,\ldots,12\}$, are shown as

$$\begin{aligned}
\overline{\Phi}_7 &= \left[\left\|\widehat{\Theta}_1 \Phi_1^{-1} \Phi_2\right\| + \left\|\widehat{\Theta}_2\right\| \left\|\Psi_1 \Phi_1^{-1} \Phi_2 + \Psi_2\right\| \ \ \left\|\widehat{\Theta}_1 \Phi_1^{-1} \Phi_3\right\| + \left\|\widehat{\Theta}_2\right\| \left\|\Psi_1 \Phi_1^{-1} \Phi_3 + \Psi_3\right\| \right], \\
\overline{\Phi}_8 &= \left\|I_{sN_1} + \widehat{\Theta}_1 \Phi_1^{-1} \Phi_4\right\| + \left\|\widehat{\Theta}_2\right\| \left\|\Psi_1 \Phi_1^{-1} \Phi_4 + \Psi_4\right\|, \\
\overline{\Phi}_9 &= \left\|\widehat{\Theta}_1 \Phi_1^{-1} \Phi_5\right\| + \left\|\widehat{\Theta}_2\right\| \left\|\Psi_1 \Phi_1^{-1} \Phi_5 + \Psi_5\right\|, \\
\overline{\Phi}_{10} &= \left\|\widehat{\Theta}_1 \Phi_1^{-1} \Phi_6\right\| + \left\|\widehat{\Theta}_2\right\| \left\|\Psi_1 \Phi_1^{-1} \Phi_6 + \Psi_6\right\|, \\
\overline{\Phi}_{11} &= \left\|\widehat{\Theta}_1 \Phi_1^{-1} \Phi_7\right\| + \left\|\widehat{\Theta}_2\right\| \left\|\Psi_1 \Phi_1^{-1} \Phi_7 + \Psi_7\right\|, \\
\overline{\Phi}_{12} &= \left\|\widehat{\Theta}_2\right\| \left\|\Psi_8\right\|.
\end{aligned} \tag{62}$$

From Assumption 1, we obtain $\|\delta z_{k+1}^1(0,n_2)\|$, $\|\delta z_{k+1}^2(0,n_2)\|$, and $\delta Z_{k+1}(0)$ are bounded. Using Lemma 2 in [22], if $\overline{\Phi}_8 < 1$, there is

$$\limsup_{k \longrightarrow +\infty} \left\|\widehat{E}_k(n_2+1)\right\| \le \beta_{\widehat{E}}, \quad n_2 \in \{0,1,\ldots,N_2-1\}. \tag{63}$$

From (51), we obtain

$$\limsup_{k \longrightarrow +\infty} \left\|e_k(n_1,n_2)\right\| \le \beta_e', \tag{64}$$

where $n_1 \in \{1,2,\ldots,N_1\}$ and $n_2 \in \{1,2,\ldots,N_2\}$. Theorem 2 is completed. □

Corollary 2. *For 2-D LDSFM (44) under Assumption 1 with $\Lambda_1 = 0$ and $\Lambda_2 = 0$ and Assumptions 2-3, use the P-type ILC law (45). If there exists the learning gain K satisfying (46), then the tracking error $e_k(n_1,n_2)$ converges to zero, i.e.,*

$$\lim_{k \longrightarrow +\infty} e_k(n_1,n_2) = 0, \tag{65}$$

where $n_1 \in \{1,2,\ldots,N_1\}$ and $n_2 \in \{1,2,\ldots,N_2\}$.

Remark 2. It is noted that 2-D LDSFM contains a 2-D linear discrete singular Attasi model as a special case. Also, a 2-D singular Roesser model under some specified coefficient matrix requirements can be converted into 2-D LDSFM [32]. Consequently, depending on the ILC results obtained from 2-D LDSFM, it is easy to extend the 2-D singular Roesser model and singular Attasi model.

5. Illustrative Examples

In this section, to demonstrate the feasibility and effectiveness of the designed ILC algorithm for 2-D LDSFM, computer simulation is conducted for mathematical systems.

Example 1. Consider the ILC issue for 2-D LDSFM (44) with the following parameters:

$$\begin{aligned}
E_g &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \\
A_1 &= \begin{bmatrix} 0.1 & 0.01 \\ 0.01 & 0.03 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.1 & 0.02 \\ 0.01 & 0.04 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} 0.01 & 0.02 \\ 0.1 & 0.3 \end{bmatrix}, \\
B &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\
C &= \begin{bmatrix} -1 & -0.5 \end{bmatrix}.
\end{aligned} \tag{66}$$

Taking $N_1 = N_2 = 20$, the desired surface trajectory $y_d(n_1,n_2)$ is described by $y_d(n_1,n_2) = \sin[0.2\pi(n_1+n_2)]$ for $n_1 \in \{0,1,\ldots,20\}$ and $n_2 \in \{0,1,\ldots,20\}$, which is shown in Figure 1. We select two nonsingular transformation matrices $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & -0.2 \end{bmatrix}$ and $Q = \begin{bmatrix} 0.5 & 0.1 \\ 0.5 & -0.1 \end{bmatrix}$. In ILC law

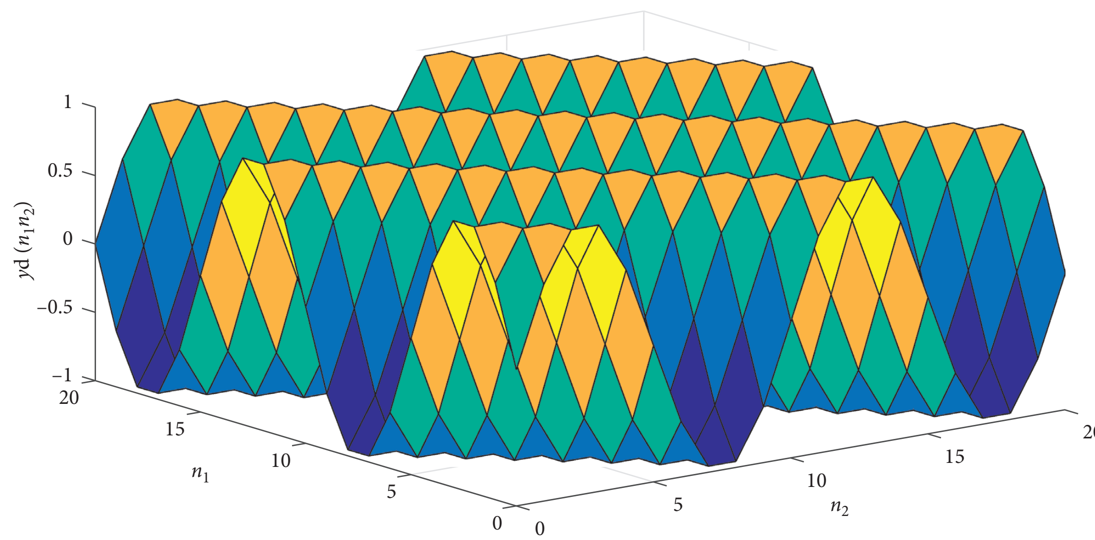

Figure 1: The desired surface trajectory $y_d(n_1, n_2)$ for $n_1 \in \{0, 1, \ldots, 20\}$ and $n_2 \in \{0, 1, \ldots, 20\}$.

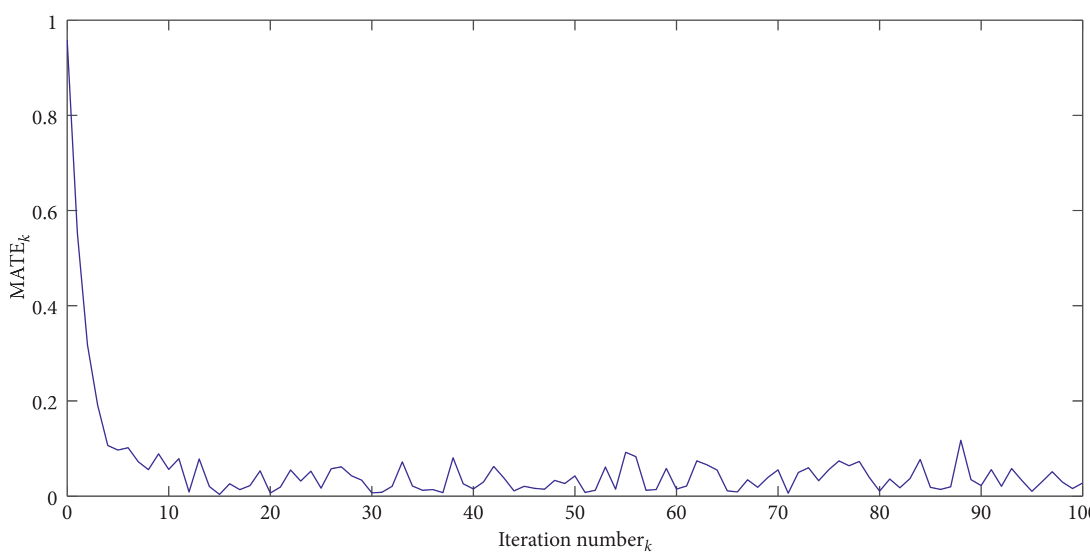

Figure 2: Under the ILC law (38), the profile of tracking error index MATE_k with iteration number k.

(45), let the initial control input be given as $u_0(n_1, n_2) = 0$, where $n_1 \in \{0, 1, \ldots, 19\}$ and $n_2 \in \{0, 1, \ldots, 19\}$, and the learning gain K be selected as $K = -0.4$, which satisfies convergence condition (46) in Theorem 2 and Corollary 2. The maximum absolute tracking error index MATE_k is adopted to evaluate the ILC tracking performance, which is given as follows:

$$\mathrm{MATE}_k = \max_{n_1 \in \{1,2,\ldots,20\}} \max_{n_2 \in \{1,2,\ldots,20\}} \left| y_d(n_1, n_2) - y_k(n_1, n_2) \right|. \tag{67}$$

Case 1. Corresponding to Assumption 1 with iteration-varying boundary states, let the boundary states $x_k(0, n_2)$, $n_2 \in \{1, 2, \ldots, 20\}$, and $x_k(n_1, 0)$, $n_1 \in \{0, 1, \ldots, 20\}$, be given as

$$\begin{aligned} x_k(0, n_2) &= \begin{bmatrix} 0 \\ \sin(0.2\pi n_2) \end{bmatrix} + \begin{bmatrix} m_{1,k} \\ m_{2,k} \end{bmatrix}, \\ x_k(n_1, 0) &= \begin{bmatrix} -0.5\sin(0.2\pi n_1) \\ 0 \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n_{2,k} \end{bmatrix}, \end{aligned} \tag{68}$$

where $m_{1,k}$, $m_{2,k}$, $n_{1,k}$, and $n_{2,k}$ vary randomly at $(-0.5, 0.5)$, $(-0.1, 0.1)$, $(-0.5, 0.5)$, and $(-0.1, 0.1)$ along the iteration k. Figure 2 displays the profile of MATE_k with iteration number k by using ILC law (45). Obviously, robust boundedness of ultimate ILC tracking error is validated from Figure 2.

Case 2. Corresponding to Assumption 1 with $\Lambda_1 = 0$ and $\Lambda_2 = 0$, let the boundary states satisfy

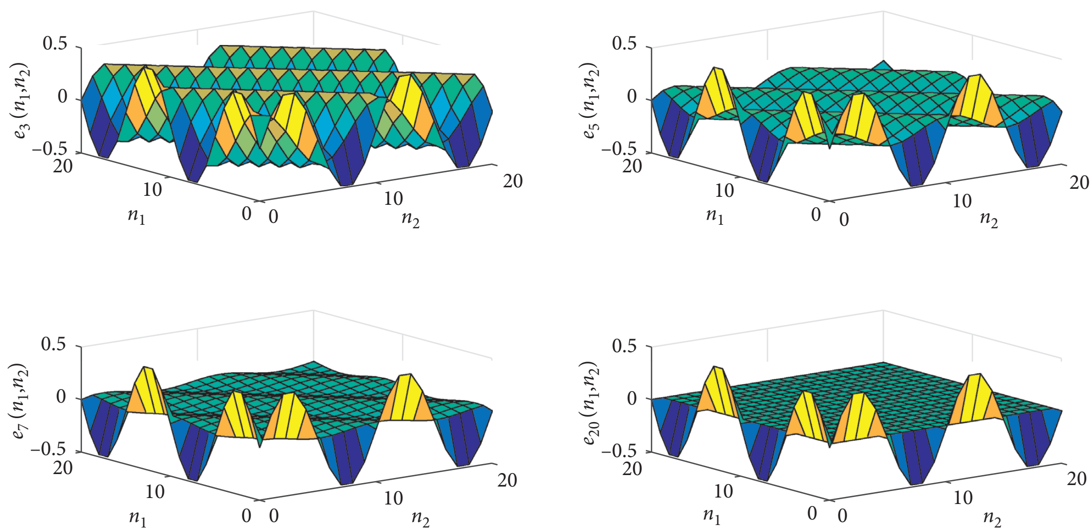

FIGURE 3: Under ILC law (45), the tracking error surface $e_k(n_1, n_2)$ at $k = 3, 5, 7, 20$.

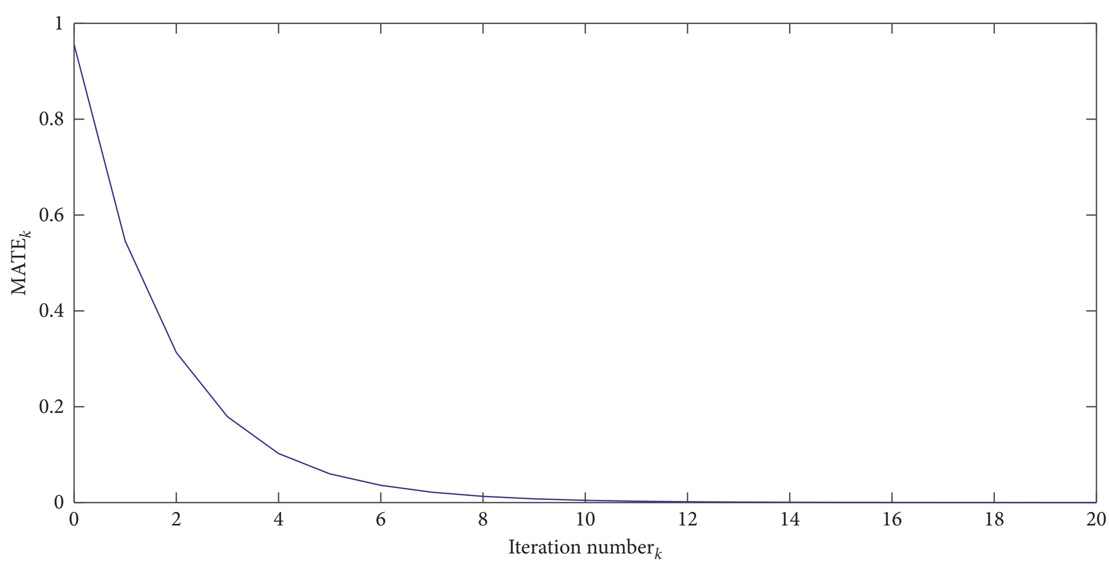

FIGURE 4: Under ILC law (45), the profile of tracking error index MATE_k with iteration number k.

$$x_k(0, n_2) = \left[\,0 \quad \sin(0.2\pi n_2)\,\right]^T, \quad n_2 \in \{1, 2, \ldots, 20\},$$
$$x_k(n_1, 0) = \left[\,-0.5\sin(0.2\pi n_1) \quad 0\,\right]^T, \quad n_1 \in \{0, 1, \ldots, 20\}. \tag{69}$$

Figure 3 displays the profile of MATE_k with iteration number k by using ILC law (45). The tracking error surface $e_k(n_1, n_2)$ at $k = 3, 5, 7, 20$ is shown in Figure 4. Apparently, a perfect tracking to repetitive reference trajectory $y_d(n_1, n_2)$ except the boundaries $n_1 = 0$ and $n_2 = 0$ can be observed from Figures 3-4, and the effectiveness of the presented ILC algorithm is illustrated.

6. Conclusions

At present, the ILC tracking issue for 2-D LDSFM under iteration-varying boundary states has not been studied. This paper first introduces singular value decomposition theory into ILC investigations on 2-D LDSFM. In addition, 3-D linear inequality stability theory is first proposed to analysis the convergence property on ILC algorithms, which is a novel analysis method. In the future work, more robust ILC uncertainties for 2-D nonlinear singular systems will be considered.

Data Availability

All the data used to support the study have been included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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