

## *Retraction*

# **Retracted: Application of Algorithms of Constrained Fuzzy Models in Economic Management**

### **Complexity**

Received 23 January 2024; Accepted 23 January 2024; Published 24 January 2024

Copyright © 2024 Complexity. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

### **References**

- [1] L. Meng and D. Zhu, "Application of Algorithms of Constrained Fuzzy Models in Economic Management," *Complexity*, vol. 2021, Article ID 9912534, 12 pages, 2021.

## Research Article

# Application of Algorithms of Constrained Fuzzy Models in Economic Management

Lingyan Meng<sup>1</sup> and Dishi Zhu <sup>2</sup>

<sup>1</sup>Department of Economics and Management, Jining University, Qufu 273115, Shandong, China

<sup>2</sup>School of Economics and Law, University of Science and Technology Liaoning, Anshan 114051, Liaoning, China

Correspondence should be addressed to Dishi Zhu; zhudishi@ustl.edu.cn

Received 16 March 2021; Revised 2 April 2021; Accepted 8 April 2021; Published 16 April 2021

Academic Editor: Zhihan Lv

Copyright © 2021 Lingyan Meng and Dishi Zhu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Stochasticity and ambiguity are two aspects of uncertainty in economic problems. In the case of investments in risky assets, this uncertainty is manifested in the uncertainty of future returns. On the contrary, the complexity of the economic phenomenon itself and the ambiguity inherent in human thinking and judgment are characterized by indistinct boundaries. For the same problem, research from different perspectives can often provide us with more comprehensive and systematic information. Currently, the expected value of return or the variance representing risk is still used as a rational investment criterion for both single-stage portfolios and multistage portfolios. However, in general, the greater the expected return of an investor, the greater the risk he should take. Different investors have different requirements for profitability, but regardless of their expected return, they always hope to find a set of portfolios that maximize the probability of achieving the expected rate of return. In this paper, after analyzing the development of portfolio investment theory research, we take fuzzy information processing as the entry point and systematically discuss the theory and methods of fuzzy modeling of portfolio investment decision-making from the perspective of fuzziness around the portfolio investment decision-making process. The results of the empirical analysis show that the existence of basis constraints affects investors' investment strategies as well as their final returns, but there is a limit to the influence of basis constraints on portfolio performance, and investors can obtain optimal investment returns by selecting a reasonable number of securities to form a portfolio based on the characteristics of different securities.

## 1. Introduction

The financial market is a high-risk and volatile market, which is influenced by many factors such as the market's own laws, policy changes, and national economic regulations and is full of unpredictable factors [1]. How to use portfolio optimization models to obtain high-yield, low-risk investment strategies in the financial market is an important direction of modern financial theory research [2]. The basic idea of the portfolio is to diversify assets to hedge some of the risks and to study how to allocate limited assets to maximize returns and minimize risks in the face of future uncertainty. From the perspective of financial research, the starting and ending point of financial research is financial decision-making and management; then, for investors, the essence of

financial activity is portfolio selection [3]. Therefore, it can be said that portfolio management is the origin and driving force of modern financial theory research [4]. In 1952, researchers used the variance of securities returns to portray risk, considered the correlation between the securities in a portfolio, and put forward the classical portfolio theory [5]. The first application of mathematical and statistical methods to the study of portfolio selection opened the prelude of modern portfolio theory, and since then scholars' research on portfolio optimization models have been developed on the basis of the classical MV model [6].

Although the MV model depends on the strict assumptions' base, the financial market changes frequently, and the difference in investor's risk preferences can affect investment decisions [7]. Investment decisions are often

done in a case by case, in addition to being dependent on average and variance. At this stage, it is not desirable for the investment activity to include a lot of securities assets in the portfolio with the related expenses such as stamp tax and fees [8]. The higher the amount of securities or assets, the higher the associated costs. However, if the number of assets in a portfolio is small, there is no risk hedging across the portfolio, and if one asset is lost, it affects the whole portfolio, so the number of assets in the portfolio is too many or too small [9]. As an important constraint of the portfolio selection model, the base constraint has attracted the attention of scholars, and selecting a reasonable number of securities to build a portfolio can reduce the cost to a certain extent and also reduce the complexity of management [10], as shown in Figure 1. The securities market itself is an extremely complex system, and the returns and risks of securities are often uncertain. Investors need to make investment decisions in an uncertain environment, and uncertainty is what makes investment decisions difficult [11]. Uncertainty of events has two different forms of expression; one is the uncertainty of whether the event occurs, which is commonly referred to as randomness; the other is the uncertainty of the state of the event itself, which is ambiguity [12]. Randomness is only about the amount of information, while vagueness is about the meaning of information. It can be said that vagueness is a kind of uncertainty that is more profound than randomness. In real life, the existence of vagueness is more extensive than that of randomness, especially in the field of subjective perception, and the influence of vagueness on the construction of portfolio selection models is more profound than that of randomness [13]. With the development of the market, the fuzzy uncertainty in the securities market has been gradually paid attention to, and researchers have started to pay attention to the fuzziness in the securities market and use the methodological theory of fuzzy mathematics to study portfolio models, and fuzzy investment decision-making is becoming a frontier direction with extreme research significance [14]. The presence of the base constraint increases the complexity of the portfolio model and affects the shape of the effective frontier, making the effective frontier of the solution discontinuous [15]. In view of this, many foreign experts have studied several methods for solving portfolio models with basis constraints. For the problem of the portfolio model with basis constraints, some scholars use various exact algorithms, such as the Lagrangian decomposition method, branch delimitation method, and branch shearing method [16]. However, when the investment scale is large, the computational efficiency of these exact algorithms will be greatly reduced, while intelligent algorithms have the advantages of high computational efficiency and better accuracy in solving this kind of quadratic programming problems, so many scholars focus on the study of solving this. Therefore, many scholars focus on intelligent algorithms to solve this type of nonlinear programming, such as genetic algorithms and particle swarm optimization algorithms.

With the rapid changes in financial markets, investors often need to adjust their investment strategies according to the changes in the market environment. Dynamic

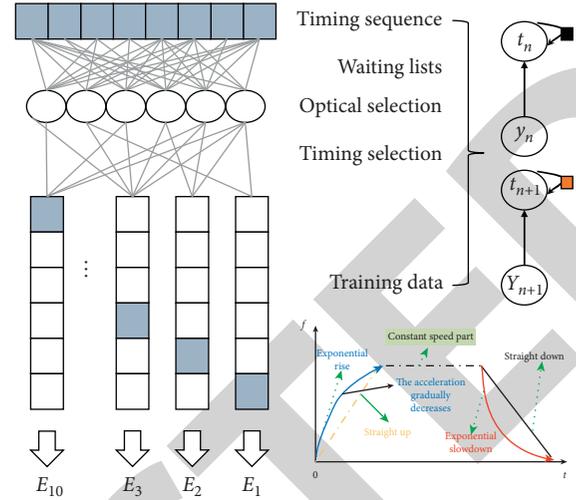


FIGURE 1: Base constraint model.

multistage portfolio models focus on how to dynamically adjust the portfolio to maximize the final wealth or utility. Compared with single-period investment, multiperiod investment is a stochastic nonlinear dynamic planning process, and its optimization solution is much more complex than that of the single-period portfolio. As the investment phase changes and the real investment environment becomes more complex, it becomes more difficult to construct an investment model that meets investors' expectations, and even the model is built without access to the real frontier of the portfolio, and the multistage portfolio model is less efficient. So far, most of the studies on multistage portfolios are based on stronger assumptions, and the multistage investment problem that meets the actual investment characteristics needs urgent progress exploration and research. Therefore, multistage base-constrained portfolio selection models are extremely promising and of research interest. In order to have a deeper understanding of the fuzzy portfolio model with basis constraints and a broader application of the proposed model, this paper uses a combination of theoretical research and empirical analysis. Two different portfolio models are constructed based on Markowitz's portfolio research theory, and a detailed theoretical description of the model construction is given, followed by a study of the model application using a large amount of empirical data.

## 2. Related Work

The securities market is an extremely complex system, full of many uncertainties, and the participants of portfolio construction are investors, so the influence of human factors on investment decisions cannot be ignored, such as the subjective will of investors, the knowledge and experience of experts, and the lack of institutional regulations in the emerging markets have a great impact on the portfolio [17]. In this case, how to integrate the knowledge and experience of experts with the historical data of securities is the problem faced by every investor. With the increasing application of

fuzzy set theory proposed by researchers, people have started to realize that fuzzy set theory can be used to deal with the ambiguity in the securities market. So far, more research studies on single-period portfolio problems in fuzzy environments have been achieved [18]. Foreign scholars started their research on fuzzy portfolio models earlier, and the scope and depth of the research are worthy of our study. Researchers refined and developed the fuzzy decision theory, which provided a solid theoretical foundation for solving this class of fuzzy decision problems [19]. After that, many scholars have conducted a lot of research on fuzzy portfolio problems using fuzzy decision theory as the basic framework. Researchers have used fuzzy decision theory to study the dynamic portfolio problem with one risk-free asset and multiple risky assets and proposed a fuzzy control investor selection model [20].

The researchers studied the portfolio optimization model when the security returns are fuzzy and extended the MV model in a fuzzy environment [21]. The researchers constructed the mean-semivariance fuzzy portfolio model by assuming that the security returns are approximated as LR fuzzy numbers and evaluating expected returns and risks by interval-valued means, while considering the downside risk of the portfolio. The researchers constructed a portfolio optimization (APO) synthesis model by extending the mean-variance optimization (MVO) portfolio model to a mean-semiabsolute deviation (SAD) model and applying multicriteria decision-making (MCDM) through fuzzy mathematical planning. The researchers optimized the two constrained fuzzy AHP methods to obtain the modified constrained fuzzy AHP (RCFAHP) method and used both methods to construct two portfolio models. The researchers proposed a multicriteria plausibility framework model for the investment rebalancing problem, in which return, risk, and liquidity are represented by fuzzy parameters, respectively [22]. The researchers combined fuzzy theory with portfolio matrices, and the proposed model construction method is able to accurately describe investors' subjective preferences [23–26]. Using a single-input type fuzzy inference method, the researchers investigated the robust stochastic fuzzy MV-based portfolio selection problem [27]. The researchers proposed a fuzzy multicriteria model for portfolio selection that includes the classical return-risk biobjective model, additionally allows portfolio construction based on expert advice, and uses fuzzy tools to incorporate the concept of social responsibility into the portfolio selection model.

Researchers have studied the portfolio selection model based on fuzzy double random variables, constructed a portfolio selection model based on fuzzy double random VAR (FBVAR-PSM), and designed a particle swarm optimization algorithm (FBS-PSO) to solve the model to find the approximate optimal solution. There are many MV extension models with constraints, such as base constraints, cost constraints, and transaction cost constraints, which make it difficult to obtain the exact solution of the model by conventional solution methods, so scholars have studied algorithms for solving the model from different perspectives, especially intelligent algorithms. Researchers

studied the subset selection problem in regression analysis and the portfolio selection problem in asset management and proposed a branch-and-bound algorithm to solve portfolio models with basis constraints based on the special structure of the problem. The researchers proposed a new bootstrap variational operator and combined it with a multiobjective evolutionary algorithm to solve the base-constrained portfolio optimization problem. The researchers solved the base-constrained portfolio problem by a relaxation process and obtained better results. The researchers used a neural network algorithm to solve the base-constrained portfolio model and compared the results obtained with the researchers' proposed solution, which showed that the effective frontier obtained using the neural network algorithm is closer to the true frontier of the mean-variance model. The researchers introduced a basis constraint on the classical mean-variance model, established a basis-constrained mean-absolute deviation model, and used DC programming and differentiation algorithms to solve this model problem. The researchers proposed a particle swarm optimization algorithm and applied it to a mixed quadratic programming portfolio optimization problem.

The researchers considered the base constraint and marginal constraint downside risk cases and then constructed an NP-hard mixed-integer quadratic programming model. Penalty functions are used in the convex difference evolution process to improve the algorithm accuracy, and then, the model is solved using the DC algorithm (DCA) to obtain the results. The researchers used the memetic algorithm to solve the Markowitz extended model with constraints such as base constraints, transaction costs, and threshold constraints. The researchers compared five multiobjective evolutionary algorithms in solving the mean-variance model in the presence of nonlinear objective functions and discrete constraints with cardinality constraints. The results show that SPEA2 has a clear advantage and that NSGA-II and SPEA2 can solve large-scale problems with up to 2196 assets. The researchers compare the results of genetic algorithms, forbidden search, and simulated annealing algorithms in finding finite frontiers for base-constrained portfolio problems and analyzing the strengths and weaknesses of each algorithm. The researchers studied the subset selection problem in regression analysis and the portfolio selection problem in asset management and proposed branch-and-bound algorithms to solve portfolio models with basis constraints based on the particular structure of the problem. The researchers proposed a new bootstrap variational operator and combined it with a multiobjective evolutionary algorithm to solve the base-constrained portfolio optimization problem. The researchers solved the base-constrained portfolio problem by a relaxation process and obtained better results. The researchers used a neural network algorithm to solve the base-constrained portfolio model and compared the results obtained with the researchers' proposed solution, which showed that the effective frontier obtained using the neural network algorithm is closer to the true frontier of the mean-variance model.

The researchers introduced a basis constraint on the classical mean-variance model, developed a basis-constrained mean-absolute deviation model, and used DC programming and differentiation algorithms to solve this model problem. The researchers proposed a particle swarm optimization algorithm and applied it to a hybrid quadratic portfolio optimization problem. The researchers considered the base constraint and marginal constraint downside risk cases and then constructed an NP-hard mixed-integer quadratic programming model. Penalty functions are used in the convex difference evolution process to improve the accuracy of the algorithm, and then, the model is solved using the DC algorithm (DCA) to obtain the results. The researchers used the memetic algorithm to solve the Markowitz extended model with constraints such as base constraints, transaction costs, and threshold constraints. The researchers compared five multiobjective evolutionary algorithms in solving the mean-variance model in the presence of nonlinear objective functions and discrete constraints with cardinality constraints. The results show that SPEA2 has a clear advantage and that NSGA-II and SPEA2 can solve large-scale problems with up to 2196 assets. The researchers compare the results of genetic algorithms, forbidden search, and simulated annealing algorithms in finding finite frontiers for base-constrained portfolio problems and analyzing the advantages and disadvantages of each algorithm.

### 3. Fuzzy Portfolio Selection Model with Base Constraints

**3.1. Trapezoidal Fuzzy Numbers and Operations.** The LR-type fuzzy numbers include triangular fuzzy numbers, trapezoidal fuzzy numbers, and various irregular fuzzy numbers. Triangular numbers are easy to calculate, are useful to explain, and are widely used in recent studies. However, trapezoidal fuzzy numbers are superior to other types of fuzzy numbers, especially in terms of conformity and accuracy, based on sufficient hysteresis data in securities. Trapezoidal fuzzy numbers can be more fully accounted for multiple properties of the securities return, so the trapezoidal fuzzy numbers become larger. The utility of trapezoidal fuzzy numbers in the field of financial research is huge. This section describes the securities data with trapezoidal fuzzy numbers. Before introducing trapezoidal fuzzy numbers, we briefly understand the definition and manipulation of the LR-type fuzzy numbers.

Assume that a fuzzy number  $A$  is a regular fuzzy set with fuzzy convexity and continuous affiliation function in the set of real numbers  $R$ , where the family of fuzzy numbers is denoted as  $F$ . Assume that  $A$  is a fuzzy set with  $\gamma$ -level intercept:

$$[A]^\alpha = [x(\beta), y(\beta)], (\beta \in [0, 1]). \quad (1)$$

Then, the exact form of the mean value of the probability of  $A$  is

$$Q(A) = \int_{i=0}^n (x(\beta) + y(\beta))dx. \quad (2)$$

Assume that the platform parameters and personalization parameters of the product family are  $x_1$  and  $x_2$  and the mapping of platform and personalization functions are  $1/2f$  and  $f$ , respectively, and the expressions of the mapping are linear and contain fuzzy coefficients; the optimization directions of platform and personalization functions are minimal and maximal, respectively, and the constraints are also linear and contain fuzzy coefficients; the optimization mechanism is based on the platform functions, and the optimization is carried out collaboratively. The optimization mechanism is based on the platform function and is performed collaboratively, i.e., firstly, the platform parameter is guaranteed to be taken within its feasible range, then the personalization function is optimized under the given platform parameter value, and finally the platform function is optimized; then, a fuzzy two-layer linear programming model for the product family parameter design can be established, as shown in the following equation:

$$\begin{cases} \min f_i(x_1, x_2) = C_{ij}x_1 + C_{ij}x_2 \\ Bx_1 \leq d \\ x \neq 2 \\ \text{Poss}(f_i(x, y)) \leq \beta_i \\ x_2 \leq 1 \end{cases}, \quad (3)$$

where  $C_{ij} = (1, 2)$  denotes the fuzzy coefficients in the platform and personalization function mapping, respectively,  $B$  and  $d$  denote the fuzzy coefficients in the platform function optimization constraint, and  $A_j$  ( $i = 1, 2$ ) and  $b$  denote the fuzzy coefficients in the personalization function optimization constraint, both of which are fuzzy numbers obeying a certain likelihood distribution. Although the model considers only the simplest case with one platform parameter and one personalization parameter, it can be easily extended to more general cases with multiple parameters. Several papers have studied this model and proposed a method to transform it into an ordinary two-level linear programming solution. In this paper, we will propose a method to transform it into an ordinary (single-level) linear programming solution from another perspective. The fuzzy coefficients are transformed into interval numbers by the threshold (level) of the possibility distribution, and the interval constraints are transformed into deterministic linear constraints by using the interval constraint satisfaction degree, while the lower bound of the lower-level objective and its satisfaction  $u$ , which forms the objective interval as the upper-level optimization constraints, are transformed into deterministic single-level planning problems by introducing the reconciliation coefficient  $\lambda$ , and finally, the satisfactory solutions of the product family codesign are obtained.

**3.2. Methods of Acquiring Customer Requirements.** In today's highly developed science and technology, especially information technology, the business environment of enterprises has undergone fundamental changes. Customer satisfaction with enterprise products and services will be the decisive factor for enterprise survival and development, and customer satisfaction will be the source of enterprise benefits. Systematic and comprehensive understanding of customer needs is a necessary condition to meet customer needs. It is also the starting point of mass customization and the problem that needs to be considered from the beginning to the end. In this case, enterprises can no longer simply seek to maximize profits; to achieve sustainable development, we must establish and maintain a good relationship with customers in the long term and truly grasp the needs of customers.

Current customer needs will change over time, and customer needs are expanding from the core layer to the form and extension layers, mainly in the following aspects, as shown in Figure 2.

- (1) Consumer demand extends from the use value of products to product-related services. In the modern society, product services are occupying an increasingly important position, and manufacturing companies are producing tangible products and providing more and more services for customers around tangible products.
- (2) From the economy of goods to the economy of feelings, customers pay more and more attention to spiritual enrichment and satisfaction, and their demand for goods has gone beyond the level of price and quality and beyond the limits of image and brand, and they are very interested in whether the goods have the charm of activating the soul and pay more attention to the pursuit of spiritual satisfaction in the process of purchasing and consuming goods.
- (3) Expansion from functional and utility needs to emotional needs: previously, it was pointed out that consumers first look at goods with their eyes before they deepen their impressions in their minds. Modern consumers do not buy a product just for its physical function and utility, nor for the purpose of acquiring ownership of the product, but rather for a series of psychological satisfaction and pleasure through the purchase of the product, and the outstanding personality of the product can satisfy this desire. Goods and services that reflect the concept of self and satisfy the feeling of "self" are very popular among customers.
- (4) Expansion from basic and expectation demand to excitement demand: basic demand is the demand for products that are essential to customers, expectation demand is the demand for product quality, which customers generally consider more, and excitement demand refers to the product features that customers do not expect. When the desired demand is popularized, it may become the basic demand. When the excitement-type demand appears, it may be

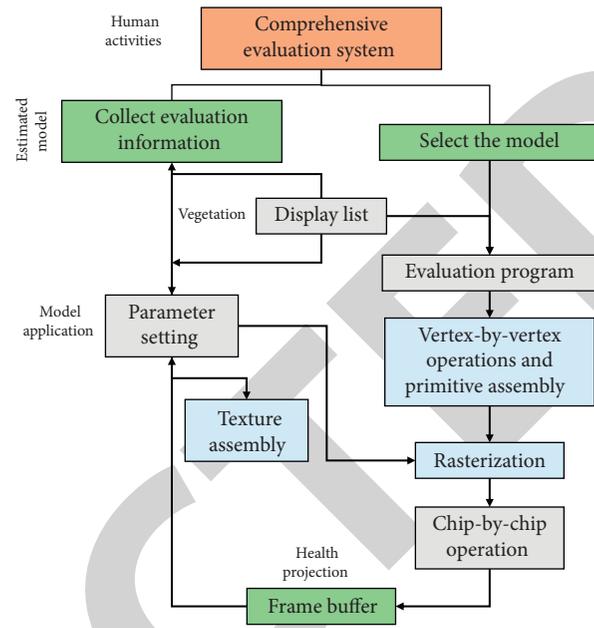


FIGURE 2: Changes in customer needs.

transformed into the expectation-type demand. The key to mass customization is to satisfy the excitement of the customer as much as possible based on the basic and desired needs.

Understanding the reasons for changes in customer needs helps to correctly grasp customer needs. The main reasons for the change of customer needs are the change of customer values and the change of customer income.

- (1) Changes in customer values: with the development of society and the progress of human beings, customer values have also changed accordingly. In industrially developed countries, the change in customer values is mainly due to the rise of the New Humanity. They are well-educated, exposed to a lot of external information, good at independent thinking, and like to be innovative and independent.
- (2) Changes in the market: with the development of the market into the stage of oversupply, that is, from the seller's market to the buyer's market, the customer is in the dominant position in the market, and the consumer demand then expands beyond the use value of the product. What customers care about is not only the use value of the product. It also includes a variety of services related to the product, such as respect in the purchase process, guidance in the consumption process, timely maintenance after the failure of the product, and product recycling and replacement services.
- (3) Increase in customer income: customer needs are increasingly expanding from the traditional practical scope to spiritual enjoyment and so on. Increasing incomes have led them to consider more about so-called quality, which increasingly refers to the atmosphere and the association of the environment-in-

fact, the psychological meaning of the product. Increasing incomes have made price and quality less of a priority for customers when buying products, and design, branding, and performance are more important, and products are judged by their “likes and dislikes.” Customers tend to focus on whether the product can bring vitality, comfort, and beauty to their lives, and they demand not only the function and brand of the product but also a certain charm of the product.

In response to changes in customer demand, companies have taken various countermeasures, such as expanding their business scope to service industries, giving due consideration to customers’ feelings when using products, increasing the human touch and personalization of products, and adopting countermeasures for the buyer’s market, as shown in Figure 3.

- (1) Expansion of manufacturing companies into service industries: expanding the scope of business into service industries helps create business differences from competitors. This is because enterprises are in direct contact with customers, providing them with personalized products and services, and adapting to changes in the market with changes in technology, products, and services. Enterprises should consciously shift the core of competition from production or sales to service quality, and the increase of enterprise profits depends first of all on the improvement of service quality. The basic criterion of business operation is to make customers satisfied. Relying on the advantages of service to win customers has become the consensus of more and more excellent enterprises. In a sense, the enterprise that makes customers feel satisfied is invincible, and such an enterprise can even have customers who are loyal to its brand products at a higher price than its competitors, and such an enterprise can often have more profit margin.
- (2) Fully consider the feelings of customers when using the product: when designing and manufacturing products, enterprises must fully consider the feelings of customers when using the product.
- (3) Increase the personalization of products and human touch: products with distinctive personality characteristics are conducive to winning market opportunities, and presenting emotional needs, showing self-assertion, and branding self-existence of products will be increasingly favored by the majority of consumers.
- (4) Adopting countermeasures for the buyer’s market: companies need to adopt countermeasures for customer demand in the buyer’s market in order to achieve higher profits.

Current product family design research focuses on improving product deformability and rapid product configuration through modularization. Design methods guided

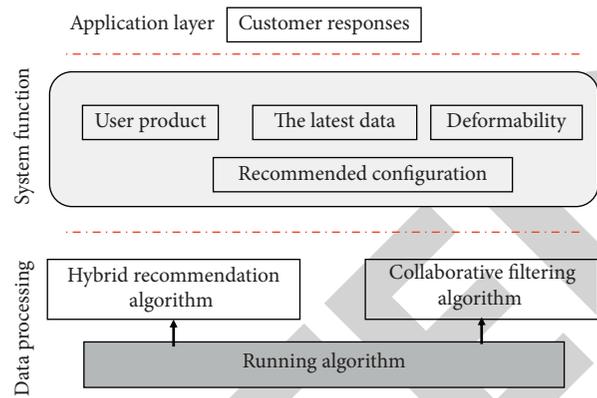


FIGURE 3: Responses to customer needs.

by this idea include, for example, core platform-based design methods and robust design methods. The core of mass customization is rapidity and customization, and the product family design mainly caters to the rapidity. Within a particular time frame within a product plan, it is possible to adapt to the user needs changing over time to achieve a deeper serial product family development, thereby improving the adaptability of the company’s products. Product development usually passes the process of mapping user requirements, engineering metrics, and product families. There are currently widely used ways to link user requirements to product development. It is important to consider similarities and differences in the design of product families. Similarity refers to the similarity of the members constituting the product family from different sides of the structure, function, etc., or combined, and its main means is modularity and core platform. The current product family design mainly focuses on the consideration of similarity and less on the study of difference, which is an important characteristic for product families to distinguish different family members and respond to different user needs. An important idea of the mass customization design is to achieve a shift from reactive to predetermined design thinking and therefore must require the design of a family of products by planning to meet this requirement. A family of products includes a family of products over time and a family of products at a point in time.

Therefore, the product family design must be carried out in response to dynamic changes in user requirements so that the designed product family may be highly adaptable to changing needs. At the same time, the product family design is a design that is under numerous target constraints. Dynamic changes in user requirements include mainly changes over time and changes at a point in time that will be transmitted downstream and affect the composition and structure of the product. The design of product families to realize user requirements and the mapping between user requirements, product parameters, and product functions need to be carried out in two steps; the first step is the mapping between user requirements and product parameters, and the second step is the mapping between product parameters and product functions. The relationship between user requirements and product parameters is generally

ambiguous at the beginning of the design, and the mapping between the two is the core and foundation of mass customization. The final planned product family is planned and designed based on a comprehensive consideration of all the above changes and the product family as a whole, so the result of the mapping of user requirements and product parameters is a collection of product parameters for the whole product family rather than for a single product. This collection is rationally analyzed, and the changes of each product member in different periods as well as within the same period are divided by index allocation, thus realizing the planning of the whole product family. Changes in demand cause changes in the values of the corresponding product parameters, and the degree of such changes is different for different user needs. Changes in product parameters bring about changes in the selection of specific component parts. The two levels of correlation mapping realize the impact of changing user requirements on the final parameters of a functional model of a product family and finally obtain the overall requirements of the product family building blocks with respect to the series of products (at different points in time and over time), on the basis of which the design of the product family is carried out.

### 3.3. Hierarchical Optimization Model with Fuzzy Coefficients.

In this paper, we study the algorithm of lower-level associative two-level multiobjective planning with fuzzy coefficients; the object of the study is an upper-level decision maker and its  $m$  subordinates;  $x$  and  $y_i$  are the decision variables of the upper-level decision maker and the  $i$ th subordinate, respectively;  $g$  is the fuzzy coefficient, and  $F(x, y_1, \dots, y_m, g)$  and  $f_i(x, y_1, \dots)$  are the objective functions of the high-level decision-maker and the  $i$ -th subordinate, respectively.  $y_m$  and  $y_i$  are the objective functions of the high-level decision-maker and the  $i$ -th subordinate, respectively. The two-level multiobjective programming problem model formed is as follows:

$$\begin{cases} \max_x F(x, y_i, g) \neq 0 \\ s, t, G(x) \leq 0 \\ x \leq 1, y_i \neq 1, i = 1, 2, \dots, q \end{cases}, \quad (4)$$

where the decision variable  $x$  is an  $n$ -dimensional row vector and  $y_i$  is a  $N$ -dimensional row vector. For the fuzzy two-layer multiobjective decision problem, the most important problem is the conceptual problem of the solution and how to define the solution to be meaningful and to be applied to the actual decision environment, followed by the problem of how to solve it. This section focuses on the problem of solving the fuzzy two-layer multiobjective problem. The problem of how to solve will be discussed based on the concept of the solution.

Consider the following mathematical planning problem:

$$\begin{cases} \text{Max}[F(x, y_i, g)] \neq 0 \\ s, t, G(x) \leq \frac{b}{q} \\ x \leq q \end{cases}. \quad (5)$$

Based on the above definition, the concept of satisfactory solution for (a, b) is extended to two-level multiobjective planning to form a satisfactory solution for the two-level multiobjective planning problem with fuzzy coefficients, where  $\omega$  denotes the likelihood of the upper-level decision maker ( $q, m$ ) and  $\delta$  denotes the likelihood of the lower-level decision maker:

$$\begin{cases} \text{Max} F \neq 0 \\ s, t, (x, y) \in S^\alpha = \{(x, y) \in S\}. \\ \text{Poss}(f_i(x, y)) \leq \beta_i \end{cases} \quad (6)$$

In this section, the concept of satisfaction solution for fuzzy two-layer multiobjective programming is proposed by extending the concept of satisfaction solution from general fuzzy multiobjective programming. The proposed satisfaction solution  $(x, y)^q$  has a clear meaning, and the satisfaction contains two parts, fuzzy constraint satisfaction and fuzzy objective satisfaction, which makes the decision maker's decision preferences for both parts well reflected. At the same time, the decision maker's preference is divided into feasibility under the possibility level of constraint  $\alpha$  and validity under the possibility level of objective function  $\beta$ . The feasibility of the upper-level constraint and the validity of the objective function are combined to form  $\omega = (\beta, \alpha)$ , and the feasibility of the lower-level constraint and the validity of the objective function are combined to form  $\delta = (\beta, \alpha)$ , which reflects the decision maker's preference well, which is consistent with the actual environment of decision-making. This section discusses the problem of two-layer planning with fuzzy coefficients.

The basic treatment is to take the  $\alpha$ -intercept set of the affiliation degree function of fuzzy coefficients, convert the fuzzy coefficients into an interval number, and then process the interval number to form a parametric  $\alpha$ -planning model, as shown in Figure 4. The form  $\gamma$  of the satisfaction solution proposed in the previous paper is also used for the fuzzy two-layer planning model with fuzzy coefficients, where  $\alpha$  denotes the possibility, which is expressed as the  $\alpha$  intercept set, and  $\gamma$  denotes the satisfaction, which includes two aspects: the satisfaction of the upper and lower levels of constraints and the satisfaction of the upper and lower levels of objectives. The satisfaction for the two-layer planning solution with fuzzy coefficients for constraints has only the constraint satisfaction of the upper and lower layers, and the satisfaction for the two-layer planning solution with fuzzy coefficients for goals has only the goal satisfaction of the upper and lower layers, while the satisfaction for the two-layer planning solution with fuzzy coefficients for both constraints and goals contains both the constraint satisfaction of the upper and lower layers and the goal satisfaction of the upper and lower layers, but the constraint satisfaction is fused in the respective. The constraint satisfaction is integrated in the respective goal satisfaction.

The expectation value operator is still handled by fuzzy simulation in the calculation process. For general fuzzy two-

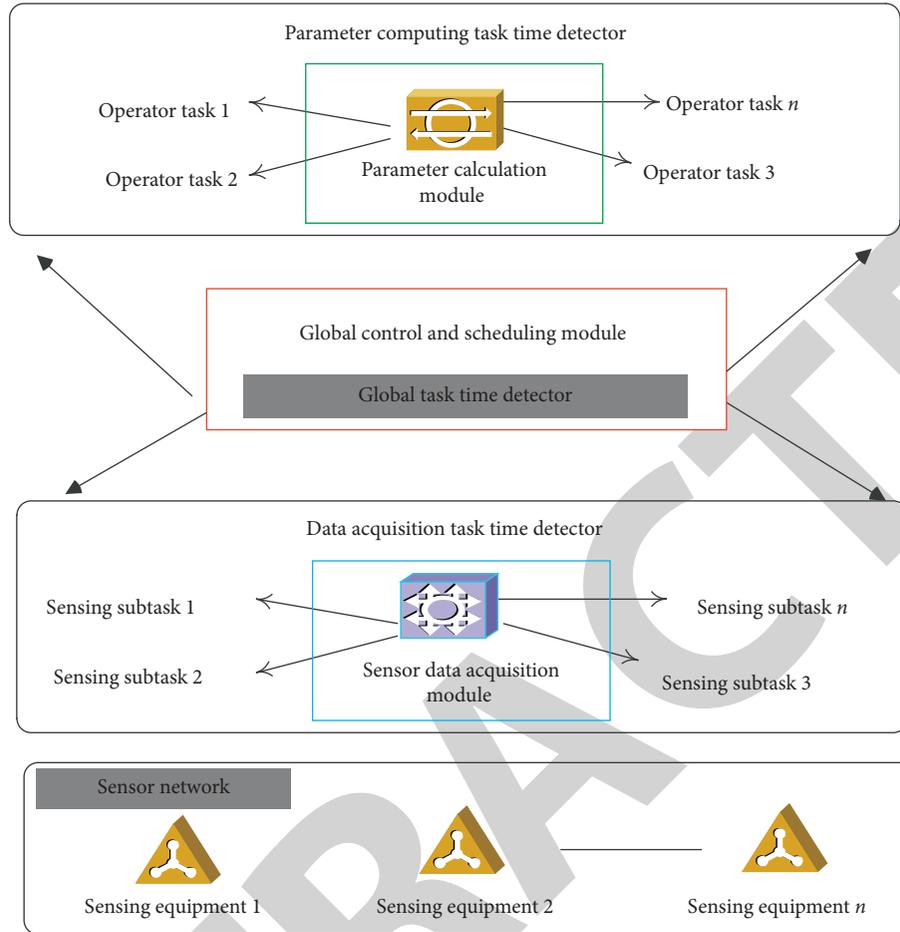


FIGURE 4: Planning model processing flow.

level planning, the particle swarm algorithm in this section can also be used to solve the fuzzy planning as long as it can be transformed into deterministic planning, as shown in Figure 5.

Let the population size be  $m$ , and  $fitness[i]$  is the fitness of the  $i$ <sup>th</sup> particle.

Basic steps of particle swarm.

Step 1: initialize the particle swarm, including the population size, the position, and velocity of each particle

Step 2: calculate the fitness  $fitness[i]$  for each particle

Step 3: for each particle  $i$ , compare its fitness with the individual extremum  $ip$ -best, and replace  $ip$ -best if it is better

Step 4: for each particle  $i$ , compare its fitness with the global extremum  $g$ -best, and replace  $ig$ -best if it is better

Step 5: updating the velocity and position of the particle according to it

Step 6: if the end condition is met (error is good enough or the maximum number of cycles is reached), exit; otherwise, go back to Step 2

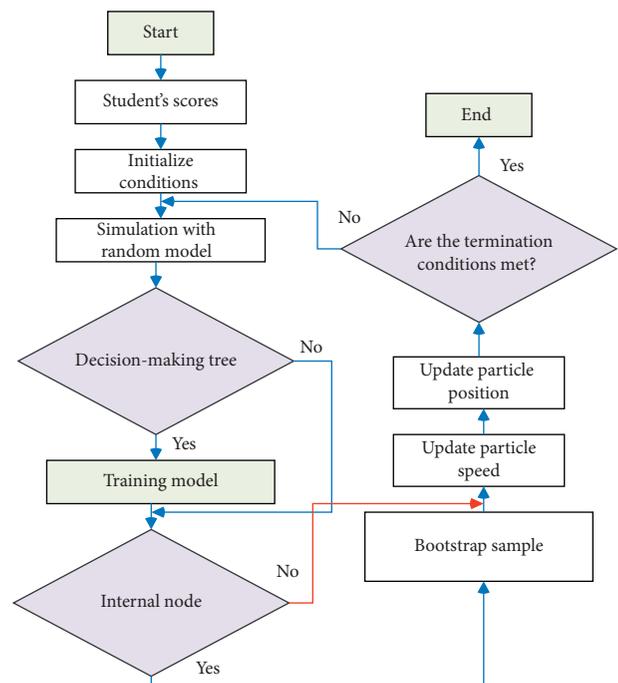


FIGURE 5: Flow chart of the particle swarm algorithm.

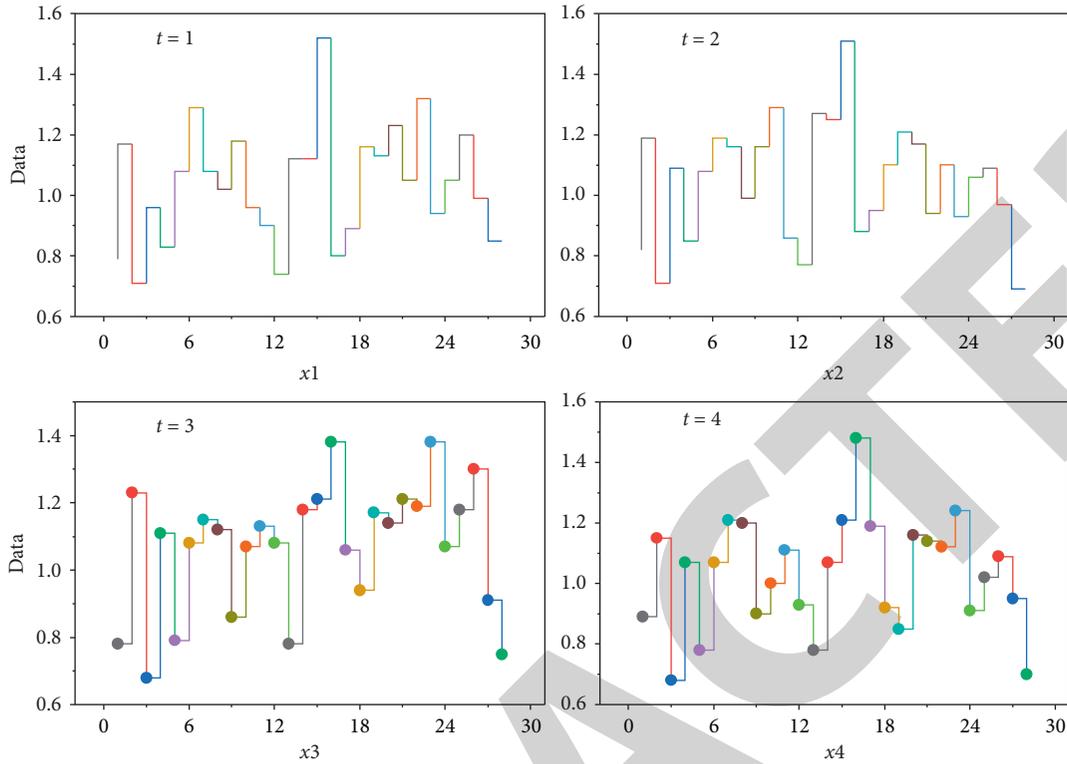


FIGURE 6: Optimal investment strategies and returns for different degrees of likelihood.

#### 4. System Optimization Test

With the base constraint  $Y=6$ , the optimal investment strategy is obtained using a genetic algorithm to solve the model, and the results are shown in Figure 6. If  $Y=6$ , the best investment strategy for the first period is  $X = 0.200, x_{61} = 0.200, j_{c71} = 0.200; Q_{41} = 0.200, x_{181} = 0.200$ . This means that investors need to allocate initial asset capital to securities 1, 6, 7, 11, and 14 in proportions of 20%, 20%, 20%, 20%, and 20%. At this point, the total of the investment to maximize the investment revenue is greater than one and borrowing funds at the beginning of the period. In the second stage, the type of securities changes, but the investment ratio remains the same, and securities 7 and 14 come out of the portfolio, and securities 16 and 17 enter the portfolio. This shows that securities 16 and 17 are superior to securities 7 and 14 at this stage. From Figure 3, it is also possible to analyze the composition of the portfolio in the remaining three periods. For the fourth period of the portfolio, the wealth at the end of the third period is allocated to the six securities 1, 6, 11, 14, 16, and 18 with 19.3%, 16.8%, 17.8%, 19.7%, 20%, and 20% of the wealth at the end of the third period. With a security base constraint of 6, the genetic algorithm finally obtains the maximum wealth value of 1.9880 at the end of the fifth period after 2000 iterations.

From Figure 7, we can see that, in the model with the base constraint and threshold constraint, the final return is 11927.56, which is greater than the final return of the model. In terms of return alone, the presence of the base constraint and threshold constraint obviously affects the final return. If

there are basic constraints and threshold constraints, initial funds will be invested in securities 1, 4, 5, 6, and 7 at 29.5%, 1.5%, 29.9%, 29.8%, and 13.3%, respectively. In the second term, the first term funds are invested in 30%, 0.5%, 29.2%, 9.8%, 0.9%, and 29.7%, respectively. There is no change to the type of securities. In the third period, securities 2 enter the portfolio, the securities 4 comes out of the portfolio, and the securities 2 has some effect on the securities 4. As a result of the model, there are no two constraints of basic constraints and threshold constraints, so the funds are assigned to seven securities in the first period, and the investment ratios of securities 5 and 6 both exceed 0.3 in the first period. The period and the investment ratio of securities 1 and 5 both reach about 0.4 in the second and third periods. This means that securities 1, 5, and 6 are functioning very well at these three stages, but due to the model's threshold constraints, the rate of investment cannot exceed 0.3. The proportion of investment in securities 1 and 3 has been approaching 0.3 in the model; when in the model, there is no threshold constraint, the proportion of purchase of security 1 increases significantly, and some of the proportion of investment exceeds. The greater the return of the securities, the greater the risk, and without the base and threshold constraints, the risk of the entire portfolio increases when investing in several high-yielding securities, and in order to balance the return and risk, the return of the entire portfolio decreases, so the final return of the model is lower than the model, which also shows that the introduction of the base and threshold constraints can optimize the portfolio strategy with comprehensive consideration of risk.

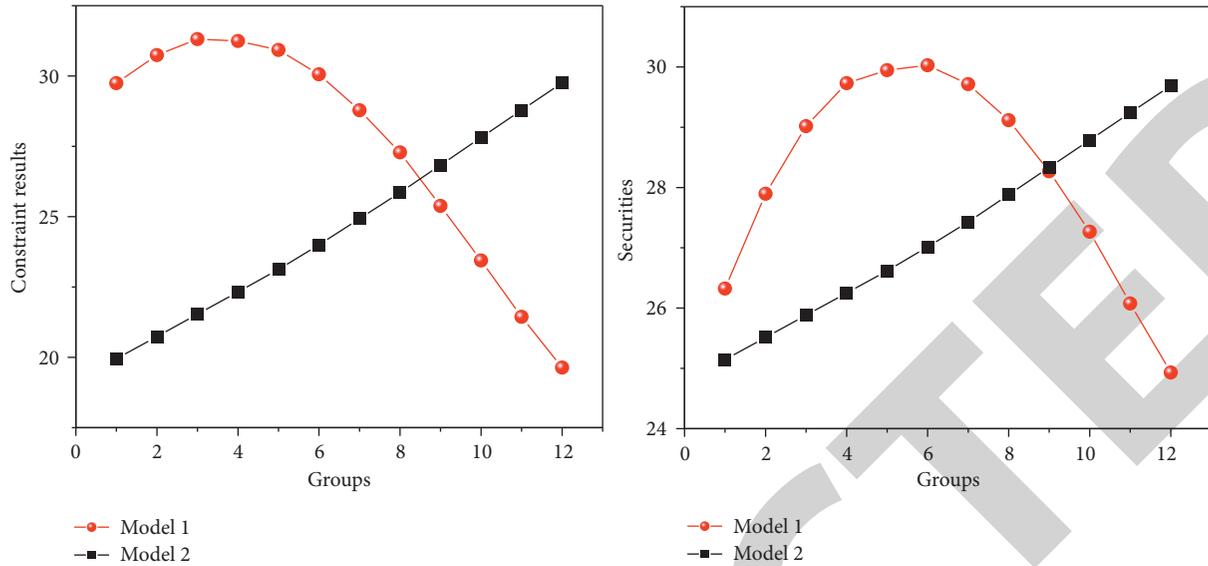


FIGURE 7: Comparison of the final results obtained by the two models.

From the analysis in Figure 8, it is clear that most of the optimal investment securities are the same for  $K=6$  and  $K=9$ . There are four securities identical in the first period investment process, securities 1, 6, 11, and 18, which all appear in the first period investment strategy in the condition of  $K=6$  and  $K=9$ ; five securities are identical in the second period, securities 1, 6, 11, 17, and 18; five securities also appear simultaneously in the third period portfolio, securities 1, 6, 9, 17, and 18. In the fourth investment period, four securities are the same, securities 1, 6, 11, and 18; in the fifth investment period, six securities, securities 1, 6, 9, 11, 17, and 18, appear in the portfolio at the same time when  $K=6$  and  $K=9$ , and it is also these 6 securities that constitute the portfolio in the case of  $K=6$ . The results show that the investment strategy in the multiperiod portfolio has the same characteristics under different base constraints, which indicates that the performance of these securities appearing at the same time in this test phase is good and has a certain return potential to guide investors to purchase. Since the number of investments is limited during the investment process and the new securities affect the covariance matrix when calculating the risk, the risk changes significantly, which leads to a different degree of investment bias in the different investment phases. In addition, the purpose of the study is to find the optimal investment strategy and obtain the optimal basis constraints; therefore, the basis constraints are set to  $K=1$  and 2, respectively, using the above data, and the final wealth values were calculated based on the model, and the final wealth values of the multiperiod portfolios are shown in the figure. It can be seen from the figure that the corresponding  $K > 10$  has the same solution as when  $K=10$ , that is, the investment strategy obtained at  $K=10$  is optimal, and when  $K=0$ , the portfolio is allowed to contain more securities, but based on the optimality of the objective and the iteration of the algorithm, some of the securities are invested. As it can be seen from the figure that the final wealth obtained increases as the value of  $K$  increases, but

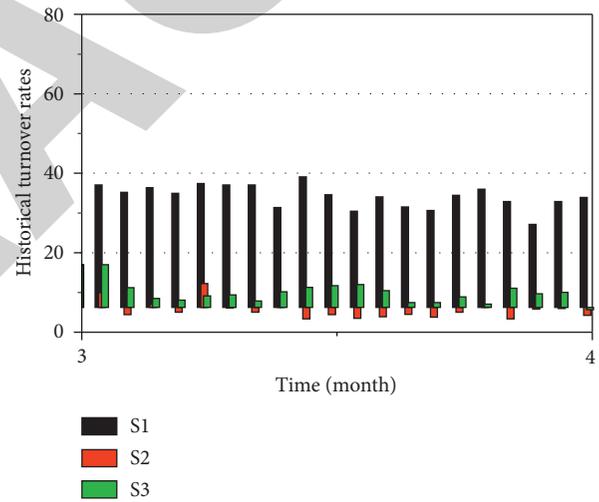


FIGURE 8: Historical turnover rates for economic management.

when  $K$  increases to a certain value, the final wealth value will not change again, which shows that the base constraint  $K$  has an impact in the multiperiod portfolio. Within a certain range, as the basic constraints increase, the return on the investment portfolio also increases, but after reaching a certain level, the base constraint will not have an impact on the optimal investment strategy.

### 5. Conclusion

In order to more accurately describe the return and risk of a portfolio and other factors, this paper uses fuzzy theory to study the construction of a portfolio model. In actual investment, the investment environment is often not ideal and there are various practical constraints. Too many securities lead to large transaction costs and affect returns, and a small number leads to a larger risk for the whole portfolio. The

basis constraint serves as an important constraint to limit the number of securities in a portfolio, reduce unnecessary transaction costs, and, at the same time, reduce investment risk. This paper focuses on multistage portfolio selection in a fuzzy investment environment with uncertain returns and risks of securities. There are many uncertainties in the financial market; we describe the return and risk of securities by trapezoidal fuzzy numbers and use the likelihood mean and likelihood semivariance to represent the return and wind face of the portfolio, while considering the practical application of the portfolio model; we introduce the base constraint as a constraint to limit the number of securities in the portfolio and focus on the impact of the base constraint on the investment strategy. The fuzzy analysis method is used to deal with fuzzy data in the financial market, and a fuzzy multiperiod portfolio selection model is constructed. Then, the fuzzy decision technique is used to transform the fuzzy optimization problem into an exact problem solution, and the multistage problem is decomposed to simplify the computational difficulty. The empirical results of the data show that the presence of basis constraints affects the portfolio returns and that different investment strategies can be obtained by adjusting the quantitative limits of the assets in the portfolio.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References

- [1] C. Zhang, B. A. Engel, and P. Guo, "An interval-based fuzzy chance-constrained irrigation water allocation model with double-sided fuzziness," *Agricultural Water Management*, vol. 210, pp. 22–31, 2018.
- [2] S. Zandkarimkhani, H. Mina, M. Biuki, and K. Govindan, "A chance constrained fuzzy goal programming approach for perishable pharmaceutical supply chain network design," *Annals of Operations Research*, vol. 295, no. 1, pp. 425–452, 2020.
- [3] F. Chen, G. H. Huang, Y. R. Fan, and J. P. Chen, "A copula-based fuzzy chance-constrained programming model and its application to electric power generation systems planning," *Applied Energy*, vol. 187, pp. 291–309, 2017.
- [4] J. Xu, G. Huang, Z. Li, and J. Chen, "A two-stage fuzzy chance-constrained water management model," *Environmental Science and Pollution Research*, vol. 24, no. 13, pp. 12437–12454, 2017.
- [5] J. Krejčí, O. Pavlaáčka, and J. Talašová, "A fuzzy extension of analytic hierarchy process based on the constrained fuzzy arithmetic," *Fuzzy Optimization and Decision Making*, vol. 16, no. 1, pp. 89–110, 2017.
- [6] Y. Gao, S. Tong, and Y. Li, "Observer-based adaptive fuzzy output constrained control for MIMO nonlinear systems with unknown control directions," *Fuzzy Sets and Systems*, vol. 290, pp. 79–99, 2016.
- [7] B. Xu, X. Yin, X. Yin, Y. Wang, and S. Pang, "Fault diagnosis of power systems based on temporal constrained fuzzy petri nets," *IEEE Access*, vol. 7, pp. 101895–101904, 2019.
- [8] Y. Li, Z. Ma, and S. Tong, "Adaptive fuzzy output-constrained fault-tolerant control of nonlinear stochastic large-scale systems with actuator faults," *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2362–2376, 2017.
- [9] Y. Li and S. Tong, "Adaptive fuzzy output constrained control design for multi-input multioutput stochastic nonstrict-feedback nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 12, pp. 4086–4095, 2017.
- [10] S. M. Mohseni-Bonab and A. Rabiee, "Optimal reactive power dispatch: a review, and A new stochastic voltage stability constrained multi-objective model at the presence of uncertain wind power generation," *IET Generation Transmission & Distribution*, vol. 11, no. 4, pp. 815–829, 2017.
- [11] J. Wu, B. Su, J. Li, X. Zhang, X. Li, and W. Chen, "Adaptive fuzzy control for full states constrained systems with non-strict-feedback form and unknown nonlinear dead zone," *Information Sciences*, vol. 376, pp. 233–247, 2017.
- [12] Y. Cai, X. Lin, W. Yue, and P. Zhang, "Inexact fuzzy chance-constrained programming for community-scale urban stormwater management," *Journal of Cleaner Production*, vol. 182, pp. 937–945, 2018.
- [13] C. Zhang and P. Guo, "A generalized fuzzy credibility-constrained linear fractional programming approach for optimal irrigation water allocation under uncertainty," *Journal of Hydrology*, vol. 553, pp. 735–749, 2017.
- [14] C. Zhang, F. Zhang, S. Guo, X. Liu, and P. Guo, "Inexact nonlinear improved fuzzy chance-constrained programming model for irrigation water management under uncertainty," *Journal of Hydrology*, vol. 556, pp. 397–408, 2018.
- [15] W.-J. Chang, C.-C. Ku, and B.-J. Huang, "Multi-constrained fuzzy intelligent control for uncertain discrete systems with complex noises: an application to ship steering systems," *Journal of Marine Engineering & Technology*, vol. 16, no. 1, pp. 11–21, 2017.
- [16] A.-T. Nguyen, K. Tanaka, A. Dequidt, and M. Dambrine, "Static output feedback design for a class of constrained Takagi-Sugeno fuzzy systems," *Journal of the Franklin Institute*, vol. 354, no. 7, pp. 2856–2870, 2017.
- [17] X. Li, Y. Liu, Y. Wang, and Z. Gao, "Evaluating transit operator efficiency: an enhanced DEA model with constrained fuzzy-AHP cones," *Journal of Traffic and Transportation Engineering (English Edition)*, vol. 3, no. 3, pp. 215–225, 2016.
- [18] A.-T. Nguyen, T. Laurain, R. Palhares, J. Lauber, C. Sentouh, and J.-C. Popieul, "LMI-based control synthesis of constrained Takagi-Sugeno fuzzy systems subject to L<sub>2</sub>/L<sub>∞</sub> disturbances," *Neurocomputing*, vol. 207, pp. 793–804, 2016.
- [19] L. Zhang and G.-H. Yang, "Adaptive fuzzy output constrained decentralized control for switched nonlinear large-scale systems with unknown dead zones," *Nonlinear Analysis: Hybrid Systems*, vol. 23, pp. 61–75, 2017.
- [20] X. Zheng, X. Wang, Y. Yin, and L. Hu, "Stability analysis and constrained fuzzy tracking control of positive nonlinear systems," *Nonlinear Dynamics*, vol. 83, no. 4, pp. 2509–2522, 2016.
- [21] S. M. Mohseni-Bonab, A. Rabiee, and B. Mohammadi-Ivatloo, "Voltage stability constrained multi-objective optimal reactive power dispatch under load and wind power uncertainties: a stochastic approach," *Renewable Energy*, vol. 85, pp. 598–609, 2016.
- [22] J. Wen, J. Yang, B. Jiang, H. Song, and H. Wang, "Big data driven marine environment information forecasting: a time

- series prediction network,” *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 1, pp. 4–18, 2021.
- [23] Y. Ji, G. Huang, W. Sun, and Y. Li, “Water quality management in a wetland system using an inexact left-hand-side chance-constrained fuzzy multi-objective approach,” *Stochastic Environmental Research and Risk Assessment*, vol. 30, no. 2, pp. 621–633, 2016.
- [24] Q. Jiang, F. Shao, W. Gao et al., “Unified no-reference quality assessment of singly and multiply distorted stereoscopic images,” *IEEE Transactions on Image Processing*, vol. 28, no. 4, pp. 1866–1881, 2018.
- [25] B. Zhu, S. Ma, R. Xie, J. Chevallier, and Y.-M. Wei, “Hilbert spectra and empirical mode decomposition: a multiscale event analysis method to detect the impact of economic crises on the European carbon market,” *Computational Economics*, vol. 52, no. 1, pp. 105–121, 2018.
- [26] J. Zhu, Q. Shi, P. Wu et al., “Complexity analysis of pre-fabrication contractors’ dynamic price competition in mega projects with different competition strategies,” *Complexity*, vol. 2018, Article ID 5928235, , 2018.
- [27] J. Yang, J. Wen, Y. Wang et al., “Fog-based marine environmental information monitoring toward ocean of things,” *IEEE Internet of Things Journal*, vol. 7, no. 5, pp. 4238–4247, 2019.