

Research Article

Evolutionary Game Analysis of Capital-Constrained Supplier's and Manufacturer's Financing Schemes

Suyong Zhang ^{1,2}, Panos. M. Pardalos,² and Xiaodan Jiang¹

¹College of Transport and Communication, Shanghai Maritime University, Shanghai, China

²Center for Applied Optimization, Department of Industrial and Systems Engineering, University of Florida, Gainesville, FL 32611, USA

Correspondence should be addressed to Suyong Zhang; soonchueng@foxmail.com

Received 23 September 2020; Revised 8 February 2021; Accepted 23 October 2021; Published 18 December 2021

Academic Editor: M. De Aguiar

Copyright © 2021 Suyong Zhang Panos. M. Pardalos and Xiaodan Jiang. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Purchase order financing (POF) and buyer direct financing (BDF) are both innovative financing schemes aiming to help financial constrained suppliers secure financing for production. In this paper, we investigate the interaction mechanism between suppliers' financing strategy selection and manufacturers' loans offering strategy adoption under two innovative financing schemes. We developed an evolutionary game model to effectively investigate the interaction mechanism between suppliers and manufacturers and analyzed the evolutionary stable strategies of the game model. Then we used system dynamics to present the performance of the evolutionary game model and took a sensitivity analysis to verify the theoretical results. The main conclusions are as follows: in the supply chain, to deal with the noncooperation among suppliers and manufacturers on innovative financing schemes, the revenue of manufacturers, the rate of manufacturer loan, and the proper financial risk factor should be relatively high.

1. Introduction

To reduce operating costs, many manufacturers in developed regions purchase source from small suppliers. But the limited working capital of small suppliers affects production decisions, and enterprises desire to obtain finance from the bank to execute their production. However, small and startup suppliers have obstruction accessing from the bank, because of their lack of credit history. According to the British Commercial Bank survey, nearly 100,000 SMEs are rejected by banks each year when financing from the bank [1]. These suppliers often rely on two ways for loans. One of the ways is the loans offered by banks and secured by suppliers' assets, such as factories [2]. The other way is turning down the orders from reputable manufacturers because these manufacturers can offer direct financing or guarantee for suppliers to obtain the bank loans [3]. Inevitably, suppliers without financing directly affect the

operation of supply chain, such as fail to deliver, higher retail price, and even worse end products to consumers.

To satisfy the financing needs of the small suppliers, two innovative nonasset-based financing schemes have recently come out. The first is purchase order financing (POF) which indicates that the bank lends to suppliers based on purchase order issued by reputable manufacturers [4, 5]. Since POF loans are only granted based on purchase orders issued by reputable buyers, the main risk associated with POF is not the buyer's credit risk, but the supplier's performance risk; that is, the supplier may not be able to deliver the order quality as specified by the buyer [6]. The second scheme is buyer direct financing (BDF) which indicates that the manufacturer acts as both the buyer and the lender, and finances suppliers for production [3]. BDF has been adopted by manufacturers and suppliers in both developed and developing markets. Rolls-Royce has provided loans of more than 500 million pounds to small suppliers who cannot obtain sufficient financing through other channels.

Similarly, GSK has lent billions of pounds to its small suppliers [7]. As both purchase order financing (POF) and buyer direct financing (BDF) are still taking shape, researchers are investigating whether banks or manufacturers are in a better position to finance suppliers.

In the complex and ever-changing economic environment, it brings various risks to the supply chain. However, financial risk has become increasingly prominent and critical in today's economic world. According to the survey result shown by "McKinsey & Company" [8]; the financial uncertainty is a top factor that could influence supply chain decisions. Thus, it is meaningful for us to investigate the supply chain members financing strategies under the consideration of financial risk.

To investigate the abovementioned topic, we investigate the interaction mechanism between suppliers' financing strategy selection and manufacturers' loans offering strategy adoption under two innovative financing schemes, by using evolutionary game theory (EGT). In the proposed game theory method, the population of suppliers and manufacturers are named as players. To adopt the optimal strategy at each time, manufacturers and suppliers choose the strategy that they have the utmost expected utility. We all know EGT is used in many fields, such as strategy interaction [9], mobile health [10], players cooperation [11], and social rumor control [11]. Due to the emergence of two innovative financial schemes for manufacturers, small size and setup suppliers have subsequently gained two ways to solve their financial problems to ensure the continued stability of the capital chain. By using EGT to develop a novel model under two innovative financing schemes, manufacturers have two types of choices—BDF (buyer direct financing) and POF (purchase order financing)—and suppliers also have two strategies—BF (bank financing) and MF (manufacturer financing). In reality, rational manufacturers seek to maximize their profits. When suppliers choose BF (bank financing) strategy, they will obtain loan from the bank which offers a variety of financing options for suppliers' business to purchase inventory and materials. When suppliers choose MF (manufacturer financing) strategy, they will obtain loan from the manufacturer which offers two innovative financing schemes (BDF and POF).

Firstly, under the two innovative financing schemes, the model of evolutionary game between manufacturers and suppliers is developed. Secondly, the payoff matrix of suppliers and manufacturers under two innovative financing schemes was analyzed and the evolutionary stable strategies were obtained. By using the evolutionary game, this study is for analysis of the long-term behaviors of the two innovative financing schemes of populations of manufacturers and suppliers considering the financial risk.

The remainder of this paper is organized as follows. Section 2 introduces a review of the literature. In Section 3, an evolutionary game model under two innovative financing schemes (BDF and POF) is developed, and the evolutionary stable strategies are obtained. In Section 4, a numerical example is examined with system dynamics simulation, presenting the performance of the evolutionary game model. Section 5 summarizes the main results and describes the future research.

2. Literature Review

Our research is related to three research streams: supply chain finance scheme, the financial risk management, and the application of evolutionary game theory.

2.1. The Influence of Financing Scheme on Supply Chain Management. The supply chain finance is an important research topic in supply chain operations. Modigliani and Miller [12] pointed out that operational and financial decisions can be made separately in a perfect capital market. Pakhira et al. [13] consider the financial scheme under the retailer's budget constraint in a supplier-retailer supply chain. Johari et al. [14] coordinated the supply chain by considering the financial scheme. Kaur [15] investigates the financial scheme for a two-echelon SC considering default risk. Babich and Sobel [16] investigated how to coordinate the operational and financial decisions with the increase likelihood of a successful initial public offering (IPO). Lin and He [17] considered three financing strategies, bank financing (BF), a guaranteed contract with a buy-back clause (GBB), a guaranteed contract with a wholesale price discount clause (GWD), and investigated the influence of supplier's asset structure in financing strategies on the supply chain consist of one retailer, one capital-constraint supplier, and one bank. Buzacott and Zhang [2] incorporate financing into supply chain operation decisions and indicate financing scheme is profitable for the supply chain partners. Babich and Tang [18] studied the financing mechanism for dealing with product adulteration problems. Rui and Lai [19] studied the deferred payment and inspection mechanisms for mitigating supplier product adulteration. Tang et al. [3] examined the relative efficiency of POF and BDF under both endogenous supplier performance risk and information asymmetry. These researches are not based on different financial schemes (i.e., POF and BDF). Although Tang et al. considered both POF and BDF as we do, they did not investigate the difference between POF and BDF by using the evolutionary game theory. Our paper studies the interaction mechanism between suppliers' financing strategy selection and manufacturers' loans offering strategy adoption under two innovative financing schemes (POF and BDF).

2.2. The Financial Risk Measurement Methods. Many papers focused on the risk measurement methods. Liu and Cruz [20] studied the impact of financial risk on optimal supply chain decisions by using the net present value method (NPV). Applequist et al. [21] proposed a new method to evaluate the risk and uncertainty of chemical manufacturing supply chains. Soni and Kodali [22] developed PROMETHEE-II model to assess the global supply chain risk.

Krystofik et al. [23] developed a framework for the risk assessment of delaying the delivery of shipments to customers in the presence of incomplete information. Bandalay et al. [24] developed an integrated risk management model to test the level of supply chain risk management. Munir et al. [25] explored the association between supply chain

integration (SCI) and supply chain risk management (SCRM) based on the information processing view of risk management to improve operational performance.

Zeng and Yen [26] applied topology theory and Markov chain to measure the resilience of supply chain financial risk from the perspective of partnership. Cardoso et al. [27] proposed a mixed integer linear programming model to measure the financial risk in the design of closed-loop supply chains. The above literature shows that multicriteria risk measurement methods, such as NPV and MILP, all belong to static analysis methods. However, we all know that the supply chain's response strategy to financial risk is dynamically changing with time, which is a continuous learning process. After realizing this issue, we develop evolutionary game models to analyze the impact of financial risks on supply chain financing strategy adoption.

2.3. The Evolutionarily Game Theory to Examine the Long-Term Behaviors of Populations. Wang et al. [28] explore the long-term evolutionary behavior of sustainability. Hosseini-Motlagh et al. [29] investigate sustainable financing for a financially constrained manufacturer using a one-population evolutionary game. Wu et al. [30] established an evolutionary game model between government and enterprises in the context of low carbon and studied the impact of government incentive policies on enterprises' emission reduction strategies. Fan et al. [31] developed the evolutionary game model of government and enterprises under the conditions of government supervision and no-government supervision, respectively. Zhang et al. [32] used evolutionary game to study the interaction mechanism between manufacturer's green technology strategy selection and government regulation. To obtain more insight on dynamic supply chain financing strategy, our research investigates the interaction mechanism between suppliers' financing strategy selection and manufacturers' loans offering strategy adoption under two innovative financing schemes by using evolutionary game theory (EGT).

The contributions of this paper are reflected in the following, many scholars have employed general game theory to discuss financial decisions in supply chain management, such as signaling game and Stackelberg game. They usually focus on static problems. To obtain more insight on dynamic supply chain financing strategy, our research is based on the evolutionary game to investigate the interaction mechanism between suppliers' financing strategy selection and manufacturers' loans offering strategy adoption under two innovative financing schemes. By theoretical analysis and numerical study, this paper can offer references for the construction of financing market.

3. Evolutionary Game Model

3.1. Model Description. We assume both players of the game are bounded rationality. Two strategies for each of the players are considered in this evolutionary game model. Suppliers strategy space is defined as $S = (BF, MF)$, BF (bank

financing) represents the supplier's loan from the bank which offers a variety of financing options for suppliers' business to purchase inventory and materials. MF (manufacturer financing) represents the supplier's loan from the manufacturer which offers two innovative financing schemes. Manufacturers' strategy space is defined as $M = (BDF, POF)$. BDF (buyer direct financing) indicates that the manufacturer acts as both the buyer and the lender and finances suppliers for production [3]. POF (purchase order financing) indicates that the bank lends suppliers based on purchase order issued by reputable manufacturers [4, 5]. In the long-term environment, we model four different scenarios of two populations under four strategy profiles: (1) (BDF, MF): when the manufacturer directly issued a loan to the supplier and the supplier happened to accept this financial scheme; (2) (POF, MF): when the manufacturer bank lends suppliers based on purchase order issued by reputable manufacturers and the supplier is willing to accept this financial scheme; (3) (BDF, BF): when the manufacturer directly issued a loan to the supplier, the supplier happened to refuse this financial scheme, and the supplier switches to bank loans directly. (4) (POF, BF): when the manufacturer bank lends suppliers based on purchase order issued by reputable manufacturers, the supplier happened to refuse this financial scheme, and the supplier switches to bank loans directly. The difference between this scenario and scenario (BDF, BF) is that the manufacturer's willingness to provide financial guarantees for the supplier often depends on the supplier's reliability.

x is the probability that suppliers adopt MF strategy, and then $1 - x$ represents the probability that suppliers adopt BF strategy. y is the probability that manufacturers adopt BDF strategy, and then $1 - y$ represents the probability that manufacturers adopt POF strategy. Hence, the evolutionary game model is defined as $f = \{(x, 1 - x), (y, 1 - y)\}$. In the long term, the financing issues of the two populations are modeled under four strategies which are played between manufacturers and suppliers. The major notations related to our research are listed in Table 1. Table 2 indicates payoffs of main manufacturers and suppliers under different strategies. We have

$$\pi_{SM1} = V_s - C_s - L_S(1 + \gamma_m), \quad (1)$$

$$\pi_{SM2} = V_s - C_s - L_S(1 + \gamma_b)e^{-\alpha}, \quad (2)$$

$$\pi_{SB3} = V_s - C_s - L_S(1 + \gamma_b), \quad (3)$$

$$\pi_{SB4} = V_s - C_s - L_S(1 + \gamma_b), \quad (4)$$

$$\pi_{MB1} = L_S(1 + \gamma_m) - C_m, \quad (5)$$

$$\pi_{MP2} = R_m - C_m - L_S(1 + \gamma_b)(1 - e^{-\alpha}), \quad (6)$$

$$\pi_{MB3} = L_S - C_m, \quad (7)$$

$$\pi_{MP4} = R_m - C_m. \quad (8)$$

TABLE 1: Major notations.

Notations	Explanation
π_{SMi}	The profits of the supplier who takes MF strategy
π_{SBi}	The profits of the supplier who takes BF strategy
π_{MBi}	The profits of the manufacturer with BDF strategy
π_{MPi}	The profits of the manufacturer with POF strategy
V_s	The revenue of supplier
γ_m	Interest rate of manufacturer loans. $\gamma_m \in [0, 1]$
γ_b	Interest rate of bank loans $\gamma_b \in [0, 1]$
L_S	The suppliers' loans
C_m	The production costs of the manufacturer
C_s	The production costs of the supplier
R_m	The revenue of the manufacturer
α	Financial risk factor. $\alpha > 0$

TABLE 2: Payoff matrix.

		Manufacturers	
		BDF	POF
Suppliers	MF	π_{SM1}	π_{SM2}
	BF	π_{SB3}	π_{SB4}
		π_{MB1}	π_{MP2}
		π_{MB3}	π_{MP4}

The proportion of suppliers who choose strategy MF over time is noted by y , which is $0 \leq y \leq 1$; moreover, the proportion of manufacturers who select strategy BDF is indicated by x , which is $0 \leq x \leq 1$. Let U_{MF} be the expected utility of the supplier who takes MF strategy, U_{BF} the expected utility of the supplier who takes BF strategy, and E_{BDF}

and E_{POF} the expected utility of the manufacturer who takes divergent strategies (BDF or POF, respectively). According to Table 2 and equations (1)–(8), we can obtain the mixed strategy utilities of suppliers and manufacturers as follows:

$$U_{MF} = y(V_s - C_s - L_S(1 + \gamma_m)) + (1 - y)(V_s - C_s - L_S(1 + \gamma_b)e^{-\alpha}), \quad (9)$$

$$U_{BF} = y(V_s - C_s - L_S(1 + \gamma_b)) + (1 - y)(V_s - C_s - L_S(1 + \gamma_b)), \quad (10)$$

$$E_{BDF} = x(L_S(1 + \gamma_m) - C_m) + (1 - x)(L_S - C_m), \quad (11)$$

$$E_{POF} = x(R_m - C_m - L_S(1 + \gamma_b)(1 - e^{-\alpha})) + (1 - x)(R_m - C_m). \quad (12)$$

Therefore, the mean utility of suppliers and manufacturers is obtained. Let \bar{U} and \bar{E} represent suppliers' mean utility and manufacturers' mean utility, respectively, and \bar{U} and \bar{E} are, respectively, presented as the following:

$$\bar{U} = xU_{MF} + (1 - x)U_{BF}, \quad (13)$$

$$\bar{E} = yE_{BDF} + (1 - y)E_{POF}. \quad (14)$$

According to Zhang et al. [32], the replicator dynamic equations of financing strategy MF adopted by suppliers

$F(x)$ and strategy BDF selected by manufacturers $F(y)$ are determined as

$$F(x) = \frac{dx}{dt} = x(U_{MF} - \bar{U}), \quad (15)$$

$$F(y) = \frac{dy}{dt} = y(E_{GT} - \bar{E}). \quad (16)$$

By substituting (9), (11), (13), and (14) into (15) and (16), the replicator dynamic equations can be obtained as

$$F(x) = e^{-\alpha}(-1+x)xL_s((-1+e^\alpha)(-1+y) - (-1+e^\alpha+y)\gamma_b + e^\alpha y\gamma_m). \quad (17)$$

$$F(y) = -e^{-\alpha}(-1+y)y(-e^\alpha R_m + L_s(e^\alpha - x + e^\alpha x + (-1+e^\alpha)x\gamma_b + e^\alpha x\gamma_m)). \quad (18)$$

Let $F(x) = 0$ and $F(y) = 0$; a solution that does not change over time will be equilibrium.

Proposition 1. For the above model,

(a) $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$ are the fixed equilibrium points.

(b) If $0 < (e^\alpha(R_m - L_s))/(L_s(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_b + e^\alpha\gamma_m)) < 1$, $0 < ((-1 + e^\alpha)(1 + \gamma_b))/(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_m) < 1$, (x^*, y^*) is the equilibrium point and (x^*, y^*) are shown as follows:

$$(x^*, y^*) = \left(\frac{e^\alpha(R_m - L_s)}{L_s(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_b + e^\alpha\gamma_m)}, \frac{(-1 + e^\alpha)(1 + \gamma_b)}{-1 + e^\alpha - \gamma_b + e^\alpha\gamma_m} \right). \quad (19)$$

Proof. Proposition 1 (a) can easily be obtained. Referring to, the equilibrium points must satisfy $F(x) = 0$, $F(y) = 0$. As $0 < x < 1$, $0 < y < 1$, (x^*, y^*) can be obtained.

Proposition 1 shows the equilibrium points from the evolutionary game model, whether these points are the ESS that should be discussed next.

Proposition 2. For the above equilibrium points, the stability analysis of evolutionary game is as follows:

- (i) The fixed equilibrium points are all unstable, except the point $(0, 1)$; the evolutionary game cannot achieve stability at these four fixed points
- (ii) The center point (x^*, y^*) is the saddle point

Proof. According to Lyapunov stability analysis, the Jacobian matrix J (Hofbauer and Sigmund, 1998) is

$$J = \begin{bmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} \end{bmatrix} \quad (20)$$

$$= \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix},$$

where

$$\begin{aligned} \alpha_{11} &= e^{-\alpha}(-1+2x)L_s((-1+e^\alpha)(-1+y) \\ &\quad - (-1+e^\alpha+y)\gamma_b + e^\alpha y\gamma_m), \\ \alpha_{12} &= e^{-\alpha}(-1+x)xL_s(-1+e^\alpha - \gamma_b + e^\alpha\gamma_m), \\ \alpha_{21} &= -e^{-\alpha}(-1+y)yL_s(-1+e^\alpha + (-1+e^\alpha)\gamma_b + e^\alpha\gamma_m), \\ \alpha_{22} &= -e^{-\alpha}(-1+2y)(-e^\alpha R_m \\ &\quad + L_s(e^\alpha - x + e^\alpha x + (-1+e^\alpha)x\gamma_b + e^\alpha x\gamma_m)). \end{aligned} \quad (21)$$

The det J and tr J of five strategies are shown in Table 3, where det $J = \alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}$ and tr $J = \alpha_{11} + \alpha_{22}$.

As shown in Table 3, it is uncertain whether det J and tr J of $(0, 1)$, $(1, 0)$, $(1, 1)$ strategies are bigger or smaller than zero. The asymptotically stable strategy pair must satisfy det $J \geq 0$ and tr $J < 0$ and should be disturbance rejection, which meets the condition $\partial F(x)/\partial x < 0$, $\partial F(y)/\partial y < 0$. By calculating, $(0, 1)$ is the ESS for this game model. Proposition 2 (i) is proven.

The central point (x^*, y^*) adapts to the basic conditions of evolutionary stable strategy. (x^*, y^*) is the Lyapunov stability. Then we should investigate the stability nature of point (x^*, y^*) . If it is the asymptotically stability, (x^*, y^*) is the evolutionary stable strategy.

Considering the center point, the Jacobian Matrix J' is

$$J' = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad (22)$$

where

$$\begin{aligned} b_{11} &= \frac{(L_s - R_m)(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_m)(-e^\alpha R_m + L_s(-1 + 2e^\alpha + (-1 + e^\alpha)\gamma_b + e^\alpha\gamma_m))}{L_s(-1 + e^\alpha + (-1 + e^\alpha)\gamma_b + e^\alpha\gamma_m)^2}, \\ b_{12} &= b_{21} = 0, \\ b_{22} &= \frac{(-1 + e^\alpha)L_s(1 + \gamma_b)(\gamma_b - \gamma_m)(-1 + e^\alpha + (-1 + e^\alpha)\gamma_b + e^\alpha\gamma_m)}{(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_m)^2}. \end{aligned} \quad (23)$$

TABLE 3: The det J and tr J of five strategies.

Strategy	det J	tr J
(0, 0)	$e^{-\alpha} L_s ((1 - e^\alpha)(1 + \gamma_b))(R_m - L_s)$	$-e^{-\alpha} L_s ((1 - e^\alpha)(1 + \gamma_b)) + (L_s - R_m)$
(0, 1)	0	$L_s(\gamma_b - \gamma_m) + (L_s - R_m)$
(1, 0)	$e^{-\alpha} L_s ((1 - e^\alpha)(1 + \gamma_b))(-R_m + L_s(2 - e^{-\alpha} + (-e^{-\alpha} + 1)\gamma_b + \gamma_m))$	$e^{-\alpha} L_s ((1 - e^\alpha)(1 + \gamma_b)) + (-R_m + L_s(2 - e^{-\alpha} + (-e^{-\alpha} + 1)\gamma_b + \gamma_m))$
(1, 1)	$L_s(\gamma_m - \gamma_b)(R_m + L_s(2 - e^{-\alpha} + (-e^{-\alpha} + 1)\gamma_b + \gamma_m))$	$L_s(\gamma_m - \gamma_b) + (R_m + L_s(2 - e^{-\alpha} + (-e^{-\alpha} + 1)\gamma_b + \gamma_m))$
(x^*, y^*)	+	0

The characteristic equation of Jacobian Matrix J' is $J' = \lambda E - J'$, $\det|J'| = 0$, the characteristic roots $\lambda_{1,2}$ can be deduced as

$$\lambda_{1,2} = \pm \frac{i\sqrt{A * B}}{\sqrt{L_s(-1 + e^\alpha + (-1 + e^\alpha)\gamma_b + e^\alpha\gamma_m)(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_m)}}, \quad (24)$$

where

$$\begin{aligned} A &= (L_s - R_m)(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_m)(-e^\alpha R_m \\ &\quad + L_s(-1 + 2e^\alpha + (-1 + e^\alpha)\gamma_b + e^\alpha\gamma_m)) \\ B &= (-1 + e^\alpha)L_s(1 + \gamma_b)(\gamma_b - \gamma_m)(-1 + e^\alpha \\ &\quad + (-1 + e^\alpha)\gamma_b + e^\alpha\gamma_m) \end{aligned} \quad (25)$$

Matrix J' causes virtual characteristic roots, so the point (x^*, y^*) is not asymptotically stable. And it is not the evolutionary stable strategy. Proposition 2 (ii) is proven.

As Proposition 2 shown, $(0, 1)$ is the only ESS in this dynamic system, so every subtle change may not have an impact on the behaviors of suppliers and manufacturers.

The phase diagram of the evolution path is shown in Figure 1. Point $O(0, 1)$ stands for the stable strategy among manufacturers and suppliers. And it proves suppliers are willing to adopt the BF strategy while manufacturers are willing to take BDF strategy. So the dynamic system will evolve to the equilibrium point $O(0, 1)$.

We define the area of $OA_4A_2A_3$ as $S_{OA_4A_2A_3}$, and $S_{OA_4A_2A_3}$ depicts the evolutionary proportion of (BF, BDF) strategy. From the coordinates of A_4 , we can easily obtain

$$S_{OA_4A_2A_3} = \frac{1}{2} \left(\frac{e^\alpha(R_m - L_s)}{L_s(-1 + e^\alpha - \gamma_b + e^\alpha\gamma_b + e^\alpha\gamma_m)} + 1 - \frac{(-1 + e^\alpha)(1 + \gamma_b)}{-1 + e^\alpha - \gamma_b + e^\alpha\gamma_m} \right). \quad (26)$$

Corollary 1. *The number of manufacturers and suppliers who adopt strategy pair (BF, BDF) increases with increasing of α .*

Proof. The first-order partial derivation of $S_{OA_4A_2A_3}$ with respect to financial risk factor α is determined by the following:

$$\frac{\partial S_{OA_4A_2A_3}}{\partial \alpha} = \frac{e^\alpha \text{Log}[e](L_s - R_m)(1 + \gamma_b)}{L_s(1 + \gamma_b - e^\alpha(1 + \gamma_b + \gamma_m))^2} < 0. \quad (27)$$

Thus, $S_{OA_4A_2A_3}$ is decreasing with the increase of financial risk factor. That means the point A_4 evolves into point A_3 and the probability that the replicator dynamic system will converge to strategy pair (BF, BDF).

Corollary 2. *It implies the impacts of financial risk factor on the probability that game players adopt strategy pair (BF, BDF). As the financial risks grow, suppliers tend to choose BF strategy and manufacturers are willing to adopt BDF strategy. The game players choose this strategy pair to maintain stability in the financial risks. When the financial risk is relatively high, the small supplier access to manufacturer financing will be hazardous. Manufacturers choose the buyer direct financing scheme to suppliers who were unable to give suppliers a guarantee. Apparently, strategy pair (BF, BDF) can ensure the profits of manufacturers and suppliers to a large extent.*

Corollary 3. *The number of manufacturers and suppliers who adopt strategy pair (BF, BDF) increases with increasing of γ_b .*

Proof. The first-order partial derivation of $S_{OA_4A_2A_3}$ with respect to interest rate of bank loans γ_b is determined by the following:

$$\frac{\partial S_{OA_4A_2A_3}}{\partial \gamma_b} = \frac{e^\alpha(-1 + e^\alpha)(L_s - R_m)}{L_s(1 + \gamma_b - e^\alpha(1 + \gamma_b + \gamma_m))^2} < 0. \quad (28)$$

Therefore, $S_{OA_4A_2A_3}$ is decreasing with the increase of bank loans rate. That means the point A_4 evolves into point A_3 ; the probability that the replicator dynamic system will evolve to strategy pair (BF, BDF) increases with the increase of γ_b .

Corollary 2 shows the impacts of interest rate of the bank loans on the probability that game players adopt strategy pair (BF, BDF). However, the game players adopt strategy pair (BF, BDF) to maintain stability in the high-level interest rate of the bank loans. As we all know that strategy pair (BF, BDF) can ensure the profits of game players to a large extent. Thus it is beneficial to the profit of manufacturers and suppliers when the interest rate of the bank loans is relatively high.

Corollary 4. *The number of manufacturers and suppliers who adopt strategy pair (BF, BDF) increases with increasing of γ_m .*

Proof. The first-order partial derivation of $S_{OA_4A_2A_3}$ with respect to interest rate of the manufacturer loans γ_m is determined by the following:

$$\frac{\partial S_{OA_4A_2A_3}}{\partial \gamma_m} = \frac{e^{2\alpha}(L_s - R_m)}{L_s(1 + \gamma_b - e^\alpha(1 + \gamma_b + \gamma_m))^2} < 0. \quad (29)$$

Therefore, $S_{OA_4A_2A_3}$ is decreasing with the increase of interest rate of the manufacturer loans. That means the point A_4 evolves into point A_3 ; the probability that the replicator dynamic system will evolve to strategy pair (BF, BDF) increases with the increase of γ_m .

Corollary 3 indicates the impacts of interest rate of the manufacturer loans on the probability that game players adopt strategy pair (BF, BDF). As the interest rate of the manufacturer loans increases, the probability that the

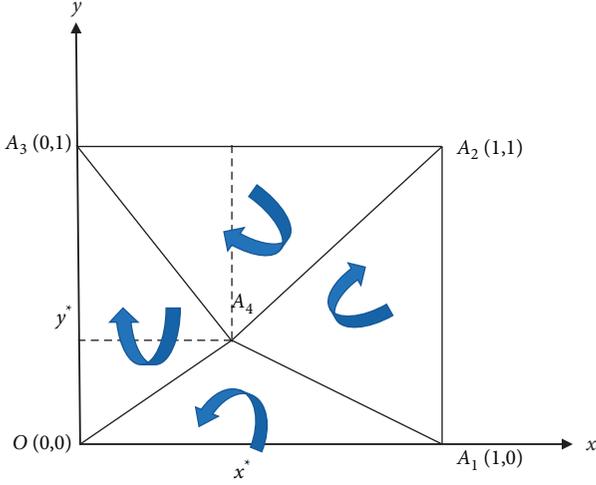


FIGURE 1: Manufacturers' evolutionary stable strategy and suppliers' evolutionary stable strategy evolve to $(0, 1)$.

replicator dynamic system will evolve to strategy pair (BF, BDF) increases. The game players adopt this strategy pair to maintain stability in the high-level interest rate of the bank loans. Clearly, strategy pair (BF, BDF) always can ensure the profits of game players to a large extent. Therefore, $S_{OA_4A_2A_3}$ is decreasing with the increase of interest rate of the manufacturer loans, which means $S_{OA_4A_2A_1}$ increases with the increase of interest rate of the manufacturer loans and point A_4 gets far away from point A_3 . Thus, it is not beneficial for the system evolving to (MF, POF) when the interest rate of the manufacturer loans is relatively high.

Corollary 5. *The number of manufacturers and suppliers who adopt strategy pair (BF, BDF) increases with increasing of L_s .*

Proof. The first-order partial derivation of $S_{OA_4A_2A_3}$ with respect to the suppliers' loans L_s is determined by the following:

$$\frac{\partial S_{OA_4A_2A_3}}{\partial L_s} = -\frac{e^\alpha R_m}{L_s^2(-1 - \gamma_b + e^\alpha(1 + \gamma_b + \gamma_m))} < 0. \quad (30)$$

Therefore, $S_{OA_4A_2A_3}$ is decreasing with the increase of the supplier loans. That means the point A_4 evolves into point A_3 ; the probability that the replicator dynamic system will evolve to strategy pair (BF, BDF) increases with the suppliers' loans L_s .

Corollary 4 shows the impacts of the suppliers' loans on the probability that game players adopt strategy pair (BF, BDF). As the supplier loans increase, suppliers tend to choose BF strategy and manufacturers are not willing to adopt POF strategy. The game players adopt (BF, BDF) strategy pair to maintain stability when the suppliers' loans are relatively high. $S_{OA_4A_2A_3}$ decrease as the L_s increases; it means manufacturers and suppliers are not willing to adopt strategy pair (MF, POF). Clearly, strategy pair (BF, BDF) can ensure the profits of game players to a large extent; thus it is

harmful for the system evolve to (MF, POF) when the suppliers' loans are relatively high.

Corollary 6. *The number of manufacturers and suppliers who adopt strategy pair (MF, POF) increases with increasing of R_m .*

Proof. The first-order partial derivation of $S_{OA_4A_2A_3}$ with respect to the revenue of the manufacturer R_m is determined by the following:

$$\frac{\partial S_{OA_4A_2A_3}}{\partial R_m} = \frac{1}{L_s(1 + \gamma_b - e^{-\alpha}(1 + \gamma_b) + \gamma_m)} > 0. \quad (31)$$

Therefore, $S_{OA_4A_2A_3}$ is increasing with the increase of the revenue of the manufacturer. That means the point A_4 evolves into point A_1 ; the probability that the replicator dynamic system will evolve to strategy pair (MF, POF) increases with the revenue of the manufacturer R_m .

Corollary 5 shows the impacts of R_m on the probability that game players adopt strategy pair (MF, POF). As the manufacturers' revenue increases, suppliers tend to choose MF strategy and manufacturers are not willing to adopt BDF strategy. The game players adopt (MF, POF) strategy pair to maintain stability when R_m is relatively high. Thus, it is harmful for the system to evolve to (MF, POF) when the revenue of the manufacturer is relatively low.

4. Numerical Study

We use Vensim PLE to test the performance of evolutionary game model. The system dynamics model (SD model) is developed before we use Vensim PLE to show the dynamics behavior among suppliers and manufacturers. A series of numerical studies are implemented by a financial chain with groups of suppliers and manufacturers. The two sections are considered: Section 1 discussed how game players respond to different initial values; Section 2 assessed the influencing factors of strategy pair (MF, POF).

4.1. SD Model. The system dynamics model of the evolutionary game among suppliers and manufacturers is shown in Figure 2.

The system dynamics model consists of two flow rates, eight intermediate variables, four flows, and eight external variables. Two flow rates try to depict the change in the probability of suppliers (manufacturers) adopting MF (BDF) strategy. The four flows are used to indicate the probability of suppliers (manufacturers) choosing MF (BDF) strategy or BF (POF) strategy. Eight external variables correspond to eight variables in the game payoff matrix.

4.2. The Evolution Tendency of Game Players. To simulate the evolutionary game model, three sets of numerical experiments are carried out. Numerical studies are obtained by solving the replicator dynamics equation by using SD method. This section is for observation of evolution tendency among the different initial values. To make this purpose, we set up a

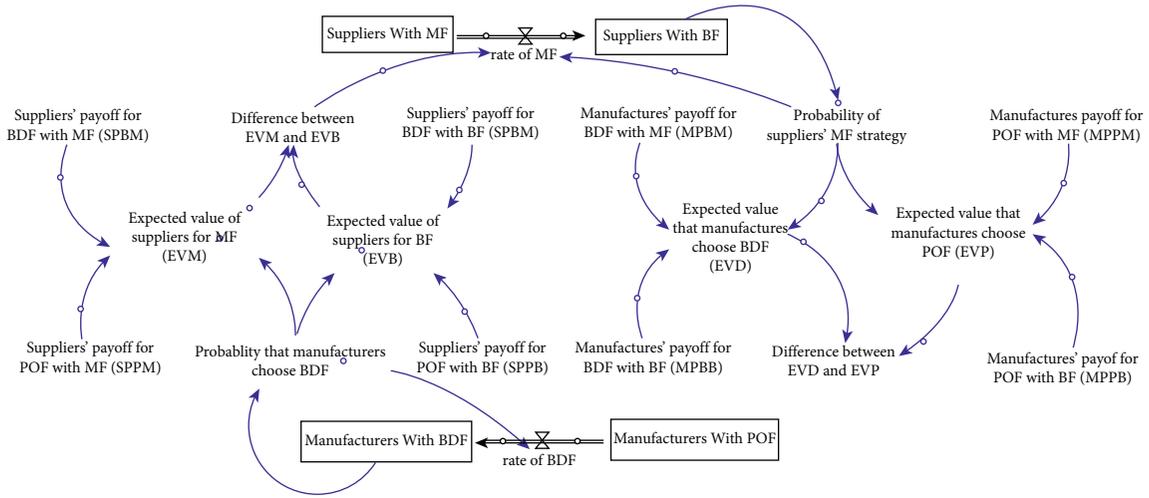


FIGURE 2: System dynamics model of suppliers and manufacturers behavior.

SD model with three scenarios; the initial values are determined in Table 4. If we change the initial value, we can get two ESS (0, 1) and (1, 0), but the dynamic system will never evolve to the points (0, 0) and (1, 1).

Figure 3 shows the probability of game players adopting (BF, BDF) strategy, where the blue lines represent the probability of suppliers adopting MF strategy and the red lines show the manufacturers' strategy tendency. The numerical experiments are depicted in the scenario (i)–(iii). After 100 simulations, the evolutionary game can catch the ESS in scenarios (i) and (iii). In scenario (i), the dynamic system converges to the strategy pair (BF, BDF) when the revenue of the manufacturer $R_m = 2.1$. In scenario (ii), the dynamic system cannot converge to the strategy pair (MF, BDF) and (BF, POF) when the revenue of the manufacturer $R_m = 1.4$. In scenario (iii), the dynamic system converges to the strategy pair (MF, POF) when the revenue of the manufacturer $R_m = 3$, and the cost of manufacturer $C_m = 0.15$ is relatively high. The manufacturers want to choose POF strategy and the suppliers are willing to adopt MF strategy when the manufacturers' revenue and cost are relatively high.

Figure 3 indicates that the evolution results are related to the manufacturers' revenue. As shown in Figure 3(a), if the manufacturer sells low premium items, the dynamic system cannot reach stability in the evolution of financial strategies. This is because the manufacturer who without competitiveness cannot provide a convincing guarantee for the supplier is even more unable to fund suppliers directly. Therefore, suppliers should establish supply chain relationships with manufacturers with high brand influence, to obtain a more stable capital chain and gradually strengthen the cooperative relationship among supply chain members. When the two parties establish a mutually beneficial cooperative relationship, the company can help suppliers find ways to reduce costs, thereby reducing prices. Furthermore, when the two parties have established a good cooperative relationship, many tasks can be simplified, such as ordering, statistics, and quality inspection, thereby reducing cost.

TABLE 4: The external variable values in simulation.

Group	i	ii	iii
V_s	2	2	2
γ_m	0.05	0.05	0.05
γ_b	0.04	0.04	0.04
L_s	1.5	1.5	1.5
C_m	0.1	0.05	0.15
C_s	0.2	0.2	0.2
R_m	2.1	1.4	3
α	1	1	1

As shown in Figure 3(b), if the manufacturer produces proper premium items, the dynamic system converges to the strategy pair (BF, BDF). The manufacturer has insufficient cash flows and is unwilling to provide financing guarantees for suppliers funding from the bank, suppliers are more willing to choose bank financing strategy to ensure that their capital chain does not break. Manufacturers with proper premium products are more willing to provide partners with BDF financial strategies, directly provide sources of funds to suppliers, further cooperate with suppliers, increase product innovation and transformation, and gradually expand market share.

As shown in Figure 3(c), if the manufacturer sells high-premium products, the dynamic system converges to the strategy pair (MF, POF). The manufacturer has sufficient cash flows and is willing to provide financing guarantees for suppliers funding from the bank, and suppliers are more willing to choose manufacturer financing strategy to ensure that their capital chain does not break. Small size and startup suppliers often have difficulty accessing financing from the bank because of their lack of credit history. Thus, when high-premium product manufacturers cooperate with suppliers, suppliers prefer to choose manufacturer financing strategy, and it is more profitable for manufacturers to provide financial guarantees for suppliers instead of directly lending suppliers.

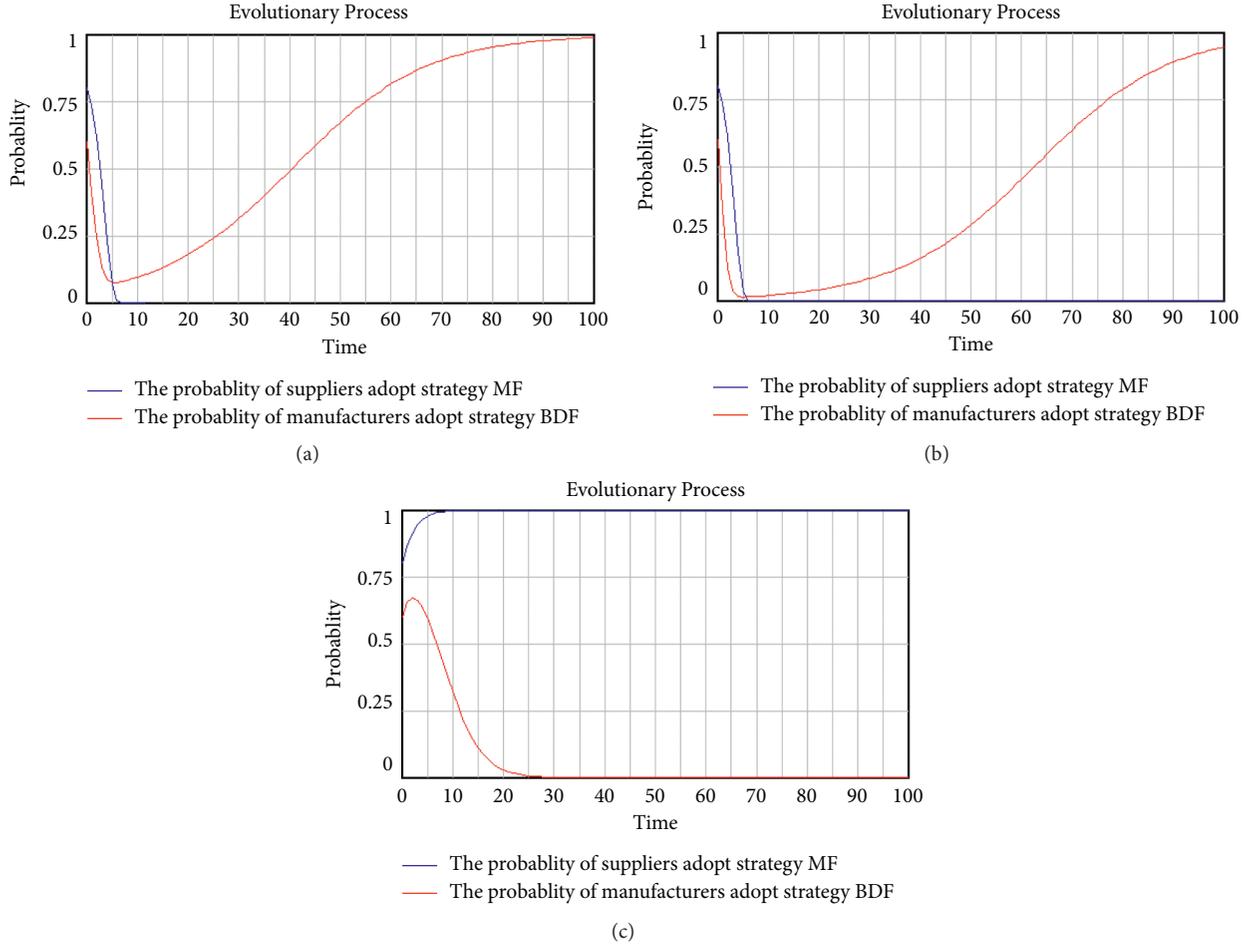


FIGURE 3: The evolutionary process.

4.3. *The Influencing Factors of Strategy Pair (BF, BDF).* In this section, a sensitivity analysis is performed to better understand the above theoretical results. The sensitivity analysis based on $S_{OA_4A_2A_3}$ is decided by various parameters.

Figure 4 shows the number of manufacturers and suppliers who adopt strategy pair (BF, BDF) increases with increasing of financial risk factor α . Given a fixed value of financial risk factor, the high value of manufacturers' revenue brings about the high proportion of selecting strategy pair (MF, POF). However, with increasing of financial risk factor, the changes in the proportion of selecting strategy pair (BF, BDF) tend to be flat. Thus, when $\alpha < 3$, reducing the financial risk factor can promote the dynamic system evolving to the strategy pair (MF, POF).

Figure 5 illustrates the impacts of the bank loan rate γ_b and financial risk factor α on the evolutionary proportion of selecting strategy pair (BF, BDF). The sensitivity analysis results certificate Corollary 2. It reveals the probability that the replicator dynamic system will evolve to strategy pair (BF, BDF) increases with the increase of γ_b . Given a fixed value of the bank loan rate, the high value of financial risk factor brings about the high proportion of selecting strategy pair (BF, BDF).

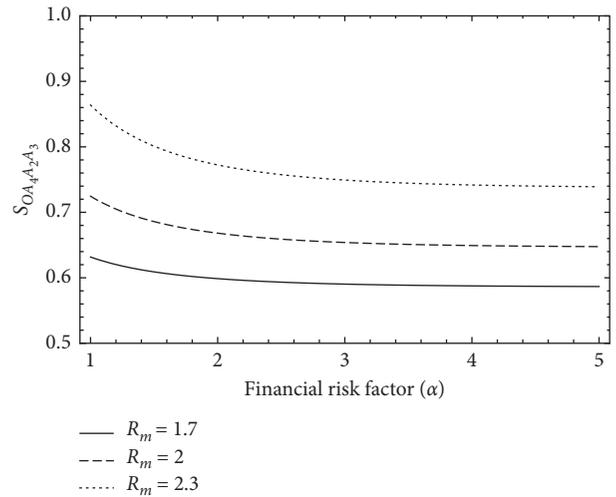


FIGURE 4: The impacts of financial risk factor on $S_{OA_4A_2A_3}$ with various manufacturers' revenue.

Figure 6 depicts the impacts of the manufacturer loan rate γ_m and financial risk factor α on the evolutionary proportion of selecting strategy pair (BF, BDF). It reveals

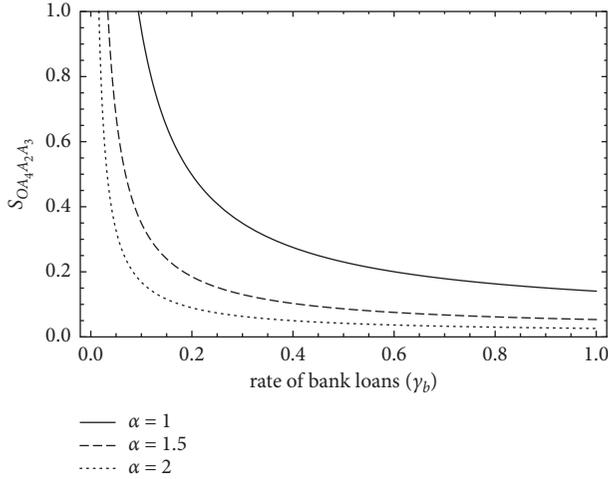


FIGURE 5: The impacts of the bank loan rate on $S_{OA_4A_2A_3}$ with various financial risk factors.

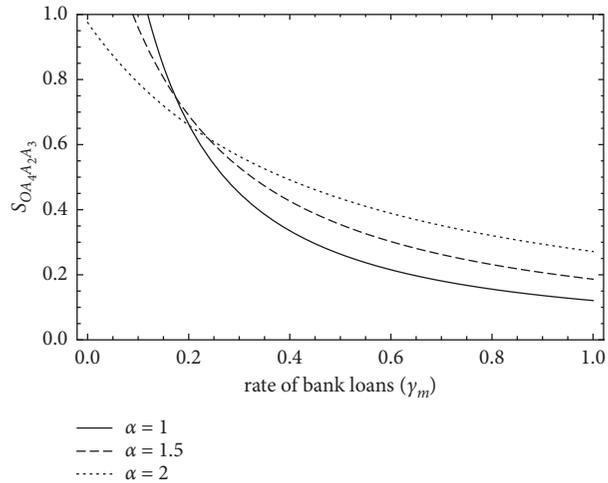


FIGURE 6: The impacts of the manufacturer loan rate on $S_{OA_4A_2A_3}$ with various financial risk factors.

the probability that the replicator dynamic system will evolve to strategy pair (BF, BDF) increases with the increase of γ_m . When $\gamma_m > 0.2$, given a fixed value of the manufacturer loan rate, the high value of financial risk factor brings about the high proportion of selecting strategy pair (BF, BDF). When $\gamma_m > 0.2$, given a fixed value of the manufacturer loan rate, the high value of financial risk factor brings about the high proportion of selecting strategy pair (MF, POF).

Figure 7 reveals the impacts of the suppliers' loans L_s and manufacturers' revenue R_m on the evolutionary proportion of selecting strategy pair (BF, BDF). The sensitivity analysis results certificate Corollary 4. The $S_{OA_4A_2A_3}$ decrease as the L_s increases, it means manufacturers and suppliers are not willing to adopt strategy pair (MF, POF) when L_s is relatively high. In addition, the suppliers' loan L_s increases as manufacturers' revenue R_m .

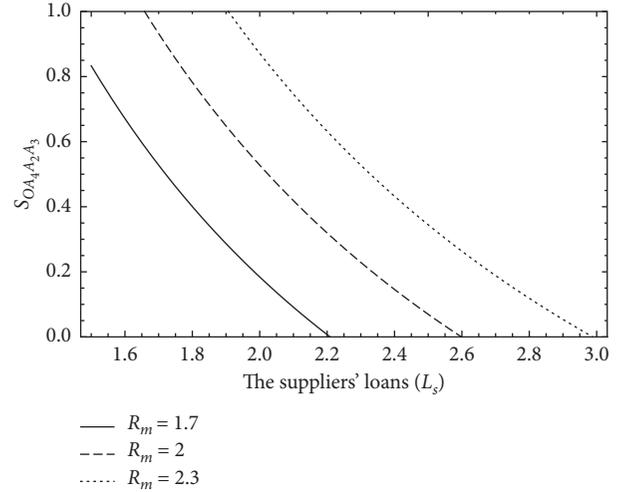


FIGURE 7: The impacts of the suppliers' loans on $S_{OA_4A_2A_3}$ with various manufacturers' revenue.

5. Conclusions

POF and BDF are both innovative financing schemes aiming to help financial constrained suppliers secure financing for production. In this paper, we investigate the interaction mechanism between suppliers' financing strategy selection and manufacturers' loans offering strategy adoption under two innovative financing schemes. Firstly, we proposed an evolutionary game model to effectively investigate the interaction mechanism between suppliers and manufacturers. Secondly, the payoff matrix of suppliers and manufacturers under two innovative financing schemes was analyzed and the evolutionary stable strategies were obtained. Finally, we use system dynamics to present the performance of the evolutionary game model and took a sensitivity analysis to verify the theoretical results.

We can draw the following conclusions: Firstly, the evolutionary model can reach the stability strategy pair (BF, BDF); if we change the initial value, we can get another stability strategy pair (MF, POF) when manufacturers' revenue and cost are relatively high. Secondly, when high-premium product manufacturers cooperate with suppliers, suppliers prefer to choose manufacturer financing strategy, and it is more profitable for manufacturers to provide financial guarantees for suppliers instead of directly lending suppliers. Thirdly, for manufacturers to participate in the game, the initial conditions need to be changed to make sure the evolutionary game evolves to the strategy pair (MF, POF). When the rate of manufacturer loan is relatively high, the high value of financial risk factor brings about the high proportion of selecting strategy pair (MF, POF). And the financial risk factor becomes larger in a moderate range, which can accelerate the evolution of the system to (MF, POF) much faster.

As the first attempt at understanding the interaction mechanism between suppliers' financing strategy selection and manufacturers' loans offering strategy adoption under BDF and POF financing schemes, our research is not

without limitations though; we take the evolutionary game model to encourage manufacturers and suppliers to reach financial cooperation and obtain the approaches to promote the dynamic system evolving to the strategy pair (MF, POF). Nevertheless, we assume that the information between players is symmetry. We should consider information between players is asymmetry in the future research. We only focus on the static rate of loans in our model; it is interesting to explore how the dynamic rate of loans affects the evolutionary system. Furthermore, how to implement the dynamic rate of loans should be further discussed.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was supported by MOE (Ministry of Education in China) Project of Humanities and Social Sciences (project no. 21YJCZH223) and also sponsored by Shanghai Sailing Program (no. 21YF1416500).

References

- [1] G. Liu, W. Zhao, and X. Zhao, "A new firm entry under capital constraint," 2018, <https://ssrn.com/abstract=3324336>.
- [2] J. A. Buzacott and R. Q. Zhang, "Inventory management with asset-based financing," *Management Science*, vol. 50, no. 9, pp. 1274–1292, 2004.
- [3] C. S. Tang, S. A. Yang, and J. Wu, "Sourcing from suppliers with financial constraints and performance risk," *Manufacturing & Service Operations Management*, vol. 20, no. 1, pp. 70–84, 2018.
- [4] A. Martin, *The Places They Go when Banks Say No*, New York Times, New York, NY, USA, 2010, <http://www.nytimes.com/2010/01/31/business/smallbusiness/31order.html?dbk>.
- [5] C. Tice, *Can a Purchase Order Loan Keep Your Business Growing?* *Entrepreneur*, <http://www.entrepreneur.com/article/207058>, 2010.
- [6] Gustin D. (2014) Purchase Order Finance, the Tough Nut to Crack. Trade Financing Matters (April 28), <http://spendmatters.com/tfismatters/purchase%20order-finance-the-tough-nut-to-crack/>.
- [7] S. Watkins, *Top UK Firms Lend Billions to Struggling Suppliers as They Are Forced to Act as Banks*, Daily Mail, London, UK, 2012, <http://www.dailymail.co.uk/money/news/article-2220517/Top-UK-firms-lend%20billions-suppliers-forced-act-bank-s.html>.
- [8] McKinsey & Company, *Managing Global Supply Chains: McKinsey Global Survey Results*, McKinsey Quarterly, Shanghai, China, 2008.
- [9] L. Liu, J. J. Lv, J. Y. Li, and Y. Z. Wang, "Evolutionary game model of information interaction in social network," *Journal of Information and Computational Science*, vol. 12, no. 6, pp. 2375–2388, 2015.
- [10] Y. Chen, S. Ding, H. Zheng, Y. Zhang, and S. Yang, "Exploring diffusion strategies for mHealth promotion using evolutionary game model," *Applied Mathematics and Computation*, vol. 336, pp. 148–161, 2018.
- [11] M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, and A. Szolnoki, "Statistical physics of human cooperation," *Physics Reports*, vol. 687, pp. 1–51, 2017.
- [12] F. Modigliani and M. H. Miller, "The cost of capital, corporation finance and the theory of investment," *The American Economic Review*, vol. 48, no. 3, pp. 261–297, 1958.
- [13] N. Pakhira, M. K. Maiti, and M. Maiti, "Uncertain multi-item supply chain with two level trade credit under promotional cost sharing," *Computers & Industrial Engineering*, vol. 118, pp. 451–463, 2018.
- [14] M. Johari, S.-M. Hosseini-Motlagh, M. Nematollahi, M. Goh, and J. Ignatius, "Bi-level credit period coordination for periodic review inventory system with price-credit dependent demand under time value of money," *Transportation Research Part E: Logistics and Transportation Review*, vol. 114, pp. 270–291, 2018.
- [15] A. Kaur, "Two-level trade credit with default risk in the supply chain under stochastic demand," *Omega*, vol. 88, pp. 4–23, 2019.
- [16] V. Babich and M. J. Sobel, "Pre-IPO operational and financial decisions," *Management Science*, vol. 50, no. 7, pp. 935–948, 2004.
- [17] Q. Lin and J. He, "Supply chain contract design considering the supplier's asset structure and capital constraints," *Computers & Industrial Engineering*, vol. 137, Article ID 106044, 2019.
- [18] V. Babich and C. S. Tang, "Managing opportunistic supplier product adulteration: deferred payments, inspection, and combined mechanisms," *Manufacturing & Service Operations Management*, vol. 14, no. 2, pp. 301–314, 2012.
- [19] H. Rui and G. Lai, "Sourcing with deferred payment and inspection under supplier product adulteration risk," *Production and Operations Management*, vol. 24, no. 6, pp. 934–946, 2015.
- [20] Z. Liu and J. M. Cruz, "Supply chain networks with corporate financial risks and trade credits under economic uncertainty," *International Journal of Production Economics*, vol. 137, no. 1, pp. 55–67, 2012.
- [21] G. E. Applequist, J. F. Pekny, and G. V. Reklaitis, "Risk and uncertainty in managing chemical manufacturing supply chains," *Computers & Chemical Engineering*, vol. 24, no. 9–10, pp. 2211–2222, 2000.
- [22] G. Soni and R. Kodali, "A decision framework for assessment of risk associated with global supply chain," *Journal of Modelling in Management*, vol. 8, no. 1, pp. 25–53, 2013.
- [23] M. Krystofik, C. J. Valant, J. Archbold, P. Bruessow, and N. G. Nenadic, "Risk assessment framework for outbound supply-chain management," *Information*, vol. 11, no. 9, p. 417, 2020.
- [24] D. Bandaly, A. Satir, and L. Shanker, "Integrated supply chain risk management via operational methods and financial instruments," *International Journal of Production Research*, vol. 52, no. 7, pp. 2007–2025, 2014.
- [25] M. Munir, M. S. Sadiq Jajja, K. A. Chatha, and S. Farooq, "Supply chain risk management and operational performance: the enabling role of supply chain integration," *International Journal of Production Economics*, vol. 227, Article ID 107667, 2020.
- [26] B. Zeng and B. P.-C. Yen, "Rethinking the role of partnerships in global supply chains: a risk-based perspective,"

- International Journal of Production Economics*, vol. 185, pp. 52–62, 2017.
- [27] S. R. Cardoso, A. P. Barbosa-Póvoa, and S. Relvas, “Integrating financial risk measures into the design and planning of closed-loop supply chains,” *Computers & Chemical Engineering*, vol. 85, pp. 105–123, 2016.
- [28] Z. Wang, Q. Wang, B. Chen, and Y. Wang, “Evolutionary game analysis on behavioral strategies of multiple stakeholders in E-waste recycling industry,” *Resources, Conservation and Recycling*, vol. 155, Article ID 104618, 2020.
- [29] S.-M. Hosseini-Motlagh, M. Nematollahi, M. Johari, and B. R. Sarker, “A collaborative model for coordination of monopolistic manufacturer’s promotional efforts and competing duopolistic retailers’ trade credits,” *International Journal of Production Economics*, vol. 204, pp. 108–122, 2018.
- [30] B. Wu, P. Liu, and X. Xu, “An evolutionary analysis of low-carbon strategies based on the government-enterprise game in the complex network context,” *Journal of Cleaner Production*, vol. 141, pp. 168–179, 2017.
- [31] R. Fan, L. Dong, W. Yang, and J. Sun, “Study on the optimal supervision strategy of government low-carbon subsidy and the corresponding efficiency and stability in the small-world network context,” *Journal of Cleaner Production*, vol. 168, pp. 536–550, 2017.
- [32] S. Zhang, C. Wang, and C. Yu, “The evolutionary game analysis and simulation with system dynamics of manufacturer’s emissions abatement behavior under cap-and-trade regulation,” *Applied Mathematics and Computation*, vol. 355, pp. 343–355, 2019.