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Research Article

Statistics and Calculation of Entropy of Dominating David Derived Networks

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A graph's entropy is a functional one, based on both the graph itself and the distribution of probability on its vertex set. In the theory of information, graph entropy has its origins. Dominating David derived networks have a variety of important applications in medication store, hardware, and system administration. In this study, we discuss dominating David derived network of type 1, 2, and 3 written as $D_1(n)$, $D_2(n)$, and $D_3(n)$, respectively of order n. We also compute some degree-based entropies such as Randić, ABC, and GA entropy of $D_1(n)$, $D_2(n)$, and $D_3(n)$.

1. Introduction and Preliminary Results

A graph G is a tuple $G = (V, E)$, where V is the set of vertices and E is the set of edges. A graph can be represented by a numerical quantity which is known as topological index. These indices have a vast number of application in various fields biology, computer science, information technology, and chemistry. Topological indices are used in QSAR/QSPR studies.

To comprehend the properties and data contained in the network example of graphs, there are a number of mathematical values, known as structure invariants, topological indices, or topological descriptors, which have been determined and concentrated in the course of recent many years. The topological indices have tremendous number of uses in the chemical graph which is the uncommon part of numerical science.

The combination of mathematics, information technology, and chemistry is a new division known as cheminformatics. It deals with QSAR and QSPR studies which predict the bio and physical chemical activities of compounds. The theory of topological indices was started by Wiener [17], when he was working on the boiling point of paraffins. The Wiener index is stated as

$$W(G) = \sum_{(u,v)\subseteq V(G)} d(u, v). \quad (1)$$

A number of problems that occur in discrete mathematics, statistics, biology, computer science, chemistry, information theory, etc., investigate the entropy of structures to deal with them. Shannon, in 1948, gave the concept of entropy [12]. The entropy of a graph G is defined as follows.

Let G be a graph and $V(G) = \{1, 2, \ldots, n\}$ be the vertex set of G. Let $P = (p_1, p_2, \ldots, p_n)$ be the probability density of $V(G)$ and $VP(G)$ be the vertex packing polytope of G. Then, entropy of G with respect to P is

$$H(G, P) = \min_{a\in VP(G)} \sum_{i=1}^{n} p_i \log\left(\frac{1}{a_i}\right). \quad (2)$$

Graph entropy has been utilized broadly to portray the structure of graph-based frameworks in numerical science

[6]. Trucco gave the idea of graph entropy in 1955 [15]. He said, the graph entropy is dependent on order of vertices.

The construction of dominating David derived networks is the same as the David derived networks. The David derived network is obtained from Star of David network SD (n) by splitting each edge of it into two by installing another vertex. In the same way, dominating David derived network, DDD (n), is obtained from honeycomb network HC (n). For the detailed construction, the reader can refer to [8, 9].

1.1. Degree-Based Topological Indices. The first degree-based topological index was presented by Milan Randic [11] and generalised by Bollabas and Erdos [2] and Amic et al. [1], in 1998:

$$R_\alpha(G) = \sum_{e=uv\in E(G)} (d_u \times d_v)^\alpha, \tag{3}$$

where $\alpha = 1, -1, (1/2), -(1/2)$.

ABC index was introduced in 1998, by Estrada et al. [7]. It has the formula

$$ABC(G) = \sum_{e=uv\in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}}. \tag{4}$$

Vukicevic and Furtula studied this index for the first time [16]. It is written as the GA index:

$$\mathrm{GA}(G) = \sum_{e=uv\in E(G)} \frac{2\sqrt{d_u \times d_v}}{(d_u + d_v)}. \tag{5}$$

1.2. Degree-Based Entropy of Graph. The entropy of a graph G is defined as

$$\mathrm{ENT}_\psi(G) = -\sum_{i=1}^{p} \frac{d(u_i)}{\sum_{j=1}^{p} d(u_j)} \log\left[\frac{d(u_i)}{\sum_{j=1}^{p} d(u_j)}\right], \tag{6}$$

where d_{u_i} is the degree or vertex u_i.

$$\begin{aligned}
\mathrm{ENT}_d(G) &= -\sum_{i=1}^{p} \frac{d(u_i)}{\sum_{j=1}^{p} d(u_j)}\left[\log d(u_i) - \log \sum_{j=1}^{p} d(u_j)\right],\\
\Longrightarrow \mathrm{ENT}_d(G) &= \log \sum_{j=1}^{p} d(u_j) - \frac{1}{\sum_{j=1}^{p} d(u_j)} \sum_{i=1}^{p} [d(u_i) \log d(u_i)],\\
\Longrightarrow \mathrm{ENT}_d(G) &= \log \sum_{j=1}^{p} d(u_j) - \frac{1}{\sum_{j=1}^{p} d(u_j)} \sum_{i=1}^{p} \left[\log d(u_i)^{d(u_i)}\right],
\end{aligned} \tag{7}$$

by using the Hand Shaking Lemma, and we have $\sum_{j=1}^{p} d(u_j) = 2q$. So,

$$\mathrm{ENT}_d(G) = \log(2q) - \frac{1}{2q} \sum_{i=1}^{p} \left[\log \ (d(u_i))^{d(u_i)}\right]. \tag{8}$$

1.3. Edge Weight-Based Entropy of Graph. The edge weight entropy of a graph G was introduced in 2014 by Chen et al. [5]:

$$\mathrm{ENT}_d(G) = -\sum_{u'v'\in E(G)} \frac{d(u'v')}{\sum_{uv\in E(G)} d(uv)} \log\left[\frac{d(u'v')}{\sum_{uv\in E(G)} d(uv)}\right]. \tag{9}$$

1.3.1. Randić Entropy. Using equation (3), equation (9) reduced to

$$\mathrm{ENT}_{R_\alpha}(G) = \log(R_\alpha) - \frac{1}{R_\alpha} \sum_{i=1}^{m} \sum_{uv\in E_i(G)} \left[(d(u) \times d(v))^\alpha\right]^{(d(u)\times d(v))^\alpha}. \tag{10}$$

1.3.2. Atom Bond Connectivity Entropy. Using equation (4), equation (9) reduced to

$$\mathrm{ENT}_{ABC}(G)=\log(ABC)-\frac{1}{ABC}\sum_{i=1}^{m}\sum_{uv\in E_i(G)}\log\left[\sqrt{\frac{d(u)+d(v)-2}{d(u)\times d(v)}}\right]^{\sqrt{(d(u)+d(v)-2)/(d(u)\times d(v))}}. \tag{11}$$

1.3.3. The Geometric Arithmetic Entropy. Using equation (5), equation (9) reduced to

$$\mathrm{ENT}_{\mathrm{GA}}(G)=\log(\mathrm{GA})-\frac{1}{\mathrm{GA}}\sum_{i=1}^{m}\sum_{uv\in E_i(G)}\log\left[\frac{2\sqrt{d(u)\times d(v)}}{d(u)+d(v)}\right]^{(2\sqrt{d(u)\times d(v)})/(d(u)+d(v))}. \tag{12}$$

2. Main Results

Simonraj and George [13] computed the metric dimension of David network and Imran et al. [8] computed the degree-based topological indices of dominating David derived networks; also, Song et al. [14] computed the entropy-based indices of Hex derived networks. Here, we discuss the dominating David derived networks in this work and calculate the exact results for entropies based on edges.

2.1. Results on Dominating David Derived Network of Type 1. Here, we calculate certain degree-based entropies of dominating David derived network of type 1. $D_1(n)$ is shown in Figure 1. The edge partition of $D_1(n)$ is shown in Table 1. We compute Randic entropy, ABC entropy, and GA entropy for $D_1(n)$.

2.2. Randic Entropy of $D_1(n)$. If $H\cong D_1(n)$, then, by using Table 1 and equation (3), we have

$$\begin{aligned}R_\alpha(H)&=4n\times(4)^\alpha+(4n-4)\times(6)^\alpha+(28n-16)\times(8)^\alpha+\left(9n^2-13n+5\right)\times(9)^\alpha\\&\quad+\left(36n^2-56n+24\right)\times(12)^\alpha+\left(36n^2-52n+20\right)\times(16)^\alpha,\end{aligned} \tag{13}$$

for $\alpha=1$. Randic index is

$$\Longrightarrow R_1(H)=1089n^2-1357n+501. \tag{14}$$

For $\alpha=-1$,

$$\Longrightarrow R_{-1}(H)=\frac{25}{4}n^2-\frac{151}{36}n+\frac{41}{46}. \tag{15}$$

For $\alpha=(1/2)$,

$$\Longrightarrow R_{(1/2)}(H)=295.707658n^2-343.995772n+123.085646. \tag{16}$$

For $\alpha=-(1/2)$,

$$\Longrightarrow R_{-(1/2)}(H)=22.392305n^2-19.966653n+6.305022. \tag{17}$$

Using equation (10) and Table 1, the Randic entropy is

$$\begin{aligned}\mathrm{ENT}_{(R_\alpha)}(H)&=\log(R_\alpha)-\frac{1}{R_\alpha}\Big[4n\times\log\left(4^\alpha\right)^{4^\alpha}+(4n-4)\times\log\left(6^\alpha\right)^{6^\alpha}\\&\quad+(28n-16)\times\log\left(8^\alpha\right)^{8^\alpha}+\left(9n^2-13n+5\right)\times\log\left(9^\alpha\right)^{9^\alpha}\\&\quad+\left(36n^2-56n+24\right)\times\log\left(12^\alpha\right)^{12^\alpha}\\&\quad+\left(36n^2-52n+20\right)\times\log\left(16^\alpha\right)^{16^\alpha}\Big].\end{aligned} \tag{18}$$

For $\alpha=1$,

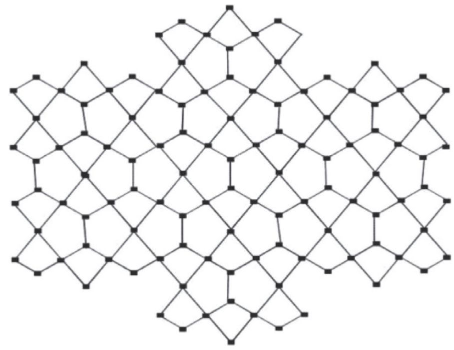

Figure 1: Dominating David derived network of type 1 $(D_1(2))$.

Table 1: Edge partition of $D_1(n)$.

$(d(u), d(v))$, where $uv \in E(D_1(n))$	Cardinality of edges
$(2, 2)$	$4n$
$(2, 3)$	$4n - 4$
$(2, 4)$	$28n - 16$
$(3, 3)$	$9n^2 - 13n + 5$
$(3, 4)$	$36n^2 - 56n + 24$
$(4, 4)$	$36n^2 - 52n + 20$

$$\begin{aligned} \mathrm{ENT}_{(R_1)}(H) &= \log(R_1) - \frac{1}{R_1}\Big[4n \times \log\ (4)^4 + (4n-4) \times \log\ (6)^6 \\ &\quad + (28n-16) \times \log\ (8)^8 + \left(9n^2 - 13n + 5\right) \times \log\ (9)^9 \\ &\quad + \left(36n^2 - 56n + 24\right) \times \log\ (12)^{12} + \left(36n^2 - 52n + 20\right) \times \log\ (16)^{16}\Big], \\ \Longrightarrow \mathrm{ENT}_{(R_1)}(H) &= \log(R_1) - \frac{1}{R_1}\left(1237.073052n^2 - 1608.08325n + 604.792358\right). \end{aligned} \tag{19}$$

For $\alpha = (1/2)$,

$$\begin{aligned} \mathrm{ENT}_{(R_{(1/2)})}(H) &= \log\left(R_{(1/2)}\right) - \frac{1}{R_{(1/2)}}\Big[4n \times \log\ (\sqrt{4})^{\sqrt{4}} + (4n-4) \times \log\ (\sqrt{6})^{\sqrt{6}} \\ &\quad + (28n-16) \times \log\ (\sqrt{8})^{\sqrt{8}} + \left(9n^2 - 13n + 5\right) \times \log\ (\sqrt{9})^{\sqrt{9}} \\ &\quad + \left(36n^2 - 56n + 24\right) \times \log\ (\sqrt{12})^{\sqrt{12}} \\ &\quad + \left(36n^2 - 52n + 20\right) \times \log\ (\sqrt{16})^{\sqrt{16}}\Big], \\ \Longrightarrow \mathrm{ENT}_{(R_{(1/2)})}(H) &= \log\left(R_{(1/2)}\right) - \frac{1}{R_{(1/2)}}\left(166.869996n^2 - 206.530299n + 75.935599\right). \end{aligned} \tag{20}$$

For $\alpha = -1$,

$$
\begin{aligned}
\mathrm{ENT}_{\left(R_{-1}\right)}(H) = \log\left(R_{-1}\right) - \frac{1}{R_{-1}}\Bigg[& 4n \times \log\left(\frac{1}{4}\right)^{(1/4)} + (4n-4) \times \log\left(\frac{1}{6}\right)^{(1/6)} \\
&+ (28n-16) \times \log\left(\frac{1}{8}\right)^{(1/8)} + \left(9n^2 - 13n + 5\right) \times \log\left(\frac{1}{9}\right)^{(1/9)} \\
&+ \left(36n^2 - 56n + 24\right) \times \log\left(\frac{1}{12}\right)^{(1/12)} \\
&+ \left(36n^2 - 52n + 20\right) \times \log\left(\frac{1}{16}\right)^{(1/16)}\Bigg], \\
\Longrightarrow \mathrm{ENT}_{\left(R_{-1}\right)}(H) = \log\left(R_{-1}\right) - \frac{1}{R_{-1}}&\left(-6.901056n^2 + 6.046277n - 1.8687\right).
\end{aligned}
\tag{21}
$$

For $\alpha = -(1/2)$,

$$
\begin{aligned}
\mathrm{ENT}_{\left(R_{-(1/2)}\right)}(H) = \log\left(R_{-(1/2)}\right) - \frac{1}{R_{-(1/2)}}\Bigg[& 4n \times \log\left(\frac{1}{\sqrt{4}}\right)^{(1/\sqrt{4})} + (4n-4) \times \log\left(\frac{1}{\sqrt{6}}\right)^{(1/\sqrt{6})} \\
&+ (28n-16) \times \log\left(\frac{1}{\sqrt{8}}\right)^{(1/\sqrt{8})} + \left(9n^2 - 13n + 5\right) \times \log\left(\frac{1}{\sqrt{9}}\right)^{(1/\sqrt{9})} \\
&+ \left(36n^2 - 56n + 24\right) \times \log\left(\frac{1}{\sqrt{12}}\right)^{(1/\sqrt{12})} \\
&+ \left(36n^2 - 52n + 20\right) \times \log\left(\frac{1}{\sqrt{16}}\right)^{(1/\sqrt{16})}\Bigg], \\
\Longrightarrow \mathrm{ENT}_{\left(R_{-(1/2)}\right)}(H) = \log\left(R_{-(1/2)}\right) - \frac{1}{R_{-(1/2)}}&\left(-12.457494n^2 + 12.909738n - 4.354213\right),
\end{aligned}
\tag{22}
$$

where R_α for $\alpha = 1, -1, (1/2), -(1/2)$ is written in equations (14)–(17), respectively.

2.3. *The Atom Bond Entropy of $D_1(n)$.*

If $H \cong D_1(n)$, then, by using equation (4) and Table 1, the ABC index is

$$
\begin{aligned}
ABC(H) = (4n) \times \sqrt{\frac{2+2-2}{2\times 2}} &+ (4n-4) \times \sqrt{\frac{2+3-2}{2\times 3}} \\
&+ (28n-16) \times \sqrt{\frac{2+4-2}{2\times 4}} + \left(9n^2 - 13n + 5\right) \times \sqrt{\frac{3+3-2}{3\times 3}} \\
&+ \left(36n^2 - 56n + 24\right) \times \sqrt{\frac{3+4-2}{3\times 4}} + \left(36n^2 - 52n + 20\right) \times \sqrt{\frac{4+4-2}{4\times 4}}, \\
\Longrightarrow ABC(H) = 51.283308n^2 &- 51.202034n + 16.93058.
\end{aligned}
\tag{23}
$$

Using equation (11) and Table 1, the ABC entropy is

$$\begin{aligned}
\mathrm{ENT}_{ABC}(H) &= \log(ABC) - \frac{1}{ABC}\left[(4n)\times\log\left(\sqrt{\frac{2+2-2}{2\times 2}}\right)^{\sqrt{(2+2-2)/(2\times 2)}}\right.\\
&+ (4n-4)\times\log\left(\sqrt{\frac{2+3-2}{2\times 3}}\right)^{\sqrt{(2+3-2)/(2\times 3)}}\\
&+ (28n-16)\times\log\left(\sqrt{\frac{2+4-2}{2\times 4}}\right)^{\sqrt{(2+4-2)/(2\times 4)}}\\
&+ \left(9n^2-13n+5\right)\times\log\left(\sqrt{\frac{3+3-2}{3\times 3}}\right)^{\sqrt{(3+3-2)/(3\times 3)}}\\
&+ \left(36n^2-56n+24\right)\times\log\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{\sqrt{\frac{3+4-2}{3\times 4}}}\\
&\left.+ \left(36n^2-52n+20\right)\times\log\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{\sqrt{(4+4-2)/4\times 4}}\right],\\
\Longrightarrow \mathrm{ENT}_{ABC}(H) &= \log(ABC) - \frac{1}{ABC}\left(-10.16953n^2 + 11.348686n - 4.011986\right),
\end{aligned} \tag{24}$$

where ABC index of $D_1(n)$ is written in equation (23).

2.4. The Geometric Arithmetic Entropy of $D_1(n)$. If $H \cong D_1(n)$, then, by using equation (5) and Table 1, the GA index is

$$\begin{aligned}
\mathrm{GA}(H) &= (4n)\times\frac{2\sqrt{2\times 2}}{2+2} + (4n-4)\times\frac{2\sqrt{2\times 3}}{2+3}\\
&+ (28n-16)\times\frac{2\sqrt{2\times 4}}{2+4} + \left(9n^2-13n+5\right)\times\frac{2\sqrt{3\times 3}}{3+3}\\
&+ \left(36n^2-56n+24\right)\times\frac{2\sqrt{3\times 4}}{3+4} + \left(36n^2-52n+20\right)\times\frac{2\sqrt{4\times 4}}{4+4},\\
\Longrightarrow \mathrm{GA}(H) &= 80.630759n^2 - 86.107789n + 29.749711.
\end{aligned} \tag{25}$$

Using equation (12) and Table 1, we have

$$\begin{aligned}\mathrm{ENT}_{\mathrm{GA}}(H) = \log(\mathrm{GA}) - \frac{1}{\mathrm{GA}}\Bigg[&(4n)\times\log\left(\frac{2\sqrt{2\times 2}}{2+2}\right)^{(2\sqrt{2\times 2})/(2+2)}\\
&+(4n-4)\times\log\left(\frac{2\sqrt{2\times 3}}{2+3}\right)^{(2\sqrt{2\times 3})/(2+3)} + (28n-16)\times\log\left(\frac{2\sqrt{2\times 4}}{2+4}\right)^{(2\sqrt{2\times 4})/(2+4)}\\
&+\left(9n^2-13n+5\right)\times\log\left(\frac{2\sqrt{3\times 3}}{3+3}\right)^{(2\sqrt{3\times 3})/(3+3)}\\
&+\left(36n^2-56n+24\right)\times\log\left(\frac{2\sqrt{3\times 4}}{3+4}\right)^{(2\sqrt{3\times 4})/(3+4)}\\
&+\left(36n^2-52n+20\right)\times\log\left(\frac{2\sqrt{4\times 4}}{4+4}\right)^{(2\sqrt{4\times 4})/(4+4)}\Bigg],\\
\Longrightarrow \mathrm{ENT}_{\mathrm{GA}}(H) = \log(\mathrm{GA}) - \frac{1}{\mathrm{GA}}&\left(-0.159534n^2-0.461756n+0.314202\right),\end{aligned} \tag{26}$$

where GA index of $D_1(n)$ is written in equation (25).

2.5. Results on Dominating David Derived Network of Type 2. Here, we calculate certain degree-based entropies of Dominating David Derived network of type 2. The $D_2(n)$ shown in Figure 2, and edge partition is shown in Table 2. We compute Randic entropy, *ABC* entropy and GA entropy for $D_2(n)$.

2.6. Randic Entropy of $D_2(n)$. If $H \cong D_2(n)$, then, by using equation (3) and Table 2, we have

$$\begin{aligned}R_\alpha(H) = (4n)\times(4)^\alpha + \left(18n^2-22n+6\right)\times(6)^\alpha + (28n-16)\times(8)^\alpha\\
+\left(36n^2-56n+24\right)\times(12)^\alpha + \left(36n^2-52n+20\right)\times(16)^\alpha.\end{aligned} \tag{27}$$

For $\alpha = 1$,

$$\Longrightarrow R_1(H) = 1116n^2 - 1396n + 516. \tag{28}$$

For $\alpha = -1$,

$$\Longrightarrow R_{-1}(H) = \frac{33}{4}n^2 - \frac{85}{12}n + \frac{9}{4}. \tag{29}$$

For $\alpha = (1/2)$,

$$\Longrightarrow R_{(1/2)}(H) = 312.798474n^2 - 368.682505n + 132.580543. \tag{30}$$

For $\alpha = -(1/2)$,

$$\Longrightarrow R_{-(1/2)}(H) = 26.740774n^2 - 26.247775n + 8.720839. \tag{31}$$

Using equation (10) and Table 2, we have

$$\begin{aligned}\mathrm{ENT}_{(R_\alpha)}(H) = \log(R_\alpha) - \frac{1}{R_\alpha}\Big[&(4n)\times\log\left(4^\alpha\right)^{4^\alpha} + \left(18n^2-22n+6\right)\times\log\left(6^\alpha\right)^{6^\alpha}\\
&+(28n-16)\times\log\left(8^\alpha\right)^{8^\alpha} + \left(36n^2-56n+24\right)\times\log\left(12^\alpha\right)^{12^\alpha}\\
&+\left(36n^2-52n+20\right)\times\log\left(16^\alpha\right)^{16^\alpha}\Big].\end{aligned} \tag{32}$$

For $\alpha = 1$,

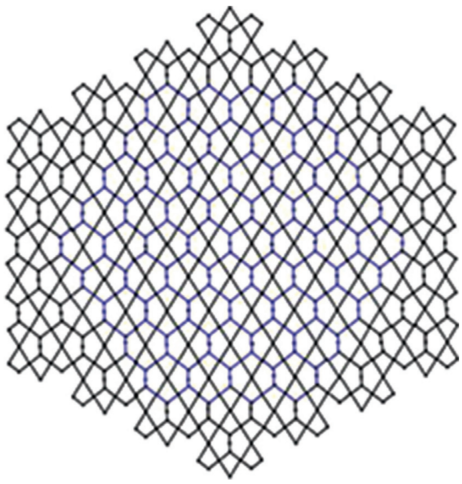

Figure 2: Dominating david derived network of type 2 $D_2(4)$.

Table 2: Edge partition of $D_2(n)$.

$d(u), d(v)$ where $uv \in E(D_2(n))$	Cardinality of edges
(2, 2)	$4n$
(2, 3)	$18n^2 - 22n + 6$
(2, 4)	$28n - 16$
(3, 4)	$36n^2 - 56n + 24$
(4, 4)	$36n^2 - 52n + 20$

$$\begin{aligned} \mathrm{ENT}_{(R_1)}(H) &= \log(R_1) - \frac{1}{R_1}\Big[(4n)\times\log\ (4)^4 + \left(18n^2 - 22n + 6\right)\times\log\ (6)^6 \\ &\quad + (28n-16)\times\log\ (8)^8 + \left(36n^2 - 56n + 24\right)\times\log\ (12)^{12} \\ &\quad + \left(36n^2 - 52n + 20\right)\times\log\ (16)^{16}\Big], \\ \Longrightarrow \mathrm{ENT}_{(R_1)}(H) &= \log(R_1) - \frac{1}{R_1}\left(1243.819743n^2 - 1617.828471n + 608.54052\right). \end{aligned} \tag{33}$$

For $\alpha = (1/2)$,

$$\begin{aligned} \mathrm{ENT}_{(R_{(1/2)})}(H) &= \log\left(R_{(1/2)}\right) - \frac{1}{R_{(1/2)}}\Big[(4n)\times\log\ (\sqrt{4})^{\sqrt{4}} + \left(18n^2 - 22n + 6\right)\times\log\ (\sqrt{6})^{\sqrt{6}} \\ &\quad + (28n-16)\times\log\ (\sqrt{8})^{\sqrt{8}} + \left(36n^2 - 56n + 24\right)\times\log\ (\sqrt{12})^{\sqrt{12}} \\ &\quad + \left(36n^2 - 52n + 20\right)\times\ \log\ (\sqrt{16})^{\sqrt{16}}\Big], \\ \Longrightarrow \mathrm{ENT}_{(R_{(1/2)})}(H) &= \log\left(R_{(1/2)}\right) - \frac{1}{R_{(1/2)}}\left(171.142383n^2 - 212.701526n + 78.309148\right). \end{aligned} \tag{34}$$

For $\alpha = -1$,

$$\begin{aligned} \mathrm{ENT}_{(R_{-1})}(H) &= \log(R_{-1}) - \frac{1}{R_{-1}}\left[(4n)\times\log\left(\frac{1}{4}\right)^{(1/4)} + \left(18n^2 - 22n + 6\right)\times\log\left(\frac{1}{6}\right)^{(1/6)}\right. \\ &\quad + (28n-16)\times\log\left(\frac{1}{8}\right)^{(1/8)} + \left(36n^2 - 56n + 24\right)\times\log\left(\frac{1}{12}\right)^{(1/12)} \\ &\quad \left. + \left(36n^2 - 52n + 20\right)\times\log\left(\frac{1}{16}\right)^{(1/16)}\right], \\ \Longrightarrow \mathrm{ENT}_{(R_{-1})}(H) &= \log(R_{-1}) - \frac{1}{R_{-1}}\left(-8.281267n^2 + 8.039915n - 2.635484\right). \end{aligned} \tag{35}$$

For $\alpha = -(1/2)$,

$$\begin{aligned}
\mathrm{ENT}_{\left(R_{-(1/2)}\right)}(H) = \log\left(R_{-(1/2)}\right) - \frac{1}{R_{-(1/2)}}\Bigg[&(4n)\times\log\left(\frac{1}{\sqrt{4}}\right)^{(1/\sqrt{4})}\\
&+\left(18n^2-22n+6\right)\times\log\left(\frac{1}{\sqrt{6}}\right)^{(1/\sqrt{6})}+(28n-16)\times\log\left(\frac{1}{\sqrt{8}}\right)^{(1/\sqrt{8})}\\
&+\left(36n^2-56n+24\right)\times\log\left(\frac{1}{\sqrt{12}}\right)^{(1/\sqrt{12})}\\
&+\left(36n^2-52n+20\right)\times\log\left(\frac{1}{\sqrt{16}}\right)^{(1/\sqrt{16})}\Bigg],\\
&\Longrightarrow \mathrm{ENT}_{\left(R_{-(1/2)}\right)}(H) = \log\left(R_{-(1/2)}\right) - \frac{1}{R_{-(1/2)}}\left(-13.88524n^2+14.972039n-5.147406\right),
\end{aligned} \tag{36}$$

where R_α for $\alpha = 1, -1, (1/2), -(1/2)$ is written in equations (28)–(31), respectively.

2.7. The Atom Bond Entropy of $D_2(n)$. If $H \cong D_2(n)$, then, by using equation (4) and Table 2, we have

$$\begin{aligned}
ABC(H) = (4n)\times\sqrt{\frac{2+2-2}{2\times 2}} &+ \left(18n^2-22n+6\right)\times\sqrt{\frac{2+3-2}{2\times 3}}\\
&+(28n-16)\times\sqrt{\frac{2+4-2}{2\times 4}}+\left(36n^2-56n+24\right)\times\sqrt{\frac{3+4-2}{3\times 4}}\\
&+\left(36n^2-52n+20\right)\times\sqrt{\frac{4+4-2}{4\times 4}},\\
&\Longrightarrow ABC(H) = 58.01123n^2-60.920143n+20.668314.
\end{aligned} \tag{37}$$

Using equation (11) and Table 2, we have

$$\begin{aligned}
\mathrm{ENT}_{ABC}(H) = \log(ABC) - \frac{1}{ABC}\Bigg[&(4n)\times\log\left(\sqrt{\frac{2+2-2}{2\times 2}}\right)^{\sqrt{(2+2-2)/(2\times 2)}}\\
&+\left(18n^2-22n+6\right)\times\log\left(\sqrt{\frac{2+3-2}{2\times 3}}\right)^{\sqrt{(2+3-2)/(2\times 3)}}\\
&+(28n-16)\times\log\left(\sqrt{\frac{2+4-2}{2\times 4}}\right)^{\sqrt{((2+4-2))/(2\times 4)}}\\
&+\left(36n^2-56n+24\right)\times\log\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{\sqrt{(3+4-2)/(3\times 4)}}
\end{aligned}$$

$$+\left(36n^2-52n+20\right)\times\log\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{\sqrt{(4+4-2)/(4\times 4)}}\Bigg],$$
$$\Longrightarrow \mathrm{ENT}_{ABC}(H)=\log(ABC)-\frac{1}{ABC}\left[-11.028726n^2+12.589746n-4.489317\right], \tag{38}$$

where ABC index of $D_2(n)$ is written in equation (37).

2.8. The Geometric Arithmetic Entropy of $D_2(n)$. If $H \cong D_2(n)$, then, by using equation (5) and Table 2, we have

$$\begin{aligned}\mathrm{GA}(H)&=(4n)\times\frac{2\sqrt{2\times 2}}{2+2}+\left(18n^2-22n+6\right)\times\frac{2\sqrt{2\times 3}}{2+3}\\&\quad+(28n-16)\times\frac{2\sqrt{2\times 4}}{2+4}+\left(36n^2-56n+24\right)\times\frac{2\sqrt{3\times 4}}{3+4}\\&\quad+\left(36n^2-52n+20\right)\times\frac{2\sqrt{4\times 4}}{4+4},\\&\Longrightarrow \mathrm{GA}(H)=89.267086n^2-98.582482n+34.54767.\end{aligned} \tag{39}$$

Using equation (12) and Table 2, we have

$$\begin{aligned}\mathrm{ENT}_{\mathrm{GA}}(H)&=\log(\mathrm{GA})-\frac{1}{\mathrm{GA}}\Bigg[(4n)\times\log\left(\frac{2\sqrt{2\times 2}}{2+2}\right)^{(2\sqrt{2\times 2})/(2+2)}\\&\quad+\left(18n^2-22n+6\right)\times\log\left(\frac{2\sqrt{2\times 3}}{2+3}\right)^{(2\sqrt{2\times 3})/(2+3)}\\&\quad+(28n-16)\times\log\left(\frac{2\sqrt{2\times 4}}{2+4}\right)^{(2\sqrt{2\times 4})/(2+4)}\\&\quad+\left(36n^2-56n+24\right)\times\log\left(\frac{2\sqrt{3\times 4}}{3+4}\right)^{(2\sqrt{3\times 4})/(3+4)}\\&\quad+\left(36n^2-52n+20\right)\times\log\left(\frac{2\sqrt{4\times 4}}{4+4}\right)^{(2\sqrt{4\times 4})/(4+4)}\Bigg],\\\mathrm{ENT}_{\mathrm{GA}}(H)&=\log(\mathrm{GA})-\frac{1}{\mathrm{GA}}\left(-0.315869n^2-0.235939n+0.227349\right),\end{aligned} \tag{40}$$

where the GA index of $D_2(n)$ is written in equation (39).

2.9. Results on Dominating David Derived Network of Type 3. Here, we calculate certain degree-based entropies of dominating David derived network of type 3. $D_3(n)$ is shown in Figure 3, and edge partition is shown in Table 3. We compute Randic entropy, ABC entropy, and GA entropy for $D_3(n)$.

2.10. Randic Entropy of $D_3(n)$. If $H \cong D3(n)$, then, by using equation (3) and Table 3, we have

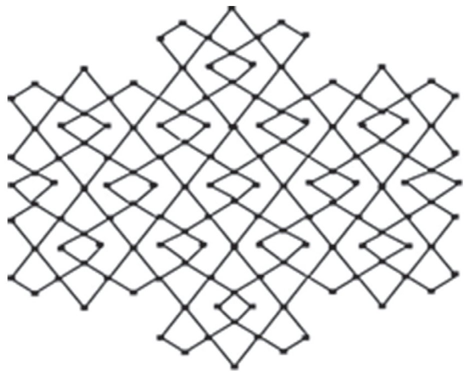

FIGURE 3: Dominating David derived network of type 3 $D_3(2)$.

TABLE 3: Edge partition of $D_3(n)$.

$d(u), d(v)$, where $uv \in E(D3(n))$	Cardinality of edges
(2, 2)	$4n$
(2, 4)	$36n^2 - 20n$
(4, 4)	$72n^2 - 108n + 44$

$$R_\alpha(H) = (4n) \times (4)^\alpha + \left(36n^2 - 20n\right) \times (8)^\alpha + \left(72n^2 - 108n + 44\right) \times (16)^\alpha. \tag{41}$$

For $\alpha = 1$,

$$\Longrightarrow R_1(H) = 1440n^2 - 1872n + 704. \tag{42}$$

For $\alpha = -1$,

$$\Longrightarrow R_{-1}(H) = 9n^2 - \frac{33}{4}n + \frac{11}{4}. \tag{43}$$

For $\alpha = (1/2)$,

$$\Longrightarrow R_{(1/2)}(H) = 389.823376n^2 - 480.568542n + 176. \tag{44}$$

For $\alpha = -(1/2)$,

$$\Longrightarrow R_{-(1/2)}(H) = 30.727922n^2 - 32.071068n + 11. \tag{45}$$

Using equation (10) and Table 2, we have

$$\begin{aligned} \mathrm{ENT}_{(R_\alpha)}(H) = \log(R_\alpha) - \frac{1}{R_\alpha}\Big[&(4n) \times \log\left(4^\alpha\right)^{4^\alpha} + \left(36n^2 - 20n\right) \times \log\left(8^\alpha\right)^{8^\alpha} \\ &+ \left(72n^2 - 108n + 44\right) \times \log\left(16^\alpha\right)^{16^\alpha}\Big]. \end{aligned} \tag{46}$$

For $\alpha = 1$,

$$\begin{aligned} \mathrm{ENT}_{(R_1)}(H) &= \log(R_1) - \frac{1}{R_1}\Big[(4n) \times \log(4)^4 + \left(36n^2 - 20n\right) \times \log(8)^8 \\ &\quad + \left(72n^2 - 108n + 44\right) \times \log(16)^{16}\Big], \\ \Longrightarrow \mathrm{ENT}_{(R_1)}(H) &= \log(R_1) - \frac{1}{R_1}\left(1647.236136n^2 - 2215.580768n + 847.700468\right). \end{aligned} \tag{47}$$

For $\alpha = (1/2)$,

$$\begin{aligned} \mathrm{ENT}_{(R_{(1/2)})}(H) &= \log\left(R_{(1/2)}\right) - \frac{1}{R_{(1/2)}}\Big[(4n) \times \log\left(\sqrt{4}\right)^{\sqrt{4}} + \left(36n^2 - 20n\right) \times \log\left(\sqrt{8}\right)^{\sqrt{8}} \\ &\quad + \left(72n^2 - 108n + 44\right) \times \log\left(\sqrt{16}\right)^{\sqrt{16}}\Big], \\ \Longrightarrow \mathrm{ENT}_{(R_{(1/2)})}(H) &= \log\left(R_{(1/2)}\right) - \frac{1}{R_{(1/2)}}\left(219.371113n^2 - 283.224918n + 105.962558\right). \end{aligned} \tag{48}$$

For $\alpha = -1$,

$$\begin{aligned}\mathrm{ENT}_{(R_{-1})}(H) &= \log(R_{-1}) - \frac{1}{R_{-1}}\left[(4n)\times\log\left(\frac{1}{4}\right)^{(1/4)} + \left(36n^2-20n\right)\times\log\left(\frac{1}{8}\right)^{(1/8)}\right. \\ &\quad \left. + \left(72n^2-108n+44\right)\times\ \log\left(\frac{1}{16}\right)^{(1/16)}\right], \\ \Longrightarrow \mathrm{ENT}_{(R_{-1})}(H) &= \log(R_{-1}) - \frac{1}{R_{-1}}\left(-9.482445n^2 + 9.783475n - 3.31133\right).\end{aligned} \tag{49}$$

For $\alpha = -(1/2)$,

$$\begin{aligned}ENT_{(R_{-(1/2)})}(H) &= \log\left(R_{-(1/2)}\right) - \frac{1}{R_{-(1/2)}}\left[(4n)\times\ \log\left(\frac{1}{\sqrt{4}}\right)^{1/\sqrt{4}} + \left(36n^2-20n\right)\times\ \log\left(\frac{1}{\sqrt{8}}\right)^{1/\sqrt{8}}\right. \\ &\quad \left. + \left(72n^2-108n+44\right)\times\ \log\left(\frac{1}{\sqrt{16}}\right)^{1/\sqrt{16}}\right], \\ \Longrightarrow ENT_{(R_{-(1/2)})}(H) &= \log\left(R_{-(1/2)}\right) - \frac{1}{R_{-(1/2)}}\left(-16.584309n^2 + 18.846465n - 6.62266\right),\end{aligned} \tag{50}$$

where R_α for $\alpha = 1, -1, (1/2), -(1/2)$ is written in equations (42)–(45), respectively.

2.11. The Atom Bond Entropy of $D_3(n)$. If $H \cong D_2(n)$, then, by using equation (4) and Table 3, we have

$$\begin{aligned}ABC(H) &= (4n)\times\sqrt{\frac{2+2-2}{2\times 2}} + \left(36n^2-20n\right)\times\sqrt{\frac{2+4-2}{2\times 4}} \\ &\quad + \left(72n^2-108n+44\right)\times\sqrt{\frac{4+4-2}{4\times 4}}, \\ \Longrightarrow ABC(H) &= 69.546659n^2 - 77.449932n + 26.944387.\end{aligned} \tag{51}$$

Using equation (11) and Table 3, we have

$$\begin{aligned}\mathrm{ENT}_{ABC}(H) &= \log(ABC) - \frac{1}{ABC}\left[(4n)\times\ \log\left(\sqrt{\frac{2+2-2}{2\times 2}}\right)^{\sqrt{(2+2-2)/(2\times 2)}}\right. \\ &\quad + \left(36n^2-20n\right)\times\ \log\left(\sqrt{\frac{2+4-2}{2\times 4}}\right)^{\sqrt{(2+4-2)/(2\times 4)}} \\ &\quad \left. + \left(72n^2-108n+44\right)\times\ \log\left(\sqrt{\frac{4+4-2}{4\times 4}}\right)^{\sqrt{(4+4-2)/(4\times 4)}}\right], \\ \Longrightarrow \mathrm{ENT}_{ABC}(H) &= \log(ABC) - \frac{1}{ABC}\left[-13.222141n^2 + 15.788864n - 5.738733\right],\end{aligned} \tag{52}$$

Table 4: Comparison table of entropies for $D_1(n)$.

n	ENT_{R_1}	$ENT_{R_{(1/2)}}$	$ENT_{R_{-1}}$	$ENT_{R_{-(1/2)}}$	ENT_{ABC}	ENT_{GA}
1	1.3640	1.3889	1.3935	1.3879	1.3973	1.3977
2	2.2405	2.2563	2.2361	2.2547	2.2617	2.2623
3	2.6849	2.6969	2.6777	2.6956	2.7009	2.7015
4	2.9793	2.9895	2.9723	2.9884	2.9929	2.9934
5	3.1992	3.2084	3.1928	3.2075	3.2115	3.2118
6	3.3747	3.3833	3.3689	3.3824	3.3861	3.3865
7	3.5207	3.5288	3.5154	3.5281	3.5315	3.5318
8	3.6458	3.6535	3.6408	3.6528	3.6560	3.6564
9	3.7550	3.7625	3.7505	3.7619	3.7649	3.7653
10	3.8521	3.8594	3.8478	3.8588	3.8617	3.8620

Table 5: Comparison table of entropies for $D_2(n)$.

n	ENT_{R_1}	$ENT_{R_{(1/2)}}$	$ENT_{R_{-1}}$	$ENT_{R_{-(1/2)}}$	ENT_{ABC}	ENT_{GA}
1	1.3791	1.4056	1.3756	1.4051	1.4143	1.4148
2	2.2668	2.2884	2.2574	2.2872	2.2958	2.2966
3	2.7136	2.7328	2.7025	2.7314	2.7394	2.7403
4	3.0090	3.0271	2.9972	3.0255	3.0335	3.0342
5	3.2295	3.2469	3.2174	3.2453	3.2531	3.2537
6	3.4053	3.4223	3.3930	3.4207	3.4284	3.4291
7	3.5516	3.5683	3.5392	3.5666	3.5743	3.5749
8	3.6767	3.6932	3.6643	3.6916	3.6992	3.6998
9	3.7862	3.8025	3.7736	3.8008	3.8083	3.8089
10	3.8834	3.8995	3.8707	3.8978	3.9053	3.9059

where the ABC index of $D_3(n)$ is written in equation (51).

2.12. The Geometric Arithmetic Entropy of $D_3(n)$. If $H \cong D_3(n)$, then, by using equation (5) and Table 3, we have

$$\begin{aligned} GA(H) &= (4n) \times \frac{2\sqrt{2\times 2}}{2+2} + \left(36n^2 - 20n\right) \times \frac{2\sqrt{2\times 4}}{2+4} + \left(72n^2 - 108n + 44\right) \times \frac{2\sqrt{4\times 4}}{4+4}, \\ \Longrightarrow GA(H) &= 105.941125n^2 - 122.856181n + 44. \end{aligned} \tag{53}$$

Using equation (12) and Table 2, we have

$$\begin{aligned} ENT_{GA}(H) = \log(GA) - \frac{1}{GA}\Bigg[&(4n) \times \log\left(\frac{2\sqrt{2\times 2}}{2+2}\right)^{(2\sqrt{2\times 2})/(2+2)} \\ &+ \left(36n^2 - 20n\right) \times \log\left(\frac{2\sqrt{2\times 4}}{2+4}\right)^{(2\sqrt{2\times 4})/(2+4)} \\ &+ \left(72n^2 - 108n + 44\right) \times \log\left(\frac{2\sqrt{4\times 4}}{4+4}\right)^{(2\sqrt{4\times 4})/(4+4)}\Bigg], \\ ENT_{GA}(H) = \log(GA) - \frac{1}{GA}&\left(-0.868087n^2 + 0.482271n\right), \end{aligned} \tag{54}$$

where the GA index of $D_3(n)$ is written in equation (53).

Table 6: Comparison table of entropies for $D_3(n)$.

n	ENT_{R_1}	$\mathrm{ENT}_{R_{(1/2)}}$	$\mathrm{ENT}_{R_{-1}}$	$\mathrm{ENT}_{R_{-(1/2)}}$	ENT_{ABC}	$\mathrm{ENT}_{\mathrm{GA}}$
1	1.4075	1.4368	1.4042	1.4364	1.4463	1.4469
2	2.3296	2.3502	2.3215	2.3491	2.3568	2.3577
3	2.7842	2.8020	2.7759	2.8010	2.8077	2.8087
4	3.0827	3.0994	3.0748	3.0984	3.1047	3.1057
5	3.3049	3.3209	3.2972	3.3199	3.3261	3.3269
6	3.4819	3.4975	3.4744	3.4965	3.5024	3.5033
7	3.6289	3.6442	3.6215	3.6432	3.6490	3.6499
8	3.7546	3.7697	3.7473	3.7687	3.7745	3.7754
9	3.8645	3.8794	3.8572	3.8784	3.8841	3.8849
10	3.9619	3.9767	3.9547	3.9758	3.9814	3.9823

3. Discussion

Since entropy plays a vital role in various fields of science such as software engineering, medication, and pharmaceutical, its numerical values and graphical representation is very much important for researchers. Here, we calculate some exact values of degree-based entropies of dominating David derived networks $D_1(n)$, $D_2(n)$, and $D_3(n)$. Furthermore, we construct Tables 4–6 to estimate the degree-based entropies for various values of n. From Tables 4–6, we can see that, as n increases, the degree-based entropies of these networks also increase.

4. Conclusion

In this study, taking into account the definition of Shannon and Chen entropy, we studied the classifications of $\mathrm{DDD}(n)$ and also computed the entropies of them. We discuss the degree-based topological indices such as Randic, *ABC*, and GA index and find their closed formulae of entropy for dominating David derived network. We, in like manner, enlisted the mathematical assessments of these entropies in Tables 4–6. We gave the relation of these entropies which help us to know the physio-chemical activity of these networks [10, 3, 4].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed to this article equally.

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