

Research Article

Evaluation of the Number of Visits to Chinese Medical Institutions Using a Logistic Differential Equation Model

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In this study, we established a two-dimensional logistic differential equation model to study the number of visits in Chinese PHCIs and hospitals based on the behavior of patients. We determine the model's equilibrium points and analyze their stability and then use China medical services data to fit the unknown parameters of the model. Finally, the sensitivity of model parameters is evaluated to determine the parameters that are susceptible to influence the system. The results indicate that the system corresponds to the zero-equilibrium point, the boundary equilibrium point, and the positive equilibrium point under different parameter conditions. We found that, in order to substantially increase visits to PHCIs, efforts should be made to improve PHCI comprehensive capacity and maximum service capacity.

1. Introduction

The medical and health service system of China, established through long-term development, consists of hospitals, primary health care institutions (PHCIs), and professional public health institutions, and covers both urban and rural areas [1]. Of these, hospitals and PHCIs are the main types of China's medical institutions. PHCIs are generally responsible for the treatment of frequently occurring diseases, as well as for rehabilitation and nursing for some diseases. The treatment of diseases that cannot be addressed in PHCIs is undertaken by hospitals. In China, the number of PHCIs accounts for as high as 95% of the whole medical system. However, in comparison with hospitals, resource utilization and the number of visits are fairly low [2, 3]. The number of visits is an important measure of the service efficiency of medical institutions and is also the main symbol of whether the development levels of different services of these institutions are balanced.

Looking back at the progress of the development of China's medical healthcare system, the main reasons for variation in the development level are the following: (1) compared with hospitals, the comprehensive level of medical

services in PHCIs is poor due to reasons such as shortages of medical staff, beds, advanced equipment, etc. [4, 5]; (2) under the influence of the current market economy, there is a lack of an appropriate division of labor and cooperative mechanism among China's medical institutions [6]; (3) the family-medicine physician system in the United States and Canada is such that residents should first contact their family physician when they are sick, except for emergency treatment. If the disease in question is beyond the treatment capabilities of the family physician, the family doctor will issue a referral form to the hospital or specialist for treatment. In contrast, Chinese residents can freely choose medical institutions for treatment. Thus, many patients select hospitals as their primary choice regardless of the severity of the disease, resulting in an increased workload in hospitals, while the resource utilization efficiency of PHCIs remains extremely low. This leads to an imbalance in the development level between medical institutions [7–9].

In order to improve the above situation, the State Council launched a new round of healthcare reforms in 2009. While these reforms have been pursued for more than ten years, no significant change has been achieved in the number of visits to PHCIs. According to statistics for 2019,

the average number of visits to hospitals in China was nearly three times that of visits to PHCIs [10, 11].

To date, many researchers have paid close attention to the study of patient visits and have achieved promising results. Li et al. took the time of the promulgation of China's key medical policies as the node and constructed a segmental regression model to analyze the impact of these policies on visits to medical institutions at all levels in the country. Their results showed that visits to different medical institutions were influenced by different medical policies [12]. Jin and Song et al. used descriptive statistics to analyze changes in the number of visits to medical institutions in Guizhou province and showed that the residents' habit of going to high-grade hospitals did not change following the reform of the medical and healthcare system [13]. Wang et al. predicted the visiting rate of medical institutions in Chinese hospitals from 2020 to 2030 under different modes of division of labor and cooperation by constructing a micro-simulation model and found that improving the treatment rate of PHCIs was conducive to the sustainable development of medical resources [14]. Xie et al. used the Verhulst model to predict the slow growth rate of visits to community health service centers. The prediction accuracy of this model was better than that of the GM (1,1) model [15]. Novikov et al. studied the relationship between temperature and the number of emergency department visits and reported that temperature can increase this number. Subsequently, they established a Poisson regression model to predict the number of emergency room visits in the next two days [16]. To predict the emergency treatment data of the consecutive day so that medical institutions could reasonably allocate medical resources in advance, Ekström used regression analysis to study the data of website visits on the Internet and found that the number of visits between 6 p.m. and midnight was significantly correlated with emergency room visits the next day. Thus, it was considered highly effective to use Internet data to predict emergency visits [17]. CulM used regression models and neural network models to analyze the data of public hospital emergency rooms demonstrating that the artificial neural network model could more accurately predict the number of emergency patients in the mid-to-long term, allowing the hospital to allocate resources ahead of time [18]. José et al. divided the age of patients in the pediatric emergency department of hospitals into two groups—0–2 years and 3–14 years—and conducted a retrospective study on pediatric emergency departments in hospitals to assess the effect of population growth on the number of visits. Their results showed that the number of emergency visits by the younger age group increased sharply with the population growth rate, whereas the number of emergency visits by the senior age group began to decline as the population increased to its highest value [19].

The above studies mainly used statistical methods to analyze historical data or used common prediction models to forecast the number of future patient visits. However, the number of patient visits is affected by many factors including the behavior of patients seeking medical treatment, population dynamics, or the level of medical services, and its fluctuation has obvious nonlinear and complex

characteristics. In contrast to previous studies, the research purpose and innovations of this paper are mainly reflected in the following aspects:

- (1) We use a logistic differential equation model to study the development trend of the number of visits to medical institutions in the absence of strong government intervention. The differential equation model is an excellent method of describing the process of a system, and it can transform complex social problems into an intuitive mathematical model. At present, mathematicians continue to expand the new field of differential equation research, thus promoting the development of differential equations [20–22]. The differential equation model has been widely used in studying epidemic diseases [23–27]. Especially, since the outbreak of pneumonia in the novel coronavirus infection, many scientists have established models to analyze and predict the development trend of infectious diseases and have achieved promising results [28–34]. This model has also been applied in the fields of sustainable science [35], economics [36], and in other areas, but few studies have focused on the study of the study of visits to medical institutions. The differential equation model of this paper is similar to the plaque model of the cross-regional transmission of infectious diseases [37]. Its topological structure is nonreversing, and it focuses on describing the phenomenon of the impact of patient behaviors on medical institutions, that is, higher-than-normal service demand. Thus, we provide a reference for future research by applying the two-way input model with regulatory measures.
- (2) A simulation is conducted to analyze the sensitivity of the model parameters, and the sensitivity factors that have an important effect on the system are identified from multiple uncertain factors. If a small change in a parameter can lead to a larger change in the system, this parameter is called a sensitive factor. Conversely, a parameter with considerable change leading to small change is called a nonsensitive factor. Since most decisions are made under uncertain circumstances, this type of analysis is a common method used by decision makers to solve problems, as it can provide a scientific basis for decision-making by a government to regulate the allocation of medical resources from the perspective of system engineering and to assist the balanced development of medical institutions.

The rest of the paper is organized as follows. Section 2 comprises the background of the field of study, the assumptions, and the process of establishing the model. Section 3 describes the process of solving the model and discusses the qualitative behavior and asymptotic properties of the solutions. Section 4 analyzes the sensitivity of parameter values to identify how parameter changes affect the system. Section 5 concludes the paper.

2. Establishment of the Mathematical Model

As mentioned above, China's medical system is a complex, nonlinear system. In this section, we establish a two-dimensional differential equation model based on the present situation to analyze the process of change of the number of visits in PHCIs and hospitals over time.

We assume that the medical system is closed, that is, there are no cross-regional medical treatment cases. We also do not consider cases where patients should be treated in hospitals but choose PHCIs for treatment. Additionally, according to the division of labor in Chinese medical institutions, common diseases should be treated in PHCIs first, while rare diseases or major diseases should be treated in hospitals. However, many patients with common diseases prefer to choose hospitals directly because of the hospitals' better medical conditions, as shown in Figure 1.

We use $x(t)$ and $y(t)$ to represent the number of visits to PHCIs and hospitals at time t . They are nonnegative, continuous, and differentiable functions. Let x_0 and y_0 denote the number of visits to PHCIs and hospitals at the initial time.

Since medical resources are limited, we hold the opinion that the growth rate of visits to PHCIs and hospitals conforms to the logistic block growth law if patients are reasonable in choosing a medical institution according to the severity of their conditions [38]. We use $r(x)$ and $r(y)$ to represent the growth-rate function of visits to PHCIs and hospitals, which means that they will not grow indefinitely when the number of visits to medical institutions reaches a certain level.

Therefore, the change of the number of visits per unit time in China's medical system can be expressed as

$$\begin{cases} \frac{dx(t)}{dt} = r(x)x(t), \\ \frac{dy(t)}{dt} = r(y)y(t), \end{cases} \quad (1)$$

where

$$\begin{cases} x(0) = x_0, \\ y(0) = y_0. \end{cases} \quad (2)$$

We use the following basic linear minus function to reflect the retarded growth law:

$$r(x) = r_1 - s_1 x(t), \quad (3)$$

$$r(y) = r_2 - s_2 y(t), \quad (4)$$

where r_1 and r_2 represent their inherent increase rate, which is positively correlated with the population growth rate and the aging rate. In order to determine the meaning of s_1 and s_2 , we use k_1 and k_2 to represent the maximum visiting capacity of PHCIs and hospitals, which is positively correlated with the comprehensive capabilities of the respective medical institutions.

If $x(t) = k_1$ and $y(t) = k_2$, then $r_1 = 0$ and $r_2 = 0$, which indicate that the number of visits will not continue to

increase. By substituting these into equations (3) and (4), we obtain

$$\begin{cases} s_1 = \frac{k_1}{r_1}, \\ s_2 = \frac{k_2}{r_2}. \end{cases} \quad (5)$$

Thus,

$$\begin{cases} r(x) = r_1 \left(1 - \frac{x(t)}{k_1}\right), \\ r(y) = r_2 \left(1 - \frac{y(t)}{k_2}\right). \end{cases} \quad (6)$$

Then, substituting them into system (1),

$$\begin{cases} \frac{dx(t)}{dt} = r_1 x(t) \left(1 - \frac{x(t)}{k_1}\right), \\ \frac{dy(t)}{dt} = r_2 y(t) \left(1 - \frac{y(t)}{k_2}\right). \end{cases} \quad (7)$$

We found that the left factors $r_1 x(t)$ and $r_2 y(t)$ reflect the growth trend of patient visits, but the right factors $(1 - x(t)/k_1)$ and $(1 - y(t)/k_2)$ indicate the blocking effect of the limited resources of medical institutions on the growth of patient visits.

Taking into account that deaths occur in the population and some people give up treatment due to difficulties in seeking medical treatment, this reduces the number of visits to medical institutions, so we use d_1 and d_2 to represent the churn rate of visits to PHCIs and hospitals, which positively correlates with the above situations. Thus, system (1) takes the form

$$\begin{cases} \frac{dx(t)}{dt} = r_1 x(t) \left(1 - \frac{x(t)}{k_1}\right) - d_1 x(t), \\ \frac{dy(t)}{dt} = r_2 y(t) \left(1 - \frac{y(t)}{k_2}\right) - d_2 y(t). \end{cases} \quad (8)$$

However, many patients with common diseases prefer to directly choose hospitals that feature better medical conditions for treatment, which results in an increased number of visits to hospitals and fewer visits to PHCIs, as shown in Figure 1. We use m to represent this leapfrog medical treatment rate that is related to patient behavior, and then, mx represents the number of visits from PHCIs to hospitals per unit time. So, system (1) can be rewritten as follows:

$$\begin{cases} \frac{dx(t)}{dt} = r_1 x(t) \left(1 - \frac{x(t)}{k_1}\right) - d_1 x(t) - mx(t), \\ \frac{dy(t)}{dt} = r_2 y(t) \left(1 - \frac{y(t)}{k_2}\right) - d_2 y(t) + mx(t). \end{cases} \quad (9)$$

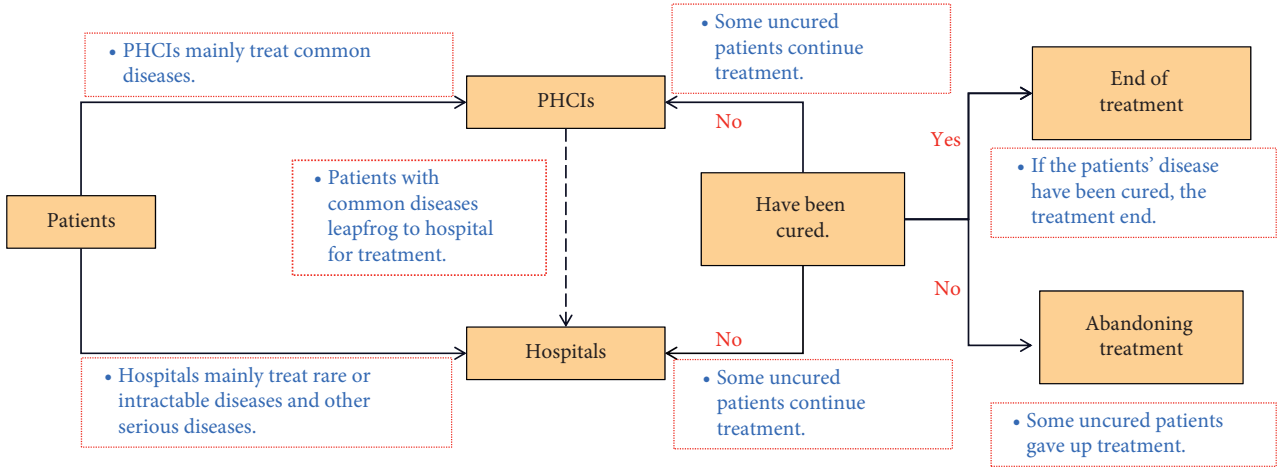


FIGURE 1: Dynamic flowchart of patients' visits to the medical system.

The initial conditions are

$$\begin{cases} x_0, y_0 > 0, \\ r_{1,2}, k_{1,2}, d_{1,2}, m, n > 0. \end{cases} \quad (10)$$

3. Qualitative Analysis

3.1. Equilibrium Points. The main purpose of our study of differential equations is not to analyze their behavior at each moment, but to study the future state of the system by discussing the solutions of differential equations and their various properties. For most differential equations, their general solutions cannot be found. Thus, we usually study the special solution where the derivative is zero, also called the equilibrium point, which is the point where the trend of motion change is zero. Then, according to the structure of the differential equation, we study the properties of the equilibrium point or investigate the distribution of the curve determined by the differential equation. Next, we analyze the equilibrium points of this system to understand the possible future state of the medical system in China.

Theorem 1. *The system has three different equilibrium points under different parameter conditions:*

- (1) $E_1^* = (0, 0)$ always exists in any condition
- (2) $E_2^* = (0, k_2(r_2 - d_2)/r_2)$ exists only if $r_2 - d_2 > 0$
- (3) $E_3^* = (k_1(r_1 - d_1 - m)/r_1, k_2/2r_2(r_2 - d_2 + \sqrt{(r_2 - d_2)^2 + 4r_2k_1m(r_1 - d_1 - m)/r_1k_2}))$ exists only if $r_1 - d_1 - m > 0$

Proof. The zero solutions of a system of differential equations (10) are its equilibrium points. In order to unify the subsequent expressions, let $dx(t)/dt = f_1(x, y)$ and $dy(t)/dt = f_2(x, y)$, and then, we solve the below algebraic equations:

$$\begin{cases} f_1(x, y) \equiv r_1x(t)\left(1 - \frac{x(t)}{k_1}\right) - d_1x(t) - mx(t) = 0, \\ f_2(x, y) \equiv r_2y(t)\left(1 - \frac{y(t)}{k_2}\right) - d_2y(t) + mx(t) = 0. \end{cases} \quad (11)$$

Subsequently, four solutions are obtained:

$$\begin{aligned} E_1^* &= (x_1^*, y_1^*), \\ E_2^* &= (x_2^*, y_2^*), \\ E_3^* &= (x_3^*, y_3^*), \\ E_4^* &= (x_4^*, y_4^*), \end{aligned} \quad (12)$$

where $x_1^* = 0, y_1^* = 0, x_2^* = 0, y_2^* = k_2(r_2 - d_2)/r_2, x_3^* = k_1(r_1 - d_1 - m)/r_1, y_3^* = k_2/2r_2(r_2 - d_2 + \sqrt{(r_2 - d_2)^2 + 4r_2k_1m(r_1 - d_1 - m)/r_1k_2})$, and $x_4^* = k_1(r_1 - d_1 - m)/r_1, y_4^* = k_2/2r_2(r_2 - d_2 - \sqrt{(r_2 - d_2)^2 + 4r_2k_1m(r_1 - d_1 - m)/r_1k_2})$.

Because the number of visits is always nonnegative, it is in line with the actual situation if the equilibrium point is greater than or equal to zero. Therefore,

- (i) Equilibrium point E_1^* always exists, and the practical significance of this point is that there will be no patients in the medical system, such that the number of visits will be zero.
- (ii) Equilibrium point E_2^* exists only if $r_2 - d_2 > 0$, which means that the system has this equilibrium point if the natural growth rate of the visits to hospitals is bigger than the churn rate. The practical significance of this point is that there will be no patients in PHCIs, but all are concentrated in hospitals, and the number is y_2^* .
- (iii) Equilibrium point E_3^* exists only if $r_1 - d_1 - m > 0$, which means that the system has this equilibrium

point if the inherent growth rate of the number of visits to PHCIs is greater than the sum of the rate of inherent attrition and the leapfrog medical treatment. The practical significance of this point is that the number of visits concentrated in PHCIs and hospitals is x_3^* and y_3^* .

- (iv) Equilibrium point E_4^* is illogical because $y_4^* < 0$. Thus, it will not be discussed in this paper.

We have established the possible future state of the system through the above equilibrium points. Next, we need to determine in which state China's medical system will be in the future, which requires further discussion of the stability of the equilibrium points. Since differential equations describe the motion process of the system, the stability of the system is not only determined by the structure and parameters of this system but is also related to the initial conditions and the magnitude of external disturbances. If the equilibrium point is unstable, a small error or disturbance of the initial value will change the topological structure of the system. The stability of the equilibrium points will be discussed in the next section. \square

3.2. Local Stability Analysis. One of the classic methods of judging the stability of the equilibrium point of a nonlinear differential equation is to determine the stability according to linearization. Lyapunov pointed out that, for nonlinear differential equations, if the linearized characteristic equation has no root of zero root or zero real part, then the stability state of its equilibrium point is the same as that of the equilibrium point of nonlinear differential equations [39].

- (i) When the roots of linearized characteristic equations are all negative roots or reals, the equilibrium point of the nonlinear differential equations is locally asymptotically stable.
- (ii) When a linearized characteristic equation has the root of the positive real part or the root of the positive root, the equilibrium point of the nonlinear differential equations is unstable.
- (iii) When a linearized characteristic equation has zeros or zero real part root, the nonlinear differential equations belong to the critical situation, and the stability state of the equilibrium point cannot be judged by the stability state of the linear approximate equations but should be analyzed by other means.

Theorem 2. Under different parameter conditions, the stable states of the three equilibrium points of the system will be different:

- (1) If $r_1 - d_1 - m \leq 0$ and $r_2 - d_2 \leq 0$, then E_1^* is a locally asymptotically stable equilibrium point
- (2) If $r_1 - d_1 - m \leq 0$ and $r_2 - d_2 > 0$, then E_2^* is a locally asymptotically stable equilibrium point
- (3) If $r_1 - d_1 - m > 0$, then E_3^* is a locally asymptotically stable equilibrium point

Proof. The coefficient matrix of the approximate equations of the nonlinear equations are obtained after Taylor's expansion of (13) at each equilibrium point:

$$A|_{E_i^*} = \begin{bmatrix} \frac{\partial f_1(x, y)}{\partial x} & \frac{\partial f_1(x, y)}{\partial y} \\ \frac{\partial f_2(x, y)}{\partial x} & \frac{\partial f_2(x, y)}{\partial y} \end{bmatrix} \Big|_{E_i^*} = \begin{bmatrix} -\frac{2r_1}{k_1}x(t) + r_1 - d_1 - m & 0 \\ m & -\frac{2r_2}{k_2}y(t) + r_2 - d_2 \end{bmatrix} \Big|_{E_i^*} \quad i = 1, 2, 3. \quad (13)$$

Then, the characteristic equation becomes

$$\det(A - \lambda I)|_{E_i^*} = \begin{bmatrix} -\frac{2r_1}{k_1}x(t) + r_1 - d_1 - m - \lambda & 0 \\ m & -\frac{2r_2}{k_2}y(t) + r_2 - d_2 - \lambda \end{bmatrix} \Big|_{E_i^*} = 0. \quad (14)$$

The above expression can be written in a more explicit form as follows:

$$(\lambda^2 + p\lambda + q)|_{E_i^*} = 0, \quad (15)$$

where

$$p|_{E_i^*} = -\left(\frac{\partial f_1(x, y)}{\partial x} + \frac{\partial f_2(x, y)}{\partial y}\right) = \frac{2r_1}{k_1}x(t) - r_1 + d_1 + m + \frac{2r_2}{k_2}y(t) - r_2 + d_2, \quad (16)$$

$$\begin{aligned} q|_{E_i^*} &= \det A \\ &= \left(\frac{2r_1}{k_1}x(t) - r_1 + d_1 + m\right)\left(\frac{2r_2}{k_2}y(t) - r_2 + d_2\right). \end{aligned} \quad (17)$$

The characteristic roots are denoted by $\lambda_{1,2}$; thus,

$$\lambda_{1,2} = \frac{(-p \pm \sqrt{p^2 - 4q})}{2}. \quad (18)$$

Therefore, E_i^* is locally asymptotically stable if $p|_{E_i^*} > 0$ and $q|_{E_i^*} > 0$, and E_i^* is unstable if $p|_{E_i^*} < 0$ or $q|_{E_i^*} < 0$.

(1) Substituting $E_1^* = (x_1^*, y_1^*)$ into equations (16) and (17) yields

$$\begin{aligned} p|_{E_1^*} &= (-r_1 + d_1 + m) + (-r_2 + d_2), \\ q|_{E_1^*} &= (r_1 - d_1 - m)(r_2 - d_2). \end{aligned} \quad (19)$$

- (a) $p|_{E_1^*} > 0$ and $q|_{E_1^*} > 0$ only when $r_1 - d_1 - m < 0$ and $r_2 - d_2 < 0$ are satisfied at the same time. Thus, $\lambda_{1,2} < 0$.
- (b) There is a special case: if $r_1 - d_1 - m = 0$ and $r_2 - d_2 \leq 0$, the characteristic roots are zero. As mentioned earlier at the beginning of this section, this is the critical state, so we cannot infer the stability using the linearized equations, but we can solve them graphically.

Since there is no variable y in the first equation of system (10), we can consider it a one-dimensional equation and analyze the stability of its equilibrium point through its curves [40].

We let $f_1(x, y) = f_1(x) = r_1x(t)(1 - x(t)/k_1) - d_1x(t) - mx(x)$, and we found that $f_1(x)$ has two zero solutions when $r_1 - d_1 - m = 0$. As shown in Figure 2(a), $f_1(x)$ goes to the left and approaches the origin with the increase of x , so $x = 0$ is the stable equilibrium point of $f_1(x)$.

Next, we substitute $x = 0$ into in the second equation of system (10), which is also a one-dimensional equation at that time.

We let $f_2(x, y) = f_2(y) = r_2y(t)(1 - y(t)/k_2) - d_2y(t)$, and we found that if $r_2 - d_2 \leq 0$, then

$f_2(y)$ has a zero solution and a negative solution. The negative solution is illogical and so will not be further discussed. If $r_2 - d_2 = 0$, then $f_2(y)$ has two zero solutions. We can see in Figure 2(b) that, with the increase of y , $f_2(y)$ goes to the left and approaches the origin, so $y = 0$ is the equilibrium point of $f_2(y)$. Obviously, if $r_1 - d_1 - m = 0$ and $r_2 - d_2 \leq 0$, then E_1^* is also a stable equilibrium point. Summarizing the above discussion, E_1^* is a locally stable equilibrium point when $r_1 - d_1 - m \leq 0$ and $r_2 - d_2 \leq 0$.

(2) Substituting $E_2^* = (x_2^*, y_2^*)$ into equations (16) and (17) yields

$$\begin{aligned} p|_{E_2^*} &= (-r_1 + d_1 + m) + (r_2 - d_2), \\ q|_{E_2^*} &= (-r_1 + d_1 + m)(r_2 - d_2). \end{aligned} \quad (20)$$

- (a) $p|_{E_2^*} > 0$ and $q|_{E_2^*} > 0$ only when $r_1 - d_1 - m < 0$ and $r_2 - d_2 > 0$ are satisfied at the same time; thus, $\lambda_{1,2} < 0$.
- (b) However, if $r_1 - d_1 - m = 0$ and $r_2 - d_2 > 0$, the characteristic roots have a zero root. As mentioned at the beginning of this section, this is also the critical state, and thus, it can be solved graphically. In this situation, $f_2(y)$ has a zero solution and a positive solution. We can see in Figure 3 that, with the increase of y , $f_2(y)$ departs from the origin and tends to a positive solution, which is the equilibrium point of $f_2(y)$. Obviously, E_2^* is also a stable equilibrium point when $r_1 - d_1 - m = 0$ and $r_2 - d_2 > 0$.

In summary, E_2^* is a locally stable equilibrium point when $r_1 - d_1 - m \leq 0$ and $r_2 - d_2 > 0$.

(3) Substituting $E_3^* = (x_3^*, y_3^*)$ into equations (16) and (17) yields

$$\begin{aligned} p|_{E_3^*} &= (r_1 - d_1 - m) + \sqrt{(r_2 - d_2)^2 + \frac{4r_2k_1m(r_1 - d_1 - m)}{r_1k_2}}, \\ q|_{E_3^*} &= (r_1 - d_1 - m)\sqrt{(r_2 - d_2)^2 + \frac{4r_2k_1m(r_1 - d_1 - m)}{r_1k_2}}. \end{aligned} \quad (21)$$

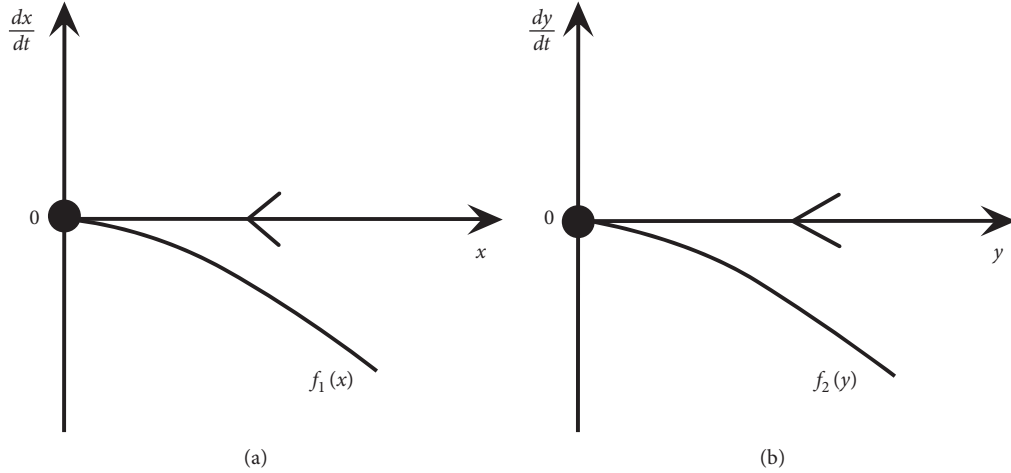


FIGURE 2: The curves of (a) $f_1(x)$ and (b) $f_2(y)$. (a) $r_1 - d_1 - m = 0$. (b) $r_2 - d_2 \leq 0$.

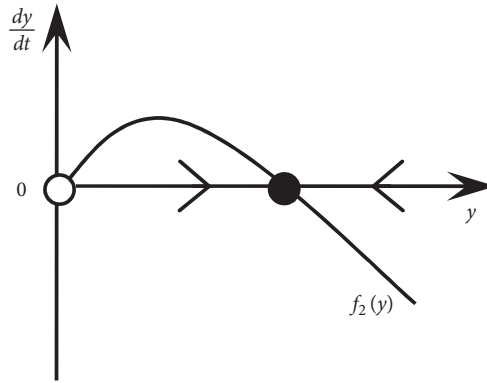


FIGURE 3: The curve of $f_2(y)$.

Therefore, if $r_1 - d_1 - m > 0$, then $p|_{E_3^*} > 0$ and $q|_{E_3^*} > 0$; thus, $\lambda_{1,2} < 0$, and E_3^* is locally asymptotically stable.

The three previously discussed equilibrium points represent different practical meanings:

- (i) If the growth rate of visits to PHCIs is lower than or equal to the sum of the churn rate and the leapfrog medical treatment rate, and the growth rate of visits to hospitals is lower than or equal to the churn rate, no patients will ultimately seek medical treatment in the system.
- (ii) If the growth rate of visits to PHCIs is lower than or equal to the sum of the churn rate and the leapfrog medical treatment rate, and the growth rate of visits to hospitals is greater than the churn rate, patients will eventually concentrate in hospitals, and the number of visits will be y_2^* , but no patients in PHCIs.
- (iii) If the growth rate of the number of visits to PHCIs is greater than the sum of the churn rate and the leapfrog medical treatment rate, patients will be evenly distributed between PHCIs and hospitals in numbers of x_3^* and y_3^* , respectively. \square

3.3. Sector Field and Solution Curve Analysis. In this section, we draw the slope field and the solution curves of the system to verify the above conclusions. The solution curves consist of the graph of solutions of the equation and the projection of the integral curves onto the Oxy -plane, which describes the approximate graph of the integral curve. Each tangent line of any point on the solution curves has a slope, and the slopes of all points make up the slope field, which is the graphical solution of the differential equation. We can intuitively see the patterns of change and properties of all the solutions of the differential equations [41, 42].

According to the realistic significance of the model, the Chinese population level, and the current visit data, we assume that $x \in [0.5 \times 10^5]$, $y \in [0.2 \times 10^6]$, $k_1 = 1 \times 10^6$, $k_2 = 2 \times 10^6$, $m = 0.1$, $r_1 = 0.8$, and $r_2 = 0.3$.

Subsequently, we can change the condition for the existence of the equilibrium points by changing the values of d_1 and d_2 . Next, we use *MATLAB*(R2018a) to draw the slope field and solution curves of the system under different parameter conditions.

In the slope field, the blue circular points represent the equilibrium point, and the direction and size of the arrow indicate the direction and speed of the point, respectively.

In the solution curves, each curve is a solution of the system and the arrows on each line show where they go over time.

Scenario 1. If $d_1 = 0.7$ and $d_2 = 0.3$, then $r_1 - d_1 - m \leq 0$ and $r_2 - d_2 \leq 0$. According to Theorem 1, the system has only one equilibrium point, E_1^* .

We can see from the slope field (Figure 4(a)) that all the arrows point to the origin, the speed changes from fast to slow, and the speed of the arrow at the origin is reduced to zero. It is clear from the solution curve (Figure 4(b)) that, as time elapses, all solution curves in the end point to the origin. Therefore, E_1^* is a locally asymptotically stable equilibrium point.

Consequently, if the growth rate of visits to PHCIs is less than or equal to the sum of the churn rate and the leapfrog medical treatment rate and the growth rate of visits to hospitals is less than or equal to the churn rate, the system will converge to the origin, no matter what the initial value or the interference is, and no patients will seek medical treatment in the end.

Scenario 2. If $d_1 = 0.7$ and $d_2 = 0.05$, then $r_1 - d_1 - m \leq 0$ and $r_2 - d_2 > 0$. According to Theorem 1, the system has two equilibrium points, E_1^* and E_2^* .

We can see from the slope field (Figure 5(a)) that only the vertical arrow passes through E_1^* , and the arrows in the other directions are far away from E_1^* and point to E_2^* . The speed changes from fast to slow, and the speed of the arrow at E_2^* gradually decreases to zero. The solution curve indicates (Figure 5(b)) that, as time elapses, all solution curves finally converge at point E_2^* . This means that, under this parameter condition, E_1^* is an unstable saddle point and E_2^* is a locally asymptotically stable node, that is, the system remains stable at E_2^* .

Consequently, if the growth rate of visits to PHCIs is less than or equal to the sum of the churn rate and the leapfrog medical treatment rate and the growth rate of visits to hospitals is greater than the churn rate, patients will eventually concentrate on hospitals, and the number will be y_2^* , but none will visit PHCIs.

Scenario 3. If $d_1 = 0.5$ and $d_2 = 0.05$, then $r_1 - d_1 - m > 0$ and $r_2 - d_2 \leq 0$. According to Theorem 1, the system has two equilibrium points, E_1^* and E_3^* .

It can be inferred from the slope field (Figure 6(a)) that only the vertical arrow passes through E_1^* , and the arrows in the other directions point to E_3^* . The speed changes from fast to slow, and the speed of the arrow at E_3^* gradually decreases to zero. It can be derived from the solution curve (Figure 6(b)) that, as time elapses, all solution curves finally converge at E_3^* . This shows that, under this parameter condition, E_1^* is an unstable equilibrium point, and E_3^* is a locally asymptotically stable node, that is, the system remains stable at E_3^* .

Scenario 4. If $d_1 = 0.5$ and $d_2 = 0.1$, then $r_1 - d_1 - m > 0$ and $r_2 - d_2 > 0$. According to Theorem 1, the system has three equilibrium points, E_1^* , E_2^* , and E_3^* .

The slope field demonstrates (Figure 7(a)) that only the vertical arrow passes through E_1^* and E_2^* , and the arrows in the other directions point to E_3^* . The speed changes from fast to slow, and the speed of the arrow at E_3^* gradually decreases to zero. It can be inferred from the solution curve (Figure 7(b)) that, as time elapses, all solution curves finally converge at E_3^* . This shows that, under this parameter condition, E_1^* and E_2^* are unstable equilibrium points, and E_3^* is a locally asymptotically stable node, that is, the system remains stable at E_3^* .

Scenarios 3 and 4 indicate that only if the growth rate of the number of visits to PHCIs is greater than the sum of the churn rate and leapfrog medical treatment rate, patients will be evenly distributed between PHCIs and hospitals in numbers x_3^* and y_3^* . This is the state of the development of an effective medical system pursued by most countries.

Based on the results of the above analysis, the following questions arise. What is the future of medical institutions in China? Is the usage efficiency of medical resource allocation reasonable or unreasonable? Does a change of parameters have an impact on the system, and if so, to what extent? A discussion providing possible answers continues in Section 4.

4. Simulation Analysis

4.1. Parameter Fitting and Model Validation. This study selects two sets of time-series data for visits of PHCIs and hospitals from the statistical data of the National Health Commission of the People's Republic of China, which are reported each December. The time span of data is from January 2011 to November 2018. The date of January 2011 is used as the initial value, and data for the period of January 2011 to May 2018 are used to perform a least-squares optimal fitting of the unknown parameters of the model and then to obtain the values $m = 0.0011$, $k_1 = 51960$, $k_2 = 69990$, $r_1 = 0.1019$, $r_2 = 0.0810$, $d_1 = 0.0298$, and $d_2 = 0.0480$. The data obtained meet the stability condition of equilibrium point E_3^* , and it is possible to calculate the number of consultations for the steady state in PHCIs and hospitals, which is $x_3^* = 36214$ and $y_3^* = 29654$.

The determination coefficient R^2 shows that the fitting degree of the regression line to the observed value is better. Its maximum value is 1, and the closer it is to 1, the better the fitting degree is. The obtained $R^2 = 0.978$, which is close to 1. Thus, the fitting is close to the real value.

In order to further verify the fitting effect, we use the data from the period of June to November 2018. As shown in Figure 8(a), the fitting effect for the number of consultations in PHCI is better, which shows that the current increase in the number of consultations has gradually slowed down and is close to a steady state. The maximum visiting capacity still has much room for consultations and treatments, which means that the efficiency of resource allocation is low.

Since the number of hospital visits shows obvious seasonal periodic changes [43], this paper does not consider seasonal factors in the modeling, so the fitting curve of hospital visits (Figure 8(b)) shows the true value of the number of visits, and the difference in fitted values for

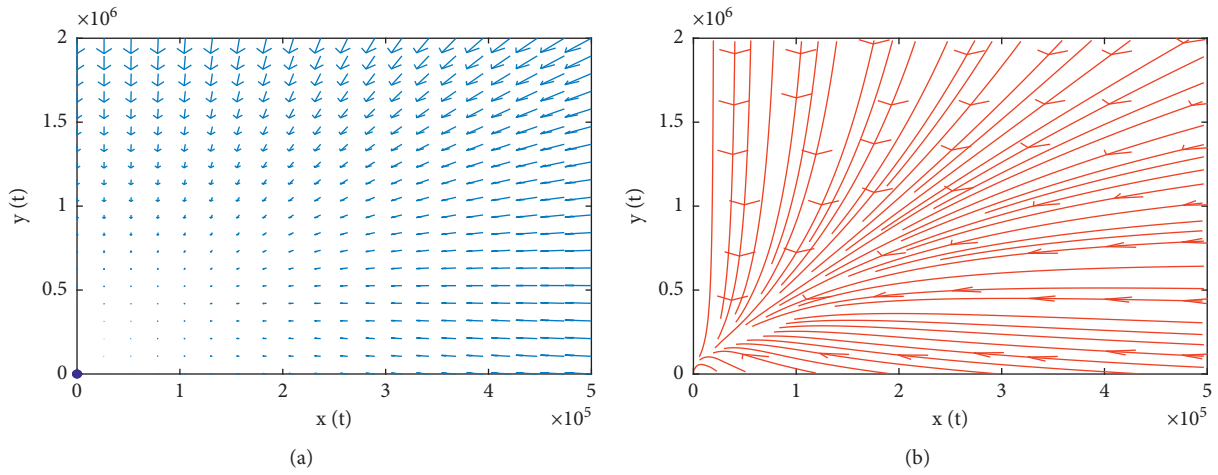


FIGURE 4: (a) Slope field and (b) solution curves ($r_1 - d_1 - m \leq 0$ and $r_2 - d_2 \leq 0$).

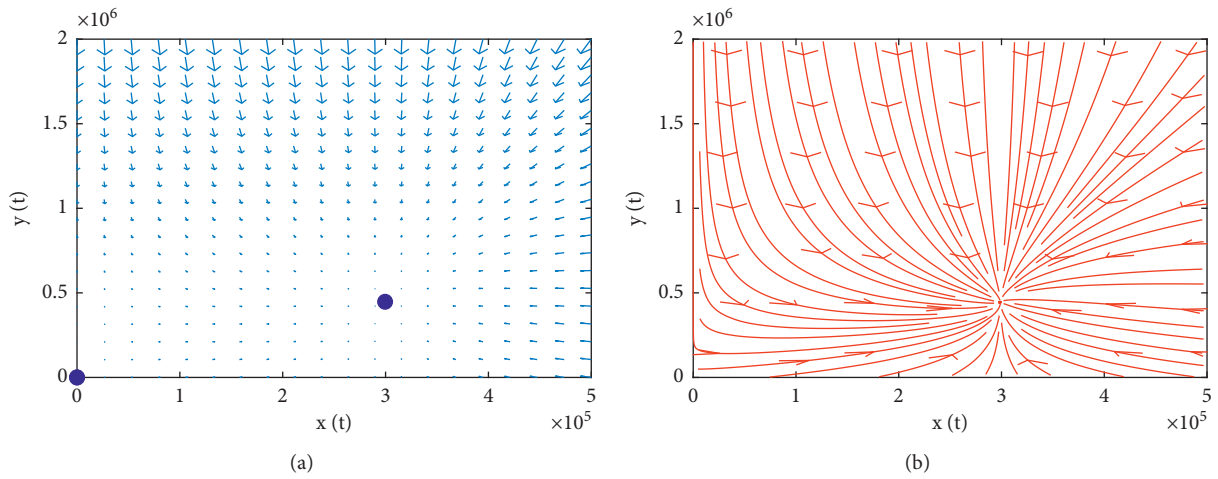


FIGURE 5: (a) Slope field and (b) solution curves ($r_1 - d_1 - m \leq 0$ and $r_2 - d_2 > 0$).

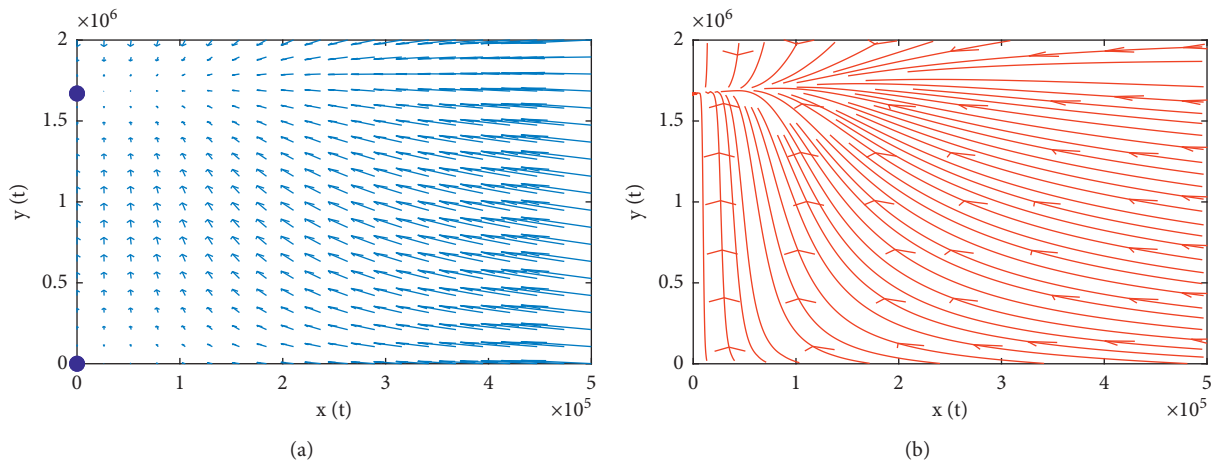


FIGURE 6: (a) Slope field and (b) solution curves ($r_1 - d_1 - m > 0$ and $r_2 - d_2 \leq 0$).

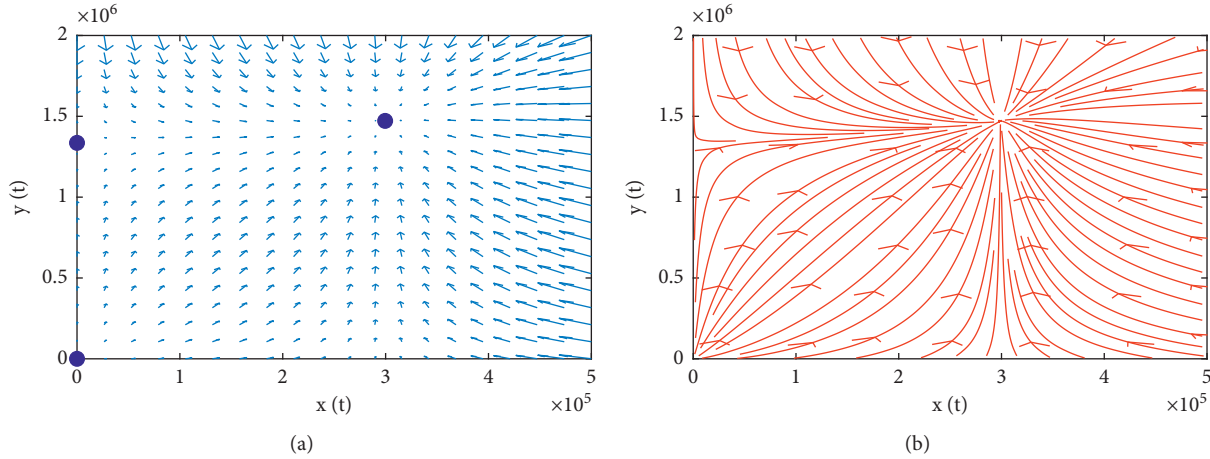


FIGURE 7: (a) Slope field and (b) solution curves ($r_1 - d_1 - m > 0$ and $r_2 - d_2 > 0$).

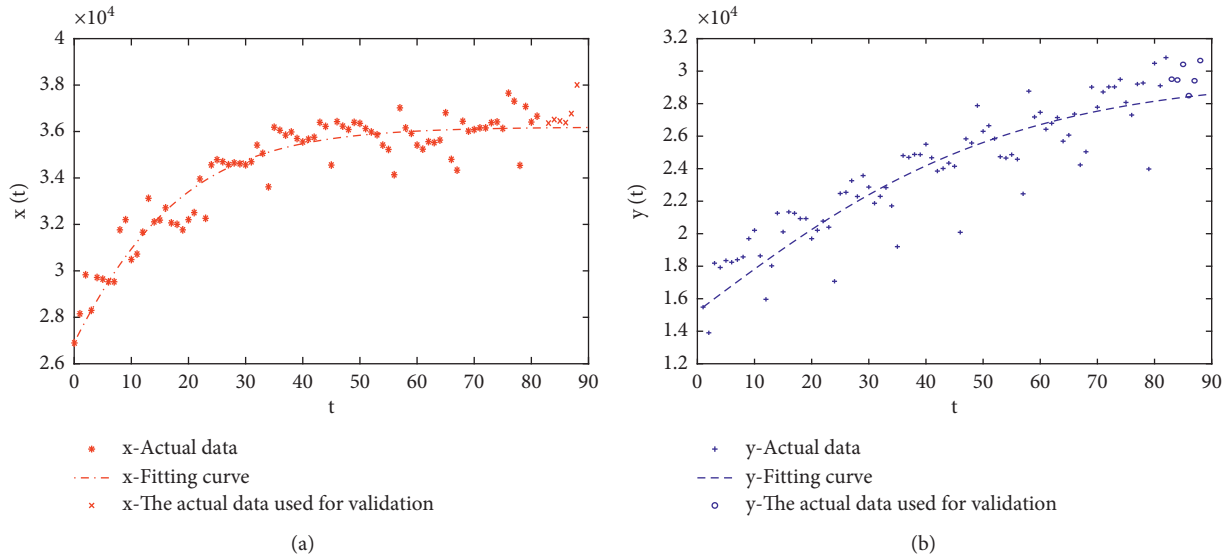


FIGURE 8: Scatter diagram and fitting curve of real data of (a) PHCIs and (b) hospitals.

certain months is normal. We can see from the figure that the number of hospital visits is still growing rapidly, and it is expected to reach a stable state around 2025.

4.2. Parameter Sensitivity Analysis

4.2.1. *Impact of m .* In order to analyze the impact of different parameter values on the number of visits in the medical system, this paper simulates the system model by taking three cases that are less than, equal to, or greater than the parameter fitting value.

Figure 9 presents a $x(t)$ and $y(t)$ change when in the orderly medical treatment status ($m = 0$), the current medical treatment status ($m = 0.0011$), and greater than the current status ($m = 0.0021$). Other parameters are fixed.

Through simulation, we can see that changes in the value of m will affect $x(t)$ and $y(t)$ values at the same time. This

reveals that if other parameters are fixed, the reduction in m value can decrease the number of visits to hospitals while increasing the number of visits to PHCIs, that is, patients can be reasonably reoriented. However, even if there is no phenomenon of leapfrog medical treatment, it is difficult to substantially increase the number of consultations in PHCIs. Therefore, simply changing the patient's medical treatment habits does not have a significant effect on increasing the number of visits to PHCIs.

4.2.2. *Impact of k_1 and k_2 .* Figures 10 and 11, respectively, show the effects of different values of maximum patient capacity on the system.

We find that a change of the k_1 value will positively affect the values of $x(t)$ and $y(t)$ at the same time, but it has a much greater effect on $x(t)$ and a very small impact on $y(t)$. There is no influence on the value of $x(t)$, and there is a

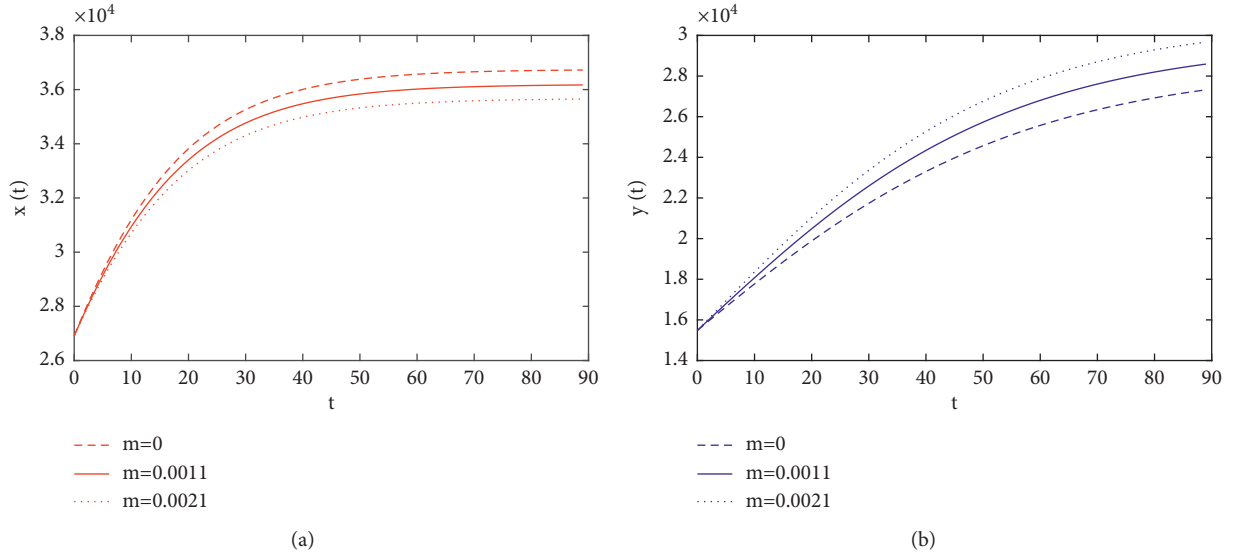


FIGURE 9: Effect of parameter value m on the number of visits to (a) PHCIs and (b) hospitals.

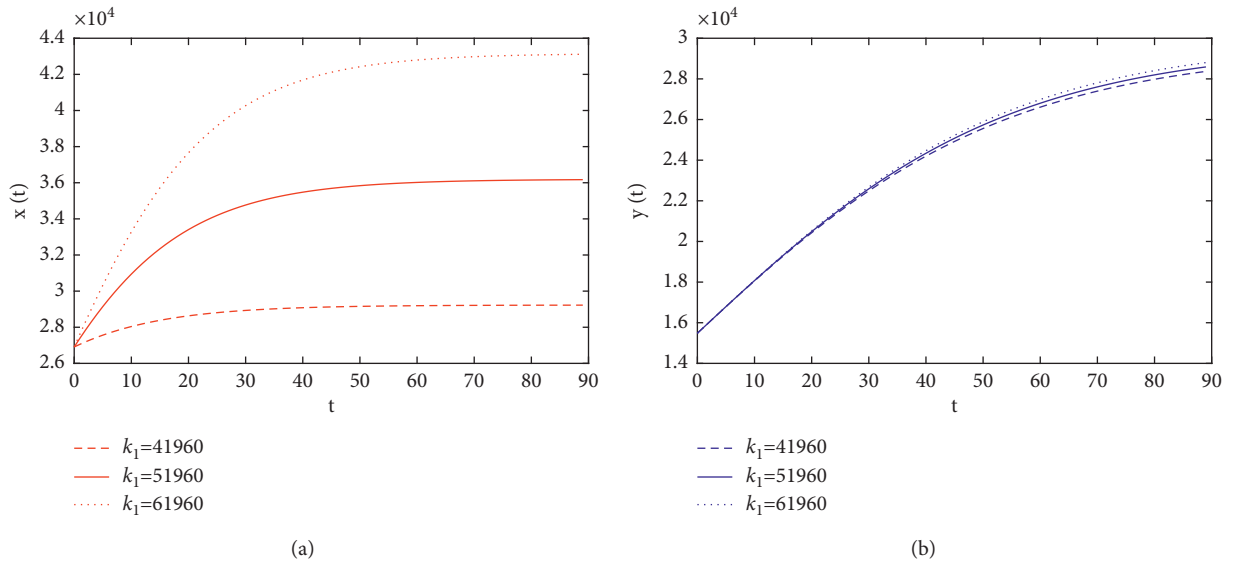


FIGURE 10: Effect of parameter value k_1 on the number of visits to (a) PHCIs and (b) hospitals.

positive influence on the value of $y(t)$. The value change of $y(t)$ when k_2 changes is less than that of $y(t)$ when k_1 changes by the same magnitude. Therefore, improving the maximum number of patients in each type of medical institution can increase their number of visits and have more influence on PHCIs.

4.2.3. Impact of r_1 and r_2 . Figures 12 and 13, respectively, show the effects of different values of maximum patient capacity on the system.

We find that a change of r_1 value will positively affect the values of $x(t)$ and $y(t)$ at the same time, but it has much greater effect on $x(t)$ and a very small impact on $y(t)$, which can almost be ignored. There is no influence on the value of $x(t)$ and a positive influence on the value of $y(t)$ when the r_2

value is changed. A change of the $y(t)$ value when r_2 changes is greater than that of $x(t)$ when r_1 changes by the same magnitude.

The above result indicates that a growing population or aging rate can cause an inherent increase in the rate of visits and enhance the number of visits to clinics in various institutions. Thus, improving the intrinsic increase rate for each type of medical institution can increase their number of visits and have more influence on hospitals.

4.2.4. Impact of d_1 and d_2 . Figures 14 and 15, respectively, show the effects of different churn rate values on the system.

We find that the change in the d_1 value will negatively affect the values of $x(t)$ and $y(t)$ at the same time, but it has

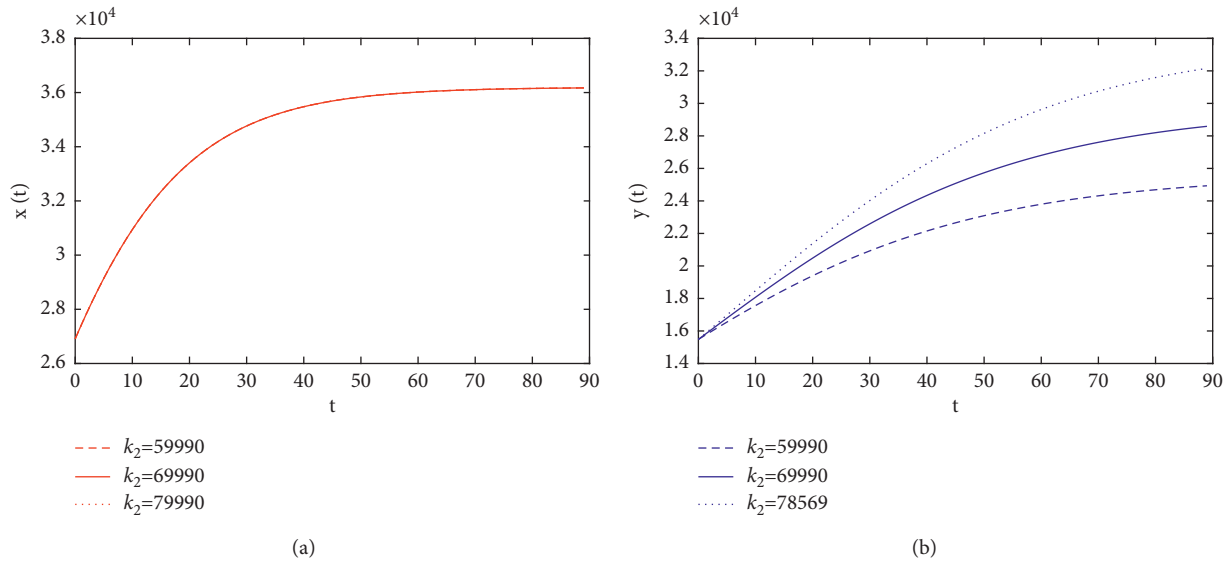


FIGURE 11: Effect of parameter value k_2 on the number of visits to (a) PHCIs and (b) hospitals.

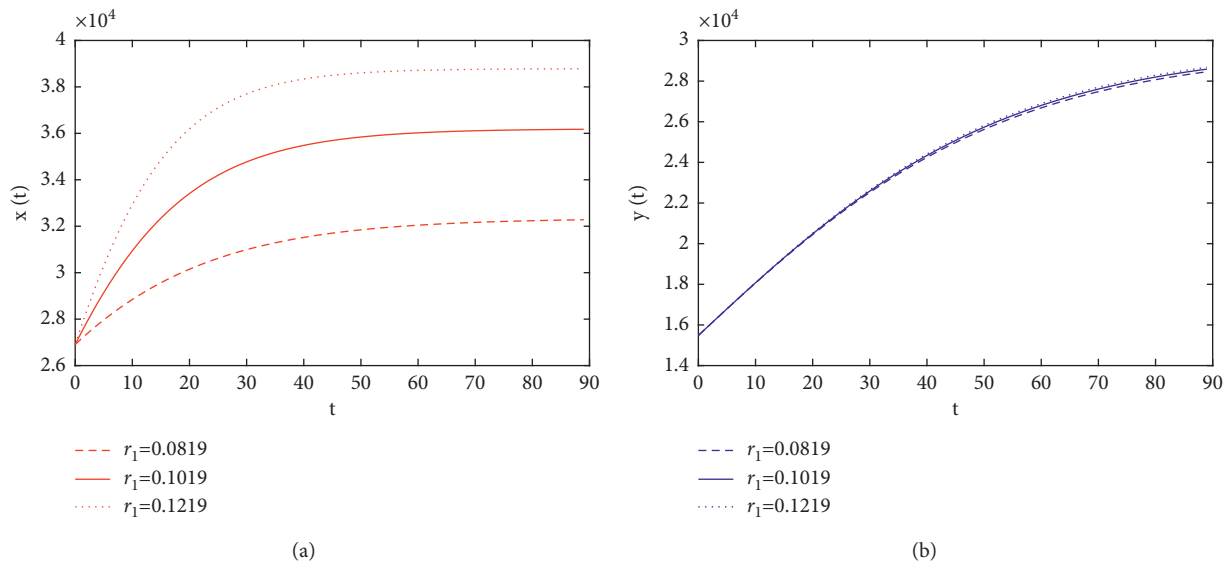


FIGURE 12: Effect of parameter value r_1 on the number of visits to (a) PHCIs and (b) hospitals.

a much greater effect on $x(t)$ and a very small impact on $y(t)$, which can almost be ignored. There is no influence on the value of $x(t)$ and a positive influence on the value of $y(t)$ when the d_2 changes. The change of value $y(t)$ when d_2 changes is greater than that of $x(t)$ when d_1 changes by the same magnitude.

This indicates that the churn rate rises with the increase of human mortality and abandonment rate, and the number of visits to clinics in various institutions is reduced. Thus, the reduction of the churn rate in each level of medical institutions can increase their number of visits and have more influence on hospitals.

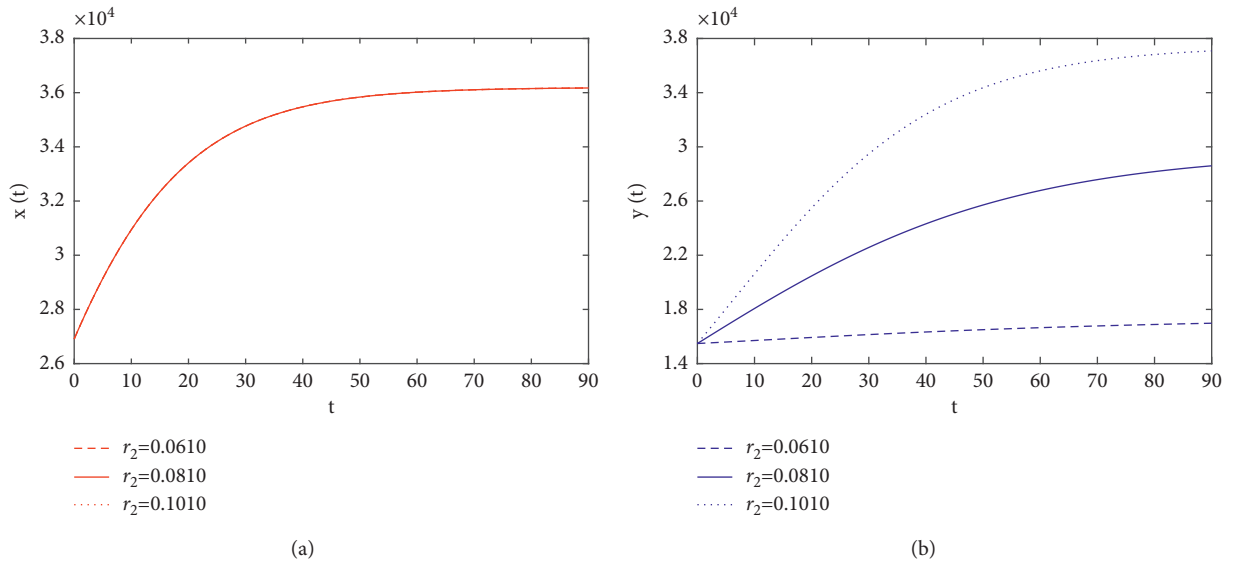


FIGURE 13: Effect of parameter value r_2 on the number of visits to (a) PHCIs and (b) hospitals.

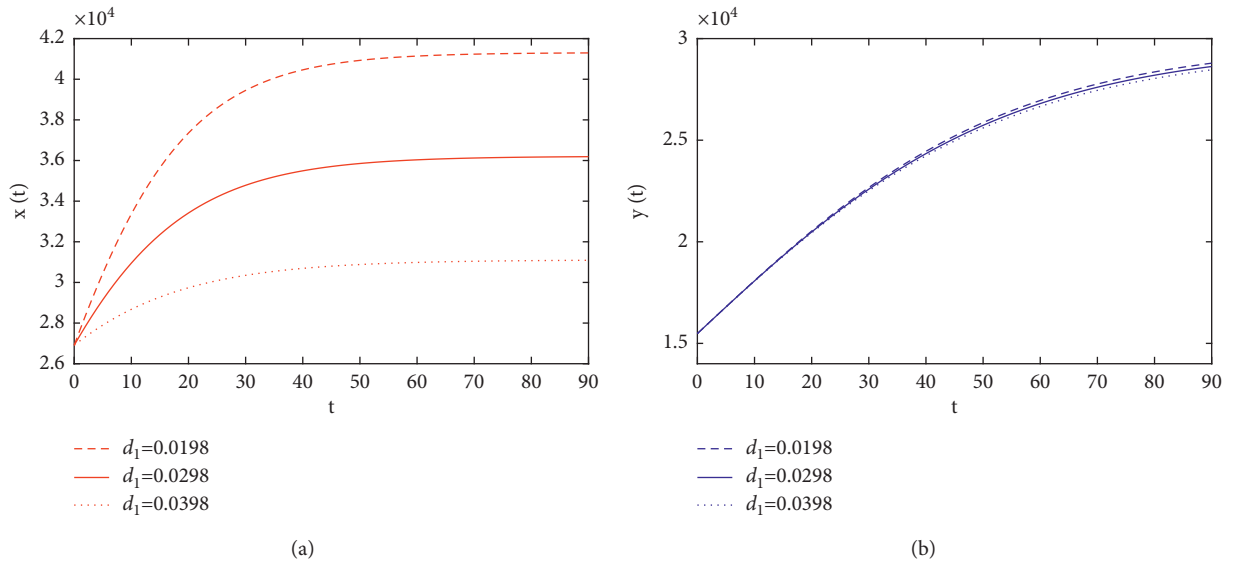


FIGURE 14: The effect of parameter value d_1 on the number of visits to (a) PHCIs and (b) hospitals.

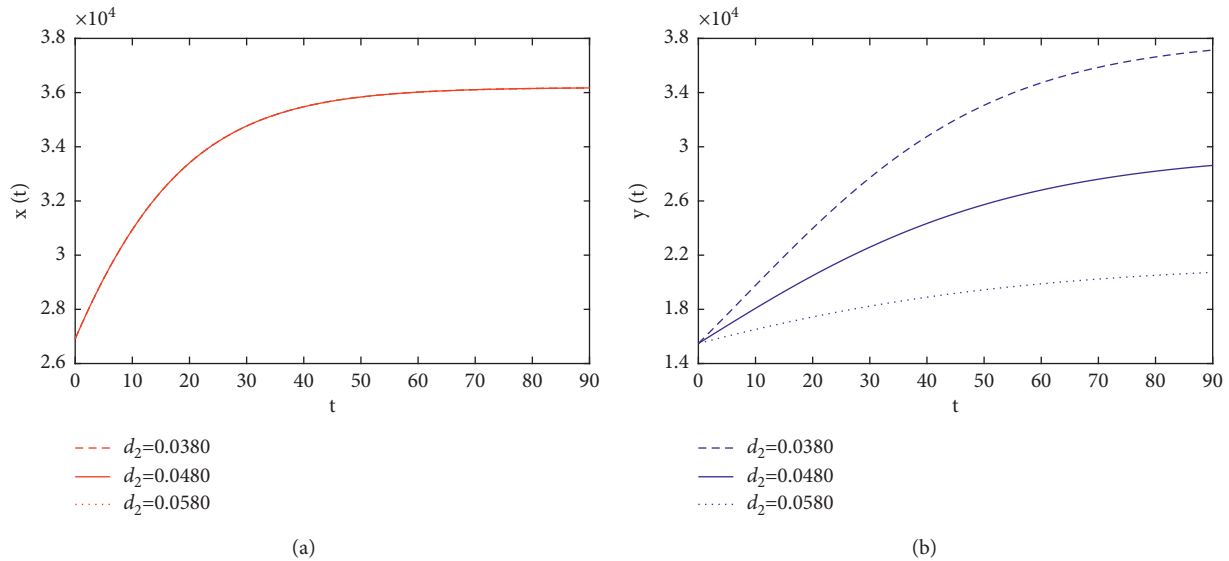


FIGURE 15: The effect of parameter value d_2 on the number of visits to (a) PHCIs and (b) hospitals.

5. Conclusions and Suggestions

This paper analyzes the change of the number of visits over time to PHCIs and hospitals in China based on the logistic differential equation model and evaluates the dynamic behavior and parameter sensitivity of the system. Our results show that

- (1) The system corresponds to three different types of locally asymptotically stable equilibrium points under different parameter conditions, namely, a zero-equilibrium point, boundary equilibrium point, and positive equilibrium point. Only the positive equilibrium point forms a basis for the balanced development of the medical system. Thus, the relationship between the inherent growth rate of the number of visits to PHCIs should be greater than the sum of the churn rate and the leapfrog medical treatment rate, and the development status of PHCIs plays a significant role in achieving a balanced development.
- (2) At present, China's PHCIs and hospitals are characterized by certain numbers of visits, but the resource utilization rate of PHCIs is fairly low. If other external factors remain unchanged, this trend will continue until the system reaches a stable state. The number of visits to PHCIs is about to reach a stable state, while the number of visits to hospitals is growing rapidly and is expected to reach a stable state in 2025. Therefore, with the growth and intensified aging of the population, there is a very urgent need to increase the consultation rate of primary medical institutions.
- (3) Reducing the rate of leapfrog medical treatment will increase the number of visits to PHCIs, as well as reduce the number of visits to hospitals, but the overall change would not be large. Increasing PHCIs' maximum visiting capacity or the inherent rate of

visits or reducing the churn rate of visits will both greatly boost the number of visits to PHCIs and hospitals, while the impact on hospitals is small enough to be ignored. Since the topological structure of this model is a one-way input, increasing hospitals' maximum visiting capacity or the inherent rate of visits or reducing the churn rate of visits will greatly enhance the number of visits to hospitals, but will not affect the number of visits to PHCIs.

Therefore, it can be concluded that, to increase the rate of visits to PHCIs in China and to improve the status quo of the unreasonable use of higher-level medical resources, not only should we formulate policies in terms of changing patient habits (the effect is not obvious) but also we must not restrict the expansion and development of hospitals. This finding is different from some previous research conclusions [44]. Instead, attention should be paid to improving the comprehensive diagnostic and treatment capabilities of PHCIs, such as enhancing the training of medical staff, increasing the number of beds, and increasing the amount and quality of equipment and medication, i.e., expanding the consultation capacity and further increasing the consultation rate of primary medical institutions. This conclusion further verifies that the main reason for the low rate of consultation in China's PHCIs is their low diagnostic and treatment capabilities. The inherent growth rate of the number of visits is also related to the loss rate and population changes, although these relationships are not enough within the scope of the main discussion of this article to formulate improved health policies and thus were not discussed in it.

In the process of model building, we simplified some actual realities. For example, we simplified the leapfrog medical treatment rate to a constant, even though it is affected by a variety of factors. At the same time, the number of visits in medical institutions changes periodically every year, something we did not consider in this study. Therefore,

in follow-up research, we will optimize the model according to the characteristics of China's medical system. Finally, global stability analysis is not only very important but also challenging, so global stability analysis will also be the focus of our follow-up research [45], so as to provide the Chinese government with a more effective and relatively more accurate decision-making basis for the allocation of medical resources.

Data Availability

All the data are included in the article.

Conflicts of Interest

There are no conflicts of interest regarding the publication of the paper.

Authors' Contributions

All authors contributed equally to the writing of this paper.

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Supplementary Materials

1. MatLab-codes of phase diagram. 2. MatLab-codes of the influence of m value changes on the system. 3. MatLab-codes of the influence of k_1 value changes on the system. 4. MatLab-codes of the influence of k_2 value changes on the system. 5. MatLab-codes of the influence of r_1 value changes on the system. 6. MatLab-codes of the influence of r_2 value changes on the system. 7. MatLab-codes of the influence of d_1 value changes on the system. 8. MatLab-codes of the influence of d_2 value changes on the system. (*Supplementary Materials*)

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