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Research Article

Robust Constrained Model Predictive Control for T-S Fuzzy Uncertain System with Data Loss and Data Quantization

Hongchun Qu ,[1] Yu Li,[2] and Wei Liu[2]

[1]*College of Information Science and Technology, Zaozhuang University, Zaozhuang 277000, Shandong, China*
[2]*College of Automation, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*

Correspondence should be addressed to Hongchun Qu; hcchyu@gmail.com

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This paper addresses the robust constrained model predictive control (MPC) for Takagi-Sugeno (T-S) fuzzy uncertain quantized system with random data loss. To deal with the quantization error and the data loss over the networks, the sector bound approach and the Bernoulli process are introduced, respectively. The fuzzy controller and new conditions for stability, which are written as the form of linear matrix inequality (LMI), are presented based on nonparallel distributed compensation (non-PDC) control law and an extended nonquadratic Lyapunov function, respectively. In addition, slack and collection matrices are provided for reducing the conservativeness. Based on the obtained stability results, a model predictive controller which explicitly considers the input and state constraints is synthesized by minimizing an upper bound of the worst-case infinite horizon quadratic cost function. The developed MPC algorithm can guarantee the recursive feasibility of the optimization problem and the stability of closed-loop system simultaneously. Finally, the simulation example is given to illustrate the effectiveness of the proposed technique.

1. Introduction

Model predictive control (MPC) is an effective advanced control method that has aroused extensive attention among the academic and industrial communities in the past decades [1–3]. The main idea of MPC is online solution of an optimization problem to obtain a sequence of optimal control inputs; then, only the first one is implemented and it repeats this procedure with new measurements at the next sampling time [4]. The defining feature of MPC comes from its ability to handle the multivariable systems with hard constraints. A large amount of results on MPC of ideal systems without model uncertainties were reported in [5–7]. However, model uncertainty exists in practical industrials, and hence robust MPC is of theoretical and practical significance and need to be considered. To solve this problem, the authors in [8] firstly proposed a robust MPC synthesis approach for the uncertain system by solving a linear matrix inequalities (LMIs) optimization problem which explicitly considered the input and state constraints. The key point of [8] was to utilize the concept of robust invariant set to guarantee the recursive feasibility of the LMI optimization problem. Since then, much progress has been made in the research of MPC synthesis approach. There are mainly two aspects: one is to reduce the computational burden (see [9, 10]) and the other is to improve the control performance (see [11]).

Meanwhile, networked control systems (NCSs) have been a hot research topic in the last decade which combines sensors, controllers, and actuators via a shared communication network. Compared with traditional point-to-point control systems, NCSs bring incomparable advantages, such as high reliability, easy installation and maintenance, and excellent flexibility [12, 13]. However, some phenomenons, such as data loss and quantization, inevitably exist in the communication channels which would deteriorate the performance or even destroy the stability of the systems. There have emerged many works about the controller design and stability analysis for the NCSs with packet loss and/or quantization (see, e.g., [14–16]). Works on the stabilization problem for NCSs with packet loss can be found in [17–20].

Some nice results about the quantized feedback control problem for NCSs were reported in [21, 22]. Further, most recently, some researchers considered more complex network environment where packet loss and quantization were coexisted in NCSs [23–27].

Some interesting results are presented in the aforementioned documents; however, all of the results are under the assumption that the controlled plant is a linear system without system uncertainty. Unfortunately, almost all the systems in industry are nonlinear with uncertainty. The research studies on the uncertain system have been made by many researchers, and several common descriptions of uncertain systems can be found in [8, 28, 29], such as polytopic paradigm and structured uncertainty. In addition, Takagi–Sugeno (T-S) fuzzy model is employed in this paper to describe the nonlinear system. As we all know that the T-S fuzzy model can transform the nonlinear system into a set of linear submodels by IF-THEN rules instead of handling it directly. In the analysis of T-S fuzzy systems, the controller is designed based on the linear submodels, while the implementation of the controller clearly depends on the membership functions which explicitly consider the nonlinearity of the system. The local linearities of the T-S fuzzy system provide a bridge between the analysis of nonlinear systems and the fruitful linear control theory results [30–35]. For the T-S fuzzy system, some interesting technologies are introduced for the purpose of reducing conservatism, such as nonquadratic Lyapunov function [36, 37], extended nonquadratic Lyapunov function [38, 39] with nonparallel distributed compensation (non-PDC) law, and slack matrices [40]. Recently, some researchers begin applying the T-S fuzzy model to describe complex nonlinear plant in NCSs, and more investigations have been carried out. The controller design and stability analysis for T-S fuzzy model-based NCSs with data losses were considered in [41, 42], and for NCSs with quantization, the same problems were addressed in [44, 45].

There also have been some nice works that introduce MPC to deal with the problem for NCSs represented by the T-S fuzzy system (see, e.g., [43, 46, 47]). The authors in [43] used Bernoulli random binary distribution to model data loss occurring intermittently between the controller and the physical plant. Meanwhile, piecewise approach is more effective for fuzzy systems with trapezoidal membership functions. Consequently, the fuzzy predictive control problem of nonlinear NCSs subject to parameter uncertainties and data loss was considered in [46]. The difference between [43, 46] is that the authors in [46] employed slack matrices to develop less conservative conditions for stability analysis of the closed-loop system. Yu et al. [47] considered more intricate network where data loss and quantization were existed simultaneously. In [48], a fuzzy predictive controller which guaranteed the stability of the closed-loop system was designed in terms of sector bound approach. Tang et al. [49] investigated the output feedback model predictive control for networked control systems with packet loss and data quantization. Although some remarkable results are presented in the foregoing documents, most of the T-S fuzzy systems are described without uncertainty. In addition, the same problem for NCSs, packet loss, data quantization, and the model predictive control strategy, were considered in both of this paper and [49]. However, this paper is different from [49]. First, the interval-type 2 T-S fuzzy model is introduced to describe the nonlinear NCSs, while the type 1 T-S fuzzy model is applied in this paper and the system uncertainty is also considered which is not involved in [49]; second, the result for [49] was obtained based on the assumption that the system states were not measurable and the output feedback MPC strategy was addressed, while this paper assumed that the system states can be sampled and state feedback MPC approach was investigated. It is noted that different assumptions resulted in completely different solutions. Indeed, few works are focused on the problem of reducing the conservatism for fuzzy MPC for NCSs which motivates our research. To illustrate this research more clearly, we provide Table 1 to compare published works with this paper.

This paper focuses on the synthesis approach of the robust constrained MPC with quantization and data loss for the uncertain fuzzy system. The quantization error is regarded as sector bound uncertainties by using the sector bound approach, and the data loss process is described by Bernoulli distributed sequence. A new stability condition for NCSs is achieved based on extended nonquadratic Lyapunov function and additional slack and collection matrices. By optimizing an objective function in the infinite time horizon at each sampling time, a robust MPC optimization problem which explicitly considers the data loss and quantization is presented. Furthermore, the optimization problem is proved to be recursive feasible, and the networked control system turns out to be asymptotically stable.

The remainder of this paper is summarized as follows. The problem formulation is given in Section 2. Section 3 provides the new stability results for fuzzy NCSs. Online synthesis of MPC for fuzzy NCSs is presented in Section 4. Section 5 gives the analysis of feasibility and stability. Section 6 presents a simulation example, and Section 7 draws the conclusion.

Notations. For any vector x and positive-definite matrix W, $\|x\|_W^2 = x^T W x$. R^n and $R^{n\times m}$ denote n-dimensional Euclidean space and the set of $n \times m$ real matrices, respectively. I is the identity matrix with appropriate dimension. For any vector, $x(k+s|k)$ is the value of x at sampling instant $k+s$. The superscript T denotes the transpose of matrix or vector. The symbol $*$ stands for a symmetric element or submatrix. For simplicity, $\mu_t, t \in \{i, j\}$ and $\mu_z^+, z \in \{l, m\}$ are used in place of $\mu_t(f(k))$, $\mu_z(f(k+1))$ and $\mu_t(f(k+s|k))$, $\mu_z(f(k+s+1|k))$, respectively. $\chi_h = \sum_{i=1}^r \mu_i(\cdot)\chi_i$, $\chi_{h+} = \sum_{l=1}^r \mu_l(\cdot)\chi_l$, $\chi_{hh} = \sum_{i=1}^r \sum_{j=1}^r \mu_i(\cdot)\mu_j(\cdot)\chi_{ij}$, and $\chi_{hh+} = \sum_{l=1}^r \sum_{m=1}^r \mu_l(\cdot)\mu_m(\cdot)\chi_{lm}$, with $\chi \in \{A, B, E, K, Q, S\}$.

2. Problem Formulation

Consider a discrete-time T-S fuzzy model with r rules and describe its ith rule as

Table 1: Comparisons of published papers and this paper.

	[13]	[21]	[22]	[25]	[31]	[33]	[35]	[40]	[45]	This paper
Data loss	Yes	No	No	Yes	No	No	No	Yes	No	Yes
Data quantization	No	Yes	Yes	Yes	No	No	No	No	Yes	Yes
Uncertainties	No	No	Yes	No	No	Yes	Yes	No	Yes	Yes
T-S fuzzy model	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Non-PDC control law	No	No	No	No	No	No	Yes	No	No	Yes

$$\begin{aligned}&\text{Rule } i\text{: IF } f_1(k) \text{ is } \xi_1^i, f_2(k) \text{ is } \xi_2^i, \ldots, \text{ and } f_\theta(k) \text{ is } \xi_\theta^i, \text{ THEN}\\&x(k+1) = (A_i + \Delta A_i)x(k) + (B_i + \Delta B_i)u(k), \quad i = \{1, 2, \ldots, r\},\end{aligned} \tag{1}$$

where $\xi_g^i\,(g = 1, 2, \ldots, \theta)$ stands for the fuzzy set of rule i and $f(k) = [f_1(k), \ldots, f_g(k)]$ is the corresponding premise variable which depends on the states of the system; $x(k) \in \Re^n$ and $u(k) \in \Re^m$ denote the system state and control input, respectively; $A_i \in \Re^{n\times n}$ and $B_i \in \Re^{n\times m}$ are known matrices; and ΔA_i and ΔB_i are introduced to describe the system uncertainty, which satisfy

$$[\Delta A_i \;\; \Delta B_i] \in \Omega \coloneqq \left\{[DF(k)E_{1i}\, DF(k)E_{2i}]F(k)^T F(k) \le I, \quad i = 1, 2, \ldots, r, k \ge 0\right\}, \tag{2}$$

where $F(k)$ is an uncertain matrix and D, E_{1i}, and E_{2i} are known matrices.

According to the above discussion, the definition of the inferred T-S fuzzy model can be represented as follows:

$$x(k+1) = \sum_{i=1}^{r} \mu_i(f(k))[(A_i + \Delta A_i)x(k) + (B_i + \Delta B_i)u(k)], \tag{3}$$

where

$$\begin{aligned}\mu_i(f(k)) &= \frac{\varpi_i(f(k))}{\sum_{i=1}^{r}\varpi_i(f(k))},\\ \varpi_i(f(k)) &= \prod_{g=1}^{\theta}\xi_g^i\left(f_g(k)\right), \quad i \in \{1, 2, 3 \ldots, r\},\end{aligned} \tag{4}$$

in which $\varpi_i(x(k))$ represents the weight of the ith rule and $\mu_i(f(k))$ is the membership grade of $f(k)$ in ξ_j^i and satisfies

$$\begin{aligned}&\mu_i(f(k)) \ge 0,\\&\sum_{i=1}^{r}\mu_i(f(k)) = 1, \quad i \in \{1, 2, 3 \ldots, r\}.\end{aligned} \tag{5}$$

In this paper, we assume that the communication network exists in sensor-to-controller (S/C) and controller-to-actuator (C/A) links, and the configuration of quantized NCSs with data loss is shown in Figure 1. At time k, the output of controller is quantized as

$$u_c(k) = \varphi(v(k)), \tag{6}$$

where $\varphi(\cdot)$ denotes a logarithmic quantizer which is defined as

$$\varphi(v) = \begin{cases}\varepsilon_i, & \text{if } \dfrac{1}{1+\tau}\varepsilon_i < v \le \dfrac{1}{1-\tau}\varepsilon_i, v > 0,\\ 0, & \text{if } v = 0,\\ -\varphi(-v), & \text{if } v < 0,\end{cases} \tag{7}$$

in which $\tau = (1 - \rho/1 + \rho)$ and ρ is the quantization density. The set of quantization levels is presented as follows: $v = \{\pm\varepsilon_i, \varepsilon_i = \rho^i\varepsilon_0, i = \pm 1, \pm 2, \ldots\} \cup \{\pm\varepsilon_0\} \cup \{0\}, \quad 0 < \rho < 1$, $\varepsilon_0 > 0$. Each quantization level corresponds to a segment such that the quantizer maps the whole segment. Based on the sector bound approach in [48], the control input can be expressed as follows:

$$u_c(k) = \varphi(v(k)) = (I + \varsigma(k))v(k), \quad \varsigma(k) \in [-\tau, \tau], \tag{8}$$

where $\varsigma(k) = \text{diag}\{\varsigma_1(k), \varsigma_2(k), \ldots, \varsigma_m(k)\}$, $|\varsigma_i(k)| \le \tau_i$.

When $x(k)$ and $u_c(k)$ are transmitted in the communication channel, it may be lost due to the poor network traffic. Hence, two Bernoulli processes are introduced to model the data loss in NCSs.

$$\begin{aligned}x_c(k) &= \alpha(k)x(k),\\ u(k) &= \beta(k)u_c(k),\end{aligned} \tag{9}$$

where $x_c(k)$ is input of controller. $\{\alpha(k)\}$ and $\{\beta(k)\}$ are two mutually independent Bernoulli processes. $\{\alpha(k), \beta(k)\} \in \{0, 1\}$ (1 means that the data are transmitted successfully and 0 indicates that data packet is lost).

Then, we assume that $\{\alpha(k)\}$ and $\{\beta(k)\}$ satisfy the following conditions:

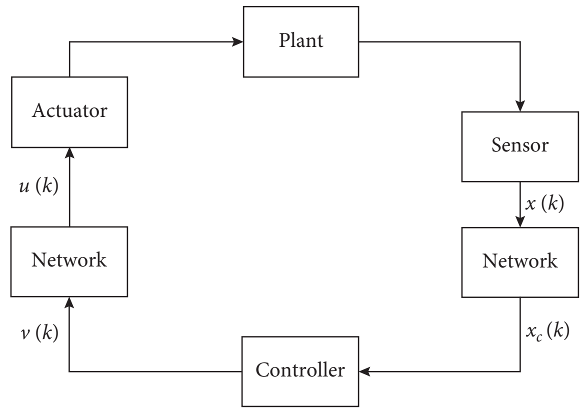

Figure 1: Configuration of quantized NCSs with data loss.

$$\begin{aligned}
&\Pr\{\alpha(k)=1\}=E\{\alpha(k)\}=\overline{\alpha},\\
&\Pr\{\alpha(k)=0\}=1-\overline{\alpha},\\
&\Pr\{\beta(k)=1\}=E\{\beta(k)\}=\overline{\beta},\\
&\Pr\{\beta(k)=0\}=1-\overline{\beta}.
\end{aligned} \quad (10)$$

In the following, the fuzzy controller is presented.

$$\begin{aligned}
&\text{Rule } R^i\text{: IF } f_1(x(k)) \text{ is } \xi_1^i,\ f_2(x(k)) \text{ is } \xi_2^i, \dots, \text{ and } f_\theta(x(k)) \text{ is } \xi_\theta^i, \text{ THEN}\\
&\qquad v(k)=K_iQ_i^{-1}x_c(k),\quad i\in\{1,2,\dots,r\},
\end{aligned} \quad (11)$$

where $K_i\in\mathfrak{R}^{m\times n}$ in which $i\in\{1,2,\dots,r\}$ and $Q_i\in\mathfrak{R}^{n\times n}$ in which $i\in\{1,2,\dots,r\}$ are the controller gains. Then, the state feedback fuzzy controller is defined by

$$\begin{aligned}
v(k)&=\left(\sum_{i=1}^{r}\mu_i(f(k))K_i\right)\left(\sum_{i=1}^{r}\mu_i(f(k))Q_i\right)^{-1}x_c(k)\\
&=K_hQ_h^{-1}x_c(k),\quad i\in\{1,2,\dots,r\}.
\end{aligned} \quad (12)$$

Based on (9) and (12), the following state feedback fuzzy controller is achieved:

$$u(k)=(I+\varsigma(k))\alpha(k)\beta(k)K_hQ_h^{-1}x(k),\quad i\in\{1,2,\dots,r\}. \quad (13)$$

Let $\{\omega(k)\}=\alpha(k)\beta(k)$. It is obvious that if $\alpha(k)=1$ and $\beta(k)=1$, $\omega(k)=1$; otherwise, $\omega(k)=0$. Thus, we have

$$\begin{aligned}
&\Pr\{\omega(k)=1\}=E\{\omega(k)\}=\overline{\omega},\\
&\Pr\{\omega(k)=0\}=1-\overline{\omega}.
\end{aligned} \quad (14)$$

According to (3), (8), and (13), the overall uncertain closed-loop fuzzy system is described as follows:

$$\begin{aligned}
x(k+1)&=\sum_{i=1}^{r}\mu_i(f(k))\left[\left(A_i+\Delta A_i\right)+\left(B_i+\Delta B_i\right)(I+\varsigma(k))\omega(k)K_hQ_h^{-1}\right]x(k)\\
&=\left[\widetilde{A}_h+\overline{\omega}\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}+\widetilde{\omega}(k)\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}\right]x(k),
\end{aligned} \quad (15)$$

where $A_h=\sum_{i=1}^{r}\mu_i(f(k))A_i$, $\Delta A_h=\sum_{i=1}^{r}\mu_i(f(k))\Delta A_i$, $B_h=\sum_{i=1}^{r}\mu_i(f(k))B_i$, $\Delta B_h=\sum_{i=1}^{r}\mu_i(f(k))\Delta B_i$, $K_j=\sum_{i=1}^{r}\mu_i(f(k))K_h$, and $Q_j=\sum_{i=1}^{r}\mu_i(f(k))Q_h$. $\widetilde{A}_h=A_h+\Delta A_h$, $\widetilde{B}_h=B_h+\Delta B_h$, and $\widetilde{\omega}(k)=\omega(k)-\overline{\omega}$. Obviously, $E\{\widetilde{\omega}(k)\}=0$ and $E\{\widetilde{\omega}(k)\widetilde{\omega}(k)\}=\overline{\omega}(1-\overline{\omega})$.

3. New Stability Results for Fuzzy NCSs

Compared with the conditions of stability for traditional control fuzzy systems, a new stability condition which can reduce the conservatism is presented in this section by introducing the notation of slack matrices. This new stability condition is the foundation of the MPC for fuzzy NCSs which will be provided in the next section.

First, the following two lemmas that play a significant role in the proof of Theorem 1 are introduced.

Lemma 1 (see [50]). *For vectors a and b and matrices N, X, Y, and Z of any appropriate dimensions, if* $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$, *then the following inequality holds:*

$$-2a^T N b \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}. \quad (16)$$

$$\begin{aligned} -2a^T N b &\leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} 0 & -N \\ -N^T & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \\ &\leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} 0 & -N \\ -N^T & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \\ &\leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}. \end{aligned} \quad (17)$$

This inequality is generally called Moon inequality. It can be seen that if $N = Y = I$ and $Z = X^{-1}$, then $-2a^T b \leq a^T X a + b^T X^{-1} b$. For any $w, z \in R^n$ and any positive-definite matrix $c \in R^{n\times n}$, the following inequality holds:

$$2w^T z \leq w^T c^{-1} w + z^T c z. \quad (18)$$

Lemma 2 (see [51]). *Symmetric matrix L_1 and matrices L_2 and L_3 satisfy the following inequality:*

$$L_1 + L_2 F(k) L_3 + L_3^T F(k)^T L_2^T < 0, \quad (19)$$

for $F(k)^T F(k) \leq I$, if there exists a scalar $\eta > 0$ such that

$$L_1 + \eta L_2 L_2^T + \eta^{-1} L_3^T L_3 < 0. \quad (20)$$

Next, the following theorem is given to obtain the conditions of the stability for NCSs.

Theorem 1. *Considering the T-S fuzzy system (3), the asymptotic stability is guaranteed if there exist scalars η_{ij} and $\overline{\eta}_{ij}$, matrices K_j, Q_j, and Y, symmetric matrices G_{ii}^l and G_{ij}^l, and $S_i > 0, S_j > 0$ such that*

$$\Psi_{ii}^l \leq -G_{ii}^l, \quad i, l \in \{1, 2, \ldots, r\}, \quad (21)$$

$$\Psi_{ij}^l + \Psi_{ji}^l \leq -G_{ij}^l, \quad j > i, i, j, l \in \{1, 2, \ldots, r\}, \quad (22)$$

$$\begin{bmatrix} 2G_{11}^l & * & \cdots & * \\ G_{12}^l & 2G_{22}^l & \cdots & * \\ \vdots & \vdots & \ddots & * \\ G_{1r}^l & \cdots & G_{(r-1)r}^l & 2G_{rr}^l \end{bmatrix} \geq 0, \quad l \in \{1, 2, \ldots, r\}, \quad (23)$$

where

$$\begin{aligned} \Psi_{ij}^l &= \begin{bmatrix} \overline{d}_1 & * & * \\ \overline{d}_2 & \overline{d}_3 & * \\ \overline{d}_4 & \overline{d}_5 & \overline{d}_6 \end{bmatrix}, \quad i, j, l \in \{1, 2, \ldots, r\}, \\ \overline{d}_1 &= \begin{bmatrix} S_i - Q_j - Q_j^T & * & * \\ A_i Q_j + \overline{\omega} B_i K_j & -S_{l+} + \overline{\omega}^2 B_i \tau Y \tau B_i^T & * \\ \kappa B_i K_j & \kappa\overline{\omega} B_i \tau Y \tau B_i^T & -S_{l+} + \kappa^2 B_i \tau Y \tau B_i^T \end{bmatrix}, \\ \overline{d}_2 &= \begin{bmatrix} E_{1i} Q_j + \overline{\omega} E_{2i} K_j & \overline{\omega}^2 E_{2i} \tau Y \tau B_i^T & \overline{\omega}\kappa E_{2i} \tau Y \tau B_i^T \\ 0 & \eta_{ij} D^T & 0 \\ \kappa E_{2i} Q_j K_j & \overline{\omega}\kappa E_{2i} \tau Y \tau B_i^T & \kappa^2 E_{2i} \tau Y \tau B_i^T \end{bmatrix}, \\ \overline{d}_3 &= \begin{bmatrix} -\eta_{ij} I + \overline{\omega}^2 E_{2i} \tau Y \tau E_{2i}^T & * & * \\ 0 & -\eta_{ij} I & * \\ \overline{\omega}\kappa E_{2i} \tau Y \tau E_{2i}^T & 0 & -\overline{\eta}_{ij} I + \kappa^2 E_{2i} \tau Y \tau E_{2i}^T \end{bmatrix}, \\ \overline{d}_4 &= \begin{bmatrix} 0 & 0 & -\overline{\eta}_{ij} D^T \\ K_j & 0 & 0 \end{bmatrix}, \\ \overline{d}_5 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \overline{d}_6 &= \begin{bmatrix} -\overline{\eta}_{ij} I & * \\ 0 & -Y \end{bmatrix}. \end{aligned} \quad (24)$$

Proof. See Appendix A for details. □

4. Online Synthesis of MPC for Fuzzy NCSs

Based on the stability condition achieved in the above section, the online synthesis of MPC is derived for fuzzy NCSs with the quantization and data loss. Furthermore, the model uncertainty and physical constraints are considered.

The following non-PDC fuzzy predictive controller is used for the T-S fuzzy system:

$$\begin{aligned} u(k+s \mid k) &= \alpha(k)\beta(k)(I + \varsigma(k)) \left(\sum_{i=1}^{r} \mu_i (f(k+s \mid k)) K_i \right) \\ &\quad \times \left(\sum_{i=1}^{r} \mu_i (f(k+s|k)) Q_i \right)^{-1} x(k+s|k) \\ &= \omega(k)(I + \varsigma(k)) K_h Q_h^{-1} x(k+s|k). \end{aligned} \quad (25)$$

In view of (25), we can get the predictive model of uncertain T-S fuzzy NCSs:

$$x(k+s+1|k) = \left[\widetilde{A}_h + \overline{\omega}\widetilde{B}_h (I+\varsigma(k))K_h Q_h^{-1} + \widetilde{\omega}(k)\widetilde{B}_h (I+\varsigma(k))K_h Q_h^{-1}\right] x(k+s|k), \tag{26}$$

where $\widetilde{A}_h = A_h + \Delta A_h$, $\widetilde{B}_h = B_h + \Delta B_h$, and $\widetilde{\omega}(k) = \omega(k) - \overline{\omega}$.

The following is the optimization problem for the online constrained robust fuzzy MPC, which is optimized in the infinite time horizon at each time:

$$\min_{u(k+s|k), s\geq 0} \max_{\varsigma(k), \left[\Delta A_i(k+s)\Delta B_i(k+s)\right]\in\Omega, i\in\{1,2,\ldots,r\}, s\geq 0} J_0^{\infty}(k), \tag{27}$$

where $J_0^{\infty}(k) = \sum_{s=0}^{\infty} E\left\{[x(k+s|k)^T Wx(k+s|k) + u(k+s|k)^T Ru(k+s|k)]\right\}$, with $W>0$ and $R>0$ the weighting matrices. It is obvious that this objective function is related to the model uncertainty, quantization density, and expected values of stochastic variables.

The following extended nonquadratic Lyapunov function is considered:

$$\begin{aligned} V(x(k+s|k)) &= x(k+s|k)^T \left(\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(f(k+s|k))\mu_j(f(k+s|k))S_{ij}\right)^{-1} \\ &\quad \times x(k+s|k) \\ &= x(k+s|k)^T S_{hh}^{-1} x(k+s|k), \quad S_{ij}>0, s\geq 0. \end{aligned} \tag{28}$$

Then, imposing the following stability constraint at each time,

$$\begin{aligned} &E\{V(x(k+s+1|k)) - V(x(k+s|k))\} \\ &\quad \leq -E\left\{\left[x(k+s|k)^T Wx(k+s|k) + u(k+s|k)^T Ru(k+s|k)\right]\right\}. \end{aligned} \tag{29}$$

Summing up both sides of (29) from $s=0$ to $s=\infty$, we have

$$E\{V(x(k+\infty|k)) - V(x(k|k))\} \leq -E\sum_{s=0}^{\infty}\left[x(k+s|k)^T Wx(k+s|k) + u(k+s|k)^T Ru(k+s|k)\right]. \tag{30}$$

As we all know, when $s \longrightarrow \infty$, $E\{x(k+s|k)\}=0$. Then, we can obtain $\lim_{s\longrightarrow\infty} E\{V(x(k+s|k))\}=0$. Thus, $J_0^{\infty}(k) \leq E\{V(x(k|k))\}$, which is equal to

$$\max_{\left[\Delta A_i(k+s)\Delta B_i(k+s)\right]\in\Omega, i\in\{1,2,\ldots,r\}} J_0^{\infty}(k) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(k|k))\mu_j(\xi(k|k))\|x(k|k)\|_{S_{ij}^{-1}}^2, \tag{31}$$

where $s\geq 0$.

Hence, the problem of minimizing J_0^{∞} is transformed into minimizing a scalar γ satisfying

$$x(k|k)^T \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(k|k))\mu_j(\xi(k|k))S_{ij}^{-1}x(k|k) \leq \gamma. \tag{32}$$

Moreover, input and state constraints are

$$E\left\{\left\|u_a(k+s|k)\right\|_2\right\} \leq \bar{u}, \quad a=1,2,\ldots,n_u, \tag{33}$$

$$E\left\{\left\|(\zeta x(k+s+1|k))_a\right\|_2\right\} \leq \bar{x}, \quad a=1,2,\ldots,n_x, \tag{34}$$

where $u_a(k+s|k)$ and $x_a(k+s+1|k)$ are the *ath* component of the vector $u(k+s|k)$ and $x(k+s+1|k)$, respectively.

Can be solved by the following theorem.

Theorem 2. *Consider the uncertain fuzzy NCS (26) and assume that the communication link parameter $\bar{\omega}$ and quantization density ρ are known. The non-PDC state feedback control law (25) can guarantee the robust asymptotically stability of the closed-loop fuzzy system if there exist scalars γ, η_{ij}, $\bar{\eta}_{ij}$, η_{ijlm}, and $\bar{\eta}_{ijlm}$, positive-definite matrices $\widetilde{S}_{ij}$, $\widetilde{S}_{lm}$, and G_{ij}^{lm}, and any matrices K_j, Q_j, $G_{ij}^{lm}=(G_{ji}^{lm})^T$, and $U_{lm}=(U_{ml})^T\,(m>l)$ such that the following minimization problem is feasible:*

$$\min_{\gamma,\eta_{ij},\eta_{ijlm},\bar{\eta}_{ij},\bar{\eta}_{ijlm},\widetilde{S}_{ij},\widetilde{S}_{lm},G_{ii}^{lm},G_{ij}^{lm},Q_j,K_j,U_{ll},U_{lm},N,O,T} \gamma, \tag{35}$$

subject to

$$\begin{bmatrix} 1 & * \\ x(k|k) & \widetilde{S}_{ij} \end{bmatrix} \geq 0, \quad i,j \in \{1,2,\ldots,r\}, \tag{36}$$

$$\Psi_{ii}^{lm} \leq -G_{ii}^{lm}, \quad i,l,m \in \{1,2,\ldots,r\}, \tag{37}$$

$$\Psi_{ij}^{lm} + \Psi_{ji}^{lm} \leq -G_{ij}^{lm} - G_{ji}^{lm}, \quad j>i,\; i,j,l,m \in \{1,2,\ldots,r\}, \tag{38}$$

$$\phi_{ll} \geq 0, \quad l \in \{1,2,\ldots,r\}, \tag{39}$$

$$\phi_{lm} + \phi_{ml} \geq 0, \quad m>l,\; l,m \in \{1,2,\ldots,r\}, \tag{40}$$

where

$$\Psi_{ij}^{lm} = \begin{bmatrix} m_1 & * & * \\ m_2 & m_3 & * \\ m_4 & m_5 & m_6 \end{bmatrix}, \tag{41}$$

$$\phi_{lm} = \begin{bmatrix} G_{11}^{lm}-U_{lm} & G_{12}^{lm}-U_{lm} & \cdots & G_{1r}^{lm}-U_{lm} \\ G_{21}^{lm}-U_{lm} & G_{22}^{lm}-U_{lm} & \cdots & G_{2r}^{lm}-U_{lm} \\ \vdots & \vdots & \ddots & \vdots \\ G_{r1}^{lm}-U_{lm} & \cdots & \cdots & G_{rr}^{lm}-U_{lm} \end{bmatrix}, \quad l,m \in \{1,2,\ldots,r\}, \tag{42}$$

$$m_1 = \begin{bmatrix} \bar{m}_1 & * & * \\ A_iQ_j + \bar{\omega}B_iK_j & \bar{m}_2 & * \\ \kappa B_iK_j & \bar{\omega}\kappa B_i\tau N\tau B_i^T & \bar{m}_3 \end{bmatrix},$$

$$m_2 = \begin{bmatrix} E_{1i}Q_j + \bar{\omega}E_{2i}K_j & \bar{\omega}^2 E_{2i}\tau N\tau B_i^T & \bar{\omega}\kappa E_{2i}\tau N\tau B_i^T \\ 0 & \eta_{ijlm}D^T & 0 \\ \kappa E_{2i}K_j & \bar{\omega}\kappa E_{2i}\tau N\tau B_i^T & \kappa^2 E_{2i}\tau N\tau B_i^T \end{bmatrix},$$

$$m_3 = \begin{bmatrix} \bar{m}_4 & * & * \\ 0 & -\eta_{ijlm}I & * \\ \bar{\omega}\kappa E_{2i}\tau N\tau E_{2i}^T & 0 & \bar{m}_5 \end{bmatrix},$$

$$m_4 = \begin{bmatrix} 0 & 0 & \bar{\eta}_{ijlm}D^T \\ W^{1/2}Q_j & 0 & 0 \\ \varrho K_j & \bar{\omega}\varrho\tau N\tau B_i^T & \kappa\varrho\tau N\tau B_i^T \\ K_j & 0 & 0 \end{bmatrix},$$

$$m_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \bar{\omega}\varrho\tau N\tau E_{2i}^T & 0 & \kappa\varrho\tau N\tau E_{2i}^T \\ 0 & 0 & 0 \end{bmatrix},$$

$$m_6 = \begin{bmatrix} -\overline{\eta}_{ijlm}I & * & * & * \\ 0 & -\gamma I & * & * \\ 0 & 0 & -\gamma I + \overline{\varrho}^2 \tau N \tau & * \\ 0 & 0 & 0 & -N \end{bmatrix},$$

$$\overline{m}_1 = \widetilde{S}_{ij} - Q_j - Q_j^T,$$

$$\overline{m}_2 = -\widetilde{S}_{lm+} + \overline{\omega}^2 B_i \tau N \tau B_i^T,$$

$$\overline{m}_3 = -\widetilde{S}_{lm+} + \kappa^2 B_i \tau N \tau B_i^T,$$

$$\overline{m}_4 = -\eta_{ijlm} I + \overline{\omega}^2 E_{2i} \tau N \tau E_{2i}^T,$$

$$\overline{m}_5 = -\overline{\eta}_{ijlm} I + \kappa^2 E_{2i} \tau N \tau E_{2i}^T,$$

$$\begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1r} \\ U_{21} & U_{22} & \cdots & U_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ U_{r1} & U_{r2} & \cdots & U_{rr} \end{bmatrix} \geq 0. \tag{43}$$

Proof. See Appendix B for details. □

Then, we handle the input constraint (33). For any $[\Delta A_i \;\; \Delta B_i] \in \Omega, i \in \{1, 2, \ldots, r\}$, we have

$$\begin{aligned} &\max_{s\geq 0} E\left\{\left|u_a(k+s|k)\right|^2\right\} \\ &= \max_{s\geq 0} E\left\{\left|\left((\overline{\omega} + \omega - \overline{\omega})(I + \varsigma(k)) K_h Q_h^{-1} x(k+s|k)\right)_a\right|^2\right\} \\ &\leq \max_{x(k+s|k)\in\varepsilon(\widetilde{S}_{hh})} \left\|\left(\overline{\omega}(I + \varsigma(k)) K_h Q_h^{-1} \widetilde{S}_{hh}^{1/2}\right)_a\right\|_2^2 \left\|\left(\widetilde{S}_{hh}^{-(1/2)} x(k+s|k)\right)_a\right\|_2^2 \\ &\leq \left\|\left(\overline{\omega}(I + \varsigma(k)) K_h Q_h^{-1} \widetilde{S}_{hh}^{1/2}\right)_a\right\|_2^2, \quad a \in \{1, 2, \ldots, n_u\}. \end{aligned} \tag{44}$$

If there exists a symmetric matrix X that satisfies

$$\begin{aligned} &X - \overline{\omega}^2 (I + \varsigma(k)) K_h Q_h^{-1} \widetilde{S}_{hh} \left[(I + \varsigma(k)) K_h Q_h^{-1}\right]^T \geq 0 \\ &X_{aa} \leq u_{a,\max}^2, \quad a \in \{1, 2, \ldots, n_u\}. \end{aligned} \tag{45}$$

By applying the Schur complement and $Q_h^T S_h^{-1} Q_h \geq Q_h + Q_h^T - S_h$ [52], (45) becomes

$$\begin{bmatrix} \widetilde{S}_{ij} - Q_j - Q_j^T & * \\ \overline{\omega}(I + \varsigma(k)) K_j & -X \end{bmatrix} \leq 0, \quad X_{aa} \leq u_{a,\max}^2, a \in \{1, 2, \ldots, n_u\}, \tag{46}$$

which equals to

$$\begin{bmatrix} \widetilde{S}_{ij} - Q_j - Q_j^T & * \\ \overline{\omega} K_j & -X \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{\omega} \end{bmatrix} \varsigma(k) \begin{bmatrix} K_j & 0 \end{bmatrix} + \begin{bmatrix} K_j^T \\ 0 \end{bmatrix} \varsigma(k) \begin{bmatrix} 0 & \overline{\omega} \end{bmatrix} \leq 0. \tag{47}$$

Using Lemma 2 and Schur complement, it yields

$$\begin{bmatrix} \widetilde{S}_{ij} - Q_j - Q_j^T & * & * \\ \overline{\omega} K_j & -X + \overline{\omega}\tau O \tau \overline{\omega} & * \\ K_j & 0 & -O \end{bmatrix} \leq 0, \tag{48}$$

where O is a positive-definite matrix. In consequence, (33) is guaranteed by (49) through the convex feature of μ_i and μ_j.

In addition, we deal with the state constraint (34). For any $\begin{bmatrix} \Delta A_i & \Delta B_i \end{bmatrix} \in \Omega, i \in \{1, 2, \ldots, r\}$, we can achieve

$$\begin{aligned} &\max_{s\geq 0} E\left\{ \left| (\zeta x(k+s+1|k))_a \right|^2 \right\} \\ &= \max_{s\geq 0} E\left\{ \left| \zeta_a \left(\left[\widetilde{A}_h + \overline{\omega} \widetilde{B}_h (I + \varsigma(k)) K_h Q_h^{-1} + \widetilde{\omega}(k+s|k) \times \widetilde{B}_h (I + \varsigma(k)) K_h Q_h^{-1} \right] x(k+s|k) \right)_a \right|^2 \right\} \\ &\leq \max_{x(k+s|k)\in\varepsilon(\widetilde{S}_{hh})} \left\| \zeta_a \left(\left[\widetilde{A}_h + \overline{\omega} \widetilde{B}_h (I + \varsigma(k)) K_h Q_h^{-1} \right] \widetilde{S}_{hh}^{1/2} \right)_a \right\|_2^2 \left\| \left(\widetilde{S}_{hh}^{-(1/2)} x(k+s|k) \right)_a \right\|_2^2 \\ &\leq \left\| \zeta_a \left(\left[\widetilde{A}_h + \overline{\omega} \widetilde{B}_h (I + \varsigma(k)) K_h Q_h^{-1} \right] \widetilde{S}_{hh}^{1/2} \right)_a \right\|_2^2. \end{aligned} \tag{49}$$

If there exists a symmetric matrix Z that satisfies

$$\begin{aligned} &Z - \zeta \left(\widetilde{A}_h Q_h + \overline{\omega} \widetilde{B}_h (I + \varsigma(k)) K_h \right) Q_h^{-1} \widetilde{S}_{hh} Q_h^{-T} \left(\widetilde{A}_h Q_h + \overline{\omega} \widetilde{B}_h (I + \varsigma(k)) K_h \right)^T \zeta^T \geq 0 \\ &Z_{aa} \leq x_{a,\max}^2, \quad a \in \{1, 2, \ldots, n_x\}. \end{aligned} \tag{50}$$

Using Lemmas 1 and 2 and Schur complement, (50) holds if

$$\begin{aligned} &\begin{bmatrix} \widetilde{S}_{ij} - Q_j - Q_j^T & * & * & * \\ \zeta \left(A_i Q_j + \overline{\omega} B_i (I + \varsigma(k)) K_j \right) & -Z & * & * \\ E_{1i} Q_j + \overline{\omega} E_{2i} (I + \varsigma(k)) K_j & 0 & -\eta_{ij} I & * \\ 0 & \eta_{ij} D^T \zeta^T & 0 & -\eta_{ij} I \end{bmatrix} \leq 0, \\ &Z_{aa} \leq x_{a,\max}^2, \quad a \in \{1, 2, \ldots, n_x\}, \end{aligned} \tag{51}$$

for all $F(k+s|k)$ satisfying $F(k+s|k)^T F(k+s|k) \leq I$. Equation (52) can be transformed into

$$\begin{aligned} &\begin{bmatrix} \widetilde{S}_{ij} - Q_j - Q_j^T & * & * & * \\ \zeta \left(A_i Q_j + \overline{\omega} B_i K_j \right) & -Z & * & * \\ E_{1i} Q_j + \overline{\omega} E_{2i} K_j & 0 & -\eta_{ij} I & * \\ 0 & \eta_{ij} D^T \zeta^T & 0 & -\eta_{ij} I \end{bmatrix} + \begin{bmatrix} 0 \\ \zeta \overline{\omega} B_i \\ \overline{\omega} E_{2i} \\ 0 \end{bmatrix} \varsigma(k) \begin{bmatrix} K_j^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \\ &+ \begin{bmatrix} K_j & 0 & 0 & 0 \end{bmatrix}^T \varsigma(k) \begin{bmatrix} 0 & (\zeta \overline{\omega} B_i)^T & (\overline{\omega} E_{2i})^T & 0 \end{bmatrix} \leq 0. \end{aligned} \tag{52}$$

According to Lemma 1 and utilizing Schur complement, the following inequality is obtained:

$$\begin{bmatrix} \widetilde{S}_{ij} - Q_j - Q_j^T & * & * & * & * \\ \zeta\left(A_i Q_j + \overline{\omega} B_i K_j\right) & m_{18} & * & * & * \\ E_{1i} Q_j + \overline{\omega} E_{2i} K_j & \overline{\omega} E_{2i} \tau T \tau \left(\zeta \overline{\omega} B_i\right)^T & -\eta_{ij} I + \overline{\omega} E_{2i} \tau T \tau \left(\overline{\omega} E_{2i}\right)^T & * & * \\ 0 & \eta_{ij} D^T \zeta^T & 0 & -\eta_{ij} I & * \\ K_j & 0 & 0 & 0 & -T \end{bmatrix} \leq 0, \quad (53)$$

where $m_{18} = -Z + \zeta \overline{\omega} B_i \tau T \tau (\zeta \overline{\omega} B_i)^T$ and T is a positive-definite matrix. We can conclude that (34) is guaranteed by (53). The proof is completed.

Based on the above discussions, the solution of (35) can be obtained by solving the following optimization problem:

$$\begin{aligned} &\min_{\gamma,\Omega} \gamma \\ &\text{subject to} \, (27) - (34) \text{ and } (44), \end{aligned} \quad (54)$$

where $\Omega = \{\eta_{ij}, \overline{\eta}_{ij}, \eta_{ijlm}, \overline{\eta}_{ijlm}, \widetilde{S}_{ij}, \widetilde{S}_{lm}, G_{ii}^{lm}, G_{ij}^{lm}, Q_j, K_j, U_{ll}, U_{lm}, X, Z, N, O, T\}$. At time instant $k \geq 0$, optimization problem (54) is solved online and we can get optimal control sequence $u(k+s|k), s \geq 0$. Then, only the first control input $u(k|k)$ is implemented.

The implementation steps of designing RMPC are summarized in Algorithm 1.

The complexity of solving the LMI optimization problem is polynomial time, which (regarding the fastest interior-point algorithms) is proportional to $K^3 L$, where K is the total number of scalar variables and L is the total row size of the LMI system. For the optimization problem which is described by (54), the total number of scalar variables is$K = 1 + 2r^2 + 2r^4 + n_x(n_x+1)\ r^2 + rn_x^2 + rn_x n_u + (1/2)n_x\ (n_x+1) + 2n_u\ (n_u + 1) + (8n_x + 2n_u)(8n_x + 2n_u + 1)(r^4 + r^2)$ and the total row size of the LMI system is $L = (8n_x + 2n_u)(3/4)r^4 + (8n_x + 2n_u)r^3 + (6n_x + 3n_u + 1)r^2$, where r is the number of inference rules. It can be seen that the increase in r, n_x, and n_u will increase the computational complexity exponentially. Let $r = n_x = n_u$ and take the most influential parts of K and L, respectively. Then, the computational complexity is $(15/2)n_x^{24}$.

5. The Analysis of Feasibility and Stability

5.1. The Analysis of Feasibility

Theorem 3. *Consider the uncertain fuzzy NCS (26), if any feasible solution is achieved from the optimization problem (55) at time k, then the optimization problem (55) is feasible for all time $t > k$.*

Proof. See Appendix C for details. □

5.2. The Analysis of Stability

Theorem 4. *Consider the uncertain fuzzy NCS (26). The control input $u(k)$, which is applied in a receding horizon way by solving optimization problem (55), robustly asymptotically stabilizes the closed-loop fuzzy NCSs.*

Proof. See Appendix D for details. □

6. Simulation Example

In this section, the following example of a continuous stirred tank reactor (CSTR) is given to illustrate the effectiveness of the proposed method.

Example 1. As shown in Figure 2, consider the nonlinear model of CSTR which has been described in [11].

Assuming constant liquid volume, the CSTR liquid volume, the CSTR for an exothermic, irreversible reaction, i.e., $A \longrightarrow B$, is presented by the following dynamic model based on a component for reactant A and an energy balance:

$$\begin{aligned} \dot{C}_A &= \frac{q}{V}\left(C_{Af} - C_A\right) - k_0 \exp\left(-\frac{E/R}{T}\right) C_A, \\ \dot{T} &= \frac{q}{V}\left(T_f - T\right) - \frac{(-\Delta H)}{\rho C_p} k_0 \exp\left(-\frac{E/R}{T}\right) C_A + \frac{UA}{V \rho C_p}\left(T_c - T\right), \end{aligned} \quad (55)$$

where C_A, T, and T_c denote the concentration of A in the reactor, the reactor temperature, and temperature of the coolant stream, respectively.

The aim of using CSTR is to regulate T by manipulating T_c, satisfying the constraint $T_c^l \leq T_c \leq T_c^u$. The system state and input variables are set as $x = [C_A - C_A^{\text{eq}} T - T^{\text{eq}}]^T$, $u = T_c - T_c^{\text{eq}}$. In addition, let $y = x_2$. In this example, we choose x_2 as the premise variable of the fuzzy system and $\underline{x}_2 \leq x_2 \leq \overline{x}_2$, where $\underline{x}_2 = T^l - T^{\text{eq}}$ and $\overline{x}_2 = T^u - T^{\text{eq}}$. Then, the following equations are defined:

(1) Set the communication link parameter $\overline{\omega}$ and quantization density ρ
(2) At time $k \geq 0$, solve the optimization problem (54) and get $\gamma, \eta_{ij}, \overline{\eta}_{ij}, \eta_{ijlm}, \overline{\eta}_{ijlm}, \widetilde{S}_{ij}, \widetilde{S}_{lm}, G_{ii}^{lm}, G_{ij}^{lm}, Q_j, K_j, U_{ll}, U_{lm}, X, Z, N, O, T$
(3) Using (25), calculate the optimal control sequence $u(k+s|k), s \geq 0$
(4) Quantize and transmit the first control input $u(k|k)$ into the communication network
(5) Update time to $k+1$ and then return to step (2)

Algorithm 1: Robust constrained model predictive control.

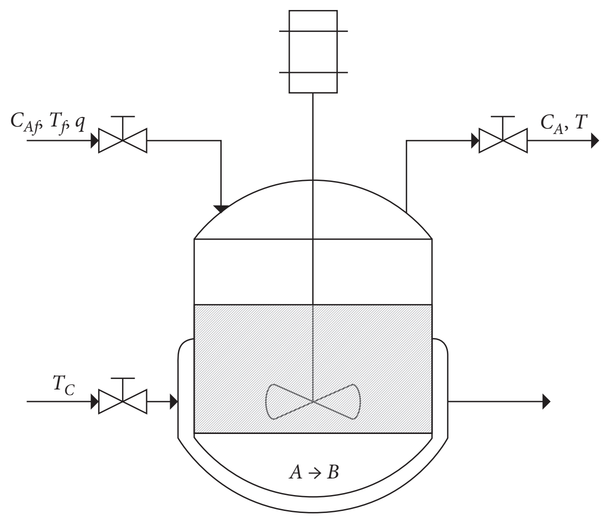

Figure 2: Structure of the continuous stirred tank reactor.

$$
\begin{aligned}
g_1(x_2) &= k_0 \exp\left(-\frac{E/R}{x_2 + T^{\mathrm{eq}}}\right),\\
g_1^0 &= \frac{\left[g_1(\underline{x}_2) + g_1(\overline{x}_2)\right]}{2},\\
g_2(x_2) &= k_0 \left[\exp\left(-\frac{E/R}{x_2 + T^{\mathrm{eq}}}\right) - \exp\left(-\frac{E/R}{T^{\mathrm{eq}}}\right)\right] C_A^{\mathrm{eq}} \frac{1}{x_2},\\
g_2^0 &= \frac{\left[g_2(\underline{x}_2) + g_2(\overline{x}_2)\right]}{2},\\
\varpi_1(x_2) &= g_1(x_2) - g_1^0,\\
\varpi_2(x_2) &= g_2(x_2) - g_2^0.
\end{aligned} \tag{56}
$$

The membership functions are described as

$$
\begin{aligned}
\overline{\mu}_1(x_2) &= \frac{1}{2}\,\frac{\varpi_1(x_2) - \varpi_1(-\underline{x}_2)}{\varpi_1(\overline{x}_2) - \varpi_1(\underline{x}_2)},\\
\overline{\mu}_2(x_2) &= \frac{1}{2}\,\frac{\varpi_1(\overline{x}_2) - \varpi_1(x_2)}{\varpi_1(\overline{x}_2) - \varpi_1(\underline{x}_2)},\\
\overline{\mu}_3(x_2) &= \frac{1}{2}\,\frac{\varpi_2(x_2) - \varpi_2(\underline{x}_2)}{\varpi_2(\overline{x}_2) - \varpi_2(\underline{x}_2)},\\
\overline{\mu}_4(x_2) &= \frac{1}{2}\,\frac{\varpi_2(\overline{x}_2) - \varpi_2(x_2)}{\varpi_2(\overline{x}_2) - \varpi_2(\underline{x}_2)}.
\end{aligned} \tag{57}
$$

Thus, (55) can be transformed into the following T-S fuzzy model:

$$A_{1c} = \begin{bmatrix} -\frac{q}{V} - g_1^0 - 2\varpi_1(\overline{x}_2) & -g_2^0 \\ \frac{(-\Delta H)}{\rho C_p} g_1^0 + 2\frac{(-\Delta H)}{\rho C_p}\varpi_1(\overline{x}_2) & -\frac{q}{V} - \frac{UA}{V\rho C_p} + \frac{(-\Delta H)}{\rho C_p} g_2^0 \end{bmatrix},$$

$$A_{2c} = \begin{bmatrix} -\frac{q}{V} - g_1^0 - 2\varpi_1(\underline{x}_2) & -g_2^0 \\ \frac{(-\Delta H)}{\rho C_p} g_1^0 + 2\frac{(-\Delta H)}{\rho C_p}\varpi_1(\underline{x}_2) & -\frac{q}{V} - \frac{UA}{V\rho C_p} + \frac{(-\Delta H)}{\rho C_p} g_2^0 \end{bmatrix},$$

$$A_{3c} = \begin{bmatrix} -\frac{q}{V} - g_1^0 & -g_2^0 - 2\varpi_2(\overline{x}_2) \\ \frac{(-\Delta H)}{\rho C_p} g_1^0 & -\frac{q}{V} - \frac{UA}{V\rho C_p} + \frac{(-\Delta H)}{\rho C_p} g_2^0 + 2\frac{(-\Delta H)}{\rho C_p}\varpi_2(\overline{x}_2) \end{bmatrix},$$

$$A_{4c} = \begin{bmatrix} -\frac{q}{V} - g_1^0 & -g_2^0 - 2\varpi_2(\underline{x}_2) \\ \frac{(-\Delta H)}{\rho C_p} g_1^0 & -\frac{q}{V} - \frac{UA}{V\rho C_p} + \frac{(-\Delta H)}{\rho C_p} g_2^0 + 2\frac{(-\Delta H)}{\rho C_p}\varpi_2(\underline{x}_2) \end{bmatrix},$$

$$B_{1c} = B_{2c} = B_{3c} = B_{4c} = \begin{bmatrix} 0 \\ \frac{UA}{V\rho C_p} \end{bmatrix}. \quad (58)$$

Choose sampling period $T_s = 0.06$ min. The values and meaning of other parameters are given in Table 2.

$$\begin{aligned} A_{1d} &= \begin{bmatrix} 0.7899 & -0.0020 \\ 7.1638 & 0.9232 \end{bmatrix}, \\ A_{2d} &= \begin{bmatrix} 0.9588 & -0.0022 \\ -0.8035 & 0.9326 \end{bmatrix}, \\ A_{3d} &= \begin{bmatrix} 0.8678 & -0.0035 \\ 3.4932 & 0.9951 \end{bmatrix}, \\ A_{4d} &= \begin{bmatrix} 0.8728 & -0.0007 \\ 3.2519 & 0.8650 \end{bmatrix}, \\ B_{1d} &= \begin{bmatrix} -0.0001 & 0.1208 \end{bmatrix}^T, \\ B_{2d} &= \begin{bmatrix} -0.0001 & 0.1212 \end{bmatrix}^T, \\ B_{3d} &= \begin{bmatrix} -0.0002 & 0.1254 \end{bmatrix}^T, \\ B_{4d} &= \begin{bmatrix} -0.000048 & 0.1169 \end{bmatrix}^T. \end{aligned} \quad (59)$$

In this section, for simplicity, we adopt the following T-S fuzzy model:

$$x(k+1) = \sum_{i=1}^{2} \mu_i(x_2(k))(A_i x(k) + B_i u(k)), \quad (60)$$

with

Table 2: The operating parameters of CSTR.

Parameter	Description	Value	Unit
q	Process flow rate	100	L/min
T_f	Actual feed temperature	350	K
C_{Af}	Feed concentration	1	mol/L
V	Reactor volume	100	L
ρ	Liquid density	1000	g/L
C_p	Heat capacity	0.239	J/g K
ΔH	Heat of reaction	−120,000	J/mol
E/R	Activation energy	8750	K
k_0	Reaction rate constant	7.2×10^{10}	$\min^{-1}$
UA	Heat transfer coefficient	50,000	J/min K

$$\begin{aligned} \mu_1(x_2(k)) &= \overline{\mu}_1(x_2(k)) + \overline{\mu}_3(x_2(k)), \\ \mu_2(x_2(k)) &= \overline{\mu}_2(x_2(k)) + \overline{\mu}_4(x_2(k)), \\ A_1 &= \frac{1}{2}(A_{1d} + A_{3d}), \\ A_2 &= \frac{1}{2}(A_{2d} + A_{4d}), \\ B_1 &= \frac{1}{2}(B_{1d} + B_{3d}), \\ B_2 &= \frac{1}{2}(B_{2d} + B_{4d}). \end{aligned} \quad (61)$$

According to the above discussion, the T-S fuzzy model can be summarized as

$$\begin{aligned} &\text{Rule 1: IF } x_2(k) \text{ is } \Gamma_1^1, \text{ THEN} \\ &\quad x(k+1) = (A_1 + \Delta A_1(k))x((k)(B_1 + \Delta B_1(k)))u(k), \\ &\text{Rule 2: IF } x_2(k) \text{ is } \Gamma_1^2, \text{ THEN} \\ &\quad x(k+1) = (A_2 + \Delta A_2(k))x((k)(B_2 + \Delta B_2(k)))u(k), \end{aligned} \quad (62)$$

with

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.8288 & -0.0027 \\ 5.3285 & 0.9592 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0.9158 & -0.0014 \\ 1.2242 & 0.8988 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} -0.0002 & 0.1231 \end{bmatrix}^T, \\ B_2 &= \begin{bmatrix} -0.0001 & 0.1191 \end{bmatrix}^T. \end{aligned} \quad (63)$$

Considering the uncertainty of the T-S fuzzy system, the following conditions must be satisfied:

$$\begin{aligned} \Delta A_1(k) &= DF(k)E_{11}, \\ \Delta A_2(k) &= DF(k)E_{12}, \\ \Delta B_1(k) &= DF(k)E_{21}, \\ \Delta B_2(k) &= DF(k)E_{22}, \end{aligned} \quad (64)$$

Table 3: The comparison of different methods.

Methods	Sate feedback gain at $k = 1$	Performance costs J
Theorem 2, $\delta = 0.0191$	$[-6.90906, -0.47676]$	0.6686
Theorem 2, $\delta = 0.6567$	$[-6.94362, -0.48018]$	0.6575
Corollary 7 in [43]	$[-3.2213, -0.169]$	0.7075

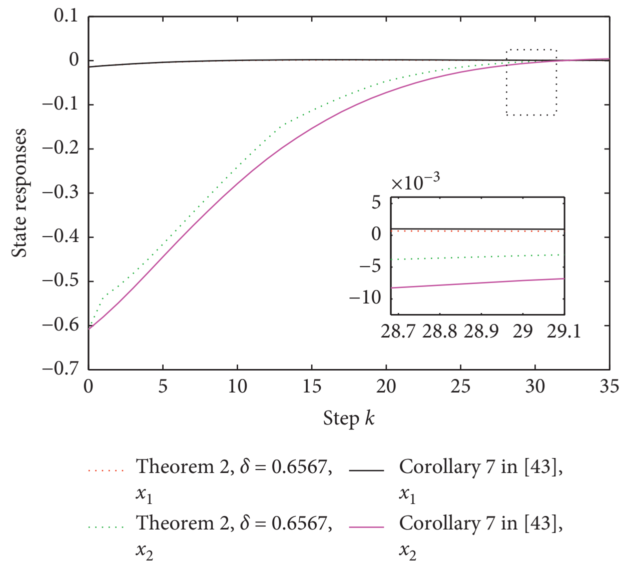

Figure 3: State trajectory.

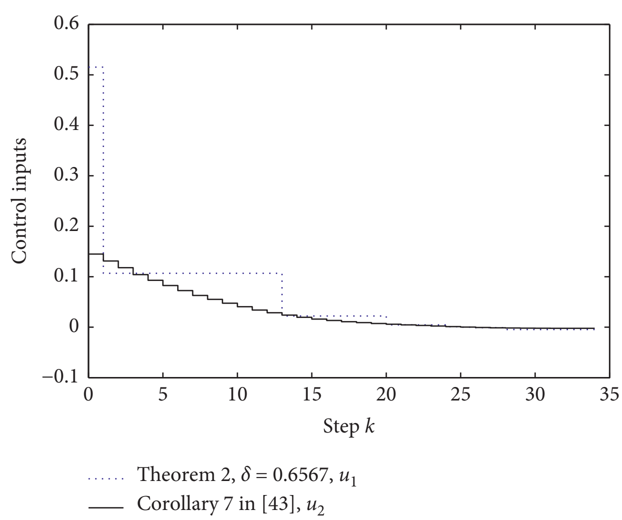

Figure 4: Control input.

with $D = I$, $E_{11} = \mathrm{diag}\{0.01; 0.001\}A_1$, $E_{12} = \mathrm{diag}\{0.01; 0.001\}A_2$, $E_{21} = \mathrm{diag}\{0.05; 0.005\}B_1$, and $E_{22} = \mathrm{diag}\{0.01; 0.001\}B_2$. For more rational analysis, we assume that $F(k)$ is norm-bounded and satisfies $F(k) = \sin(k)$, $F(k)^T F(k) \le I$.

The control input and state constraints must satisfy $|u(k)| \le 3.5K$, $|x_2(k)| \le 10K$, $s \ge 0$. We choose the initial conditions as $x(0) = [-0.01425; -0.60872]$ and $u(0) = -0.14$. Let $W = \mathrm{diag}\{0.5; 0.5\}$ and $R = 1I$. Assume

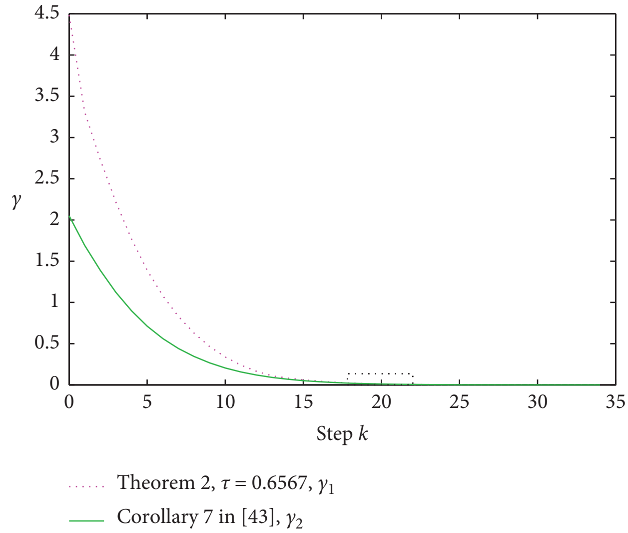

Figure 5: $\gamma(k)$ trajectory.

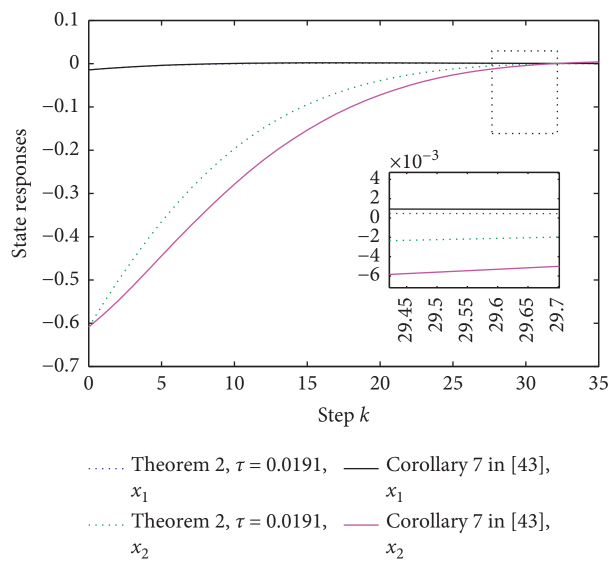

Figure 6: State trajectory.

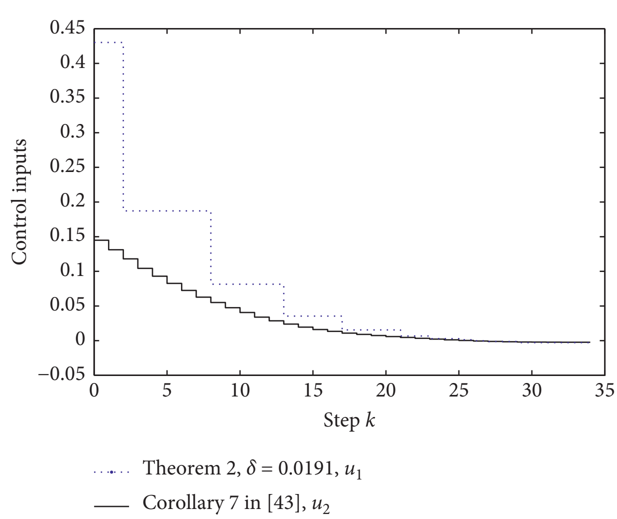

Figure 7: Control input.

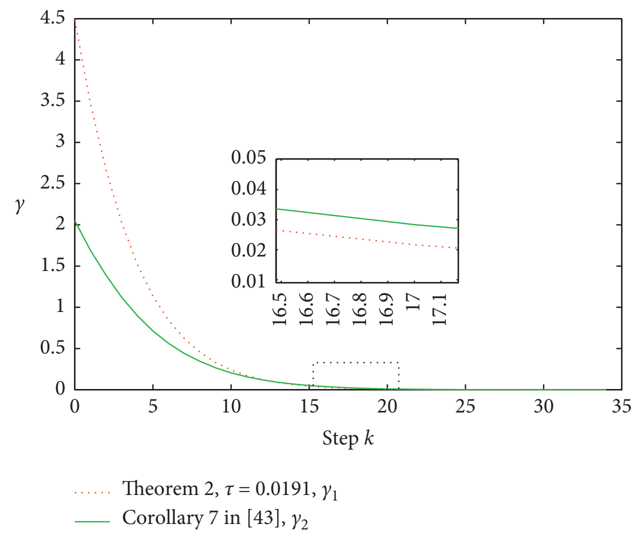

Figure 8: $\gamma(k)$ trajectory.

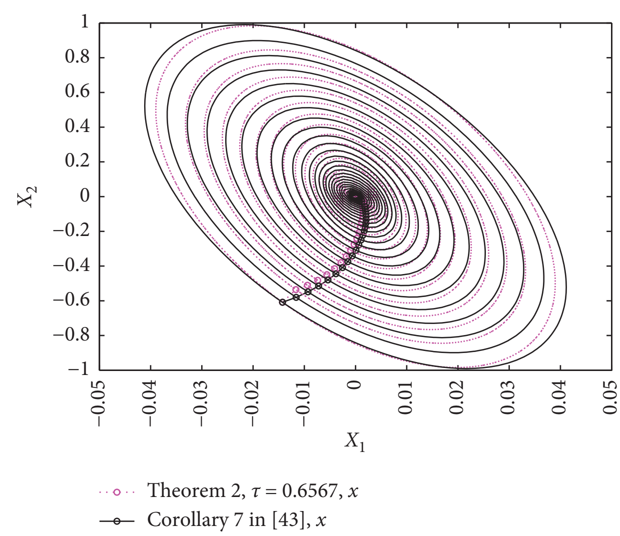

Figure 9: Invariant sets.

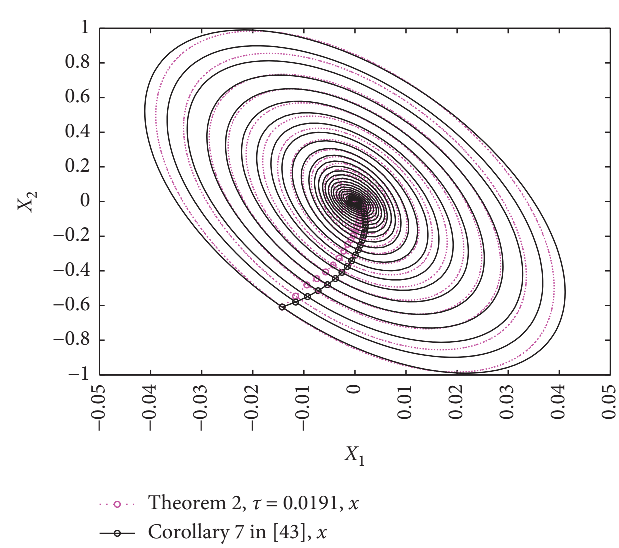

Figure 10: Invariant sets.

$\overline{\varpi} = 0.3$. The comparison in this example is made in the view of Theorem 2 with quantization density $\tau = 0.6567$ and $\tau = 0.0191$ and the method in [43], which provided a designing approach of MPC for fuzzy NCSs with data loss. The state feedback gains at $k = 1$, and the performance costs for different methods are shown in Table 3. As we can see from Figures 3–5 and Figures 6–8, the state, upper bound γ of the objective function and control input all converge to zero as $k \longrightarrow \infty$. However, some differences can be found from these figures. The convergence speed of Theorem 2 in this paper is faster than that of the method in [43]. Further, the control performance of the system in this paper is better than that in [43]. Figures 9 and 10 show the state trajectories and invariant sets of these two methods which show the superiority of the presented approach. It is obvious that the recursive feasibility and asymptotic stability of the closed-loop T-S fuzzy system can be guaranteed, which illustrates the effectiveness of our method.

7. Conclusion

In this paper, the robust constrained MPC scheme with quantization and data loss for the uncertain fuzzy system has been studied. Compared with other documents, the key idea of our approach is that the non-PDC control law and nonquadratic Lyapunov function are employed in the case of taking the problem of NCSs into consideration. Based on the nonquadratic Lyapunov function, the new conditions for stability related to slack and collection matrices are obtained. Further, the fuzzy controller is designed, and online synthesis of MPC for fuzzy NCSs is presented by solving the optimization problem in view of non-PDC control law and an extended nonquadratic Lyapunov function, respectively. In future work, we will focus on reducing the impact of network-induced delay through the MPC method or study the problem of reducing computational burden of the networked MPC method.

Appendix

A. Proof of Theorem 1

Consider the nonquadratic Lyapunov function

$$\begin{aligned} V(x(k)) &= x(k)^T \left(\sum_{i=1}^{r} \mu_i (f(k)) S_i \right)^{-1}, \\ x(k) &= x(k)^T S_h^{-1} x(k), \end{aligned} \tag{A.1}$$

where $S_i > 0$ and $\mu(f(k)) > 0$ are in a convex sum property for all $f(k), i \in \{1, 2, \ldots, r\}$. Then, the following condition must be satisfied:

$$\begin{aligned} &E\{V(x(k+1)) - V(x(k))\} \\ &= E\Big\{x^T(k)\Big(\big[\widetilde{A}_h + \overline{\varpi}\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1} + \widetilde{\omega}(k) \\ &\quad \times \widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}\big]^T S_{h+}^{-1}\big[\widetilde{A}_h + \overline{\varpi}\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1} \\ &\quad + \widetilde{\omega}(k)\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}\big] - S_h^{-1}\Big)x(k)\Big\} \leq 0. \end{aligned} \tag{A.2}$$

The following condition is put forward, so the asymptotic stability of (3) is guaranteed:

$$\begin{aligned} &\big[\widetilde{A}_h + \overline{\varpi}\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}\big]^T S_{h+}^{-1}\big[\widetilde{A}_h + \overline{\varpi}\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}\big] \\ &\quad + \kappa^2\big[\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}\big]^T S_{h+}^{-1}\big[\widetilde{B}_h(I+\varsigma(k))K_jQ_j^{-1}\big] - S_h^{-1} \leq 0, \end{aligned} \tag{A.3}$$

where $\kappa^2 = E\{\widetilde{\omega}(k)\widetilde{\omega}(k)\} = \overline{\varpi}(1-\overline{\varpi})$. Premultiplying and postmultiplying (A.3) by Q_h^T and Q_h and utilizing the inequality $Q_h^T S_h^{-1} Q_h \geq Q_h + Q_h^T - S_h$, inequality (A.3) is turned into

$$\begin{aligned} &\big[\widetilde{A}_hQ_h + \overline{\varpi}\widetilde{B}_h(I+\varsigma(k))K_h\big]^T S_{h+}^{-1}\big[\widetilde{A}_hQ_h + \overline{\varpi}\widetilde{B}_h(I+\varsigma(k))K_h\big] \\ &\quad + \kappa^2\big[\widetilde{B}_h(I+\varsigma(k))K_h\big]^T S_{h+}^{-1}\big[\widetilde{B}_h(I+\varsigma(k))K_h\big] - \big(Q_h + Q_h^T - S_h\big) \leq 0. \end{aligned} \tag{A.4}$$

Using Schur complement, we can see that (A.4) becomes

$$\begin{bmatrix} S_h - Q_h - Q_h^T & * & * \\ \widetilde{A}_hQ_h + \overline{\varpi}\widetilde{B}_h(I+\varsigma(k))K_h & -S_{h+} & * \\ \kappa\widetilde{B}_h(I+\varsigma(k))K_h & 0 & -S_{h+} \end{bmatrix} \leq 0. \tag{A.5}$$

Using (2), we have

$$
\begin{aligned}
&\begin{bmatrix} S_h - Q_h - Q_h^T & * & * \\ A_h Q_h + \overline{\omega} B_h (I + \varsigma(k)) K_h & -S_{h+} & * \\ \kappa B_h (I + \varsigma(k)) K_h & 0 & -S_{h+} \end{bmatrix}\begin{bmatrix} 0 \\ D \\ 0 \end{bmatrix} F(k) \\
&\times \left[E_{1h} Q_h + E_{2h} \overline{\omega} (I + \varsigma(k)) K_h \;\; 0 \;\; 0 \right] + \left[E_{1h} Q_h + \overline{\omega} E_{2h} (I + \varsigma(k)) K_h \;\; 0 \;\; 0 \right]^T \\
&\times F(k)^T \begin{bmatrix} 0 \\ D \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix} F(k) \left[\kappa E_{2h} (I + \varsigma(k)) K_h \;\; 0 \;\; 0 \right] \\
&+ \left[\kappa E_{2h} Q_h (I + \varsigma(k)) K_h \;\; 0 \;\; 0 \right]^T F(k)^T \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix}^T \leq 0.
\end{aligned} \tag{A.6}
$$

Applying Lemma 2, inequality (A.6) holds for all admissible uncertainties $F(k)$ satisfying $F(k)^{\mathrm{T}} F(k) \leq I$ if and only if there exist some constants η_{ij} and $\overline{\eta}_{ij}$ such that

$$
\begin{aligned}
&\begin{bmatrix} S_h - Q_h - Q_h^T & * & * \\ A_h Q_h + \overline{\omega} B_h (I + \varsigma(k)) K_h & -S_{h+} & * \\ \kappa B_h (I + \varsigma(k)) K_h & 0 & -S_{h+} \end{bmatrix} + \eta_{ij} \begin{bmatrix} 0 \\ D \\ 0 \end{bmatrix} \left[0 \;\; D^T \;\; 0 \right] \\
&+ \eta_{ij}^{-1} \begin{bmatrix} \left(E_{1h} Q_h + E_{2h} \overline{\omega} (I + \varsigma(k)) K_h \right)^T \\ 0 \\ 0 \end{bmatrix} \left[\left(E_{1h} Q_h + E_{2h} \overline{\omega} (I + \varsigma(k)) K_h \right) \;\; 0 \;\; 0 \right] \\
&+ \overline{\eta}_{ij} \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix} \left[0 \;\; 0 \;\; D^T \right] + \overline{\eta}_{ij}^{-1} \begin{bmatrix} \left(\kappa E_{2h} Q_h (I + \varsigma(k)) K_h \right)^T \\ 0 \\ 0 \end{bmatrix} \\
&\times \left[\left(\kappa E_{2h} Q_h (I + \varsigma(k)) K_h \right) \;\; 0 \;\; 0 \right] \leq 0.
\end{aligned} \tag{A.7}
$$

Applying Schur complement, we have

$$
\begin{bmatrix}
S_h - Q_h - Q_h^T & * & * & * & * & * & * \\
A_h Q_h + \overline{\omega} B_h (I + \varsigma(k)) K_h & -S_{h+} & * & * & * & * & * \\
\kappa B_h (I + \varsigma(k)) K_h & 0 & -S_{h+} & * & * & * & * \\
E_{1h} Q_h + \overline{\omega} E_{2h} (I + \varsigma(k)) K_h & 0 & 0 & -\eta_{ij} I & * & * & * \\
0 & \eta_{ij} D^T & 0 & 0 & -\eta_{ij} I & * & * \\
\kappa E_{2h} Q_h (I + \varsigma(k)) K_h & 0 & 0 & 0 & 0 & -\overline{\eta}_{ij} I & * \\
0 & 0 & -\overline{\eta}_{ij} D^T & 0 & 0 & 0 & -\overline{\eta}_{ij} I
\end{bmatrix} \leq 0, \tag{A.8}
$$

which is equivalent to

$$\begin{bmatrix} S_h - Q_h - Q_h^T & * & * & * & * & * & * \\ A_hQ_h + \overline{\omega}B_hK_h & -S_{h+} & * & * & * & * & * \\ \kappa B_hK_h & 0 & -S_{h+} & * & * & * & * \\ E_{1h}Q_h + \overline{\omega}E_{2h}K_h & 0 & 0 & -\eta_{ij}I & * & * & * \\ 0 & \eta_{ij}D^T & 0 & 0 & -\eta_{ij}I & * & * \\ \kappa E_{2h}Q_hK_h & 0 & 0 & 0 & 0 & -\overline{\eta}_{ij}I & * \\ 0 & 0 & -\overline{\eta}_{ij}D^T & 0 & 0 & 0 & -\overline{\eta}_{ij}I \end{bmatrix} + n_1\varsigma(k)n_2 + n_2^T\varsigma(k)n_1^T \leq 0, \tag{A.9}$$

where $n_1 = [0\,(\overline{\omega}B_h)^T\,(\kappa B_h)^T\,(\overline{\omega}E_{2h})^T\,0\,(\kappa E_{2h})^T\,0]^T$ and $n_2 = [K_h\,0\,0\,0\,0\,0\,0]$.

Utilizing Lemma 1, one can get

$$n_1\varsigma(k)n_2 + n_2^T\varsigma(k)n_1^T \leq n_1\tau Y\tau n_1^T + n_2^TY^{-1}n_2. \tag{A.10}$$

Condition (A.9) holds if

$$\begin{bmatrix} S_h - Q_h - Q_h^T & * & * & * & * & * & * \\ A_hQ_h + \overline{\omega}B_hK_h & -S_{h+} & * & * & * & * & * \\ \kappa B_hK_h & 0 & -S_{h+} & * & * & * & * \\ E_{1h}Q_h + \overline{\omega}E_{2h}K_h & 0 & 0 & -\eta_{ij}I & * & * & * \\ 0 & \eta_{ij}D^T & 0 & 0 & -\eta_{ij}I & * & * \\ \kappa E_{2h}Q_hK_h & 0 & 0 & 0 & 0 & -\overline{\eta}_{ij}I & * \\ 0 & 0 & -\overline{\eta}_{ij}D^T & 0 & 0 & 0 & -\overline{\eta}_{ij}I \end{bmatrix} + n_1\tau Y\tau n_1^T + n_2^TY^{-1}n_2 \leq 0. \tag{A.11}$$

By applying Schur complement, the following is achieved:

$$\begin{aligned} \begin{bmatrix} d_1 & * & * \\ d_2 & d_3 & * \\ d_4 & d_5 & d_6 \end{bmatrix} &= \sum_{l=1}^{r}\mu_l^+\left(\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Psi_{ij}^l\right) \leq 0, \quad i,j,l \in \{1,2,\ldots,r\}, \\ d_1 &= \begin{bmatrix} S_h - Q_h - Q_h^T & * & * \\ A_hQ_h + \overline{\omega}B_hK_h & -S_{h+} + \overline{\omega}^2B_h\tau Y\tau B_h^T & * \\ \kappa B_hK_h & \kappa\overline{\omega}B_h\tau Y\tau B_h^T & -S_{h+} + \kappa^2B_h\tau Y\tau B_h^T \end{bmatrix}, \\ d_2 &= \begin{bmatrix} E_{1h}Q_h + \overline{\omega}E_{2h}K_h & \overline{\omega}^2E_{2h}\tau Y\tau B_h^T & \overline{\omega}\kappa E_{2h}\tau Y\tau B_h^T \\ 0 & \eta_{ij}D^T & 0 \\ \kappa E_{2h}Q_hK_h & \overline{\omega}\kappa E_{2h}\tau Y\tau B_h^T & \kappa^2E_{2h}\tau Y\tau B_h^T \end{bmatrix}, \\ d_3 &= \begin{bmatrix} -\eta_{ij}I + \overline{\omega}^2E_{2h}\tau Y\tau E_{2h}^T & * & * \\ 0 & -\eta_{ij}I & * \\ \overline{\omega}\kappa E_{2h}\tau Y\tau E_{2h}^T & 0 & -\overline{\eta}_{ij}I + \kappa^2E_{2h}\tau Y\tau E_{2h}^T \end{bmatrix}, \\ d_4 &= \begin{bmatrix} 0 & 0 & -\overline{\eta}_{ij}D^T \\ K_h & 0 & 0 \end{bmatrix}, \\ d_5 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ d_6 &= \begin{bmatrix} -\overline{\eta}_{ij}I & * \\ 0 & -Y \end{bmatrix}. \end{aligned} \tag{A.12}$$

Equation (A.12) guarantees (24) on account of the convex sum property of membership functions. Resorting to (21) and (22), the above inequality yields

$$\sum_{i}^{r}\mu_i^2\Psi_{ii}^l+\sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_j\left(\Psi_{ij}^l+\Psi_{ji}^l\right)\leq-\left(\sum_{i=1}^{r}\mu_i^2G_{ii}^l+\sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_jG_{ij}^l\right). \tag{A.13}$$

Therefore, there always exists

$$\sum_{i=1}^{r}\mu_i^2G_{ii}^l+\frac{1}{2}\sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_jG_{ij}^l\geq 0,\quad l\in 1,2,\ldots,r, \tag{A.14}$$

or equivalently

$$\begin{bmatrix}\mu_1 I\\ \vdots\\ \mu_r I\end{bmatrix}^T\begin{bmatrix}2G_{11}^l & \cdots & G_{1r}^l\\ \vdots & \ddots & \vdots\\ G_{1r}^l & \cdots & 2G_{rr}^l\end{bmatrix}\begin{bmatrix}\mu_1 I\\ \vdots\\ \mu_r I\end{bmatrix}\geq 0. \tag{A.15}$$

It is obvious that the uncertain fuzzy system in (1) is asymptotically stable via the fuzzy controller (13), and the proof is completed.

B. Proof of Theorem 2

Set $\widetilde{S}_{ij}=\gamma S_{ij}$. According to (32) and Schur complement, one can get

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(k|k))\mu_j(\xi(k|k))\begin{bmatrix}1 & *\\ x(k|k) & \widetilde{S}_{ij}\end{bmatrix}\geq 0,\quad i,j\in\{1,2\ldots,r\}. \tag{B.1}$$

Due to $\mu(f(k|k))\geq 0$, (37) is achieved.
Considering (26), (28), and (29), we have

$$\begin{aligned}&E\Big\{x(k+s|k)^T\Big[\left(\widetilde{\mathscr{A}}_h+\overline{\omega}\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_hQ_h^{-1}+\widetilde{\omega}(k+s|k)\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_hQ_h^{-1}\right)^T\\&\times S_{hh+}^{-1}\left(\widetilde{\mathscr{A}}_h+\overline{\omega}\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_hQ_h^{-1}\widetilde{\omega}(k+s|k)\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_hQ_h^{-1}\right)-S_{hh}^{-1}\Big]\\&\times\ x(k+s|k)\Big\}\leq E\Big\{-x(k+s|k)^T\Big(W+\left[\omega(k)(I+\varsigma(k))K_hQ_h^{-1}\right]^T\\&\times R\left[\omega(k)(I+\varsigma(k))K_hQ_h^{-1}\right]\Big)x(k+s|k)\Big\}.\end{aligned} \tag{B.2}$$

Premultiplying and postmultiplying both sides of (B.2) by Q_h^T and Q_h and using the inequality $Q_h^TS_h^{-1}Q_h\geq Q_h+Q_h^T-S_h$, we have

$$\begin{aligned}&\left(\widetilde{\mathscr{A}}_hQ_h+\overline{\omega}\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_h\right)^TS_{hh+}^{-1}\left(\widetilde{\mathscr{A}}_hQ_h+\overline{\omega}\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_h\right)\\&\quad+\kappa^2\left(\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_h\right)^TS_{hh+}^{-1}\left(\widetilde{\mathscr{B}}_h(I+\varsigma(k))K_h\right)-\left(Q_h+Q_h^T-S_{hh}\right)\\&\leq-\left(Q_h^TWQ_h+\left(\overline{\omega}^2+\kappa^2\right)\left((I+\varsigma(k))K_h\right)^TR(I+\varsigma(k))K_h\right),\end{aligned} \tag{B.3}$$

where $\kappa^2=E\{\widetilde{\omega}(k)\widetilde{\omega}(k)\}=\overline{\omega}(1-\overline{\omega})$. Because of the uncertainty $\Delta A_h,\Delta B_h$ of the fuzzy system and $\widetilde{S}_{hh}=\gamma S_{hh}$, the following inequality is obtained:

$$\begin{bmatrix} \widetilde{S}_{hh} - Q_h - Q_h^T & * & * \\ A_h Q_h + \overline{\omega} B_h (I + \varsigma(k)) K_h & -\widetilde{S}_{hh+} & * \\ \kappa B_h (I + \varsigma(k)) K_h & 0 & -\widetilde{S}_{hh+} \end{bmatrix} + \begin{bmatrix} 0 \\ D \\ 0 \end{bmatrix} F(k+s|k)$$
$$\times \begin{bmatrix} E_{1h} Q_h + \overline{\omega} E_{2h} (I + \varsigma(k)) K_h & 0 & 0 \end{bmatrix} + \begin{bmatrix} E_{1h} Q_h + \overline{\omega} E_{2h} (I + \varsigma(k)) K_h & 0 & 0 \end{bmatrix}^T$$
$$\times F(k+s|k)^T \begin{bmatrix} 0 \\ D \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix} F(k+s|k) \begin{bmatrix} \kappa E_{2h} (I + \varsigma(k)) K_h & 0 & 0 \end{bmatrix} \tag{B.4}$$
$$+ \begin{bmatrix} \left(\kappa E_{2h} (I + \varsigma(k)) K_h\right)^T \\ 0 \\ 0 \end{bmatrix} F(k+s|k)^T \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix}^T .$$
$$\leq -\frac{1}{\gamma} \left(Q_h^T W Q_h + \left(\overline{\omega^2} + t\kappa^2 \right) \left((I + \varsigma(k)) K_h \right)^T R (I + \varsigma(k)) K_h \right).$$

Using Lemma 2 and Schur complement, if and only if there are η_{ijlm} and $\overline{\eta}_{ijlm}$ satisfying

$$\begin{bmatrix} m_7 & * \\ m_8 & m_9 \end{bmatrix} \leq 0, \tag{B.5}$$

where

$$m_7 = \begin{bmatrix} \widetilde{S}_{hh} - Q_h - Q_h^T & * & * & * \\ \overline{m}_6 & -\widetilde{S}_{hh+} & * & * \\ \kappa B_h (I + \varsigma(k)) K_h & 0 & -\widetilde{S}_{hh+} & * \\ \overline{m}_7 & 0 & 0 & -\eta_{ijlm} I \end{bmatrix},$$
$$\overline{m}_6 = A_h Q_h + \overline{\omega} B_h (I + \varsigma(k)) K_h,$$
$$\overline{m}_7 = E_{1h} Q_h + \overline{\omega} E_{2h} (I + \varsigma(k)) K_h, \tag{B.6}$$
$$m_8 = \begin{bmatrix} 0 & \eta_{ijlm} D^T & 0 & 0 \\ \kappa E_{2h} (I + \varsigma(k)) K_h & 0 & 0 & 0 \\ 0 & 0 & \overline{\eta}_{ijlm} D^T & 0 \\ W^{1/2} Q_h & 0 & 0 & 0 \\ R^{1/2} \left(\overline{\omega}^2 + \kappa^2 \right)^{1/2} (I + \varsigma(k)) K_h & 0 & 0 & 0 \end{bmatrix},$$
$$m_9 = \operatorname{diag}\left\{ -\eta_{ijlm} I, -\overline{\eta}_{ijlm} I, -\overline{\eta}_{ijlm} I, -\gamma I, -\gamma I \right\}.$$

Employing the similar method to deal with quantization density in Theorem 1, the following inequality is established:

$$n_3 \tau N \tau n_3^T + n_4^T N^{-1} n_4 + \begin{bmatrix} m_{10} & * \\ m_{11} & m_9 \end{bmatrix} \leq 0, \tag{B.7}$$

where

$$m_{10} = \begin{bmatrix} \widetilde{S}_{hh} - Q_h - Q_h^T & * & * & * \\ \overline{m}_8 & -\widetilde{S}_{hh+} & * & * \\ \kappa B_h K_h & 0 & -\widetilde{S}_{hh+} & * \\ \overline{m}_9 & 0 & 0 & -\eta_{ijlm} I \end{bmatrix},$$

$$m_{11} = \begin{bmatrix} 0 & \eta_{ijlm} D^T & 0 & 0 \\ \kappa E_{2h} K_h & 0 & 0 & 0 \\ 0 & 0 & \overline{\eta}_{ijlm} D^T & 0 \\ W^{1/2} Q_h & 0 & 0 & 0 \\ R^{1/2} \left(\overline{\omega}^2 + \kappa^2\right)^{1/2} K_h & 0 & 0 & 0 \end{bmatrix}, \tag{B.8}$$

$$n_4 = \begin{bmatrix} K_h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$n_3 = \begin{bmatrix} 0 & (\overline{\omega} B_h)^T & (\kappa B_h)^T & (\overline{\omega} E_{2h})^T & 0 & (\kappa E_{2h})^T & 0 & 0 & R^{1/2} \left(\overline{\omega}^2 + \kappa^2\right)^{1/2} \end{bmatrix}^T$$

$$\overline{m}_8 = A_h Q_h + \overline{\omega} B_h K_h,$$

$$\overline{m}_9 = E_{1h} Q_h + \overline{\omega} E_{2h} K_h.$$

By utilizing Schur complement, one can get

$$\begin{bmatrix} m_{12} & * & * \\ m_{13} & m_{14} & * \\ m_{15} & m_{16} & m_{17} \end{bmatrix} = \sum_{l=1}^{r} \sum_{m=1}^{r} \mu_l^+ \mu_m^+ \left(\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \Psi_{ij}^{lm} \right) \leq 0, \tag{B.9}$$

where

$$m_{12} = \begin{bmatrix} \overline{m}_{10} & * & * \\ \overline{m}_{11} & \overline{m}_{12} & * \\ \kappa B_h K_h & \overline{\omega} \kappa B_h \tau N \tau B_h^{\mathrm{T}} & \overline{m}_{13} \end{bmatrix},$$

$$\overline{m}_{10} = \widetilde{S}_{hh} - Q_h - Q_h^{\mathrm{T}},$$

$$\overline{m}_{12} = -\widetilde{S}_{hh+} + \overline{\omega}^2 B_h \tau N \tau B_h^{\mathrm{T}},$$

$$\overline{m}_{13} = -\widetilde{S}_{hh+} + \kappa^2 B_h \tau N \tau B_h^{\mathrm{T}}$$

$$m_{13} = \begin{bmatrix} \overline{m}_{14} & \overline{\omega}^2 E_{2h} \tau N \tau B_h^{\mathrm{T}} & \overline{\omega} \kappa E_{2h} \tau N \tau B_h^{\mathrm{T}} \\ 0 & \eta_{ijlm} D^{\mathrm{T}} & 0 \\ \kappa E_{2h} K_h & \overline{\omega} \kappa E_{2h} \tau N \tau B_h^{\mathrm{T}} & \kappa^2 E_{2h} \tau N \tau B_h^{\mathrm{T}} \end{bmatrix},$$

$$\overline{m}_{11} = A_h Q_h + \overline{\omega} B_h K_h,$$

$$m_{14} = \begin{bmatrix} \overline{m}_{15} & * & * \\ 0 & -\eta_{ijlm} I & * \\ \overline{\omega} \kappa E_{2h} \tau N \tau E_{2h}^{\mathrm{T}} & 0 & \overline{m}_{16} \end{bmatrix},$$

$$\begin{aligned}
\overline{m}_{15} &= -\eta_{ijlm}I + \overline{\omega}^2 E_{2h}\tau N\tau E_{2h}^{\mathrm{T}},\\
m_{15} &= \begin{bmatrix} 0 & 0 & \overline{\eta}_{ijlm}D^T \\ W^{1/2}Q_h & 0 & 0 \\ \overline{\varrho}K_h & \overline{\omega}\overline{\varrho}\tau N\tau B_h^{\mathrm{T}} & \kappa\overline{\varrho}\tau N\tau B_h^{\mathrm{T}} \\ K_h & 0 & 0 \end{bmatrix},\\
\overline{m}_{16} &= -\overline{\eta}_{ijlm}I + \kappa^2 E_{2h}\tau N\tau E_{2h}^{\mathrm{T}},\\
m_{16} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \overline{\omega}\overline{\varrho}\tau N\tau E_{2h}^{\mathrm{T}} & 0 & \kappa\overline{\varrho}\tau N\tau E_{2h}^{\mathrm{T}} \\ 0 & 0 & 0 \end{bmatrix},\\
\overline{m}_{14} &= E_{1h}Q_h + \overline{\omega}E_{2h}K_h,\\
m_{17} &= \operatorname{diag}\left\{-\overline{\eta}_{ijlm}I, \gamma I, -\gamma I + \overline{\varrho}^2\tau N\tau, -N\right\},
\end{aligned} \tag{B.10}$$

where $\overline{\varrho} = R^{1/2}(\overline{\omega}^2 + \kappa^2)^{1/2}$. Owing to $\sum_{l=1}^{r}\sum_{m=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_l^+\mu_m^+\mu_i\mu_j = 1$, (42) is guaranteed. Using the same method which is described in [53, 54], based on (B.9), one can get

$$\begin{aligned}
&\sum_{l=1}^{r}\sum_{m=1}^{r}\mu_l^+\mu_m^+\left(\sum_{i=1}^{r}\mu_i^2\Psi_{ii}^{lm} + \sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_j\left(\Psi_{ij}^{lm} + \Psi_{ji}^{lm}\right)\right)\\
&\quad\le -\sum_{l=1}^{r}\sum_{m=1}^{r}\mu_l^+\mu_m^+\left(\sum_{i=1}^{r}\mu_i^2 G_{ii}^{lm} + \sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_j\left(G_{ij}^{lm} + G_{ji}^{lm}\right)\right)\\
&\quad= -\sum_{l=1}^{r}\mu_l^{+2}G^{ll} - \sum_{l=1}^{r}\sum_{m>l}^{r}\mu_l^+\mu_m^{l+}\left(G^{lm} + G^{ml}\right),
\end{aligned} \tag{B.11}$$

where $G^{ll} = \sum_{i=1}^{r}\mu_i^2 G_{ii}^{ll} + \sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_j(G_{ij}^{ll} + G_{ji}^{ll})$, $G^{lm} = \sum_{i=1}^{r}\mu_i^2 G_{ii}^{lm} + \sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_j(G_{ij}^{lm} + G_{ji}^{lm})$, and $G^{ml} = \sum_{i=1}^{r}\mu_i^2 G_{ii}^{ml} + \sum_{i=1}^{r}\sum_{j>i}^{r}\mu_i\mu_j(G_{ij}^{ml} + G_{ji}^{ml})$.

Introducing matrices U_{ll}, U_{lm}, and U_{ml}, it yields

$$\begin{aligned}
G^{ll} &\ge U_{ll},\\
G^{lm} + G^{ml} &\ge U_{lm} + U_{ml}.
\end{aligned} \tag{B.12}$$

From (B.11) and (B.12), we can get

$$\begin{aligned}
&\sum_{l}\mu_l^{+2}U_{ll} + \sum_{l}\sum_{m>l}\mu_l^+\mu_m^+(U_{lm} + U_{ml})\\
&\quad= \begin{bmatrix}\mu_1^+ I\\ \mu_2^+ I\\ \vdots\\ \mu_r^+ I\end{bmatrix}^T \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1r}\\ U_{21} & U_{22} & \cdots & U_{2r}\\ \vdots & \vdots & \ddots & \vdots\\ U_{r1} & U_{r2} & \cdots & U_{rr}\end{bmatrix}\begin{bmatrix}\mu_1^+ I\\ \mu_2^+ I\\ \vdots\\ \mu_r^+ I\end{bmatrix} \ge 0.
\end{aligned} \tag{B.13}$$

Under the above discussion, (B.9) is established and the stability of the T-S fuzzy system (15) is guaranteed. Obviously, (B.12) equals to

$$\begin{aligned}
&\left[\mu_1^+ I \;\; \mu_2^+ I \;\; \cdots \;\; \mu_r^+ I\right]\phi_{ll}\left[\mu_1^+ I \;\; \mu_2^+ I \;\; \cdots \;\; \mu_r^+ I\right]^T \ge 0,\\
&\left[\mu_1^+ I \;\; \mu_2^+ I \;\; \cdots \;\; \mu_r^+ I\right](\phi_{lm} + \phi_{ml})\left[\mu_1^+ I \;\; \mu_2^+ I \;\; \cdots \;\; \mu_r^+ I\right]^T \ge 0.
\end{aligned} \tag{B.14}$$

Therefore, we can get (40) and (41). The proof is completed.

C. Proof of Theorem 3

It is obvious that only (37) is related to state $x(k|k)$; in other words, we only need to prove that this LMI is still feasible for all future measurable system states $x(k+s|k+s), s \ge 1$.

At time k, we assume that γ_k^* and Ω_k^* are the optimal solution which are derived from the optimization problem (55). Since (37) is satisfied, we have

$$E\left\{x(k|k)^T S^*(k|k)x(k|k)\right\} \le \widetilde{\gamma}, \tag{C.1}$$

where $S^*(k|k) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(f(k|k))\mu_j(f(k|k))\left\{S_{ij}^{*-1}\right\}_k$. Then, a feasible solution $\{\gamma, \Omega\}_{k+1} = \{\gamma^*, \Omega^*\}_k$ is established at time $k+1$. Referring to Theorem 3, we can conclude that $x(k+1|k+1) \in \varepsilon(\widetilde{S}_{hh}^*)$ and

$$E\left\{x(k+1|k+1)^T S^*(k|k)x(k+1|k+1)\right\} \le \widetilde{\gamma}. \tag{C.2}$$

We infer that (37) is feasible at time $k+1$. Thus, it can be summarized that the optimization problem (55) is also feasible for all times $k+s, s \ge 1$, and the proof is completed.

D. Proof of Theorem 4

In this paper, in order to guarantee the asymptotic stability of the fuzzy system, we choose the extended nonquadratic Lyapunov function. Thus, if we can guarantee that the extended nonquadratic Lyapunov function is strictly decreasing, the asymptotic stability of the fuzzy system is guaranteed.

According to Theorem 3, the feasibility of optimization is guaranteed. At time $k+1$, if we obtain the optimal solution

$\{\gamma^*, S_{ij}^{*-1}\}_{k+1}$, it means that the solution at time $k+1$ must satisfy

$$E\{x(k+1|k+1)^T S^*(k+1|k+1)x(k+1|k+1)\} \leq E\{x(k+1|k+1)^T S(k+1|k+1)x(k+1|k+1)\}, \quad (D.1)$$

where $S^*(k+1|k+1) = \sum_{l=1}^r \sum_{m=1}^r \mu_l(x(k+1|k+1))\mu_m(x(k+1|k+1))\{S_{ij}^{*-1}\}_{k+1}$ and $S(k+1|k+1) = \sum_{l=1}^r \sum_{m=1}^r \mu_l(x(k+1|k+1))\mu_m(x(k+1|k+1))\{S_{ij}^{*-1}\}_k$. Then, considering (29) and setting $s=0$, we have

$$E\{x(k+1|k)^T S^*(k+1|k)x(k+1|k)\} < E\{x(k|k)^T S^*(k|k)x(k|k)\}, \quad (D.2)$$

with $S(k+1|k) = \sum_{l=1}^r \sum_{m=1}^r \mu_l(x(k+1|k))\mu_m(x(k+1|k))\{S_{ij}^{*-1}\}_k$ and $S^*(k|k) = \sum_{i=1}^r \sum_{j=1}^r \mu_i(x(k|k))\mu_j(x(k|k))\{S_{ij}^{*-1}\}_k$. Since $x(k+1|k+1) = x(k+1|k)$, we can get

$$E\{x(k+1|k+1)^T S^*(k+1|k+1)x(k+1|k+1)\} < E\{x(k|k)^T S^*(k|k)x(k|k)\}. \quad (D.3)$$

In summary, the strictly decreasing of the extended nonquadratic Lyapunov function is proofed except $x(k|k) = 0$. Therefore, the closed-loop fuzzy system is robust asymptotically stable, and the proof is completed.

Data Availability

Data availability and data simulation program used to support the findings of this study have been deposited in the figshare repository https://doi.org/10.6084/m9.figshare.12947621.v1.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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