Research Article

A Novel Pythagorean Group Decision-Making Method Based on Evidence Theory and Interactive Power Averaging Operator

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Since Pythagorean fuzzy sets can better reflect the cognition of the decision objects for experts, researchers have begun to pay increasingly more attention to them in recent years. The majority of the research on Pythagorean fuzzy environment assumes that the decision maker is completely rational and does not consider the correlation among the attribute variables. In view of the above, this paper proposes a method to solve the multiple attribute group decision-making problem based on D-S theory and interactive power averaging operator. First, the new Pythagorean fuzzy interactive weighted power average operator is designed to aggregate the attribute evaluation information given by experts one by one, and the comprehensive evaluation information of each expert is obtained. Then, the expert comprehensive evaluation information is aggregated by the rule of evidence combination to obtain the comprehensive evidence information and confidence interval of each candidate. Then, the decision-making method for candidate alternatives is performed by the possibility discriminant rule. The design method considers not only the decision makers' bounded rationality but also the correlation among the attribute variables. Finally, the selection of the energy exploitation plan illustrates the feasibility and effectiveness of the proposed group decision method.

1. Introduction

Since Bellman and Zadeh formally proposed the concept of fuzzy decision in 1970, it has attracted a large number of scholars to study this field of decision-making due to its wide application [1]. With the deepening of research, traditional fuzzy sets can no longer meet the needs of modern decision science. Some extended fuzzy set concepts have been proposed, such as interval fuzzy sets, triangular fuzzy sets, trapezoidal fuzzy sets, type-2 fuzzy sets, and type-n fuzzy sets. However, these fuzzy sets have not broken through the limitations of the dichotomy and cannot represent the psychological characteristics of negation and hesitation that people show in the decision-making process. For this reason, Atanassov expands fuzzy sets and proposes the concept of intuitionistic fuzzy sets (IFS), which uses membership, nonmembership, and hesitation to represent information

more delicately [2]. In view of the fact that intuitionistic fuzzy sets can flexibly represent decision-making information from multiple perspectives of membership, nonmembership, and hesitation, it has attracted the attention of many scholars and has been widely used to solve fuzzy decision problems. In 2014, Yager proposed Pythagorean fuzzy sets on the basis of intuitionistic fuzzy sets, and the constraint conditions of intuitionistic fuzzy sets were reduced such that the sum of the squares of membership and nonmembership was less than 1 [3].

After several years of development, Pythagorean fuzzy set theory and its application research have obtained some achievements [4–7]. Zhang et al. defined some similarity measures of the Pythagorean fuzzy sets and discussed their properties [8]. They also proposed a Pythagorean fuzzy decision-making method to solve the intelligent medical diagnostic problem [8]. Wan et al. further proposed a new order relation

for elements of Pythagorean fuzzy sets and then used it to solve decision-making problems [9]. Ejegwa defined the triparametric correlation coefficients of Pythagorean fuzzy sets and verified that they can capture the three orthodox parameters of Pythagorean fuzzy sets [10]. In order to measure the closeness between Pythagorean fuzzy information units, Thao analyzed the existing defects of intuitionistic fuzzy similarity calculation methods, proposed a new similarity calculation formula to measure the Pythagorean fuzzy numbers, and used it to solve the multiattribute decision-making problem [11]. Chinnadurai et al. proposed the Pythagorean fuzzy soft sets and defined its basic operations such as addition, intersection, union, and complement. Based on this, a multiple attribute decision-making method using the integration operator was designed to solve the human resource management problem [12]. Zhang et al. introduced Pythagorean fuzzy preference relations and its additive consistency, which expanded the application scope of preference relations [13]. Given that the Muirhead mean operator can capture the correlation between multiple decision variables, Liu et al. discussed the relevant properties of the Muirhead mean operator in the Pythagorean fuzzy linguistic environment and proposed a Pythagorean linguistic multiple attribute decision-making method [14]. Considering the cross relationship between the degree of membership and the degree of nonmembership, Wang et al. proposed the interactive algorithm of Pythagorean fuzzy number and some collections based on interactive operations and used it to solve the multiple attribute decision-making problem of enterprise resource planning [15]. Combining the dual hesitant fuzzy sets and Pythagorean fuzzy sets, Ji et al. proposed probabilistic dual hesitant Pythagorean fuzzy sets and designed a new decision-making method to solve the multiple attribute decision-making problem in portfolio selection [16]. Wan et al. developed a Pythagorean fuzzy group decision method based on the degree of truth and information entropy [17]. Akram et al. proposed a novel approach based on the Hamacher operator for complex Pythagorean fuzzy decision-making [18]. Considering group utility measures and individual regret measures, Ma et al. developed a method to solve Pythagorean fuzzy multiple attribute group decision-making [19]. Wan et al. developed a novel TOPSIS method for Pythagorean fuzzy multiple attribute group decision-making [20]. Akram et al. proposed Pythagorean fuzzy Yager aggregation operators and then used them to solve the multicriteria decision-making problem [21].

However, the abovementioned Pythagorean fuzzy decision-making problems are solved under the assumption that the decision maker is completely rational, and this assumption is difficult to satisfy in actual decision-making. Evidence theory, also known as Dempster–Shafer theory, or D-S theory for short, is based on decision makers' understanding of objective things; based on the evidence and knowledge that decision makers possess, D-S theory provides uncertainty measures for fuzzy decision information. This way of portraying uncertainty is closer to human thinking habits and can reflect the bounded rational behaviour of experts in the decision-making process. This feature makes evidence theory widely used to solve fuzzy decision-making problems. However, to date, no evidence has been used to synthesize Pythagorean fuzzy

information. In addition, the power average operator [22] is a classic fuzzy information gathering tool. Since it was proposed, it has received attention from many scholars and has been extended to different fuzzy environments. The common power average operators are the hesitant fuzzy power aggregation operator, intuitionistic fuzzy power average operator, and triangular intuitionistic fuzzy power average operator; they are used to solve different ambiguities in the decision-making information aggregation problem in a fuzzy environment. However, these power average operators can gather fuzzy information only from the perspective of the overall equilibrium and cannot capture the relevant information between attributes. He et al. stated that interactive computing can capture the relevant information between attributes and the information between membership and nonmembership [24]. In view of the above analysis, this paper proposes an improved power averaging operator, the Pythagorean interactive power averaging operator, combined with evidence theory, and a Pythagorean fuzzy group decision method under bounded rationality.

2. Evidence Theory

The recognition framework refers to a limited and complete set of all possible results of a certain proposition. The construction of the recognition framework can transform abstract propositional operations into set operations. Important concepts such as the trust function, likelihood function, and evidence combination function of D-S theory are all based on the recognition framework.

Definition 1 (see [25]). F is set as the DS identification framework, and $F = \{F_1, F_2, \dots, F_n\}$ which is a set of mutually independent conclusions. Then, the basic probability assignment (BPA) function of the framework, also known as the mass function, is the mapping: $m: 2^F \rightarrow [0, 1]$, which satisfies the following conditions: (1) $m(\emptyset) = 0$; (2) and (2) $\sum_{A \in 2^F} m(A) = 1$. Among the components, $\emptyset$ is the empty set, A is any subset of F , and $2^F = \{\emptyset, \{F_1\}, \dots, \{F_n\}, \{F_1, F_2\}, \dots, \{F_1, F_n\}, \dots, F\}$ is the set consisting of all subsets of F .

Definition 2 (see [25]). Let F be a D-S recognition framework, $F = \{F_1, F_2, \dots, F_n\}$ be a set of mutually independent conclusions, and A and B be subsets of F . Then, the belief measure (Bel) function and the plausibility measure (Pl) function are as follows:

$$\begin{aligned} \text{Bel}(A) &= \sum_{B \subseteq A} m(B), \\ \text{Pl}(A) &= \sum_{B \cap A \neq \emptyset} m(B), \end{aligned} \quad (1)$$

where $\text{Bel}(A)$ represents the credibility that the evidence A must occur and $\text{Pl}(A)$ represents the credibility that the evidence A may occur. Obviously, there is $\text{Pl}(A) \geq \text{Bel}(A)$ for any subset A of F , so the support interval of A can be expressed as $[\text{Bel}(A), \text{Pl}(A)]$, also known as the confidence interval of A . In order to fuse multiple pieces of evidentiary

information to implement optimal decision-making, Dempster proposed the following orthogonal sum rule.

$$m(A) = m_1(A_1) \oplus m_2(A_2) \oplus \dots \oplus m_n(A_n) = \begin{cases} 0, & A = \emptyset, \\ \frac{\sum_{A_1 \cap A_2 \cap \dots \cap A_n = A} m_1(A_1) m_2(A_2) \dots m_n(A_n)}{1 - \sum_{A_1 \cap A_2 \cap \dots \cap A_n = \emptyset} m_1(A_1) m_2(A_2) \dots m_n(A_n)}, & A \neq \emptyset. \end{cases} \quad (2)$$

3. Pythagorean Interactive Power Average Operator

3.1. Pythagorean Fuzzy Sets

Definition 3 (see [7]). Suppose that X is a given universe of discourse. Then, a Pythagorean fuzzy set A on X is $A = \{\langle x, r(x), d(x) \rangle | x \in X\}$, where $r(x)$ is the strength of the commitment given x and $d(x)$ is the bias value of the commitment, $r(x) \in [0, 1]$ and $d(x) \in [0, 1]$.

Zhang and Xu further proposed the concept of a Pythagorean fuzzy number. That is, the binary array of $\alpha = P(\mu_\alpha, v_\alpha)$ is called a Pythagorean fuzzy number, where μ_α and v_α satisfy the constraint conditions of $(\mu_\alpha)^2 + (v_\alpha)^2 \leq 1$. The following algorithms and binary relations are given to open the application space of Pythagorean fuzzy sets.

Definition 4 (see [8]). Let $\alpha_i = P(\mu_{\alpha_i}, v_{\alpha_i})$ ($i = 1, 2$) and $\alpha = P(\mu_\alpha, v_\alpha)$ be three Pythagorean fuzzy numbers. Then,

- (1) $\alpha_1 \oplus \alpha_2 = P(\sqrt{\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2 \mu_{\alpha_2}^2}, v_{\alpha_1} v_{\alpha_2})$
- (2) $\alpha_1 \otimes \alpha_2 = P(\mu_{\alpha_1} \mu_{\alpha_2}, \sqrt{v_{\alpha_1}^2 + v_{\alpha_2}^2 - v_{\alpha_1}^2 v_{\alpha_2}^2})$
- (3) $\lambda \alpha = P(\sqrt{1 - (1 - \mu_\alpha^2)^\lambda}, v_\alpha^\lambda); \lambda > 0$
- (4) $\alpha^\lambda = P(\mu_\alpha^\lambda, \sqrt{1 - (1 - v_\alpha^2)^\lambda}); \lambda > 0$

However, the above algorithm does not consider the interaction between the degree of membership and the degree of nonmembership, and abnormal phenomena may occur in the calculation.

Example 1. Suppose $\alpha_1 = P(0.6, 0.4)$, $\alpha_2 = P(0.5, 0.6)$, and $\alpha_3 = P(0.7, 0.0)$ are three Pythagorean fuzzy numbers; the corresponding weight vector is $w = (w_1, w_2, w_3)^T = (0.25, 0.4, 0.35)^T$. Then, the arithmetic weighted average of α_i ($i = 1, 2, 3$) obtained by Definition 4 is

$$\alpha = w_1 \alpha_1 \oplus w_2 \alpha_2 \oplus w_3 \alpha_3 = P(0.6045, 0). \quad (3)$$

This means that when calculating the arithmetic weighted average of α_i ($i = 1, 2, 3$), the nonmembership of α_1 and α_2 has no effect, which is puzzling. For this reason, Wei and others stipulated the following interactive Pythagorean fuzzy number algorithm.

Definition 5 (see [27]). Let $\alpha_i = P(\mu_{\alpha_i}, v_{\alpha_i})$ ($i = 1, 2$) and $\alpha = P(\mu_\alpha, v_\alpha)$ be three Pythagorean fuzzy numbers. Then,

Theorem 1 (see [26]). Suppose that $m_1, m_2, \dots, m_n$ is the mass function under the same recognition framework, and the corresponding evidence is $A_1, A_2, \dots, A_n$. Then,

- (1) $\alpha_1 \oplus \alpha_2 = P(\sqrt{\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2 \mu_{\alpha_2}^2}, \sqrt{v_{\alpha_1}^2 + v_{\alpha_2}^2 - v_{\alpha_1}^2 v_{\alpha_2}^2 - \mu_{\alpha_1}^2 v_{\alpha_2}^2 - v_{\alpha_1}^2 \mu_{\alpha_2}^2})$
- (2) $\alpha_1 \otimes \alpha_2 = P(\sqrt{\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2 \mu_{\alpha_2}^2 - v_{\alpha_1}^2 \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2 v_{\alpha_2}^2}, \sqrt{v_{\alpha_1}^2 + v_{\alpha_2}^2 - v_{\alpha_1}^2 v_{\alpha_2}^2})$
- (3) $\lambda \alpha = P(\sqrt{1 - (1 - \mu_\alpha^2)^\lambda}, \sqrt{(1 - \mu_\alpha^2)^\lambda - (1 - (\mu_\alpha^2 + v_\alpha^2)^\lambda)}); \lambda > 0$
- (4) $\alpha^\lambda = P(\sqrt{(1 - v_\alpha^2)^\lambda - (1 - (\mu_\alpha^2 + v_\alpha^2)^\lambda)}, \sqrt{1 - (1 - v_\alpha^2)^\lambda}); \lambda > 0$

Using the interactive algorithm, recalculate Example 1 to obtain

$$\alpha = w_1 \alpha_1 \oplus w_2 \alpha_2 \oplus w_3 \alpha_3 = P(0.6084, 0.4226). \quad (4)$$

$\min(v_{\alpha_1}, v_{\alpha_2}, v_{\alpha_3}) \leq v_\alpha = 0.4226 \leq \max(v_{\alpha_1}, v_{\alpha_2}, v_{\alpha_3})$ shows that the interactive algorithm is closer to the actual situation.

Huang et al. stipulated the following Pythagorean fuzzy number ranking criteria and distance measure.

Definition 6 (see [30]). Set $\alpha = P(\mu_\alpha, v_\alpha)$ as a Pythagorean fuzzy number. Then, the score function of α is $s_\alpha = (\mu_\alpha)^2 - (v_\alpha)^2$. For any two Pythagorean fuzzy numbers α_1 and α_2 , the following sorting criteria exist:

- (1) If $s_{\alpha_1} > s_{\alpha_2}$, then $\alpha_1 > \alpha_2$
- (2) If $s_{\alpha_1} = s_{\alpha_2}$, then $\alpha_1 = \alpha_2$
- (3) If $s_{\alpha_1} < s_{\alpha_2}$, then $\alpha_1 < \alpha_2$

Definition 7 (see [30]). Let $\alpha_i = P(\mu_{\alpha_i}, v_{\alpha_i})$ ($i = 1, 2$) be two Pythagorean fuzzy numbers. Then, the distance between α_1 and α_2 is

$$d(\alpha_1, \alpha_2) = \frac{1}{2} \left(|\mu_{\alpha_1} - \mu_{\alpha_2}| + |v_{\alpha_1} - v_{\alpha_2}| + |\pi_{\alpha_1} - \pi_{\alpha_2}| \right), \quad (5)$$

where $\pi_{\alpha_i} = \sqrt{1 - (\mu_{\alpha_i})^2 - (v_{\alpha_i})^2}$ ($i = 1, 2$) is the hesitation of α_i ($i = 1, 2$).

3.2. Pythagorean Interactive Power Average Operator. Based on the integrity of the data, Yager proposed a power average operator that can reduce the impact of abnormal information.

Definition 8 (see [22]). Let a_i ($i = 1, 2, \dots, n$) be a sequence of real numbers. If

$$PA(a_1, a_2, \dots, a_n) = \sum_{i=1}^n \frac{1 + T(a_i)}{\sum_{t=1}^n (1 + T(a_t))} a_i, \quad (6)$$

then PA is called the power average operator. Among them, $T(a_i) = \sum_{j=1, j \neq i}^n \text{Sup}(a_i, a_j)$; $\text{Sup}(a_i, a_j)$ represents the degree of support of a_i and a_j , and it meets the following conditions:

- (1) $\text{Sup}(a_i, a_j) \in [0, 1]$
- (2) $\text{Sup}(a_i, a_j) = \text{Sup}(a_j, a_i)$
- (3) If $d(a_i, a_j) \leq d(a_i, a_k)$, then $\text{Sup}(a_i, a_j) \geq \text{Sup}(a_i, a_k)$

However, the power average operator can only capture the balance of the attribute information, not the associated information. For this reason, this paper proposes an improved power average operator based on the Pythagorean interactive operation: the Pythagorean interactive power average operator.

Definition 9. Let $\alpha_i = P(\mu_{\alpha_i}, v_{\alpha_i})$ ($i = 1, 2, \dots, n$) be a list of Pythagorean fuzzy numbers with weights of w_i ($i = 1, 2, \dots, n$), where $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Then, the Pythagorean fuzzy interactive weighted power average operator is PFIWPA:

$$\text{PFIWPA}(\alpha_1, \alpha_2, \dots, \alpha_n) = \bigoplus_{i=1}^n \frac{w_i (1 + T(\alpha_i))}{\sum_{t=1}^n w_t (1 + T(\alpha_t))} \alpha_i, \quad (7)$$

where $T(\alpha_i) = \sum_{j=1, j \neq i}^n \text{Sup}(\alpha_i, \alpha_j)$ and $\text{Sup}(\alpha_i, \alpha_j)$ is the degree of support of α_i and α_j , which satisfies the constraints:

- (1) $\text{Sup}(\alpha_i, \alpha_j) \in [0, 1]$
- (2) $\text{Sup}(\alpha_i, \alpha_j) = \text{Sup}(\alpha_j, \alpha_i)$
- (3) If $d(\alpha_i, \alpha_j) \leq d(\alpha_i, \alpha_k)$, then $\text{Sup}(\alpha_i, \alpha_j) \geq \text{Sup}(\alpha_i, \alpha_k)$

Theorem 2. Let $\alpha_i = P(\mu_{\alpha_i}, v_{\alpha_i})$ ($i = 1, 2, \dots, n$) be a list of Pythagorean fuzzy numbers with weights of w_i ($i = 1, 2, \dots, n$), where $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Then, the aggregation value obtained by the PFIWPA operator is also a Pythagorean fuzzy number, and

$$\text{PFIWPA}(\alpha_1, \alpha_2, \dots, \alpha_n) = P \left(\sqrt{1 - \prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{w_j (1+T(\alpha_j))} \sum_{t=1}^n w_t (1+T(\alpha_t))}, \sqrt{\prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{w_j (1+T(\alpha_j))} \sum_{t=1}^n w_t (1+T(\alpha_t)) - \prod_{j=1}^n \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)^{w_j (1+T(\alpha_j))} \sum_{t=1}^n w_t (1+T(\alpha_t))} \right). \quad (8)$$

Proof. (1) When $(\alpha_1, \alpha_2) = \bigoplus_{i=1}^2 (w_i (1 + T(\alpha_i)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))) \alpha_i$,

$$\text{PFIWPA}(\alpha_1, \alpha_2) = \bigoplus_{i=1}^2 \frac{w_i (1 + T(\alpha_i))}{\sum_{t=1}^2 w_t (1 + T(\alpha_t))} \alpha_i. \quad (9)$$

It can be seen from Definition 5 that

$$\begin{aligned} \frac{w_1 (1 + T(\alpha_1))}{\sum_{t=1}^2 w_t (1 + T(\alpha_t))} \alpha_1 &= P \left(\sqrt{1 - (1 - \mu_{\alpha_1}^2)^{w_1 (1+T(\alpha_1))} \sum_{t=1}^2 w_t (1+T(\alpha_t))}, \right. \\ &\quad \left. \sqrt{(1 - \mu_{\alpha_1}^2)^{w_1 (1+T(\alpha_1))} \sum_{t=1}^2 w_t (1+T(\alpha_t)) - (1 - (\mu_{\alpha_1}^2 + v_{\alpha_1}^2))^{w_1 (1+T(\alpha_1))} \sum_{t=1}^2 w_t (1+T(\alpha_t))} \right), \\ \frac{w_2 (1 + T(\alpha_2))}{\sum_{t=1}^2 w_t (1 + T(\alpha_t))} \alpha_2 &= P \left(\sqrt{1 - (1 - \mu_{\alpha_2}^2)^{w_2 (1+T(\alpha_2))} \sum_{t=1}^2 w_t (1+T(\alpha_t))}, \right. \\ &\quad \left. \sqrt{(1 - \mu_{\alpha_2}^2)^{w_2 (1+T(\alpha_2))} \sum_{t=1}^2 w_t (1+T(\alpha_t)) - (1 - (\mu_{\alpha_2}^2 + v_{\alpha_2}^2))^{w_2 (1+T(\alpha_2))} \sum_{t=1}^2 w_t (1+T(\alpha_t))} \right). \end{aligned} \quad (10)$$

There is

$$\begin{aligned}
\text{PFIWPA } (\alpha_1, \alpha_2) &= \bigoplus_{i=1}^2 \frac{w_i (1 + T(\alpha_i))}{\sum_{t=1}^2 w_t (1 + T(\alpha_t))} \alpha_i \\
&= P \left(\sqrt{1 - (1 - \mu_{\alpha_1}^2)^{\left(w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)}} \right. \\
&\quad \left. \sqrt{\left((1 - \mu_{\alpha_1}^2)^{\left(w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)} - (1 - (\mu_{\alpha_1}^2 + v_{\alpha_1}^2))^{\left(w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)} \right)} \right) \\
&= P \left(\sqrt{1 - (1 - \mu_{\alpha_1}^2)^{w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))}} \right. \\
&\quad \left. \sqrt{\left((1 - \mu_{\alpha_1}^2)^{\left(w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)} - (1 - (\mu_{\alpha_1}^2 + v_{\alpha_1}^2))^{\left(w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)} \right)} \right) \\
&\oplus P \left(\sqrt{1 - (1 - \mu_{\alpha_2}^2)^{w_2 (1+T(\alpha_2)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))}} \right. \\
&\quad \left. \sqrt{\left((1 - \mu_{\alpha_2}^2)^{w_2 (1+T(\alpha_2)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))} - (1 - (\mu_{\alpha_2}^2 + v_{\alpha_2}^2))^{\left(w_2 (1+T(\alpha_2)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)} \right)} \right) \\
&= P \left(\sqrt{1 - (1 - \mu_{\alpha_1}^2)^{w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))} (1 - \mu_{\alpha_2}^2)^{w_2 (1+T(\alpha_2)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))}} \right. \\
&\quad \left. \sqrt{\left((1 - \mu_{\alpha_1}^2)^{w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))} (1 - \mu_{\alpha_2}^2)^{w_2 (1+T(\alpha_2)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t))} - (1 - (\mu_{\alpha_1}^2 + v_{\alpha_1}^2))^{\left(w_1 (1+T(\alpha_1)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)} (1 - (\mu_{\alpha_2}^2 + v_{\alpha_2}^2))^{\left(w_2 (1+T(\alpha_2)) / \sum_{t=1}^2 w_t (1 + T(\alpha_t)) \right)} \right)} \right)
\end{aligned}$$

$$= P \left(\sqrt{1 - \prod_{j=1}^2 \left(1 - \mu_{\alpha_j}^2\right)}^{\left(w_j(1+T(\alpha_j))/\sum_{t=1}^n w_t(1+T(\alpha_t))\right)} \right. \\ \left. \sqrt{\prod_{j=1}^2 \left(1 - \mu_{\alpha_j}^2\right)}^{\left(w_j(1+T(\alpha_j))/\sum_{t=1}^n w_t(1+T(\alpha_t))\right)} - \prod_{j=1}^2 \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)}^{\left(w_j(1+T(\alpha_j))/\sum_{t=1}^n w_t(1+T(\alpha_t))\right)} \right). \quad (11)$$

Therefore, when $(\alpha_1, \alpha_2, \dots, \alpha_k) \oplus_{i=1}^k (w_i(1+T(\alpha_i))/\sum_{t=1}^n w_t(1+T(\alpha_t)))\alpha_i$, formula (8) is established.

- (2) Assuming $(\alpha_1, \alpha_2, \dots, \alpha_k) = \oplus_{i=1}^k (w_i(1+T(\alpha_i))/\sum_{t=1}^n w_t(1+T(\alpha_t)))\alpha_i$ $((\alpha_1, \alpha_2, \dots, \alpha_k) = \oplus_{i=1}^k (w_i(1+T(\alpha_i))/\sum_{t=1}^n w_t(1+T(\alpha_t)))\alpha_i)$, formula (8) is still established:

$(1+T(\alpha_i))/\sum_{t=1}^n w_t(1+T(\alpha_t)))\alpha_i$, formula (8) is established; considering $(\alpha_1, \alpha_2, \dots, \alpha_k) = \oplus_{i=1}^k (w_i(1+T(\alpha_i))/\sum_{t=1}^n w_t(1+T(\alpha_t)))\alpha_i$, formula (8) is still established:

$$\text{PFIWPA } (\alpha_1, \alpha_2, \dots, \alpha_k) = \bigoplus_{i=1}^k \frac{w_i(1+T(\alpha_i))}{\sum_{t=1}^n w_t(1+T(\alpha_t))} \alpha_i \\ = P \left(\sqrt{1 - \prod_{j=1}^k \left(1 - \mu_{\alpha_j}^2\right)}^{\left(w_j(1+T(\alpha_j))/\sum_{t=1}^n w_t(1+T(\alpha_t))\right)} \right. \\ \left. \sqrt{\prod_{j=1}^k \left(1 - \mu_{\alpha_j}^2\right)}^{\left(w_j(1+T(\alpha_j))/\sum_{t=1}^n w_t(1+T(\alpha_t))\right)} - \prod_{j=1}^k \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)}^{\left(w_j(1+T(\alpha_j))/\sum_{t=1}^n w_t(1+T(\alpha_t))\right)} \right). \quad (12)$$

Then,

$$\text{PFIWPA } (\alpha_1, \alpha_2, \dots, \alpha_{k+1}) = \bigoplus_{i=1}^{k+1} \frac{w_i(1+T(\alpha_i))}{\sum_{t=1}^n w_t(1+T(\alpha_t))} \alpha_i = \bigoplus_{i=1}^k \frac{w_i(1+T(\alpha_i))}{\sum_{t=1}^n w_t(1+T(\alpha_t))} \alpha_i \oplus \frac{w_i(1+T(\alpha_i))}{\sum_{t=1}^n w_t(1+T(\alpha_t))} \alpha_{k+1}$$

$$\begin{aligned}
&= P \left(\sqrt{1 - \prod_{j=1}^k \left(1 - \mu_{\alpha_j}^2\right)^{w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))}} \right. \\
&\quad \left. \sqrt{\prod_{j=1}^k \left(1 - \mu_{\alpha_j}^2\right)^{w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))} - \prod_{j=1}^k \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)^{w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))}} \right) \\
&\oplus P \left(\sqrt{1 - \left(1 - \mu_{\alpha_{k+1}}^2\right)^{w_{k+1}(1+T(\alpha_{k+1})) / \sum_{t=1}^{k+1} w_t(1+T(\alpha_t))}} \right. \\
&\quad \left. \sqrt{\left(1 - \mu_{\alpha_{k+1}}^2\right)^{w_{k+1}(1+T(\alpha_{k+1})) / \sum_{t=1}^{k+1} w_t(1+T(\alpha_t))} - \left(1 - (\mu_{\alpha_{k+1}}^2 + v_{\alpha_{k+1}}^2)\right)^{w_{k+1}(1+T(\alpha_{k+1})) / \sum_{t=1}^{k+1} w_t(1+T(\alpha_t))}} \right) \\
&= P \left(\sqrt{1 - \prod_{j=1}^{k+1} \left(1 - \mu_{\alpha_j}^2\right)^{w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))}} \right. \\
&\quad \left. \sqrt{\prod_{j=1}^{k+1} \left(1 - \mu_{\alpha_j}^2\right)^{w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))} - \prod_{j=1}^{k+1} \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)^{w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))}} \right). \tag{13}
\end{aligned}$$

Therefore, when $n \geq 2$, formula (8) is also established.

In summary, formula (8) is established when $(n \geq 2)$ from the mathematical induction; obviously, the aggregation

value of the PFIWPA operator is still a Pythagorean fuzzy number.

Note that if $T(\alpha_i) = c (i = 1, 2, \dots, n)$, then

PFIWPA $(\alpha_1, \alpha_2, \dots, \alpha_n)$

$$\begin{aligned}
&= P \left(\sqrt{1 - \prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{\left(w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))\right)}} \right. \\
&\quad \left. \sqrt{\prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{\left(w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))\right)} - \prod_{j=1}^n \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)^{\left(w_j(1+T(\alpha_j)) / \sum_{t=1}^n w_t(1+T(\alpha_t))\right)}} \right) \\
&= P \left(\sqrt{1 - \prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{w_j(1+c) / \sum_{t=1}^n n w_t(1+c)}} \right. \\
&\quad \left. \sqrt{\prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{w_j(1+c) / \sum_{t=1}^n n w_t(1+c)} - \prod_{j=1}^n \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)^{w_j(1+c) / \sum_{t=1}^n n w_t(1+c)}} \right) \\
&= P \left(\sqrt{1 - \prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{w_j}} \right. \\
&\quad \left. \sqrt{\prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{w_j} - \prod_{j=1}^n \left(1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2)\right)^{w_j}} \right) \\
&= \text{PFIWA}(\alpha_1, \alpha_2, \dots, \alpha_n).
\end{aligned} \tag{14}$$

That is, the PFIWPA operator degenerates into the Pythagorean fuzzy interactive weighted average (PFIWA) operator. $\square$

Theorem 3. Let $\alpha_i = P(\mu_{\alpha_i}, v_{\alpha_i})$ be a list of Pythagorean fuzzy numbers. If $\alpha_i = \alpha = P(\mu_{\alpha}, v_{\alpha})$ ($i = 1, 2, \dots, n$), then PFIWPA $(\alpha_1, \alpha_2, \dots, \alpha_n) = \alpha$

Proof.

$$\begin{aligned}
 \text{PFIWPA}(\alpha_1, \alpha_2, \dots, \alpha_n) &= P \left(\sqrt{1 - \prod_{j=1}^n (1 - \mu_{\alpha}^2)^{\left(w_j (1+T(\alpha)) / \sum_{t=1}^n w_t (1+T(\alpha)) \right)}}, \right. \\
 &\quad \left. \sqrt{\prod_{j=1}^n (1 - \mu_{\alpha}^2)^{\left(w_j (1+T(\alpha)) / \sum_{t=1}^n w_t (1+T(\alpha)) \right)} - \prod_{j=1}^n (1 - (\mu_{\alpha}^2 + v_{\alpha}^2))^{\left(w_j (1+T(\alpha)) / \sum_{t=1}^n w_t (1+T(\alpha)) \right)}} \right) \\
 &= P \left(\sqrt{1 - \prod_{j=1}^n (1 - \mu_{\alpha}^2)^{w_j}}, \sqrt{\prod_{j=1}^n (1 - \mu_{\alpha}^2)^{w_j} - \prod_{j=1}^n (1 - (\mu_{\alpha}^2 + v_{\alpha}^2))^{w_j}} \right) \\
 &= P \left(\sqrt{1 - (1 - \mu_{\alpha}^2) \sum_{j=1}^n n w_j}, \sqrt{(1 - \mu_{\alpha}^2) \sum_{j=1}^n n w_j - (1 - (\mu_{\alpha}^2 + v_{\alpha}^2)) \sum_{j=1}^n n w_j} \right) \\
 &= P \left(\sqrt{1 - (1 - \mu_{\alpha}^2)}, \sqrt{(1 - \mu_{\alpha}^2) - (1 - (\mu_{\alpha}^2 + v_{\alpha}^2))} \right).
 \end{aligned} \tag{15}$$

$\square$

Theorem 4. Let α_i, α'_i ($i = 1, 2, \dots, n$) can be two columns of Pythagorean fuzzy numbers.

If $\alpha'_1, \alpha'_2, \dots, \alpha'_n$ is any permutation of $\alpha_1, \alpha_2, \dots, \alpha_n$, then PFIWPA $(\alpha_1, \alpha_2, \dots, \alpha_n) = \text{PFIWPA}(\alpha'_1, \alpha'_2, \dots, \alpha'_n)$.

Proof.

$$\begin{aligned}
 \text{PFIWPA}(\alpha_1, \alpha_2, \dots, \alpha_n) &= P \left(\sqrt{1 - \prod_{j=1}^n (1 - \mu_{\alpha_j}^2)^{w_j (1+T(\alpha_j)) / \sum_{t=1}^n n w_t (1+T(\alpha_t))}}, \right. \\
 &\quad \left. \sqrt{\prod_{j=1}^n (1 - \mu_{\alpha_j}^2)^{w_j (1+T(\alpha_j)) / \sum_{t=1}^n n w_t (1+T(\alpha_t))} - \prod_{j=1}^n (1 - (\mu_{\alpha_j}^2 + v_{\alpha_j}^2))^{w_j (1+T(\alpha_j)) / \sum_{t=1}^n n w_t (1+T(\alpha_t))}} \right) \\
 &= P \left(\sqrt{1 - \prod_{j=1}^n (1 - \mu_{\alpha'_j}^2)^{w'_j (1+T(\alpha'_j)) / \sum_{t=1}^n n w'_t (1+T(\alpha'_t))}}, \right. \\
 &\quad \left. \sqrt{\prod_{j=1}^n (1 - \mu_{\alpha'_j}^2)^{w'_j (1+T(\alpha'_j)) / \sum_{t=1}^n n w'_t (1+T(\alpha'_t))} - \prod_{j=1}^n (1 - (\mu_{\alpha'_j}^2 + v_{\alpha'_j}^2))^{w'_j (1+T(\alpha'_j)) / \sum_{t=1}^n n w'_t (1+T(\alpha'_t))}} \right) \\
 &= \text{PFIWPA}(\alpha'_1, \alpha'_2, \dots, \alpha'_n).
 \end{aligned} \tag{16}$$

$\square$

4. Pythagorean Fuzzy Multiple Attribute Group Decision-Making Method

The problem of the multiple attribute group method in Pythagorean fuzzy environment can be described as follows: the existing decision plan set $Y = \{Y_1, Y_2, \dots, Y_n\}$, $C = \{c_1, c_2, \dots, c_m\}$ is the m attributes of the plan, the attribute weight is $w = \{w_1, w_2, \dots, w_m\}$, $\{D_1, D_2, \dots, D_l\}$ is a group of decision makers, the weight vector is $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)$, and $h_{ij}^{(k)}$ represents the Pythagorean fuzzy evaluation value given by the k th decision expert D_k to the attribute c_j of the scheme Y_i . The method matrix provided by the decision expert D_k can be expressed as $H^{(k)} = (\alpha_{ij}^{(k)})_{n \times m}$. The steps of the group decision-making method of the interactive power average operator are as follows:

Step 1: convert the Pythagorean fuzzy decision matrix $H^{(k)}$ into a canonical matrix $\bar{H}^{(k)} = (\bar{\alpha}_{ij}^{(k)})_{n \times m}$, where

$$\begin{aligned} \alpha_i^k &= \text{PFIWPA}(\bar{\alpha}_{i1}^{(k)}, \bar{\alpha}_{i2}^{(k)}, \dots, \bar{\alpha}_{im}^{(k)}), i = 1, 2, \dots, n \\ &= P \left(\sqrt{1 - \prod_{j=1}^m \left(1 - \mu_{\bar{\alpha}_{ij}^{(k)}}^2\right)^{w_j(1+T(\alpha_{ij}^{(k)}))}} \sum_{t=1}^m 2w_t(1+T(\alpha_{it}^{(k)})) \right), \\ &\quad \sqrt{\prod_{j=1}^m \left(1 - \mu_{\bar{\alpha}_{ij}^{(k)}}^2\right)^{w_j(1+T(\alpha_{ij}^{(k)}))}} \sum_{t=1}^m 2w_t(1+T(\alpha_{it}^{(k)})) - \prod_{j=1}^m \left(1 - \left(\mu_{\bar{\alpha}_{ij}^{(k)}}^2 + v_{\bar{\alpha}_{ij}^{(k)}}^2\right)\right)^{w_j(1+T(\alpha_{ij}^{(k)}))}} \sum_{t=1}^m 2w_t(1+T(\alpha_{it}^{(k)})) \right). \end{aligned} \quad (17)$$

Step 5: for each expert's standardized comprehensive evaluation information on the plan, use the evidence synthesis method to gather the results.

$\bar{\alpha}_{ij}^{(k)} = \{ \alpha_{ij}^{(k)} g_j; \text{benefit attribute} N(\alpha_{ij}^{(k)}) = P(v_{\alpha_{ij}^{(k)}}), \mu_{\alpha_{ij}^{(k)}} g_j; \text{cost attribute} \}$.

Step 2: calculate the degree of support $\text{Sup}(\bar{\alpha}_{li}^{(k)}, \bar{\alpha}_{lj}^{(k)}) = 1 - d(\bar{\alpha}_{li}^{(k)}, \bar{\alpha}_{lj}^{(k)})$, $i, j = 1, 2, \dots, m$;

$l = 1, 2, \dots, n, d(\bar{\alpha}_{li}^{(k)}, \bar{\alpha}_{lj}^{(k)}) = 1/2(|\mu_{\bar{\alpha}_{li}^{(k)}} - \mu_{\bar{\alpha}_{lj}^{(k)}}| + |v_{\bar{\alpha}_{li}^{(k)}} - v_{\bar{\alpha}_{lj}^{(k)}}| + |\pi_{\bar{\alpha}_{li}^{(k)}} - \pi_{\bar{\alpha}_{lj}^{(k)}}|)$ between variables.

Step 3: calculate the support $T(\bar{\alpha}_{li}^{(k)}) = T(\bar{\alpha}_{li}^{(k)}) = \sum_{j=1, j \neq i}^m w_j \text{Sup}(\bar{\alpha}_{li}^{(k)}, \bar{\alpha}_{lj}^{(k)})$ of the variable and the remaining variables and the support indices $\eta_{li}^{(k)} = w_i(1 + T(\bar{\alpha}_{li}^{(k)}))/w_t(1 + T(\bar{\alpha}_{lt}^{(k)}))$, $i = 1, 2, \dots, m$ and $l = 1, 2, \dots, n$ of the variable.

Step 4: use the PFIWPA operator to gather the attribute values $\bar{\alpha}_{ij}^{(k)}$, $j = 1, 2, \dots, m$, of the candidate solutions to obtain the comprehensive attribute values of the experts D_k about the candidate solutions x_i :

First, standardize the comprehensive evaluation information $\alpha_i^{(k)}$ solved in step 3 to obtain standardized comprehensive evaluation information $\beta_i^{(k)}$:

$$\beta_i^{(k)} = P(\mu_{\beta_i^{(k)}}, v_{\beta_i^{(k)}}) = \lambda_k P(\mu_{\alpha_i^{(k)}}, v_{\alpha_i^{(k)}}), \quad i = 1, 2, \dots, n, k = 1, 2, \dots, l. \quad (18)$$

Regarding the comprehensive evaluation information $\beta_i^{(k)}$ of Y_i about the expert D_k as a special piece of evidence, $\beta_i^{(k)} = P(\mu_{\beta_i^{(k)}}, v_{\beta_i^{(k)}})$ contains three conclusions, namely, the degree of satisfaction, the degree of dissatisfaction, and the degree of uncertainty of Y_i . If A indicates the degree of satisfaction of D_k regarding Y_i , then $m_{ki}(A) = \mu_{\beta_i^{(k)}}/r$; If R represents the degree of dissatisfaction of D_k regarding Y_i , then $m_{ki}(R) = v_{\beta_i^{(k)}}/r$; If

(A, R) represents the degree of uncertainty of D_k regarding Y_i , then $m_{ki}(A, R) = \sqrt{1 - \mu_{\beta_i^{(k)}}^2 - v_{\beta_i^{(k)}}^2}/r$, where $r = \sqrt{1 - \mu_{\beta_i^{(k)}}^2 - v_{\beta_i^{(k)}}^2} + v_{\beta_i^{(k)}} + \mu_{\beta_i^{(k)}}$. Evidence synthesis rules are used to calculate the comprehensive evidence M_i of the plan Y_i :

$$M_i(A) = m_{1i}(A_1) \oplus m_{2i}(A_2) \oplus \dots \oplus m_{li}(A_l) = \frac{\sum_{A_1 \cap A_2 \cap \dots \cap A_l = A} m_{1i}(A_1) m_{2i}(A_2) \dots m_{li}(A_l)}{1 - \sum_{A_1 \cap A_2 \cap \dots \cap A_l = \emptyset} m_{1i}(A_1) m_{2i}(A_2) \dots m_{li}(A_l)}, \quad (19)$$

where $A, A_1, A_2, \dots, A_l \in \{A, R, (A, R)\}$.

Step 6: give the confidence interval I_i of the candidate plan Y_i regarding the comprehensive evidence $M_i = P(\mu_{M_i}, v_{M_i})$, i.e., $I_i = \text{Bel}[(x_i) = \mu_{M_i}(x_i) = \mu_{M_i} + \sqrt{1 - \mu_{M_i}^2 - v_{M_i}^2}]$. Then, give the probability matrix $P = (p_{ij})_{n \times n}$ between the candidate plans according to the confidence interval, where p_{ij} is the probability that the candidate plan Y_i is superior to Y_j . The calculation formula is $p_{ij} = \min\{[Pl(x_i) - \text{Bel}(x_i)] + [Pl(x_j) - \text{Bel}(x_j)], \max(Pl(x_i) - \text{Bel}(x_j), 0)] / ([Pl(x_i) - \text{Bel}(x_i)] + [Pl(x_j) - \text{Bel}(x_j)])\}$.

Step 7: calculate the ranking vector $\{P_1, P_2, \dots, P_n\}$ of the possibility matrix $P = (p_{ij})_{n \times n}$ where $P_i = 1/n(n-1)(\sum_{j=1}^n p_{ij} - 1 + (n/2))$, $\{i = 1, 2, \dots, n\}$.

According to the size of P_i , the candidate scheme is selected for the optimal decision; the larger the value P_i , the better the scheme.

5. Case Analysis

Example 2. Energy issues have always plagued mankind. Many countries and regions are committed to the development and utilization of new energy technologies. Nea-Kessani is a rural community in northern Greece. The region's economy is in a recession. The local government has decided to use clean energy and advanced manufacturing to provide new jobs to society and realize the transformation from traditional agriculture to modern industry. The locals decided to develop low-enthalpy geothermal resources, evaluated five mining methods as $x_i (i = 1, 2, \dots, 5)$, and formulated the following decision attributes based on its own characteristics: c_1 is the net present value of investment, c_2 is the number of jobs created, c_3 is the resource utilization, c_4 is safety, and c_5 is social impact force. Its weight vector is $w = (0.2, 0.4, 0.1, 0.1, 0.2)$. The expert group evaluated the above five attributes with real numbers between 0 and 1 and formed a consensus evaluation opinion, giving the following evaluation matrix $H^{(i)} (i = 1, 2, 3)$. In order to achieve scientific and reasonable decision-making effects, the local government decided to hire three industry experts $D_i (i = 1, 2, 3)$ to evaluate the mining plan. In addition, weights were assigned according to their professional influence, and the weight vector is $\lambda = (0.35, 0.3, 0.35)$. After investigation, Pythagorean fuzzy decision matrix provided by the experts is represented in Tables 1–3.

Next, the decision-making method of this paper is used to select the best mining plan for the local area.

Step 1: since the five evaluation indicators specified by the company are all benefit-based, there is no need to standardize, that is, $\bar{H}^{(k)} = H^{(k)} = (\alpha_{ij}^{(k)})_{n \times m}$, $k = 1, 2, 3$.

Step 2: calculate the degree of support between variables $\text{Sup}(\bar{\alpha}_{li}^{(k)}, \bar{\alpha}_{lj}^{(k)}) = 1 - d(\bar{\alpha}_{li}^{(k)}, \bar{\alpha}_{lj}^{(k)})$, $i, j = 1, 2, \dots, m$, $l = 1, 2, \dots, n$, and $k = 1, 2, 3$. Taking the evaluation matrix $H^{(1)}$ given by Expert 1 as an example, the following support matrix can be obtained:

TABLE 1: Pythagorean decision matrix $H^{(1)}$.

	c_1	c_2	c_3	c_4	c_5
x_1	$P(0.4, 0.8)$	$P(0.7, 0.6)$	$P(0.6, 0.7)$	$P(0.3, 0.8)$	$P(0.5, 0.6)$
x_2	$P(0.7, 0.5)$	$P(0.9, 0.2)$	$P(0.8, 0.5)$	$P(0.3, 0.6)$	$P(0.6, 0.5)$
x_3	$P(0.3, 0.4)$	$P(0.3, 0.7)$	$P(0.7, 0.4)$	$P(0.6, 0.4)$	$P(0.4, 0.5)$
x_4	$P(0.6, 0.6)$	$P(0.7, 0.5)$	$P(0.7, 0.2)$	$P(0.4, 0.6)$	$P(0.3, 0.7)$
x_5	$P(0.5, 0.7)$	$P(0.6, 0.4)$	$P(0.9, 0.3)$	$P(0.6, 0.7)$	$P(0.1, 0.7)$

TABLE 2: Pythagorean decision matrix $H^{(2)}$.

	c_1	c_2	c_3	c_4	c_5
x_1	$P(0.3, 0.9)$	$P(0.7, 0.6)$	$P(0.5, 0.8)$	$P(0.3, 0.6)$	$P(0.3, 0.6)$
x_2	$P(0.7, 0.4)$	$P(0.9, 0.2)$	$P(0.8, 0.1)$	$P(0.3, 0.5)$	$P(0.3, 0.5)$
x_3	$P(0.3, 0.6)$	$P(0.7, 0.7)$	$P(0.7, 0.6)$	$P(0.4, 0.4)$	$P(0.4, 0.3)$
x_4	$P(0.4, 0.8)$	$P(0.7, 0.5)$	$P(0.6, 0.2)$	$P(0.4, 0.7)$	$P(0.4, 0.7)$
x_5	$P(0.2, 0.7)$	$P(0.8, 0.2)$	$P(0.8, 0.4)$	$P(0.6, 0.6)$	$P(0.6, 0.6)$

TABLE 3: Pythagorean decision matrix $H^{(3)}$.

	c_1	c_2	c_3	c_4	c_5
x_1	$P(0.6, 0.7)$	$P(0.7, 0.6)$	$P(0.5, 0.8)$	$P(0.5, 0.5)$	$P(0.1, 0.6)$
x_2	$P(0.6, 0.5)$	$P(0.9, 0.2)$	$P(0.8, 0.1)$	$P(0.3, 0.5)$	$P(0.3, 0.4)$
x_3	$P(0.4, 0.7)$	$P(0.7, 0.5)$	$P(0.6, 0.1)$	$P(0.2, 0.9)$	$P(0.6, 0.5)$
x_4	$P(0.2, 0.9)$	$P(0.5, 0.6)$	$P(0.6, 0.2)$	$P(0.1, 0.6)$	$P(0.4, 0.7)$
x_5	$P(0.1, 0.6)$	$P(0.8, 0.2)$	$P(0.9, 0.2)$	$P(0.6, 0.5)$	$P(0.4, 0.6)$

$$\begin{aligned}
 S^{(1)}(1) &= \begin{pmatrix} 1 & 0.67 & 0.80 & 0.93 & 0.72 \\ 0.67 & 1 & 0.87 & 0.60 & 0.76 \\ 0.80 & 0.87 & 1 & 0.73 & 0.76 \\ 0.93 & 0.60 & 0.73 & 1 & 0.72 \\ 0.72 & 0.76 & 0.76 & 0.72 & 1 \end{pmatrix}, \\
 S^{(1)}(2) &= \begin{pmatrix} 1 & 0.68 & 0.85 & 0.60 & 0.87 \\ 0.68 & 1 & 0.79 & 0.28 & 0.55 \\ 0.85 & 0.79 & 1 & 0.45 & 0.72 \\ 0.60 & 0.28 & 0.45 & 1 & 0.73 \\ 0.87 & 0.55 & 0.72 & 0.73 & 1 \end{pmatrix}, \\
 S^{(1)}(3) &= \begin{pmatrix} 1 & 0.67 & 0.60 & 0.73 & 0.84 \\ 0.67 & 1 & 0.60 & 0.67 & 0.76 \\ 0.60 & 0.60 & 1 & 0.87 & 0.67 \\ 0.73 & 0.67 & 0.87 & 1 & 0.80 \\ 0.84 & 0.76 & 0.67 & 0.80 & 1 \end{pmatrix}, \\
 S^{(1)}(4) &= \begin{pmatrix} 1 & 0.87 & 0.68 & 0.80 & 0.73 \\ 0.87 & 1 & 0.79 & 0.67 & 0.60 \\ 0.68 & 0.79 & 1 & 0.67 & 0.55 \\ 0.80 & 0.67 & 0.67 & 1 & 0.87 \\ 0.73 & 0.60 & 0.55 & 0.87 & 1 \end{pmatrix}, \\
 S^{(1)}(5) &= \begin{pmatrix} 1 & 0.67 & 0.44 & 0.89 & 0.76 \\ 0.67 & 1 & 0.55 & 0.67 & 0.65 \\ 0.44 & 0.55 & 1 & 0.55 & 0.20 \\ 0.89 & 0.67 & 0.55 & 1 & 0.65 \\ 0.76 & 0.65 & 0.20 & 0.65 & 1 \end{pmatrix}.
 \end{aligned} \tag{20}$$

Step 3: calculate the support $T(\bar{\alpha}_{li}^{(k)})$ of the variable and the remaining variables and the support index of the variables $\eta_{li}^{(k)}$, $i = 1, 2, \dots, m$, $l = 1, 2, \dots, n$, and $k = 1, 2, 3$. Then, the following support matrix $T^{(k)}$ and support index matrix $\eta^{(k)}$ can be obtained:

$$\begin{aligned}
 T^{(1)} &= \begin{pmatrix} 3.12 & 2.90 & 3.16 & 2.98 & 2.96 \\ 3.00 & 2.30 & 2.81 & 2.06 & 2.87 \\ 2.84 & 2.70 & 2.74 & 3.07 & 3.07 \\ 3.08 & 2.93 & 2.69 & 3.01 & 2.75 \\ 2.76 & 2.54 & 1.74 & 2.76 & 2.26 \end{pmatrix}, \\
 \eta^{(1)} &= \begin{pmatrix} 0.21 & 0.39 & 0.11 & 0.10 & 0.20 \\ 0.23 & 0.36 & 0.11 & 0.08 & 0.22 \\ 0.20 & 0.38 & 0.10 & 0.11 & 0.22 \\ 0.21 & 0.40 & 0.09 & 0.10 & 0.19 \\ 0.22 & 0.41 & 0.07 & 0.11 & 0.18 \end{pmatrix}, \\
 T^{(2)} &= \begin{pmatrix} 2.48 & 2.47 & 2.67 & 2.71 & 2.71 \\ 2.73 & 2.04 & 2.55 & 2.33 & 2.33 \\ 2.60 & 1.95 & 2.34 & 2.54 & 2.33 \\ 2.71 & 2.61 & 2.16 & 3.07 & 3.07 \\ 2.16 & 2.64 & 2.72 & 3.08 & 3.08 \end{pmatrix}, \\
 \eta^{(2)} &= \begin{pmatrix} 0.19 & 0.39 & 0.10 & 0.11 & 0.21 \\ 0.24 & 0.35 & 0.11 & 0.10 & 0.20 \\ 0.23 & 0.35 & 0.10 & 0.11 & 0.21 \\ 0.20 & 0.38 & 0.08 & 0.11 & 0.23 \\ 0.16 & 0.39 & 0.10 & 0.11 & 0.23 \end{pmatrix}, \\
 T^{(3)} &= \begin{pmatrix} 2.89 & 2.76 & 2.66 & 2.67 & 2.28 \\ 2.64 & 1.91 & 2.42 & 2.37 & 2.28 \\ 2.63 & 2.61 & 2.11 & 1.76 & 2.83 \\ 1.98 & 2.86 & 2.11 & 2.65 & 2.82 \\ 2.07 & 2.44 & 1.93 & 2.72 & 2.52 \end{pmatrix}, \\
 \eta^{(3)} &= \begin{pmatrix} 0.22 & 0.41 & 0.10 & 0.10 & 0.17 \\ 0.24 & 0.34 & 0.11 & 0.11 & 0.20 \\ 0.21 & 0.41 & 0.08 & 0.07 & 0.22 \\ 0.15 & 0.44 & 0.08 & 0.10 & 0.22 \\ 0.18 & 0.41 & 0.08 & 0.12 & 0.21 \end{pmatrix}.
 \end{aligned} \tag{21}$$

Step 4: use the PFIWPA operator to gather the attribute values $\bar{\alpha}_{ij}^{(k)}$, $j = 1, 2, \dots, m$, of the candidate solutions to obtain expert D_k 's comprehensive evaluation value of the candidate solutions x_i , as shown in Table 4.

TABLE 4: Comprehensive evaluation information.

	D_1	D_2	D_3
x_1	$P(0.58, 0.68)$	$P(0.54, 0.71)$	$P(0.59, 0.66)$
x_2	$P(0.79, 0.39)$	$P(0.77, 0.31)$	$P(0.75, 0.33)$
x_3	$P(0.43, 0.56)$	$P(0.56, 0.72)$	$P(0.60, 0.57)$
x_4	$P(0.61, 0.55)$	$P(0.57, 0.62)$	$P(0.43, 0.67)$
x_5	$P(0.61, 0.56)$	$P(0.70, 0.45)$	$P(0.69, 0.38)$

Step 5: standardize the comprehensive evaluation value of the expert's candidate program, and then, use the evidence synthesis method to fuse the data. The specific results are obtained, as shown in Table 5.

Step 6: the comprehensive evidence information of the alternatives shows that the confidence intervals of the alternatives are $I_1 = [0.30, 0.44]$, $I_2 = [0.57, 0.75]$, $I_3 = [0.31, 0.50]$, $I_4 = [0.32, 0.51]$, and $I_5 = [0.46, 0.65]$. Based on this, the possibility matrix $P = (p_{ij})_{5 \times 5}$ between candidate schemes can be obtained:

$$P = \begin{pmatrix} \sim & 0 & 0 & 0 & 0 \\ 1 & \sim & 1 & 1 & 1 \\ 1 & 0 & \sim & 0.81 & 0.22 \\ 1 & 0 & 0.19 & \sim & 0 \\ 1 & 0 & 0.78 & 1 & \sim \end{pmatrix}. \tag{22}$$

Step 7: from the ranking vector of the degree of possibility matrix $\{0.08, 0.28, 0.18, 0.13, 0.21\}$, the ranking of candidate solutions can be obtained as $x_2 \succ x_5 \succ x_3 \succ x_4 \succ x_1$.

Therefore, the best green supplier should is x_2 .

6. Method Comparison and Discussion

Research on Pythagorean fuzzy group decision-making has just started, and there are few related documents. In this section, we compare the proposed method with the Pythagorean fuzzy TOPSIS group decision-making method [29] and the group decision-making method based on the WA operator [27]. Tables 6 and 7 show that the optimal choices given by the three group decision-making methods in Example 2 are both x_2 , which shows that the group decision-making method in this paper is feasible. In-depth analysis from [27, 29] clearly shows that the differences between the candidate solutions are magnified. This is mainly based on the following reasons:

- (1) The group decision-making methods proposed by [27, 29] assume that experts involved in decision-making are considered to be fully rational individuals. However, this assumption is not always true because of the complicated decision environment and finiteness of humans' cognitive abilities.
- (2) The PFIWPA operator used in this paper can take into account the balance and relevance of the information when gathering decision information,

TABLE 5: Results of evidence synthesis.

	Standardized comprehensive evaluation information			Comprehensive evidence information
	D_1	D_2	D_3	
x_1	$P(0.22, 0.33)$	$P(0.19, 0.33)$	$P(0.23, 0.32)$	$(0.30, 0.56, 0.14)$
x_2	$P(0.32, 0.21)$	$P(0.31, 0.16)$	$P(0.31, 0.17)$	$(0.57, 0.25, 0.17)$
x_3	$P(0.17, 0.25)$	$P(0.20, 0.34)$	$P(0.24, 0.26)$	$(0.31, 0.50, 0.18)$
x_4	$P(0.24, 0.26)$	$P(0.21, 0.28)$	$P(0.17, 0.30)$	$(0.32, 0.49, 0.19)$
x_5	$P(0.24, 0.26)$	$P(0.27, 0.21)$	$P(0.28, 0.18)$	$(0.46, 0.35, 0.19)$

TABLE 6: Calculation results of [27].

Candidate plan	Comprehensive evaluation value	Score value	Sort result
x_1	$P(0.60, 0.71)$	-0.11	5
x_2	$P(0.78, 0.22)$	0.56	1
x_3	$P(0.55, 0.45)$	0.10	4
x_4	$P(0.55, 0.39)$	0.16	3
x_5	$P(0.67, 0.32)$	0.34	2

TABLE 7: Calculation results of [29].

Candidate plan	$D(x_i, x^+)$	$D(x_i, x^-)$	The relative closeness of the alternative	Sort result
x_1	0.259	0.154	0.374	5
x_2	0.1	0.342	0.774	1
x_3	0.224	0.253	0.53	3
x_4	0.247	0.216	0.466	4
x_5	0.172	0.265	0.607	2

while the group decision-making methods used in literatures [27, 29] use independent attributes by default. Moreover, the model calculation of [27, 29] is based on Definition 4. As mentioned above, this type of calculation rule has defects, which can easily distort decision information.

- (3) When constructing the decision-making model, this paper uses evidence theory to fuse evaluation information among experts, which can be closer to human cognition than a collective settlement in a bounded rational environment [28].
- (4) The decision method given in [27, 29] uses the PFWA operator and TOPSIS when aggregating the evaluation information of different attributes and the evaluation information between experts, respectively. This paper uses the PFIWPA operator and the evidence theory to deal with the characteristics of information aggregation at different stages. The proposed method integrates the advantages of multiple methods to comprehensively solve the problem of aggregating of group decision information.

7. Conclusion

Aiming at the problem of multiple attribute group decision-making in Pythagorean fuzzy environment, this paper uses evidence theory and the Pythagorean fuzzy interactive weighted power average operator to propose a group decision-making method. The new method uses the Pythagorean

fuzzy interactive weighted power average operator to aggregate experts' attribute evaluation information and gives the comprehensive evaluation information of each expert. The operator used is better than the commonly used power average operator and not only satisfies idempotence and exchangeability but also mines attribute associations. Then, the rules of evidence synthesis are used to gather the comprehensive evaluation information of experts and give the comprehensive evidence information of the candidate program. Finally, the evidence information is used to give the best ranking of the candidate solutions. However, this method also has its shortcomings. The situation in which the attribute evaluation value is the interval Pythagorean fuzzy number and the attribute weight is unknown is not discussed. The next step will focus on the Pythagorean fuzzy group decision-making problem in these situations.

Data Availability

Data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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