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Research Article

Some Fixed-Point Theorems on Generalized Cyclic Mappings in *B*-Metric-Like Spaces

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In this paper, it is concerned with the cyclic mapping in b-metric-like spaces. The definition of W-type cyclic mappings is proposed, and then, the existence-uniqueness of the fixed points of these cyclic mappings and the corresponding fixed point theorems are studied. In b-metric-like spaces, the promotion of the concept of cyclic mapping is an interesting topic; then, it is worthy to continue to this part of the promotion. On this basis, the concept of φ-type cyclic mapping is proposed in this article, and the existence-uniqueness of fixed-point problems and the corresponding fixed-point theorem are considered and studied. The results of this paper further generalize and extend some previous results.

1. Introduction

The problem of solving the equation is involved in many research areas of mathematics; in addition, there is more than just one way to solve the equation. The fixed-point theory is a kind of general theory which is closely related to solving equations. Algebraic equations, functional equations, differential equations, and so on have various forms. These different kinds of equations, although they have different forms, can often be rewritten in terms of $f(x) = x$. So, x here is a point in some appropriate space X, and f is a mapping or movement from X to X, moving every point x to the point $f(x)$. The solution of equation $f(x) = x$ is exactly the point that is left in place under the action of f, so it is called the fixed point. So, the problem of solving the equation becomes a geometric problem of finding a fixed point.

It is a very interesting work and a hot topic to find out whether fixed points of many mappings exist and are unique. There are a lot of papers on this aspect. Mathematical researchers study the fixed-point problems in a metric space, which generally involve two aspects; on the one hand, it was to the definition of a metric space, that is, the three basic properties of metric space in the definition have been widely discussed, then all kinds of generalized metric spaces are obtained, and in the generalized metric spaces, the classic Banach mapping contraction principle is frequently extended [1]. On the other hand, based on the study of related problems of generalized metric space, the extension of the original some mappings, and the proposal of some new mappings, through the hard work of many mathematical researchers, many conclusions have been obtained. Next, we will properly elaborate on these two aspects.

The concept of a metric space is introduced as follows.

Definition 1. Let X be a nonempty set and $d: X \times X \longrightarrow [0, \infty)$ be a function such that, for all $x, y \in X$, the following three conditions hold true:

(i) $d(x, y) \geq 0, d(x, y) = 0 \Leftrightarrow x = y$
(ii) $d(x, y) = d(y, x)$
(iii) $d(x, y) \leq d(x, z) + d(z, y)$

Then, the pair (X, d) is called a metric space.

In fact, letting $d(x, y) = |x - y|$, it is easy to know that it is a metric.

Next, let us see how each of the three properties of metric spaces has been investigated.

In 2012, Amini-Harandi [2] considered the first property of metric space and made appropriate modifications, and thus they proposed the definition of metric-like space.

Definition 2. Let X be a nonempty set and $\varphi\colon X \times X \longrightarrow R_+$ be a function such that, for all $x, y, z \in X$, the following three conditions hold true:

(i) $\varphi(x, y) = 0 \Rightarrow x = y$

(ii) $\varphi(x, y) = \varphi(y, x)$

(iii) $\varphi(x, y) \leq \varphi(x, z) + \varphi(z, y)$

Then, the pair (X, φ) is called a metric-like space.

In 1993, Czerwik [3] considered the third property of metric space and thus proposed the definition of b-metric space.

Definition 3. A mapping $D\colon X \times X \longrightarrow [0, \infty)$, where X is a nonempty set, is said to be a b-metric on X if, for any $x, y, z \in X$ and $K \geq 1$, the following three conditions hold true:

(1) $D(x, y) = 0 \Leftrightarrow x = y$

(2) $D(x, y) = D(y, x)$

(3) $D(x, z) \leq K(D(x, y) + D(y, z))$

The pair (X, D) is then called a b-metric-like space.

In 2013, Alghamdi et al. [4] considered the first and third properties of metric space and thus proposed the definition of b-metric-like space.

Definition 4. Letting a b-metric-like on a nonempty set X be a function $D\colon X \times X \longrightarrow [0, +\infty)$ such that, for all $x, y, z \in X$ and a constant $K \geq 1$, the following three conditions hold true:

(**D**1) if $D(x, y) = 0 \Rightarrow x = y$

(**D**2) $D(x, y) = D(y, x)$

(**D**3) $D(x, y) \leq K(D(x, z) + D(z, y))$

The pair (X, D) is called a b-metric-like space.

It is important to note that the symbols d, D, φ, r in Definitions 1 to 4 all represent a measure symbol. In essence, the meaning is the same. In fact, there are many generalizations of the definition of metric space, such as Definitions 5–9.

Definition 5 (see [5]). A mapping $p\colon X \times X \longrightarrow R_+$, where X is a nonempty set, is said to be a partial metric on X if, for any $x, y, z \in X$, the following four conditions hold true:

(**P**1) $x = y$ if and only if $p(x, x) = p(y, y) = p(x, y)$

(**P**2) $p(x, x) \leq p(x, y)$

(**P**3) $p(x, y) = p(y, x)$

(**P**4) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$

The pair (X, p) is then called a partial metric space.

In 2017, Kamran et al. [6] obtained some fixed-point theorems in a generalized b-metric space.

Definition 6. Consider the set $F \neq \varnothing$ and a function $\hbar\colon F \times F \longrightarrow [1, \infty)$. Suppose that a function $\zeta\colon F \times F \longrightarrow R^+$ satisfies the following conditions, for all $g, h, w \in F$:

(1) $\zeta(g, h) = 0 \Leftrightarrow g = h$

(2) $\zeta(g, h) = \zeta(h, g)$

(3) $\zeta(g, h) \leq \hbar(g, h)[D(g, w) + D(w, h)]$

The pair (F, ζ) is called an extended b-metric space.

Some results for b-metric space and extended b-metric space are in [7–14]. The definition of controlled metric-type space [15] is presented as follows.

Definition 7. Given a nonempty set F and a function $\varpi\colon F^2 \to [1, \infty)$. Suppose that a function $\rho\colon F^2 \to [0, \infty)$ satisfies the following conditions, for all $g, h, w \in F$:

(ρ1) $\rho(g, h) = 0 \Leftrightarrow g = h$

(ρ2) $\rho(g, h) = \zeta(h, g)$

(ρ3) $\rho(g, h) \leq \varpi(g, w)\rho(g, w) + \varpi(w, h)\rho(w, h)$

The pair (F, ρ) is called a controlled metric-type space.

In fact, the definition of a generalization of controlled metric-type spaces to double controlled metric-type spaces is given as follows.

Definition 8 (see [16]). Consider a set $F \neq \varnothing$ and noncomparable functions $\hbar, \varepsilon\colon F \times F \longrightarrow [1, \infty)$. Suppose that a function $\zeta\colon F \times F \longrightarrow [0, \infty)$ satisfies the following conditions for all $g, h, w \in F$:

(1) $\zeta(g, h) = 0$ if only and if $g = h$

(2) $\zeta(g, h) = \zeta(h, g)$

(3) $\zeta(g, h) \leq \varpi(g, w)\zeta(g, w) + \varepsilon(w, h)\zeta(w, h)$

The pair (F, ζ) is called a double controlled metric-type space.

Next, the generalization of the double controlled metric-type space [17] is given as follows.

Definition 9. Consider a set $F \neq \varnothing$ and noncomparable functions $\hbar, \varepsilon\colon F \times F \longrightarrow [1, \infty)$. Suppose that a function $\zeta\colon F \times F \longrightarrow [0, \infty)$ satisfies the following conditions for all $g, h, w \in F$:

(1) $\zeta(g, h) = 0 \Rightarrow g = h$

(2) $\zeta(g, h) = \zeta(h, g)$

(3) $\zeta(g, h) \leq \varpi(g, w)\zeta(g, w) + \varepsilon(w, h)\zeta(w, h)$

The pair (F, ζ) is called a double controlled metric-like space.

We have explored the development of generalized metric spaces above; for these generalized metric spaces, many mappings in metric spaces and generalization of generalized metric spaces have produced many results. Next, we will focus on the generalization of related mappings in b-metric-like spaces. However, we are going to start with a classic result from metric space. In a metric space, Banach contractions' principle is a classical result in fixed-point theory. Here, we state the concept of contractive mapping as follows.

Definition 10. Let X be a metric space and $T\colon X \longrightarrow X$ be a mapping. If there exists a constant $\alpha \in (0,1)$ such that, for all $x, y \in X$, the following condition is satisfied,

$$d(Tx, Ty) \leq \alpha\, d(x, y), \tag{1}$$

then the mapping T is called a contractive mapping.

It is natural that Banach contractions principle [1] is extended in kinds of metric space, for example, partial metric space, b-metric space, and b-metric-like space.

In 2017, Lei and Wu [18] proposed the concept of the LW-type cyclic mapping in a complete b-metric-like space and obtained the corresponding fixed-point theorem as follows.

Definition 11. Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$ and there exists some nonnegative real constants γ, δ, t such that, for all $x \in G_1$, $y \in G_2$ satisfy the following condition,

$$\gamma r(x, Bx) + \delta r(y, Sy) + t r(Bx, Sy) \leq r(x, y), \tag{2}$$

then (B, S) is called as a LW-type cyclic mapping.

Theorem 1. *Let (X, r) be a complete b-metric-like space, and (B, S) be a LW-type cyclic mapping in (X, r). Suppose that G_1, G_2 are nonempty closed sets in (X, r) and $G_1 \cap G_2 \neq \varnothing$, if $\gamma + \delta + t > s$, $t > 1$. Then, there exists a unique $z^* \in G_1 \cap G_2$ such that $Bz^* = z^* = Sz^*$, that is, B and S have a unique common fixed point.*

In 2018, Weng et al. [19] proposed the concept of LW-type Lipschitz cyclic mapping in a complete b-metric-like space as follows.

Definition 12. Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$ and there exists some nonnegative real constants γ, δ, t, L and $L < \gamma + \delta + t$ such that, for all $x \in G_1$, $y \in G_2$ satisfy the following condition,

$$\gamma r(x, Bx) + \delta r(y, Sy) + t r(Bx, Sy) \leq L r(x, y), \tag{3}$$

then (B, S) is called a LW-type Lipschitz cyclic mapping.

Here, the definition of a pair semicyclic mapping and the definition of a complete b-metric-like space are given in Section 2. Next, we give the fixed-point theorem for LW-type Lipschitz cyclic mapping as follows.

Theorem 2. *Let (X, r) be a complete b-metric-like space, and (B, S) be a LW-type Lipschitz cyclic mapping in (X, r). Suppose that G_1, G_2 are nonempty closed sets in (X, r) and $G_1 \cap G_2 \neq \varnothing$ if $L \leq t$, $L = \max\{\gamma, \delta\}$, $\gamma \neq \delta$, and $s \in [1, ((L+t)/|\gamma - \delta|))$. Then, there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.*

In fact, based on Definition 12 and Theorem 2, in 2019, S. Weng [20] proposed the concept of an extended mapping for the above cyclic mapping and named it as a general LW-type cyclic mapping.

Definition 13. Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$ and there exists some nonnegative functions β, δ, t such that, for all $x \in G_1$, $y \in G_2$ satisfy the following condition,

$$\begin{aligned} &\beta(r(x, y)) r(x, Bx) + \delta(r(x, y)) r(y, Sy) \\ &\quad + t(r(x, y)) r(Bx, Sy) \leq r(x, y), \end{aligned} \tag{4}$$

where $\beta, \delta\colon [0, +\infty) \longrightarrow [0, +\infty)$, $t\colon [0, +\infty) \longrightarrow [1, +\infty)$, then (B, S) is called as a general LW-type cyclic mapping.

At the same time, a fixed-point theorem for a general LW-type cyclic mapping was obtained as follows.

Theorem 3. *Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a general LW-type cyclic mapping and G_1,G_2 be nonempty closed sets in (X, r) and $G_1 \cap G_2 \neq \varnothing$. Consider there exists λ such that the following conditions are satisfied:*

(1) $\max\{((1 - \beta(z))/(\delta(z) + t(z))), ((1 - \delta(z))/(\beta(z) + t(z))), z \in [0, +\infty)\}) \leq \lambda < 1$

(2) $\lambda s < 1$

Then, there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

In 2020, Weng et al. [21] proposed the concept of a general LW-type Lipschitz cyclic mapping and obtained some fixed-point theorems as follows.

Definition 14. Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$ and there exists some nonnegative functions β, δ, t and nonnegative real number L such that, for all $x \in G_1$, $y \in G_2$ satisfy the following condition,

$$\begin{aligned} &\beta(r(x, y)) r(x, Bx) + \delta(r(x, y)) r(y, Sy) + t(r(x, y)) \\ &\quad r(Bx, Sy) \leq L r(x, y), \end{aligned} \tag{5}$$

where $\beta, \delta\colon [0, +\infty) \longrightarrow [0, +\infty)$, $t\colon [0, +\infty) \longrightarrow [1, +\infty)$, then (B, S) is called as a general LW-type Lipschitz cyclic mapping.

Theorem 4. *Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a general LW-type cyclic mapping and*

G_1, G_2 be nonempty closed sets in (X, r) and $G_1 \cap G_2 \neq \varnothing$. Consider there exists some λ such that the following conditions are satisfied:

(1) $\max\{((L-\beta(z))/(\delta(z)+t(z))), ((L-\delta(z))/(\beta(z)+t(z))), z \in [0,+\infty\}) \leq \lambda < 1$

(2) $\lambda s < 1$

(3) $L \in [1, t(z)]$

Then, there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

In 2021, Weng and Liang [22] proposed the concept of W-type cyclic mapping in b-metric-like spaces and obtained a fixed-point theorem as follows.

Definition 15. Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$ and there exists some nonnegative numbers γ, δ, t, L and $L < \gamma + \delta$ such that, for all $x \in G_1$, $y \in G_2$ satisfy the following condition,

$$r(Bx, Sy) \leq L[r(x, y) - M(x, y)], \tag{6}$$

where $M(x, y) = \{\gamma r(x, Bx), \delta r(y, Sy)\}$, then (B, S) is called a W-type cyclic mapping.

Theorem 5. *Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a W-type cyclic mapping and G_1, G_2 be nonempty closed sets in (X, r) and $G_1 \cap G_2 \neq \varnothing$. If the following conditions are satisfied:*

(1) $\delta + \gamma = 1$, $\delta, \gamma \in (0, 1)$

(2) $sp < 1$, $p = \max\{(L/(1+L\delta)), L\delta, L\gamma, (L/(1+L\gamma))\}$

Then, there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

Our work in b-metric-like space is continuous and is a series of work. It is interesting to generalize the results of cyclic mappings in b-metric-like spaces. Let $L = 1$; it is easy to know that a LW-type Lipschitz cyclic mapping is a LW-type cyclic mapping, a general LW-type Lipschitz cyclic mapping is a general LW-type cyclic mapping, and so on. Recently, we have reviewed the previous research work, and we will propose the concept of φ-type cyclic mapping in this paper and study the existence and uniqueness of its fixed point.

2. Preliminaries

In this part, some necessary definitions that will be used in Section 3 are given.

Definition 16 (see [4]). Let (X, r) be a b-metric-like space, and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{n\longrightarrow\infty} r(x, x_n) = r(x, x)$, and we say that the sequence $\{x_n\}$ is convergent to x and denotes it by $x_n \longrightarrow x$ as $n \longrightarrow \infty$.

Definition 17 (see [4]). Let (X, r) be a b-metric-like space.

(i) A sequence $\{x_n\}$ is called Cauchy if and only if $\lim_{n\longrightarrow\infty} r(x_n, x_m)$ exists and is finite.

(ii) A b-metric-like space (X, r) is said to be complete if and only if every Cauchy sequence $\{x_n\}$ in X converges to $x \in X$ so that

$$\lim_{m,n\longrightarrow\infty} r(x_n, x_m) = r(x, x) = \lim_{n\longrightarrow\infty} r(x_n, x). \tag{7}$$

Definition 18 (see [4]). Let G_1, G_2 be nonempty sets of metric space; if $B(G_1) \subset G_2$ and $S(G_2) \subset G_1$, then the mapping (B, S): $G_1 \times G_2 \longrightarrow G_2 \times G_1$ is called as a pair semicyclic mapping, where B is said to be a lower semicyclic mapping and S is said to be a upper semicyclic mapping. If $B = S$, then B is said to be a cyclic mapping.

Definition 19 (see [12]). If $\prec$ is partially ordered in b-metric-like spaces (X, r), then $(X, r, \prec)$ is a partially ordered b-metric-like space.

3. Main Results

Now, we give the results for φ-type cyclic mapping proposed by us in this section.

Definition 20. Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semicyclic mapping in $G_1 \times G_2$ and there exists some nonnegative real constants β, δ, t, and a nonnegative bounded function $\varphi(z)$ such that, for all $x \in G_1$, $y \in G_2$ satisfy the following condition,

$$\beta r(x, Bx) + \delta r(y, Sy) + t r(Bx, Sy) \leq \varphi(r(x, y)) r(x, y), \tag{8}$$

then (B, S) is called as a φ-type cyclic mapping.

Theorem 6. *Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a φ-type cyclic mapping and G_1, G_2 be nonempty closed sets in (X, r), and $G_1 \cap G_2 \neq \varnothing$, $h = \max\{\gamma, \delta\}$, and $P = \max\{(1/(\gamma+t)), (1/(\delta+t))\}$. Consider the following conditions are satisfied:*

(1) $\varphi(z)$: $[0, \infty) \longrightarrow [h, (1/sP)]$

(2) $hsP < 1$

Then, there exists a $z^* \in G_1 \cap G_2$ such that $Bz^* = z^* = Sz^*$, that is, B and S have the common fixed point.

Proof. The sequence $\{z_n\}$ is defined as follows:

$$\begin{aligned} z_0 \in G_1, z_1 = Bz_0, z_2 = Sz_1, z_3 = Bz_2, z_4 = Sz_3, \ldots, z_{2n+1} \\ = Bz_{2n}, z_{2n+2} = Sz_{2n+1}, \ldots, n \geq 0. \end{aligned} \tag{9}$$

Step 1 : prove that the sequence $\{z_n\}$ is a Cauchy sequence. Because (B, S) is a φ-type cyclic mapping, thus, we have

$$\gamma r(z_0, Bz_0) + \delta r(z_1, Sz_1) + tr(Bz_0, Sz_1) \leq \varphi(r(z_0, z_1))r(z_0, z_1), \tag{10}$$

that is,

$$\gamma r(z_0, z_1) + \delta r(z_1, z_2) + tr(z_1, z_2) \leq \varphi(r(z_0, z_1))r(z_0, z_1). \tag{11}$$

Because of $h = \max\{\gamma, \delta\}, \varphi(x) \geq h$, then we have

$$r(z_1, z_2) \leq \frac{\varphi(r(z_0, z_1)) - \gamma}{\delta + t} r(z_0, z_1) \leq \frac{1}{\delta + t} \varphi(r(z_0, z_1))r(z_0, z_1). \tag{12}$$

Continue with the above steps; then, we obtain

$$\gamma r(z_2, Bz_2) + \delta r(z_1, Sz_1) + tr(Bz_2, Sz_1) \leq \varphi(r(z_2, z_1))r(z_2, z_1), \tag{13}$$

that is,

$$\gamma r(z_2, z_3) + \delta r(z_1, z_2) + tr(z_3, z_2) \leq \varphi(r(z_2, z_1))r(z_2, z_1). \tag{14}$$

Because of $h = \max\{\gamma, \delta\}, \varphi(x) \geq h$ and (12) and (13), then we have

$$r(z_2, z_3) \leq \frac{\varphi(r(z_2, z_1)) - \delta}{\gamma + t} r(z_1, z_2) \leq \frac{1}{\gamma + t} \varphi(r(z_2, z_1)) \frac{1}{\delta + t} \varphi(r(z_0, z_1))r(z_0, z_1). \tag{15}$$

Let $Q = r(z_0, z_1)$; because of $P = \max\{(1/(\gamma + t)), (1/(\delta + t))\}$, from the definition of the φ-type cyclic mapping, it shows that $\varphi(x)$ is bounded, and $\varphi(x) \leq M = (1/sp)$; then, according to conditions (1) and (2), it implies that

$$r(z_2, z_3) \leq PMr(z_1, z_2) \leq P^2 M^2 Q. \tag{16}$$

Continue the above steps; then, we have

$$r(z_n, z_{n+1}) \leq P^n M^n Q. \tag{17}$$

Let $m > n$, $\forall m, n \in N$; this shows

$$\begin{aligned}
r(z_n, z_m) &\leq s(r(z_n, z_{n+1}) + r(z_{z+1}, z_m)) \\
&\leq sr(z_n, z_{n+1}) + s^2 r(z_{n+1}, z_{n+2}) + \cdots + s^{m-n} r(z_{m-1}, z_m) \\
&\leq s(PM)^n r(z_0, z_1) + s^2 (PM)^{n+1} r(z_0, z_1) + \cdots \\
&\quad + s^{m-n} (PM)^{m-1} r(z_0, z_1) \\
&\leq \left[s(PM)^n + s^2 (PM)^{n+1} + \cdots + s^{m-n} (PM)^{m-1}\right] Q \\
&= \frac{1 - (sPM)^{m-n}}{1 - sPM} sP^n Q \\
&\leq \frac{1}{1 - sPM} (PM)^{n-1} Q.
\end{aligned} \tag{18}$$

Since $P = \max\{(1/(\gamma+t)), (1/(\delta+t))\}$, $s \geq 1, M = (1/sP), 0 \leq PM = (1/s) \leq 1$, from (18), and $n \longrightarrow \infty$, we get that

$$\lim_{n\longrightarrow\infty} r(z_m, z_n) = 0. \tag{19}$$

This implies from (19) that the sequence $\{z_n\}$ is a Cauchy sequence.

Step 2 : ecause (X, r) is a complete b-metric-like space, $G_1 \cap G_2 \neq \varnothing$, then there exists a point $z \in X$ such that

$$z_n \longrightarrow z \ (n \longrightarrow \infty). \tag{20}$$

Therefore,

$$\begin{aligned} z_{2n} &\longrightarrow z, \\ z_{2n+1} &\longrightarrow z \ (n \longrightarrow \infty). \end{aligned} \tag{21}$$

Because $\{z_{2n}\} \subset G_1$, $\{z_{2n+1}\} \subset G_2$, and G_1, G_2 is closed, then

$$z \in G_1 \cap G_2. \tag{22}$$

Step 3 : prove that z is a fixed point of the mapping B and S, that is, $Bz = z = Sz$. Because (B, S) is a φ-type cyclic mapping, then we get that

$$\gamma r(z, Bz) + \delta r(z, Sz) + tr(Bz, Sz) \leq \varphi(r(z, z)) r(z, z). \tag{23}$$

Because $r(z, z) = \lim_{n,m\longrightarrow\infty} r(z_n, z_m)$, then, by (23), we have

$$\begin{aligned} r(z, Bz) &= 0, \\ r(z, Sz) &= 0. \end{aligned} \tag{24}$$

It implies that

$$Bz = z = Sz. \tag{25}$$

This completes the proof. □

Remark 1. With regard to the nonnegative bounded function in Definition 20, we give a concrete example here and give a concrete expression of the nonnegative bounded function. The nonnegative bounded function is defined as

$$\varphi(x) = h\frac{x}{1+x} + \left(\frac{1}{sP} - h\right)\frac{1}{1+x}. \tag{26}$$

It is definitely a nonnegative bounded function that conforms to the definition of $\varphi(x)$, that is, $\varphi(x)$: $[0, +\infty) \longrightarrow [h, (1/sP)]$.

Corollary 1. *Suppose that (X, r) is a complete partially ordered b-metric-like space. Let (B, S) be a φ-type cyclic mapping and G_1, G_2 be nonempty closed sets in (X, r), and $G_1 \cap G_2 \neq \varnothing$, $h = \max\{\gamma, \delta\}$, and $P = \max\{(1/(\gamma+t)), (1/(\delta+t))\}$. Consider the following conditions are satisfied:*

(1) $\varphi(x)$: $[0, \infty) \longrightarrow [h, (1/sP)]$

(2) $hsP < 1$

Then, there exists a $z^* \in G_1 \cap G_2$ such that $Bz^* = z^* = Sz^*$, that is, B and S have the common fixed point.

4. Conclusion

B-metric-like space is a kind of generalized metric space. The existence-uniqueness problem of fixed points in generalized metric spaces is an important problem in the field of fixed point theory. In this paper, we study the generalization of LW-type Lipschitz cyclic mapping. By using the definition of φ-type mapping, we establish the existence-uniqueness of its fixed point and a fixed-point existence theorem of φ-type mapping. In a word, the category of fixed point theory is very broad. The development of the theory of fixed points in generalized metric space is very significant. At the same time, it is beneficial to promote the study of stability of stochastic systems in [23–30] by applying our theory.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally.

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