Research Article

Intermittent Sampled Data Control for Time-Varying Formation-Containment of the Multiagent System with/without Time Delay

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Time-varying formation-containment problems for a second-order multiagent system (SOMAS) are studied via pulse-modulated intermittent control (PMIC) in this paper. A distributed control framework utilizing the neighbors' positions and velocities is designed so that leaders in the multiagent system form a formation, and followers move to the convex hull formed by each leader. Different from the traditional formation-containment problems, this paper applies the PMIC framework, which is more common and more in line with the actual control scenarios. Based on the knowledge of matrix theory, algebraic graph theory, and stability theory, some sufficient conditions are given for the time-varying formation-containment problem of the second-order multiagent system. Some numerical simulations are proposed to verify the effectiveness of the results presented in this paper.

1. Introduction

Many scholars start to pay attention to the multiagent systems (MASs) with the rapid development of complex network system theory. In recent decades, many major breakthroughs have been made in this field [1], and those results are also widely used in various fields of production and life, such as UAV cruise system [2], smart grid [3, 4], economic dispatching [5], and multiple underactuated surface vessels [6]. The most studied collaboration problems of MASs include swarm, consensus, formation, and distributed optimization. In very recent years, the clustering behavior of MASs has also attracted widespread interest, including but not limited to consensus [7–9], tracking, and formation [10].

In these cooperative control problems, many important advances have been made in the field of formation control and containment control. Huang et al. [11] studied the containment control problem of MASs via intermittent control-based sampled data information. Wang et al. [12]

investigated the containment control problem of first-order MASs in the noisy communication environments. Rahimi et al. [13] studied time-varying formation control of collaborative heterogeneous MASs. However, most of the existing works are carried out on formation control and containment control separately. In many applications, both these cooperative behaviors often require simultaneous implementation. For example, in the coordination of multiple tanker airplanes and multiple UAVs, the tanker airplanes will form a specific formation in advance and wait for the UAVs to reach the area they surround. In order to solve this kind of problem, Dong et al. designed a continuous control strategy [14] for second-order MASs with multiple leaders and multiple followers, which can ensure the formation control of the leaders and containment of the followers. It is worth noting that the above work [14] adopted a continuous-time control which may be difficult to implement in some cases due to sampled measurement. The traditional zero-order sampling control adopts the same amplitude control input in the whole sampling interval,

which can also lead to application difficulties. For instance, in a driverless system, it is difficult for the sensors on the vehicle to work all the time, allowing for fuel economy and other reasons.

Aiming at solving the aforementioned problems, the formation-containment problem of the second-order multi-agent system (SOMAS) is studied in this paper, and the control framework of pulse-modulated intermittent control (PMIC) is adopted. The main contributions are given as follows. (i) The formation and the containment are achieved simultaneously in second-order MASs, where the leaders form a formation and the followers are contained in this formation. (ii) The PIMC is a framework that can unify impulsive control [8, 15] and zero-order sampling control [16]. It can be applied to a wider range of real-world scenarios. (iii) In this paper, some sufficient conditions are given for the parameters of the control strategy under with/without time delay cases.

The remainder sections of this paper are described as follows. Section 2 lists the basic preliminary knowledge and a model of the problem to be studied. Section 3 introduces the formation-containment analysis without time delay. Section 4 obtains and provides the results when MASs contain time delay. Section 5 gives several simulations to verify the theorems are correct. Section 6 draws the conclusion.

2. Preliminaries

2.1. Graph Theory and Some Lemmas. Let $G = (V, E, A)$ be a weighted digraph, where $V = \{1, \dots, N\}$ is a vertex set, $E \subseteq V \times V$ is a link set, and $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ is a nonnegative weighted adjacency matrix. The information flow from vertex j to vertex i is represented by a directed link $e_{ij} = (j, i)$. The elements of matrix A are described as follows: $a_{ij} > 0$ if $e_{ij} \in E$, and $a_{ij} = 0$, otherwise. Furthermore, $a_{ii} = 0$ for all $i \in V$, and $a_{ij} = a_{ji}$ in an undirected topology. Denote $N_i = \{v_j \in V : (v_j, v_i) \in E\}$ as the set of neighbors of node v_i . The Laplacian matrix L is described in terms of $L = D - A$, where $D = \text{diag}\{D_1, D_2, \dots, D_N\}$ is the in-degree matrix with $D_i = \sum_{j=1}^N a_{ij}$. A sequence of edges with different nodes of V as $\{(v_i, v_{i1}), (v_i, v_{i2}), \dots, (v_i, v_j)\}$ can be obtained if there is a directed path from the i th agent to the j th agent.

In a MAS, a SOMAS with N agents is considered, and there are M followers and $N - M$ leaders. Assume that followers can receive messages from the leaders or followers, while the leaders can only receive messages from the leaders. Let $F = \{1, 2, \dots, M\}$ and $E = \{M + 1, M + 2, \dots, N\}$ denote the sets of followers and leaders, respectively. And the Laplacian matrix L_N is described as

$$L_N = \begin{bmatrix} L_1 & L_2 \\ 0 & L_3 \end{bmatrix}, \quad (1)$$

where $L_1 \in \mathbb{R}^{M \times M}$, $L_2 \in \mathbb{R}^{M \times (N-M)}$, and $L_3 \in \mathbb{R}^{(N-M) \times (N-M)}$.

Lemma 1 (see [17]). *A complex characteristic polynomial $R(z) = z^2 + sz + r$ is Hurwitz stable if and only if $\text{Re}(s) > 0$ and $\text{Re}(s)\text{Im}(s)\text{Im}(r) + \text{Re}^2(s)\text{Re}(r) - \text{Im}^2(r) > 0$.*

Lemma 2 (see [18]). *If directed graph G contains a spanning tree, then the Laplacian matrix $L \in \mathbb{R}^{N \times N}$ of G has a simple zero eigenvalue with $\vec{1}_N$ as the associated eigenvector, and all the other $N - 1$ eigenvalues have positive real parts.*

Assumption 1. Each follower of graph G has at least one directed path from one leader.

Assumption 2. Let G_E be the graph associated with the leaders in the MAS, and G_E contains a spanning tree.

Lemma 3 (see [19]). *By Assumptions 1 and 2, it is obtained that the eigenvalues of L_1 have positive real parts, each row of $-L_1^{-1}L_2$ has a sum equal, and each entry of $-L_1^{-1}L_2$ is nonnegative.*

2.2. Model Formulation and Some Definitions. The control input of the i th agent is denoted by $u_i \in \mathbb{R}^n$, position by $p_i \in \mathbb{R}^n$, and velocity by $v_i \in \mathbb{R}^n$, respectively. Consider a SOMAS as

$$\begin{aligned} \dot{p}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t). \end{aligned} \quad (2)$$

In the following, for the sake of description, we assume that $n = 1$. However, more cases such as $n > 1$ can be derived by using the Kronecker product.

Let, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $x_i(t) = [p_i(t), v_i(t)]^T$ ($i = 1, 2, \dots, N$). Then, the dynamic equation of the i th agent can be described in a neat form as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t). \quad (3)$$

Definition 1. With the bounded initial state of each agent being chosen arbitrarily, the leaders in MAS (3) are said to realize time-varying formation if there exists a vector function $c(t) \in \mathbb{R}^2$ such that

$$\lim_{t \rightarrow \infty} (x_i(t) - h_i(t) - c(t)) = 0 \quad (i \in E), \quad (4)$$

where $h_i(t) = [h_{ip}(t), h_{iv}(t)]^T$ is a piecewise continuously differentiable vector. Then, we can denote the time-varying formation as $h_E(t) = [h_{M+1}^T(t), h_{M+2}^T(t), \dots, h_N^T(t)]^T \in \mathbb{R}^{2(N-M)}$.

Definition 2. Similarly, with the bounded initial state of each agent being chosen arbitrarily, MAS (3) is said to achieve containment if there exist nonnegative constants $\tau_{k,j}$ ($k \in F, j \in E$) such that $\sum_{j=M+1}^N \tau_{k,j} = 1$ and

$$\lim_{t \rightarrow \infty} \left(x_k(t) - \sum_{j=M+1}^N \tau_{k,j} x_j(t) \right) = 0. \quad (5)$$

3. Formation-Containment Analysis without Time Delay

In this section, we mainly study how to design the PMIC protocol to make MAS (3) achieve the time-varying formation-containment and propose some sufficient and necessary conditions for parameters. We will investigate the time-varying formation-containment problem in two steps. The first step is to transform the formation-containment problem into a stability problem. The second step is to solve the stability problem according to the related theory.

3.1. Problem Transformation. Consider the following PMIC protocols:

$$\begin{aligned} u_i(t) &= a(t - t_k) K_1 \sum_{j \in N_i} a_{ij} ((x_i(t_k) - h_i(t_k)) \\ &\quad - (x_j(t_k) - h_j(t_k))), \quad i \in E, \\ u_i(t) &= a(t - t_k) K_2 \sum_{j \in N_i} a_{ij} (x_i(t_k) - x_j(t_k)), \quad i \in F, \end{aligned} \quad (6)$$

where $K_1 = [k_{11}, k_{12}] \in \mathbb{R}^{1 \times 2}$ and $K_2 = [k_{21}, k_{22}] \in \mathbb{R}^{1 \times 2}$ are feedback gain matrices and $t_k < t < t_{k+1}$. The pulse function $a(t)$ is described as

$$a(t) = \begin{cases} \hat{a}(t), & t \in (0, d], \\ 0, & t \notin (0, d], \end{cases} \quad (7)$$

where $\hat{a}(t)$ is a piecewise continuous function. Let $T = t_{k+1} - t_k$ and $d < T$ be the control duration within a complete sampling cycle. $(t_k + d, t_{k+1}]$ is the rest interval, and $(t_k, t_k + d]$ is the control interval [20].

Under the control framework (6), MAS (3) can be described in a compact form as follows:

$$\begin{aligned} \dot{x}_E(t) &= (I_{N-M} \otimes A) x_E(t) + a(t - t_k) ((L_3 \otimes BK_1) x_E(t_k) \\ &\quad - (L_3 \otimes BK_1) h_E(t_k)), \\ \dot{x}_F(t) &= (I_M \otimes A) x_F(t) + a(t - t_k) ((L_1 \otimes BK_2) x_F(t_k) \\ &\quad + (L_2 \otimes BK_2) x_E(t_k)), \end{aligned} \quad (8)$$

where $x_E(t) = [x_{M+1}^T(t), x_{M+2}^T(t), \dots, x_N^T(t)]^T$ and $x_F(t) = [x_1^T(t), x_2^T(t), \dots, x_M^T(t)]^T$.

Let $\theta_i(t) = x_i(t) - h_i(t)$ ($i \in E$) and $\theta_E(t) = [\theta_{M+1}^T(t), \theta_{M+2}^T(t), \dots, \theta_N^T(t)]^T$. Then, system (8) can be written as

$$\begin{aligned} \dot{\theta}_E(t) &= (I_{N-M} \otimes A) \theta_E(t) + a(t - t_k) (L_3 \otimes BK_1) \theta_E(t_k) \\ &\quad + (I_{N-M} \otimes A) h_E(t) - (I_{N-M} \otimes I_2) \dot{h}_E(t). \end{aligned} \quad (9)$$

The eigenvalues of L_3 relating to G_E are denoted by λ_i ($i \in E$), where $\lambda_{M+1} = 0$ with the eigenvector as

$p_{M+1} = (\mathbf{1}_{N-M} / \sqrt{N-M})$, and $0 < \text{Re}(\lambda_{M+2}) \leq \dots \leq \text{Re}(\lambda_N)$. Let $P_E^{-1} L_3 P_E = J_E$, where J_E is the Jordan canonical form of L_3 , $P_E = [p_{M+1}, p_{M+2}, \dots, p_N]$, and $P_E^{-1} = [\tilde{p}_{M+1}, \tilde{p}_{M+2}, \dots, \tilde{p}_N]^H$. By Lemma 2, one has $J_E = \text{diag}\{0, \tilde{J}_E\}$, where $\tilde{J}_E$ consists of the Jordan blocks relating to λ_i ($i = M+2, M+3, \dots, N$). Let $\tilde{\theta}_E(t) = (P_E^{-1} \otimes I_2) \theta_E(t)$; then, system (9) can be rewritten as

$$\begin{aligned} \dot{\tilde{\theta}}_E(t) &= (I_{N-M} \otimes A) \tilde{\theta}_E(t) + a(t - t_k) (P_E^{-1} L_3 P_E \otimes BK_1) \tilde{\theta}_E(t_k) \\ &\quad + (P_E^{-1} \otimes A) h_E(t) - (P_E^{-1} \otimes I_2) \dot{h}_E(t). \end{aligned} \quad (10)$$

Let $\tilde{P}_E = [\tilde{p}_{M+2}, \tilde{p}_{M+3}, \dots, \tilde{p}_N]^H$, $\alpha_E(t) = (\tilde{P}_{M+1}^H \otimes I_2) \tilde{\theta}_E(t)$, and $\beta_E(t) = (\tilde{P}_E \otimes I_2) \tilde{\theta}_E(t)$. Then, system (10) can be divided as

$$\begin{aligned} \dot{\alpha}_E(t) &= A \alpha_E(t) + (\tilde{P}_{M+1}^H \otimes A) h_E(t) - (\tilde{P}_{M+1}^H \otimes I_2) \dot{h}_E(t), \\ \dot{\beta}_E(t) &= (I_{N-M-1} \otimes A) \beta_E(t) + a(t - t_k) (\tilde{J}_E \otimes BK_1) \beta_E(t_k) \\ &\quad + (\tilde{P}_E \otimes A) h_E(t) - (\tilde{P}_E \otimes I_2) \dot{h}_E(t). \end{aligned} \quad (11)$$

Let $\delta_i = \sum_{j \in N_i} w_{ij} (x_i(t) - x_j(t))$, $i \in F$, and $\delta_F(t) = [\delta_1(t), \delta_2(t), \dots, \delta_M(t)]$. Then, the following lemma is used to transform the formation-containment problem.

Lemma 4. MAS (3) under the PMIC framework (6) can achieve time-varying formation-containment if

$$\begin{cases} \lim_{t \rightarrow \infty} \beta_E(t) = 0_{2(N-M-1)}, \\ \lim_{t \rightarrow \infty} \delta_F(t) = 0_{2M}. \end{cases} \quad (12)$$

Proof. It is able to be proved by a similar way in [14]. $\square$

Remark 1. Lemma 4 is proved because the following results in [14] are worked out:

$$\begin{aligned} \lim_{t \rightarrow \infty} \left(x_E(t) - h_E(t) - \left(\frac{\mathbf{1}_{N-M}}{\sqrt{N-M}} \otimes I_2 \right) \alpha_E(t) \right) &= 0_{2(N-M)}, \\ \lim_{t \rightarrow \infty} \left(x_F(t) - (-L_1^{-1} L_2 \otimes I_2) x_E(t) \right) &= 0_{2M}. \end{aligned} \quad (13)$$

Considering Lemma 3, we can conclude that equation (13) satisfies Definitions 1 and 2, respectively. In other words, Lemma 4 can be proved by the above two equations.

3.2. Control Design. By means of Lemma 4, the time-varying formation-containment problem can be transformed into the convergence analysis of $\beta_E(t)$ and $\delta_F(t)$. This section presents the conditions for the parameters in control protocol (6) when $\beta_E(t)$ and $\delta_F(t)$ converge to 0.

Assumption 3. For $\forall i \in \{M+1, M+2, \dots, N\}$,

$$\lim_{t \rightarrow \infty} (A(h_i(t) - h_j(t)) - (\dot{h}_i(t) - \dot{h}_j(t))) = 0, \quad j \in N_i. \quad (14)$$

Lemma 5. *Let*

$$\dot{\tilde{\beta}}_E(t) = (I_{N-M-1} \otimes A)\tilde{\beta}_E(t) + a(t - t_k)(\bar{J}_E \otimes BK_1)\tilde{\beta}_E(t_k). \quad (15)$$

Under Assumption 3, $\lim_{t \rightarrow \infty} \beta_E(t) = 0_{2(N-M-1)}$ is satisfied if

$$\lim_{t \rightarrow \infty} \tilde{\beta}_E(t) = 0_{2(N-M-1)}. \quad (16)$$

Proof. If Assumption 3 holds, one has

$$\lim_{t \rightarrow \infty} ((L_3 \otimes A)h_E(t) - (L_3 \otimes I_2)\dot{h}_E(t)) = 0. \quad (17)$$

Substitute $L_3 = P_E J_E P_E^{-1}$ into (17), and then premultiply both sides of (17) by $P_E^{-1} \otimes I_2$. One can obtain that

$$\lim_{t \rightarrow \infty} ((\bar{J}_E \bar{P}_E \otimes A)h_E(t) - (\bar{J}_E \bar{P}_E \otimes I_2)\dot{h}_E(t)) = 0. \quad (18)$$

Under Assumption 2 and Lemma 2, $\bar{J}_E$ is nonsingular, obviously. By premultiplying both sides of (18) by $\bar{J}_E^{-1} \otimes I_2$, we have

$$\lim_{t \rightarrow \infty} ((\tilde{P}_E \otimes A)h_E(t) - (\tilde{P}_E \otimes I_2)\dot{h}_E(t)) = 0. \quad (19)$$

Considering equation (19), Lemma 5 is set up. $\square$

Lemma 6. $\forall i \in \{M+2, M+3, \dots, N\}$, equation (16) holds if the following inequalities are satisfied:

$$\frac{k_{12}}{k_{11}} > \frac{d_2}{d_1}, \quad (20)$$

$$\left(\frac{2k_{12}}{k_{11}T} - \frac{2d_2}{d_1T} \right)^2 \left(\frac{2d_2}{d_1T} - \frac{2k_{12}}{k_{11}T} - \frac{4\text{Re}(\lambda_i)}{k_{11}d_1|\lambda_i|^2T} - 1 \right) - \frac{16\text{Im}^2(\lambda_i)}{k_{11}^2d_1^2|\lambda_i|^4T^2} > 0, \quad (21)$$

where $d_1 = \int_0^d \hat{a}(s)ds$ and $d_2 = \int_0^d s\hat{a}(s)ds$.

Proof. Equation (16) holds if and only if $\tilde{\beta}_E$ in (15) is asymptotically stable. The solution of (15) can be written as

$$\tilde{\beta}_E(t) = e^{C(t-t_0)}\tilde{\beta}_E(t_0) + e^{Ct} \int_{t_0}^t e^{-Cs} a(s - t_k) D \tilde{\beta}_E(t_k) ds, \quad (22)$$

where $C = I_{N-M-1} \otimes A$ and $D = \bar{J}_E \otimes BK_1$. Let $t_k = t_0$ and $t_k + d = t$ in (22); then, we have

$$\begin{aligned} \tilde{\beta}_E(t_k + d) &= e^{Cd} \tilde{\beta}_E(t_k) + e^{C(t_k+d)} \int_{t_k}^{t_k+d} e^{-Cs} a(s - t_k) \\ &\quad D \tilde{\beta}_E(t_k) ds. \end{aligned} \quad (23)$$

According to $C^2 = 0a$ and the theory of matrix function, one can obtain that

$$\tilde{\beta}_E(t_k + d) = (I_{2(N-M-1)} + dC + d_1D + (dd_1 - d_2)CD)\tilde{\beta}_E(t_k). \quad (24)$$

Notice that $\tilde{\beta}_E(t_{k+1}) = \tilde{\beta}_E(t_k + T)$; then, the system can be discretely expressed as

$$\tilde{\beta}_E(t_{k+1}) = (I_{2(N-M-1)} + TC + d_1D + (Td_1 - d_2)CD)\tilde{\beta}_E(t_k). \quad (25)$$

Let $\Gamma = I_{2(N-M-1)} + TC + d_1D + (Td_1 - d_2)CD$ and $\det(sI_{2(N-M-1)} - \Gamma) = 0$. It yields

$$s^2 - [2 + k_{12}d_1\lambda_i + k_{11}\lambda_i(Td_1 - d_2)]s + 1 + k_{12}d_1\lambda_i - k_{11}d_2\lambda_i = 0. \quad (26)$$

Considering $\lambda_i \neq 0$, then it is focused on the conditions that the eigenvalues of Γ are encircled by the unit circle. By applying a bilinear transformation, $z = ((s+1)/(s-1))$, an updated polynomial can be found as

$$\begin{aligned} R(z) &= z^2 + \frac{2(k_{12}d_1\lambda_i - k_{11}\lambda_id_2)}{k_{11}\lambda_id_1T}z \\ &\quad + \frac{2k_{11}\lambda_id_2 - 2k_{12}d_1\lambda_i - k_{11}d_1\lambda_iT - 4}{k_{11}\lambda_id_1T}. \end{aligned} \quad (27)$$

$\|s\| < 1$ in (26) can be obtained if and only if $R(z)$ in (27) is Hurwitz stable. Based on Lemma 1, if (27) is stable, then both equations (20) and (21) hold. $\square$

Lemma 7. $\lim_{t \rightarrow \infty} \delta_F(t) = 0_{2M}$ holds if and only if $\forall i \in \{1, 2, \dots, M\}$,

$$\frac{k_{22}}{k_{21}} > \frac{d_2}{d_1}, \quad (28)$$

$$\left(\frac{2k_{22}}{k_{21}T} - \frac{2d_2}{d_1T} \right)^2 \left(\frac{2d_2}{d_1T} - \frac{2k_{22}}{k_{21}T} - \frac{4\text{Re}(\lambda_i)}{k_{21}d_1|\lambda_i|^2T} - 1 \right) - \frac{16\text{Im}^2(\lambda_i)}{k_{21}^2d_1^2|\lambda_i|^4T^2} > 0, \quad (29)$$

where $d_1 = \int_0^d \hat{a}(s)ds$ and $d_2 = \int_0^d s\hat{a}(s)ds$.

Proof. It is obtained that

$$\delta_F(t) = (L_2 \otimes I_2)x_E(t) + (L_1 \otimes I_2)x_F(t). \quad (30)$$

Substituting (8) into (30), we can obtain that

$$\begin{aligned} \dot{\delta}_F(t) &= (I_M \otimes A)\delta_F(t) + a(t - t_k)(L_1 \otimes BK_2)\delta_F(t_k) \\ &\quad a(t - t_k)(L_2L_3 \otimes BK_1)(x_E(t_k) - h_E(t_k)). \end{aligned} \quad (31)$$

By Lemmas 4 and 5, MAS (8) can achieve formation $h_E(t)$. Considering Definition 1, when the leaders' formation $h_E(t)$ is achieved, then

$$\begin{aligned} \lim_{t \rightarrow \infty} (L_2 L_3 \otimes BK_1)(x_E(t_k) - h_E(t_k)) \\ = \lim_{t \rightarrow \infty} (L_2 L_3 \otimes BK_1)(\mathbf{1} \otimes c(t_k)). \end{aligned} \quad (32)$$

Since $L_3 \mathbf{1} = 0$, one has

$$\lim_{t \rightarrow \infty} (L_2 L_3 \otimes BK_1)(x_E(t_k) - h_E(t_k)) = 0. \quad (33)$$

Then, consider the following system:

$$\dot{\delta}_F(t) = (I_M \otimes A)\delta_F(t) + a(t - t_k)(L_1 \otimes BK_2)\delta_F(t_k). \quad (34)$$

$\lim_{t \rightarrow \infty} \delta_F(t) = 0_{2M}$ means that system (34) is asymptotically stable. Similar to the analysis of Lemma 5, the conditions that make system (34) asymptotically stable are worked out with (28) and (29). $\square$

Theorem 1. MAS (3) under Assumption 3 and PMIC framework (6) realizes formation-containment if the following conditions simultaneously hold:

$$(i) \forall i \in \{1, 2, \dots, M\},$$

$$\frac{k_{22}}{k_{21}} > \frac{d_2}{d_1},$$

$$\left(\frac{2k_{22}}{k_{21}T} - \frac{2d_2}{d_1T} \right)^2 \left(\frac{2d_2}{d_1T} - \frac{2k_{22}}{k_{21}T} - \frac{4\text{Re}(\lambda_i)}{k_{21}d_1|\lambda_i|^2T} - 1 \right) - \frac{16\text{Im}^2(\lambda_i)}{k_{21}^2d_1^2|\lambda_i|^4T^2} > 0. \quad (35)$$

$$(ii) \forall i \in \{M+2, M+3, \dots, N\},$$

$$\frac{k_{12}}{k_{11}} > \frac{d_2}{d_1},$$

$$\left(\frac{2k_{12}}{k_{11}T} - \frac{2d_2}{d_1T} \right)^2 \left(\frac{2d_2}{d_1T} - \frac{2k_{12}}{k_{11}T} - \frac{4\text{Re}(\lambda_i)}{k_{11}d_1|\lambda_i|^2T} - 1 \right) - \frac{16\text{Im}^2(\lambda_i)}{k_{11}^2d_1^2|\lambda_i|^4T^2} > 0, \quad (36)$$

where $d_1 = \int_0^d \hat{a}(s)ds$ and $d_2 = \int_0^d s\hat{a}(s)ds$.

Proof. It is proved naturally by Lemmas 4, 6, and 7. $\square$

Remark 2. When selecting parameters in the control rules, we generally give K_1, K_2, d_1 , and d_2 that meet the conditions first. According to the values of these parameters and conditions (35) and (36) in Theorem 1, we can solve parameter T as follows:

$$(i) \forall i \in \{1, 2, \dots, M\},$$

$$T < \min \frac{M_1^3|\lambda_i|^4 + 4\text{Re}(\lambda_i)|\lambda_i|^2M_1^2}{16k_{21}d_1\text{Im}^2(\lambda_i) + k_{21}d_1M_1^2|\lambda_i|^4}, \quad (37)$$

where $M_1 = 2k_{21}d_2 - 2k_{22}d_1$.

$$(ii) \forall i \in \{M+2, M+3, \dots, N\},$$

$$T < \min \frac{M_1^3|\lambda_i|^4 + 4\text{Re}(\lambda_i)|\lambda_i|^2M_1^2}{16k_{11}d_1\text{Im}^2(\lambda_i) + k_{11}d_1M_1^2|\lambda_i|^4}, \quad (38)$$

where $M_2 = 2k_{11}d_2 - 2k_{12}d_1$.

4. Formation-Containment Analysis with Time Delay

We know that the time delay cannot be ignored due to its widely existence in the real world. Then, a control protocol with considering time delay is proposed as follows:

$$\begin{aligned} u_i(t) &= a(t - t_k)K_1 \sum_{j \in N_i} w_{ij}((x_i(t_k - \tau) - h_i(t_k)) - (x_j(t_k - \tau) - h_j(t_k))), \quad i \in E, \\ u_i(t) &= a(t - t_k)K_2 \sum_{j \in N_i} w_{ij}(x_i(t_k - \tau) - x_j(t_k - \tau)), \quad i \in F. \end{aligned} \quad (39)$$

The control protocol (39) is substituted into MAS (3), and we can obtain

$$\begin{aligned}\dot{x}_E(t) &= (I_{N-M} \otimes A)x_E(t) + a(t-t_k)((L_3 \otimes BK_1)x_E(t_k - \tau) - (L_3 \otimes BK_1)h_E(t_k)), \\ \dot{x}_F(t) &= (I_M \otimes A)x_F(t) + a(t-t_k)((L_1 \otimes BK_2)x_F(t_k - \tau) + (L_2 \otimes BK_2)x_E(t_k - \tau)).\end{aligned}\quad (40)$$

Considering Assumption 3, when there is a time delay in the system, similar to Section 3, we have

$$\begin{aligned}\dot{\tilde{\beta}}_E(t) &= (I_{N-M-1} \otimes A)\tilde{\beta}_E(t) + a(t-t_k)(\bar{J}_E \otimes BK_1)\tilde{\beta}_E(t_k - \tau), \\ \dot{\delta}_F(t) &= (I_M \otimes A)\delta_F(t) + a(t-t_k)(L_1 \otimes BK_2)\delta_F(t_k - \tau).\end{aligned}\quad (41)$$

It is not hard to see that Lemma 4 also applies to cases with time delay. Therefore, we still transform the formation-containment problem of the SOMAS into the stability problem of the system.

Remark 3. Considering that the time delay of the actual system is not long and the intermittent control has the

advantage of selectively adjusting the control time interval d , we assume that the time delay $\tau < \min\{d, T - d\}$.

Lemma 8. $\lim_{t \rightarrow \infty} \tilde{\beta}_E(t) = 0_{2(N-M-1)}$ holds with time delay if $\forall i \in \{M+2, M+3, \dots, N\}$,

$$-\frac{2\tau}{T} - \frac{2d_2}{d_1 T} + \frac{2k_{12}}{k_{11} T} > 0, \quad (42)$$

$$\left(-\frac{2\tau}{T} - \frac{2d_2}{d_1 T} + \frac{2k_{12}}{k_{11} T}\right)^2 \left[-\frac{4\text{Re}(\lambda_i)}{|\lambda_i|^2 k_{11} d_1 T} - 1 + \frac{2\tau}{T} + \frac{2d_2}{d_1 T} - \frac{2k_{12}}{k_{11} T}\right] - \frac{16\text{Im}^2(\lambda_i)}{|\lambda_i|^4 k_{11}^2 d_1^2 T^2} > 0, \quad (43)$$

where $d_1 = \int_0^d \hat{a}(s)ds$, $d_2 = \int_0^d s\hat{a}(s)ds$, and τ is the time delay.

Proof. Similarly as Lemma 6, differential equation (41) is solved. Since $\tau < \min\{d, T - d\}$, when $t \in [t_k, t_{k+1})$, we can obtain that

$$\tilde{\beta}_i(t) = M(t-t_k)\tilde{\beta}_i(t_k) + N(t-t_k)\tilde{\beta}_i(t_k - \tau), \quad (44)$$

where $M(\chi) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$ and $N(\chi) = \begin{bmatrix} (td_1 - d_2)\lambda_i k_{11} & (td_1 - d_2)\lambda_i k_{12} \\ \lambda_i k_{11} d_1 & \lambda_i k_{12} d_1 \end{bmatrix}$. To solve the stability problem, we need to consider the iterative problem of the system. When $t = t_k$, one has

$$\begin{aligned}\tilde{\beta}_i(t_k) &= M(T)\tilde{\beta}_i(t_{k-1}) + N(T)\tilde{\beta}_i(t_{k-1} - \tau), \\ \tilde{\beta}_i(t_k - \tau) &= M(T - \tau)\tilde{\beta}_i(t_{k-1}) + N(T - \tau)\tilde{\beta}_i(t_{k-1} - \tau).\end{aligned}\quad (45)$$

Let $\omega_i(t) = \begin{bmatrix} \tilde{\beta}_i(t) \\ \tilde{\beta}_i(t - \tau) \end{bmatrix}$; combined with (43), it is obtained that

$$\omega_i(t) = \begin{bmatrix} \tilde{\beta}_i(t) \\ \tilde{\beta}_i(t - \tau) \end{bmatrix} = \Gamma_1 \Gamma_2^k \begin{bmatrix} \tilde{\beta}_i(t_0) \\ \tilde{\beta}_i(t_0 - \tau) \end{bmatrix}, \quad (46)$$

where $\Gamma_1 = \begin{bmatrix} M(t-t_k) & N(t-t_k) \\ M(t-t_k - \tau) & N(t-t_k - \tau) \end{bmatrix}$ and

$\Gamma_2 = \begin{bmatrix} M(T) & N(T) \\ M(T - \tau) & N(T - \tau) \end{bmatrix}$. It is easy to get that

$\omega_i(t) \rightarrow 0$ if the eigenvalues of Γ_2 meet $\|s\| < 1$. Let $|sI_4 - \Gamma_2| = 0$. It yields

$$s^4 + a_1 s^3 + a_0 s^2 = 0, \quad (47)$$

where $a_1 = -\lambda_i k_{11} d_1 T + \lambda_i k_{11} d_1 \tau + \lambda_i k_{11} d_2 - \lambda_i k_{12} d_1 - 2$ and $a_0 = -\lambda_i k_{11} d_1 \tau - \lambda_i k_{11} d_2 + \lambda_i k_{12} d_1 + 1$. We can obtain that two eigenvalues of Γ_2 satisfy $s_1 = s_2 = 0$. Then, the remaining proof process has been omitted, which is similar to the proof of Lemma 6. $\square$

Lemma 9. $\lim_{t \rightarrow \infty} \delta_F(t) = 0_{2M}$ holds with time delay if $\forall i \in \{1, 2, \dots, M\}$,

$$-\frac{2\tau}{T} - \frac{2d_2}{d_1 T} + \frac{2k_{22}}{k_{21} T} > 0, \quad (48)$$

$$\left(\frac{2\tau}{T} - \frac{2d_2}{d_1 T} + \frac{2k_{22}}{k_{21} T} \right)^2 \left[-\frac{4\operatorname{Re}(\lambda_i)}{|\lambda_i|^2 k_{21} d_1 T} - 1 + \frac{2\tau}{T} + \frac{2d_2}{d_1 T} - \frac{2k_{22}}{k_{21} T} \right] - \frac{16\operatorname{Im}^2(\lambda_i)}{|\lambda_i|^4 k_{21}^2 d_1^2 T^2} > 0, \quad (49)$$

where $d_1 = \int_0^d \hat{a}(s)ds$, $d_2 = \int_0^d s\hat{a}(s)ds$, and τ is the time delay.

Proof. This proof is analogous to Lemma 8 and is therefore omitted. $\square$

Theorem 2. MAS (3) under Assumption 3 and control framework (39) realizes formation-containment if conditions (42), (43), (48), and (49) hold.

Proof. It is not hard to prove by applying Lemmas 4, 8, and 9. $\square$

5. Simulation

In this section, a two-dimensional formation-containment case of the MAS will be shown by numerical simulations. The simulations illustrate the effectiveness of the results in this paper. In this two-dimensional formation-containment case, suppose there are three leaders and two followers in the MAS. The Laplace matrix of the directed topology is given as

$$L = \begin{bmatrix} 2 & -\frac{2}{3} & -\frac{1}{3} & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \quad (50)$$

Note that $p_i(t) \in \mathbb{R}^2$, $v_i(t) \in \mathbb{R}^2$, and $x_i(t) = [p_{iX}(t), v_{iX}(t), p_{iY}(t), v_{iY}(t)]^T$, $i \in \{1, 2, \dots, 5\}$. Then, matrices A and B in dynamic equation (3) should be extended as

$$A = I_2 \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = I_2 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (51)$$

The formation function for leaders is given by

$$h_i(t) = \begin{bmatrix} \sin\left(t + \frac{2(i-3)\pi}{3}\right) \\ \cos\left(t + \frac{2(i-3)\pi}{3}\right) \\ \cos\left(t + \frac{2(i-3)\pi}{3}\right) \\ -\sin\left(t + \frac{2(i-3)\pi}{3}\right) \end{bmatrix} \quad (i = 3, 4, 5). \quad (52)$$

The eigenvalues of L are $\lambda_1 = 2.8165$, $\lambda_2 = 1.1835$, $\lambda_3 = 0$, $\lambda_4 = 1.5 + 0.866i$, and $\lambda_5 = 1.5 - 0.866i$. The function $\hat{a}(t) = (t/d)$ is set, and the control duration $d = 0.2$ is chosen; then, $d_1 = 0.1$ and $d_2 = 0.01333$.

When we do not care about time delays, it follows from (20) and (28) that $k_{12}/k_{11} > 0.1333$ and $k_{22}/k_{21} > 0.1333$. The feedback gains are chosen as $K_1 = I_2 \otimes [-1, -2]$ and $K_2 = I_2 \otimes [-1, -2]$. According to inequalities (21) and (29), $T < 1.5396$ is obtained, and $T = 0.6$ is chosen. When time delays are taken into account, we assume the time delay $\tau = 0.1$. According to Lemmas 8 and 9, we also choose the feedback gains as $K_1 = I_2 \otimes [-1, -2]$ and $K_2 = I_2 \otimes [-1, -2]$ and set $T = 0.6$.

The simulation results are shown in figures. Among them, Figures 1 and 2 are the trajectories of the position and velocity of each agent changing without considering the time delay, in which the initial state is marked with a circle and the final state is marked with a triangular row. Figures 3 and 4 show the trajectories when time delay is taken into account. The same initial state is indicated by a circle, and the final state is indicated by a triangular row. The initial states of all agents are randomly selected.

During the simulation, the five agents begin with arbitrary initial states. As time goes on, the three leaders reach a time-varying circular formation, and two followers move into the convex hull formed by leaders. At the same time, it can be seen that the system with time delay converges more slowly with the same sampling period.

FIGURE 1: The trajectory of agents' positions over time without time delay.

FIGURE 2: The trajectory of agents' velocities over time without time delay.

FIGURE 3: The trajectory of agents' positions over time with time delay.

FIGURE 4: The trajectory of agents' velocities over time with time delay.

6. Conclusion

Time-varying formation-containment problems of the SOMAS were studied in this paper. Based on the fundamental theorems of graph theory and matrix theory, the formation-containment problem is transformed into the stability problem of the SOMAS. The PMIC protocols are designed, and the sufficient conditions for the time-varying formation-containment of the SOMAS are given by proving three theorems. The simulations show the correctness of the proposed theoretical results. It is expected that the future work will be carried out on the nonlinear MAS, and the conclusion of this paper will be applied to the actual multi-intelligent vehicle experimental platform.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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