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Research Article

Competition Equilibrium Analysis of China's Luxury Car Market Based on Three-Dimensional Grey Lotka–Volterra Model

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An enterprise must be able to conduct in-depth analysis of the existing data as the information has certain grey characteristics, if it wants to occupy a dominant position in the fierce market competition. In this paper, a compound three-dimensional grey Lotka–Volterra model is developed to carry out the grey transformation of the original data, so that the data can have better simulation accuracy, and the observation noise of the original data can be reduced. The competitive situation analysis based on the three-dimensional grey Lotka–Volterra model can help enterprises better understand the market situation. This paper takes the luxury brand automobile market in mainland China as an example to conduct a competitive analysis and a balanced development simulation. It can be found that, based on Three Species System Analysis, there is a symbiotic relationship among automobile enterprises and that the three species model can be adopted in analyzing the competition and cooperation among enterprises. Through balanced development of a Symbiotic System Analysis, the results of symbiotic optimization under the achievable equilibrium state of three populations are obtained and they show that the proposed method can be used effectively to conduct the market competition analysis. It is thus of great importance to study the relationships among enterprises as it is helpful for enterprises to make strategic policies.

1. Introduction

After the 1990s, the process of economic globalization has been accelerated significantly. The globalization of automobile industry, one of the leading and typically global industries, is mainly reflected with two distinct and interrelated characteristics. First is the automobile industry chain, including investment, production, procurement, sales, after-sales service, *R&D*, and other major links of the increasingly global allocation. For example, in the past, multinational corporations established and maintained research and development institutions in their own countries, investing in target country markets in a replicated way, while now they have developed a way of allocating functional activities and capabilities to global markets. This leads to the emergence of new specialized divisions of labor cooperation mode, especially the separation trend between vehicle assembly and parts manufacturers. More and more multinational companies are involved in manufacturing of parts. The network organization structure between parts manufacturers and vehicle assembly companies with contracts as the link is becoming increasingly apparent. The global purchase of parts and components in the automobile manufacturing enterprises and the internationalization of the parts industry blur the "national characteristics" of automobile products, which makes automobile products a typical global product. Second is the large-scale reorganization between giant automobile enterprises. Since the 1990s, due to the global excess of automobile production capacity and the increasingly strict regulations on safety, emission, and energy conservation, the pace of global industrial structure adjustment in automobile industry has been accelerated significantly. Many developed countries' auto companies have strengthened their competitiveness by means of expansion, integration, and merger. The trend of automobile industry globalization has a profound impact on the development of automobile industry and industrial policies of such developing countries as China. As a world populous country with rising economy, China has become the most potential emerging market in the world with the continuous improvement of people's income level and the upgrading of consumption structure. Improving the consumption environment will become an important measure to transform the potential demand of the public for cars into real demand to promote economic growth. At present, China has initially formed a relatively independent automobile production system. The market advantages, labor quality and cost advantages, and scale advantages of industrial support of large countries are gradually emerging. With the entry of global automobile manufacturing multinational companies and the development of domestic automobile enterprises, China has become an important automobile manufacturing base in the world.

By the end of 2020, China's car ownership has reached 281 million, with 24.24 million newly registered vehicles in 2020, a decrease of 1.53 million, or 5.95% compared with 2019. Among them, 4.16 million trucks are newly registered, an increase of 650,000 or 18.43% over 2019, reaching a new high in the past decade. The number of newly registered motorcycles is 8.26 million, an increase of 2.49 million or 43.07% over 2019. A rapid growth has been maintained in recent two years, and a significant growth trend would occur in 2020 due to the impact of the epidemic. There are 70 cities in China, each of which has more than 1 million cars, an increase of 4% over the same period. Each of 31 cities in China has more than 2 million cars. The number exceeds 3 million in 13 cities. There are more than 5 million vehicles each in Beijing, Chengdu, and Chongqing, more than 4 million each in Suzhou, Shanghai, and Zhengzhou and more than 3 million each in seven cities, including Xi'an, Wuhan, Shenzhen, Dongguan, Tianjin, Qingdao, and Shijiazhuang. By the end of 2020, the national new energy vehicle ownership has reached 4.92 million, accounting for 1.75% of the total automobile, an increase of 1.11 million and 29.18% over 2019. Among them, the total number of pure electric vehicles is 4 million, accounting for 81.32% of the total new energy vehicles. The increment of new energy vehicles has exceeded 1 million for three consecutive years, showing a continuous high-speed growth trend.

In 2020, China's traffic management department has handled a total of 25.21 million motor vehicle transfer and registration services. Among them, 24.81 million are transfer business. In the past five years, the proportion of car transfer and registration business volume has increased from 59% to 102%, reflecting an increasingly active second-hand car trading market. In 2020, the number of motor vehicle drivers in China has reached 456 million. Affected by the novel coronavirus pneumonia, the number of new drivers in China (less than 1 year of driving experience) reached 22,310,000 in 2020, accounting for 4.90% of the total number of motorists in the country, a decrease of 7,120,000 and 24.19% compared with 2019. From the perspective of driver gender, there are 308 million male drivers, accounting for 67.57%, and 148 million female drivers, accounting for 32.43%. From the perspective of driver age, there are 327 million drivers aged 26 to 50, accounting for 71.79%; 60.86 million drivers aged 51 to 60, accounting for 13.36%. Although China's auto market is booming, the luxury car market is mainly monopolized by foreign manufacturers. The analysis of the competitive situation and equilibrium of luxury automobile market is helpful for a better understanding of the operation mechanism of China's automobile market.

The development of any industrial system will be restricted by its own growth ability and resource environment, so the evolution process of any industrial system is limited and regular. Almost all industries will obey the law of cycle and will go through the process from birth to growth to maturity and then to recession. Similarly, the development of the industry should not be unlimited. Due to the constraints of the industry itself and external conditions, there is a problem of limited growth. If an industrial system is regarded as an ecosystem, the enterprises in the industry can be regarded as its population. The enterprise population dynamics model mainly focuses on the change of population quantity, and its law of change is based on the law of nonlinear growth in biological population quantity. Many species in nature grow nonlinearly, and the nonlinear growth of population is also very common. Under the influence of market environment, industrial policy, and development resources in a certain region, the enterprise population may change rapidly.

The competition and coordination mechanism within the population is also an important factor. This setting is based on the principle of intraspecies competition of a biological population. There is competition in the natural biological population, the function of which is to regulate the population size. The larger the population is, the more competition there is. There are also certain competition mechanisms in the innovation population, which will restrain the overexpansion of the innovation population to a certain extent. As a result, intraspecies competition is actually one of the processes for the innovation population to achieve the survival of the fittest. Therefore, this mechanism should also be taken as an important component of the growth model of the entrepreneurial population. There is also competition or synergy among the innovation groups of enterprises in neighboring regions. Lotka–Volterra model is the main method to study the interaction mechanism among populations.

The Lotka–Volterra model was successfully spread to the research fields of social and economic system to interpret the competition behavior between organizations [1]. The model is widely used in the research fields of industrial competition [2], enterprise competition [3], market competition [4], and product competition [5]. Some researchers use the Lotka–Volterra model to analyze innovation activities among competitors [6] and technology substitution among competitors [7]. Some researchers focus on the transformation path [8].

In terms of data characteristics, social economic system is also a grey system. Grey system theory is a method to study and solve a certain system with uncertain information, that is, a grey system. Grey system is one between black system and white system, which contains both known and unknown information. The information contained in a grey system is called grey information. In the abstract systems of society, economy, and ecology, researchers should extend the viewpoints and methods of general cybernetics, systems theory, and information theory to those abstract systems and make reasonable explanations. The grey system theory establishes a set of theories and methods to solve the related problems of incomplete information system, which has great development potential in practical application. This theory is a new one, which takes the grey system as the research object and uses a specific method to describe and control the grey system. In fact, the theory is to study the whitening problem of grey system, that is, to study how to use the existing information to predict the unknown information in the future from a systematical perspective. The essence is to use the idea and method of the theory to quantify the abstract phenomenon, analyze the relevant data, and make quantitative prediction and control for the future, so as to complete the system analysis. GM (1, 1) model is widely used in the grey system theory.

GM (1, 1) model proposed by Deng [9] has been widely applied in many fields [10]. It has been applied as a forecasting approach in various fields, including wafer fabrication [11], opto-electronics industry forecasting [12], electricity forecasting [13], integrated circuit industry forecasting [14], product profit forecasting [15], and vehicle fatality risk forecasting [16]. As a single variable forecasting model, traditional GM (1, 1) cannot be used to analyze the long-term relationship between the two variables.

The classical GM (1, 1) model cannot be used in bivariate social and economic system. It is necessary to analyze complicated relationships as they influence each other. Therefore, Wu et al. used the grey Lotka–Volterra model to analyze the relationship between two variables and used the discrete grey Lotka–Volterra model to forecast the values of two variables [17]. Wang et al. applied grey forecast theory with the Lotka–Volterra competition model to explore the dynamic competition between smart TVs and flat panel TVs. After comparing forecast accuracy among the model proposed in this study, they found the proposed model has the best accuracy [5]. Mao et al. [18] established direct grey Lotka–Volterra model with adjustable background value. However, the current application of the grey Lotka–Volterra model mainly discusses the relationship between the two populations and the prediction of population size and does not involve the equilibrium analysis of the Lotka–Volterra model.

In order to make up for the defects of the current research, the research objectives of this paper are as follows: (1) a more accurate grey time series model is constructed for data transformation or prediction; (2) three species grey Lotka–Volterra model is used to analyze the competition of grey time series data; (3) a competitive and equilibrium analysis is made taking three brands of cars in China's luxury car market as an example; and (4) the accuracy of grey time series model and the robustness of the three species grey Lotka–Volterra model are verified based on case data.

The contents and arrangements of the research are as follows: (1) a compound grey transformation system based on GM (1, 1) prediction model is constructed to transform the original data; (2) the influence mechanism among enterprise populations is analyzed by the model of three-dimensional grey Lotka–Volterra system; (3) the multichoice goal programming is built to analyze the balanced competition of enterprises. Highlights of this paper are as follows: (1) an improved grey GM (1, 1) prediction model is provided; (2) the competition among three enterprises is analyzed based on three-dimensional grey Lotka–Volterra system; and (3) multichoice goal programming approach is introduced in the equilibrium analysis of the grey Lotka–Volterra model.

2. Method and Material

This study explores competition relationship among grey Lotka–Volterra system. The methods used in this study are described as follows.

As shown in Figure 1, this paper uses three steps to achieve the research goal. Firstly, a compound grey model is constructed to transform the observation data. Then Lotka–Volterra model is used to calculate the symbiotic relationship of the grey transformed data. Finally, MCGP model is used to analyze population growth, population competition, and competitive equilibrium.

2.1. Grey Transformation of Data Based on Compound Grey Model. Grey forecast is an approach based on the GM (1, 1) basic model to predict uncertain and incomplete information systems to determine the elements' future dynamic situation among a certain sequence of numbers. The processes of the GM (1, 1) model are shown as follows.

The original data series $X^{(0)}$ is

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}. \tag{1}$$

The accumulated generating operation (AGO) $X^{(1)}(k)$ is

$$\begin{cases} X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i), k = 1, 2, 3, \cdots n \\ X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n)\right\} \end{cases}. \tag{2}$$

The original form of GM (1, 1) model is

$$X^{(0)}(k) + aX^{(1)}(k) = b. \tag{3}$$

The parameter estimation based on the original form of the model and equation (3) is called the original difference grey model (ODGM) [10]. In the GM (1, 1) model, the parameters a and b should be calculated first with the method of ordinary least square. The following model is an even GM (1, 1) system or even grey model (EGM). Practice shows that the simulation effect of the model is better, and it is also a commonly used grey model.

$$\begin{cases} X^{(0)}(k) + az^{(1)}(k) = b \\ z^{(1)}(k) = \dfrac{x^{(1)}(k) + x^{(1)}(k+1)}{2} \\ k = 1, 2, 3, \cdots, n-1 \end{cases}. \tag{4}$$

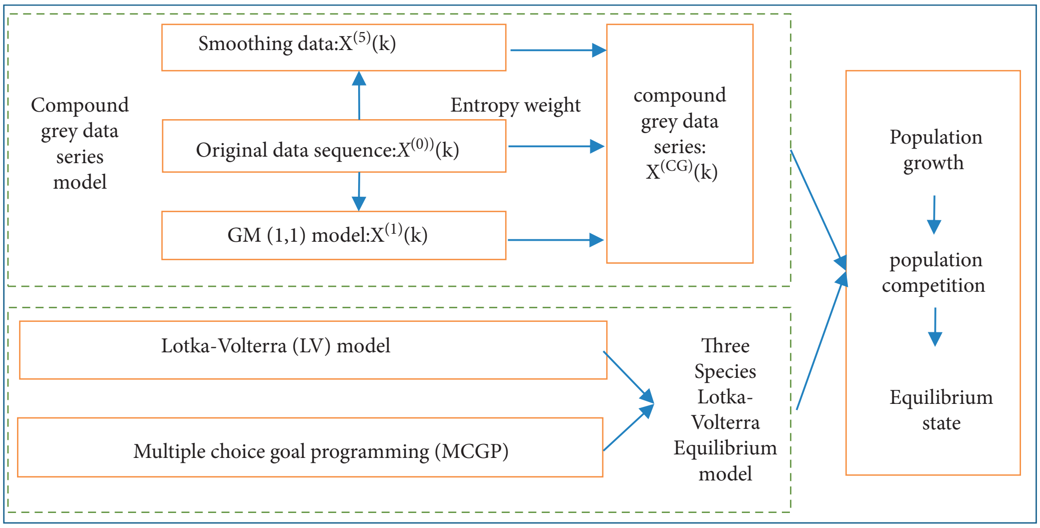

Figure 1: Research process.

After estimation of a and b, the grey prediction equation can be solved as follows:

$$\hat{X}^{(1)}(k+1) = \left[X^{(0)}(1) - \frac{b}{a}\right]e^{-ak} + \frac{b}{a}. \tag{5}$$

The following can be obtained based on inverse accumulated generating operation (IAGO):

$$\begin{aligned} X^{(0)}(k+1) &= X^{(1)}(k+1) - X^{(1)}(k) \\ &= \left[X^{(0)}(1) - \frac{b}{a}\right]e^{-ak}\left(1-e^{a}\right), \quad k = 1,2,3,\cdots,n-1. \end{aligned} \tag{6}$$

The new information priority principle is one of the basic principles of the grey system. Many scholars use this principle to optimize the grey GM (1, 1) model, but theoretical proof is needed for the priority of new information. Because the priority of the latest information in the traditional grey GM (1, 1) prediction model cannot be compared intuitively, this paper constructs a composite GM (1, 1) grey data model based on the weight of information entropy in order to make full use of the original data. This paper studies the quarterly sales of three major manufacturers in the luxury car market in mainland China and the grey model is used to transform the observation data in order to obtain more abundant research information. This paper aims at the characteristics of automobile sales data, considering the grey of original data and seasonal characteristics of sales data to construct the compound grey data series $X^{(CG)}(k)$.

$$X^{(CG)}(k) = w_1 X^{(0)}(k) + w_2 X^{(1)}(k) + w_3 X^{(S)}(k). \tag{7}$$

Among them, $X^{(S)}(k)$ is smoothing data series for seasons, and w_i is entropy weight. Entropy weight is an objective method [19]. It is a popular method and is always combined with other methods. For example, entropy combined with fuzzy VIKOR [20], with GIS and AHP [21], or with subjective weight [22]. Entropy weight method is provided as follows [23]. Supposing there are m alternatives (A_1, A_2,...,A_m) and n criteria (C_1, C_2,...,C_n) for a decision problem, then the initial decision matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \left[a_{ij}\right]_{m\times n}. \tag{8}$$

Step 1 : normalize the evaluation matrix:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}. \tag{9}$$

Step 2 : compute entropy:

$$e_j = -\frac{1}{\ln m}\sum_{i=1}^{m} r_{ij}\ln r_{ij}, \quad j = 1,2,\cdots,n. \tag{10}$$

Step 3 : calculate weights:

$$w_j = \frac{1-e_j}{\sum_{i=1}^{n}\left(1-e_j\right)}, \quad j = 1,2,\cdots,n \tag{11}$$

In order to further test the prediction accuracy of the compound grey model, this paper compares the prediction results of grey prediction model and autoregression moving average (ARMA) model that is commonly used in time series analysis.

2.2. Competitive Situation and Equilibrium Analysis Based on the Lotka–Volterra Model. The paper constructs an internal dynamic system of population 1 (P_1) based on the logistic model.

$$g_1(t) = \frac{dN_1(t)}{dt} = \alpha_1 N_1\left(1 - \frac{N_1}{K_1}\right), \tag{12}$$

where $g_1(t)$ is the growth rate in phase t and $N_1(t)$ is population size of period t. K_1 is the maximum population scale; α_1 is the intrinsic growth rate, and $(1 - N_1/K_1)$ is the retardation of growth.

The measurement model is as follows.

Because $dN_1(t) \approx \Delta N_1(t)$, $\Delta N_1(t) = N_1(t) - N_1(t-1)$, $dt \approx \Delta t = t - (t-1) = 1$.

So,

$$g_1(t) \approx \Delta N(t) = \gamma_1 N_1(t-1) + \gamma_2 N_1^2(t-1). \tag{13}$$

Among them, $\gamma_1 = \alpha_1$, usually $\gamma_1 > 1$. It usually represents the synergy within a population. $\gamma_2 = -\alpha_1/K_1$, usually, $\gamma_2 < 0$. It refers to the competition effect within a population. It is called the internal competition coefficient or population density inhibition coefficient.

Similarly, the internal relation model of population 2 (P_2) can be obtained:

$$g_2(t) = \frac{dN_2(t)}{dt} = \alpha_2 N_2\left(1 - \frac{N_2}{K_2}\right). \tag{14}$$

The following system shows the impact of P_2 on P_1:

$$g_1(t) = \frac{dN_1(t)}{dt} = \alpha_1 N_1\left(1 - \frac{N_1}{K_1} + \frac{\beta_{12}N_2}{K_2}\right), \tag{15}$$

where β_{12} is the influence of population 2 on population 1 ($\beta_{12} > 0$, synergistic effect; $\beta_{12} < 0$, competitive effect). Similarly, the following system shows the impact of P_1 on P_2:

$$g_2(t) = \frac{dN_2(t)}{dt} = \alpha_2 N_2\left(1 - \frac{N_2}{K_2} + \frac{\beta_{21}N_1}{K_1}\right), \tag{16}$$

where β_{21} is the impact factor of population 1 on population 2. The dynamic system of P_1 and P_2 is

$$\begin{cases} g_1(t) = \dfrac{dN_1(t)}{dt} = \alpha_1 N_1\left(1 - \dfrac{N_1}{K_1} + \dfrac{\beta_{12}N_2}{K_2}\right) \\ g_2(t) = \dfrac{dN_2(t)}{dt} = \alpha_2 N_2\left(1 - \dfrac{N_2}{K_2} + \dfrac{\beta_{21}N_1}{K_1}\right) \end{cases}. \tag{17}$$

Equation (17) is called the Lotka–Volterra (LV) system, which is based on the logistic model of a single species, and considers the dynamic growth of competition and symbiosis of two or more entities simultaneously in the ecosystem [24]. LV model can not only accurately describe the competition and symbiosis between corporate populations, but also determine the influence of the core population in the evolution of the entire ecosystem [25]. The LV system has better data fitting and prediction expression [26].

The classical LV model is used to simulate the dynamic relationship of populations in an ecological system. It was introduced into the fluctuation of macro-economic growth and the market competition of medium scale and scope. According to the principle of biology, there are two kinds of functional relationships among biological populations: promotion or inhibition. For their own survival and development, there is also such a relationship between market competition subjects: the existence of one subject can promote or inhibit the diffusion process of another subject. Based on the numerical value of β_{ij}, the type of interaction between species can be judged [27]. In a symbiotic system composed of population 1 (P_1), population 2 (P_2), and population 3 (P_3), the LV model is

$$\begin{cases} g_1(t) = \dfrac{dN_1(t)}{dt} = \alpha_1 N_1\left(1 - \dfrac{N_1}{K_1} + \dfrac{\beta_{12}N_2}{K_2} + \dfrac{\beta_{13}N_3}{K_3}\right) \\ g_2(t) = \dfrac{dN_2(t)}{dt} = \alpha_2 N_2\left(1 - \dfrac{N_2}{K_2} + \dfrac{\beta_{21}N_1}{K_1} + \dfrac{\beta_{23}N_3}{K_3}\right) \\ g_3(t) = \dfrac{dN_3(t)}{dt} = \alpha_3 N_3\left(1 - \dfrac{N_3}{K_3} + \dfrac{\beta_{31}N_1}{K_1} + \dfrac{\beta_{32}N_2}{K_2}\right) \end{cases}. \tag{18}$$

Among them, β_{ij} $(i = 1, 2, 3. j = 1, 2, 3.)$ is the interaction coefficient between populations. When β_{ij} is greater than zero, it indicates that there is a synergetic relationship among populations. When β_{ij} is less than zero, it means that there is competition among populations.

The following is econometric model (19) and regression analysis:

$$\begin{cases} N_1(t) = \gamma_{11}N_1(t-1) + \gamma_{12}N_1^2(t-1) + \gamma_{13}N_1(t-1)N_2(t-1) + \gamma_{14}N_1(t-1)N_3(t-1) \\ N_2(t) = \gamma_{21}N_2(t-1) + \gamma_{22}N_2^2(t-1) + \gamma_{23}N_2(t-1)N_1(t-1) + \gamma_{24}N_2(t-1)N_3(t-1) \\ N_3(t) = \gamma_{31}N_3(t-1) + \gamma_{32}N_3^2(t-1) + \gamma_{33}N_3(t-1)N_1(t-1) + \gamma_{34}N_3(t-1)N_2(t-1) \end{cases}. \tag{19}$$

Among them, $\gamma_{11} = \alpha_1 + 1$, $\gamma_{12} = -\alpha_1/K_1$, $\gamma_{13} = \alpha_1\beta_{12}/K_2$, and $\gamma_{14} = \alpha_1\beta_{13}/K_3$. Generally, $\gamma_{11} > 1, \alpha_1 > 0$, it means the synergy effect within the population 1, and $\gamma_{12} < 0$, it means the competition effect within the population. It is called the internal competition coefficient or the population density inhibition coefficient. In the same way: $\gamma_{21} = \alpha_2 + 1$, $\gamma_{22} = -\alpha_2/K_2$, $\gamma_{23} = \alpha_2\beta_{21}/K_1$, $\gamma_{24} = \alpha_2\beta_{23}/K_3$, $\gamma_{31} = \alpha_3 + 1$, $\gamma_{32} = -\alpha_3/K_3$, $\gamma_{33} = \alpha_3\beta_{31}/K_1$, and $\gamma_{34} = \alpha_3\beta_{32}/K_2$.

2.3. Population Growth under Constraints of Three Species Lotka–Volterra Equilibrium. Objective programming is an effective way to solve the problem of multiobjective programming. The basic idea is to determine the expected value of each objective function of the multiobjective programming problem. The goal programming method is more flexible, effective, and easy to use and implement in dealing with multiobjective problems. In recent years, multichoice goal programming (MCGP) has been widely used to solve many practical decision problems [28]. Chang [29] proposed the multiple choice goal programming (MCGP) method as

$$\left\{\begin{array}{l} Objective\ function, \\ Min \sum_{i=1}^{n} (d_i^{+} + d_i^{-}) + \sum_{i=1}^{n} (e_i^{+} + e_i^{-}) \\ Constraints, \\ g_i = f_i(x) - d_i^{+} + d_i^{-}, \quad i = 1, 2, \cdots, n \\ x \in X = \{x_1, x_2, \cdots, x_m\} \\ X \in F, \quad (F\ is\ the\ set\ of\ feasible\ solutions) \\ g_{i,\max} = g_i - e_i^{+} + e_i^{-}, \quad i = 1, 2, \cdots, n \\ g_{i,\max} \geq g_i \geq g_{i,\min}, \quad i = 1, 2, \cdots, n \\ e_i^{+}, e_i^{-}, d_i^{+}, d_i^{-} \geq 0, \quad i = 1, 2, \cdots, n \end{array}\right. . \tag{20}$$

Here, d_i^{+}, d_i^{-} indicate the value of the i-th goal exceeding and not reaching the expected value of the goal. $f_i(x)$ is the formula of the i-th objective. X is the decision variable. g_i indicates the expected goal of i-th. e_i^{+} are close to positive deviation of $|g_i - g_{i,\max}|$. e_i^{-} are close to negative deviation of $|g_i - g_{i,\max}|$. $g_{i,\min}$ and $g_{i,\max}$ are the lower and upper limits for the goal of g_i. Having learned from the relevant research [30, 31], the paper has embedded the equilibrium condition of LV system into the MCGP model to obtain the Lotka–Volterra-MCGP model:

$$Objective\ function, Min \sum_{i=1}^{n} (d_i^{+} + d_i^{-}) + \sum_{i=1}^{n} (e_i^{+} + e_i^{-})$$

$$\left\{\begin{array}{l} Constraints \\ g_i = f_i(x) + d_i^{-} - d_i^{+}, \quad i = 1, 2, \cdots, n \\ x \in X, X = \{x_1, x_2, \cdots, x_m\} \\ X \in F \quad (F\ is\ the\ set\ of\ feasible\ solutions) \\ g_{i,\max} = g_i + e_i^{-} - e_i^{+}, \quad i = 1, 2, \cdots, n \\ g_{i,\min} \leq g_i, g_i \leq g_{i,\max}, \quad i = 1, 2, \cdots, n \\ d_i^{+}, d_i^{-}, e_i^{+}, e_i^{-} \geq 0, \quad i = 1, 2, \cdots, n \\ g_1(t) = \dfrac{dN_1(t)}{dt} = \alpha_1 N_1 \left(1 - \dfrac{N_1}{K_1} + \dfrac{\beta_{12} N_2}{K_2} + \dfrac{\beta_{13} N_3}{K_3}\right) \\ g_2(t) = \dfrac{dN_2(t)}{dt} = \alpha_2 N_2 \left(1 - \dfrac{N_2}{K_2} + \dfrac{\beta_{21} N_1}{K_1} + \dfrac{\beta_{23} N_3}{K_3}\right) \\ g_3(t) = \dfrac{dN_3(t)}{dt} = \alpha_3 N_3 \left(1 - \dfrac{N_3}{K_3} + \dfrac{\beta_{31} N_1}{K_1} + \dfrac{\beta_{32} N_2}{K_2}\right) \\ -1 < \beta_{ij} < 1, i = 1, 2, 3. j = 1, 2, 3. \\ \sum g_i = K, (K\ is\ the\ sum\ of\ market\ capacity) \end{array}\right. . \tag{21}$$

As a linear form for objective programming, multichoice goal programming embedded with Lotka–Volterra equilibrium (LV-MCGP) can be easily resolved by any common software.

2.4. Empirical Analysis. The sales volume of luxury brand cars in mainland China has achieved strong growth again in 2020. According to the domestic automobile wholesale data provided by China passenger car federation, the sales volume of luxury car market in 2020 has been 2.79 million, with a year-on-year increase of 19.9%. The sales volume of China's automobile market has declined in the three consecutive years from 2018 to 2020 by 3.9%, 9.2%, and 6.3% respectively. However, sales of luxury cars have risen by 17.6%, 11.7%, and 19.9%, respectively, in the same period. In the passenger car federation's wholesale list, Audi won the sales champion. Audi, Mercedes Benz, and BMW sell 638,000, 619,000, and 609,000 domestic models, respectively, in 2020. ABB (representing Audi, Benz, and BMW) has contributed 1.866 million in total, accounting for 67% of the total luxury car market. The competition among the three luxury brands in first group is increasingly fierce. This situation will continue in 2021 and in the next three years, and the competition will be more intense. In 2020, China's luxury car consumption market has achieved strong growth and has become the focus of global car market. In 2021, global automobile luxury brand manufacturers will pay more attention to and rely more on the Chinese market. With the positive progress of vaccine delivery, the global economy is expected to enter the repair period in 2021, and the uncertainty of China's economic development will be greatly reduced compared with 2020. In 2021, China's luxury car market is expected to maintain a strong growth. This paper will analyze the market competition of the three main luxury brands based on the data obtained by the composite grey transformation.

Two years ago, BMW completed the change of its share ratio in the BMW Brilliance joint venture. The 75% shareholding means that BMW will get a higher profit than Mercedes Benz and Audi whenever it sells a BMW, so it will be in a more comfortable position in the competition. The high 75% shareholding ratio also means that BMW's investment in the Chinese market is guaranteed and the speed of model introduction is accelerated. Since 2021, BMW's production capacity in mainland China will rapidly rise to the level of an annual output of 800,000 vehicles after the expansion of BMW Brilliance's third factory. In addition, BMW's i-series new products can basically achieve synchronous sales with the international market since the news of pure electric vehicles.

As shown in Table 1, the process of grey transformation is easy to realize. Entropy weight is calculated based on the data in the example: $w_1 = 0.333$, $w_2 = 0.333$, and $w_3 = 0.333$. It shows that there is no difference between the original data, seasonal smoothing data, and GM (1, 1) data in the evaluation of information entropy. It also shows that the information fidelity of grey transformation is relatively high.

Mercedes Benz will launch a new generation of S-class in 2021. The S-class is always the leader of the new design language, which means that Mercedes Benz will start the wave of product upgrading again from 2021. In 2021, the C-class car will also be upgraded, which is obviously faster than in previous years.

As shown in Table 2, the Benz sales data of grey transformation is easy to realize. Similar to BMW's data, $w_1 = 0.333$, $w_2 = 0.333$, and $w_3 = 0.333$. It shows that there is no difference between the original data, seasonal smoothing data, and GM (1, 1) data in the evaluation of information entropy. It also shows that the information fidelity of grey transformation is relatively high.

Audi is trying its best to consolidate its leading position in China's luxury car market. By the end of 2020, SAIC Audi has officially announced that Audi will become the first luxury brand with two major vehicle production and sales joint ventures in China, which means that Audi will speed up again in the sales campaign of luxury brand vehicles. In 2021, SAIC Audi will release its first domestic Audi a7l, which is expected to be officially launched in 2022, and is likely to speed up the process. Audi and China First Automobile Group's new energy vehicle joint venture company in Jilin Province also signed a contract in January 2021. The first phase plant plans to invest 30 billion yuan, while Audi and Volkswagen group hold 60% of the equity of the new company. New investment and new share ratio help Audi lock in its competitive advantage in the next 10 years.

As shown in Table 3, the Audi sales data of grey transformation is realized. In this paper, the mean absolute percentage error (MAPE) index is used to evaluate the effect of grey transformation, Among them,

$$\mathrm{MAPE} = \frac{1}{n}\sum_{k=1}^{n}\left|\frac{A_k - F_k}{A_k}\right| \times 100\%, \tag{22}$$

where A_k is the actual value and F_k is the predicted value.

As shown in Table 4, the prediction effect of composite prediction model is very good. The ARMA model has the greatest error. The compound grey model has the smallest error. The practice shows that among the four prediction models, the compound grey model has the best prediction accuracy.

As shown in Figure 2, the radar chart shapes of different prediction models of the three enterprises are similar, which shows the stability of the prediction accuracy of the model. The ARMA model has the worst prediction accuracy. The compound grey system has more variety and more practical advantages. For example, (1) the compound grey model can be used to explain population dynamics. Compound grey model is an improvement of traditional population dynamics model, which is suitable for the research of ecology related fields. (2) It can be constructed as a generalized prediction model with time as the independent variable. (3) The application scenarios are more extensive than ARMA model, especially in the case of insufficient or missing original data.

Table 1: Grey transformation data of BMW sales volume.

Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA	Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA
2016-1	27663										
2016-2	13103		24175		27077	2018-8	36698	34054	37913	36222	33225
2016-3	24324	21697	24541	23520	12538	2018-9	37638	36052	38486	37392	36098
2016-4	21736	19721	24912	22123	23743	2018-10	46925	40420	39067	42138	37037
2016-5	24192	23417	25288	24299	21159	2018-11	51460	45341	39658	45486	46310
2016-6	25693	23874	25670	25079	23611	2018-12	41815	46733	40257	42935	50839
2016-7	24802	24896	26058	25252	25110	2019-1	53289	48855	40865	47670	41208
2016-8	28124	26206	26452	26927	24220	2019-2	32929	42678	41483	39030	52665
2016-9	30709	27878	26852	28480	27537	2019-3	49482	45233	42110	45608	32335
2016-10	25276	28036	27257	26857	30118	2019-4	49945	44119	42746	45603	48863
2016-11	31125	29037	27669	29277	24693	2019-5	46602	48676	43392	46223	49326
2016-12	26461	27621	28087	27390	30534	2019-6	42315	46287	44048	44217	45988
2017-1	31586	29724	28512	29941	25877	2019-7	45710	44876	44713	45100	41707
2017-2	26287	28111	28943	27780	30994	2019-8	47644	45223	45389	46085	45097
2017-3	32279	30051	29380	30570	25703	2019-9	49631	47662	46075	47789	47028
2017-4	34835	31134	29824	31931	31686	2019-10	48862	48712	46771	48115	49012
2017-5	31505	32873	30275	31551	34238	2019-11	53951	50815	47478	50748	48244
2017-6	27566	31302	30732	29867	30913	2019-12	52577	51797	48195	50856	53326
2017-7	30050	29707	31197	30318	26980	2020-1	48432	51653	48924	49670	51954
2017-8	32888	30168	31668	31575	29460	2020-2	8039	36349	49663	31350	47815
2017-9	34496	32478	32147	33040	32294	2020-3	43263	33245	50414	42307	7482
2017-10	30420	32601	32632	31885	33900	2020-4	56204	35835	51175	47738	42654
2017-11	38097	34338	33126	35187	29830	2020-5	57903	52457	51949	54103	55576
2017-12	35548	34688	33626	34621	37495	2020-6	46597	53568	52734	50966	57272
2018-1	43524	39056	34134	38905	34950	2020-7	63596	56032	53531	57720	45983
2018-2	24968	34680	34650	31433	42914	2020-8	65558	58584	54340	59494	62957
2018-3	35569	34687	35174	35143	24386	2020-9	56350	61835	55161	57782	64916
2018-4	36233	32257	35705	34732	34971	2020-10	47166	56358	55994	53173	55721
2018-5	35516	35773	36245	35844	35634	2020-11	61219	54912	56840	57657	46551
2018-6	31644	34464	36792	34300	34918	2020-12	54834	54406	57699	55647	60583
2018-7	33820	33660	37348	34943	31052	2021-1	73333	63129	58571	65011	54208

Table 2: Grey transformation data of Mercedes Benz sales.

Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA	Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA
2016-1	29256										
2016-2	17716		27301		28759	2018-8	40005	41685	40322	40671	40900
2016-3	23837	23603	27659	25033	17240	2018-9	43970	41798	40849	42206	39488
2016-4	23304	21619	28020	24314	23350	2018-10	31922	38632	41384	37313	43445
2016-5	25970	24370	28387	26242	22818	2018-11	15419	30437	41925	29260	31420
2016-6	27693	25656	28758	27369	25479	2018-12	49530	32290	42474	41431	14948
2016-7	25009	26224	29135	26789	27199	2019-1	56231	40393	43030	46551	48995
2016-8	25205	25969	29516	26897	24520	2019-2	39595	48452	43593	43880	55683
2016-9	24333	24849	29902	26361	24715	2019-3	55157	50328	44163	49883	39078
2016-10	24279	24606	30293	26393	23845	2019-4	46252	47001	44741	45998	54611
2016-11	29394	26002	30690	28695	23791	2019-5	49952	50454	45326	48577	45723
2016-12	34582	29418	31091	31697	28896	2019-6	47544	47916	45919	47126	49416
2017-1	42077	35351	31498	36309	34075	2019-7	49561	49019	46520	48367	47013
2017-2	33862	36840	31910	34204	41556	2019-8	50366	49157	47129	48884	49026
2017-3	31042	35660	32327	33010	33356	2019-9	51327	50418	47745	49830	49829
2017-4	34613	33172	32750	33512	30541	2019-10	47790	49828	48370	48662	50788
2017-5	37440	34365	33179	34995	34106	2019-11	49653	49590	49003	49415	47258
2017-6	34555	35536	33613	34568	36927	2019-12	50396	49280	49644	49773	49118
2017-7	37205	36400	34053	35886	34048	2020-1	44411	48153	50293	47619	49859
2017-8	35521	35760	34498	35260	36693	2020-2	11295	35367	50951	32538	43885
2017-9	37495	36740	34950	36395	35012	2020-3	48589	34765	51618	44991	10831
2017-10	34484	35833	35407	35241	36982	2020-4	52806	37563	52293	47554	48056
2017-11	36936	36305	35870	36370	33977	2020-5	58431	53275	52977	54895	52265
2017-12	41518	37646	36340	38501	36424	2020-6	52510	54582	53671	53588	57879

Table 2: Continued.

Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA	Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA
2018-1	56306	44920	36815	46014	40998	2020-7	55720	55554	54373	55215	51969
2018-2	33564	43796	37297	38219	55758	2020-8	55241	54490	55084	54939	55173
2018-3	43051	44307	37785	41714	33059	2020-9	63008	57990	55805	58934	54695
2018-4	44006	40207	38279	40831	42528	2020-10	53964	57404	56535	55968	62448
2018-5	44589	43882	38780	42417	43481	2020-11	62451	59808	57275	59844	53421
2018-6	43631	44075	39287	42331	44063	2020-12	60624	59013	58024	59220	61892
2018-7	41420	43213	39801	41478	43107	2021-1	68412	63829	58783	63675	60068

Table 3: Grey transformation data of Audi car sales.

Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA	Year-month	$X^{(0)}(k)$	$X^{(S)}(k)$	$X^{(1)}(k)$	$X^{(CG)}(k)$	ARMA
2016-1	55285										
2016-2	32994		41006		54906	2018-8	55781	45431	49093	50102	38383
2016-3	49537	45939	41253	45576	32627	2018-9	61015	51850	49388	54084	55401
2016-4	47440	43324	41501	44088	49161	2018-10	46988	54595	49685	50423	60632
2016-5	46644	47874	41751	45423	47065	2018-11	51842	53282	49984	51703	46613
2016-6	44011	46032	42002	44015	46269	2018-12	52881	50570	50285	51245	51465
2016-7	43746	44800	42255	43600	43638	2019-1	50987	51903	50588	51159	52503
2016-8	45512	44423	42509	44148	43373	2019-2	30295	44721	50892	41969	50610
2016-9	46075	45111	42765	44650	45138	2019-3	48821	43368	51198	47796	29930
2016-10	43193	44927	43022	43714	45701	2019-4	40334	39817	51506	43886	48445
2016-11	54789	48019	43281	48696	42820	2019-5	37320	42158	51816	43765	39963
2016-12	27063	41682	43542	37429	54410	2019-6	50666	42773	52128	48522	36951
2017-1	51069	44307	43804	46393	26699	2019-7	53808	47265	52442	51171	50289
2017-2	28694	35609	44067	36123	50692	2019-8	55166	53213	52757	53712	53429
2017-3	41506	40423	44333	42087	28329	2019-9	62283	57086	53075	57481	54787
2017-4	42444	37548	44599	41530	41134	2019-10	63301	60250	53394	58982	61900
2017-5	44406	42785	44868	44020	42072	2019-11	65684	63756	53715	61052	62917
2017-6	43717	43522	45138	44126	44033	2019-12	60546	63177	54039	59254	65299
2017-7	48232	45452	45409	46364	43344	2020-1	53888	60039	54364	56097	60164
2017-8	52377	48109	45683	48723	47857	2020-2	8105	40846	54691	34547	53509
2017-9	59382	53330	45957	52890	51999	2020-3	34753	32249	55020	40674	7752
2017-10	51994	54584	46234	50937	59000	2020-4	55626	32828	55351	47935	34385
2017-11	57294	56223	46512	53343	51616	2020-5	56473	48951	55684	53703	55246
2017-12	30889	46726	46792	41469	56914	2020-6	56588	56229	56019	56279	56093
2018-1	62741	50308	47074	53374	30523	2020-7	58893	57318	56356	57522	56208
2018-2	32883	42171	47357	40804	62357	2020-8	60629	58703	56695	58676	58512
2018-3	53557	49727	47642	50309	32516	2020-9	72298	63940	57037	64425	60247
2018-4	46422	44287	47929	46213	53179	2020-10	63998	65642	57380	62340	71909
2018-5	45402	48460	48217	47360	46048	2020-11	66756	67684	57725	64055	63614
2018-6	41759	44528	48507	44931	45028	2020-12	50301	60352	58072	56242	66370
2018-7	38753	41971	48799	43174	41387	2021-1	80008	65688	58422	68039	49924

As shown in Table 5, the regression effect is good. Among them, $\alpha_1 = -0.385$, $K_1 = 63605$. $\alpha_2 = -0.448$, $K_2 = 50707$. $\alpha_3 = 0.757$, and $K_3 = 34489$.

As shown in Table 6, the regression effect is good. The regression results of grey system and original data are different, but the positive and negative sign of their influence coefficient and the symbiotic relationship reflected are the same. Among them, $\alpha_1 = 0.212$, $K_1 = 75601$, $\alpha_2 = 0.297$, $K_2 = 43347$, $\alpha_3 = 0.407$, and $K_3 = 29156$. The competition system based on the Lotka–Volterra model is as follows:

$$\begin{cases} g_1(t) = 0.212 N_1\left(1 - \dfrac{N_1}{75601} + \dfrac{\beta_{12}N_2}{43347} + \dfrac{\beta_{13}N_3}{29156}\right) \\ g_2(t) = 0.297 N_2\left(1 - \dfrac{N_2}{43347} + \dfrac{\beta_{21}N_1}{75601} + \dfrac{\beta_{23}N_3}{29156}\right) \\ g_3(t) = 0.407 N_3\left(1 - \dfrac{N_3}{29156} + \dfrac{\beta_{31}N_1}{75601} + \dfrac{\beta_{32}N_2}{43347}\right) \end{cases}. \quad (23)$$

Table 4: Grey transformation error (MAPE).

Enterprise	$X^{(S)}(k)$ (%)	$X^{(1)}(k)$ (%)	$X^{(CG)}(k)$ (%)	ARMA(1, 1) (%)
BMW	14.970	7.647	5.786	25.933
Benz	12.908	10.586	6.629	20.720
Audi	17.627	11.735	8.400	30.735

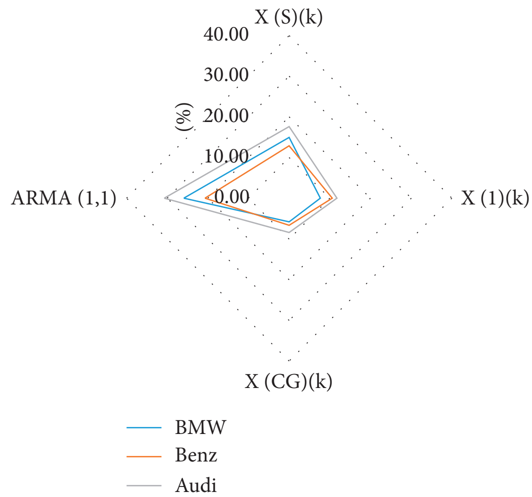

Figure 2: Comparison of error levels of four prediction models.

Table 5: Competitive analysis of raw data based on the Lotka–Volterra model.

$N_i(t)$	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}
$N_1(t)$	1.385 $(P \leq 0.001)$	$-6.059 * 10^{-6}$ $(P = 0.176)$	$5.580 * 10^{-6}$ $(P = 0.184)$	$-7.078 * 10^{-6}$ $(P = 0.087)$
$N_2(t)$	1.448 $(P \leq 0.001)$	$-8.841 * 10^{-6}$ $(P = 0.145)$	$7.653 * 10^{-6}$ $(P = 0.173)$	$-7.316 * 10^{-6}$ $(P = 0.051)$
$N_3(t)$	1.757 $(P \leq 0.001)$	$-2.196 * 10^{-5}$ $(P \leq 0.001)$	$5.881 * 10^{-6}$ $(P = 0.225)$	$3.164 * 10^{-6}$ $(P = 0.494)$

Table 6: Competitive analysis of grey transformation data based on the Lotka–Volterra model.

$N_i(t)$	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}
$N_1(t)$	1.212 $(P \leq 0.001)$	$-2.814 * 10^{-6}$ $(P = 0.462)$	$5.048 * 10^{-6}$ $(P = 0.174)$	$-5.885 * 10^{-6}$ $(P = 0.057)$
$N_2(t)$	1.297 $(P \leq 0.001)$	$-6.856 * 10^{-6}$ $(P = 0.102)$	$8.649 * 10^{-6}$ $(P = 0.037)$	$-6.810 * 10^{-6}$ $(P = 0.017)$
$N_3(t)$	1.407 $(P \leq 0.001)$	$-1.398 * 10^{-5}$ $(P \leq 0.001)$	$4.073 * 10^{-6}$ $(P = 0.331)$	$3.232 * 10^{-6}$ $(P = 0.420)$

The equilibrium point of the three species symbiosis system is as follows:

$$\begin{cases} g_1(t) = 0.212N_1 - \dfrac{0.212N_1^{\,2}}{75601} + \dfrac{0.212\beta_{12}N_1N_2}{43347} + \dfrac{0.212\beta_{13}N_1N_3}{29156} = 0 \\ g_2(t) = 0.297N_2 - \dfrac{0.297N_2^{\,2}}{43347} + \dfrac{0.297\beta_{21}N_1N_2}{75601} + \dfrac{0.297\beta_{23}N_3N_2}{29156} = 0 \\ g_3(t) = 0.407N_3 - \dfrac{0.407N_3^{\,2}}{29156} + \dfrac{0.407\beta_{31}N_1N_3}{75601} + \dfrac{0.407\beta_{32}N_2N_3}{43347} = 0 \end{cases} . \quad (24)$$

The equivalent change can be obtained as follows:

$$\Rightarrow\begin{cases} N_1 - \dfrac{{N_1}^2}{75601} + \dfrac{\beta_{12}N_1N_2}{43347} + \dfrac{\beta_{13}N_1N_3}{29156} = 0 \\ N_2 - \dfrac{{N_2}^2}{43347} + \dfrac{\beta_{21}N_1N_2}{75601} + \dfrac{\beta_{23}N_3N_2}{29156} = 0 \\ N_3 - \dfrac{{N_3}^2}{29156} + \dfrac{\beta_{31}N_1N_3}{75601} + \dfrac{\beta_{32}N_2N_3}{43347} = 0 \end{cases},$$

$$\Rightarrow\begin{cases} 1 - \dfrac{N_1}{75601} + \dfrac{\beta_{12}N_2}{43347} + \dfrac{\beta_{13}N_3}{29156} = 0 \\ 1 - \dfrac{N_2}{43347} + \dfrac{\beta_{21}N_1}{75601} + \dfrac{\beta_{23}N_3}{29156} = 0 \\ 1 - \dfrac{N_3}{29156} + \dfrac{\beta_{31}N_1}{75601} + \dfrac{\beta_{32}N_2}{43347} = 0 \end{cases}, \tag{25}$$

$$\Rightarrow\begin{cases} N_1 = 75601 + \dfrac{75601\beta_{12}N_2}{43347} + \dfrac{75601\beta_{13}N_3}{29156} \\ N_2 = 43347 + \dfrac{43347\beta_{21}N_1}{75601} + \dfrac{43347\beta_{23}N_3}{29156} \\ N_3 = 29156 + \dfrac{29156\beta_{31}N_1}{75601} + \dfrac{29156\beta_{32}N_2}{43347} \end{cases},$$

$$\Rightarrow\begin{cases} N_1 = 75601 + 1.744\beta_{12}N_2 + 2.593\beta_{13}N_3 \\ N_2 = 43347 + 0.573\beta_{21}N_1 + 1.487\beta_{23}N_3 \\ N_3 = 29156 + 0.386\beta_{31}N_1 + 0.672\beta_{32}N_2 \end{cases}$$

By substituting the above symbiotic relationship into the MCGP model [28], the following results are obtained:

$$Objective\ function, Min \sum_{i=1}^{n}\left(d_i^+ + d_i^-\right) + \sum_{i=1}^{n}\left(e_i^+ + e_i^-\right)$$

$$\begin{cases} Constraints, \\ g_i = f_i(x) + d_i^- - d_i^+, \quad i = 1, 2, \cdots, n \\ x \in X, X = \{x_1, x_2, \cdots, x_m\} \\ X \in F, \quad (F\ is\ the\ set\ of\ feasible\ solutions) \\ g_{i,\max} = g_i + e_i^- - e_i^+, \quad i = 1, 2, \cdots, n \\ g_{i,\min} \leq g_i, g_i \leq g_{i,\max}, \quad i = 1, 2, \cdots, n \\ e_i^+, e_i^-, d_i^+, d_i^- \geq 0, \quad i = 1, 2, \cdots, n \\ N_1 = 75601 + 1.744\beta_{12}N_2 + 2.593\beta_{13}N_3 \\ N_2 = 43347 + 0.573\beta_{21}N_1 + 1.487\beta_{23}N_3 \\ N_3 = 29156 + 0.386\beta_{31}N_1 + 0.672\beta_{32}N_2 \\ -1 < \beta_{ij} < 1, i = 1, 2, 3. j = 1, 2, 3. \\ \sum g_i = K, (K\ is\ the\ sum\ of\ market\ capacity) \end{cases}. \tag{26}$$

The problem is solved using the LINGO [32] software. The results of β_{ij} value optimization of equilibrium symbiosis model are shown in Table 7.

As shown in Table 7, with the expansion of market scale, more cooperation behaviors among populations are required. Cooperation behavior is comprehensive, and the intensity of cooperation is similar: the greater, the better.

2.5. Sensitivity Analysis of β_{Ij}. This paper analyzes the change range of the model optimization results from the situation of β_{ij} with equal increase and simultaneous reverse change. The greater the change in the optimization result caused by the change of β_{ij} per unit, the stronger the sensitivity of β_{ij} in the model. In order to analyze the sensitivity of β_{ij} facilitate, here β_{ij} is set in the following table.

As shown in Table 8, β_{ij} (β_{12}, $\beta_{13}\beta_{21}$, β_{23},$\beta_{31,}$ and β_{32}) has significant sensitivity in the model. When β_{ij} changes in the same direction and the same amount, the model optimizer can still run normally. The coordination change of β_{ij} significantly affects the model optimization results. The optimization result changes even more because of the coordination change of β_{ij}. Therefore, β_{ij} is the most sensitive factor in the model.

3. Results and Discussion

In order to analyze the relationship between the variables in economic system and improve the performance of GM (1, 1) prediction, the three-dimensional grey Lotka–Volterra system is presented. The empirical results indicate that the numerical aspects of the grey transformation effect of the three-dimensional grey Lotka–Volterra's MAPE values are highly accurate. As compared with the GM (1, 1), the three-dimensional grey Lotka–Volterra model used in this study offers more accurate prediction performance. The combination of grey transformation data and three-dimensional Lotka–Volterra model can mine the relationship of populations in a certain system; accordingly, this method is suitable when the data is limited for predicting the relationship between competitive products. The empirical analysis of China's luxury car market fully demonstrates the effectiveness and adaptability of this method.

Compared with the traditional grey model [9–16], the three-dimensional grey Lotka–Volterra model can well describe the mechanism of population growth. Three-dimensional grey Lotka–Volterra model is more suitable for the research of social and economic ecosystem. Compared with the traditional Lotka–Volterra model [1–4], the three-dimensional grey Lotka–Volterra model has higher accuracy. Compared with the grey Lotka–Volterra model [5, 17, 18], the Grey-Logistic model [30], and the three-dimensional Lotka–Volterra Model [31–34], the three-dimensional grey Lotka–Volterra model is more applicable. The case analysis based on three-dimensional grey Lotka–Volterra model fully demonstrates the accuracy and applicability of this method. Three-dimensional grey Lotka–Volterra model can effectively deal with the problem

Table 7: Balanced point of symbiotic growth.

Market size (K)	150000	180000	210000	240000	270000	300000
β_{12}	0.000	0.079	0.316	0.167	0.212	0.237
β_{13}	0.000	0.182	0.181	0.181	0.216	0.252
β_{21}	0.000	0.049	0.181	0.211	0.257	0.287
β_{23}	0.000	0.181	0.181	0.229	0.259	0.297
β_{31}	0.000	0.000	0.181	0.184	0.204	0.220
β_{32}	0.000	0.000	0.181	0.185	0.207	0.226

Table 8: Sensitivity analysis of the proposed model by changing β_{ij}.

System state	1	2	3	4	5	6	7
β_{12}	0	0.1	0.2	0.3	0.4	0.45	0.5
β_{13}	0	0.1	0.2	0.3	0.4	0.45	0.5
β_{21}	0	0.1	0.2	0.3	0.4	0.45	0.5
β_{23}	0	0.1	0.2	0.3	0.4	0.45	0.5
β_{31}	0	0.1	0.2	0.3	0.4	0.45	0.5
β_{32}	0	0.1	0.2	0.3	0.4	0.45	0.5
Market size (K)	150000	185126	246826	370212	740263	1479880	1.69×10^9

of insufficient data and missing data. When it is difficult to carry out regression analysis, the three-dimensional grey Lotka–Volterra model can be used to deal with it.

However, the model given in this study has its limitations in application, for example, (1) it does not reflect the life cycle mechanism of population growth, (2) the existing models cannot be used to analyze the symbiotic network, and (3) the model proposed in this paper cannot be used when studying the characteristics of data volatility. The above defects need to be remedied by future research.

4. Conclusion

The three-dimensional Lotka–Volterra model has been successfully built with the sales data of three local automobile enterprises in China. Based on Three Species System Analysis, it can be found that there is a symbiotic relationship among automobile enterprises and that the three species model can be adopted in analyzing the competition and cooperation among enterprises. Through balanced development of Symbiotic System Analysis, the result of symbiotic optimization under the equilibrium state of three populations is obtained. Equilibrium is possible in the growth of three populations; Three Species Evolutionary Analysis shows that cooperative strategy is better than competitive strategy and that this method is more practical in the analysis of enterprise competitive strategy. The findings reveal that this approach is feasible and effective to analyze competition, evolution, and balanced development of enterprise population. It is of great importance to study the relationships among enterprises as it is helpful for enterprises to make policies of strategy. Future studies could analyze the life cycle of competition among enterprises with three species Lotka–Volterra model.

This paper has a wide range of application scenarios. Grey three species Lotka–Volterra model can be used to analyze the symbiotic relationship between elements in natural and social systems. For example, (1) analysis of the relationship between competition and symbiosis of enterprise population; (2) analysis of symbiotic relationship among product populations within enterprises; (3) interaction analysis of investment portfolio; (3) prediction of input factors and output indicators in macro and regional economic systems; (4) analysis of symbiotic relationship of social ecosystem; and (5) symbiotic relationship analysis and population size prediction of biological populations in nature.

The research prospects of this paper are as follows: (1) the life cycle mechanism of population growth is introduced into the grey three species Lotka–Volterra model; (2) symbiotic network analysis models and methods can be developed based on existing models; and (3) the population dynamics method proposed in this paper can be combined with the machine learning method considering Chaotic Systems dynamics [35–37]. The new compound method can make full use of the new progress of machine learning research to transform population dynamics.

Data Availability

The experimental data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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