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Research Article

The Risk Priority Number Evaluation of FMEA Analysis Based on Random Uncertainty and Fuzzy Uncertainty

Xiaojun Wu and Jing Wu

School of Economics and Management, Tongji University, Shanghai 200092, China

Correspondence should be addressed to Jing Wu; wujing@tongji.edu.cn

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The risk priority number (RPN) calculation method is one of the critical subjects of failure mode and effects analysis (FMEA) research. Recently, RPN research under a fuzzy uncertainty environment has become a hot topic. Accordingly, increasing studies have ignored the important impact of the random sampling uncertainty in the FMEA assessment. In this study, a fuzzy beta-binomial RPN evaluation method is proposed by integrating fuzzy theory, Bayesian statistical inference, and the beta-binomial distribution. This model can effectively realize real-time, dynamic, and long-term evaluation of RPN under the condition of continuous knowledge accumulation. The major contribution of the proposed model is to use the random uncertainty and fuzzy uncertainty in an integrated model and provide a Markov Chain Monte Carlo (MCMC) method to solve the complex integrated model. The study presented a case study, which presented how to apply this model in practice and indicated the significant influence on the measurement error caused by ignoring the random uncertainty caused by expert evaluation in RPN calculations.

1. Introduction

FMEA was initially developed as a formal design methodology in the 1960s in the aerospace industry [1]. At present, it has been widely adopted to improve the security and reliability of systems [2] and for continuous improvements in product or process design [3] in various fields, for instance, wind power [4], food [5], healthcare [6], fabrics [7], construction [8], healthcare [9], and mining [10].

The traditional FMEA analysis comprises five steps. First, a group of experts are to identify all possible potential failure modes of the product or system. Second, three risk factors are taken into account for each potential failure mode: the occurrence/probability of the failure (O), the severity of the consequences (S), and the chance/probability of the failure going undetected (D) [11]. In the third step, the three factors of severity, occurrence, and detectability are multiplied together to calculate the so-called risk priority number (RPN): $\mathrm{RPN} = S \times O \times D$. Fourth, the critical failure modes are identified based on RPN rankings. Finally, the continuous improvement activities are implemented to reduce the risk of failure modes.

Three risk factors were assessed using a 10-point scale to obtain RPNs for potential failure modes. For more information, please refer to the article [12].

Although the FMEA has been studied for nearly 60 years, the theory and method still have many shortcomings (see Literature Review). Thus, researchers have attempted to improve the traditional FMEA method from various aspects and have adopted different methods to make it more adaptable. The most important extension is the studies that consider the linguistic fuzzy uncertainty of expert evaluation in the RPN calculation. However, few studies have examined the effect of random uncertainty and linguistic fuzzy uncertainty in expert assessment on the results of FMEA evaluation when they act together.

This study presents a novel method that incorporates fuzzy and probabilistic theories to compute RPN in fuzzy and stochastic uncertainty environments. It regards the evaluation process of all experts for a specific factor (such as factor S) as a stochastic process that conforms to the beta-binomial distribution with $n = 10$. Furthermore, an expert's scoring result for this specific factor comes from a fuzzy linguistic evaluation. In this way, the fuzzy uncertainty and random uncertainty in the expert scoring process can be considered simultaneously to establish an integrated evaluation method.

The major contributions of the study are (1) to enrich the research literature on FMEA by considering both stochastic and fuzzy uncertainties simultaneously; (2) a fuzzy beta-binomial distribution evaluation method is proposed to precisely describe the experts' evaluation; (3) this study presents a method for solving complex RPN models containing random uncertainty and fuzzy uncertainty with Markov Chain Monte Carlo (MCMC) method. The theoretical and practical contributions of the study are discussed in detail in the final section.

The rest of this paper is organized as follows. Section 2 describes the main shortcomings of traditional FMEA and its improvements. In Section 3, we explain the theoretical underpinnings of the study, such as the linguistic fuzzy method, Bayes' theorem, and the beta-binomial distribution. Section 4 introduces the integrated approach, and Section 5 gives a case study. The results of the case study are discussed in detail in Section 6. Finally, we draw conclusions and make recommendations for future research in Section 7.

2. Literature Review

2.1. Traditional FMEA and Its Improvements. At present, FMEA is usually used in some service and manufacturing sectors to eliminate failures and potential problems by evaluating failure modes of new or existing products, processes, or systems [13].

Among all the review articles on FMEA research, one of the most representative articles [12] summarizes the main findings of FMEA during 2009–2012. Many scholars [14–17] have since reviewed the development of FMEA research in recent years. The major shortcomings of the traditional FMEA method [8, 12, 17–19] are shown in Table 1. In response to these shortcomings, researchers have proposed many improved methods, which are also shown in Table 1.

Five major categories were used in the article [12]: multicriteria decision making (MCDM), artificial intelligence (AI), mathematical programming, hybrid approaches, and others. Many later scholars [17] have also used this classification. By contrast, this study starts from the nine shortcomings of the traditional FMEA and gives a brief literature review.

To solve the first and second shortcomings of traditional FMEA research, scholars have improved the traditional FMEA method by adding weights to the three factors. In the article [28], the data envelopment analysis (DEA) was used in the study to determine the relative importance of risk factor weights. The study [20] used ordered weights and proposed a method to reprioritize the failure modes in FMEA by combining fuzzy OWA and DEMATEL methods. Similar studies also include the articles [2, 18, 22]. Fuzzy theory has recently attracted increasing attention in weight determination research. For instance, the intuitionistic fuzzy weighted averaging (IFWA) operator [21], the fuzzy AHP method in [29], the triangular intuitionistic fuzzy entropy method [30], the three-dimensional geometric approach to fuzzy weighted Euclidean (FWE) FMEA, the IVIF MULTIMOORA method [31], and even the integrated method by extended fuzzy AHP and fuzzy MULTIMOORA [32] are all these types of typical studies. Cost [4, 6, 7, 24], customer demand [33, 34], and quality [7] are integrated into FMEA to solve the third shortcoming in many studies. For the fourth shortcoming of traditional FMEA, the Bayesian network (BN) method [16, 35–39] is the most common solution, and other methods include fuzzy cognitive maps [1], FTA analysis [25], and DEMATEL [40–42]. To address the fifth shortcoming, scholars have proposed MCDM to evaluate the potential failure based on three criteria to avoid direct calculation of the RPN. The typical methods include TOPSIS [5, 26, 43], VIKOR [27, 44], MULTIMOORA [31], and DEA [45].

The main extensions of traditional FMEA take uncertainty into consideration to solve the sixth shortcoming. The following section presents a detailed explanation.

2.2. FMEA Research Based on Uncertainty Theory. Almost all systems cannot capture information perfectly, and some of the available information is uncertain due to limited knowledge and cognition [40]. In all kinds of engineering problems, uncertainty is inevitable [13]. Therefore, it is widely believed that risk factors O, S and D are not easy to be used for accurate evaluation [46]. Thus, in recent years, GRA, fuzzy set theory, rough set theory, Dempster-Shafer theory, and probability theory have been used in FMEA research.

2.2.1. Grey Relational Analysis (GRA). Grey relational analysis (GRA) is an important application of grey system theory pioneered by Professor Deng in 1982. GRA is a widely used uncertainty method and has been used to study the FMEA factors. For instance, the integrated GRA and the DEMATEL method to rank the risk of failure were proposed in the study [40]. MCDM in combination with grey theory was studied [18]. In another paper [47], the uncertain information called D numbers and an improved GRA method were proposed for risk evaluation in FMEA.

2.2.2. Fuzzy Set Theory. Because human beings are more accustomed to the direct usage of language variables [48], big efforts have been taken to evaluate risk factors in a linguistic manner [46, 49]. The imprecise, vague, or partially true information was addressed by using the fuzzy set theory [14]. There are so many studies [3, 50, 51] that have combined FMEA with fuzzy sets to handle the weaknesses of the traditional RPN method.

2.2.3. Integrated Fuzzy Methods. Many new methods for expanding FMEA research have emerged from combining fuzzy set theory with other research methods. For instance, the combination of fuzzy set theory and GRA is typical [52–54]. In addition, fuzzy MULTIMOORA [32], fuzzy DEMATEL [55], and an intuitionistic fuzzy approach [30]

TABLE 1: Deficiencies of traditional FMEA and measures to improve it.

No.	The shortcoming of the traditional FMEA	Improvements and representative articles
1	The relative importance between *O*, *S* and *D* was not considered. It is assumed that these three factors are of equal importance, but this may not be the case when considering the practical application of FMEA.	Weights are assigned to three factors based on various weighting methods, such as OWA [20], IFWA [21], BWM [22], and FWE [23].
2	Different *O*, *S* and *D* rating sets may produce exactly the same RPN values, but their hidden risk implications may be completely different. This issue may result in wasted resources and time, or, in some cases, high-risk failure modes were not widely known.	The introduction of factor weights reduces and avoids the confusion caused by the same RPN results in different failure modes.
3	RPN calculation considers only three risk factors, mainly safety, and ignores other important factors such as quality and cost.	Cost [4], quality [7], and other factors [24] are added to improve the theoretical basis of the RPN evaluation.
4	The RPN approach does not consider the direct/indirect relationship between failure modes and is flawed for systems with many subsystems and components. When one failure causes several other failure modes, that failure should be prioritized for corrective action.	The FTA [25], Bayesian network [16], and other methods are used to present the interactions and relationships of various failures.
5	The three risk factors *O*, *S*, and *D* are evaluated on a discrete ordinal scale. However, the multiplication is not meaningful on the ordinal scale. Thus, the results obtained are not only meaningless, but also in fact misleading.	Few articles discuss the ordinal scale and multiplication issues. Alternatively, MCDM methods, such as TOPSIS [26] and DEMATEL [27], are used to prioritize the failure modes directly.
6	The three risk factors are often difficult to determine accurately. FMEA team members often provide different types of assessment information for the same risk factor, and some of the assessment information may be inaccurate, uncertain, and incomplete due to time constraints, inexperience, and insufficient data.	Introduce uncertainty assessment methods, such as fuzzy theory, rough theory, evidence theory, and probability theory into the FMEA analysis (see Section 2.2).
7	The mathematical form used to calculate RPN is very sensitive to changes in the assessment of risk factors.	Few articles discuss this issue.
8	The rating transitions for the three components of the FM are different. The relationship between the probability table for *O* and *O* is nonlinear, whereas the relationship between the probability table for *D*(*S*) and *D*(*S*) is linear.	Few articles discuss this issue.
9	The results of RPNs are discrete, and many holes are there.	Few articles discuss this issue.

are proposed to evaluate FMEA. In another interesting study [56], *S* and *D* are obtained from fuzzy rules, and *O* is obtained from an artificial neural network.

Although popular, many controversies about the use of the fuzzy set theory method exist. For instance, appropriate member functions for risk factors and priorities are difficult to define [12]. In fuzzy set processing, when defuzzification is done to calculate the final ordering of the failure modes, decisions based on the crisp analogues of a fuzzy set ignore the entropy [50]. Therefore, such rule-based approaches are often too subjective, expensive, and time-consuming and may not be the best method [17].

2.2.4. Rough Set Theory. Rough sets are another important method for studying uncertainty. The studies [57, 58] integrated the rough set theory and the TOPSIS to evaluate the risk of failure modes. Another study [59] integrates the rough set theory and the cloud model theory in FMEA analysis.

2.2.5. Dempster-Shafer Theory (DST). Recently, an increasing number of researchers have applied DST to FMEA. In the study [13], a risk-based fuzzy evidential approach was put forward to use the interval-valued DST and fuzzy axiomatic design to assess the risk of failure modes. Similar studies also include [54, 60].

2.2.6. Probability Theory. The BN is an uncertainty reasoning method that is most effective for failure structure analysis with a discrete probability table. The method does not solve the randomness of finite sampling from a population. Specifically, in FMEA analysis, the evaluation of a factor by experts is a sampling process. Although probabilistic risk analysis is an important topic in quality management and reliability research, random uncertainty is widely ignored by FMEA researchers.

On the other hand, the FMEA results remain static and are not updated with new failure knowledge [61]. Bayesian inference provides a way to dynamically evaluate RPN. In the study [62], based on the FMEA results, the cartridge and mechanical parts components are identified as principal contributors and analyzed meticulously. Data processing techniques within the framework of Bayesian inference are developed to achieve data format consistency in data aggregation and to perform uncertainty analysis using the Monte Carlo approach. Similar studies also include [8].

Furthermore, with the development of the mathematical science, some more instructive methods are put forward to evaluate the risk. In the study [63], a novel uncertain risk index model is presented, which can be used in RPN calculation.

2.2.7. Z-Number Theory. Since Zadeh [64] proposed to use Z-numbers to express uncertainty in 2011; the Z-numbers theory has gained more and more attention [65, 66]. For example, the linguistic Z-numbers theory [67] combining linguistic term sets and Z-numbers was proposed by Wang et al. and Huang et al. The study [68], combined with Projection Method, proposes a new FMEA model. Similar studies also include [64, 69].

2.3. Summary. The above literature review shows that the shortcomings of the traditional FMEA method are partially solved by the following methods: (1) introduction of factor weights into the RPN calculation; (2) analysis of failure mode relationships and structures; (3) introduction of new factors, such as quality and cost; and (4) fuzzy expression and calculation of RPN factors and weights. However, many remaining problems require additional research. In particular, problems 7, 8, and 9 in Table 1 have received minimal attention. In addition, the linguistic fuzziness of expert scoring has been extensively and intensively studied, but the sampling randomness of expert scoring has not received sufficient attention. Therefore, a fuzzy beta-binomial RPN evaluation method that integrates fuzzy set theory, Bayesian statistical inference, and the beta-binomial distribution is proposed in the following section. This study considers both the randomness and ambiguity in expert evaluation, as well as dynamic changes, to address the shortcomings of traditional FMEA.

3. Preliminaries

Some relevant theories and methods will be introduced briefly in this section in order to derive the subsequent method.

3.1. Linguistic Variables and Defuzzification. In many real situations, data uncertainty comes from two different sources: randomness and vagueness [14]. In FMEA research, the factors O, S, and D are usually evaluated in a linguistic manner [49]. A typical example [19] of using linguistic variables to express O, S, and D is shown in Table 2.

The evaluation results of the three factors O, S, and D expressed by fuzzy numbers should be converted to a defuzzified value. Several methods are used for defuzzification. The centroid method is common, and for a triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$, the defuzzified value $\overline{x}_0(\tilde{a})$ is expressed by the following equation [19]:

$$\overline{x}_0(\tilde{a}) = \frac{1}{3}(a_1 + a_2 + a_3). \tag{1}$$

Consider a fuzzy set $\widetilde{A}$. The α-cut set can be denoted by $\widetilde{A}[\alpha]$, where $0 \leq \alpha \leq 1$. The set $\widetilde{A}[\alpha]$ is a crisp set called the α-cut set of fuzzy set $\widetilde{A}$.

Table 2: Linguistic variables for rating failure modes.

Linguistic variable	Triangular fuzzy number
Very low (VL)	(0, 0, 1)
Low (L)	(0, 1, 3)
Medium low (ML)	(1, 3, 5)
Medium (M)	(3, 5, 7)
Medium high (MH)	(5, 7, 9)
High (H)	(7, 9, 10)
Very high (VH)	(9, 10, 10)

$$\widetilde{A}[\alpha] = \{x \mid \mu_A(x) \geq \alpha\}, \quad 0 \leq \alpha \leq 1, \tag{2}$$

$\mu_A(x)$ expresses the membership function of $\widetilde{A}$. For triangular fuzzy numbers, $\widetilde{A}[1] = a_2$ and $\widetilde{A}[0] = [a_1, a_3]$.

3.2. Bayes' Theorem and Posterior Distribution. In FMEA research, the evaluation results of the three factors S, O, and D are usually obtained by expert scoring, and each evaluation result is a discrete value distributed in the interval of 0–10 (for a fuzzy related study, it may be 0–10, as shown in Table 2). This evaluation result is analogous to the beta-binomial distribution with $n = 10$. For example, an evaluation result of 3 is analogous to the success of 3 of 10 Bernoulli experiments. The binomial distribution with parameters n and θ is the discrete probability distribution of the number of successes z in a sequence of n independent Bernoulli experiments, and θ is the probability of success for each trial. The formula of the binomial distribution is

$$P(X = z \mid \theta, n) = \binom{n}{z}\theta^z(1-\theta)^{n-z}. \tag{3}$$

Since each expert is different, the θ values may differ to illustrate experts' uniqueness. If θ is not a fixed value but a random variable that conforms to the beta distribution, then the distribution is a beta-binomial distribution. The letters a and b are parameters of the Beta distribution.

$$p(\theta) = \text{Beta}(a, b) = \frac{\theta^{(a-1)}(1-\theta)^{(b-1)}}{B(a,b) \propto \theta^{(a-1)}(1-\theta)^{(b-1)}}, \tag{4}$$

where $B(a, B)$ is a simple normalized constant, which can ensure that the area under the beta density integrates to one. In other words, the normalizer for the beta distribution is the beta function.

$$B(a,b) = \int_0^1 \mathrm{d}\theta\theta^{(a-1)}(1-\theta)^{(b-1)}. \tag{5}$$

Therefore, we can use the analogy between the beta-binomial distribution and expert scoring to express the randomness of the expert scoring process.

Since the characteristics of each expert are not known in advance, the a priori information of $p(\theta)$ is lacking. Given a sufficiently large amount of data, two or more Bayesian models with different priors will tend to converge to the same result. If we have no prior knowledge, we can use flat priors that do not convey much information. Therefore, the prior distribution of such information is $\mathrm{Beta}(\alpha = 1, \beta = 1)$. After expert scoring, we obtain new information about each factor, and we can use Bayesian inference to update the knowledge and obtain the posterior probability $p(\theta \mid z, N)$. The relationship between the prior distribution and the posterior distribution can be expressed by Bayes' theorem:

$$p(\theta \mid y) = \frac{p(y \mid \theta)p(\theta)}{p(y)}. \tag{6}$$

When $p(\theta) = \mathrm{Beta}(a, b)$ and $p(y \mid \theta)$ is a binomial distribution, the posterior distribution calculation process is as follows:

$$\begin{aligned}
p(\theta \mid z, n) &= \frac{p(z, n \mid \theta)p(\theta)}{p(z, n)} \\
&= \frac{\binom{n}{z}\theta^{z}(1-\theta)^{(n-z)}\left(\theta^{(a-1)}(1-\theta)^{(b-1)}/B(a,b)\right)}{p(z,n)} \\
&= \binom{n}{z}\frac{\theta^{z}(1-\theta)^{(N-z)}\theta^{(a-1)}(1-\theta)^{(b-1)}}{p(z,n)B(a,b)} \\
&= \binom{n}{z}\frac{\theta^{(z+a-1)}(1-\theta)^{((N-z+b)-1)}}{p(z,n)B(a,b)} \\
&= \binom{n}{z}\frac{\theta^{(z+a-1)}(1-\theta)^{((n-z+b)-1)}}{B(z+a, n-z+b)} \\
&\propto \mathrm{Beta}(z+a, n-z+b).
\end{aligned} \tag{7}$$

Therefore, the posterior distribution $p(\theta \mid z, n)$ is a beta distribution with parameters $(z + a, n - z + b)$. In this way, the information about θ is updated, and we can further calculate the beta-binomial distribution with the updated θ.

3.3. Beta-Binomial Distribution. In the case of $p(\theta) = \mathrm{Beta}(a, b)$, the beta-binomial distribution is denoted as

$$f(k \mid nt, naq, hb) \sim \mathrm{Beta} \sim \mathrm{binomial}(n, a, b). \tag{8}$$

This formula indicates that the Bernoulli experiment is performed n times, and the probability of k successes is obtained.

$$\begin{aligned}
f(k \mid n, a, b) &= \int_0^1 L(\theta \mid k)p(\theta \mid a, b)\mathrm{d}\theta \\
&= \binom{n}{k}\theta^{k}(1-\theta)^{n-k}\cdot\frac{\theta^{(a-1)}(1-\theta)^{(b-1)}}{B(a,b)} \\
&= \binom{n}{k}\frac{B(k+a, n-k+b)}{B(a,b)}
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
p(\theta \mid a, b) &= \mathrm{Beta}(a, b) \\
&= \frac{\theta^{(a-1)}(1-\theta)^{(b-1)}}{B(a,b)}, \quad \text{for } 0 \le \theta \le 1, \\
P(X = k \mid \theta, n) &= L(\theta \mid k) = \binom{n}{k}\theta^{k}(1-\theta)^{n-k}.
\end{aligned} \tag{10}$$

A new posterior distribution is plugged into (9).

$$f(k \mid n, a, b) \sim \mathrm{Beta} \sim \mathrm{binomial}(n, k+z+a, N-z-k+b). \tag{11}$$

That is, the probability of the score k of a certain factor is consistent with the above distribution, where $n = 10$, $N = \text{expert number} \times 10$, and z is the sum of all experts' evaluation scores for a certain factor.

3.4. The Operation Laws of Fuzzy Power. The operation laws of fuzzy power numbers are shown as follows:

$$\begin{aligned}
\theta^{\tilde{z}}\theta^{\tilde{y}} &= \theta^{\tilde{z}+\tilde{y}}, \\
\frac{\theta^{\tilde{z}}}{\theta^{\tilde{y}}} &= \theta^{\tilde{z}-\tilde{y}}, \quad \theta \ne 0, \\
\left(\theta^{\tilde{z}}\right)^{\tilde{y}} &= \theta^{\tilde{z}\cdot\tilde{y}}, \\
(\alpha\cdot\beta)^{\tilde{z}} &= \alpha^{\tilde{z}}\beta^{\tilde{z}}.
\end{aligned} \tag{12}$$

The $\tilde{z}$ and $\tilde{y}$ are fuzzy numbers. These operation laws are referred to when the mathematical reasoning is done for the fuzzy beta-binomial distribution in the following section.

4. The Proposed Method

4.1. Fuzzy Beta-Binomial Approach for SOD Evaluation. When the experts score the S, O and D factors separately, the results are usually ambiguous and can be expressed as fuzzy numbers due to the use of linguistic variables. As mentioned earlier, the S, O, D scoring process can be analogized to a binomial experiment with $n = 10$, and the expert scoring result z is a fuzzy number $\tilde{z}$.

$$\widetilde{P}(X = \tilde{z} \mid \theta, n) = \binom{n}{z}\theta^{\tilde{z}}(1-\theta)^{n-\tilde{z}}, \quad 0 \le \alpha \le 1. \tag{13}$$

Based on the fuzzy algebra, we obtain

$$\widetilde{P}(X = \tilde{z} \mid \theta, n)[\alpha] = \left\{\binom{n}{z}\theta^{\tilde{z}}(1-\theta)^{n-\tilde{z}}\right\}. \tag{14}$$

If $\widetilde{P}(X = \tilde{z} \mid \theta, n)[\alpha] = [P_{z1}(\alpha), P_{z2}(\alpha)]$ then

$$P_{z1}(\alpha) = \min\left\{\binom{n}{z}\theta^{\tilde{z}}(1-\theta)^{n-\tilde{z}}\right\}, \tag{15}$$

and

$$P_{z2}(\alpha) = \max\left\{\binom{n}{z}\theta^{\tilde{z}}(1-\theta)^{n-\tilde{z}}\right\}. \tag{16}$$

In this case, $\tilde{z}$ is expressed as a fuzzy linguistic data shown as in Tables 3 and 4. For any specific expert, the value of $\tilde{z}$ can be substituted into the formula of $P_{z1}(\alpha)$ and $P_{z2}(\alpha)$ to calculate $\widetilde{P}(X = \tilde{z} \mid \theta, n)$.

The binomial distribution is a likelihood function. Based on the prior distribution $\mathrm{Beta}(a, b)$ and binomial likelihood functions, the posterior distribution of the expert evaluation can be written as

$$\begin{aligned}\tilde{p}(\theta \mid z, N) &= \frac{\tilde{p}(\tilde{z}, tN \mid n\theta)p(\theta)}{\tilde{p}(\tilde{z}, tN)}\\ &= \frac{\binom{n}{\tilde{z}}\theta^{\tilde{z}}(1-\theta)^{n-\tilde{z}}\left(\theta^{(a-1)}(1-\theta)^{(b-1)}/B(a,b)\right)}{\tilde{p}(\tilde{z}, N)}\\ &= \binom{n}{\tilde{z}}\frac{\theta^{\tilde{z}}(1-\theta)^{N-\tilde{z}}\theta^{(a-1)}(1-\theta)^{(b-1)}}{\tilde{p}(\tilde{z}, N)B(a,b)}\\ &= \binom{n}{\tilde{z}}\frac{\theta^{(\tilde{z}+a-1)}(1-\theta)^{((N-\tilde{z}+b)-1)}}{\tilde{p}(\tilde{z}, N)B(a,b)}.\end{aligned} \tag{17}$$

Since the conjugate distribution of the beta distribution is a binomial distribution, its posterior distribution is also a beta distribution. From (17), after discarding the irrelevant terms of θ, we obtain

$$\begin{aligned}&\tilde{p}(\theta \mid z, N)\\ &\propto \theta^{(\tilde{z}+tan-q1)}(1-\theta)^{((N-\tilde{z}+b)-1)}\\ &\propto \mathrm{Beta}(\tilde{z}+a, N-\tilde{z}+b).\end{aligned} \tag{18}$$

Under the posterior distribution θ, the final evaluation results for SOD are

$$f(k \mid n, a', b') = \binom{n}{k}\frac{B\left(k+\tilde{a}', n-k+\tilde{b}'\right)}{B(a', b')}, \tag{19}$$

where $n = 10$, $\tilde{a}' = \tilde{z} + a$, and $\tilde{b}' = \tilde{z} + b$. This result can be rewritten as

$$f(\tilde{k} \mid n, a, b) \sim \mathrm{Beta}-\mathrm{binomial}(n, \tilde{z}+k+a, N-\tilde{z}-k+b). \tag{20}$$

4.2. The Proposed Evaluation Process. The integrated proposed approach based on the fuzzy beta-binomial distribution and Bayesian inference is shown in Figure 1 and described below.

Step 1. An expert group that consists of n people is established to evaluate the SOD factors and obtain the evaluation value of the jth factor by the ith expert. The prior probability distribution is Beta (1, 1).

Step 2. According to Bayes' theorem, combined with the known prior distribution Beta (1, 1) and binomial distribution likelihood function, the posterior distribution obtained is proportional to $\mathrm{Beta}(\tilde{z}+1, N-\tilde{z}+b)$, where $N = 10n$, $\tilde{z} = \sum_{i=1}^{n}\tilde{x}_{ij}$.

Step 3. Calculate the theoretical value of the expert evaluation based on the beta-binomial distribution with θ posterior distribution $\mathrm{Beta}(\tilde{z}+1, N-\tilde{z}+1)$:

$$f(\tilde{k} \mid n, a, b) \sim \mathrm{Beta}-\mathrm{binomial}(n, \tilde{z}+k+1, N-\tilde{z}-k+1). \tag{21}$$

Thus, we can obtain SOD factors' theoretical values.

Step 4. By using MCMC, we can take samples from each of the SOD theoretical distribution and calculating the $\mathrm{RPN} = S \times O \times D$; furthermore, the combined distribution of RPN can be gotten. We can prioritize the failure modes based on the combined distribution.

Overall, the first step is to collect expert evaluations of the severity (S), occurrence (O), and probability of the failure going undetected (D) in the form of fuzzy linguistic expressions (see the case study, Tables 3 and 5). In the second step, the expert evaluation is regarded as a random process, and the randomness is presented by using beta-binomial distribution for each of the SOD factors. In the third step, MCMC method is used to get the samples from each of the distributions of S, O and D factors. Furthermore, the RPN can be calculated based on the samples from three beta-binomial distributions. The final priority of each failure is

Table 3: The original data set.

—	O					S					D				
	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5
FM1	M	M	M	MH	M	ML	ML	ML	M	M	M	ML	ML	ML	ML
FM2	H	MH	H	MH	MH	H	MH	H	H	H	M	M	ML	M	M
FM3	VH	MH	VH	VH	VH	MH	MH	MH	MH	MH	MH	M	MH	MH	M
FM4	M	M	L	M	M	M	M	ML	M	M	VL	ML	VL	ML	VL
FM5	M	ML	M	M	M	M	MH	MH	M	M	L	ML	L	L	L
FM6	MH	H	M	MH	M	H	H	H	H	H	L	M	L	L	VL
FM7	ML	L	ML	ML	ML	VH	H	H	VH	H	VL	VL	VL	L	VL

Note: DM1 means the first expert's measurement. DM2, DM3, DM4, DM5 are in the same fashion. This table is reproduced from [19], [under the Creative Commons Attribution License/public domain].

determined by comparing the parameters of the RPNs of each of the failures.

5. Case Study

The following section is a case study using four different approaches shown in Table 5 and the data set used in the article [19]. The original evaluation data set is shown in Table 3, in which there are seven failure modes and five evaluation experts.

According to the linguistic variables in Table 2, the original data set can be further expressed in fuzzy numbers, as shown in Table 4.

After calculating the θ posterior distribution, the SOD factors' fuzzy numbers of each failure mode under α-cut $\alpha = 1$ are obtained, and the corresponding beta-binomial distribution is shown in Table 6.

After sampling (number of samples = 20000), the RPN of each of the failure modes is calculated under $\alpha = 1$ and shown in Table 7.

After sampling (number of samples = 50000), the RPN result of each failure mode under $\alpha = 0$ is calculated and shown in Table 8.

A comparison with the evaluation results of the literature [19] is shown in Table 9.

The detailed comparisons of four approaches are in the following section.

6. Discussion

It can be seen from Table 8 that the ranking of the proposed method differs from that of traditional FMEA and fuzzy TOPSIS for only two failure modes, but the ranking of Liu's model approach differs considerably from the results of the proposed method. This difference is due to two main reasons. First, the proposed method and the traditional FMEA method are based on the result of $\mathrm{RPN} = S \times O \times D$, whereas Liu's model and fuzzy TOPSIS are based on an MCDM method. The second reason is the effect of weights on the results in the calculation process.

The only difference in the results of the proposed method and the traditional FMEA is the ranking of FM4 and FM7 because the traditional FMEA method's SOD factors have a scoring range of 1–10, whereas the fuzzy set data may have a value of 0. In addition, the traditional FMEA method calculates the SOD factor scores (generally taking a positive integer). In the calculation process, the data are not forced to take an integer value (for example, a fractional part may exist after the mean is calculated). The above two reasons make the calculation results of FM4 and FM7 slightly different after considering randomness and ambiguity.

The proposed approach is based on $\mathrm{RPN} = S \times O \times D$, which conforms to the traditional FMEA method, and the understanding is simple and easy. Furthermore, because O is the probability of the failure, S is the severity of the failure, and D is the probability of not detecting the failure, although SOD is not a true probability value, the values can be converted by means of a specific method from the original data. A one-to-one correspondence exists with the actual probability values, so there is a function mapping relationship between the evaluation value of the SOD and the real value. The SOD data of different dimensions and measurement scales are uniformly converted into discrete values in 1–10, just as the data of different dimensions and measurement scales are standardized and converted into data in the interval 0–1. This is simply a process of data standardization; therefore, the direct multiplication of O and D after standardization has practical meaning. If the S factor is regarded as a weighting factor of failure severity, it can be multiplied by O and D to express the severity of a failure mode. The result of the multiplication of S and $O \times D$ expresses the urgency of improving the failure based on the failure probability and detection difficulty of a certain failure mode. Thus, the calculation of $S \times O \times D$ also has practical meaning. Although there are many criticisms of the method of sorting failure mode priorities by calculating $\mathrm{RPN} = S \times O \times D$ for discrete ordinal scales of measure, the basic idea of resorting RPN by multiplying the three factors has a strong theoretical basis. In terms of the practical meaning of $\mathrm{RPN} = S \times O \times D$, the additional physical weight of each factor is not reasonable in some sense.

Fuzzy uncertainty and random uncertainty are considered in the evaluation of the SOD factors of each failure mode. From the calculation results, the fuzzy numbers of each factor are consistent with the ranking results of $\alpha = 1$ and $\alpha = 0$. From the perspective of a simplified calculation in the future, it is sufficient to do the calculation considering only $\alpha = 1$. Moreover, the graph shows that the seven failure modes can be divided into three groups. The distribution patterns of FM2 and FM3 are significantly different from

Table 4: Fuzzy expression of the original data set.

	O					S					D				
—	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5
FM1	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(1, 3, 5)	(1, 3, 5)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(1, 3, 5)	(1, 3, 5)	(1, 3, 5)	(1, 3, 5)
FM2	(7, 9, 10)	(5, 7, 9)	(7, 9, 10)	(5, 7, 9)	(5, 7, 9)	(7, 9, 10)	(5, 7, 9)	(7, 9, 10)	(7, 9, 10)	(7, 9, 10)	(3, 5, 7)	(3, 5, 7)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)
FM3	(9, 10, 10)	(5, 7, 9)	(9, 10, 10)	(9, 10, 10)	(9, 10, 10)	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)
FM4	(3, 5, 7)	(3, 5, 7)	(0, 1, 3)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)	(0, 0, 1)	(1, 3, 5)	(0, 0, 1)	(1, 3, 5)	(0, 0, 1)
FM5	(3, 5, 7)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)	(0, 1, 3)	(1, 3, 5)	(0, 1, 3)	(0, 1, 3)	(0, 1, 3)
FM6	(5, 7, 9)	(7, 9, 10)	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(7, 9, 10)	(7, 9, 10)	(7, 9, 10)	(7, 9, 10)	(7, 9, 10)	(0, 1, 3)	(3, 5, 7)	(0, 1, 3)	(0, 1, 3)	(0, 0, 1)
FM7	(1, 3, 5)	(0, 1, 3)	(1, 3, 5)	(1, 3, 5)	(1, 3, 5)	(9, 10, 10)	(7, 9, 10)	(7, 9, 10)	(9, 10, 10)	(7, 9, 10)	(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	(0, 1, 3)	(0, 0, 1)

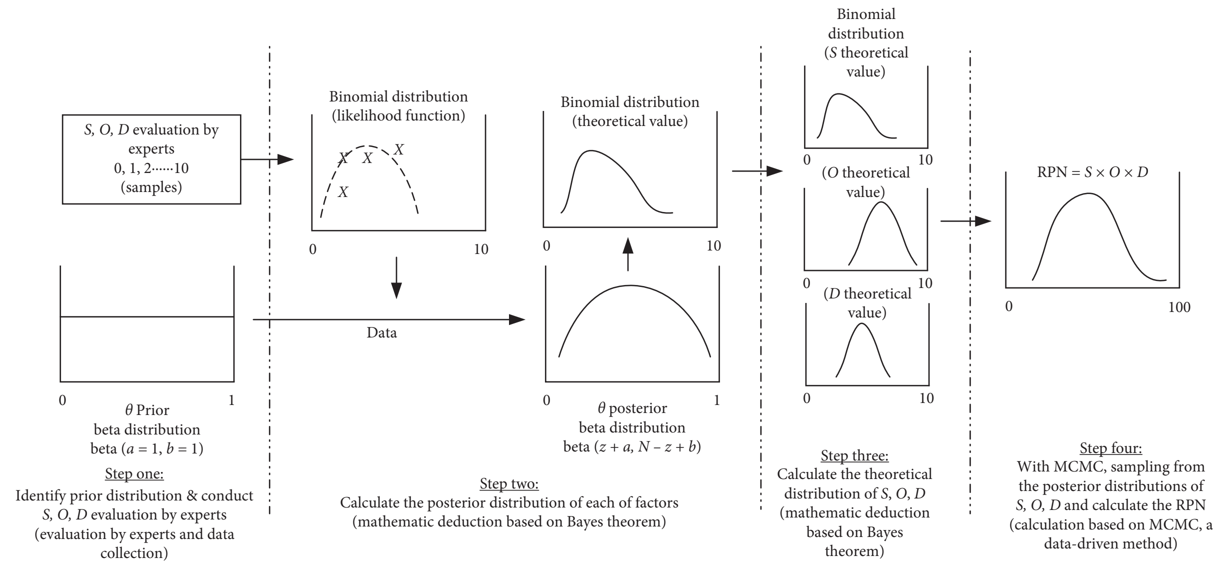


Figure 1: The evaluation process.

Table 5: The comparison between different methods.

Method or model	Expert evaluation	RPN calculation
The proposed method	Fuzzy linguistic expression and regard it as a random process	To express S, O, D with three different beta-binomial distributions; MCMC is used to get samples from three distribution and calculate the corresponding RPN
Liu's model	Fuzzy linguistic expression	Defuzzification, convert the fuzzy expressions to crisp ones and calculate the RPN
Traditional FMEA	Crisp expression	To calculate the RPN directly
Fuzzy TOPSIS	Fuzzy linguistic expression	Defuzzification, convert the fuzzy expressions to crisp ones and use the TOPSIS to evaluate the priorities of various failures

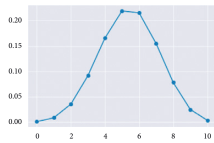
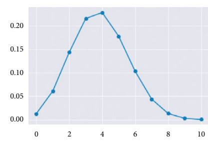
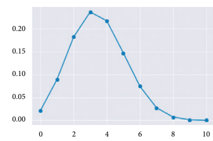
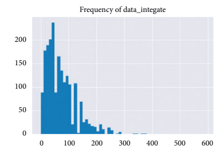
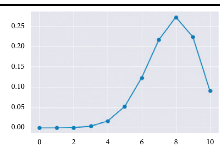
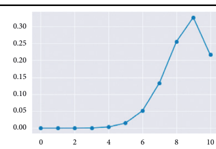
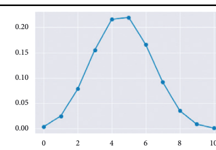
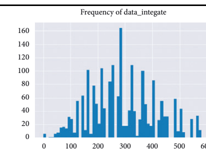
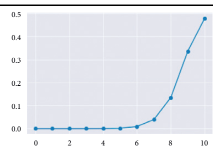
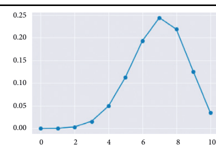
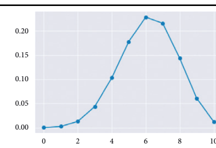
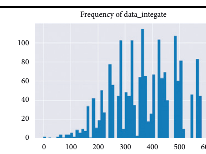
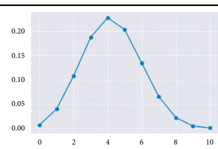
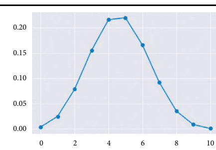
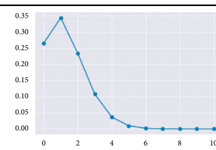
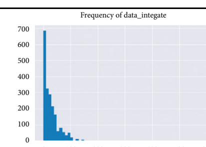
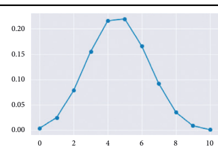
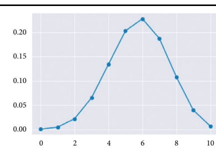
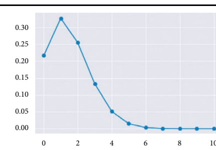
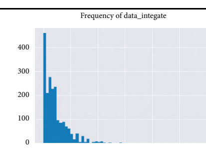
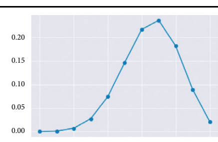
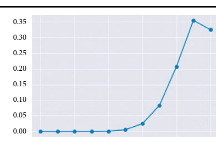
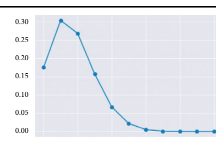
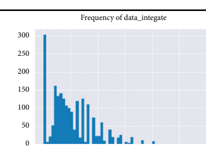
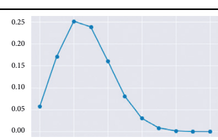
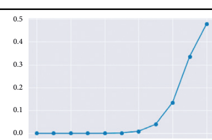
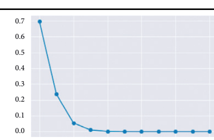
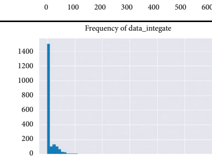
Table 6: The fuzzy beta-binomial distribution with $\alpha = 1$.

	O	$\tilde{O}[1]$	$\tilde{p}(\theta\|z,N)[1]$	S	$\tilde{S}[1]$	$\tilde{p}(\theta\|z,N)[1]$	D	$\tilde{D}[1]$	$\tilde{p}(\theta\|z,N)[1]$
FM1	(17, 27, 37)	$\tilde{O}[1] = [27]$	Beta-binomial (28, 24)	(9, 19, 29)	$\tilde{S}[1] = [19]$	Beta-binomial (20, 32)	(7, 17, 27)	$\tilde{D}[1] = [17]$	Beta-binomial (18, 34)
FM2	(29, 39, 47)	$\tilde{O}[1] = [39]$	Beta-binomial (40, 12)	(33, 43, 49)	$\tilde{S}[1] = [43]$	Beta-binomial (44, 8)	(13, 23, 33)	$\tilde{D}[1] = [23]$	Beta-binomial (24, 28)
FM3	(41, 47, 49)	$\tilde{O}[1] = [47]$	Beta-binomial (48, 4)	(25, 35, 45)	$\tilde{S}[1] = [35]$	Beta-binomial (36, 16)	(21, 31, 41)	$\tilde{D}[1] = [31]$	Beta-binomial (32, 20)
FM4	(12, 21, 31)	$\tilde{O}[1] = [21]$	Beta-binomial (22, 30)	(13, 23, 33)	$\tilde{S}[1] = [23]$	Beta-binomial (24, 28)	(2, 6, 13)	$\tilde{D}[1] = [6]$	Beta-binomial (7, 45)
FM5	(13, 23, 33)	$\tilde{O}[1] = [23]$	Beta-binomial (24, 28)	(19, 29, 39)	$\tilde{S}[1] = [29]$	Beta-binomial (30, 22)	(1, 7, 17)	$\tilde{D}[1] = [7]$	Beta-binomial (8, 44)
FM6	(23, 33, 42)	$\tilde{O}[1] = [33]$	Beta-binomial (34, 18)	(35, 45, 50)	$\tilde{S}[1] = [45]$	Beta-binomial (46, 6)	(3, 8, 17)	$\tilde{D}[1] = [8]$	Beta-binomial (9, 43)
FM7	(4, 13, 23)	$\tilde{O}[1] = [13]$	Beta-binomial (14, 38)	(39, 47, 50)	$\tilde{S}[1] = [47]$	Beta-binomial (48, 4)	(0, 1, 7)	$\tilde{D}[1] = [1]$	Beta-binomial (2, 50)

those of the other failure modes. The patterns of FM1 and FM6 are extremely similar, and the average and discrete degree of FM4, FM5, and FM7 are similar.

Overall, the proposed model in the study has two important advantages when compared with Liu's model and the fuzzy TOPSIS approach: (1) defuzzification is no longer used for the processing of fuzzy data, so more information about the original data can be preserved; and (2) even more complex models including more factors and weights can be processed to calculate the RPN. Thus, the proposed model

Table 7: The RPN under $\alpha = 1$.

	Factor O	Factor S	Factor D	RPN	Results	Ranking
FM1					Mean = 72.3 SD = 56.2 Median = 60.0	4
FM2					Mean = 302.1 SD = 135.8 Median = 288	2
FM3					Mean = 397.3 SD = 147.3 Median = 392	1
FM4					Mean = 25.9 SD = 29.8 Median = 18	6
FM5					Mean = 40.2 SD = 41.6 Median = 30	5
FM6					Mean = 102.3 SD = 84.9 Median = 81	3
FM7					Mean = 9.1 SD = 9.1 Median = 0	7

Note: SD means the standard deviation.

provides a more accurate assessment of the four different approaches in the study.

When considering the expert evaluation process as a random sampling process, the basic theory of applied statistics provides a means to evaluate the confidence of expert evaluation results. Subtle changes in the SOD evaluation values of the five experts have a considerable impact on the final RPN calculation results and the final prioritization. One of the most important reasons is that the number of experts in the case study is small. One way to solve this problem is to increase the number of experts to improve the robustness of the final evaluation results. When considering random uncertainties, the instability of the result is more easily explained by the deviation of the population mean and the sample mean. The estimation of the population mean is obtained by calculating the sample mean. When the population standard deviation is unknown, we have

$$\mu = \overline{x} \pm t \frac{s}{\sqrt{n}}, \quad (22)$$

where μ is the population mean, $\overline{x}$ is the sample mean, s is the population standard deviation $s = \sqrt{\left(\sum (x_i - \overline{x})^2 / (n-1)\right)}$, n is the sample size, and t is the standard normal value corresponding to the desired level of confidence. E is the maximum allowable error. The deviation E between the population mean and the sample mean is

$$E = t \frac{s}{\sqrt{n}}. \quad (23)$$

Thus, the number of samples n can be obtained as

$$n = \left(\frac{ts}{E}\right)^2. \quad (24)$$

Table 8: The RPN under $\alpha = 0$.

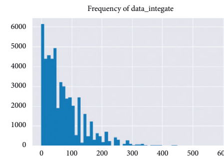
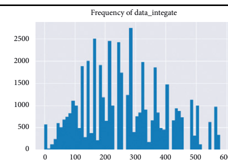
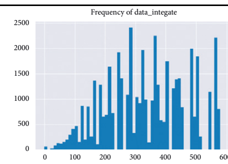
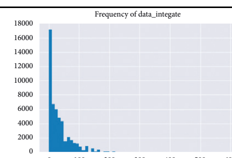
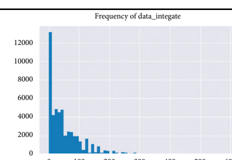
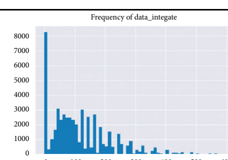
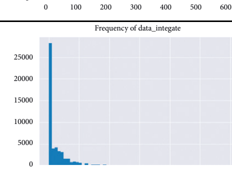
	$\alpha = 0$	RPN	Results	Ranking
FM1	$\tilde{O}[0] = [17, 37]$ $\tilde{S}[0] = [9, 29]$ $\tilde{D}[0] = [7, 27]$		Mean = 72.0 SD = 68.6 Median = 54.0	4
FM2	$\tilde{O}[0] = [29, 47]$ $\tilde{S}[0] = [33, 49]$ $\tilde{D}[0] = [13, 33]$		Mean = 280.4 SD = 156.5 Median = 252.0	2
FM3	$\tilde{O}[0] = [41, 49]$ $\tilde{S}[0] = [25, 45]$ $\tilde{D}[0] = [21, 41]$		Mean = 377.4 SD = 173.7 Median = 360.0	1
FM4	$\tilde{O}[0] = [12, 31]$ $\tilde{S}[0] = [13, 33]$ $\tilde{D}[0] = [2, 13]$		Mean = 32.7 SD = 40.1 Median = 20.0	6
FM5	$\tilde{O}[0] = [12, 33]$ $\tilde{S}[0] = [19, 39]$ $\tilde{D}[0] = [1, 17]$		Mean = 51.4 SD = 57.7 Median = 35.0	5
FM6	$\tilde{O}[0] = [23, 42]$ $\tilde{S}[0] = [35, 50]$ $\tilde{D}[0] = [3, 17]$		Mean = 114.5 SD = 100.7 Median = 90.0	3
FM7	$\tilde{O}[0] = [4, 23]$ $\tilde{S}[0] = [39, 50]$ $\tilde{D}[0] = [0, 7]$		Mean = 21.0 SD = 34.2 Median = 0.0	7

Table 9: Ranking comparison.

	Proposed approach ranking ($\alpha = 1$)	Proposed approach ranking ($\alpha = 0$)	Liu's model ($\phi = 1$)	Liu's model ($\phi = 0$)	Traditional FMEA	Fuzzy TOPSIS
FM1	4	4	6	3	4	3
FM2	2	2	1	2	2	2
FM3	1	1	3	1	1	1
FM4	6	6	7	7	7	6
FM5	5	5	5	5	5	5
FM6	3	3	2	4	3	4
FM7	7	7	4	6	5	7

For example, for the O factor of FM6 in this example (sample $s = 1.673$), within the range of evaluation values 1–10, if $E = 1$, 95% confidence level ($t = 2.776$),

$$n = \left(\frac{ts}{E}\right)^2 = \left(\frac{2.776 \times 1.673}{1}\right)^2 \approx 21.57 \approx 22. \qquad (25)$$

Another example is the S factor of FM1 (sample $s = 1.095$) for $E = 1$ and a 95% confidence level ($t = 2.776$):

$$n = \left(\frac{ts}{E}\right)^2 = \left(\frac{2.776 \times 1.095}{1}\right)^2 \approx 9.23 \approx 10. \tag{26}$$

For a 95% confidence level and an E of 1, the number of samples for each failure mode and its evaluation factors are quite different to achieve the required accuracy. In this case, the largest number of samples should be used. The final sample number can guarantee accuracy and allowable error. Taking the O factor of FM6 (sample $s = 1.673$) as an example, the deviation from the theoretical population mean is calculated as follows:

$$E = t\frac{s}{\sqrt{n}} = 2.776 \times \frac{1.673}{\sqrt{5}} \approx 2.07. \tag{27}$$

This result shows that the deviation is too large for the evaluation using a 1–10 scale. If each indicator has such a large deviation, the deviation of the overall $\text{RPN} = S \times O \times D$ will be excessive. Even if TOPSIS or VIKOR is used instead of $\text{RPN} = S \times O \times D$, the deviation of the results is still large.

The FMEA method considering random uncertainty makes it easier to understand the advantages and disadvantages of FMEA results based on a limited number of expert evaluations. (1) The greater the number of experts that participate in the evaluation, the higher the confidence of the results. (2) The result is the sample mean of a random sampling distribution. The actual population mean and this mean point may not completely coincide. The theoretical SOD factor's true value and the measured SOD factor value must be different. Regardless of whether the traditional FMEA method, fuzzy TOPSIS, or Liu's model is used, if the underlying SOD data have a large deviation from the true value, the accuracy of the results will be questionable. To ensure that the error of the evaluation results is within an acceptable range, the number of experts should be estimated according to formula (24).

One major shortcoming of traditional FMEA research is that the weight of each SOD factor is not considered. As mentioned above, the calculation of the classic $\text{RPN} = S \times O \times D$ itself implies a weighting process based on the severity of the failure. Whether including additional weights is appropriate requires further discussion. However, the important point to note is that weights are usually based on expert evaluation. As mentioned above, shortage of experts will substantially weaken the credibility of the results. Improper weighting will increase the risk of incorrect evaluation results. The ranking results of the proposed research method and fuzzy TOPSIS are extremely similar: the only difference is the ranking of FM1 and FM6. However, the difference between the proposed approach and Liu's model is considerable. One of the most important reasons is that the new weight is introduced in the calculation process of the VIKOR method in Liu's model, which dramatically changes the importance of the three SOD factors in the evaluation. The introduction of this new artificial weight vector further increases the deviation between the evaluation result and the theoretical true value of the population.

7. Conclusion

7.1. Summary. Although the traditional FMEA method has many shortcomings, prioritization of potential failure risks by calculating RPN is common in industry because the RPN calculation is simpler and easier to understand than other methods.

As for the actual evaluation process in the real world, there are at least two types of uncertainties, fuzzy uncertainty and random uncertainty. It is more beneficial to build a decision model by considering both uncertainties simultaneously for the improvement the validity of the evaluation results. After deeply understanding the impact of the two uncertainties on the evaluation results, it is possible to more clearly and accurately grasp that uncertainty is the more critical one during the evaluation process, and the result of simplified calculation only focusing on the more critical uncertainty can be closer to the true value we expect to have.

This study proposes a fuzzy beta-binomial distribution evaluation method that integrates triangular fuzzy numbers, Bayesian statistical inference, and the beta-binomial distribution. The fuzzy uncertainty of the evaluation is measured by introducing linguistic variables to make the evaluation process more humanized and more similar to natural language processing. The introduction of the measure of random uncertainty makes the expert scoring process a finite sample sampling process. Because the full sample (or large sample) sampling evaluation cannot be realized, the evaluation of limited experts is inevitably limited by the shortcomings of small sample sizes. The difference between the mean of the samples and the theoretical population mean is often outside the acceptable range, resulting in invalid conclusions, regardless of how sophisticated the data processing is or how complicated the fuzzy expression is. This aspect has been widely ignored by many studies.

In the case study, a total of five experts scored seven failure modes. Formula (25) shows that, under the conditions that error E is 1 and confidence level is 95%, at least 22 experts are required to participate in the evaluation to achieve the required evaluation requirements. However, in the case, only five experts join in the evaluation, and under a 95% confidence level, the error E is 2.07 (as shown in equation (27)), which is far from the requirement. It is found that when the sample size of experts is small, and the evaluation difference of each expert is large, random uncertainty has a greater effect on the evaluation results than fuzzy uncertainty no matter how elaborate the membership function is used in the model.

7.2. The Theoretical Contributions. The main theoretical contributions of this paper are shown as follows.

(1) This study firstly proposes a method to introduce both random and fuzzy uncertainty into FMEA to calculate RPN. It can be seen from the literature review that scholars have extended FMEA studies by introducing fuzzy uncertainty or random uncertainty into traditional FMEA analysis to compensate for the shortcomings of the traditional approach, but

these studies consider either fuzzy or random uncertainty separately and rarely consider both uncertainties simultaneously in a single model.

(2) In order to calculate the RPN with both uncertainties, this study innovatively proposes a method for describing RPN calculations under random and fuzzy uncertainty by using the beta-binomial distributions.

(3) This study pioneers a method for solving complex RPN models containing random uncertainty and fuzzy uncertainty with MCMC method. For the calculation of fuzzy numbers, this is generally achieved by defuzzification, a process that actually loses some of the information of the original equation, whereas a more efficient way to preserve the information of the original equation is through the data-driven method, such as MCMC.

7.3. The Implication to Practice. The main practical contributions of this paper are shown as follows.

(1) In traditional studies, only a single uncertainty is considered, which is not consistent with the actual practical situation. As discussed earlier, when quantifying the three factors of SOD through expert evaluation methods, both random and fuzzy uncertainties do exist, so the RPN calculation method proposed in this study with both random and fuzzy uncertainty methods is in line with the actual situation and has more practical value.

(2) The use of the MCMC method to solve the three complex beta-binomial distributed multiplication problems can effectively reduce the difficulty of solving the complex model, without losing most of the information, and the obtained risk priority evaluation results are more reliable; this method also facilitates the development of automatic solution systems through automotive computer program, thus allowing ordinary quality managers to perform more accurate RPN assessment.

(3) The numerical solution method proposed in this study provides a feasible idea for the further extended solution of the RPN calculation model, such as the calculation after further considering the weighting of the three factors of SOD and the calculation after adding new factors, which are all feasible in this method.

7.4. Limitation and Future Research. Certainly, there are still some problems that need further study: (1) this study does not consider another important dimension of uncertainty, namely, rough uncertainty in the RPN evaluation. Although studies have considered both ambiguity and roughness in the calculation of RPN, the three uncertainties have not been considered simultaneously; (2) the proposed approach uses a beta-binomial distribution to fit the expert evaluation results. In future research, the possibility of using other distributions can be further explored.

Data Availability

No other additional data were used in the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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