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Research Article

The Public Opinion Evolution under Group Interaction in Different Information Features

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Before expressing opinions, most people usually consider the standpoint of their friends nearby to avoid being isolated, which may lead to the herding effect. The words of celebrities in social networks usually attract public attention and affect the opinion evolution in the entire network. This process also causes the similar status quo. In this study, we find that the key figures play the guiding roles in public opinions who undertake the group pressure from information amount. Therefore, we build the cost function on opinion changes to study opinion evolution rules for public persons based on the spreading scope of information and information amount. Simulation analysis reveals that the information amount held by agents will affect the converging speed of public opinions, while enhancing the ability of key nodes may no more effective in guiding public opinion.

1. Introduction

The expression on individuals' opinions and spreading of information are evolutionary powers in the public opinion networks. The rapid spreading of information allows most agents in the networks to quickly evaluate information based on themselves judgments, resulting in the development of the irrespective opinion values. Opinions are developed from the information interaction and vary in public opinion networks because of the evolution on public opinion. When an agent has sufficient information about an event and has own knowledge on this specific event, his/her opinion can hardly to be influenced by the public environment. In this case, this agent can be regarded as "stubborn" one. Undoubtedly, on the Internet, the speeches of agents with various followers are also important. In public opinion networks, however, the information amount for the event held by the group, that is, the degree of their understanding of this event, will lead to varying response to opinions proposed by these agents. On the other hand, the spreading scope of information of special agents also has impacts on the overall opinion evolution in public opinion networks. Additionally, for a specific event, agents' opinions are divided into opposition, neutrality, and support, with slight differences in details. Agents tend to have preferences instead of fully supporting or opposing an event.

Opinion, as the carrier, is inseparable from information. Key agents, as one of the source of information, are bound to affect the development of public opinion. To study the influence of information changes on the opinions of different agents, the influence of information dissemination scope on group opinions, we propose an opinion evolution formula based on the cost function and public opinion pressure. In theory, our work expands the continuous opinion evolution model based on the cost function, which make us study the change rule on opinion and information specifically. In practical, our research can provide relevant departments with measurements for supervision and management on public opinion to some extent. It can effectively reduce the consumption on public resource and guide the development on public opinion.

The remainder of the paper is organized as follows: we review related literature and compare current works with our work in Section 2. In Section 3, with the growth function

on information, the pressure function on public opinion and cost function, we construct the opinion dynamic models based on BBV network for different agents. Section 4 compares simulation results with different information environment to further study the influence of information amount and information scope on public opinion evolution. Finally, conclusions and contributions are given in Section 5.

2. Literature Review

The development of social group opinions is embedded in the process of social interaction [1], and the spread of numerous opinions is accomplished by interactions between agents in the society [2, 3]. Studies of the evolution of group opinions can reflect and explain various complex social phenomena, including the aggregation of group opinions and the spread of rumors. It can also be applied to group decision-making and sociology, as well as the development of group wisdom. Examples include studying how social media and human behavior jointly influence the spread of infectious diseases through an epidemic model [1], dividing social trust into a continuous range to study how business reputation of firms can be improved in business networks [2], or introducing game theory and treating public opinion as a continuous interval within [0,1], and discussing the conformity and manipulation behavior of agents in realistic opinion networks and studying agent voting choices [3–6]. Hence, the study of the spread mechanism of group opinions can clarify various political, economic, and management phenomena, including popularity, the existence of minority opinions, consistency and diversity, and the leading role of the government [7–14]. To date, various studies of dynamics of opinion evolution have been reported. Previous studies on opinion evolution models have focused on discrete/continuous opinions, which greatly simplify the opinions expressed by agents in the social network, usually using plain interaction rules, and thus describe the most representative agent behavior and associated agent interaction characteristics. Most of the group opinions in discrete opinion models are classified as pro, con and neutral or buy and sell, left and right, usually denoted by 1, −1, 0. Discrete models include the Ising model [15–18], the Sznajd model [19–22],and the majority decision model [23–27]. Although the discrete opinion is consistent with the judgment of "right and wrong" issues, however, these models can hardly reflect the preference degree of agent opinions. In practical, agent opinions are somehow different from each other, while opinions held at a specific moment can be regarded as strong and steady. Usually, the value of an agent's opinion is normalized to a continuous closed interval [0, 1], which can reflect more information about the agent and describe the process of gradual change of the agent's opinion, such as the satisfaction evaluation of a product, the degree of confidence in a judgment, the degree of support, or opposition to an event. These characteristics can be well described by the continuous opinion evolution models, including the Deffuant model [28–31] and the HK (Hegsekmann-Krause) model [32–34] and their extended model.

A large number of extension studies have been generated based on the above-mentioned thinking approach. For example, considering agents in social networks tend to be influenced by pressures from agents nearby. Dong et al. presented a novel DW model combined with local world opinion from individuals' common friends where the opinions update depends on distance between individual opinions and network structure similarity. Finally, they analyzed the convergence of the model by simulation experiments [35]. Cheng and Yu proposed a modified HK model involving group pressure and claimed that the pressured agents can always reach a consensus infinite time [36]. Lu et al. reported that external pressure induced by public focus has negligible or weak influences [37]. Ferraioli and Ventre demonstrated that the dynamics in clique social networks always converge to consensus if the social pressure is sufficiently high [38]. For the influences of agent information on evolution of public opinions, the base agent model has a bounded confidence mode, in which information is introduced and different information releasing modes are explored [39]. Lan et al. proposed a statistical model for the influences of network rumors on the information amount of public opinion networks [40]. However, these studies did not simultaneously consider the influences of different factors.

Above studies mainly focus on the natural evolution of group opinions, but rarely study the effect of guidance from opinion leaders, and few consider the influence of the scope of opinion dissemination by opinion leaders in the evolution of group opinions. Comparison of related studies of public opinion on the impact of group pressure and agent information amount is shown in Table 1.

3. Modeling

With the in-depth study of complex network topology, numerous network models have been proposed to describe abstract social networks. Typical network models include small-world networks [41] and scale-free networks [42–44]. In most previous models, weights between nodes were not taken into account. However, in mainstream social platforms, such as Weibo or Facebook, key agents as hub nodes are not only obtain more attention, but also have a stronger impact on ordinary agents. Therefore, our work introduces to BBV network model [45] to describe the social network. In this network model, the degree, the weight and strength of agents in the network follow the power-law distribution.

We assume that social networks can be abstracted as a BBV network $G = \{\nu, \varepsilon, \delta\}$, where $\nu = \{1, 2, \ldots, k, k+1, .., k+n\}$ represents the node set, ε is the edge set, δ is the set of weight on the edge. Among where, k means the number of key agents and n means the number of normal agents. The parameter w_{ij} describes the weight between node i and its connected node j, which measure relation intensity between agents in our network model. Define $I_i \in [0, 1]$ as the information amount of agent i, the $A(0) = \{I_1(0), I_2(0), \ldots, I_k(0), I_{k+1}(0), \ldots, I_{k+n}(0)\}$ represents the information amount of all agents at initial time and the $N(0) = \{I_1(0), I_2(0), \ldots, I_n(0)\}$ represents the information amount

Table 1: Comparison of related studies of public opinion on the impact of group pressure and agent information amount.

References	Method	Opinion value	Information amount	Group pressure
Cheng and Yu [36]	Simulation	Continuous	No	Yes
Lu et al. [37]	Empirical analysis	Discrete	No	Yes
Ferraioli and Ventre [38]	Mathematical proof	Continuous	No	Yes
Zhu and Hu [39]	Empirical analysis simulation	Continuous	Yes	No
Lan et al. [40]	Simulation	Discrete	Yes	No
Current study	Simulation	Continuous	Yes	Yes

Table 2: Relevant important symbols.

Symbol	Implication
O_i	The opinion of agent i
d_i	The degree of agent i
F_i	The objective public opinion pressure on agent i
E_i	The actual public opinion pressure on agent i
KAO	The key agent's opinion value
KAI	The information amount of key agent
AAI	The maximum information amount of all agents
NAI	The maximum information amount of normal agents

of normal agents at initial time. In addition, to study the influence of information amount and group pressure on public opinion, we introduce some relevant important symbols, as shown in Table 2.

In social networks, all agents deliver their opinions and are influenced by agents nearby. Therefore, we divide agents into key agents, neighbour agents and other agents in our model. Key agents, such as opinion leaders, are the most important nodes in BBV network with the largest degree, weight, and strength. They think about not only their own ideas but also the public opinion environment before expressing their opinions. As an agent with more attention, they are under pressure from public opinion, so they convey the information on event by opinion more objectively and comprehensively. Neighbour agents are a kind of agents that directly connect with the key agents. Due to close influential key agents, their opinions are influenced by the key agents in part while insisting on themselves. Other agents are not directly connected to key agents, so they are not sensitive to information. In other words, the information sometimes cannot cover to them. Especially, the opinion evolution rule of other agents is the same as that of its neighbour agents. Finally, neighbour agents and other agents are called normal agents together.

To study the relationship between the amount of information, the scope of spread and the change of group opinion, we consider two modes on opinion spread. First, the limited spreading, that is, opinion interaction happens between key agents and neighbour agents only, which causes the change in group opinions. Second, the wide spreading, that is, when key agents and neighbour agents finish interacting, other agents are influenced by the connected neighbour agents and change their opinions unidirectionally at next time. After the above process is finished, the other agents spread the opinion to the next level other agents and influence the opinion evolution of the next level agents at next time, and so on. The opinion spread process is shown in Figure 1.

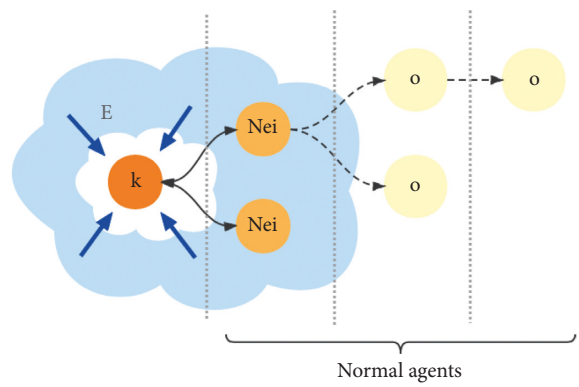

Figure 1: Information wide spreading process. (The node K represents key agents, the node Nei represents neighbour agents, the node O represents other agents, and the arrows show the direction of opinion flow.)

3.1. Growth Function of Information Amount. When each agent obtains information about a specific event first, he/she has different degrees of mastery for the information. Obviously, the closer to event an agent is, the more complete information can be obtained and the larger information amount is. As spreading distance of information increases, the degree of distortion and the degree of misinterpretation for the news increase, which means that the information and opinion deviate away from the event itself. However, with the interaction of agent opinions and information in public opinion networks, agents' information amount will increase and accumulate over time. We define $I_i \in [0, 1]$ as information amount of Agent i. For central agent i and its neighbour agent j, there are $I_i \cup I_j \leq 1, I_i \cap I_j \leq \min\{I_i, I_j\}$, that is, the maximum information amount grasped by a single agent in the network is 1, and the overlapping information amount between the two agents is less than 1; at the same time, the sum of information amount of two agents does not exceed 1 after the common information is removed. The growth rule of information amount for agent is defined as follows. This formula is applicable to the key agents and the neighbour agents of the key agents. When obtaining information unilaterally, other neighbour nodes in the surrounding area are not taken into account and only a single information source node is considered. Here, i is the central node and j is the neighbour node, k is the information growth coefficient:

If the affected agent is the key agent i,

$$I_i(t+1) = I_i(t)\cdot\left(1+k\frac{\tau}{I_i(t)}\right)\begin{cases}\in\left[0,\left|I_i(t)-\frac{\sum_{j\in\alpha i} I_j}{d_i}\right|\right], & \tau I_i \geq \frac{\sum_{j\in\alpha i} I_j}{d_i},\\ \tau = \left|I_i(t)-\frac{\sum_{j\in\alpha i} I_j}{d_i}\right|, & I_i < \frac{\sum_{j\in\alpha i} I_j}{d_i}.\end{cases} \tag{1}$$

When the affected node is neighbour agent j

$$I_j(t+1) = I_j(t)\cdot\left(1+k\frac{\tau}{I_j(t)}\right)\begin{cases}\tau\in\left[0,\left|I_j(t)-I_i\right|\right], & I_j \geq I_i,\\ \tau = \left|I_j(t)-I_i\right|, & I_j < I_i.\end{cases} \tag{2}$$

The parameter τ in the above equation is the information difference, which indicates the agent's judgment of the difference between the information amount held by neighbour and itself. For equation (1), i is the central node, at this time, we compare the information amount of average for all neighbouring nodes of i with itself. If the information of the central node is greater than the average information of the neighbour, the difference τ is randomly taken in limited range, where the minimum amount obtained by the agent i is 0 and the maximum is the difference value. Meanwhile, the coefficient k value is also smaller. On the contrary, if the information amount of node i is less than the average information amount of neighbour, the corresponding information difference τ is the difference between the two. Because node i believes that more available information is from the outside and the value of k is larger. Equation (2) considers that neighbour node j is influenced by the information from the central node i in the same way as before.

3.2. Public Opinion Pressure Function. Human beings are social, and the motivation on behavior is influenced by other people around. Before expressing their opinions, the agents will take into account the attitudes of other familiar neighbours around them. Under the influence of friends, the opinions of agents tend to the main stream. For the agent, the pressure that agents feel is inversely proportional to information amount they have, that is, the more information they have, the less likely they are to be influenced by other agents around them. In addition, they also pay attention to themselves opinions whether they are influenced by others or delivery opinions to the outside world.

Define the opinion pressure by output by all neighbour of agent i as $\gamma_i \in (0,+\infty)$, which is based on the difference in opinion and strength of the relationship between agent i and its neighbour. Considering the influence of the information amount held by itself, we define that the objective opinion pressure on agent i is $F_i \in (0,+\infty)$. The factual opinion pressure on agent i is $E_i \in [0,1]$ and is influenced by F_i and the parameter a (a refers to the "stress level," which reflects how much pressure an agent is subjected to before he or she starts to become patient with the growth of pressure from outside). In addition, because of diminishing marginal utility in human psychology, the logistic function is suitable for describing this process and this form has been proven and widely used [46–50]. In our work, as the pressure exerted by the surrounding nodes on node i increases, the factual opinion pressure E_i grows faster and slower and finally stabilizes. Therefore, the three following relationship equation is available:

$$\gamma_i = \sum_{j\in\alpha i} w_{ij}\left(O_j - O_i\right), F_i = \frac{\gamma_i}{I_i}, E_i = \frac{1}{1+e^{a-Fi}}. \tag{3}$$

3.3. Cost Function of Opinion Change and Optimal Strategy Formula. When a key agent is affected by a neighbour node, the influence and overall opinion value difference based on the relation intensity with the surrounding agents and the difference in opinion will be considered. The information amount grasped is combined to change the agent's own opinion value. And when the neighbour node receives the opinion value of the central agent, only the impact of the difference between the information amount and the opinion value and the relation intensity is considered because the information is obtained unilaterally. The interaction process of the opinion of a single agent and surrounding neighbour agents is divided into two stages: first the central agent is affected and opinions change and express, and then the opinions of neighbour change.

Referring to an opinion evolution rule based on cost function proposed by Li and Zhu [51], we construct the cost function under the external pressure from the neighbour. And then the best strategy formula describing the change on the opinion is deduced. The decision cost function for node i to change its opinion after being influenced by surrounding nodes is shown in equation (4). The two items on the right-hand side of the equation represent the cost of a change in perspective due to self-inflicted costs and the cost of external pressure, respectively,

$$J_i\left(O_i, O_{\alpha i}\right) = \frac{1}{2}I_i\left(O_i - O_i(t)\right)^2 + \frac{1}{2}E_i\sum_{j\in\alpha i}\left(O_i - O_j\right)^2. \tag{4}$$

Derivation of O_i in the formula is

$$\frac{\partial J_i}{\partial O_i} = I_i\left(O_i - O_i(t)\right) + E_i\sum_{j\in\alpha i}\left(O_i - O_j\right). \tag{5}$$

The cost minimization condition is $\partial J_i/\partial O_i = 0$. Rearrange the term

$$\begin{aligned} I_iO_i + E_id_iO_i &= I_iO_i(t) + E_i\sum_{j\in\alpha i} O_j,\\ O_i\left(I_i + E_id_i\right) &= I_iO_i(t) + E_i\sum_{j\in\alpha i} O_j,\\ O_i(t+1) &= \frac{I_i}{I_i+E_id_i}O_i(t) + \frac{E_i}{I_i+E_id_i}\sum_{j\in\alpha i} O_j. \end{aligned} \tag{6}$$

For the neighbour agent j of agent i, the equation for the cost of change on opinion regarding neighbour agent j is like

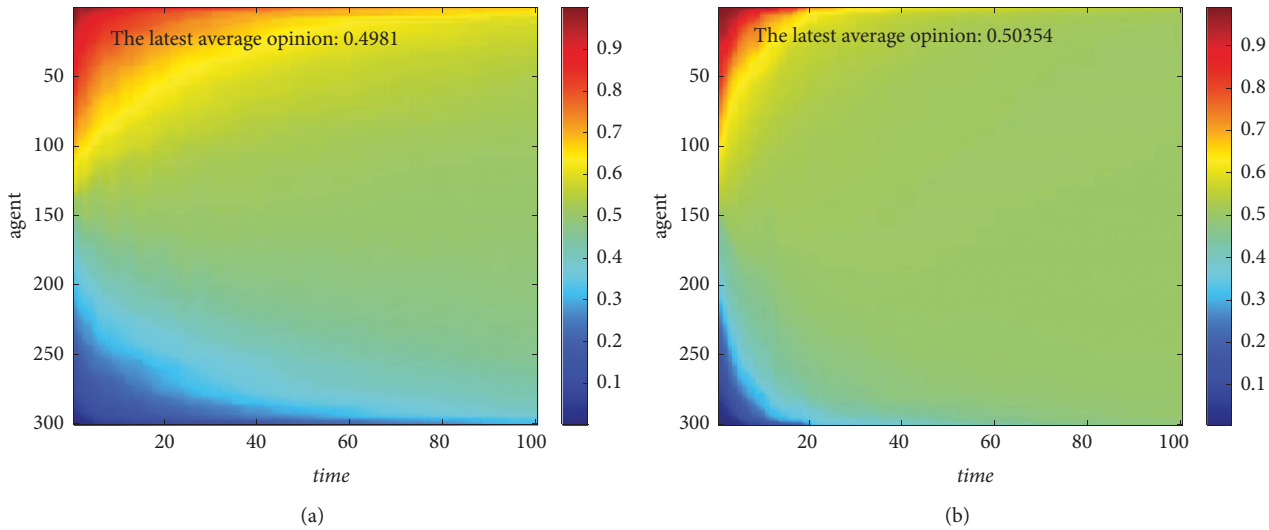

Figure 2: Comparison of the opinion distribution of the network with time. (a) $A(0) \sim U(0,1)$, (b) $A(0) \sim U(0,0.1)$.

the above, where O_i is a known number and O_j is unknown. In this case, because the agent j is influenced separately from the central agent i, the relationship and attribution between them is considered only. In equation (7), the first term on the right-hand side represents the cost function of the change on opinion held by agent j relative to the previous moment, and the second term is the cost function of the degree of influence of agent i on the opinion of agent j. The parameter w_{ij} represents the strength of the relationship between the two, and the magnitude of $|I_i - I_j|$ represents the difference in the amount of information between them.

$$J_j(O_j, O_i) = \frac{1}{2} I_j \left(O_j - O_j(t)\right)^2 + \frac{1}{2} w_{ij} |I_i - I_j| \left(O_i - O_j(t)\right)^2. \quad (7)$$

Similarly, take the derivative of O_j

$$\frac{\partial J_j}{\partial O_j} = I_j \left(O_j - O_j(t)\right) + w_{ij} |I_i - I_j| \left(O_i - O_j(t)\right). \quad (8)$$

The cost minimization condition is $\partial J_j / \partial O_j = 0$. Rearrange the term

$$O_j(t+1) = \frac{I_j}{w_{ij}|I_i - I_j| + I_j} O_j(t) + \frac{w_{ij}|I_i - I_j|}{w_{ij}|I_i - I_j| + I_j} O_i. \quad (9)$$

4. Simulation and Discussion

ABBV network with 300 nodes is established. To compare the simulation results, we set the maximum weight is 1. To discuss the influence of different information features on the public opinion, several situations are additionally set up. Additionally, we also introduce the wide spreading mechanism to explore whether opinions of Key agent sand information spreading will impact differently on the opinion evolution. Finally, we simulate the process on public opinion under different situations with the changes in information spreading scope and information amount. In the simulation, when the information amount of agent i is greater than or equal to that of agent j or neighbour average information amount, the information growth coefficient k in the formulas (1) and (2) is 0.001, otherwise it is 0.01. The coefficient a is 5 in the formula (3). To observe the evolution mechanism on group opinion and to combine with the scale-free characteristics for the network, we arrange all opinion values at each time step in descending order, which means that the agent label only represents the number instead of the serial number and does not represent the change of opinion of any agent in the horizontal axis direction overtime. Moreover, each time step iterates 60 times.

The above is the free evolution result of the BBV network with 300 nodes. Figure 2 shows the simulation results without any changes where the opinion value of all nodes is uniformly distributed in [0, 1], and the information amount is uniformly distributed in [0, 1]. In the figure, when it is more closed to the red part, the opinion value is more closed to 1; when it is more closed to the blue part, the opinion value is more closed to 0; and when it is more closed to green part, the opinion value is more closed to 0.5. It can be observed that the values of all agent opinions in the network converge to 0.5 with time in Figure 2(a). After 100 time steps, the final average opinion value of 0.4981 is close to 0.5. In Figure 3, the maximum information amount of all nodes is set to 0.1. Obviously, compared with Figure 2(a), the opinion of all agents in the network is still inclined to 0.5, but the convergence rate is faster at the same time. These two experiments are to simulate the evolution on public opinions under different information amount grasped by agents at the beginning of the event. Apparently, in the initial period of a specific event, less information the agents have, the more easily the opinions reach agreement. Imagine that in public opinion networks, each agent has a limited degree on information mastery; the agent will easily agree with the opinions of other agents under the same level of public opinion pressure from the neighbour nodes, but not vice versa. If the truth of the event was concealed deliberately and even the public did not understand the event, the public

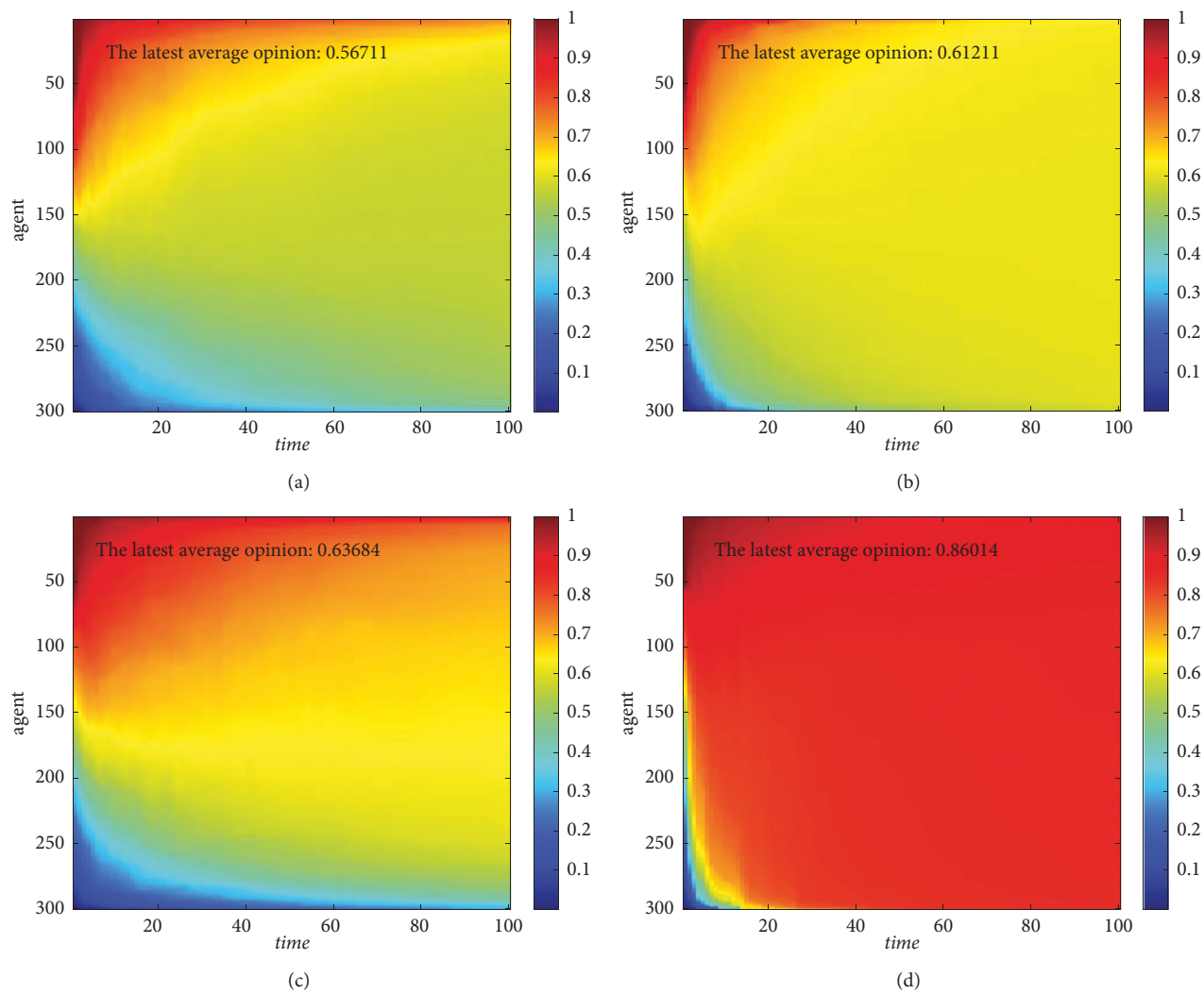

Figure 3: Comparison of guiding effect of key agents in different situation under limit spread, target opinion is 1. (a) Case 1. (b) Case 2. (c) Case 3. (d) Case 4.

would make extensive speculations and arbitrarily express their positions on the event. Even if there is insufficient evidence and information is insufficient, the public tends to believe the opinions from the surrounding agents whether they are for or against the opinions originally. After full understanding of the opinions from both sides, agents usually fall into neutrality. In the end, the group quickly reaches an agreement. In a transparent society, however, the instant information publication allows each agent to understand the full picture of the event first time. Agents will have enough information to support their own opinions and are not easily affected by other agents. As a result, the time for converge on all opinions and reach consensus will be delayed.

In the simulation, the external public opinion pressure suffered by each agent is also counted. It is found that the key agents are exposed most evidently to public opinion pressure from neighbour agents, while normal agents are not. Under insufficient information amount of key agents, the greater actual pressure of public opinions agent feels, the more easily his/her opinion value changes. With the time, the opinion value of key agents tends to be the average level of the group. At the same time, both the felt and actual pressure of public opinions will decrease and eventually converge with the group.

4.1. Simulation of the Guidance Effect of Key Agents. Aiming at exploring the influence of key agents on public opinion, the different situations are set up as follows: whether the information can be widely spread, whether the information amount is sufficient. And the influence of these situations on the evolution of network opinions is discussed. Considering the comparability on the simulation results, we set an appropriate number of iterations and 16.7% (50 nodes) keys agents in network. The set of cases to simulate is shown in Table 3.

4.1.1. Simulation Results of Different Cases under Limited Spreading. First, we strict the external spread of opinions of key agents. It means that we only consider that key agents

Table 3: The set of cases to simulate.

Case	Limited spreading	Wide spreading
KAO = 1, AAI = 1, $A(0) \sim U(0,1)$	Case 1	Case A
KAO = 1, AAI = 0.1, $A(0) \sim U(0,0.1)$	Case 2	Case B
KAO = 1, KAI = 1, NAI = 1, $N(0) \sim U(0,1)$	Case 3	Case C
KAO = 1, KAI = 1, NAI = 0.1, $N(0) \sim U(0,0.1)$	Case 4	Case D

Note: limited spreading indicates key agents have no power of spread its opinion to the outside in one time. Wide spreading indicates that key agents can propagate its opinion to the public at once, and the wide spreading level is 1 here.

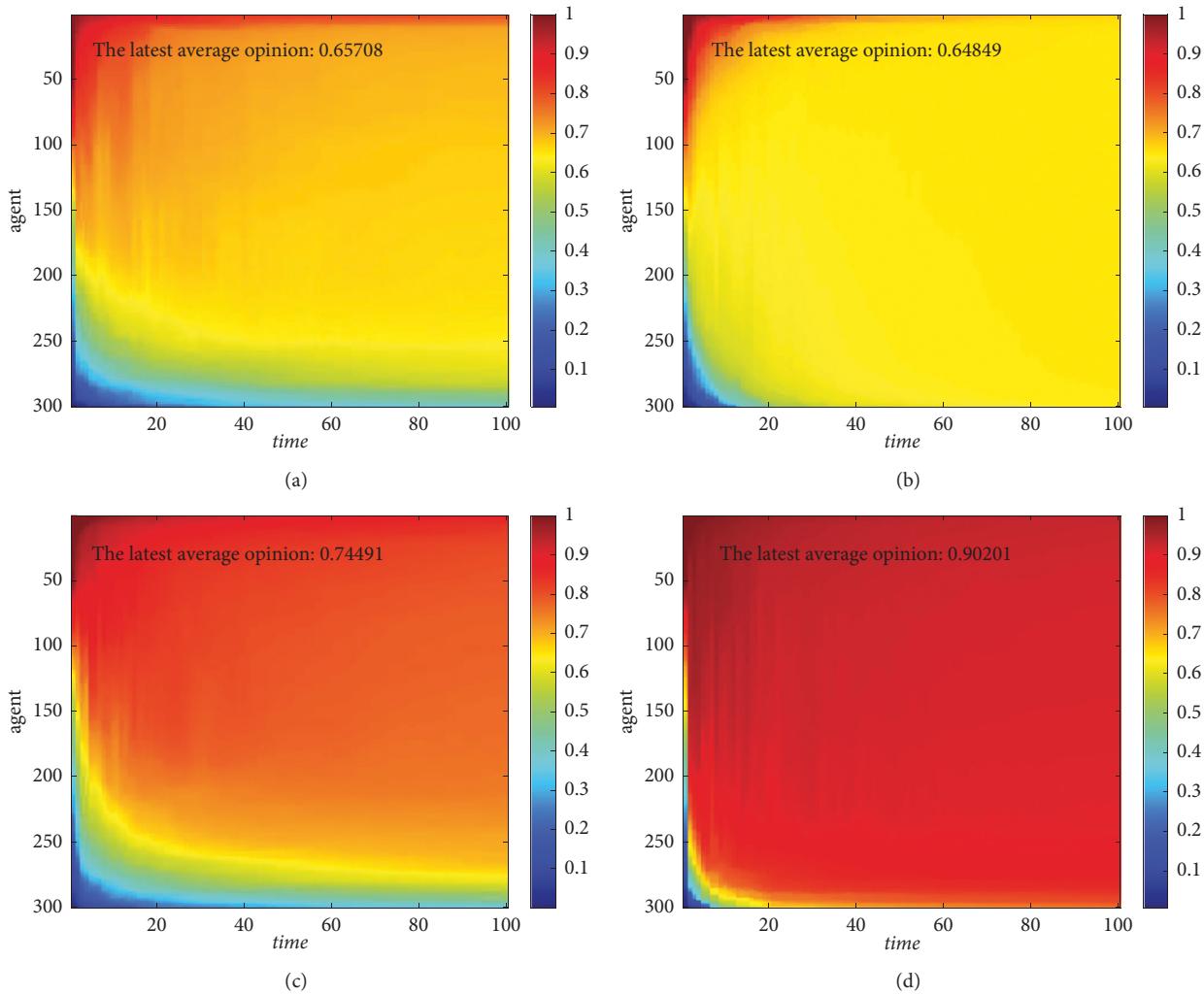

Figure 4: Comparison of guiding effect of key agents in different situation under wide spreading, target opinion is 1. (a) Case A. (b) Case B. (c) Case C. (d) Case D.

interact the opinions and information with neighbour agents directly connected to them. In Figure 3(a), the final average opinion value is 0.56711 in the same period. Obviously, the public opinion on entire networks is affected by the opinions of these 50 key agents where the group opinion trend is towards 1, but not evident. Figure 3(b) reflects the influence of key agents on the group opinion under information is insufficient in the entire networks. The final average opinion value of the group in the same period is 0.61211. Compared with Figure 3(a), the impact of key agents is relatively more obvious. Note worthily, the opinions of almost all agents here are finally close to the final average group opinion under this situation, which attribute to insufficient information amount on group. And a slight stratification phenomenon appears in Figure 3(c). The pressure on the group is relatively smaller because the agents have grasped a certain information amount at the same time, so it is difficult to reach consensus. In this case, the final average group opinion is greater than that in Figure 3(a) but smaller than that in Figure 3(b). The case describes whether the normal agents absorb the opinion on key agents, when the group has some cognition for the event and the key agents have sufficient

Table 4: The results of cases which key agents guiding the public opinion under different scenes.

Cases	Limited spreading	Wide spreading
KAO = 1, AAI = 1, $A(0) \sim U(0,1)$	0.5740 (Case 1)	0.6433 (Case A)
KAO = 1, AAI = 0.1, $A(0) \sim U(0,0.1)$	0.6175 (Case 2)	0.6522 (Case B)
KAO = 1, KAI = 1, NAI = 1, $N(0) \sim U(0,1)$	0.6374 (Case 3)	0.7692 (Case C)
KAO = 1, KAI = 1, NAI = 0.1, $N(0) \sim U(0,0.1)$	0.8687 (Case 4)	0.8996 (Case D)

Note: all values correspond to the cases in Table 3.

Figure 5: Comparison of the guidance effects of key agents in different cases, and the radar chart shows the difference in the result of limit and wide spreading.

information. Apparently, compared to the situation where all agents do not know the event very well, the impact on key agents is weaken. Under limited spreading on information of key agents, the average final opinion is 0.86014 in Figure 3(d) which is larger than that in any previous situation. When key agents have sufficient information, agents who do not know much about event information play a more effective role of guiding. These normal agents are under more public opinion pressure, while key agents are under relatively less public opinion pressure. When key agents have sufficient information and the other knows little about an event, the group opinion can be easily guided. It is worth noting that information is divided into objective and subjective. We do not rule out the case that the key agents fabricate a logical "complete story," which is extremely dangerous for some sensitive public. Because when the group is unable to fully understand the event, a seemingly perfect and flawless "story" is easily accepted by the group that knows almost nothing. In this case, the stand points conveyed by key agents are more likely to be recognized by the group, which obviously has a certain negative impact on social stability.

4.1.2. Simulation Results of Different Cases under Wide Spreading. In this simulation, we set external wide spreading on information of key agents, that is, the neighbour of key agent will spread the opinion value to other agents who are connected with them. Of course, the spread opinion value is not the original opinion by the key agents but will be slightly changed by the passed nodes after being spread again and again. And the degree of external spreading information on all key agents is 1; that is, a single spread link will pass through 1 node atmost.

In Figure 4(a), a distinct opinion stratification phenomenon finally appears. Obviously, the wide spreading of information on key agents has a guiding effect to some extent, but there are still some agents who do not recognize mainstream opinions. Additionally, the wide spreading of information on key agents will quickly make some agents have a certain degree of understanding of event information, so that the actual public opinion pressure will be reduced. This causes that the group opinion will become difficult to be unified.

Figure 4(b) shows the results of setting the maximum information amount of all agents in the network to 0.1 differing to the parameter conditions of Figure 4(a). It is unexpected that there are almost no agents with relatively large differences in opinions in the network and almost all nodes reach consensus. However, in this case, the average opinion value of the group is 0.64849, slightly lower than 0.65708. The reason for this situation can be explained by the fact that under insufficient information the actual pressure felt by agents is relatively larger, which is likely to change their opinions and normal agents hardly trust key agents without sufficient evidence. Therefore, although all agents are affected by key agents, the effect is slightly weaker than that in Figure 3(b). At this time, the wide spreading on information is even less effective than limited spreading.

In Figure 4(c), the final average opinion value of the group is higher but there still is the stratification phenomenon because some agents disagree with the main stream opinions. When the group has a certain understanding for information, key agents express opinions with sufficient information, which enable some agents who do not know enough about the event to obtain more information. And the wide spreading of opinions accelerates this process. As above, the information amount can affect the actual pressure on public opinion and determine whether the opinions are changed easily or not.

In Figure 4(d), the group has the higher acceptability for the opinion on key agents. Although there are a small number of agents who have opinions different from the mainstream opinion in the early stage, they still disappear quickly. Under sufficient information and wide spreading on information, normal agents with maximum information amount of 0.1 and random distribution of the opinion value

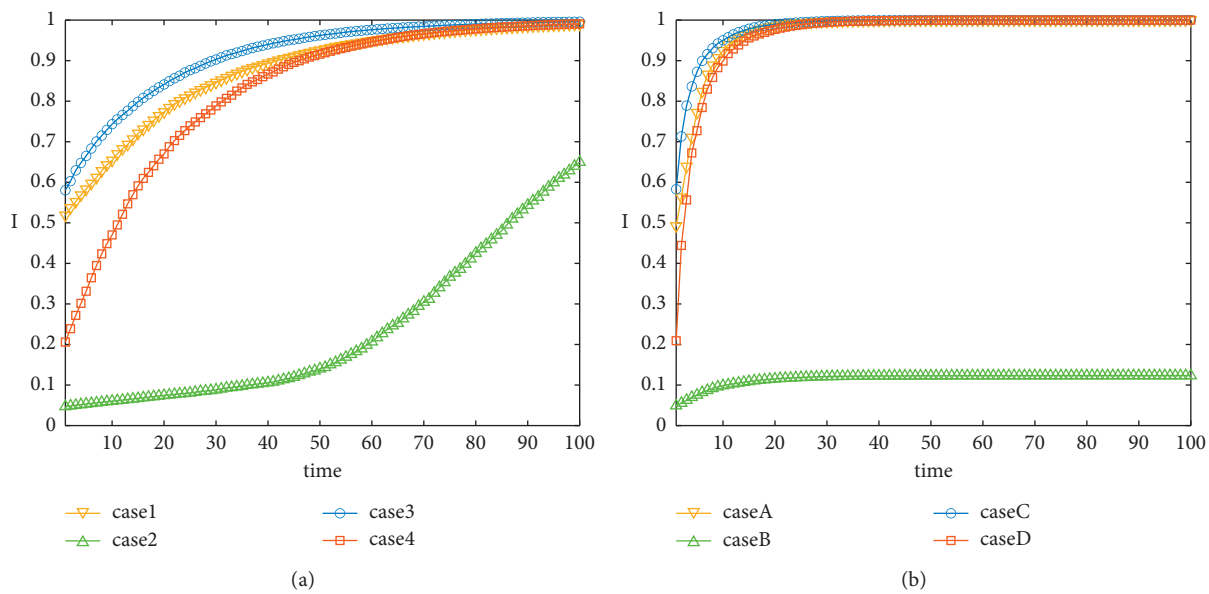

Figure 6: Comparison of the information amount in different scenes. In the long term, it shows the average information amount with time and matches the previous 8 cases.

are extremely easy to accept the opinions of key agents. After careful observation, it can be found that there is an unstable chaotic phenomenon in the initial part, which shows that the agent is hesitant when face different opinions from each side in the early stage. It is related to the limited information the agent has in the early stage and the actual pressure on public opinions.

4.1.3. Effectiveness Comparison. To reduce the influence of randomness on the conclusion, we simulate each case 20 times and calculate the final average opinion value on the group, as shown in Table 4. Also, a radar chart is drawn for easy comparison.

In general, the wide spreading of value on key agents has better effect than limited spreading. In particular, when the opinion value on key agent is 1, the information amount is 1 with wide spreading and the maximum information amount of other agents is 0.1, group opinion are the closest to the original opinion on key agents. The difference between the effects of wide spreading and limited spreading of key agents' opinions is greater when the amount of information about the group is more adequate (Case 1(A) and Case 3(C)). To analyze the results in Figure 5, we study the influence of information amount on the group, information spreading scope, and the initial information amount under different cases.

4.2. The Average Information Amount on the Network and Public Opinion Pressure under Different Circumstances. The opinion evolution is a process along with change on information amount. Here, the average information amount of the group in each case is compared. Notably, the group information amount and the opinion value are not directly related, and after the group opinion is stable, the information amount will continue to grow. Therefore, the first 20 time steps in the figure are equivalent to 100 time steps in the previous cases.

Figure 6(a) clearly shows the different changes on the information amount with time in several different situations. The network average information amount in Case 1, Case 3, and Case 4 almost reach 1 during the same period, while Case 2 does not. In Case 1 and Case 3, the information amount of the network group is relatively sufficient, so the group information amount quickly approaches to the maximum. The initial growth on information amount in Case 2 is slow; and it achieves rapid increase after reaching a certain level. In Case 4, the information amount on key agent plays an important role. Although the information amount on most agents in the network is in sufficient and the maximum information amount is limited to 0.1, the key agents have sufficient information amount. Due to the high degree characteristic on key agents, that is, key agents are connected to numerous nodes; it effectively helps with the external information spread from key agents. Thus, the information amount of the group can reach the maximum at the same time.

Figure 6(b) exhibits the results of the information wide spreading. In this case, the average opinion value of network groups in Case A, Case C, and Case D reaches 1 in a short period of time. Case B first grows slightly after a short period of time and then almost stagnates. Compared with the limited spreading in Figure 4(c), Cases A, C, and D in Figure 6(b) reach the maximum with a faster speed, and the information wide spreading strongly promotes the information acquisition speed of other nodes. However, Case B does not follow this rule, and there is a long-term stagnate, which is a huge difference compared with Case 2 in Figure 6(a). It can be an explained as following. When the information of key agents is insufficient and with wide

spreading, the normal agents accept such small information amount repeatedly and they cannot understand the event fully. It results in a stagnate on group average information amount in a long term. In this case, it can be said that key agents hinder the information acquisition for other agents to a certain extent.

Case 2 in Figure 6(a) restricts the information spreading, but it does not prevent each agent in the network from obtaining different information in various aspects. Therefore, after the information amount has accumulated to a certain extent, the information amount explodes, instead of being widely restricted by the information of key agents.

The above figures reflect the change on the average public opinion pressure from the group with time, among where E is the actual public opinion pressure that agents felt, and F is the objective public opinion pressure agents receive. Obviously, the effect on opinions convergence makes the external pressure on agents rapidly decrease. It can be found that the average level of public opinion pressure suffered by the group is the smallest in Figure 7(a) case 3 than other cases. Followed by Case 1 and Case 4, it is not difficult to find that the smaller the amount information of group is, the greater the pressure on public opinion is. It means that sufficient information on key agents can reduce the pressure of public opinion on the group to a certain extent, which is verified by Case 3 and Case 4. Comparing Figures 7(a) and 7(b), the wide spreading on information of key agents promotes the convergence on the group and makes the E approach to 0 faster.

Figures 7(e) and 7(f) show the comparison of the objective public opinion pressure and the actual public opinion pressure on each agent at the initial time and a round of opinion evolution. It can be found that agents with higher degrees may be more likely to suffer from objective public opinion pressure. It is related to the uniform distribution for opinion on neighbours, and the large weight also enlarges this. Even if the difference in opinion is small, the pressure on public opinion will increase under the influence on weight. The positive or negative of the objective public opinion pressure reflects the direction of change on the agent's opinion value. If the public opinion pressure is positive, the agent's opinion value will change towards 1, or conversely. In addition, the pressure felt by the agent is positively correlated with the objective pressure (compare Figures 7(a) and 7(c), Figures 7(b) and 7(d)). After opinion evolution, the group opinion pressure is about 0, which indicates that the final group opinion value and information amount reach a steady state and the agent is minimally affected by the opinion on neighbour.

4.3. Guiding Effectiveness of Changing Spreading Scope and Group Maximum Information Amount. As shown in Figure 8, Strategy 1 can be observed that the spreading scope changes from 0 to 1, and the guidance becomes more effective. As the spreading scope continues to increase, the guiding effect does not continue to increase but oscillates. In Strategy 2, with the increase of information spreading scope from key agents, the guiding effect does not change significantly. From limited spreading to spreading scope of 1, the guiding effect of this process becomes slightly worse. This situation occurs because insufficient information makes the group in the network easily affected by the opinions on other nodes around. In the wide spreading, due to the insufficient information amount, it is easy for the group to accept the opinions on key agents and the opinions on other agents, so the spreading effect on key agents is not dramatical. In strategy 1 where the information amount is relatively sufficient, it is not easy to change opinions under limited spreading because the agent has a certain information amount. If information from the key agents is widely spread, it can make more agents whose information amount has not reached the threshold change opinions. Taking into account the scale-free behavior in the BBV network, information spread widely will cause repeated spread. In other words, key agents with the large degree could affect most of the agents with in a shorter spreading scope. At the same time, opinions on key agents be re-affected by those agents quickly because of short spreading path. Therefore, the continuous expansion of wide spreading is difficult to improve the guiding effect of key agents in the network. Here, we can consider the cost of key agents to spread opinion information. Obviously, there is no need to blindly expand the spreading scope of key agents. It is only necessary to spread information from key agents as far as possible to reach a certain appropriate range. For relevant departments, they can save the cost that guiding the public to make comments or forward information. (For example, if you livestream your products, it is better to have more people watch it directly than to have viewers spread their product descriptions to their friends, which is more conducive to increasing sales.) Apparently, the guiding effect on key agents in Strategy 1 and Strategy 2 in Figure 5 is similar, but as the spreading scope expands, the guiding effect of Strategy 1 is generally better than that of Strategy 2 gradually. It is still caused by the difference in the information amount on the agents. The agent with insufficient information amount is impacted by neighbour agents, and it is difficult to maintain the original opinion, and vice versa.

In Figure 8, strategy 3, the wide spreading scope of 0 means that the effect of limited spreading is rather inferior to that of situation where the wide spreading scope is 1 and the spreading scope continues to expand. In the case of Strategy 3, allowing information of key agents and opinions to be widely spread when the initial information amount is relatively insufficient, so that the evolution will have an effective impact on public opinion networks from the beginning. The reason why this scenario is more effective than strategy 1 and strategy 2 is that key agents have enough information to convince surrounding neighbour. Strategy 4 is a scenario where the opinion value of key agents is 1, the information amount is 1, and the maximum information amount of other nodes is 0.1. However, it can be found that no matter it is limited spreading or wide spreading, the final guiding effect is almost the same. By comparing Figures 3(b) and 4(b), it is not difficult to find a feature. The final opinion value held by all nodes in Figure 3(b) has a small difference, while the difference in Figure 4(b) is relatively large and

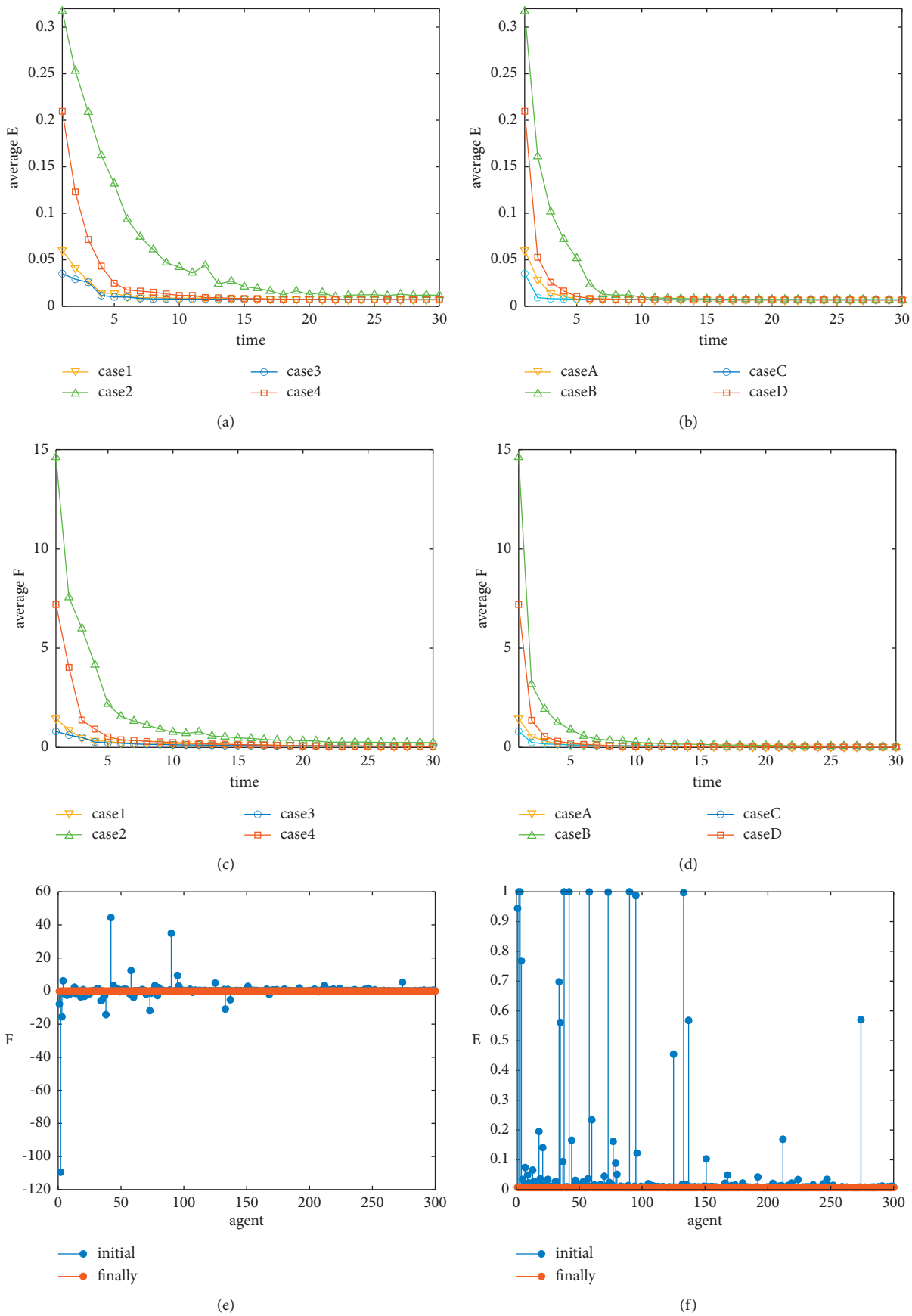

Figure 7: Comparison of the public opinion pressure (a, b). The average F of different cases (c, d). The average E of different cases (e). The F of each agent, the blue stems show the initial F, and the red stem shows the final F after once opinion evolution in case 1 (f). The E of each agent, the blue stems show the initial E, and the red stem shows the final E after once opinion evolution in case 1.

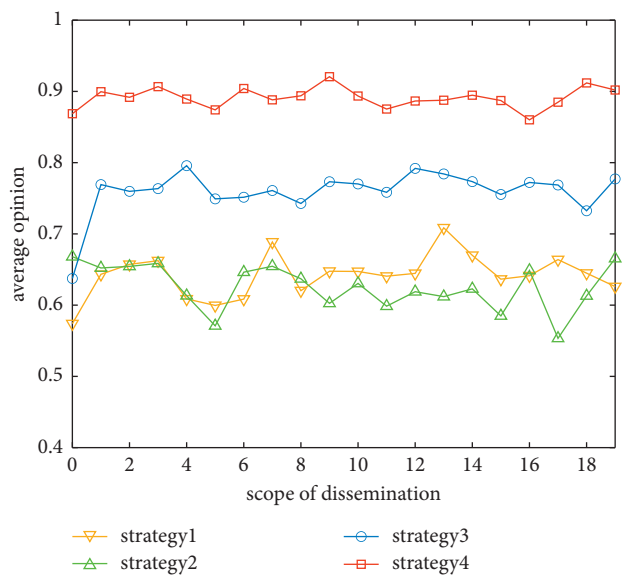

Figure 8: The distribution of the result of average opinion by the guidance from key agents. The scope of transmission represents that the scope of key agents can spread its opinion once time. Strategy 1: KAO = 1, AAI = 1; strategy 2: KAO = 1, AAI = 0.1; strategy 3: KAO = 1, KAI = 1, NAI = 1; strategy 4: KAO = 1, KAI = 1, NAI = 0.1.

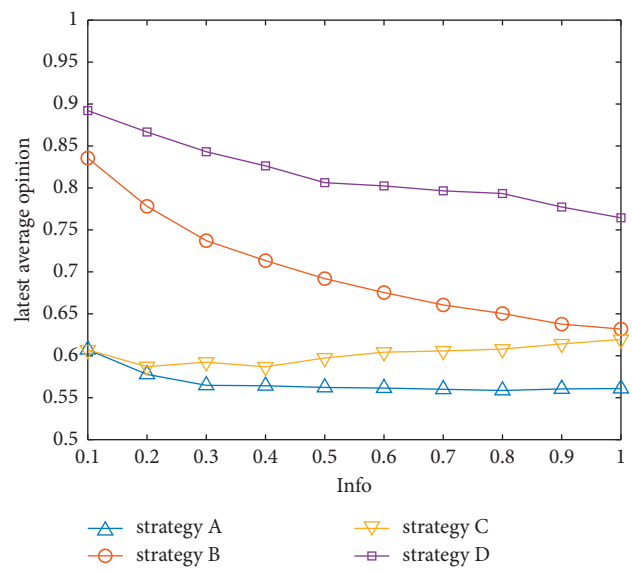

Figure 9: The distribution of the result of average opinion by the guidance from key agents. Strategy A : KAO = 1, $A(0) \sim U(0, \text{Info})$, limited spreading. Strategy B : KAO = 1, KAI = 1, $N(0) \sim U(0, \text{Info})$, limited spreading. Strategy C : KAO = 1, $A(0) \sim U(0, \text{Info})$, wide spreading. Strategy D : KAO = 1,KAI = 1, $N(0) \sim U(0, \text{Info})$, wide spreading.

there is a less obvious stratification phenomenon. It can be considered that the intention of wide spreading is to affect as many agents as possible in a short period of time, but after some agents accept the opinions of key agents, as the overall information amount in the network increases, agents actually feel less pressure on public opinion, that is, they become stubborn and it is difficult to change their opinion value. Limited spreading is a slow process; key agents affect a small number of agents and then these small agents will affect other agents next in a short time. Compared with wide spreading, this process has stronger stability, and the opinion value is easier to converge. Additionally, in contrast to the insufficient information amount of key agents, sufficient information amount of key agents has a more stable guiding effect on the public opinion networks.

The increase in the information amount means that the external public opinion pressure that the agent feels is relatively reduced. Figure 9 compares the effectiveness of different strategies under different information amount of the group. Overall, the difference in different strategies is relatively obvious. Strategy A, as the initial information amount of the group continues to increase, the guiding effect of key agents gradually deteriorates and there is a light oscillation and instability. For Strategy B, as the initial group information amount continues to increase, the guiding effect of key agents declines steadily. While for Strategy C, as the initial information amount of the group increases, the public opinion pressure felt by the agent is reduced, and the guiding effect does not change significantly, but oscillate at the same level. For Strategy D, the influencing effect of key agents slowly decreases as the group information amount increases.

Strategy C and strategy D are the results of the guiding effectiveness of key agents under the change of the information amount during wide spreading, and that in strategy A and strategy B under the information limited spreading are relatively inferior. In strategy A and strategy C, only when the information amount is extremely small, the guiding effect under wide spreading is inferior to that under limited spreading. In the case that the group information amount is quite small and the information of key agents is also very small, that is, the maximum information amount is 0.1, strategy A will have a better effect than strategy C. When the maximum information amount reaches 1, the key agents of strategy B and strategy C have the similar guiding effect. At this time, strategy B is the result when the opinion value of key agents is 1, the information amount is 1, and the maximum information amount of other nodes is 1 with limited spreading (Case C), and the final value of strategy C is the result when the key agent is 1, the maximum information amount of all nodes is 1 with wide spreading (Case A). By comparing Figures 3(a) and 3(c), it is indeed found that the final average opinion value of the two is similar. The value of Figure 3(c) is slightly higher than that of Figure 3(a), which is also inline with the situation in Figure 6(a) where strategy C is finally slightly higher than strategy B. The stable phenomenon of strategy C is because when key agents have the same information amount as other nodes, no matter how the group information amount level changes, even if the wide spreading is introduced, the guiding effect of key agents will not change much. The decline in Strategy A is because under limited spreading, as the information amount of the group grows, the opinion value of key agents becomes less and less convincing and difficult to be accepted by the group. Strategy B and strategy D also follow this rule. According to the analysis for above different strategies under the change of

the information amount, we can conduct the different strategies to achieve the goal on guiding opinions based on different understanding on group and the cost.

5. Conclusion

In our paper, according to the cost function, we construct different opinion evolution laws for different agents. And then we study the influence of information amount and information dissemination mode on group opinion by setting different information characteristics. To summarize, the main contributions of the paper are as follows:

(1) How much individuals know about information on event affect the trend of final public opinion. When an agent knows less about the event information, he/she is more likely to be influenced by others around, and the opinion is easier to change. It inspires that relevant departments should put a more comprehensive opinion as soon as possible, which could guide the development of public opinion by increasing individual cognition.

(2) Generally, it is more dramatical that public opinions are close to key agents' opinions under the wide spreading than the limited spreading. It is noticeable that the expansion on information diffusion does not necessarily mean that the aggregation effect on public opinion is better. Public opinions tend to be relatively stable when the range of information diffusion is expanded to some extent. It inspired that relevant departments could control the limited scope of information dissemination to achieve public opinion control, which avoid waste of public resources. For example, when there are COVID-19 cases, the government only needs to supervise the close contact or secondary contact to control the epidemic effectively.

Although this work only considers the least-cost decision-making methods of agents regarding information amount and opinion changes, and the public opinion pressure imposed by neighbour agents in a simple manner, all conclusions are drawn from simulation results, it still reveals some special mechanisms in some opinion evolution.

As a result, our research enriches the study of predicting the public opinion development. And at the same time, it can also provide a theoretical basis for government public opinion monitoring.

Data Availability

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] W. Wood, "Attitude change: persuasion and social influence," *Annual Review of Psychology*, vol. 51, no. 1, pp. 539–570, 2000.

[2] D. Centola, V. M. Eguíluz, and M. W. Macy, "Cascade dynamics of complex propagation," *Physica A: Statistical Mechanics and Its Applications*, vol. 374, no. 1, pp. 449–456, 2007.

[3] B. Li, Y. Feng, Z. Xiong, W. Yang, and G. Liu, "Research on AI security enhanced encryption algorithm of autonomous IoT systems," *Information Sciences*, vol. 575, no. 2021, pp. 379–398, 2021.

[4] L. Yang, Z. Xiong, G. Liu, Y. Hu, X. Zhang, and M. Qiu, "An analytical model of page dissemination for efficient big data transmission of C-ITS," *IEEE Transactions on Intelligent Transportation Systems*, pp. 1–10, 2022.

[5] W. Zheng, Y. Xun, X. Wu, Z. Deng, X. Chen, and Y. Sui, "A comparative study of class rebalancing methods for security bug report classification," *IEEE Transactions on Reliability*, vol. 70, no. 4, pp. 1658–1670, 2021.

[6] N. A. Christakis and J. H. Fowler, "The spread of obesity in a large social network over 32 years," *New England Journal of Medicine*, vol. 357, no. 4, pp. 370–379, 2007.

[7] A. Caravaggio and M. Sodini, "I love shopping. . .but what am I going to buy? Social interaction and consumption choices," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 30, no. 9, Article ID 093133, 2020.

[8] E. Du, E. Chen, J. Liu, and C. Zheng, "How do social media and individual behaviors affect epidemic transmission and control?" *The Science of the Total Environment*, vol. 761, no. 2021, Article ID 144114, 2021.

[9] M. Askarizadeh and B. Tork Ladani, "Soft rumor control in social networks: modeling and analysis," *Engineering Applications of Artificial Intelligence*, vol. 100, Article ID 104198, 2021.

[10] S.-R. Yan, X.-L. Zheng, Y. Wang, W. W. Song, and W.-Y. Zhang, "A graph-based comprehensive reputation model: exploiting the social context of opinions to enhance trust in social commerce," *Information Sciences*, vol. 318, pp. 51–72, 2015.

[11] G. Boschi, C. Cammarota, and R. Kühn, "Opinion dynamics with emergent collective memory: the impact of a long and heterogeneous news history," *Physica A: Statistical Mechanics and Its Applications*, vol. 569, Article ID 125799, 2021.

[12] F. Meng, W. Cheng, and J. Wang, "Semi-supervised software defect prediction model based on tri-training," *KSII Transactions on Internet and Information Systems*, vol. 15, pp. 4028–4042, 2021.

[13] A. Li, C. Masouros, A. L. Swindlehurst, and W. Yu, "1-bit massive MIMO transmission: embracing interference with symbol-level precoding," *IEEE Communications Magazine*, vol. 59, no. 5, pp. 121–127, 2021.

[14] A. Li, D. Spano, J. Krivochiza et al., "A tutorial on interference exploitation via symbol-level precoding: overview, state-of-the-art and future directions," *IEEE Communications Surveys & Tutorials*, vol. 22, no. 2, pp. 796–839, 2020.

[15] S. Galam, Y. Gefen Gefen (Feigenblat), and Y. Shapir, "Sociophysics: a new approach of sociological collective behaviour. I. mean-behaviour description of a strike," *Journal of Mathematical Sociology*, vol. 9, no. 1, pp. 1–13, 1982.

[16] S. Galam and S. Moscovici, "Towards a theory of collective phenomena: consensus and attitude changes in groups," *European Journal of Social Psychology*, vol. 21, no. 1, pp. 49–74, 1991.

[17] L. Li, Y. Fan, A. Zeng, and Z. Di, "Binary opinion dynamics on signed networks based on Ising model," *Physica A: Statistical Mechanics and its Applications*, vol. 525, pp. 433–442, 2019.

[18] K. M. Frahm and D. L. Shepelyansky, "Ising-PageRank model of opinion formation on social networks," *Physica A: Statistical Mechanics and Its Applications*, vol. 526, Article ID 121069, 2019.

[19] F. S. N. Karan, A. R. Srinivasan, and S. Chakraborty, "Modeling and numerical simulations of the influenced Sznajd model," *Physical Review*, vol. 96, no. 2, Article ID 022310, 2017.

[20] L. Li, A. Scaglione, A. Swami, and Q. Zhao, "Consensus, polarization and clustering of opinions in social networks," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 6, pp. 1072–1083, 2013.

[21] K. Sznajd-Weron and J. Sznajd, "Opinion evolution in closed community," *International Journal of Modern Physics C*, vol. 11, no. 06, pp. 1157–1165, 2000.

[22] K. Sznajd-Weron, "Sznajd model and its applications," vol. 36, p. 8, 2005, https://arxiv.org/abs/physics/0503239.

[23] S. Galam, "Minority opinion spreading in uniform geometry," *The European Physical Journal B*, vol. 25, no. 4, pp. 403–406, 2002.

[24] J. M. Encinas, H. Chen, M. M. de Oliveira, and C. E. Fiore, "Majority vote model with ancillary noise in complex networks," *Physica A: Statistical Mechanics and Its Applications*, vol. 516, pp. 563–570, 2019.

[25] A. L. M. Vilela and A. J. F. de Souza, "Majority-vote model with a bimodal distribution of noises in small-world networks," *Physica A: Statistical Mechanics and Its Applications*, vol. 488, pp. 216–223, 2017.

[26] K. Wang, H. Wang, and S. Li, "Renewable quantile regression for streaming datasets," *Knowledge-Based Systems*, vol. 235, no. 2022, Article ID 107675, 2022.

[27] W. Zheng, X. Tian, B. Yang et al., "A few shot classification methods based on multiscale relational networks," *Applied Sciences*, vol. 12, no. 8, p. 4059, 2022.

[28] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch, "Mixing beliefs among interacting agents," *Advances in Complex Systems*, vol. 3, no. 01n04, Article ID 01n04, pp. 87–98, 2000.

[29] J. Zhang and Y. Hong, "Opinion evolution analysis for short-range and long-range Deffuant-Weisbuch models," *Physica A: Statistical Mechanics and Its Applications*, vol. 392, no. 21, pp. 5289–5297, 2013.

[30] M. Bashari and M.-R. Akbarzadeh-T, "Controlling opinions in Deffuant model by reconfiguring the network topology," *Physica A: Statistical Mechanics and Its Applications*, vol. 544, Article ID 123462, 2020.

[31] C. Huang, Q. Dai, W. Han, H. Cheng, H. Li, and H. au, "Effects of heterogeneous convergence rate on consensus in opinion dynamics," *Physica A: Statistical Mechanics and Its Applications*, vol. 499, pp. 428–435, 2018.

[32] R. Hegselmann and U. Krause, "Opinion dynamics and bounded confidence models, analysis, and simulation," *Journal of Artificial Societies and Social Simulation*, vol. 3, 2002.

[33] W. Han, C. Huang, and J. Yang, "Opinion clusters in a modified Hegselmann-Krause model with heterogeneous bounded confidences and stubbornness," *Physica A: Statistical Mechanics and Its Applications*, vol. 531, Article ID 121791, 2019.

[34] G. Fu, W. Zhang, and Z. Li, "Opinion dynamics of modified Hegselmann-Krause model in a group-based population with heterogeneous bounded confidence," *Physica A: Statistical Mechanics and Its Applications*, vol. 419, pp. 558–565, 2015.

[35] Q. Dong, Q. Sheng, L. Martínez, and Z. Zhang, "An adaptive group decision making framework: individual and local world opinion based opinion dynamics," *Information Fusion*, vol. 78, no. 2022, pp. 218–231, 2022.

[36] C. Cheng and C. Yu, "Opinion dynamics with bounded confidence and group pressure," *Physica A: Statistical Mechanics and Its Applications*, vol. 532, Article ID 121900, 2019.

[37] P. Lu, Y. Zhang, and Y. Xiang, "Collective actions from on line to offline," *Physica A: Statistical Mechanics and Its Applications*, vol. 533, Article ID 120889, 2019.

[38] D. Ferraioli and C. Ventre, "Social pressure in opinion dynamics," *Theoretical Computer Science*, vol. 795, pp. 345–361, 2019.

[39] H. Zhu and B. Hu, "Impact of information on public opinion reversal-An agent based model," *Physica A: Statistical Mechanics and Its Applications*, vol. 512, pp. 578–587, 2018.

[40] Y. Lan, Z. Lian, R. Zeng et al., "A statistical model of the impact of online rumors on the information quantity of online public opinion," *Physica A: Statistical Mechanics and Its Applications*, vol. 541, Article ID 123623, 2020.

[41] D. J. Watts and S. H. Strogatz, "Collective dynamics of "small-world" networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[42] W. Zheng and L. Yin, "Characterization inference based on joint-optimization of multi-layer semantics and deep fusion matching network," *PeerJ Computer Science*, vol. 8, no. 2022, Article ID e908, 2022.

[43] W. Zheng, Y. Zhou, S. Liu, J. Tian, B. Yang, and L. Yin, "A deep fusion matching network semantic reasoning model," *Applied Sciences*, vol. 12, no. 7, p. 3416, 2022.

[44] A. L. Barabási, R. Albert, and H. Jeong, "Mean-field theory for scale-free uniform networks," *Physica A: Statistical Mechanics and Its Applications*, vol. 1-2, no. 1-2, pp. 173–187, 1999.

[45] A. Barrat, M. Barthélemy, and A. Vespignani, "Modeling the evolution of weighted networks," *Physical Review*, vol. 70, no. 6, Article ID 066149, 2004.

[46] Y. Zhao, T. Lu, W. Su, P. Wu, L. Fu, and M. Li, "Quantitative measurement of social repulsive force in pedestrian movements based on physiological responses," *Transportation Research Part B: Methodological*, vol. 130, pp. 1–20, 2019.

[47] I. F. Mello, L. Squillante, G. O. Gomes, A. C. Seridonio, and M. de Souza, "Epidemics, the Ising-model and percolation theory: a comprehensive review focused on Covid-19," *Physica A: Statistical Mechanics and Its Applications*, vol. 573, Article ID 125963, 2021.

[48] J. P. Tripathi, S. Bugalia, K. Burdak, and S. Abbas, "Dynamical analysis and effects of law enforcement in a social interaction model," *Physica A: Statistical Mechanics and Its Applications*, vol. 567, Article ID 125725, 2021.

[49] X. Chen, C. Huang, H. Wang, W. Wang, X. Ni, and Y. Li, "Negative emotion arousal and altruism promoting of online

public stigmatization on COVID-19 pandemic," *Frontiers in Psychology*, vol. 12, no. 2021, Article ID 652140, 2021.

[50] R. Sun, J. Wang, Q. Cheng, Y. Mao, and W. Y. Ochieng, "A new IMU-aided multiple GNSS fault detection and exclusion algorithm for integrated navigation in urban environments," *GPS Solutions*, vol. 25, no. 4, p. 147, 2021.

[51] T. Li and H. Zhu, "Effect of the media on the opinion dynamics in online social networks," *Physica A: Statistical Mechanics and Its Applications*, vol. 551, Article ID 124117, 2020.