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Research Article

Adaptive Fuzzy Cooperative Control for Nonlinear Multiagent Systems with Unknown Control Coefficient and Actuator Fault

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In this paper, an adaptive fuzzy containment condtrol is considered for nonlinear multiagent systems, in which it contains the unknown control coefficient and actuator fault. The uncertain nonlinear function has been approximated by fuzzy logic system (FLS). The unknown control coefficient and the remaining control rate of actuator fault can be solved by introducing a Nussbaum function. In order to avoid the repeated differentiations of the virtual controllers, first-order filters are added to the traditional backstepping control method. By designing the maximum norm of ideal adaptive parameters, only one adaptive parameter needs to be adjusted online for each agent itself. An adaptive fuzzy containment controller is constructed through the backstepping control technique and compensating signals. It is demonstrated that all the signals in nonlinear multiagent systems are bounded by designing adaptive fuzzy containment controller, and all followers can converge to the convex hull built by the leaders. The simulation studies can further confirm the effectiveness of the proposed control method in this paper.

1. Introduction

With the development of science and technology, more and more complex systems can be described as multiagent systems (MASs), for instance, unmanned aerial vehicle formation, sensor networks, and disaster emergency response (see [1–3]). Therefore, the cooperative control problem of MASs has been extensively concerned by scholars and gradually become one of the hot issues in the control field. According to different control objectives, the cooperative control of MASs is classified as flocking control [4], consensus control [5, 6], formation control [7], synchronous control [8], containment control [9, 10], and so on. Because of its wide applications, containment control is a fundamental and important research subject of MASs control, such as [11, 12]. For containment control, its purpose is to design a controller so that all follower agents can converge to the convex hull built by the leaders [9–18]. For example, the path-guided containment maneuvering approach was proposed for networked two-wheeled mobile robots with multiple virtual leaders moving along multiple parameterized paths in [11]. An output feedback distributed containment control method was investigated for marine vessels guided by multiple parameterized paths with unmeasured velocity in [12].

In literature [13–15], all containment control methods were investigated for linear MASs. But, for the containment problem of nonlinear MASs, the control methods in [13–15] were not feasible. In fact, nonlinear systems can better describe a practical engineering problem than linear systems, such as manipulator systems [19], inverted pendulum systems [20], and rigid robotic systems [21]. All systems models in [16–18] and practical application systems models in [19–26] are first- or second-order systems. For example, under the common assumption that each agent can only obtain the relative information of its neighbours intermittently, the containment control method was proposed for a class of second-order MASs with inherent nonlinear dynamics in [17]. However, many actual industrial systems models are often high-order systems. For instance, a jerk system and single-link flexible manipulators [23] were modeled as third-order and fourth-order nonlinear systems, respectively. So, it is of theoretical and practical value to study the containment control of high-order systems. Some containment control approaches were studied for high-order nonlinear MASs in [24–26]. Compared with low-order MASs, the containment control method for high-order nonlinear MASs is more difficult due to the complexity of high-order systems. In [25], for nonlinear nonaffine pure-feedback MASs, a distributed containment control was proposed based on directed graph topology. For strict-feedback nonlinear systems, the backstepping control technique is an effective approach to solve the cooperative control problem, such as [21, 27–29]. In [21], the finite-time containment control problem was considered for nonlinear MASs with unmeasurable states and input delay. Adaptive neural networks (NNs) backstepping control was studied for uncertain strict-feedback nonlinear systems with input delay in [29].

It is well known that adaptive intelligent control (FLSs and neural networks (NNs)) is an effective way to solve the control problem of nonlinear systems with unknown nonlinear functions, due to the fact that FLSs and NNs can approximate an unknown nonlinear function with arbitrary precision in a compact set, which are called universal approximators [21, 27–34]. In the case of full-state constraints, an adaptive fuzzy controller was designed for nonlinear strict-feedback systems in [28]. Some containment control approaches based on FLSs or NNs as an approximator were investigated for nonlinear MASs in [35–38]. In [36, 37], the containment control approaches were investigated for uncertain nonlinear systems with actuator faults. Under the condition of full-state constraints, a fuzzy adaptive containment control method was investigated for nonlinear systems in [38]. But, in [35–38], the number of adaptive laws was too many, which results in the need to adjust too many parameters online, and the control design is too complicated. When the system order is high, the complexity of the control design increases exponentially. In addition, actuator faults are inevitable in industrial systems, especially for MASs. But, so far, there have been no results on adaptive fuzzy fault-tolerant containment control method for uncertain nonlinear MASs with unknown control coefficient and external disturbance.

This article focuses on the adaptive fuzzy containment control problem for uncertain nonlinear MASs with unknown control coefficient and actuator fault. By using backstepping control technique, compensating signals, first-order filters, and Nussbaum function, some issues in the above control methods can be solved. The main contributions are summarized as follows:

(1) It is the first time to construct adaptive fuzzy containment controllers for uncertain nonlinear MASs with unknown control coefficient and actuator faults. Compared with the existing nonlinear MASs in [27, 39], assuming that there were no actuator faults and the control coefficient was 1, the nonlinear MASs considered in this paper have more practical significance. The unknown control coefficient and the remaining control rate of actuator faults in nonlinear MASs will be resolved by introducing a Nussbaum function.

(2) For the adaptive fuzzy containment control method in the literature [21], the number of designed adaptive laws was equal to or more than the order of the systems. But, in this paper, instead of estimating the optimal parameter vectors themselves but by estimating the maximum value of the norm of the optimal parameter vectors, the number of adaptive laws is greatly reduced.

2. Problem Description and Preliminaries

2.1. Problem Description. Consider nonlinear MASs with N followers and M leaders. The dynamic equations for the τ-th follower are described as follows:

$$\begin{cases} \dot{x}_{\tau,i} = F_{\tau,i}\left(\underline{x}_{\tau,i}\right) + x_{\tau,i+1} + \phi_{\tau,i}(t), \\ 1 \leq i \leq n-1, \\ \dot{x}_{\tau,n} = F_{\tau,n}\left(\underline{x}_{\tau,n}\right) + g_{\tau,n}\left(\underline{x}_{\tau,n}\right)u_{\tau}(t) + \phi_{\tau,n}(t), \\ y_{\tau} = x_{\tau,1}, \end{cases} \quad (1)$$

where $\tau = 1, 2, \ldots, N$. $\underline{x}_{\tau,i} = [x_{\tau,1}, x_{\tau,2}, \ldots, x_{\tau,i}]^T \in R^i$, and $\underline{x}_{\tau,n} = [x_{\tau,1}, x_{\tau,2}, \ldots, x_{\tau,n}]^T \in R^n$ are the state vectors. $y_{\tau} \in R$ is the output of the τ-th follower. $F_{\tau,i}(\underline{x}_{\tau,i})$ is an unknown smooth function. $\phi_{\tau,i}(t)$ is a bounded unknown external disturbance. $g_{\tau,n}(\underline{x}_{\tau,n})$ represents an unknown smooth nonlinear function. $u_{\tau}(t)$ denotes the input of the considered system. The leader's signal y_{rj} is a sufficiently smooth function; y_{rj} and $\dot{y}_{rj}$ are all bounded, where $j = N+1, \ldots, N+M$.

Remark 1. In the literature [7, 24], it is assumed that the MASs will not show actuator faults at any time. However, due to machine aging or working environment, actuator faults are inevitable in actual industrial production. Therefore, it is more practical to design the cooperative control of nonlinear MASs with actuator faults in this paper.

To model the information exchange between agents, the graph theory is introduced. The exchange of information between the followers and the leaders is portrayed by $G = (\nu, \varrho, \overline{A})$. $\nu = \{n_1, \ldots n_N, n_{N+1}, \ldots, n_{N+M}\}$ is the set of agents, where the followers are labeled as $\tau = 1, \ldots, N$, while the leaders are labeled as $\tau = N+1, \ldots, N+M$. Thus, in the set ν, the first N agents are the followers, and the last M agents are the leaders. $\varrho = \{(n_{\tau}, n_j)\} \in \nu \times \nu$ represents the edge set, and the adjacency matrix is defined as $\overline{A} = [a_{\tau j}] \in R^{(N+M)\times(N+M)}$. $(n_{\tau}, n_j) \in \varrho$ means follower j is able to gain the information of its neighbor agent τ. If $(n_{\tau}, n_j) \notin \varrho$, $a_{\tau j} = 0$; if not, $a_{\tau j} = 1$. $L = [L_{\tau j}] \in R^{(N+M)\times(N+M)} = D - \overline{A}$ stands for Laplacian matrix, and $L_{\tau j} = -a_{\tau j}$ if $\tau \neq j$; $D = \mathrm{diag}(d_1, \ldots, d_N, d_{N+1}, \ldots, d_{N+M})$ denotes degree matrix of agent τ, and $d_{\tau} = \sum_{j=1, j\neq\tau}^{N+M} a_{\tau j}$. Let

$v_f = [n_1, \ldots, n_N]$ be the follower agents' set, and let $v_L = [n_{N+1}, \ldots, n_{N+M}]$ denote the leader agents' set. Every follower agent owns at least a neighbor. On the contrary, leaders do not own neighbors. For this condition, the Laplacian L of the graph is partitioned as

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix}. \quad (2)$$

It is supposed that there exists at least a leader who possesses a directed path to every follower agent. Then, each eigenvalue of matrix L_1 has a positive real part, the sum of each line of $-L_1^{-1}L_2$ is equal to 1, and all the entries of $-L_1^{-1}L_2$ are nonnegative. Letting $y_l = [y_{(N+1)}, \ldots, y_{(N+M)}]^T$, the convex hull built by the leaders can be defined as $y_d(t) = -L_1^{-1}L_2 y_l$. Then, the containment error can be denoted as $y - y_d(t)$, where $y = [y_1, \ldots, y_N]^T$.

However, actuators may become faulty in practical engineering. Bias faults and gain faults are two kinds of actuator faults that commonly occur in the practice, expressed as [40]

$$u_\tau^f(t) = \rho_\tau\left(\underline{x}_{\tau,n}\right)u_\tau(t) + \zeta_\tau(t), \quad (3)$$

where the remaining control rate ρ_τ of gain faults satisfies $0 < \rho_\tau(\underline{x}_{\tau,n}) \leq 1$ and $\zeta_\tau(t)$ represents a bounded signal of bias faults. Here, $\rho_\tau(\underline{x}_{\tau,n})$ at the failure time instant t_f is assumed to be unknown.

Considering (3), the follower dynamic equation (1) can be rewritten as

$$\begin{cases} \dot{x}_{\tau,i} = F_{\tau,i}\left(\underline{x}_{\tau,i}\right) + x_{\tau,i+1} + \phi_{\tau,i}(t), \\ 1 \leq i \leq n-1, \\ \dot{x}_{\tau,n} = F_{\tau,n}\left(\underline{x}_{\tau,n}\right) + \Gamma_\tau u_\tau(t) + \Phi_{\tau,n}, \\ y_\tau = x_{\tau,1}, \end{cases} \quad (4)$$

where $\Gamma_\tau = g_{\tau,n}(\underline{x}_{\tau,n})\rho_\tau(\underline{x}_{\tau,n})$ and $\Phi_{\tau,n} = g_{\tau,n}(\underline{x}_{\tau,n})\zeta_\tau(t) + \phi_{\tau,n}(t)$.

Assumption 1 (see [40, 41]). There exist positive constants $\underline{g}_\tau$ and $\overline{g}_\tau$, and $\tau = 1, \ldots, N$, such that $\underline{g}_\tau \leq g_{\tau,n}(\underline{x}_{\tau,n}) \leq \overline{g}_\tau$. Without losing generality, it is assumed that $g_{\tau,n}(\underline{x}_{\tau,n})$ is positive in the following adaptive fault-tolerant containment controller design.

Assumption 2. All the signals $\zeta_\tau(t)$ and $\phi_{\tau,i}(t)$ are bounded for the actual industrial control, so it is assumed that $\zeta_\tau(t) \leq \zeta_\tau^*$ and $\phi_{\tau,i}(t) \leq \phi_{\tau,i}^*$ with ζ_τ^* and $\phi_{\tau,i}^*$ being positive constants. Because $g_{\tau,n}(\underline{x}_{\tau,n})$ is bounded, we shall assume that $\Phi_{\tau,n} \leq \Phi_{\tau,n}^*$ with $\Phi_{\tau,n}^*$ being a positive constant.

Nussbaum Type Gain: a continuous function $N(s)$ is a Nussbaum-type function if

$$\begin{aligned} &\lim_{s\longrightarrow+\infty} \sup \frac{1}{s}\int_0^s N(\varsigma)\mathrm{d}\varsigma = +\infty, \\ &\lim_{s\longrightarrow+\infty} \inf \frac{1}{s}\int_0^s N(\varsigma)\mathrm{d}\varsigma = -\infty. \end{aligned} \quad (5)$$

For example, $\varsigma^2\cos(\varsigma)$, $\varsigma^2\sin(\varsigma)$, and $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ are Nussbaum-type functions. For clarity, the even Nussbaum-type function $N(\varsigma) = e^{\varsigma^2}\cos(\pi/2\varsigma)$ is used throughout this paper.

Lemma 1 (see [42]). *For any positive semidefinite smooth function $V(t)$ and any smooth function $\varsigma(t)$ with $t \in [0, t_f)$, if, for an even smooth Nussbaum-type function $N(\varsigma)$, the inequality,*

$$0 \leq V(t) \leq c_0 + e^{-c_1 t}\int_0^t g(x(\tau))N(\varsigma)\dot{\varsigma}e^{c_1\tau}\mathrm{d}\tau + e^{-c_1 t}\int_0^t \dot{\varsigma}e^{c_1\tau}\mathrm{d}\tau, \quad \forall t \in \left[0, t_f\right), \quad (6)$$

is true with positive constants c_1 and c_0 and a function $g(x(t))$ taking values in the unknown closed intervals $I: = [l^-, l^+]$ with $0 \notin I$, then $V(t)$, $\varsigma(t)$ and $\int_0^t g(x(\tau))N(\varsigma)\dot{\varsigma}\mathrm{d}\tau$ are bounded on $[0, t_f)$.

2.2. Fuzzy Logic Systems. On the basis of [28, 30], the knowledge base for FLS comprises a collection of fuzzy if-then rules as follows:

$$R^l\text{: if } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ and} \ldots \text{and } x_n \text{ is } F_n^l, \text{then } y \text{ is } G^l, \quad l = 1, 2, \ldots, N, \quad (7)$$

where $x = [x_1, x_2, \ldots, x_n]^T$ and y are the input and output of the FLS, respectively. The fuzzy sets F_τ^l and G^l are associated with the fuzzy membership functions $\mu_{F_\tau^l}(x_\tau)$ and $\mu_{G_\tau^l}(x_\tau)$, respectively. N is the rules number.

Through the singleton fuzzification, the center average defuzzification, and product inference engine, the FLS can be represented as

$$y(x) = \sum_{l=1}^N \overline{y}_l \frac{\prod_{\tau=1}^n \mu_{F_\tau^l}(x_\tau)}{\sum_{l=1}^N\left[\prod_{\tau=1}^n \mu_{F_\tau^l}(x_\tau)\right]}, \quad (8)$$

where $\overline{y}_l = \max_{y\in R}\mu_{G_\tau^l}(y)$.

The fuzzy basis functions are designed as

$$\varphi_l = \frac{\prod_{\tau=1}^n \mu_{F_\tau^l}(x_\tau)}{\sum_{l=1}^N\left[\prod_{\tau=1}^n \mu_{F_\tau^l}(x_\tau)\right]}, \quad (9)$$

and $\theta^T = [\overline{y}_1, \ldots, \overline{y}_N] = [\theta_1, \ldots, \theta_N]$ and $\varphi(x) = [\varphi_1(x), \ldots, \varphi_N(x)]^T$. Then, FLS (7) can be further expressed as $y(x) = \theta^T\varphi(x)$.

3. Adaptive Fuzzy Containment Control Design and Stability Analysis

In this section, an adaptive fuzzy fault-tolerant containment controller will be constructed by using the backstepping control technique. Due to the fact that FLS can approximate an unknown nonlinear function in a compact set, which is called universal approximator, nonlinear function $F_{\tau,i}(\underline{x}_{\tau,i})$ can be approximated by FLS. It is assumed that

$$\widehat{F}_{\tau,i}\left(\underline{x}_{\tau,i}|\theta_{\tau,i}\right)=\theta_{\tau,i}^{T}\varphi_{\tau,i}\left(\underline{x}_{\tau,i}\right), \tag{10}$$

and the optimal parameter vector $\theta_{\tau,i}^{*}$ is designed as

$$\theta_{\tau,i}^{*}=\arg\min_{\theta_{\tau,i}\in\Omega_{\tau,i}}\left[\sup_{\underline{x}_{\tau,i}\in U_{\tau,i}}\left|\widehat{F}_{\tau,i}\left(\underline{x}_{\tau,i}|\theta_{\tau,i}\right)-F_{\tau,i}\left(\underline{x}_{\tau,i}\right)\right|\right], \tag{11}$$

where $\Omega_{\tau,i}$ is a bounded region of $\theta_{\tau,i}$ and $U_{\tau,i}$ is a bounded region of $\underline{x}_{\tau,i}$. At the same time, the approximation error $\varepsilon_{\tau,i}$ is designed as

$$\varepsilon_{\tau,i}=F_{\tau,i}\left(\underline{x}_{\tau,i}\right)-\widehat{F}_{\tau,i}\left(\underline{x}_{\tau,i}|\theta_{\tau,i}^{*}\right), \tag{12}$$

where $\varepsilon_{\tau,i}$ satisfies $|\varepsilon_{\tau,i}|\le\varepsilon_{\tau,i}^{*}$ with $\varepsilon_{\tau,i}^{*}$ being a positive constant.

According to [43], in order to reduce the number of adaptive laws, a new constant W_{τ}^{*} is designed as follows:

$$W_{\tau}^{*}=\max\left\{\left\|\theta_{\tau,i}^{*}\right\|^{2}: i=1,2,\ldots,n\right\}. \tag{13}$$

Obviously, W_{τ}^{*} is an unknown positive constant.

In order to accomplish the control objective, we design the following coordinate transformations:

$$s_{\tau,1}=\sum_{j=1}^{N}a_{\tau j}\left(y_{\tau}-y_{j}\right)+\sum_{j=N+1}^{N+M}a_{\tau j}\left(y_{\tau}-y_{rj}(t)\right), s_{\tau,i}=x_{\tau,i}-\overline{\alpha}_{\tau,i},\quad i=2,\ldots,n, \tag{14}$$

where $s_{\tau,i}$ denotes error surface and $\overline{\alpha}_{\tau,i}$ represents the output of the first-order filter.

$$\eta_{\tau,i}\dot{\overline{\alpha}}_{\tau,i}+\overline{\alpha}_{\tau,i}=\alpha_{\tau,i},\overline{\alpha}_{\tau,i}(0)=\alpha_{\tau,i}(0), \tag{15}$$

with a positive constant $\eta_{\tau,2}$, which is introduced to avoid the repeated differentiation of the virtual control signal $\alpha_{\tau,i}$.

(i) Step 1: from (14), $\dot{s}_{\tau,1}$ is expressed as

$$\dot{s}_{\tau,1}=d_{\tau}\theta_{\tau,1}^{*T}\varphi_{\tau,1}+d_{\tau}\varepsilon_{\tau,1}+d_{\tau}s_{\tau,2}+d_{\tau}\overline{\alpha}_{\tau,2}+d_{\tau}\phi_{\tau,1}-\sum_{j=1}^{N}a_{\tau j}\left(\theta_{j,1}^{*T}\varphi_{j,1}+\varepsilon_{j,1}+x_{j,2}+\phi_{j,1}\right)-\sum_{j=N+1}^{N+M}a_{\tau j}\dot{y}_{rj}(t). \tag{16}$$

The following compensating dynamics is introduced to compensate the negative impact of the filter error $\overline{\alpha}_{\tau,2}-\alpha_{\tau,2}$:

$$\dot{q}_{\tau,1}=-c_{\tau,1}q_{\tau,1}+d_{\tau}\overline{\alpha}_{\tau,2}-d_{\tau}\alpha_{\tau,2}+d_{\tau}q_{\tau,2},\quad q_{\tau,1}(0)=0, \tag{17}$$

with a positive constant $c_{\tau,1}$.

It follows from (16) and (17) that $\overline{s}_{\tau,1}=s_{\tau,1}-q_{\tau,1}$ satisfies the following differential equation:

$$\begin{aligned}\dot{\overline{s}}_{\tau,1}=&d_{\tau}\theta_{\tau,1}^{*T}\varphi_{\tau,1}+d_{\tau}\varepsilon_{\tau,1}+d_{\tau}\phi_{\tau,1}+d_{\tau}s_{\tau,2}\\&+c_{\tau,1}q_{\tau,1}-d_{\tau}q_{\tau,2}+d_{\tau}\alpha_{\tau,2}\\&-\sum_{j=1}^{N}a_{\tau j}\left(\theta_{j,1}^{*T}\varphi_{j,1}+\varepsilon_{j,1}+x_{j,2}+\phi_{j,1}\right)\\&-\sum_{j=N+1}^{N+M}a_{\tau j}\dot{y}_{rj}(t).\end{aligned} \tag{18}$$

The first virtual control $\alpha_{\tau,2}$ is designed as

$$\alpha_{\tau,2}=\frac{1}{d_{\tau}}\left(-c_{\tau,1}s_{\tau,1}+\sum_{j=N+1}^{N+M}a_{\tau j}\dot{y}_{rj}-\frac{1}{2}\overline{s}_{\tau,1}W_{\tau}\varphi_{\tau,1}^{T}\varphi_{\tau,1}+\sum_{j=1}^{N}a_{\tau j}x_{j,2}+\sum_{j=1}^{N}a_{\tau j}\theta_{j,1}^{T}\varphi_{j,1}\right), \tag{19}$$

where $c_{\tau,1}$ is a positive design parameter.

Then, $\dot{\overline{s}}_{\tau,1}$ can be further written as

$$\dot{\overline{s}}_{\tau,1}=d_{\tau}\theta_{\tau,1}^{*T}\varphi_{\tau,1}+d_{\tau}\varepsilon_{\tau,1}+d_{\tau}\overline{s}_{\tau,2}-c_{\tau,1}\overline{s}_{\tau,1}+d_{\tau}\phi_{\tau,1}-\frac{1}{2}\overline{s}_{\tau,1}W_{\tau}\varphi_{\tau,1}^{T}\varphi_{\tau,1}-\sum_{j=1}^{N}a_{\tau j}\widetilde{\theta}_{j,1}^{T}\varphi_{j,1}-\sum_{j=1}^{N}a_{\tau j}\varepsilon_{j,1}-\sum_{j=1}^{N}a_{\tau j}\phi_{j,1}, \tag{20}$$

where $\bar{s}_{\tau,2} = s_{\tau,2} - q_{\tau,2}$ and $\widetilde{\theta}_{j,1} = \theta^*_{j,1} - \theta_{j,1}$.

The Lyapunov function candidate is considered in the following:

$$V_{\tau,1} = \frac{1}{2}\bar{s}^2_{\tau,1} + \frac{1}{2}\sum_{j=1}^{N} a_{\tau,j}\frac{1}{r_{j,1}}\widetilde{\theta}^T_{j,1}\widetilde{\theta}_{j,1}, \tag{21}$$

where $r_{j,1}$ is a positive design parameter.

From equation (21), $\dot{V}_{\tau,1}$ can be expressed as

$$\begin{aligned}\dot{V}_{\tau,1} &= \bar{s}_{\tau,1}\dot{\bar{s}}_{\tau,1} - \sum_{j=1}^{N} a_{\tau j}\frac{1}{r_{j,1}}\widetilde{\theta}^T_{j,1}\dot{\theta}_{j,1}\\ &= \bar{s}_{\tau,1}\left(d_\tau\theta^{*T}_{\tau,1}\varphi_{\tau,1} + d_\tau\varepsilon_{\tau,1} + d_\tau\bar{s}_{\tau,2} - c_{\tau,1}\bar{s}_{\tau,1} + d_\tau\phi_{\tau,1} - \frac{1}{2}\bar{s}_{\tau,1}W_\tau\varphi^T_{\tau,1}\varphi_{\tau,1} - \sum_{j=1}^{N} a_{\tau j}\widetilde{\theta}^T_{j,1}\varphi_{j,1} - \sum_{j=1}^{N} a_{\tau j}\varepsilon_{j,1} - \sum_{j=1}^{N} a_{\tau j}\phi_{j,1}\right)\\ &\quad - \sum_{j=1}^{N} a_{\tau j}\frac{1}{r_{j,1}}\widetilde{\theta}^T_{j,1}\dot{\theta}_{j,1}.\end{aligned} \tag{22}$$

It can be easily verified that

$$\begin{aligned}\bar{s}_{\tau,1}d_\tau\theta^{*T}_{\tau,1}\varphi_{\tau,1} &\leq \frac{1}{2}\bar{s}^2_{\tau,1}W^*_\tau\varphi^T_{\tau,1}\varphi_{\tau,1} + \frac{1}{2}d^2_\tau,\\ \bar{s}_{\tau,1}\left(d_\tau\varepsilon_{\tau,1} + d_\tau\phi_{\tau,1} - \sum_{j=1}^{N} a_{\tau,j}\varepsilon_{j,1} - \sum_{j=1}^{N} a_{\tau j}\phi_{j,1}\right) &\leq \bar{s}^2_{\tau,1} + d^2_\tau\varepsilon^{*2}_{\tau,1} + d^2_\tau\phi^{*2}_{\tau,1} + \left(\sum_{j=1}^{N} a_{\tau j}\varepsilon^*_{j,1}\right)^2 + \left(\sum_{j=1}^{N} a_{\tau j}\phi^*_{j,1}\right)^2.\end{aligned} \tag{23}$$

From (23), $\dot{V}_{\tau,1}$ is expressed as

$$\begin{aligned}\dot{V}_{\tau,1} \leq& -\left(c_{\tau,1} - 1\right)\bar{s}^2_{\tau,1} + d_\tau\bar{s}_{\tau,1}\bar{s}_{\tau,2} + \frac{1}{2}\bar{s}^2_{\tau,1}\widetilde{W}_\tau\varphi^T_{\tau,1}\varphi_{\tau,1}\\ &- \sum_{j=1}^{N} a_{\tau j}\frac{1}{r_{j,1}}\widetilde{\theta}^T_{j,1}\left(\dot{\theta}_{j,1} + r_{j,1}\bar{s}_{\tau,1}\varphi_{j,1}\right) + \frac{1}{2}d^2_\tau + d^2_\tau\varepsilon^{*2}_{\tau,1} + d^2_\tau\phi^{*2}_{\tau,1}\\ &+ \left(\sum_{j=1}^{N} a_{\tau j}\varepsilon^*_{j,1}\right)^2 + \left(\sum_{j=1}^{N} a_{\tau j}\phi^*_{j,1}\right)^2,\end{aligned} \tag{24}$$

where $\widetilde{W}_\tau = W^*_\tau - W_\tau$.

The adaptive law is designed as

$$\dot{\theta}_{j,1} = r_{j,1}\left(-\bar{s}_{\tau,1}\varphi_{j,1} - \delta_{j,1}\theta_{j,1}\right). \tag{25}$$

Then, one has

$$\begin{aligned}\dot{V}_{\tau,1} \leq& -\left(c_{\tau,1} - 1\right)\bar{s}^2_{\tau,1} + d_\tau\bar{s}_{\tau,1}\bar{s}_{\tau,2} + \omega_{\tau,1}\\ &+ \frac{1}{2}\bar{s}^2_{\tau,1}\widetilde{W}_\tau\varphi^T_{\tau,1}\varphi_{\tau,1} + \sum_{j=1}^{N} a_{\tau j}\delta_{j,1}\widetilde{\theta}^T_{j,1}\theta_{j,1},\end{aligned} \tag{26}$$

where $\omega_{\tau,1} = 1/2d^2_\tau + d^2_\tau\varepsilon^{*2}_{\tau,1} + d^2_\tau\phi^{*2}_{\tau,1} + (\sum_{j=1}^{N} a_{\tau j}\varepsilon^*_{j,1})^2 + (\sum_{j=1}^{N} a_{\tau j}\phi^*_{j,1})^2$.

(ii) Step 2: from (14), the derivative of $s_{\tau,2}$ is expressed as

$$\dot{s}_{\tau,2} = \theta^{*T}_{\tau,2}\varphi_{\tau,2} + \varepsilon_{\tau,2} + x_{\tau,3} + \phi_{\tau,2} - \dot{\bar{\alpha}}_{\tau,2}. \tag{27}$$

The impact of the error $\bar{\alpha}_{\tau,3} - \alpha_{\tau,3}$ can be reduced by the following compensating dynamics:

$$\dot{q}_{\tau,2} = -c_{\tau,2}q_{\tau,2} + \bar{\alpha}_{\tau,3} - \alpha_{\tau,3} + q_{\tau,3}, \quad q_{\tau,2}(0) = 0, \tag{28}$$

with a positive constant $c_{\tau,2}$.

Define $\bar{s}_{\tau,3} = s_{\tau,3} - q_{\tau,3}$. Then, the derivative of $\bar{s}_{\tau,2}$ can be determined by

$$\dot{\bar{s}}_{\tau,2} = \theta^*_{\tau,2}\varphi_{\tau,2} + \varepsilon_{\tau,2} + \phi_{\tau,2} + \bar{s}_{\tau,3} - \dot{\bar{\alpha}}_{\tau,2} + \alpha_{\tau,3} + c_{\tau,2}q_{\tau,2}. \tag{29}$$

Choose the following virtual control $\alpha_{\tau,3}$:

$$\alpha_{\tau,3} = -c_{\tau,2}s_{\tau,2} + \dot{\bar{\alpha}}_{\tau,2} - \frac{1}{2}\bar{s}_{\tau,2}W_\tau\varphi^T_{\tau,2}\varphi_{\tau,2} - d_\tau\bar{s}_{\tau,1}. \tag{30}$$

From (30), the derivative of $\bar{s}_{\tau,2}$ can be rewritten as

$$\begin{aligned}\dot{\bar{s}}_{\tau,2} =& \theta^{*2}_{\tau,2}\varphi_{\tau,2} + \varepsilon_{\tau,2} + \phi_{\tau,2} + \bar{s}_{\tau,3}\\ &- c_{\tau,2}\bar{s}_{\tau,2} - \frac{1}{2}\bar{s}_{\tau,2}W_\tau\varphi^T_{\tau,2}\varphi_{\tau,2} - d_\tau\bar{s}_{\tau,1}.\end{aligned} \tag{31}$$

A simple manipulation shows that the Lyapunov function candidate,

$$V_{\tau,2}=\frac{1}{2}\bar{s}_{\tau,2}^2+V_{\tau,1}, \tag{32}$$

has the following derivative:

$$\begin{aligned}\dot{V}_{\tau,2}&\le \bar{s}_{\tau,2}\dot{\bar{s}}_{\tau,2}+\dot{V}_{\tau,1}\\ &\le \bar{s}_{\tau,2}\left(\theta_{\tau,2}^{*2}\varphi_{\tau,2}+\varepsilon_{\tau,2}+\phi_{\tau,2}+\bar{s}_{\tau,3}-c_{\tau,2}\bar{s}_{\tau,2}-\frac{1}{2}\bar{s}_{\tau,2}W_\tau\varphi_{\tau,2}^T\varphi_{\tau,2}\right)\\ &\quad -\left(c_{\tau,1}-1\right)\bar{s}_{\tau,1}^2+\omega_{\tau,1}+\frac{1}{2}\bar{s}_{\tau,1}^2\widetilde{W}_\tau\varphi_{\tau,1}^T\varphi_{\tau,1}+\sum_{j=1}^N a_{\tau j}\delta_{j,1}\widetilde{\theta}_{j,1}^T\theta_{j,1}.\end{aligned} \tag{33}$$

It is true that

$$\begin{aligned}\bar{s}_{\tau,2}\theta_{\tau,2}^{*T}\varphi_{\tau,2}&\le\frac{1}{2}\bar{s}_{\tau,2}^2W_\tau^*\varphi_{\tau,2}^T\varphi_{\tau,2}+\frac{1}{2},\\ \bar{s}_{\tau,2}\left(\varepsilon_{\tau,2}+\phi_{\tau,2}\right)&\le\bar{s}_{\tau,2}^2+\frac{1}{2}\varepsilon_{\tau,2}^{*2}+\frac{1}{2}\phi_{\tau,2}^{*2}.\end{aligned} \tag{34}$$

Then, one can have

$$\begin{aligned}\dot{V}_{\tau,2}&\le-\sum_{\sigma=1}^2\left(c_{\tau,\sigma}-1\right)\bar{s}_{\tau,\sigma}^2+\bar{s}_{\tau,2}\bar{s}_{\tau,3}+\omega_{\tau,2}\\ &\quad+\sum_{j=1}^N a_{\tau j}\delta_{j,1}\widetilde{\theta}_{j,1}^T\theta_{j,1}+\frac{1}{2}\sum_{\sigma=1}^2\bar{s}_{\tau,\sigma}^2\widetilde{W}_\tau\varphi_{\tau,\sigma}^T\varphi_{\tau,\sigma},\end{aligned} \tag{35}$$

where $\omega_{\tau,2}=\omega_{\tau,1}+1/2+1/2\varepsilon_{\tau,2}^{*2}+1/2\phi_{\tau,2}^{*2}$.

(iii) Step i $(3\le i\le n-1)$: the derivative of $s_{\tau,i}$ can be expressed as

$$\dot{s}_{\tau,i}=\theta_{\tau,i}^{*2}\varphi_{\tau,i}+\varepsilon_{\tau,i}+x_{\tau,i+1}+\phi_{\tau,i}-\dot{\bar{\alpha}}_{\tau,i}. \tag{36}$$

Similar to Step 2, to compensate the impact of $\bar{\alpha}_{\tau,i+1}-\alpha_{\tau,i+1}$, the following compensating dynamics is introduced:

$$\dot{q}_{\tau,i}=-c_{\tau,i}q_{\tau,i}+\bar{\alpha}_{\tau,i+1}-\alpha_{\tau,i+1}+q_{\tau,i+1},\quad q_{\tau,i}(0)=0, \tag{37}$$

with $c_{\tau,i}$ being a positive design parameter.

It can be verified that the derivative of $\bar{s}_{\tau,i}=s_{\tau,i}-q_{\tau,i}$ is given by

$$\dot{\bar{s}}_{\tau,i}=\theta_{\tau,i}^{*}\varphi_{\tau,i}+\varepsilon_{\tau,i}+\phi_{\tau,i}+\bar{s}_{\tau,i+1}-\dot{\bar{\alpha}}_{\tau,i}+\alpha_{\tau,i+1}+c_{\tau,i}q_{\tau,i}, \tag{38}$$

where $\bar{s}_{\tau,n+1}=s_{\tau,n+1}-q_{\tau,n+1}$.

Define the following virtual control $\alpha_{\tau,i+1}$:

$$\alpha_{\tau,i+1}=-c_{\tau,i}s_{\tau,i}+\dot{\bar{\alpha}}_{\tau,i}-\frac{1}{2}\bar{s}_{\tau,i}W_\tau\varphi_{\tau,i}^T\varphi_{\tau,i}-\bar{s}_{\tau,i-1}. \tag{39}$$

Then, the derivative of $\bar{s}_{\tau,i}$ can be rewritten as

$$\begin{aligned}\dot{\bar{s}}_{\tau,i}&=\theta_{\tau,i}^{*2}\varphi_{\tau,i}+\varepsilon_{\tau,i}+\phi_{\tau,i}+\bar{s}_{\tau,i+1}-c_{\tau,i}\bar{s}_{\tau,i}\\ &\quad-\frac{1}{2}\bar{s}_{\tau,i}W_\tau\varphi_{\tau,i}^T\varphi_{\tau,i}-\bar{s}_{\tau,i-1}.\end{aligned} \tag{40}$$

A simple manipulation shows that the Lyapunov function candidate,

$$V_{\tau,i}=\frac{1}{2}\bar{s}_{\tau,i}^2+V_{\tau,i-1}, \tag{41}$$

has the following derivative:

$$\begin{aligned}\dot{V}_{\tau,i}&\le\bar{s}_{\tau,i}\dot{\bar{s}}_{\tau,i}+\dot{V}_{\tau,i-1}\\ &\le\bar{s}_{\tau,i}\left(\theta_{\tau,i}^{*2}\varphi_{\tau,i}+\varepsilon_{\tau,i}+\phi_{\tau,i}+\bar{s}_{\tau,i+1}-c_{\tau,i}\bar{s}_{\tau,i}-\frac{1}{2}\bar{s}_{\tau,i}W_\tau\varphi_{\tau,i}^T\varphi_{\tau,i}\right)-\sum_{\sigma=1}^{i-1}\left(c_{\tau,\sigma}-2\right)\bar{s}_{\tau,\sigma}^2+\omega_{\tau,i-1}+\sum_{j=1}^N a_{\tau,j}\delta_{j,1}\widetilde{\theta}_{j,1}^T\theta_{j,1}+\frac{1}{2}\sum_{\sigma=1}^{i-1}\bar{s}_{\tau,\sigma}^2\widetilde{W}_\tau\varphi_{\tau,\sigma}^T\varphi_{\tau,\sigma}.\end{aligned} \tag{42}$$

By using Young's inequality, the following inequalities can be verified:

$$\begin{aligned}\bar{s}_{\tau,i}\theta_{\tau,i}^{*T}\varphi_{\tau,i}&\le\frac{1}{2}\bar{s}_{\tau,i}^2W_\tau^*\varphi_{\tau,i}^T\varphi_{\tau,i}+\frac{1}{2},\\ \bar{s}_{\tau,i}\left(\varepsilon_{\tau,i}+\phi_{\tau,i}\right)&\le\bar{s}_{\tau,i}^2+\frac{1}{2}\varepsilon_{\tau,i}^{*2}+\frac{1}{2}\phi_{\tau,i}^{*2}.\end{aligned} \tag{43}$$

Substituting (43) into (42), one has

$$\dot{V}_{\tau,i}\le-\sum_{\sigma=1}^i\left(c_{\tau,\sigma}-1\right)\bar{s}_{\tau,\sigma}^2+\sum_{j=1}^N a_{\tau j}\delta_{j,1}\widetilde{\theta}_{j,1}^T\theta_{j,1}+\frac{1}{2}\sum_{\sigma=1}^i\bar{s}_{\tau,\sigma}^2\widetilde{W}_\tau\varphi_{\tau,\sigma}^T\varphi_{\tau,\sigma}+\bar{s}_{\tau,i}\bar{s}_{\tau,i+1}+\omega_{\tau,i}, \tag{44}$$

where $\omega_{\tau,i} = \omega_{\tau,i-1} + 1/2 + 1/2\varepsilon_{\tau,i}^{*2} + 1/2\phi_{\tau,i}^{*2}$.

(iv) Step n: the derivative of $s_{\tau,n}$ is given by

$$\dot{s}_{\tau,n} = \theta_{\tau,n}^{*T}\varphi_{\tau,n} + \varepsilon_{\tau,n} + \Gamma_\tau u_\tau(t) + \Phi_{\tau,n} - \dot{\alpha}_{\tau,n}. \tag{45}$$

Define $\bar{s}_{\tau,n} = s_{\tau,n} - q_{\tau,n}$ with $q_{\tau,n}$ being determined by

$$\dot{q}_{\tau,n} = -c_{\tau,n}q_{\tau,n}, \quad q_{\tau,n}(0) = 0, \tag{46}$$

where $c_{\tau,n}$ is a positive design parameter.

Then, it follows from (45) and (46) that

$$\dot{\bar{s}}_{\tau,n} = \theta_{\tau,n}^{*T}\varphi_{\tau,n} + \varepsilon_{\tau,n} + \Gamma_\tau u_\tau(t) + \Phi_{\tau,n} - \dot{\alpha}_{\tau,n} + c_{\tau,n}q_{\tau,n}. \tag{47}$$

The actual adaptive fuzzy controller is chosen as

$$u_\tau = N(\varsigma_\tau)\nu_\tau, \tag{48}$$

with

$$\dot{\varsigma}_\tau = \gamma_\tau\bar{s}_{\tau,n}\nu_\tau, \tag{49}$$

and

$$\nu_\tau = c_{\tau,n}s_{\tau,n} - \dot{\alpha}_{\tau,n} + \frac{1}{2}\bar{s}_{\tau,n}W_\tau\varphi_{\tau,n}^T\varphi_{\tau,n} + \bar{s}_{\tau,n-1}, \tag{50}$$

where γ_τ is a positive design parameter.

Substituting (48) and (50) into (47), $\dot{\bar{s}}_{\tau,n}$ is obtained as

$$\begin{aligned}\dot{\bar{s}}_{\tau,n} &= \theta_{\tau,n}^{*T}\varphi_{\tau,n} + \varepsilon_{\tau,n} + \Gamma_\tau N(\varsigma_\tau)\nu_\tau + \nu_\tau + \Phi_{\tau,n}\\ &\quad - c_{\tau,n}\bar{s}_{\tau,n} - \frac{1}{2}\bar{s}_{\tau,n}W_\tau\varphi_{\tau,n}^T\varphi_{\tau,n} - \bar{s}_{\tau,n-1}.\end{aligned} \tag{51}$$

Consider the Lyapunov function in the following:

$$V_{\tau,n} = \frac{1}{2}\bar{s}_{\tau,n}^2 + \frac{1}{2b_\tau}\widetilde{W}_\tau^2 + V_{\tau,n-1}, \tag{52}$$

where b_τ is a positive design parameter.

The time derivative of $V_{\tau,n}$ becomes

$$\begin{aligned}\dot{V}_{\tau,n} &\leq \bar{s}_{\tau,n}\dot{\bar{s}}_{\tau,n} - \frac{1}{b_\tau}\widetilde{W}_\tau\dot{W}_\tau + \dot{V}_{\tau,n-1}\\ &\leq \bar{s}_{\tau,n}\left(\theta_{\tau,n}^{*T}\varphi_{\tau,n} + \varepsilon_{\tau,n} + \Gamma_\tau N(\varsigma_\tau)\nu_\tau + \nu_\tau + \Phi_{\tau,n} - c_{\tau,n}\bar{s}_{\tau,n} - \frac{1}{2}\bar{s}_{\tau,n}W_\tau\varphi_{\tau,n}^T\varphi_{\tau,n}\right) - \frac{1}{b_\tau}\widetilde{W}_\tau\dot{W}_\tau\\ &\quad - \sum_{\sigma=1}^{n-1}(c_{\tau,\sigma} - 1)\bar{s}_{\tau,\sigma}^2 + \sum_{j=1}^{N} a_{\tau j}\delta_{j,1}\widetilde{\theta}_{j,1}^T\theta_{j,1} + \frac{1}{2}\sum_{\sigma=1}^{n-1}\bar{s}_{\tau,\sigma}^2\widetilde{W}_\tau\varphi_{\tau,\sigma}^T\varphi_{\tau,\sigma} + \omega_{\tau,n-1}.\end{aligned} \tag{53}$$

We have

$$\begin{aligned}\bar{s}_{\tau,n}\theta_{\tau,n}^{*T}\varphi_{\tau,n} &\leq \frac{1}{2}\bar{s}_{\tau,n}^2W_\tau^*\varphi_{\tau,n}^T\varphi_{\tau,n} + \frac{1}{2},\\ \bar{s}_{\tau,n}\left(\varepsilon_{\tau,n} + \Phi_{\tau,n}\right) &\leq \bar{s}_{\tau,n}^2 + \frac{1}{2}\varepsilon_{\tau,n}^{*2} + \frac{1}{2}\Phi_{\tau,n}^{*2}.\end{aligned} \tag{54}$$

Then, $\dot{V}_{\tau,n}$ can be estimated by

$$\begin{aligned}\dot{V}_{\tau,n} &\leq -\sum_{\sigma=1}^{n}(c_{\tau,\sigma} - 1)\bar{s}_{\tau,\sigma}^2 + \sum_{j=1}^{N} a_{\tau j}\delta_{j,1}\widetilde{\theta}_{j,1}^T\theta_{j,1} - \frac{1}{b_\tau}\widetilde{W}_\tau\left(\dot{W}_\tau - \frac{1}{2}\sum_{\sigma=1}^{n} b_\tau\bar{s}_{\tau,\sigma}^2\varphi_{\tau,\sigma}^T\varphi_{\tau,\sigma}\right)\\ &\quad + \Gamma_\tau N(\varsigma_\tau)\nu_\tau\bar{s}_{\tau,n} + \nu_\tau\bar{s}_{\tau,n} + \omega_{\tau,n-1} + \frac{1}{2} + \frac{1}{2}\varepsilon_{\tau,n}^{*2} + \frac{1}{2}\Phi_{\tau,n}^{*2}.\end{aligned} \tag{55}$$

With the adaptive law,

$$\dot{W}_\tau = \frac{1}{2}\sum_{\sigma=1}^{n} b_\tau\bar{s}_{\tau,\sigma}^2\varphi_{\tau,\sigma}^T\varphi_{\tau,\sigma} - p_\tau W_\tau. \tag{56}$$

$\dot{V}_{\tau,n}$ can be further estimated by

$$\dot{V}_{\tau,n} \leq -\sum_{\sigma=1}^{n}(c_{\tau,\sigma} - 1)\bar{s}_{\tau,\sigma}^2 + \sum_{j=1}^{N} a_{\tau j}\delta_{j,1}\widetilde{\theta}_{j,1}^T\theta_{j,1} + \frac{p_\tau}{b_\tau}\widetilde{W}_\tau W_\tau + \Gamma_\tau N(\varsigma_\tau)\nu_\tau\bar{s}_{\tau,n} + \nu_\tau\bar{s}_{\tau,n} + \omega_{\tau,n-1} + \frac{1}{2} + \frac{1}{2}\varepsilon_{\tau,n}^{*2} + \frac{1}{2}\Phi_{\tau,n}^{*2}. \tag{57}$$

It is true that

$$\frac{p_\tau}{b_\tau}\widetilde{W}_\tau W_\tau \le \frac{p_\tau^2}{2b_\tau^2}W_\tau^{*\,2} - \left(\frac{p_\tau}{b_\tau} - \frac{1}{2}\right)\widetilde{W}_\tau^2,$$

$$\sum_{j=1}^{N} a_{\tau j}\delta_{j,1}\widetilde{\theta}_{j,1}^{T}\theta_{j,1} \le \frac{1}{2}\sum_{j=1}^{N} a_{\tau j}\delta_{j,1}^{2}\theta_{j,1}^{*T}\theta_{j,1}^{*} - \sum_{j=1}^{N} a_{\tau j}\left(\delta_{j,1} - \frac{1}{2}\right)\widetilde{\theta}_{j,1}^{T}\widetilde{\theta}_{j,1}. \tag{58}$$

Then, $\dot{V}_{\tau,n}$ is obtained as follows:

$$\dot{V}_{\tau,n} \le -\sum_{\sigma=1}^{n}\left(c_{\tau,\sigma} - 1\right)\bar{s}_{\tau,\sigma}^{2} - \left(\frac{p_\tau}{b_\tau} - \frac{1}{2}\right)\widetilde{W}_\tau^2 - \sum_{j=1}^{N} a_{\tau j}\left(\delta_{j,1} - \frac{1}{2}\right)\widetilde{\theta}_{j,1}^{T}\widetilde{\theta}_{j,1} + \Gamma_\tau N(\varsigma_\tau)v_\tau\bar{s}_{\tau,n} + v_\tau\bar{s}_{\tau,n} + \omega_\tau, \tag{59}$$

where $\omega_\tau = 1/2\sum_{j=1}^{N} a_{\tau j}\delta_{j,1}^{2}\theta_{j,1}^{*T}\ \ \theta_{j,1}^{*} + p_\tau^2/2b_\tau^2 W_\tau^{*\,2} + \omega_{\tau,n-1} + 1/2 + 1/2\varepsilon_{\tau,n}^{*2} + 1/2\Phi_{\tau,n}^{*2}$.

Remark 2. In the literature [21], by using backstepping control technique, the adaptive containment controller was designed for nonlinear MASs without actuator faults, and the number of adaptive laws was greater than or equal to the order of the system. Adaptive fuzzy containment controllers are investigated for nonlinear MASs with actuator faults in this paper. Meanwhile, compared with literature [21], the number of adaptive laws that need to be designed is less, and the amount of calculation is greatly reduced.

According to the above adaptive fuzzy fault-tolerant containment control, the main result can be summarized as a theorem as follows.

Theorem 1. *Consider the adaptive fuzzy backstepping containment controller, composed of the compensating signals (17), (28), (37), and (46), the virtual controllers (19), (30), and (39), the actual controller (48), and adaptive laws (25) and (56). If there exist the parameters $c_{\tau,\sigma}$, $\delta_{j,1}$, p_τ, and b_τ, such that $c_{\tau,\sigma} - 1 > 0$, $p_\tau/b_\tau - 1/2 > 0$, and $\delta_{j,1} - 1/2 > 0$, then all the signals in the closed-loop system are bounded and all followers can converge to the convex hull built by the leaders.*

Proof. Define $c_\tau = \min_{i=1,\ldots,n}\left\{2(c_{\tau,i} - 1), 2p_\tau - b_\tau, 2\delta_{j,1}r_{j,1} - r_{j,1}\right\}$. From (59), one has

$$\dot{V}_{\tau,n} \le -c_\tau V_{\tau,n} + \frac{\Gamma_\tau}{\gamma_\tau}N(\varsigma_\tau)\dot{\varsigma}_\tau + \frac{1}{\gamma_\tau}\dot{\varsigma}_\tau + \omega_\tau, \tag{60}$$

which has a solution of the form

$$V_{\tau,n}(t) \le c_0 + e^{-ct}\int_0^t \frac{\Gamma_\tau}{\gamma_\tau}N(\varsigma_\tau)\dot{\varsigma}_\tau e^{c\varrho}d\varrho + e^{-ct}\int_0^t \frac{1}{\gamma_\tau}\dot{\varsigma}_\tau e^{c\varrho}d\varrho, \tag{61}$$

where $c_0 = V_{\tau,n}(0) + 2\omega_\tau/c$. According to Lemma 1, all the signals for follower τ are bounded. Similarly, the whole nonlinear multiagent systems are stable.

4. Simulation Example

In order to illustrate the feasibility of the designed adaptive fuzzy backstepping fault-tolerant containment control, the nonlinear MASs are considered with two leaders and three followers. The τ-th follower is considered as

$$\begin{cases} \dot{x}_{\tau,1} = F_{\tau,1}\left(x_{\tau,1}\right) + x_{\tau,2} + \phi_{\tau,1}(t), \\ \dot{x}_{\tau,2} = F_{\tau,2}\left(\underline{x}_{\tau,2}\right) + g_{\tau,2}\left(\underline{x}_{\tau,2}\right)u_\tau(t) + \phi_{\tau,2}(t), \\ y_\tau = x_{\tau,1}, \quad \tau = 1, 2, 3, \end{cases} \tag{62}$$

where $F_{1,1} = 0.5e^{-x_{1,1}}$, $F_{1,2} = 0.3\sin(x_{1,1})\cos(x_{1,2})$, $F_{2,1} = -0.2x_{2,1}e^{-x_{2,1}}$, $F_{2,2} = x_{2,1}^2\cos(x_{2,2})$, $F_{3,1} = x_{3,1}\cos(x_{3,1})$, $F_{3,2} = 0.4x_{3,2}\sin(x_{3,1})$, $g_{1,2} = e^{\sin(x_{1,1}x_{1,2})}$, $g_{2,2} = e^{\cos(x_{2,2})\sin(x_{2,1}^4)}$, $g_{3,2} = e^{\cos(x_{3,2}x_{3,1}^4)}$, $\phi_{1,1} = 0.5\cos(t)$, $\phi_{1,2} = \sin(t)\cos(t)$, $\phi_{2,1} = 0.2\sin(t)$, $\phi_{2,2} = \sin(0.5t)$, $\phi_{3,1} = \cos(t)$, and $\phi_{3,2} = 0.5\cos(t)\sin(t)$.

The flow of information among leaders and followers is elaborated through Figure 1, in which F_1, F_2, and F_3 denote followers and L_1 and L_2 represent leaders. The actuator bias faults are chosen as $\zeta_1 = 0.2\sin(t)$, $\zeta_2 = 0.6\cos(0.5t)$, and $\zeta_3 = 0.1\sin(0.1t)$, and the actuator fault appears at $t_f = 12s$.

The unknown nonlinear remaining control rate coefficients ρ_τ are chosen as $\rho_1 = 1/1 + e^{-0.2}$, $\rho_2 = 1/1 + e^{-2(\sin(x_{2,1})^2 + \cos(x_{2,2})^2)}$, and $\rho_3 = 1/1 + e^{-0.2\sin(x_{3,2})}$. The leaders' signals are chosen as $y_{r1} = \sin(t) + 1$ and $y_{r2} = \sin(t) - 0.7$.

From the directed communication graph, the adjacency matrix is obtained as follows:

$$\overline{A} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{63}$$

The initial values are chosen as $x_1(0) = [0.2, 0.1]^T$, $x_2(0) = [0.5, 0]^T$, and $x_3(0) = [-0.2, 0.3]^T$. The other initial values are set as zero. The design parameters are tuned to be $c_{1,1} = 27$, $c_{1,2} = 28$, $c_{2,1} = 22$, $c_{2,2} = 27$, $c_{3,1} = 20$, $c_{3,2} = 27$, $\eta_{1,2} = \eta_{2,2} = \eta_{3,2} = 0.1$, and $p_1 = p_2 = p_3 = 20$ by trial and error.

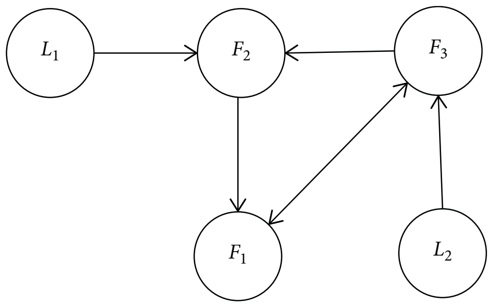

Figure 1: Directed communication graph.

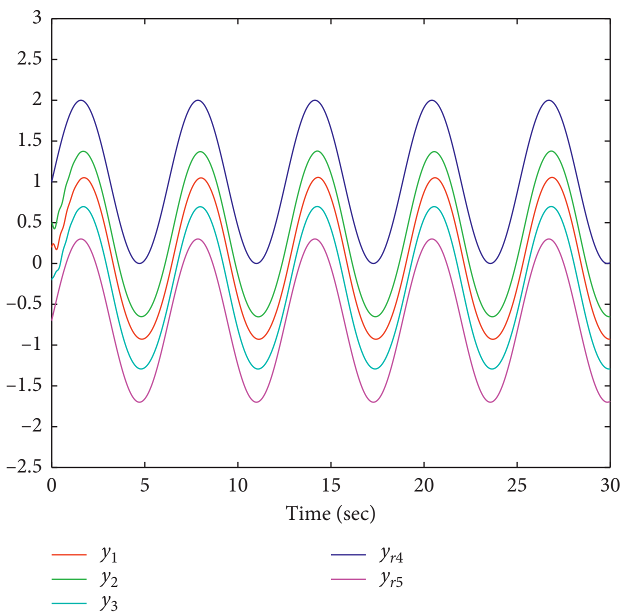

Figure 2: Curves of y_{rj} and y_τ.

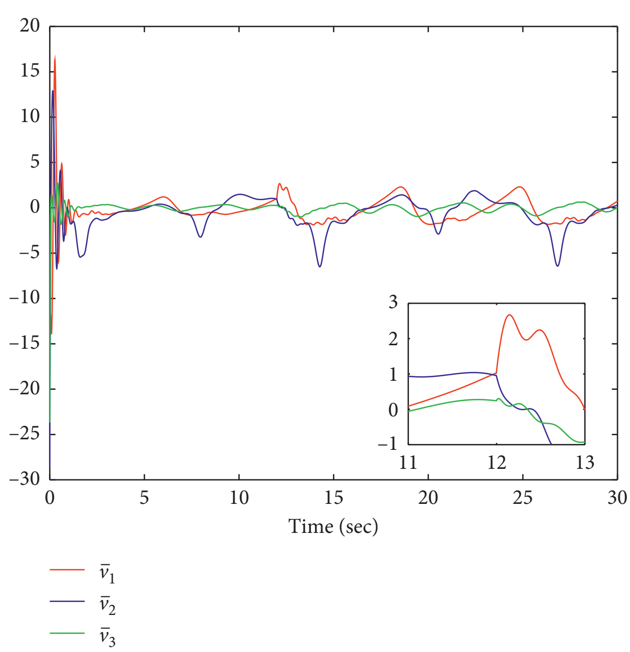

Figure 3: Curves of u_τ.

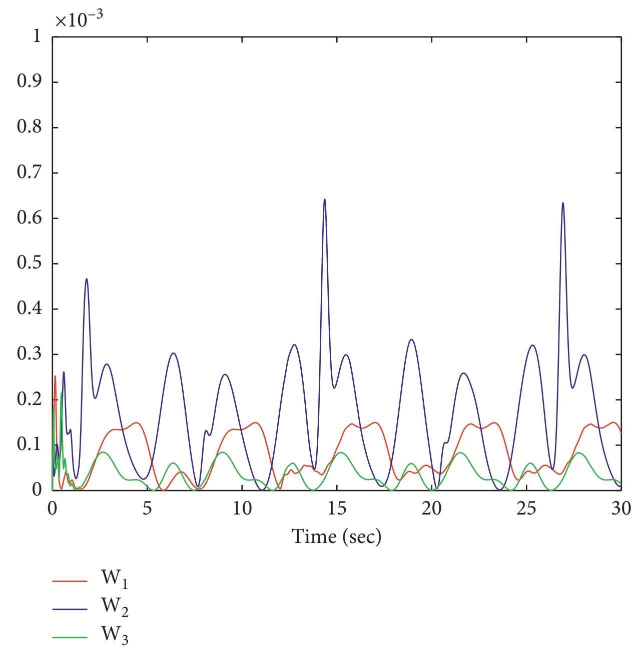

Figure 4: Curves of W_1, W_2, and W_3.

Through the proposed controller, the simulation results are given in Figures 2–4. Figure 2 denotes the curves of y_{rj} and $y_\tau\,(\tau = 1, 2, 3, j = 4, 5)$. From Figure 2, we can clearly see that all the followers F_1, F_2, and F_3 can converge to the convex hull built by the leaders L_4 and L_5. That is to say, all the followers are running among the leaders. Figure 3 depicts the curves of the controller u_τ. According to Figure 3, although the actuator fault appears at $t_f = 12$, the containment control performance of the proposed control method can still be guaranteed. Figure 4 demonstrates the curves of W_τ. Figures 2–4 show that the stability of every follower's system is guaranteed through the designed adaptive fuzzy backstepping containment controller. Besides, all followers are able to converge to the convex hull built by the leaders.

5. Conclusion

An adaptive fuzzy containment control method has been studied for nonlinear MASs with unknown control coefficient and actuator faults. The unknown control coefficient and the remaining control rate of actuator faults have been solved by introducing a Nussbaum-type function. The fuzzy logic system has been used as an approximator to approximate an unknown nonlinear function. The adaptive fuzzy containment controller has been designed by using the backstepping control technique and compensating signals. Only few adaptive parameters have been designed for each follower agent. It has been demonstrated that the designed adaptive fuzzy containment controller can ensure that all the signals in system are bounded and make all followers converge to the convex hull built by the leaders. In the future, we will investigate the adaptive cooperative controller design for practical application of nonlinear MASs. [44]

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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