Research Article

Backstepping Output Feedback Control for the Stochastic Nonlinear System Based on Variable Function Constraints with the Subsea Intelligent Electroexecution Robot System

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The output feedback controller is designed for a class of stochastic nonlinear systems that satisfy uncertain function growth conditions for the first time. The multivariate function growth condition has greatly relaxed the restrictions on the drift and diffusion terms in the original stochastic nonlinear system. Here, we cleverly handle the problem of uncertain functions in the scaling process through the function maxima theory so that the Ito differential system can achieve output stabilization through Lyapunov function design and the solution of stochastic nonlinear system objects satisfies the existence of uniqueness, ensuring that the system is globally asymptotically stable in the sense of probability. Furthermore, it is concluded that the system is inversely optimally stable in the sense of probability. Finally, we apply the theoretical results to the practical subsea intelligent electro-execution robot control system and obtain good results.

1. Introduction

In the field of control theory, research on the output feedback control of stochastic nonlinear systems has produced some valuable results. As far as the output feedback control is concerned, it has obvious advantages over the state feedback control in practical systems because it does not need to obtain all the system states' information [1–10], and as far as the stochastic nonlinear system itself is concerned, stochastic process and nonlinearity constitute the two most basic characteristics of the practical system. As a practical system, external interference is inevitable, and many interferences appear as the stochastic process. This has obvious manifestations in the fields of engineering technology and socioeconomic management and even extends to the field of large systems. Therefore, the study of

stochastic nonlinear systems has strong practical significance.

Krstić and Kanellakopoulos [11] studied the design of output feedback controllers for a class of stochastic nonlinear systems (1). The nonlinear terms in this system are all functions that can measure the output for the first time.

$$\begin{aligned} dx_i &= x_{i+1}dt + \phi_i(y)d\omega, \quad (i = 1, \dots, n-1), \\ dx_n &= udt + \phi_n(y)d\omega, \\ y &= x_1, \end{aligned} \quad (1)$$

where $\phi_i(y)$ is a smooth nonlinear function on the output, initial value $\phi_i(0) = 0$, and ω is the standard Wiener process. In this paper, authors Deng and Krstic propose a controller based on full-order observers with the help of backstepping technology. The output feedback stabilization results of the

system are given. Based on [6], a series of research results are subsequently produced in the research [11–22]. Lei and Lin [13] studied the output feedback stabilization of the system by changing the energy supply function method. Chang et al. [14] introduced a stochastic small-gain theorem to obtain a design scheme for output feedback controllers based on reduced-order observers. The above papers have one thing in common: their nonlinear terms all depend on measurable output terms.

For the controller design of high-order nonlinear systems, some research results have been obtained in recent years [23–33]. In early papers [20, 21], the authors used high-order nonlinear systems as strict feedback systems, based on which the controller design work and homogeneous dominant method have been proposed to effectively control the nonlinear terms. In subsequent papers [23, 24], the authors continued to study output tracking and disturbance suppression for higher-order systems. Regarding the form of the controller, Mazenc et al. [25] used an increased power integration method to design a state feedback controller for high-order nonlinear systems which find the upper bound of the nonlinear term can make the closed-loop system globally asymptotically stable. The authors in [26–31] gradually weakened growth conditions of the nonlinear term and accordingly constructed smooth state feedback controllers. Deng and Krstic [26] studied high-order nonlinear systems with parameter uncertainty. With adaptive control methods, a dynamic gain adaptive law has been introduced, and an adaptive state feedback controller has been designed. Aiming at the problem of designing output feedback controllers for high-order nonlinear systems, especially the unmeasurability of states, first, a state observer needs to be built. Compared with the design of state feedback controllers, the problem of the output feedback control is very difficult.

In [28], for the output feedback design process of nonlinear systems, whether stability design or tracking control design, it is necessary to restrict the growth conditions of unmeasurable states in the nonlinear term; otherwise, there is no discussion of control issues. Nonlinear growth conditions are a hypothetical limitation on the nonlinear term, which are within a certain constraint. For example, there is a constant C that makes the nonlinear term $f(x)$ satisfy $|f(x)| \leq C(|x_1| + \dots + |x_n|)$ which is a typical linear growth condition. When the states' variable is finitely changed, the states have a finite increase as a nonlinear function of the independent variable, and the rate of growth satisfies certain constraints. As the most basic linear growth condition, this growth condition has been used by quite a lot of research results as the basic assumption for controller design [33–35]. For later researchers, there is a very challenging task for nonlinear term constraints, that further weakens the constraints on nonlinear terms. The latest research hotspots focus on how to design controllers in cases where nonlinear terms satisfy the power growth condition. In the early stage, for a nonlinear system that satisfies certain growth conditions, the state variables of the system itself are mostly measurable or partially measurable. However, in the practical system, the “black box” situation of the system is ubiquitous. In the case that the input and output are measurable and the state variables are unmeasurable, the

design of the system has very practical value. In [28–32], the state feedback and output feedback control problems of nonlinear systems with unmeasurable states have been studied, and the constraints of nonlinear terms have been gradually relaxed.

Jiang and Praly [32] used homogeneous dominance theory and relaxed the growth conditions to design an output feedback controller. After introducing stochastic factors into higher-order nonlinear systems, some research results have been produced. Some of the results are mainly focused on the study of a special kind of stochastic nonlinear systems. The system is characterized by its nonlinear terms not only relying on measurable outputs but also on all unmeasurable states. Li et al. [34] studied a class of high-order stochastic nonlinear systems, which can neither feedback linearize nor affine to control inputs. The authors used the coordinate transformation method in nonlinear theory to construct a state observer for a stochastic nonlinear system, selected an appropriate Lyapunov function, and designed an output feedback controller which satisfies the system's stabilization conditions. Li et al. [35] further studied the stabilization of the output feedback for higher-order stochastic nonlinear systems with the relaxation of nonlinear term growth conditions. By using the design idea of the homogeneous dominant method, the final output feedback controller is obtained so that closed-loop stochastic nonlinear systems are globally asymptotically stable in the sense of probability. In recent years, the field of stochastic nonlinear control theory has focused on the study of a special kind of stochastic nonlinear systems, that is, the nonlinear terms of stochastic nonlinear systems depend on measurable outputs [35–48]. Liu and Xie [36] used the theory of homogeneity and the method of increasing power integration to construct an output feedback controller to ensure the global asymptotic stability of the system in the sense of probability. In [37], a state observer is constructed by introducing coordinate transformation, and the Lyapunov function is selected to study the design of output feedback controllers for stochastic nonlinear systems in a more general condition. Based on [36–41], Xie and Liu [38] removed the assumption that the observer gain is greater than one and avoided the conservative situation of the designed output feedback controller. Guo and Fang [39] introduced unknown control coefficients, and the values of the upper and lower bounds of the control coefficients are generalized by constants to allow arbitrary values to be arbitrarily large or arbitrarily small.

However, in the previous research results, the nonlinear terms (drift term and diffusion term) of the system must satisfy the growth condition which is linear and highly restrictive. This harsh condition makes the application of research results narrow. Therefore, this paper creatively proposes a more relaxed constraint condition, that is, a power growth condition $p(x)$ that has always been presented in the form of a function as follows:

$$|\varphi_i(x)| \leq d \left(|x_1|^{p_i(t,x)} + |x_2|^{p_i(t,x)} + \dots + |x_i|^{p_i(t,x)} \right). \quad (2)$$

This paper focuses on the output feedback problem of a class of stochastic nonlinear systems that satisfy the growth conditions of the form of an uncertain function. The

nonlinear term depends on the measurable output and all unmeasurable states. For the output feedback problem of such systems, there is no latest research result. We strive to design the output feedback controller to ensure that the equilibrium point of the system's origin is globally asymptotically stable in the probabilistic sense. At the same time, we further derive that the system is inversely optimally stable in the probabilistic sense. The main contributions of this paper are as follows:

- (1) The set power growth condition relaxes the restrictions on the drift and diffusion terms of the stochastic nonlinear system, and the scope of the application of the output feedback controller is also greatly expanded.
- (2) In the previous research results, most of the systems studied are for the system determined by the constant growth condition. Even for uncertain systems, the control function has the control direction and control coefficient [33–35]. Li et al. [35] focused on the output feedback practical tracking problem for a class of stochastic nonlinear systems in which the coefficient in front of the control action growth condition $p(x)$ is 1. Such system design greatly limits the scope of design and application of the system controller, especially in the actual system, because in many practical systems, such as aerospace systems, terrestrial satellite navigation systems, industrial processes, and other complex systems, the control role mostly contains uncertain factors. The uncertainty here does not only refer to the uncertainty of the system itself but also to the uncertainty of the system control. Therefore, it is urgent to study the uncertainty function growth condition. Based on this consideration, we consider the output feedback problem for a class of stochastic nonlinear systems with the uncertainty function growth condition. By introducing a new high-gain adjustment law, the design method of the global output control for such uncertain nonlinear systems is given.
- (3) In [41], the authors proposed a design idea for a high-gain observer. We use this design method to construct a state observer for a system object that satisfies the more general condition of the power growth condition. And by setting a series of Lyapunov functions and using the backstepping method, finally authors can get an output feedback controller with high-gain parameters so that the solution of the random nonlinear system object satisfies the existence of uniqueness while ensuring that the system is globally asymptotically stable in probability.

This paper is organized as follows. Section 2 provides the problem description. In Section 3, an output feedback controller is designed. In Section 4, the system is analyzed, and inverse optimal stabilization in probability is achieved. Section 5 applies theoretical results to the practical subsea electrohydraulic valve control system. Section 6 summarizes this paper.

2. Problem Description

This paper mainly studies the following types of stochastic nonlinear systems:

$$\begin{cases} dx_i = x_{i+1}dt + \varphi_i(x)d\omega, & (i = 1, \dots, n-1), \\ dx_n = udt + \varphi_n(x)d\omega, \\ y = x_1. \end{cases} \quad (3)$$

In this stochastic nonlinear system, $x = (x_1, \dots, x_n)^T \in R^n$ is the state of the system, $u \in R$ is the input of the system, and $y \in R$ is the output of the system. Observable state $x_2, \dots, x_n$ is not measurable, and output y is measurable. ω is an m -dimensional standard Brownian motion defined on the probability space (Ω, Γ, P) . The nonlinear term $\varphi_i (i = 1, \dots, n): R^n \rightarrow R^m$ is a continuous smooth function and satisfies $\varphi_i(0) = 0$.

For the above system (3), the following assumptions are given.

Assumption 1. For the nonlinear term of system (3), there exists a constant such that the following inequality holds:

$$|\varphi_i(x)| \leq d \left(|x_1|^{p_i(t,x)} + \dots + |x_i|^{p_i(t,x)} \right), \quad (4)$$

where $p_i(x, t)$ is a positive function.

Remark 1. In the previous research results, the constraints of the nonlinear term are very narrow, and the scope of the application is very small. Earlier constraints focused on linear growth conditions or piecewise linear growth conditions. In the latest part of the research, power growth conditions are applied to controller design, but there are still obvious shortcomings, that is, power growth conditions are basically definite constants, and this kind of constant is simply to increase the order which the system itself has not been greatly expanded. Then, in this paper, the determined power order value is creatively functionalized, that is, a variable function $p_i(x, t)$ is generated in a nonlinear constraint condition. Based on this, the entire stochastic nonlinear system has greatly increased adaptability.

In the following, we will introduce the design idea of the output feedback controller in detail. The final output feedback controller ensures that stochastic nonlinear system (3) is globally asymptotically stable in the sense of probability.

3. Control Design

The controller design of stochastic nonlinear system (3) is divided into two parts. The first part is to design a state observer for the system; the second part uses the backstepping method to design an output feedback controller for system (3) to meet the corresponding control performance indicators.

3.1. High-Order Gain Observer Design. Since the system is not measurable except for x_1 which is a measurable state, a set of high-order gain observers is first constructed:

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + K^i h_i (x_1 - \hat{x}_1), & (i = 1, \dots, n-1), \\ \dot{\hat{x}}_n = u + K^n h_n (x_1 - \hat{x}_1). \end{cases} \quad (5)$$

In this set of observers, the real number $K > 0$ is the high-gain parameter to be designed, $\hat{x}_i$ ($i = 1, \dots, n$) is dependent on the measurable output y , and $h_i > 0$ is a known constant that stabilizes the matrix

$$A = \begin{pmatrix} -h_1 & & \\ \cdots & I_{n-1} & \\ -h_n & 0 \cdots 0 \end{pmatrix}. \quad (6)$$

Then, there exists a positive definite matrix P that satisfies $A^T P + PA = -I$. Based on the above analysis, combined with the state observer, we define the error equation of system (3) as

$$\begin{aligned} \tilde{x} &= (\tilde{x}_1, \dots, \tilde{x}_n), \\ \tilde{x}_i &= \frac{(x_i - \hat{x}_i)}{K^{i-1}} \quad (i = 1, \dots, n). \end{aligned} \quad (7)$$

According to (3)–(7), the system error model can be obtained as

$$d\tilde{x} = KA\tilde{x}dt + \Phi(x)d\omega, \quad (8)$$

where

$$\Phi(x) = \left(\varphi_1(x), \frac{\varphi_2(x)}{K}, \dots, \frac{\varphi_n(x)}{K^{n-1}} \right)^T. \quad (9)$$

Since the growth conditions of the system are controlled by the function group $p_i(x, t)$ with uncertain parameters, the key to the design of the output feedback control system in this paper is to handle the function growth conditions, normalize them, and limit the range. First, based on the output feedback control theory and its mathematical application, a Lyapunov function is selected:

$$V_0(\tilde{x}) = (n+1)\tilde{x}^T P \tilde{x}. \quad (10)$$

Use the following inequality:

$$\begin{aligned} (a+b)^2 &\leq 2(a^2 + b^2), \\ \sum_{i=1}^n |a_i|^2 &\leq \left(\sum_{i=1}^n |a_i| \right)^2, \\ \left(\sum_{i=1}^n a_i \right)^2 &\leq n \sum_{i=1}^n a_i^2. \end{aligned} \quad (11)$$

We can get the system's Ito differential equation based on this Lyapunov function as

$$\begin{aligned} LV_0 &\leq -(n+1)K|\tilde{x}|^2 + (n+1)\|P\|d^2 \left(\sum_{i=1}^n \frac{1}{K^{i-1}} \right)^2 \left(|x_1|^{p_1(x)} + \dots + \frac{|x_n|^{p_n(x)}}{K^{n-1}} \right) \\ &\leq \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \left(\sum_{i=1}^n \left(\frac{\tilde{x}_i^{\bar{\theta}}}{K^{i-1}} \right) + \sum_{i=1}^n \left(K \tilde{x}_i^{\bar{\theta}} |\bar{p}_i(x)| \right) \right)^2 - (n+1)K|\tilde{x}|^2 \\ &\leq -\sqrt{|p_1(x)| + \dots + |p_n(x)|} \left((n+1)nK - nd^* \sum_{i=1}^n \left(K \tilde{x}_i^{\bar{\theta}} \|\bar{p}_i(x)\| \right)^2 \right) \|\tilde{x}\|_{\infty}^{2\bar{\theta}} \\ &\quad + nd \left(\tilde{x}_1^{2\bar{\theta}} + \frac{\tilde{x}_2^{2\bar{\theta}}}{K^2} + \dots + \frac{\tilde{x}_n^{2\bar{\theta}}}{K^{2n-2}} \right), \end{aligned} \quad (12)$$

where

$$\begin{aligned} \theta_i &= \left(\sqrt{p_{i\max}^2(x) + p_{i\min}^2(x)} \right), \\ \bar{\theta} &= |\theta_1| + \dots + |\theta_n|, \end{aligned} \quad (13)$$

$$\|\tilde{x}\|_{\infty} = \max_i |x_i|,$$

$$\begin{aligned} d^* &= (n+1)\|P\|d^2 \left(\sum_{i=1}^n \frac{1}{K^{i-1}} \right)^2 \\ &= \frac{(n+1)\|P\|d^2 \left(\sum_{i=0}^{n-1} (i+1)K^i + \sum_{i=n}^{2n-2} (2n-i-1)K^i \right)}{K^{2n-2}}. \end{aligned} \quad (14)$$

Here, this paper cleverly performs the extremum processing of the growth conditions and, through the normalization of the maximum and minimum results, converts the function growth conditions into uncertain parameter growth conditions.

3.2. Backstepping Method to Design an Output Feedback Controller. In this section, we will get the output feedback controller through the backstepping method. Based on the consideration of system simplification, the following coordinate transformations are introduced for stochastic nonlinear systems:

$$\begin{aligned} w_1 &= \hat{x}_1, \\ w_i &= \hat{x}_i - \beta_{i-1}(\hat{x}_1, \dots, \hat{x}_{i-1}). \end{aligned} \quad (15)$$

In the new states, the idea of virtual control is introduced, that is, β_{i-1} is the virtual controller to be designed.

Based on the observer system function $V_0(\tilde{x})$ in Section 3.1, we construct a new set of Lyapunov functions:

$$V_1(\tilde{x}, w_1) = V_0(\tilde{x}) + \sqrt{|p_1(x)| + \dots + |p_n(x)|} \frac{1}{\bar{\theta} + 1} w_1^{\bar{\theta} + 1}. \quad (16)$$

Using equations (8)–(15), one can obtain

$$\begin{aligned} LV_1 &\leq - \left((n+1)nK - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\tilde{x}\|_\infty^{2p} \\ &\quad + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \left(\hat{x}_1^{2\bar{\theta}} + \frac{\hat{x}_2^{2\bar{\theta}}}{K^2} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} \right) + w_1^{\bar{\theta}} (\hat{x}_2 + Kh_1 \tilde{x}_1) \\ &\leq - \left((n+1)nK - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\tilde{x}\|_\infty^{2p} \\ &\quad + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\left(\frac{\hat{x}_3^{2\bar{\theta}}}{K^4} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} \right) + \frac{\hat{x}_2^{2p}}{K^2} + w_1^{2p} \right) \\ &\quad + w_1^{\bar{\theta}} w_2 + w_1^{\bar{\theta}} \beta_1 + K \tilde{x}_1^2 + \frac{K}{4} h_1^2 w_1^{2\bar{\theta}}. \end{aligned} \quad (17)$$

In fact, in the iterative process of this paper, there is an idea of knowing the results before making assumptions. Therefore, we need to make certain necessary assumptions. These assumptions are not only constraints on the research object model but also our implementation. And this is a necessary condition in the controller derivation process. Due to the omission of the first draft, certain assumptions and constraints are actually required here.

Here, according to the backstepping theory, using (8)–(16) to select a specific relationship $K \geq 2^{2p} n d^*$, one can get

$$\begin{aligned} \sqrt{|p_1(x)| + \dots + |p_n(x)|} d w_1^2 &\leq \frac{K}{2} w_1^2, \\ \frac{\hat{x}_2^{2\bar{\theta}}}{K^2} &\leq \left(\frac{1}{K^2} w_2^{2\bar{\theta}} + \frac{1}{K^2} \beta_1^{2\bar{\theta}} \right). \end{aligned} \quad (18)$$

Therefore, after scaling down the inequality, we can get the first virtual controller as

$$\begin{aligned} \beta_1 &= -K \sqrt{|p_1(x)| + \dots + |p_n(x)|} v_1 w_1^{\bar{\theta}}, \\ v_1 &= \frac{1}{2} \left(1 + \frac{h_1^2}{2} + n \right). \end{aligned} \quad (19)$$

The idea of reverse thrust control needs to be explained here. Only in the continuous reverse thrust process, continuous virtual controllers will appear, and these virtual controllers have certain mutual laws. Only when the theory of virtual control is added in the amplification process of Lyapunov formulas of different orders can the final negative definite result be obtained, and the result of this item one puts forward is the result of applying virtual control.

Importing the first virtual controller into the first Ito differential equation, a new Ito differential equation can be obtained:

$$\begin{aligned}
LV_1 &\leq - \left((n+1)nK - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\tilde{x}\|_\infty^{2\bar{\theta}} \\
&\quad + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\frac{\hat{x}_3^{2\bar{\theta}}}{K^4} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} + \frac{2}{K^2} w_2^{2\bar{\theta}} \right) + w_1^{\bar{\theta}} w_2 + w_1^{\bar{\theta}} \beta_1 + K \tilde{x}_1^2 + \frac{K}{4} h_1^2 w_1^{2\bar{\theta}} \\
&\leq - \left(((n+1)n-1)K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\tilde{x}\|_\infty^{2\bar{\theta}} \\
&\quad + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\frac{\hat{x}_3^{2\bar{\theta}}}{K^4} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} + \frac{2}{K^2} w_2^{2\bar{\theta}} \right) \\
&\quad + w_1^{\bar{\theta}} w_2 + w_1^{\bar{\theta}} \beta_1 + K \sqrt{|p_1(x)| + \dots + |p_n(x)|} \left(\frac{1}{2} + \frac{h_1^2}{4} \right) w_1^{2\bar{\theta}} \tag{20} \\
&\leq - \left(((n+1)n-1)K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\tilde{x}\|_\infty^{2\bar{\theta}} \\
&\quad + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\frac{\hat{x}_3^{2\bar{\theta}}}{K^4} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} + 2\nu_1^{2p} \sqrt{\frac{w_1^{2\bar{\theta}} \cdot w_1^{2\bar{\theta}}}{2\bar{\theta}}} + \frac{2}{K^2} w_2^{2\bar{\theta}} \right) - nK w_1^{2\bar{\theta}} \\
&\leq - \left(((n+1)n-1)K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\tilde{x}\|_\infty^{2\bar{\theta}} \\
&\quad + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\frac{\hat{x}_3^{2p}}{K^4} + \dots + \frac{\hat{x}_n^{2p}}{K^{2n-2}} + \bar{\theta} n d^* \nu_1^{2\bar{\theta}} \right) - K w_1^{2p}.
\end{aligned}$$

After obtaining the first virtual controller, through the theory of coherent analogy, we can infer backwards in turn until we get the $i-1$ virtual controller.

$$\beta_1(\hat{x}_1), \dots, \beta_{i-1}(\hat{x}_1, \dots, \hat{x}_{i-1}), \tag{21}$$

$$\beta_1(\hat{x}_1) = -K \sqrt{|p_1(x)| + \dots + |p_n(x)|} \nu_1 w_1^{\bar{\theta}}$$

$$\vdots$$

$$\beta_{i-1}(\hat{x}_1, \dots, \hat{x}_{i-1}) = -K^{i-1} \sqrt{|p_1(x)| + \dots + |p_n(x)|} \nu_{i-1} w_{i-1}^{\bar{\theta}}, \tag{22}$$

where

$$\begin{aligned}
 v_1 &= \frac{1}{2} + \frac{h_1^2}{4} + n \\
 &\vdots \\
 v_{i-1} &= \frac{1}{4} (h_{i-1} + v_{i-2} h_{i-2} + v_{i-2} v_{i-3} h_{i-3} + \cdots + v_{i-2} v_{i-3} \cdots v_1 h_1)^2 + \cdots + \frac{1}{4} (v_{i-3} v_{i-2} - v_{i-2}^2)^2 + v_{i-2} + 1 + n - (i-2).
 \end{aligned} \tag{23}$$

The constant $v_j > 0$ is a real constant that does not depend on K so that the expression of using the Lyapunov function for the i time is

$$V_{i-1}(\tilde{x}, w_1, \dots, w_{i-1}) = V_0(\tilde{x}) + \sqrt{|p_1(x)| + \cdots + |p_n(x)|} \frac{1}{\bar{\theta} + 1} \sum_{j=1}^{i-1} \frac{1}{K^j} w_j^{\bar{\theta}+1}. \tag{24}$$

This Lyapunov function is satisfying

$$\begin{aligned}
 LV_{i-1} &\leq - \left(((n+1)n - (i-1))K - \sqrt{|p_1(x)| + \cdots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\tilde{x}\|_{\infty}^{2\bar{\theta}} \\
 &\quad + \sqrt{|p_1(x)| + \cdots + |p_n(x)|} d \left(\frac{\hat{x}_{i+1}^{2\bar{\theta}}}{K^{2i}} + \cdots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} \right) \\
 &\quad - \sum_{j=1}^{i-1} \frac{1}{K^{2j-2}} \left(\left(n - 4^{(\bar{\theta}-1)(j-1)} \right) K - \sqrt{|p_1(x)| + \cdots + |p_n(x)|} \bar{\theta} d v_j^{2p} \right) w_j^{2\bar{\theta}} \\
 &\quad + \sqrt{|p_1(x)| + \cdots + |p_n(x)|} d \frac{2}{K^{2(i-1)}} w_i^{2\bar{\theta}} + \frac{1}{K^{2(i-2)}} w_{i-1}^{\bar{\theta}} w_i.
 \end{aligned} \tag{25}$$

The next Lyapunov function of the recursive series is

$$\begin{aligned}
 V_i(\tilde{x}, w_1, \dots, w_{i-1}) &= V_{i-1}(\tilde{x}, w_1, \dots, w_{i-1}) \\
 &\quad + \sqrt{|p_1(x)| + \cdots + |p_n(x)|} \frac{1}{\bar{\theta} + 1} \cdot \frac{1}{K^i} w_i^{\bar{\theta}+1}.
 \end{aligned} \tag{26}$$

Whether the system's Ito differential equation based on the latest Lyapunov function satisfies negative definiteness needs to be obtained by the following proof. Using $K \geq \sqrt{|p_1(x)| + \cdots + |p_n(x)|} d$ and combining equations (18)–(25), one can get

$$\begin{aligned}
LV_i \leq & - \left(((n+1)n - (i-1))K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)p-(i-1)} \right)^2 \right) \|\tilde{x}\|_{\infty}^{2\bar{\theta}} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\frac{\hat{x}_{i+2}^{2\bar{\theta}}}{K^{2(i+1)}} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} \right) \\
& - \sum_{j=1}^{i-1} \frac{1}{K^{2j-2}} \left((n - 4^{(\theta-1)(j-1)})K - 2^{4\theta-1} p n d^* v_j^{2\theta} \right) w_j^{2\theta} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \frac{1}{K^{2i}} \hat{x}_{i+1}^{2\bar{\theta}} + \frac{1}{K^{2(i-1)}} w_i^{2\bar{\theta}} \hat{x}_{i+1} + \frac{1}{K^i} w_i^{\bar{\theta}} \hat{x}_{i+1} \\
& + w_i^{\bar{\theta}} h_i \tilde{x}_i + \frac{1}{K^{i-1}} w_i^{\bar{\theta}} v_{i-1} p w_{i-1}^{\bar{\theta}-1} (\hat{x}_i + K^{i-1} h_{i-1} \tilde{x}_1) \\
& + \frac{1}{K^{i-2}} w_i^{\bar{\theta}} v_{i-1} v_{i-2} p v_{i-2}^{\bar{\theta}-1} (\hat{x}_{i-1} + K^{i-2} h_{i-2} \tilde{x}_1) + \dots + \frac{1}{K^{i-(i-1)}} w_i^{\bar{\theta}} v_{i-1} v_{i-2} \cdot v_1 p w_1^{\bar{\theta}-1} (\hat{x}_2 + K h_1 \tilde{x}_1) \\
\leq & - \left(((n+1)n - i)K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)\bar{\theta}-(i-1)} \right)^2 \right) \|\tilde{x}\|_{\infty}^{2\bar{\theta}} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\frac{\hat{x}_{i+2}^{2\bar{\theta}}}{K^{2(i+1)}} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} \right) \\
& - \sum_{j=1}^i \frac{1}{K^{2j-2}} \left((n - 4^{(\bar{\theta}-1)j})K - p \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* v_j^{2\bar{\theta}} \right) w_j^{2\bar{\theta}} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \frac{2}{K^{2i}} w_{i+1}^{2\bar{\theta}} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \frac{2}{K^{2i}} \beta_i^{2\bar{\theta}} + \frac{1}{K^{2(i-1)}} w_i^{\bar{\theta}} w_{i+1} \\
& + \frac{1}{K^{2(i-1)}} w_i^{\bar{\theta}} \beta_i + \frac{1}{4K^{2i-3}} w_i^{2\bar{\theta}} (h_i + v_{i-1} h_{i-1} \dots + v_{i-1} v_{i-2} \cdot v_1 h_1)^2 \\
& + \dots + \frac{1}{4K^{2i-3}} w_i^{2\bar{\theta}} (v_{i-3} v_{i-2} - v_{i-2}^2)^2 + \frac{1}{K^{2i-3}} w_i^{2\bar{\theta}} (v_{i-1} + 1).
\end{aligned} \tag{27}$$

The system is differentiated by Ito microdifferentiation and needs to satisfy the following results:

$$\begin{aligned}
LV_i \leq & - \left(((n+1)n-i)K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \left(\sum_{i=1}^n K^{(i-1)\bar{\theta}-(i-1)} \right)^2 \right) \|\tilde{x}\|_{\infty}^{2\bar{\theta}} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \left(\frac{\hat{x}_{i+2}^{2\bar{\theta}}}{K^{2(i+1)}} + \dots + \frac{\hat{x}_n^{2\bar{\theta}}}{K^{2n-2}} \right) \\
& - \sum_{j=1}^i \frac{1}{K^{2j-2}} \left((n-4^{(\bar{\theta}-1)j})K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} \bar{\theta} d^* v_j^{2\bar{\theta}} \right) w_j^{2\bar{\theta}} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \frac{2}{K^{2i}} w_{i+1}^{2\bar{\theta}} + \frac{1}{K^{2(i-1)}} w_i^{\bar{\theta}} w_{i+1}.
\end{aligned} \tag{28}$$

By selecting the i virtual controller,

$$\begin{aligned}
\beta_i(\hat{x}_1, \dots, \hat{x}_i) &= -K^i \sqrt{|p_1(x)| + \dots + |p_n(x)|} v_i w_i^{\bar{\theta}}, \\
v_i &= h_i + v_{i-1} h_{i-1} + v_{i-1} v_{i-2} h_{i-2} + \dots \\
&+ v_{i-1} v_{i-2} \dots v_1 h_1.
\end{aligned} \tag{29}$$

Next, repeating the recursive process of step i , one can get

$$\begin{aligned}
LV_{n-1} \leq & - \left(((n+1)n-(n-1))K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \left(\sum_{i=1}^n K^{(i-1)\bar{\theta}-(i-1)} \right)^2 \right) \|\tilde{x}\|_{\infty}^{2\bar{\theta}} \\
& - \sum_{j=1}^{n-1} \frac{1}{K^{2j-2}} \left((n-4^{(\bar{\theta}-1)(j-1)})K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} \bar{\theta} d^* v_j^{2\bar{\theta}} \right) w_j^{2\bar{\theta}} \\
& + \sqrt{|p_1(x)| + \dots + |p_n(x)|} d^* \frac{2}{K^{2(n-1)}} w_n^{2\bar{\theta}} + \frac{1}{K^{2(n-2)}} w_{n-1}^{\bar{\theta}} w_n.
\end{aligned} \tag{30}$$

Here, the Lyapunov function is chosen as

$$\begin{aligned}
V_{n-1}(\tilde{x}, w_1, \dots, w_{n-1}) &= V_{n-2}(\tilde{x}, w_1, \dots, w_{n-2}) \\
&+ \sqrt{|p_1(x)| + \dots + |p_n(x)|} \frac{1}{K^{n-1}} \cdot \frac{1}{\bar{\theta}+1} w_n^{2\bar{\theta}}.
\end{aligned} \tag{31}$$

In the last step through backstepping theory, select the controller

$$u(\hat{x}_{[n]}) = -K^n \sqrt{|p_1(x)| + \dots + |p_n(x)|} v_n w_n^{\bar{\theta}}. \tag{32}$$

Use the last Lyapunov function

$$\begin{aligned}
V_n(\tilde{x}, w_1, \dots, w_n) &= (n+1) \tilde{x}^T P \tilde{x} \\
&+ \sqrt{|p_1(x)| + \dots + |p_n(x)|} \frac{1}{\bar{\theta}+1} \sum_{j=1}^n \frac{1}{K^j} w_j^{\bar{\theta}+1}.
\end{aligned} \tag{33}$$

One can get the last Ito system function:

$$\begin{aligned} LV_n \leq & - \left(n^2 K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} d \left(\sum_{i=1}^n K^{(i-1)\bar{\theta} - (i-1)} \right)^2 \right) \|\bar{x}\|_\infty^{2\bar{\theta}} \\ & - \sum_{j=1}^{n-1} \frac{1}{K^{2j-2}} \left(\left(n - 4^{(\bar{\theta}-1)(j-1)} \right) K - \sqrt{|p_1(x)| + \dots + |p_n(x)|} \bar{\theta} dv_j^{2\bar{\theta}} \right) w_j^{2\bar{\theta}} - \frac{1}{K^n} w_n^{2\bar{\theta}}. \end{aligned} \quad (34)$$

That is, the final output feedback controller is controlled by a high-gain term $K^n \sqrt{|p_1(x)| + \dots + |p_n(x)|}$.

4. System Stability Analysis

The stability analysis of the closed-loop system is performed below, and the main conclusions of this paper are obtained. First, according to system (3) and (28)–(31), one needs to determine the size relationship between

$$\left| \sqrt{|p_1(x)| + \dots + |p_n(x)|} \frac{\bar{\theta} dv_n^{2\bar{\theta}}}{n - 4^{(\bar{\theta}-1)(n-1)}} \right|, \quad (35)$$

$$K^n \sqrt{|p_1(x)| + \dots + |p_n(x)|}.$$

Substituting equation (34) into equation (33), it can get the following inequality:

$$\begin{aligned} K > & \sqrt{|p_1(x)| + \dots + |p_n(x)|} \cdot \frac{\bar{\theta} v_n^{2\bar{\theta}}}{n - 4^{(\bar{\theta}-1)(n-1)}} \cdot \frac{(n+1)\|P\|d^2}{K^{2n-2}} \\ & \cdot \left(\sum_{i=0}^{n-1} (i+1)K^i + \sum_{i=n}^{2n-2} (2n-i-1)K^i \right). \end{aligned} \quad (36)$$

The above formula is equivalent to

$$\begin{aligned} K^{2n-1} & > \frac{n}{n-4^{(\bar{\theta}-1)(n-1)}} \cdot \sqrt{|p_1(x)| + \dots + |p_n(x)|} \cdot \bar{\theta} \cdot v_n^{2\bar{\theta}} \\ & \cdot (n+1)\|P\|d^2 \left(\sum_{i=0}^{n-1} (i+1)K^i + \sum_{i=n}^{2n-2} (2n-i-1)K^i \right). \end{aligned} \quad (37)$$

According to the factorization theorem of the real coefficient polynomial, (36) can be further expressed as

$$\begin{aligned} & (K - K_1)^{m_1} \dots (K - K_r)^{m_r} \\ & \cdot (K^2 + c_1 K + b_1)^{n_1} \dots (K^2 + c_s K + b_s)^{n_s} > 0, \end{aligned} \quad (38)$$

where m_i and n_j are positive integers. It can be concluded that if there is at least one positive number for $K_1, \dots, K_r$ under the condition of the appropriate value of $\bar{\theta}$, one can choose $K^* = \max_{1 \leq i \leq r} \{K_i\}$. Thus, there always exists $K^* \geq 0$ so that the

closed-loop stochastic nonlinear system is globally asymptotically stable in the probability sense. Through the above analysis, one can get the following theorem.

Theorem 1. For stochastic nonlinear system (3) that satisfies function condition Assumption 1, there is a high-gain parameter $K^* \geq 0$ so that output feedback controller (31) designed in Section 3 of this paper can guarantee that, for any system initial value, closed-loop stochastic nonlinear system (3) exists almost everywhere and is unique; and for any system initial value, the closed-loop stochastic nonlinear system is globally asymptotically stable in the probability sense.

On the basis of Theorem 1, an inverse optimal controller is designed for system (3) so that it meets specific performance indicators while achieving the control objectives. First, Lemma 1 is given [37].

Lemma 1. Define the following controller:

$$u = \alpha(x) = -R_2^{-1}(L_{\phi_2} V)^T \frac{\ell \gamma \left(|L_{\phi_2} V R_2^{-1/2}| \right)}{|L_{\phi_2} V R_2^{-1/2}|^2}, \quad (39)$$

where $\gamma(\cdot)$ is a class K_∞ function, $\ell \gamma(s) = s(\gamma)^{-1}(s) - \gamma((\gamma)^{-1}(s))$, and $R_2(x)$ is a matrix value function and satisfies $R_2(x) = R_2^T(x) > 0$. It can be known from Theorem 1 that output feedback controller (31) guarantees that system (3) is globally asymptotically stable in the sense of probability, and then the following controller is constructed:

$$u^* = \beta^*(x) = -\frac{\theta}{2} R_2^{-1}(L_{\phi_2} V)^T \frac{\ell \gamma \left(|L_{\phi_2} V R_2^{-1/2}| \right)}{|L_{\phi_2} V R_2^{-1/2}|^2}, \quad \theta \geq 2, \quad (40)$$

which can minimize the performance function

$$J(u) = E \left(\int_0^\infty \left(l(x) + \theta^2 \gamma \left(\frac{2}{\theta} |R_2(x)^{1/2} u| \right) \right) d\tau \right), \quad (41)$$

to solve the inverse optimal stabilization problem of stochastic nonlinear systems (3). $l(x)$ satisfies the following equation:

$$l(x) = 2\theta \left(\ell \gamma \left(|L_{\phi_2} V R_2^{-1/2}| \right) - \frac{1}{2} \text{Tr} \left(\phi_1^T \frac{\partial^2 V}{\partial x^2} \phi_1 \right) \right) + \theta(\theta - 2) \ell \gamma \left(|L_{\phi_2} V R_2^{-1/2}| \right). \quad (42)$$

Next, the design process of the inverse optimal controller of the system studied in this paper is given. First, the differential equation between the state error and the state estimation is given:

$$\begin{pmatrix} d\tilde{x} \\ d\hat{x} \end{pmatrix} = \bar{\phi}_1(\tilde{x}, \hat{x}) d\omega + \bar{\phi}_2(\tilde{x}, \hat{x}) u dt. \quad (43)$$

Selecting

$$\gamma(s) = \sqrt{|p_1(x)| + \dots + |p_n(x)|} \frac{1}{2K^{2(n-1)}s^2}, \quad (44)$$

one can obtain $(\gamma')^{-1}(s) = K^{2(n-1)}s$. Combining Lemma 1, one obtains

$$u = \beta(\hat{x}) = -R_2^{-1}(\hat{x}) \sqrt{|p_1(x)| + \dots + |p_n(x)|} \frac{1}{K^{2(n-1)}\omega_n^2} \bar{\omega}_n^1 K^{2(n-1)}, \quad (45)$$

where defined $R_2(\hat{x}) \cdot 2K^n \omega_n = 1$. The inverse optimal controller can be obtained as

$$u^* = \beta^*(\hat{x}) = -\frac{\bar{\theta}}{2} R_2^{-1}(\hat{x}) \frac{1}{K^{2(n-1)}} \omega_n^{\bar{\theta}} K^{2(n-1)} = -\frac{\bar{\theta}}{2} R_2^{-1}(\hat{x}) \omega_n^{\bar{\theta}}. \quad (46)$$

Through the above analysis, the theorem of the inverse optimal controller is obtained.

Theorem 2. *The following controller*

$$u^* = -\mu \sqrt{|p_1(x)| + \dots + |p_n(x)|} K^n \omega_n \omega_n^{\bar{\theta}} \quad (47)$$

exists so that stochastic nonlinear system (3) can perform inverse optimal stabilization design in the sense of probability and minimize the following target performance function:

$$J(u) = E \left(\int_0^\infty \left(l(\tilde{x}, \hat{x}) + \frac{1}{K^{2(n-1)}} \frac{v_n^{-1}(\hat{x})}{K^n} u^2 \right) dr \right). \quad (48)$$

5. Simulation Example of Subsea Electroexecution Robot System

The underwater subsea intelligent electroexecution robot composite position control system is used in practical offshore engineering such as oil and gas extraction and production, especially in the field of subsea oil and gas development. In practical engineering applications, this system presents uncertain and nonlinear characteristics. The electrohydraulic composite valve position control system is composed of the water equipment part and underwater equipment part (as shown in Figure 1), where the water part includes power supply unit, hydraulic station, and other power units and other control units; the underwater part includes electromagnetic reversing valve, hydraulic actuator, and electronic control unit controlling the electromagnetic reversing valve. The electronic control unit is internally designed for the controller logic. In actual marine engineering applications, in order to ensure the stability, reliability, and safety of the system, the response speed of the valve position control needs to be stable, accurate, and timely; during the valve position opening/closing process, it is also subject to the hydraulic power unit from the surface. The system is simultaneously subjected to hydraulic driving force, spring restoring force, and the force generated by the fluid medium inside the oil and gas pipeline during the opening of the actuator.

Combining existing research results [38, 39], the model of the electrohydraulic composite valve position control system during the valve position opening process is obtained as follows:

$$p(S - S_1) + \rho g H S_1 = m \frac{d^2 \tau}{dt^2} + \left(\rho g H + \frac{128 \mu l_p Q}{\pi d_p^4} \right) (S - S_2) + k(L_0 + \tau) + f \left(\tau, \frac{d\tau}{dt} \right) + w(t), \quad (49)$$

$$Q_h = v(S - S_1) \left(2 * \frac{(P_1 - P_2)}{\rho_h} \right).$$

Due to the pressure instability of the oil and gas medium and the formation of gravel and other components, it will cause some interference during the opening of the electrohydraulic composite valve position. Therefore, there are interference factors in the pipeline of

Figure 2. At the same time, due to factors such as the change in the viscosity coefficient of the control fluid with temperature and the presence of vibrations in the hydraulic system, the resistance during the valve opening process has the characteristics of meeting the growth

FIGURE 1: Underwater electrohydraulic composite valve position control system.

FIGURE 2: The trajectory of system states.

conditions of functional constraints. Therefore, the model of the underwater valve matches the stochastic nonlinear system we studied above.

Next, we use the observer and controller designed in this paper to control the electrohydraulic valve position and use computer simulation software for simulation analysis and verification. Through the analysis of the electrohydraulic composite valve position control system, the following state variables, measurement outputs, and control output variables are defined:

$$x_1 = \tau, \quad (50)$$

$$x_2 = \frac{d\tau}{dt},$$

$$\begin{aligned} dx_1 &= x_2 dt + dx_1^{p(x)} \sin x_2 d\omega, \\ dx_2 &= u dt + d(x_1^{p(x)} + x_2^{(x)}) d\omega, \\ y &= x_1. \end{aligned} \quad (51)$$

According to the definition of the state variable in the above formula, in actual application, on the one hand, the valve displacement can be measured, but the measurement of the valve displacement speed is difficult, which will directly affect the design of the state feedback controller; on the other hand, in actual application, the valve displacement sensor has a certain transmission delay and packet loss, which also has a certain impact on the design effect of the controller. Combining the measurement output with the “control fluid inlet pressure,” since the monitoring and collection of the “control fluid inlet pressure” are located inside the control module in actual offshore oil and gas production, it has stable measurement and fast transmission effects, so it is established by combining this measurement. Observers and controllers are analyzed using the stability theorem and controller design method proposed in Section 3.sec3

Here, select $p(x) = \sin x$ where control $d = 0.5$ which satisfies the assumptions of Section 2, and select the higher-order gain observer as

FIGURE 3: The trajectory of system observer states.

FIGURE 4: The trajectory of the output feedback controller.

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + K(y - \hat{x}_1), \\ \dot{\hat{x}}_2 &= u + K^2(y - \hat{x}_1).\end{aligned}\quad (52)$$

Using the theory of the third controller design, after recursive operation,

$$\begin{aligned}d^* &= \frac{13}{4} \|\bar{\theta}\| \left(1 + \frac{1}{K}\right)^2, \quad \|\bar{\theta}\| = 20.5, K > 65.8, \\ u^* &= -\|\bar{\theta}\| R_2^{-1}(\hat{x}) w_n^p = -20.5 K^2 v_2 w_2^3.\end{aligned}\quad (53)$$

Furthermore, the parameters of the output feedback tracking controller are set according to Theorems 1 and 2. Then, the system simulation is started, and the initial value is selected. Figures 3 and 4 show the response curve of closed-loop systems (49) and (50), which can be obtained by observing the simulation diagram. The actual and estimated states of two-dimensional systems (49) and (50) are bounded, and it gradually converges to zero. At the same time, the trajectories of the output feedback controller are also given. The choice of different values and the

choice of different initial values will lead to a stable time and control amount. There are significant differences. Simulation results verify the effectiveness of the control scheme.

6. Conclusion

In this paper, for the first time, the output feedback controller is designed for a class of stochastic nonlinear systems that meet the growth conditions of uncertain functions. The multivariate function growth conditions we have assumed here have greatly relaxed the restrictions on the drift and diffusion terms in the original stochastic nonlinear system, making the system under study more general, and the scope of the application of the output feedback controller is also large. Here, we cleverly handle the problem of uncertain functions in the scaling process through the function maxima theory so that the Ito differential system can achieve output stabilization through Lyapunov function design so that the solution of stochastic nonlinear system objects satisfies the existence of uniqueness, ensuring that the system is globally asymptotically stable in the sense of probability, and it is further concluded that the system is inversely optimally stable in the sense of probability. Finally, we apply the theoretical results to the actual awake electrohydraulic valve control system and obtain good results.

There are some problems to be investigated: the systems studied in this theory and the paper are all continuous systems, so the study of control problems for the corresponding discrete systems is a future research direction. By querying the relevant literature, Wang Zidong's team, in the field of control theory, has done a lot of research work on the filtering and control of complex systems under the condition of missing information. Among them, the research on discrete systems has a greater impact on our subsequent research work.

Data Availability

All the data generated or analyzed during this research are included within this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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