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Research Article

Mean Square Consensus of General Linear Multiagent Systems with Communication Noises under Switching Topologies

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This paper investigates the distributed consensus problem of general linear multiagent systems (MASs) with communication noises under fixed and Markovian switching topologies, respectively. Each agent can obtain full state of itself and receive its neighbors' state with noises, where intensities of noises are vector functions of relative states of agents. Bearing in mind the above constrains, a consensus protocol is proposed, where the gain matrix is obtained by the algebraic Riccati equation and the coupling strength is restricted in a given interval. By using the stochastic stability theorem, we show that mean square consensus is achieved in fixed topology case and switching topologies case, respectively. Furthermore, an estimation of the exponential convergence rate of consensus is given explicitly. Finally, simulation examples are given to show the correctness of the proposed results.

1. Introduction

In system and control community, the coordination problem of MASs is one of the most concerned hotspots in the past decade, which has shown its potential in real-world applications, such as distributed sensor networks, smart grids, and multirobot formation [1–4]. Consensus is a fundamental issue in the control problem of MASs, which refers to designing a protocol such that all agents converge to a common value. Research results on the consensus problem can be extended to solve many coordination problems of MASs, including flocking, swarming, and rendezvous formation [5–7].

In real world, agents and their connections are often affected by noises, which could sometimes affect the performance or even destroy the stability of systems [8, 9]. Generally, noises can be divided into two categories: additive noise and multiplicative noise, and both kinds of noises have been considered in the study of MASs [10]. For additive noise, which destroys the signal in the form of superposition, its intensity is determined by external factors, such as lightning, and pulse. In 2009, Huang and Manton introduced the stochastic approximation technique to design a decreasing nonnegative gain function $c(t)$, which could attenuate the impact of additive noise while letting the MASs achieve consensus [11]. The idea of nonnegative gain function was then utilized by some scholars to investigate MASs with additive noise. For example, Li et al. proposed sufficient and necessary conditions for the decreasing nonnegative gain function to achieve asymptotic unbiased mean square average consensus [12]. Based on these results, leader-following consensus problem was solved in [13], containment control problem was studied in [14], and bipartite consensus problem was concerned in [15]. For multiplicative noise, its intensity depends on states of the system, e.g., measuring relative states through analog fading channels [16]. In [17], by using the small gain theorem, Zhang et al. developed necessary and sufficient conditions for mean square consensus and almost sure consensus for MASs. By using the stochastic stability theorem, heterogeneous MASs were studied in [18], and MASs with nonidentical channel fading were analyzed in [19]. In practical applications, the two kinds of noise may coexist in MASs [20]. Motivated by this phenomenon, MASs with both additive and multiplicative noises were considered in [10] for the consensus problem, and in [21], for the containment control problem. However, in the above studies, the concerned MASs were in first- or second-order dynamics under fixed topology.

Typically, agents are connected through a network, which is not only affected by noises, but also has problems of link failure or abrupt change, etc. [22–25]. Many significant results on MASs under switching topologies have been addressed [26–29]. It can be found that the switching signals in many existing results were subject to deterministic time sequences [30–32]. However, due to unpredictable changes in the communication networks, it is more significant to study the case that topologies switch randomly [33]. In [34], consensus problem of double-integrator MASs was studied under Markovian switching topologies. In [35, 36], consensus problems were investigated for MASs with semi-Markovian switching topologies. In addition, there are reports involving Markovian switching topologies and communication noises at the same time. For example, in [13], Wang et al. considered the mean square and almost sure consensus problem for MASs with Markovian switching topologies and additive noises. The results in [13] were then extended in [37], where sufficient and necessary conditions were obtained for single-integrator MASs with Markovian switching topologies and additive noises.

Inspired by the above discussions, mean square consensus of general linear MASs with communication noises under Markovian switching topologies is investigated in this paper. Consensus protocol will be designed by combining the stochastic stability theory, the Riccati equation, and some theories on matrix. The contributions of this paper are summarized as follows:

(i) Consensus problem of general linear MASs with communication noises is studied. The considered noises are induced by the communication among agents, which is a distinct feature of networked systems. Moreover, the general linear MASs include some results concerned with first-order MASs as special cases [37].

(ii) To capture the time-varying communication among agents in real, we extend the consensus problem by studying the switching topologies case. We assume that the switching signals are subject to a Markovian process, under which we merely require the combined topology rather than each underlying topology being connected.

We organize the rest of the paper in the following way. Section 2 contains some useful preliminaries and a formulation of the problem. In Sections 3 and 4, consensus results for fixed and Markovian switching topologies under communication noises are provided. Section 5 is devoted to simulation examples. Finally, a conclusion of the paper is given in Section 6.

The following notations will be used. We define a column vector that is all ones as 1, the N-dimensional column vector with the i th element being 1 and others being zero as $\eta_{N,i}$, the matrix $(1/N)11^T$ as J_N, and the N-dimensional identity matrix as I_N. For any given square matrix $\mathbf{A} \in \mathbb{R}^{N\times N}$, define $\lambda_{\min}(A) = \min_{1\le i\le N}\{|\lambda_i(A)|\}$. Denote $\mathbf{I}_a^b = \{a, a+1, \ldots, b\}$ for $a < b$. $\mathbb{E}[\cdot]$ denotes the mathematical expectation.

2. Problem Formulation

2.1. Graph Theory. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be an undirected graph, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of nodes; $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. Node i means agent i. An edge of $\mathcal{G}$ is denoted by (i, j), and it implies that the information can be exchanged between node j and node i. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ represents the structure of the graph, where $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. Assume that there are no self-loops, i.e., $a_{ii} = 0$ for all $i \in \mathcal{V}$. The set of neighbors of agent i is denoted as $N_i = \{j \mid (i, j) \in \mathcal{E}\}$. Let $\mathcal{D} = \operatorname{diag}\{d_1, d_2, \ldots, d_N\}$, where $d_i = \sum_{j\in N_i} a_{ij}$ is the degree of agent i. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined to be $L = \mathcal{D} - \mathcal{A}$.

For a positive integer m, the union of m graphs $\mathcal{G}^1 = (\mathcal{V}, \mathcal{E}^1, \mathcal{A}^1), \ldots, \mathcal{G}^m = (\mathcal{V}, \mathcal{E}^m, \mathcal{A}^m)$ is denoted by $\cup_{\mathbf{r}=1}^{\mathbf{m}} \mathbf{G}^{\mathbf{r}} = (\mathbf{V}, \cup_{\mathbf{r}=1}^{\mathbf{m}} \mathbf{E}^{\mathbf{r}}, \cup_{\mathbf{r}=1}^{\mathbf{m}} \mathbf{A}^{\mathbf{r}})$. Let $\mathcal{G}(\sigma(t)) = (\mathcal{V}, \mathcal{E}(\sigma(t)), \mathcal{A}(\sigma(t)))$ be the interaction topology of agents at time t, where the edge set $\mathcal{E}(\sigma(t))$ and the adjacency matrix $\mathcal{A}(\sigma(t))$ are time varying.

Lemma 1 (see [26]). *If $\mathcal{G}$ is a connected graph that is undirected, $L \in \mathbb{R}^{N\times N}$ is the corresponding Laplacian matrix, and its eigenvalues can be ordered in ascending order as*

$$0 = \lambda_1(L) < \lambda_2(L) \le \cdots \le \lambda_N(L), \tag{1}$$

and

$$\min_{1^T x=0, x\neq 0} \frac{x^T L x}{\|x\|^2} = \lambda_2(L), \tag{2}$$

where $\lambda_2(L)$ is called the algebraic connectivity of $\mathcal{G}$.

2.2. Problem Formulation. In this section, we consider MASs with the following dynamics:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \mathrm{i} \in \mathbf{I}_1^{\mathrm{N}}, \tag{3}$$

where $x_i(t) \in \mathbb{R}^n$ is the state of the i th agent and $u_i(t) \in \mathbb{R}^n$ is the control input. A and B are given constant matrices with appropriate dimensions satisfying that (A, B) is controllable.

Remark 1. Comparing with first-order systems, states in general linear system (3) are coupled through matrix A and the control u_i is not placed on the state x_i directly. It can be found that the general linear MASs (3) include the first-order MASs in [37] as special cases. Therefore, in this paper, we require (A, B) to be controllable and employ the Riccati equation to obtain a feasible gain matrix. Then, a kind of

Lyapunov function, which differs from that in integrator cases, will be designed to prove the stability of the consensus error.

In real MASs, each agent receives information from its neighbors with random perturbations. Hence, when agent i communicates with its neighbor agent j, agent i receives the state of agent j in the following form:

$$y_{ji}(t) = x_j(t) + g_{ij}\left(x_i(t) - x_j(t)\right)\xi_{ij}(t), \tag{4}$$

where $y_{ji}(t) \in \mathbb{R}^n$ denotes the measurement of $x_j(t)$ by agent i and $\xi_{ij}(t) \in \mathbb{R}$ denotes the communication noises. The noise intensity function $g_{ij}(\cdot)$ is a mapping from $\mathbb{R}^n$ to $\mathbb{R}^n$. There exists a constant $\varepsilon > 0$, such that $\|g_{ij}(x)\| \le \varepsilon\|x\|$, $i = 1, \ldots, N$, and $j \in N_i$, for any $x \in \mathbb{R}^n$. The noise process $\xi_{ij}(t), i, j = 1, \ldots, N$ satisfies $\int_0^t \xi_{ij}(s)ds = w_{ij}(t), \quad t \ge 0$, where $w_{ij}(t), i, j = 1, \ldots, N$ is an independent Brownian motion.

Due to the existence of communication noises, the consensus protocol is designed as

$$u_i(t) = cK\sum_{j=1}^{N} a_{ij}\left(y_{ji}(t) - x_i(t)\right), \tag{5}$$

where c is the coupling strength and K is the gain matrix to be designed later.

In this work, we also consider the consensus of MASs (3) over randomly switching topologies and the consensus protocol is modified as follows:

$$u_i(t) = cK\sum_{j=1}^{N} a_{ij}(\sigma(t))\left(y_{ji}(t) - x_i(t)\right), \tag{6}$$

where $a_{ij}(\sigma(t))$ is the element of $\mathscr{A}(\sigma(t))$ and $\mathscr{G}(\sigma(t)) = \mathscr{V}, \mathscr{E}(\sigma(t)), \mathscr{A}(\sigma(t))$ will randomly switch among m distinct topologies $\mathscr{G}(\sigma(t)) \in \{\mathscr{G}^1, \ldots, \mathscr{G}^m\}$, and $\mathscr{G}(\sigma(t)) = \mathscr{G}_r$, if and only if the random variable $\sigma(t) = r \in \mathbb{M} = \{1, \ldots, m\}$. The switching process $\{\sigma(t), t \ge 0\}$ is governed by a time-homogeneous Markov process, whose state space corresponds to all possible topologies.

For MASs (3) and distributed control protocols (5) or (6), the following questions need to be addressed. (i) Under what conditions can the mean square consensus be achieved? (ii) How to design the control gain matrix K and coupling strength c?

In this paper, the common probability space for all random variables is denoted by $(\Omega, \mathbb{F}, \mathbb{P})$, where Ω is the sample space of elementary events. $\mathbb{F}$ is the σ-field of subsets of the sample space and $\mathbb{P}$ is the probability measure on $\mathbb{F}$. Let the infinitesimal generator of the continuous-time Markov process $\{\sigma(t), t \ge 0\}$ be $\Xi = [q_{rs}]_{m\times m}$, which is given by

$$\mathbb{P}\{\sigma(t+h) = s|\sigma(t) = r\} = \begin{cases} q_{rs}h + o(h), & \text{if } \sigma(t) \text{jumps from } r \text{ to } s, \\ 1 + q_{rr}h + o(h), & \text{otherwise,} \end{cases} \tag{7}$$

where q_{rs} is the transition rate from state r to state s with $q_{rs} \ge 0$, if $r \ne s$, $q_{rr} = -\sum_{s\ne r} q_{rs}$, and $o(h)$ denote an infinitesimal of a higher order than h, that is, $\lim_{h\longrightarrow 0}(o(h)/h) = 0$. Note that Ξ is a transition rate matrix, whose row summation is zero and all off-diagonal elements are non-negative.

Definition 1. The MASs (3) with proper designed consensus protocol are said to achieve mean square consensus if for any given $x_i(0)$

$$\lim_{t\longrightarrow\infty} \mathbb{E}\left\|x_i - x_j\right\|^2 = 0, \quad \forall i, j. \tag{8}$$

Remark 2. Mean square stable is generally used to reflect the stability of a stochastic system. Due to the existence of noises, the overall MASs become stochastic systems. Therefore, the mean square consensus defined above is suitable to describe the consensus of the concerned MASs with noises.

3. Consensus on Fixed Topology

Substituting consensus protocol (5) into (3), we get

$$dx(t) = \left((I_N \otimes A) - (cL \otimes BK)\right)x(t)dt + c\sum_{i,j=1}^{N} a_{ij} \times \left(\eta_{N,i} \otimes \left(BKg_{ij}\left(x_i(t) - x_j(t)\right)\right)\right)dw_{ij}(t). \tag{9}$$

As (A, B) is controllable and let matrices $Q \in \mathbb{R}^{n\times n}$ be positive definite. The control gain matrix K is designed as

$$K = \frac{1}{2}B^T P, \tag{10}$$

where P is the unique positive definite solution to the following algebraic Riccati equation (ARE)

$$0 = A^T P + PA + Q - PBB^T P. \tag{11}$$

Theorem 1. *For the undirected connected graph, the MASs (3) with communication noises achieve mean square consensus under consensus protocol (3), if K is designed as (10) and c satisfies*

$$\frac{1}{\lambda_2^2(L)} \le c^2 < \frac{N\lambda_{\min}(Q)}{2(N-1)\varepsilon^2\lambda_{\max}(L)\lambda_{\max}\left(K^T B^T PBK\right)}. \tag{12}$$

Proof. Denote $e(t) = ((I_N - J_N) \otimes I_n)x(t)$; we have

$$de(t) = \left((I_N \otimes A) - (cL \otimes BK)\right)e(t)dt + c\sum_{i,j=1}^{N} a_{ij}\left((I_N - J_N)\eta_{N,i} \otimes \left(BKg_{ij}\left(e_i(t) - e_j(t)\right)\right)\right)dw_{ij}(t). \tag{13}$$

According to the definition of $\eta_{N,i}$ and J_N, we can get $\eta_{N,i}^T(I_N - J_N)\eta_{N,i} = (N-1)/N$. Let $V(t) = e^T(t)(I_N \otimes P)e(t)$. Using It ô's formula [38], we have

$$\begin{aligned} dV(t) &= e^T(t)\Big(((I_N\otimes A) - (cL\otimes BK))^T (I_N\otimes P) \\ &\quad + (I_N\otimes P)((I_N\otimes A) - (cL\otimes BK))\Big)e(t)dt \\ &\quad + M_1(t) + \frac{N-1}{N}c^2 \sum_{i,j=1}^{N} a_{ij}^2 g_{ij}^T\big(e_i(t) - e_j(t)\big) \\ &\quad \times K^T B^T PBK g_{ij}\big(e_i(t) - e_j(t)\big)dt \\ &\le e^T(t)\Big(((I_N\otimes A) - (cL\otimes BK))^T (I_N\otimes P) \\ &\quad + (I_N\otimes P)((I_N\otimes A) - (cL\otimes BK))\Big)e(t)dt \\ &\quad + M_1(t) + \frac{N-1}{N}c^2\lambda_{\max}\big(K^T B^T PBK\big) \\ &\quad \times \sum_{i,j=1}^{N} a_{ij}^2 g_{ij}^T\big(e_i(t) - e_j(t)\big) g_{ij}\big(e_i(t) - e_j(t)\big)dt, \end{aligned} \tag{14}$$

where $M_1(t) = 2e^T(t)c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}((I_N - J_N)\eta_{N,i} \otimes PBKg_{ij}(e_i(t) - e_j(t)))dw_{ij}(t)$.

By using K in (10) and the inequality in (12), we have

$$\begin{aligned} & e^T(t)\Big(((I_N\otimes A) - (cL\otimes BK))^T (I_N\otimes P) \\ &\quad + (I_N\otimes P)((I_N\otimes A) - (cL\otimes BK))\Big)e(t)dt \\ &= e^T(t)\Big(I_N\otimes\big(A^TP + PA\big) - cL\otimes\big(PBB^TP\big)\Big)e(t)dt \\ &\le e^T(t)\Big(I_N\otimes\big(A^TP + PA\big) - c\lambda_2 I_N\otimes PBB^TP\Big)e(t)dt \\ &\le e^T(t)\Big(I_N\otimes\big(A^TP + PA - PBB^TP\big)\Big)e(t)dt \\ &= -e^T(t)(I_N\otimes Q)e(t)dt. \end{aligned} \tag{15}$$

Combining (14) and (15), we have

$$\begin{aligned} dV(t) &= -e^T(t)(I_N\otimes Q)e(t)dt + M_1(t) \\ &\quad + \frac{N-1}{N}c^2\lambda_{\max}\big(K^TB^TPBK\big) \\ &\quad \times \sum_{i,j=1}^{N} a_{ij}^2 g_{ij}^T\big(e_i(t) - e_j(t)\big) g_{ij}\big(e_i(t) - e_j(t)\big)dt \\ &\le -\lambda_{\min}(Q)\|e(t)\|^2 dt + M_1(t) \\ &\quad + 2\frac{N-1}{N}c^2\varepsilon^2\lambda_{\max}(L)\lambda_{\max}\big(K^TB^TPBK\big)\|e(t)\|^2 dt \\ &= -\rho\|e(t)\|^2 dt + M_1(t), \end{aligned} \tag{16}$$

where

$$\rho = \lambda_{\min}(Q) - 2\frac{N-1}{N}c^2\varepsilon^2\lambda_{\max}(L)\lambda_{\max}\big(K^TB^TPBK\big) > 0. \tag{17}$$

Finally, we have

$$\frac{d\|\mathbb{E}e(t)\|^2}{dt} \le \frac{-\rho}{\lambda_{\min}(P)}\|\mathbb{E}e(t)\|^2. \tag{18}$$

Then by the comparison theorem [39], we get

$$\mathbb{E}\|e(t)\|^2 \le \|e(0)\|^2 \exp\left\{\frac{-\rho}{\lambda_{\min}(P)}t\right\}, \tag{19}$$

leading to $\lim_{t\longrightarrow\infty}\mathbb{E}\|x_i(t) - x_j(t)\|^2 = 0$. This completes the proof. □

Remark 3. Comparing with existing works concerning with noises, general linear MASs with communication noises are considered in this paper. For MASs with additive noises, stochastic approximation technique was widely adopted, which resulted in time-varying coupling strengths [12, 21]. In this paper, by employing Riccati equation and It ô's formula, the coupling strength in the consensus protocol is time-invariant, but restricted in a given interval. For some works dealt with communication noises, the concerned MASs were in first-order dynamics, which were special cases of this paper [37].

4. Consensus on Markovian Switching Topologies

In this section, we will analyze consensus of MASs (3) on Markovian switching topologies.

Theorem 2. *Assume that the union graph of $\{\mathscr{G}^r, 1\le r\le m\}$ is connected, the MASs (3) achieve mean square consensus under consensus protocol (6), if K is designed as (9) and c satisfies*

$$\frac{1}{\lambda_2^2(L_{un})} \le c^2 < \frac{N\lambda_{\min}(Q)}{2(N-1)\varepsilon^2\lambda_{\max}(L_{un})\lambda_{\max}\big(K^TB^TPBK\big)}. \tag{20}$$

Proof. The dynamics of error system in switching topologies case is

$$\begin{aligned} \frac{de(t)}{dt} &= \big[(I_N\otimes A) - (cL(\sigma(t))\otimes BK)\big]e(t) \\ &\quad + c\sum_{i,j=1}^{N} a_{ij}(\sigma(t))\big[(I_N - J_N)\eta_{N,i} \\ &\quad \otimes\big(BKg_{ij}\big(e_i(t) - e_j(t)\big)\big)\big]\xi_{ij}(t). \end{aligned} \tag{21}$$

In this case, we choose a Lyapunov function for $\sigma(t) = r$ as

$$V_r(t) = \mathbb{E}\big[e^T(t)(I_N\otimes P)e(t)1_{\{\sigma(t)=r\}}\big], \quad \forall r\in\mathbb{M}, \tag{22}$$

where $\sigma(t)$ admits a unique stationary distribution $\pi = [\pi_1, \ldots, \pi_m]^T$. Then the Lyapunov function $V(t)$ for the overall system can be expressed as $V(t) = \sum_{r=1}^m V_r(t)$.

By using the stationary distribution π, the expectation of $V(e(t), \sigma(r))$ becomes

$$\mathbb{E}[V(e(t), \sigma(r))] = \sum_{r=1}^m \mathbb{E}[V_r(e(t), \sigma(r))]\pi_r. \tag{23}$$

By employing the It $\hat{o}$'s formula, we have

$$\begin{aligned}
\frac{dV(t)}{dt} &= \sum_{r=1}^m \pi_r \frac{dV_r(t)}{dt} = \sum_{r=1}^m \mathbb{E}\left[e^T(t)\left(I_N \otimes \left(A^T P + PA\right)\right)e(t)\right]\pi_r - 2c\sum_{r=1}^m \mathbb{E}\left[e^T(t)(L_r \otimes PBK)e(t)\right]\pi_r \\
&+ c^2 \frac{N-1}{N} \sum_{r=1}^m \mathbb{E}\left[\sum_{i,j=1}^N a_{ij}^2(\sigma(r)) g_{ij}^T\left(e_i(t) - e_j(t)\right) \times K^T B^T PBK g_{ij}\left(e_i(t) - e_j(t)\right)\right]\pi_r \\
&+ \sum_{r,s=1}^m \pi_s q_{rs} V_s(t) \leq \mathbb{E}\left[e^T(t)\left(I_N \otimes \left(A^T P + PA\right)\right)e(t)\right] - 2c\mathbb{E}\left[e^T(t)(L_{un} \otimes PBK)e(t)\right] \\
&+ 2c^2 \frac{N-1}{N} \lambda_{\max}\left(K^T B^T PBK\right)\varepsilon^2 \\
&\times \mathbb{E}\left[e^T(t)(L_{un} \otimes I_n)e(t)\right],
\end{aligned} \tag{24}$$

where $L_{un} = \sum_{r=1}^m \pi_r L_r$. Similar to (10), we have

$$\begin{aligned}
& e^T(t)\left[\left(I_N \otimes \left(A^T P + PA\right)\right) - 2c(L_{un} \otimes PBK)\right]e(t) \\
&\leq e^T(t)\left[\left(I_N \otimes \left(A^T P + PA\right)\right) - 2c\lambda_2(L_{un})(I_N \otimes PBK)\right]e(t) \\
&\leq e^T(t)\left[\left(I_N \otimes \left(A^T P + PA\right)\right) - 2(I_N \otimes PBK)\right]e(t) \\
&= e^T(t) I_N \otimes \left(A^T P + PA - PBB^T P\right)e(t) \\
&= -e^T(t)(I_N \otimes Q)e(t).
\end{aligned} \tag{25}$$

By (24) and (25), it yields

$$\begin{aligned}
\frac{dV(t)}{dt} &= -\mathbb{E}\left[e^T(t)(I_N \otimes Q)e(t)\right] + 2c^2\varepsilon^2 \frac{N-1}{N} \\
&\quad \times \lambda_{\max}\left(K^T B^T PBK\right)\mathbb{E}\left[e^T(t)(L_{un} \otimes I_n)e(t)\right] \\
&\leq \left[-\lambda_{\min}(Q) + 2c^2\varepsilon^2 \frac{N-1}{N}\lambda_{\max}(L_{un})\right. \\
&\quad \left. \times \lambda_{\max}\left(K^T B^T PBK\right)\right]\mathbb{E}\|e(t)\|^2 \\
&= -\varrho \mathbb{E}\|e(t)\|^2,
\end{aligned} \tag{26}$$

where $\varrho = \lambda_{\min}(Q) - 2c^2\varepsilon^2 N - 1/N\lambda_{\max}(L_{un})\lambda_{\max}(K^T B^T PBK) > 0$.

Similar to (16), we get

$$\mathbb{E}\|e(t)\|^2 \leq \|e(0)\|^2 \exp\left\{\frac{-\varrho}{\lambda_{\min}(P)}t\right\}. \tag{27}$$

This completes the proof. □

Remark 4. In light of the assumption on the vector function g_{ij}, the intensity of noises gets weaker while achieving consensus. Specifically, when the norm of relative state between two agents decreases, the intensity of noise in their communication channel becomes smaller. Therefore, compared with additive noises, the multiplicative noises with intensities depending on relative states can better describe noises in the analog fading communication channel. If all of the communication noises $g_{ij}(\cdot) \equiv 0\,(i, j = 1, \ldots, N)$, our result can be degenerated into the noise-free case [22].

Remark 5. Compared with existing studies, we consider Markovian switching topologies and communication noises for general linear MASs. In this case, the impact of noises on the MASs is changing while the underlying topology is switching, which brings challenges for the analysis of the consensus problem. Compared with the fixed topology case, we only require the combined topology to be connected, which relax the assumption on the topology at each instant.

5. Simulation Example

In this section, we present two numerical examples to verify our theoretical results. We consider a MAS of 6 agents under fixed topology and under switching topology in Examples 1 and 2, respectively.

Example 1. Considering a MAS with the following dynamics:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),$$
$$A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \quad (28)$$

The underlying communication topology is depicted in Figure 1. The corresponding Laplacian matrix is

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 3 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix}. \quad (29)$$

We choose

$$Q = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \quad (30)$$

According to ARE (10), we have

$$K = \begin{bmatrix} 0.2061 & -0.07046 & 0.1090 \\ 0.0342 & 0.5317 & 0.3632 \\ 0.0992 & 0.3666 & -0.2323 \end{bmatrix}. \quad (31)$$

Let $g_{ij}(x_i(t) - x_j(t)) = \varepsilon(x_i(t) - x_j(t))$, $\varepsilon = 0.2$, and by simple calculation, we have $c = 2.3$, which ensures the sufficient condition (9) in Theorem 1. The noises here are subject to Brownian process and the simulation is conducted by the Euler–Maruyama method. Under these settings, the MASs achieve consensus as shown in Figure 2. According to Figure 2, we find that the process of achieving consensus is chartering due to the existence of communication noises. We generate 100 sample paths to simulate the mean square average, and Figure 3 shows the system achieves mean square consensus.

Example 2. Consider a MAS of 6 agents with the interaction topology randomly switches among $\mathcal{G}^1$, $\mathcal{G}^2$, and $\mathcal{G}^3$ in Figure 4. The Laplacian of the combined graph is $L_{un} = \pm \sum_{r=1}^{m} \pi_r L_r$. The transition rate matrix is chosen as

$$\Xi = \begin{bmatrix} -6 & 2 & 4 \\ 3 & -4 & 1 \\ 2 & 1 & -3 \end{bmatrix}. \quad (32)$$

Let $g_{ij}(x_i(t) - x_j(t)) = \varepsilon(x_i(t) - x_j(t))$, $\varepsilon = 0.3$, and by simple calculation, we have $c = 2$, which ensures the sufficient condition (19) in Theorem 2. Figure 5 shows the sample paths of 6 agents under a known generator matrix. After a realization of randomly switching topologies, the consensus is reached. Figure 6 shows the switching signals, which are subject to a Markovian process. Compared with Example 1,

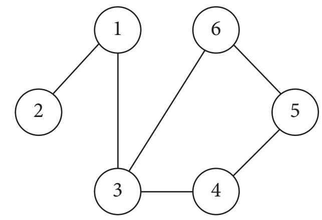

FIGURE 1: The fixed topology of Example 1.

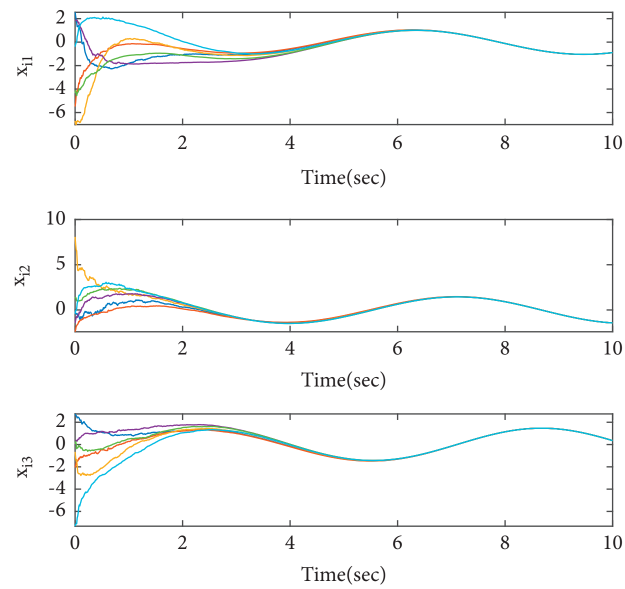

FIGURE 2: States of 6 agents of Example 1.

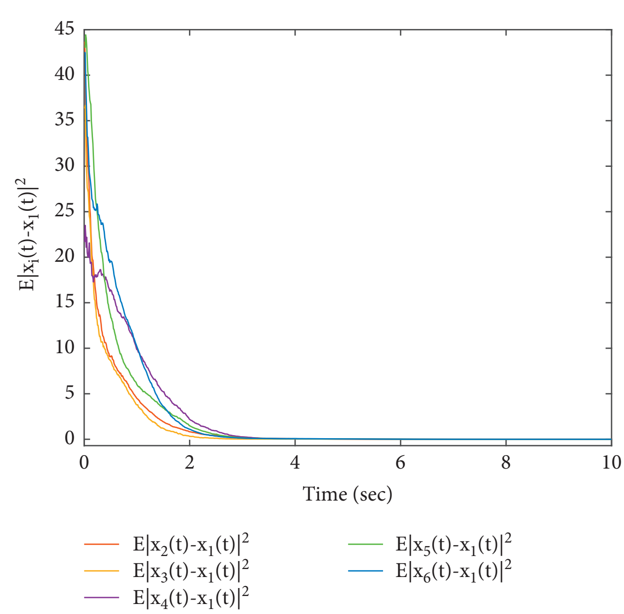

FIGURE 3: Mean square errors $\mathbb{E}|x_i(t) - x_1(t)|^2$ of Example 1.

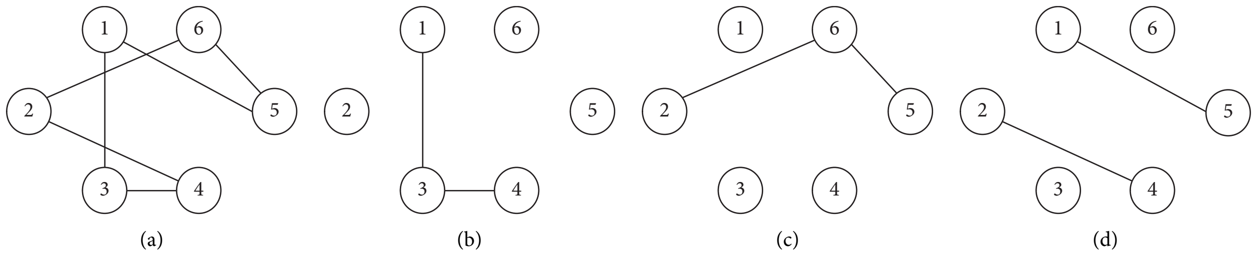

Figure 4: The switching topologies of Example 2. (a) $\mathscr{G}$. (b) $\mathscr{G}^1$. (c) $\mathscr{G}^2$. (d) $\mathscr{G}^3$.

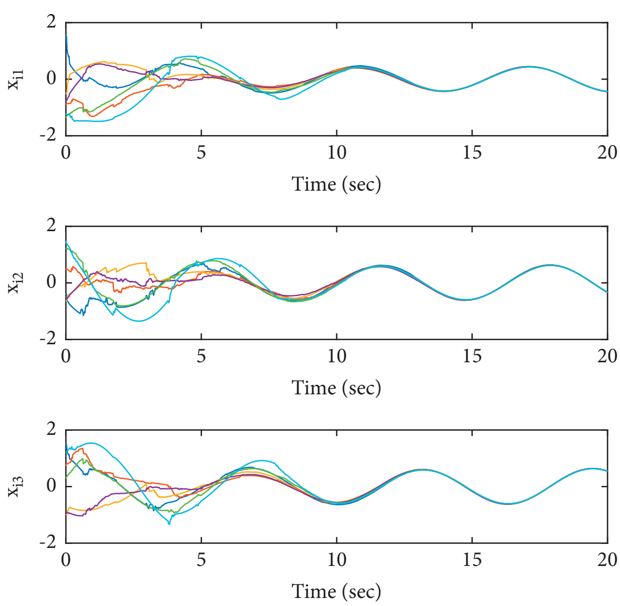

Figure 5: States of the 6 agents in Example 2.

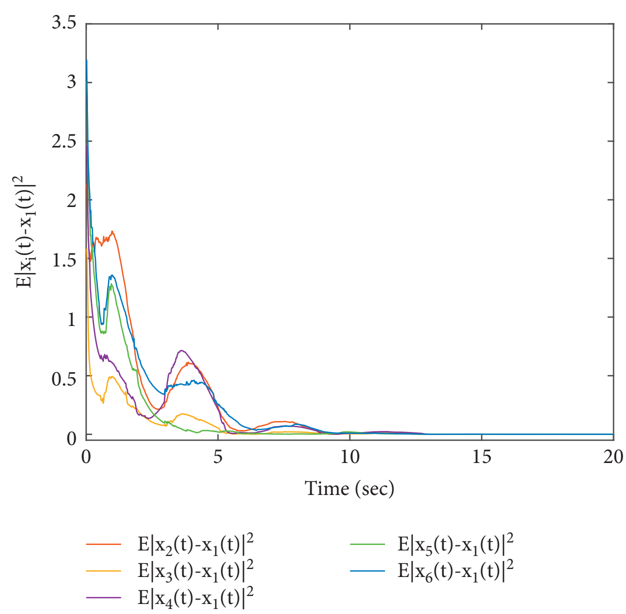

Figure 7: Mean square errors $\mathbb{E}|x_i(t) - x_1(t)|^2$ of Example 2.

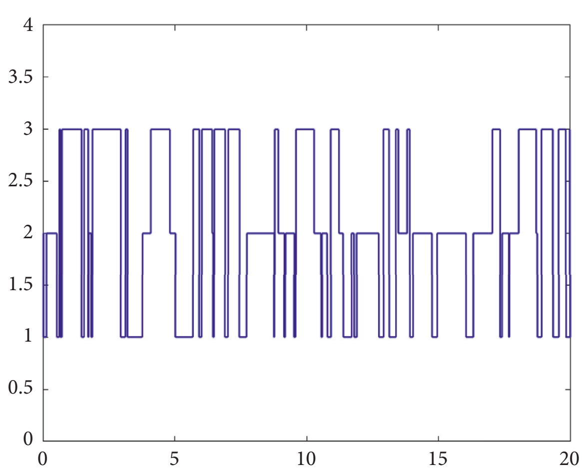

Figure 6: The Markovian switching signals.

it takes much longer for the switching topologies case to achieve consensus. We generate 100 sample paths to simulate the mean square average, and Figure 7 shows the MAS achieves mean square consensus.

6. Conclusions

Motivated by the uncertainties in real communication networks, in this article, we study the consensus problem of the general linear continuous-time MASs with communication noises. Each agent can obtain full state of itself and receive its neighbors' state information with noises, whose intensity is a vector function of agents' relative states. Research is conducted on both fixed topology and switching topologies, respectively. Mean square consensus is proved by using stochastic analysis and algebraic graph theory, and an estimation of the exponential convergence rate of consensus is given. For future research on this topic, the case of finite time consensus will be taken into account.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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