Research Article

Adaptive Fuzzy Consensus Tracking Control for Nonlinear Multiagent Systems with Time-Varying Delays and Constraints

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This paper proposes an adaptive fuzzy distributed consensus tracking control scheme for a class of uncertain nonlinear dynamic multiagent systems (MASs) with state time-varying delays and state time-varying constraints. The existing controllers with Lyapunov–Krasovskii functions (LKFs) were not suitable to address time-varying delays and time-varying constraints in nonlinear MASs simultaneously. State constraints further increase the difficulty of controller design and stability analysis, especially for nonstrict feedback systems. Fuzzy logic systems (FLSs) tackle the approximation of unknown dynamics functions and parameters. Especially when the distributed consensus tracking error is infinitely close to the origin, although there is no singular value, it would lead to the rapid growth of control rate or uncontrollability. Constructing appropriate piecewise functions can effectively avoid the above occurrence and accelerate convergence. Based on Lyapunov stability theory and algebraic graph theory, the constructed tracking control can ensure states within defined time-varying constraint bounds and eliminate the influence of time delays. All signals in closed-loop systems can be guaranteed semiglobally uniformly ultimately bounded (SUUB). Finally, the validity of the theoretical method is verified by the simulation.

1. Introduction

Over the past few decades, inspired by flocking behavior in nature, related research on multiagent systems (MASs) had developed rapidly due to potential military and civilian applications. The literature involved many aspects such as satellites, flights, distributed computing, robotics, power systems, surveillance and reconnaissance systems, multi-missile coordinated attacks, and intelligent transportation systems in [1–7]. In [8], based on the urgent needs of intelligent agriculture, the bioinspired coordination protocol was employed to achieve refinement in agricultural management. Consensus control is the most fundamental and most valuable core research of multiagent. The research directions include consensus control [9, 10], tracking control [11], aggregation control [12], formation control [13], and synchronous control [1, 14].

The consensus framework is proposed by Olfat-Saber and Murray in [14, 15]. Previous literature mainly focused

on linear MASs and acquired large numbers of breakthrough achievements. Communication security, information samples, edge maps, and other related methods have also made some achievements in [16–18]. In practical mechanical systems, the problems of inherent nonlinearity and uncertainty would be further enhanced on account of the complex cooperative tasks, the limitation or lack of system dynamic cognition, and so forth. Several remarkable consensus control approaches of linear MASs would not directly be employed to nonlinear MASs. It is indispensable to apply adaptive technology and intelligent control methods to solve uncertain nonlinear distributed MASs.

Adaptive technology had become an effective method to address various nonlinear systems in the literature [19–21]. The neural networks (NNs) [19, 20] were related to the event trigger. The fuzzy logic systems (FLSs) [21–23] solved the finite time of stochastic nonlinear systems in [21]. Distributed nonlinear MASs among the literature [24–27] had been proved distinguished approximation ability to solve the

unknown dynamics. The consensus control was discussed in [9], and consensus tracking control protocols were studied in [24–26]. The multileader consensus control method was researched in [27]. However, the above methods have not involved the problem of state time delays.

The differences in controller performance are often accompanied by state time delays which would reduce the speed of convergence, affect system performance, and lead to instability or ineffectiveness. Meanwhile, once time delays exceed the threshold, they would affect the whole systems. Therefore, time delays would be an important consideration. Some achievements had been made for time delays in uncertain nonlinear systems in [28–31]. Particularly, [30, 31] solved the time delayed switched systems. In [32, 33], LKFs were employed to eliminate time delays in cooperative tracking control with visual leader and leader-following, respectively. Nevertheless, it is obvious that the above omitted the constraints. In particular, it is unavoidable to consider the physical limits and safety performance of each agent, the mutual internal collision avoidance, and other operation restrictions. If ignoring constraints, system performance would be damaged or reduced and even would cause fatal accidents. From a practical point of view, the theoretical study of constraints becomes crucially important in MASs.

In the design of constrained controllers, reliable methods needed to be applied. The Barrier Lyapunov functions have been preferred as the primary candidate functions for designing controllers with constraints. The study in [34] presented the method to indirectly address the constraint by constructing barrier functions. Many BLFs, integral BLF (IBLF), and ABLF based adaptive approximate controllers were designed for different categories of the nonlinear systems in [34–41], and nonlinear MASs in [27, 42]. Integral barrier Lyapunov functions have been employed in [39–41]. The study in [27] proposed an adaptive fuzzy containment control based on the distributed observer and distributed sliding mode estimators with state constraints via multiple BLFs. Note that the above constraint strategies are indirect methods, which are applicable to the case that the feasibility of virtual controllers needs to be satisfied and the constraint boundaries are not infinitesimal. The nonlinear coordinate transformation function was used to construct the constraint completely dependent on states in [43] and [44], and the complicated feasibility analysis was reduced. The authors of [45, 46] proposed the control strategies to solve both the state constraints and state time-varying delays. Therefore, direct constraints on output or states are worth further investigation of the constrained cooperative control.

Motivated by the above analysis, this paper schedules a novel adaptive fuzzy distributed cooperative tracking control approach for a class of uncertain nonlinear MASs with state time-varying delays and constraints. The main contributions of the paper are as follows:

- (1) Contrasting with the existing literature on time delays in [32, 33], time delays in the case of time variation are further investigated. The most important one is to overcome the failure of the decomposition theorem in

nonstrict feedback. Meanwhile, state time-varying constraints are introduced into distributed time delayed MASs since the state time-varying delays and constraints further increase the difficulty of controller design, which is an enormous challenge.

- (2) Distinguished from BLFs, the constraints construct by a nonlinear coordinate transformation in [43] that directly constrain on states, which are applicable to avoid complex feasibility analysis of virtual controllers. The constructed appropriate piecewise functions can effectively avoid the rapid growth and uncontrollability of the control rate and accelerate the convergence when the synchronization error is infinitely close to the origin.

The proposed strategy is aimed at the study of the time-varying state of the agents, which has great realistic significance and further enriches the exploration of the uncertain nonlinear MASs.

The organization of the article is as follows: the presentation and problem of the systems are described in Section 2; the adaptive fuzzy distributed cooperative control design is provided in Section 3; simulation is given in Section 4; and the conclusion is given in Section 5.

2. Systems Presentation and Problem Description

2.1. Systems Description and Problem Presentation. Consider a class of uncertain nonlinear distributed consensus systems with time-varying delay and the i^{th} dynamic is described as

$$\dot{x}_i(t) = f_i(x_i(t)) + \rho_i(x_i(t - \tau_i(t))) + u_i(t), \quad (1)$$

where $x_i(t) = [x_{i1}, x_{i2}, \dots, x_{im}]^T \in R^m$ represents the state vector of i^{th} agent, $i = 1, 2, \dots, n$, and n is the number of agents; $f_i(x_i)$ and $\rho_i(x_i(t - \tau_i(t)))$: $R^m \rightarrow R^m$ are unknown nonlinear continuous vector functions; $\tau_i(t)$ represents the uncertain time-varying delay; and $u_i(t) \in R^m$ is the control input vector.

Assumption 1. $\forall t \geq 0$, the i^{th} unknown smooth nonlinear time delayed function $\rho_i(x_i(t - \tau_i(t)))$, satisfies

$$\|\rho_i(x_i(t - \tau_i(t)))\| \leq q_i(x_i(t - \tau_i(t))), \quad (2)$$

where $q_i(x_i(t - \tau_i(t)))$ is the known positive smooth function.

Remark 1. The unknown time-varying delays function $\rho(x_i(t - \tau_i(t)))$ contains the time-varying delayed state. The separation technique is used to decompose the unknown with all the time-varying delays into positive continuous functions in order to compensate for the time delays by LKFs in [45].

Assumption 2. The unknown time-varying $\tau_i(t)$ should satisfy the following: (1) the $\dot{\tau}_i(t)$ is bounded by a known constant $\tau_{d\max}$ and $\dot{\tau}_i(t) \leq \tau_{d\max} \leq 1$ and (2) the continuous

$\tau_i(t)$ should be uniformly bounded by a known constant $\tau_{\max}$ and $\tau_i(t) \leq \tau_{\max}$ in [33, 45, 46].

The desired reference leader signal is described by the following dynamic function:

$$\dot{x}_l(t) = g_l(t), \quad (3)$$

where $x_l(t) \in R^m$ is the state of reference leader and $g_l(t) \in R^m$ is a bounded smooth vector function.

The time-varying state constraint is considered, and the constraint boundary of i^{th} agent meets requirements as follows:

$$-k_{c_j}(t) < x_{ij}(t) < k_{c_j}(t), \quad (4)$$

where x_{ij} represents the j^{th} state of the i^{th} agent; $j = 1, 2, \dots, m$; $k_{c_j}(t)$ is a user-defined time-varying bound function in j^{th} state; and $k_c(t) \in R^m$. The initial value should satisfy $-k_{c_j}(0) < x_{ij}(0) < k_{c_j}(0)$.

The requirement of (4) can be satisfied by choosing a nonlinear coordinate transformation as follows:

$$s_{ij}(t) = \frac{x_{ij}(t)}{(k_{c_j}(t) + x_{ij}(t))(k_{c_j}(t) - x_{ij}(t))}, \quad (5)$$

where $s_{ij}(t)$ is a constructed state transition that converts the original $x_{ij}(t)$ bounded within $(-k_{c_j}(t), k_{c_j}(t))$ into a nonconstrained state, $s_i(t) = [s_{i1}, s_{i2}, \dots, s_{im}]^T \in R^m$.

Then, (5) satisfies

$$\begin{cases} \lim_{x_i(t) \rightarrow -k_c(t)} s_i(t) = -\infty, \\ \lim_{x_i(t) \rightarrow k_c(t)} s_i(t) = +\infty. \end{cases} \quad (6)$$

Take the derivative of $s_{ij}(t)$ as follows:

$$\dot{s}_{ij}(t) = \frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} \dot{x}_{ij}(t) + \frac{\partial s_{ij}(t)}{\partial k_{c_j}(t)} \dot{k}_{c_j}(t), \quad (7)$$

where $\partial s_{ij}(t)/\partial x_{ij}(t) = k_{c_j}^2(t) + x_{ij}^2(t)/(k_{c_j}^2(t) - x_{ij}^2(t))^2$ and $\partial s_{ij}(t)/\partial k_{c_j}(t) = -2x_{ij}(t)k_{c_j}(t)/(k_{c_j}^2(t) - x_{ij}^2(t))^2$.

The state of the leader in (3) can be transformed by the same form of (5); it can be expressed as

$$\alpha_{lj}(t) = \frac{x_{lj}(t)}{(k_{c_j}(t) + x_{lj}(t))(k_{c_j}(t) - x_{lj}(t))}, \quad (8)$$

where $\alpha_{lj}(t)$ is a state transition to transform the constrained $x_{lj}(t)$ within $(-k_{c_j}(t), k_{c_j}(t))$ into an unconstrained state, $\alpha_i(t) = [\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{im}]^T \in R^m$. Then, (8) satisfies

$$\begin{cases} \lim_{x_l(t) \rightarrow -k_c(t)} \alpha_l(t) = -\infty, \\ \lim_{x_l(t) \rightarrow k_c(t)} \alpha_l(t) = +\infty. \end{cases} \quad (9)$$

Take the derivative of (8) as follows:

$$\dot{\alpha}_{lj}(t) = \frac{\partial \alpha_{lj}(t)}{\partial x_{lj}(t)} \dot{x}_{lj}(t) + \frac{\partial \alpha_{lj}(t)}{\partial k_{c_j}(t)} \dot{k}_{c_j}(t), \quad (10)$$

where $\partial \alpha_{lj}(t)/\partial x_{lj}(t) = k_{c_j}^2(t) + x_{lj}^2(t)/(k_{c_j}^2(t) - x_{lj}^2(t))^2$ and $\partial \alpha_{lj}(t)/\partial k_{c_j}(t) = -2x_{lj}(t)k_{c_j}(t)/(k_{c_j}^2(t) - x_{lj}^2(t))^2$.

Remark 2. The states x_i and x_l are converted into s_i and α_l , respectively; $\partial s_{ij}(t)/\partial x_{ij}(t) > 0$ and $\partial \alpha_{lj}(t)/\partial x_{lj}(t) > 0$ are invertible functions; and $s_{ij}(t)$ and $\alpha_{lj}(t)$ are monotonically increasing. Just proving $s_{ij}(t)$ and $\alpha_{lj}(t)$ are bounded, by criteria of monotone bounded, the original states x_i and x_l would converge and satisfy the constraint region.

Assumption 3. The time-varying bound vector functions $k_c(t)$ and $\dot{k}_c(t)$ satisfy the following: $\forall t \geq 0$; there exist positive constant vectors $K_c \in R^m$ and $K_c^1 \in R^m$ that satisfy $|k_{c_j}(t)| \leq K_{c_j}$ and its time derivative $\dot{k}_{c_j}(t)$ satisfies $|\dot{k}_{c_j}(t)| \leq K_{c_j}^1$.

Remark 3. In numbers of practical cooperative control of multiagent systems, inevitably, constraints need to be addressed due to physical limitations or performance. If the constraints were ignored, the systems may be degraded or damaged by interagent influences, and even fatal accidents would occur. State constraints are more effective in solving states of transient or steady real systems than only with output constraints. The control process is time-varying, so it is significant to study the time-varying full state constraints.

Remark 4. The constraints based on BLFs (or IBLFs) require additional complex stability analysis to ensure the feasibility conditions of virtual controllers. The state-dependent transition method in [43, 44] which can circumvent the above problems, flexibly solve initial conditions, and reduce the difficulty of stability analysis.

Remark 5. The nonlinear coordinate transformation function is constructed and purely dependent on the constraint states. Introducing a new coordinate transformation for the backstepping design of MASs, it directly responses to state constraints and completely avoids the harsh feasibility conditions. This will get rid of the tedious feasibility verification. Let the designer have more freedom and be humanized to choose the designed parameters in implementations. Apply a wider range of initial conditions.

This paper only shows the feasibility and effectiveness of constructing state time-varying constraints to distributed nonlinear and uncertain time-varying delayed MASs. The asymmetric forms would be further discussed in other papers.

2.1.1. Control Objective. Devise an adaptive fuzzy decentralized tracking controller u_i that, for the dynamic of agents described in (1) and (3), can guarantee the following:

(i) All followers converge to the desired small neighborhood around the origin by tracking the leader. (ii) The full states would not violate the time-varying constraint boundaries in (5) and (8). (iii) All the signals in the closed-loop systems are SUUB.

The following assumptions and lemmas should be satisfied in order to establish a cooperative control objective.

2.2. Algebraic Graph Theory and Notations. An undirected connected topology expresses as $G = (V, E, A)$, where

$V = \{v_1, v_2, \dots, v_n\}$ denotes a set of nodes and v_i is i^{th} node. $E \subseteq V \times V$ denotes the set of edges, $\varphi_{ij} = (v_i, v_j) \in E$ refers to the edge from node i to node j , and $\varphi_{ij} = \varphi_{ji}$. $A = [a_{ij}] \in R^{n \times n}$ is a weighted adjacency matrix that a weight corresponds to the edge, and when $\varphi_{ij} \in R^{n \times n}$, $a_{ij} = 1$. Otherwise, $a_{ij} = 0$. In particular, define $a_{ii} = 0$.

The graph Laplacian matrix L is given in the following definition:

$$L = D - A, \quad (11)$$

where $L = [l_{ij}] \in R^{n \times n}$; $D = \text{diag}\{d_1, d_2, \dots, d_n\} \in R^{n \times n}$ defines an in-degree matrix; $d_i = \sum_{j \in N_i} a_{ij}$ represents the in-degree of node i ; $N_i = \{j: (j, i) \in E\}$; L is an irreducible matrix; and $[1, 1, \dots, 1]^T \in R^n$ is a right eigenvector with the eigenvalue zero.

Lemma 1. *The graph Laplacian matrix L is a semidefinite symmetric matrix; all eigenvalues are nonnegative real; and its n real eigenvalues can be listed in ascending order as follows:*

$$0 = \lambda_1 < \lambda_2 \leq \lambda_3 \leq \dots \leq \lambda_n \leq C, \quad (12)$$

where $C = 2(\max_{1 \leq i \leq n} d_i)$ is algebraic connectivity and it is used to analyze the rate of consensus convergence.

Lemma 2. *$\forall t > 0$, the continuous function $V(t)$ satisfies $V(t) > 0$, and $V(0)$ is bounded. If it can prove the inequality $\dot{V}(t) \leq -\ell V(t) + \hbar$ where $\ell > 0$, $\hbar > 0$, then*

$$\dot{V}(t) \leq V(0)e^{-\ell t} + \frac{\hbar}{\ell}(1 - e^{-\ell t}). \quad (13)$$

Assumption 4. In the graph topology G , for the leader, there is at least one directed path from the root node to follower nodes.

2.3. Distributed Controller. The proposed distributed control algorithm can make arbitrary i^{th} agent feedback to itself and communicate with neighbor agents via adjacent matrix. Since fixed topology graphs are the basis for different types of switching connection topology and are widely applied such as [27, 28], this paper is based on the fixed topological form.

The synchronization error $e_i(t) \in R^m$ consists of the i^{th} local consensus error and the tracking error among the leader and followers, and that can be defined as follows:

$$e_i(t) = \sum_{j=1}^{N_i} a_{ij}(s_i(t) - s_j(t)) + b_i(s_i(t) - \alpha_i(t)), \quad (14)$$

where a_{ij} represents the element in adjacency matrix A . b_i is the communication weight when only i^{th} agent exchanges information with leader $b_i > 0$; otherwise $b_i = 0$, and $B = \text{diag}\{b_1, b_2, \dots, b_n\}$ is the communication weight matrix and satisfies $b_1 + b_2 + \dots + b_n > 0$:

Let

$$\tilde{L} = L + B, \quad (15)$$

where $\tilde{L}$ is a symmetrical positive definite matrix and its real eigenvalues $\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_n$ satisfy Lemma 1.

Based on the matrix theory and graph theory, it can be readily concluded that zero is an m -multiplicity eigenvalue of $\tilde{L} \otimes I_m$; $\otimes$ and I_m , respectively, stand for the Kronecker product and identity matrix of dimension $m \times m$. Let $\varphi_{11}, \varphi_{12}, \dots, \varphi_{1m}, \dots, \varphi_{2m}, \dots, \varphi_{nm}$ be the eigenvectors and $\lambda_2, \dots, \lambda_n$ eigenvalues of $\tilde{L} \otimes I_m$ such there can be a set of orthogonal bases of R^{nm} . The definition is given by graph theory:

$$\tilde{L} \otimes I_m = P \Lambda P^T, \quad (16)$$

where $P^T P = P P^T = I_{nm}$, I_{nm} denoting identity matrix of dimension $nm \times nm$ and $P = [\varphi_{11}, \varphi_{12}, \dots, \varphi_{nm}] \in R^{nm \times nm}$.

Define the tracking error vector as the following form:

$$\varsigma_i(t) = s_i(t) - \alpha_i(t), \quad (17)$$

where $\varsigma_i(t) = [\varsigma_{i1}(t), \varsigma_{i2}(t), \dots, \varsigma_{im}(t)]^T \in R^m$ represents the tracking error between the i^{th} follower agent and the leader agent. Then, (14) can be rewritten as

$$e_i(t) = \sum_{j=1}^{N_i} a_{ij}(\varsigma_i(t) - \varsigma_j(t)) + b_i \varsigma_i(t), \quad (18)$$

where $e_i(t) = [e_{i1}(t), e_{i2}(t), \dots, e_{im}(t)]^T$.

The unknown functions and variables in multiagent systems can be approximated by FLSs as follows:

$$F_{ij}(X_{ij}) = \omega_{ij}^T \Psi_{ij}(X_{ij}) + \varepsilon_{ij}, \quad (19)$$

where $\omega_{ij} \in R^l$ is the optimal fuzzy weight vector, l_j denotes the number of fuzzy rules, $\Psi_{ij}(X_{ij})$ is the fuzzy basis vector function, ε_{ij} shows estimation error satisfied $\|\varepsilon_{ij}\| \leq \bar{\varepsilon}_{ij}$, and $\bar{\varepsilon}_{ij}$ is an arbitrary indeterminate positive constant.

3. Adaptive Fuzzy Distributed Cooperative Control Design

In this section, an adaptive fuzzy distributed cooperative control algorithm will be designed for (1) and (3). Define the following candidate function:

$$V_\omega = \frac{1}{2} \varsigma(t)^T Q \varsigma(t) + \frac{1}{2} \sum_{i=1}^n \text{tr}[\tilde{\omega}_i^T \Gamma_i^{-1} \tilde{\omega}_i], \quad (20)$$

where $\varsigma(t) = (\varsigma_1^T(t), \varsigma_2^T(t), \dots, \varsigma_n^T(t))^T \in R^{nm}$ is the consensus error vector that $\varsigma_i(t)$ has defined in (17), then $\varsigma(t) = (\tilde{L} \otimes I_m) e(t)$, $Q = P \tilde{\Lambda}^{-1} P^T$, $\tilde{\omega}_{ij} = \omega_{ij} - \hat{\omega}_{ij}$, $\hat{\omega}_{ij}$ is the estimate of adaptive laws, and ω_{ij} and $\hat{\omega}_{ij}$ will be given later.

The inequality can be described as follows:

$$\frac{\lambda_{\min}(Q)}{2} \leq \frac{1}{2} \varsigma(t)^T Q \varsigma(t) \leq \frac{\lambda_{\max}(Q)}{2}, \quad (21)$$

where $\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ are the smallest and largest eigenvalues of Q , respectively. From (16), it yields

$$\begin{aligned} \frac{1}{2}(\zeta(t)^T Q \zeta(t)) &= \frac{1}{2} e^T(t) (\tilde{L} \otimes I_m)^T P \hat{\Lambda}^{-1} P^T (\tilde{L} \otimes I_m) e(t) = \frac{1}{2} e^T(t) P \Lambda P^T P \hat{\Lambda}^{-1} P^T P \Lambda P^T e(t) \\ &= \frac{1}{2} e^T(t) (\tilde{L} \otimes I_m) e(t) = \frac{1}{2} \zeta(t)^T e(t) = \frac{1}{2} \sum_{i=1}^n \zeta_i^T(t) e_i(t), \end{aligned} \quad (22)$$

where $\Lambda = \text{diag}\{0I_m, \tilde{\lambda}_2 I_m, \dots, \tilde{\lambda}_n I_m\}$.

Redefine the Lyapunov candidate function as follows:

$$V_\omega = \frac{1}{2} \sum_{i=1}^n \zeta_i^T(t) e_i(t) + \frac{1}{2} \sum_{i=1}^n \text{tr}[\tilde{\omega}_i^T \Gamma_i^{-1} \tilde{\omega}_i]. \quad (23)$$

Take the derivative of (23). Then, $\dot{V}_\omega$ can yield

$$\dot{V}_\omega = \frac{1}{2} \dot{\zeta}(t)^T Q \zeta(t) + \frac{1}{2} \zeta(t)^T Q \dot{\zeta}(t) + \frac{1}{2} \sum_{i=1}^n \text{tr}[\dot{\tilde{\omega}}_i^T \Gamma_i^{-1} \tilde{\omega}_i] + \frac{1}{2} \sum_{i=1}^n \text{tr}[\tilde{\omega}_i^T \Gamma_i^{-1} \dot{\tilde{\omega}}_i] = \sum_{i=1}^n e_i^T(t) \dot{\zeta}_i(t) - \sum_{i=1}^n \text{tr}[\tilde{\omega}_i^T \Gamma_i^{-1} \dot{\tilde{\omega}}_i]. \quad (24)$$

$\dot{V}_\omega$ can be written as follows:

$$\dot{V}_\omega = \sum_{j=1}^m \dot{V}_{\omega j} = \sum_{j=1}^m \sum_{i=1}^n [e_{ij}^T(t) \dot{\zeta}_{ij}(t) - \tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\tilde{\omega}}_{ij}]. \quad (25)$$

The derivative of $\zeta_{ij}(t)$ in the form of (17) can be described as

$$\dot{\zeta}_{ij}(t) = \frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} \dot{x}_{ij}(t) - \frac{\partial \alpha_{ij}(t)}{\partial x_{ij}(t)} \dot{x}_{lj}(t) + \left(\frac{\partial s_{ij}(t)}{\partial k_{c_j}(t)} - \frac{\partial \alpha_{lj}(t)}{\partial k_{c_j}(t)} \right) \dot{k}_{c_j}(t). \quad (26)$$

Combining systems (1), (3), and (26) into $\dot{V}_{\omega j}$ yields

$$\dot{V}_{\omega j} \leq \sum_{i=1}^n e_{ij}^T(t) \left\{ \frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} [u_{ij}(t) + \rho_{ij}(x_{ij}(t - \tau_i(t))) + f_{ij}(x_{ij}(t))] + K_{ij}(t) - \frac{\partial \alpha_{lj}(t)}{\partial x_{ij}(t)} g_{lj}(t) \right\} - \sum_{i=1}^n (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\tilde{\omega}}_{ij}), \quad (27)$$

where $K_{ij}(t)$ is i^{th} time-varying gain described as follows:

$$K_{ij}(t) = \sup \sqrt{\left(\frac{\partial s_{ij}(t)}{\partial k_{c_j}(t)} \dot{k}_{c_j}(t) \right)^2 + \left(\frac{\partial \alpha_{lj}(t)}{\partial k_{c_j}(t)} \dot{k}_{c_j}(t) \right)^2} + \beta_{ij}. \quad (28)$$

β_{ij} is a positive parameter to be given.

Using Young's inequality and Assumption 1, in order to separate e_{ij} and uncertainties $\rho_{ij}(x_{ij}(t - \tau_i(t)))$, the inequality gets

$$e_{ij}^T(t) \frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} \rho_{ij}(x_{ij}(t - \tau_i(t))) \leq \frac{1}{2\sqrt{\nu}} \|e_{ij}(t)\|^2 M_{ij}^2 + \frac{1}{2} \nu q_{ij}^2(x_{ij}(t - \tau_i(t))), \quad (29)$$

where $\nu = 1 - \tau_{d\max}$ has been illustrated in Remark 4 and $M_{ij} = \|\partial s_{ij}(t)/\partial x_{ij}(t)\|$.

The unknown smooth function $F_{ij}(X_{ij})$ is approximated by FLSs to tackle the unknown dynamics functions and parameters. $F_{ij}(X_{ij})$ is denoted as

$$F_{ij}(X_{ij}) = f_{ij}(x_{ij}(t)) + \frac{1}{2} \frac{1}{\sqrt{\nu}} e_{ij}(t) \frac{\partial s_{ij}(t)}{\partial x_{ij}(t)}, \quad (30)$$

where $X_{i1} = [x_{i1}, x_{i2}, k_{c_i}, e_{i1}]^T$, $X_{i2} = [x_{i1}, x_{i2}, k_{c_i}, e_{i2}]^T$, and $\|e_{ij}(t)\|^2 M_{ij}^2$ takes the norm of $e_{ij}(t) \partial s_{ij}(t)/\partial x_{ij}(t)$ in (29).

Equation (30) can be estimated by (19) as follows:

$$F_{ij}(X_{ij}) = \omega_{ij}^T \Psi_{ij}(X_{ij}) + \varepsilon_{ij}. \quad (31)$$

Combining (28), (29), and (30) by FLSs, then (27) can yield

$$\dot{V}_{\omega j} \leq \sum_{i=1}^n e_{ij}^T(t) \left[\frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} (u_{ij}(t) + \omega_{ij}^T \Psi_{ij}(X_{ij}) + \varepsilon_{ij}) + U_{ij}(t) \right] + \frac{1}{2} \sum_{i=1}^n v q_{ij}^2(x_{ij}(t - \tau_i(t))) - \sum_{i=1}^n (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\hat{\omega}}_{ij}), \quad (32)$$

where $U_{ij}(t) = K_{ij}(t) - \partial \alpha_{ij}(t) / \partial x_{ij}(t) g_{lj}(t)$.

By using Young's inequality, we obtain the following:

$$\sum_{i=1}^n e_{ij}^T(t) \frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} \varepsilon_{ij} \leq \frac{1}{2} \sum_{i=1}^n \|e_{ij}(t)\|^2 M_{ij}^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\varepsilon}_{ij}^2. \quad (33)$$

Combining (33) into (32), we obtain

$$\begin{aligned} \dot{V}_{\omega j} \leq & \sum_{i=1}^n e_{ij}^T(t) \left[\frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} (u_{ij}(t) + \omega_{ij}^T \Psi_{ij}(X_{ij})) + U_{ij}(t) \right] + \frac{1}{2} \sum_{i=1}^n v q_{ij}^2(x_{ij}(t - \tau_i(t))) \\ & - \sum_{i=1}^n (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\hat{\omega}}_{ij}) + \frac{1}{2} \sum_{i=1}^n \|e_{ij}(t)\|^2 M_{ij}^2 + \sum_{i=1}^n \frac{1}{2} \tilde{\varepsilon}_{ij}^2. \end{aligned} \quad (34)$$

The Lyapunov–Krasovskii function is constructed to compensate uncertain time-varying delays as follows:

$$V_{\rho}(t) = \sum_{i=1}^m V_{\rho j}(t) = \frac{1}{2} \sum_{i=1}^m \sum_{i=1}^n \int_{t-\tau_i(t)}^t \beta_{ij}(x_{ij}(s)) ds, \quad (35)$$

where $\beta_{ij}(x_{ij}(s)) \leq q_{ij}^2(x_{ij}(s))$ has been defined under Assumption 1.

Take the derivative of $V_{\rho j}(t)$. It obtains

$$\dot{V}_{\rho j}(t) \leq \frac{1}{2} \sum_{i=1}^n q_{ij}^2(x_{ij}(t)) - \frac{1}{2} v \sum_{i=1}^n q_{ij}^2(x_{ij}(t - \tau_i(t))). \quad (36)$$

The Lyapunov function candidate is chosen by combining (23) and (35) as follows:

$$\begin{aligned} V(t) &= V_{\omega} + V_{\rho}(t), \\ \sum_{j=1}^m V_{ij}(t) &= \sum_{j=1}^m V_{\omega j} + \sum_{i=1}^m V_{\rho j}(t), \\ \sum_{j=1}^m V_{ij}(t) &= \sum_{j=1}^m \left(\frac{1}{2} \sum_{i=1}^n \varsigma_{ij}^T(t) e_{ij}(t) + \frac{1}{2} \sum_{i=1}^n (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\hat{\omega}}_{ij}) \right) + \frac{1}{2} \sum_{j=1}^m \sum_{i=1}^n \int_{t-\tau_i(t)}^t \beta_{ij}(x_{ij}(s)) ds. \end{aligned} \quad (37)$$

According to (34) and (36), the derivative of $V_{ij}(t)$ can obtain

$$\dot{V}_{ij} \leq \sum_{i=1}^n e_{ij}^T(t) \left[\frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} (u_{ij}(t) + \omega_{ij}^T \Psi_{ij}(X_{ij})) + U_{ij}(t) \right] + \frac{1}{2} \sum_{i=1}^n q_{ij}^2(x_{ij}(t)) - \sum_{i=1}^n (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\hat{\omega}}_{ij}) + \frac{1}{2} \sum_{i=1}^n \|e_{ij}(t)\|^2 M_{ij}^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\varepsilon}_{ij}^2. \quad (38)$$

The designed decentralized controller has been described as follows:

$$u_{ij}(t) = \begin{cases} \left(\frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} \right)^{-1} \left[-k_{ij}(t) e_{ij}(t) - \hat{\omega}_{ij}^T \Psi_{ij}(X_{ij}) \right. \\ \left. - \frac{1}{2} \frac{\|e_{ij}(t)\| q_{ij}^2(x_{ij}(t))}{\|e_{ij}(t)\|^2 + \chi_{ij}} - U_{ij}(t) \right], & e_{ij}(t) \in \Omega_{\delta_i}^c, \\ 0, & e_{ij}(t) \in \Omega_{\delta_i}, \end{cases} \quad (39)$$

where $\partial s_{ij}(t)/\partial x_{ij}(t)$ is a monotonically increasing function and the inverse must exist in Remark 1. $k_{ij}(t)$ is the controller gain that will be defined later. $\Omega_{\delta_i} = \{e_{ij}(t) \mid \|e_{ij}(t)\| < \delta_i\}$ is a compact set, $\Omega_{\delta_i}^c$ is its complement set, δ_i is an arbitrarily positive real number, and its definition will be given later. Once in Ω_{δ_i} near the origin, there is no need to impose on control. The piecewise function χ_{ij} satisfies the following:

$$\chi_{ij} = \begin{cases} 1, & e_{ij}(t) \in \Omega_{\delta_i}^c, \\ 0, & e_{ij}(t) \in \Omega_{\delta_i}. \end{cases} \quad (40)$$

Remark 6. Particularly, singularities may occur when $e_i(t) = [0]_m$ in cooperative control strategy in $1/2\|e_{ij}(t)\|^{-1} q_{ij}^2(x_{ij}(t))$. The function $\lim_{e_{ij}(t) \rightarrow 0} e_{ij}^{-1}(t) = \infty$ means the limit of synchronization error function infinitely approaches the sphere near the origin. Even though it is not equal to zero and there is no singular value, it is still uncontrollable. Therefore, (40) is designed to prevent the occurrence and accelerate the convergence rate.

Figure 1 is the flow diagram of the control approach for a class of uncertain nonlinear MASs with state time-varying constraints and delays, which can clearly guide the construction process of the control strategy.

From Remark 3, the adaptive law for $\hat{w}_{ij}$ is designed as follows:

$$\dot{\hat{w}}_{ij} = \Gamma_{ij} \{e_{ij}(t) \Psi_{ij}(X_{ij}) - \kappa_{ij} \hat{w}_{ij}\}, \quad (41)$$

where Γ_{ij} is a positive symmetric gain. κ_{ij} is the positive constant to improve the approximation error robustness of FLSS.

When $e_{ij}(t) \in \Omega_{\delta_i}^c$, combining (39), (40), and (41) into (38) yields

$$\dot{V}_{ij}(t) \leq \sum_{i=1}^n -k_{ij}(t) \|e_{ij}(t)\|^2 + \sum_{i=1}^n \kappa_{ij} \tilde{w}_{ij}^T \tilde{w}_{ij} + \frac{1}{2} \sum_{i=1}^n \|e_{ij}(t)\|^2 M_{ij}^2 + \frac{1}{2} \sum_{i=1}^n \tilde{e}_{ij}^2. \quad (42)$$

Using Young's inequality, it follows that

$$\tilde{w}_{ij}^T \tilde{w}_{ij} = \tilde{w}_{ij}^T (\omega_{ij} - \tilde{\omega}_{ij}) \leq \frac{1}{2} \omega_{ij}^2 - \frac{1}{2} \tilde{w}_{ij}^T \tilde{\omega}_{ij}. \quad (43)$$

According to (43) into (42), then $\dot{V}_{ij}(t)$ obtains

$$\dot{V}_{ij}(t) \leq \sum_{i=1}^n \left[-k_{ij}(t) \|e_{ij}(t)\|^2 + \frac{1}{2} \sum_{i=1}^n M_{ij}^2 \|e_{ij}(t)\|^2 \right] - \frac{1}{2} \kappa_{ij} \tilde{w}_{ij}^T \tilde{\omega}_{ij} + \eta_{j2}, \quad (44)$$

where $\eta_{j2} = 1/2 \sum_{i=1}^n \kappa_{ij} \omega_{ij}^2 + 1/2 \sum_{i=1}^n \tilde{e}_{ij}^2$.

The definition of controller gain $k_{ij}(t)$ is described briefly as follows:

$$k_{ij}(t) = k_{ij}^0 + k_{ij}^1(t), \quad (45)$$

where $k_{ij}^0 \geq 1/2 M_{ij}^2$, and $k_{ij}^1(t)$ satisfies

$$k_{ij}^1(t) = \frac{\gamma_{ij}}{2} \left[\lambda_{\max}(Q) + \frac{1}{\|e_{ij}(t)\|^2 + \chi_{ij}} \int_{t-\tau_{\max}}^t \beta_{ij}(x_{ij}(s)) ds \right]. \quad (46)$$

Substitute (45) and (46) into (44); it yields

$$\dot{V}_{ij}(t) \leq -\frac{1}{2} \sum_{i=1}^n \gamma_{ij} \lambda_{\max}(Q) \|e_{ij}(t)\|^2 - \frac{1}{2} \sum_{i=1}^n \frac{\kappa_{ij}}{\Gamma_{ij}^{-1}} (\tilde{w}_{ij}^T \Gamma_{ij}^{-1} \tilde{w}_{ij}) - \frac{1}{2} \sum_{i=1}^n \gamma_{ij} \int_{t-\tau_{\max}}^t \beta_{ij}(x_{ij}(s)) ds + \eta_{j2}, \quad (47)$$

where $\beta_{ij}(x_{ij}(s)) > 0$ has been defined; therefore, the integral inequality satisfies

$$\frac{1}{2} \sum_{i=1}^n \int_{t-\tau_{\max}}^t \beta_{ij}(x_{ij}(s)) ds \geq \frac{1}{2} \sum_{i=1}^n \int_{t-\tau_i(t)}^t \beta_{ij}(x_{ij}(s)) ds. \quad (48)$$

By (48), $\dot{V}_{ij}(t)$ can obtain

$$\dot{V}_{ij}(t) \leq -\frac{1}{2} \sum_{i=1}^n \gamma_{ij} \lambda_{\max}(Q) \|e_{ij}(t)\|^2 - \frac{1}{2} \sum_{i=1}^n \frac{\kappa_{ij}}{\Gamma_{ij}^{-1}} (\tilde{w}_{ij}^T \Gamma_{ij}^{-1} \tilde{w}_{ij}) - \frac{1}{2} \sum_{i=1}^n \gamma_{ij} \int_{t-\tau_i(t)}^t \beta_{ij}(x_{ij}(s)) ds + \eta_{j2}. \quad (49)$$

FIGURE 1: The block diagram illustrating the flow of the proposed control approach.

From (21), $\dot{V}_{ij}(t)$ yields

$$\dot{V}_{ij}(t) \leq -\eta_{j1} V_{ij}(t) + \eta_{j2}, \quad (50)$$

where $\eta_{j1} = \min_{1 \leq i \leq n} \{\gamma_{ij}, \kappa_{ij}/\Gamma_{ij}^{-1}\}$.

Then, compute this sum and integral of $\dot{V}_{ij}(t)$ obtained that

$$V(t) \leq V(0)e^{-\eta_1 t} + \left[\frac{\eta_2}{\eta_1} (1 - e^{-\eta_1 t}) \right], \quad (51)$$

where $\eta_1 = \min_{1 \leq j \leq m} \eta_{j1}$, $\eta_2 = \sum_{j=1}^m \eta_{j2}$.

Theorem 1. For nonlinear dynamics (1) and (3) with Assumptions 1–3 and the controller gains in (39), the adaptive law is given by (41). By selecting appropriate parameters, the

adaptive fuzzy cooperative control strategy can guarantee the following:

- (1) The synchronization error converges to the desired small neighborhood near the origin
- (2) The state time-varying constraints for agents given by (5) and (8) would not be violated
- (3) The time-varying delays can be compensated by LKFs. The signals in the closed-loop systems can be verified SUUB

Proof. When $e_{ij}(t) \in \Omega_{\delta_i}^c$, choosing the Lyapunov function in (37), the derivative of $V(t)$ is as follows:

$$\begin{aligned} \dot{V}(t) \leq & \sum_{j=1}^m \left\{ \sum_{i=1}^n \bar{e}_{ij}^T(t) \left\{ \frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} [u_{ij}(t) + \rho_{ij}(x_{ij}(t - \tau_i(t)) + f_{ij}(x_{ij}t))] + U_{ij}(t) \right\} \right. \\ & \left. - \sum_{i=1}^n (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\tilde{\omega}}_{ij}) + \sum_{i=1}^n q_i^2(x_i(t)) - \frac{1}{2} \nu \sum_{i=1}^n q_i^2(x_i(t - \tau_i(t))) \right\}. \end{aligned} \quad (52)$$

Applying the results in (28), (29), (30), (33), and (36) into (52), it obtains

$$\dot{V}(t) \leq \sum_{j=1}^m \left\{ \sum_{i=1}^n e_{ij}^T(t) \left[\frac{\partial s_{ij}(t)}{\partial x_{ij}(t)} (u_{ij}(t) + \omega_{ij}^T \Psi_{ij}(X_{ij})) + U_{ij}(t) \right] + \sum_{i=1}^n \frac{1}{2} q_{ij}^2(x_{ij}(t)) - \sum_{i=1}^n (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\tilde{\omega}}_{ij}) + \frac{1}{2} \sum_{i=1}^n M_{ij}^2 \|e_{ij}(t)\|^2 + \sum_{i=1}^n \frac{1}{2} \bar{e}_{ij}^2 \right\}. \quad (53)$$

Substitute (39), (41), controller gain (45), (46) into (53); then, it yields

$$\dot{V}(t) \leq \sum_{j=1}^m \left\{ -\frac{1}{2} \sum_{i=1}^n \gamma_{ij} \lambda_{\max}(Q) \|e_{ij}(t)\|^2 - \frac{1}{2} \sum_{i=1}^n \gamma_{ij} \int_{t-\tau_{\max}}^t \beta_{ij}(x_{ij}(s)) ds - \frac{1}{2} \sum_{i=1}^n \frac{\kappa_{ij}}{\Gamma_{ij}} (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \dot{\tilde{\omega}}_{ij}) + \eta_{j2} \right\}. \quad (54)$$

From (21), given the integral inequality in (48), $\dot{V}(t)$ yields

$$\dot{V}(t) \leq \sum_{j=1}^m \sum_{i=1}^n \left[-\frac{1}{2} \gamma_{ij} \lambda_{\max}(Q) \|e_{ij}(t)\|^2 - \frac{\gamma_{ij}}{2} \int_{t-\tau_i(t)}^t \beta_{ij}(x_{ij}(s)) ds - \frac{1}{2} \frac{\kappa_{ij}}{\Gamma_{ij}^{-1}} (\tilde{\omega}_{ij}^T \Gamma_{ij}^{-1} \tilde{\omega}_{ij}) + \eta_{j2} \right] \leq -\eta_1 V(t) + \eta_2. \quad (55)$$

For $e_{ij} \in \Omega_{\delta_i}^c$, integrate the above $\dot{V}(t)$ over $[0, t]$; then, it can obtain

$$0 < V(t) \leq \frac{\eta_2}{\eta_1} + \left(V(0) - \frac{\eta_2}{\eta_1} \right) e^{-\eta_1 t}. \quad (56)$$

From (21), (37), and (56), the inequality obtains

$$\sum_{j=1}^m \sum_{i=1}^n \frac{\lambda_{\min}(Q)}{2} \|e_{ij}(t)\|^2 \leq V(t) \leq \frac{\eta_2}{\eta_1} + \left(V(0) - \frac{\eta_2}{\eta_1} \right) e^{-\eta_1 t} \leq \frac{\eta_2}{\eta_1} + V(0) e^{-\eta_1 t} \sum_{j=1}^m \sum_{i=1}^n \|e_{ij}(t)\|^2 \leq \frac{2\eta_2}{\lambda_{\min}(D)\eta_1} + \frac{2}{\lambda_{\min}(D)} V(0) e^{-\eta_1 t}. \quad (57)$$

Given $\delta_i > \sqrt{2\eta_2/\lambda_{\min}(Q)\eta_1}$, there exists $T > 0$, for $i = 1, \dots, n$, and all $t > T$:

$$\|e_{ij}(t)\| < \delta_i, \quad (58)$$

where δ_i is a small positive constant which determines tracking performance. The inequality can be obtained bounded by the Lyapunov Theory and selecting the appropriate parameters in the closed-loop MASs.

The proof is completed. $\square$

Remark 8. For the uncertain nonlinear MASs (1) and (3), Assumptions 1–3, the actual controller is designed in (39) and the adaptive law (41) guarantees that all the signals in the closed-loop systems are SUUB. The parameters selection guidelines for controller design with state time-varying constraints and delays in MASs are demonstrated:

- (i) Select appropriately designed parameters such that k_{ij}^0 , ω_{ij}^T , β_{ij} , δ_i , Γ_{ij} , κ_{ij} , γ_{ij} , $\bar{e}_{ij}$. Meanwhile, a positive matrix Q is needed to satisfy $Q = P\hat{\Lambda}^{-1}P^T$ and $P\hat{\Lambda}P^T = \tilde{L} \otimes I_m$.
- (ii) Decrease the control gain Γ_{ij} , ω_{ij}^2 , and $\bar{e}_{ij}$. Increasing the adaptive gain γ_{ij} and k_{ij}^0 would result in better consensus tracking performance. Other parameters are selected according to experience. Therefore, the parameter design of the controller should be adjusted according to the system performance and the tradeoff between tracking performance and constraints between agents such as collision avoidance.

4. Simulation Example

The following example is given to illustrate the effectiveness of the proposed adaptive fuzzy tracking control scheme for distributed uncertainties nonlinear MASs containing one

leader with three followers. Assume that i^{th} agent with time-varying delays is as follows:

$$\begin{cases} \dot{x}_{i1} = x_{i2}(t) \sin(D_{i1}x_{i1}(t)) + \rho_{i1}(x_{i1}(t - \tau_i(t))) + u_i(t), \\ \dot{x}_{i2} = x_{i1}(t) \cos(D_{i2}x_{i2}(t)) + \rho_{i2}(x_{i2}(t - \tau_i(t))) + u_i(t). \end{cases} \quad (59)$$

The dynamic of the leader can be described as

$$\dot{x}_l(t) = \begin{bmatrix} 0.5 \sin(0.8t) \\ 0.1 + 0.5 \sin(t) \end{bmatrix}, \quad (60)$$

where u_i is the input of the cooperative tracking control, $x_l(t)$ is the state of the leader, the designed trajectories are $g_{l1} = 0.5 \sin(0.8t)$, and $g_{l2} = 0.1 + 0.5 \sin(t)$. The state variables $x_{i1}(t)$ and $x_{i2}(t)$ are bounded by time-varying state constraints as follows: $-k_{c1}(t) < x_{i1}(t) < k_{c1}(t)$, and $k_{c1}(t) = 1 + 0.3 \sin(2t)$; $-k_{c2}(t) < x_{i2}(t) < k_{c2}(t)$, and $k_{c2}(t) = 1.8 + 0.4 \sin(0.5t)$; C_{i1} and C_{i2} are defined in Table 1; and D_{i1} and D_{i2} are defined in Table 2, $i = 1, 2, 3$. The time-varying delays are $\tau_1(t) = 1.25 - 0.5 \sin(0.5t)$, $\tau_2(t) = 1.3 - 0.6 \sin(0.5t)\sin(t)$, $\tau_3(t) = 1.2 - 0.5 \cos(0.5t)$, $\tau_{\max} = 2$.

The information interaction between agents is shown in Figure 2.

The topology illustrates the information interaction between agents in Figure 2 and $B = \text{diag}\{1, 1, 0\}$. The Lap-

lacian matrix L is given as $L = \begin{pmatrix} 0.7 & -0.7 & 0.0 \\ -0.7 & 1.5 & -0.8 \\ 0.0 & -0.8 & 0.8 \end{pmatrix}$.

The initial of three agents are $x_1(0) = (0.05, 0.108)^T$, $x_2(0) = (-0.13, 0.1)^T$, and $x_3(0) = (0.038, -0.135)^T$. The designed parameters are chosen as $\Gamma_{i1} = \Gamma_{i2} = 1$, $\kappa_{11} = 0.7$, $\kappa_{21} = 0.4$, $\kappa_{31} = 0.4$, $\kappa_{12} = 0.6$, $\kappa_{22} = 0.6$, $\kappa_{32} = 0.6$, $k_{01} = k_{02} = 270$, $\delta_i = 10^{-7}$, and $\gamma_i = 0.9$, $i = 1, 2, 3$.

Apparently, Assumption 2 can be satisfied by choosing $q_i(x_i(t)) = \sqrt{(C_{i1}x_{i1}(t))^2 + (C_{i2}x_{i2}(t))^2}$. The adaptive FLSs

TABLE 1: Values of C_{i1} and C_{i2} , $i = 1, 2, 3$.

i	1	2	3
C_{i1}	1	3	-2
C_{i2}	1.5	2.7	0.8

TABLE 2: Values of D_{i1} and D_{i2} , $i = 1, 2, 3$.

i	1	2	3
D_{i1}	0.6	-3	6
D_{i2}	0.7	0.5	-5

controller u_i is given by (39), and the fuzzy rule numbers are $l_1 = l_2 = 10$. Choose the fuzzy membership functions as follows:

$$\begin{aligned}
 \mu_{F_{i1}^{l_1}}(X_{i1}) &= \exp \left[-\frac{(x_{i1} - 0.5l_1)^2}{8} - \frac{(x_{i2} - 0.5l_1)^2}{8} - \frac{(k_{c_1} - 0.5l_1)^2}{8} - \frac{(e_{i1} - 0.5l_1)^2}{8} \right], \\
 \mu_{F_{i2}^{l_2}}(X_{i2}) &= \exp \left[-\frac{(x_{i1} - 0.5l_2)^2}{8} - \frac{(x_{i2} - 0.5l_2)^2}{8} - \frac{(k_{c_2} - 0.5l_2)^2}{8} - \frac{(e_{i2} - 0.5l_2)^2}{8} \right], \\
 \Psi_{i1}^{l_1}(X_{i1}) &= \frac{\mu_{F_{i1}^{l_1}}(X_{i1})}{\sum_{l_1=1}^{10} \mu_{F_{i1}^{l_1}}(X_{i1})}, \\
 \Psi_{i2}^{l_2}(X_{i2}) &= \frac{\mu_{F_{i2}^{l_2}}(X_{i2})}{\sum_{l_2=1}^{10} \mu_{F_{i2}^{l_2}}(X_{i2})}.
 \end{aligned} \tag{61}$$

The simulation results are shown in Figures 3–7, and $i = 1, 2, 3$. The trajectories of the desired signal g_{i1} and full state $x_{i1}(t)$ with time-varying symmetrical barrier functions $-k_{c_1}(t)$ and $k_{c_1}(t)$ are shown in Figure 4. The full state $x_{i2}(t)$ constraints are bounded by time-varying symmetric functions $-k_{c_2}(t)$ and $k_{c_2}(t)$ and track the desired signal g_{i2} in Figure 5.

From Figures 3–5, it is easy to indicate that the tracking trajectory always holds within the time-varying constraints

FIGURE 2: The communication topology for one leader and three followers.

$-k_{c_j}(t) < x_{ij}(t) < k_{c_j}(t)$, $j = 1, 2$, and converges uniformly to the tracking signal. Figure 6 shows the synchronization errors. e_{i1} and e_{i2} fluctuate infinitely and approach the origin. Figure 7 shows the norms of the adaptive laws that can show that the effect of control is better. From the simulation, it is obvious that the proposed adaptive fuzzy distributed cooperative tracking control protocol is effective.

FIGURE 3: The three agents follow the trajectory of the leader.

FIGURE 4: The trajectories of state x_{11} with the reference signal g_{11} in the time-varying constraint boundaries $-k_{c1}(t)$ and $k_{c1}(t)$.

FIGURE 5: The trajectories of state x_{12} with the reference signal g_{12} in the time-varying constraint boundaries $-k_{c2}(t)$ and $k_{c2}(t)$.

FIGURE 6: The trajectories of errors e_{11} and e_{12} with time-varying delays and constraints.

FIGURE 7: The boundedness of the adaptive laws.

5. Conclusion

An adaptive fuzzy cooperative tracking control has been presented for a class of uncertain nonlinear MASs with state time-varying delays and time-varying constraints. LKFs have been used to compensate for unknown time-varying delays and overcome the difficulty of employing the variable separation technique in the nonfeedback systems. To guarantee that time-varying delay states would not violate the time-varying bounds by constructing nonlinear coordinate transformations, ulteriorly, additional piecewise constructors can speed up convergence and avoid uncontrollability. The simulation results can be sufficiently demonstrated effective control. This paper further enriches the study of nonlinear state constrained systems. The simulation results provide theoretical support for further practical application.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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