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Research Article

Robust H_∞ Control of DC Motor in the Presence of Input Delay and Disturbance by the Predictor-Based Method

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In this study, the speed control of a DC motor with input delay and external disturbance is investigated. A delay-dependent memory state feedback robust predictor-based H_∞ controller that formerly has presented is utilized. Sufficient LMIs conditions provide the stabilizing gain of the predictor-based controller. The first-order model of a DC motor has been studied previously using PID and STA (super twisting algorithm) methods. However, in this paper, the delay-dependent robust H_∞ controller is used on both the first and second orders of the DC motor. Finally, the predictor-based H_∞ controller is compared to the PID controller and the common H_∞ controller as well. The simulation results show that the proposed method has been more useful and efficient.

1. Introduction

In literature and references, time-delay systems are referred to by a variety of names. Some of them are after-effect or dead-time systems, hereditary systems, deviating argument equations, or differential-difference equations. These systems belong to the class of functional differential equations (FDEs), which can be infinitely dimensional in preference to ordinary differential equations (ODEs) [1].

Systems with delays occur in engineering, biology, physics, operations research, and economics. These delays are also so important in accounting for human behavior, studying and analyzing traffic-flow stability, and designing collision-free traffic flow using adaptive controllers. Some factors need to be considered in various time-delay systems. For example, in traffic-flow models, the drivers' delayed reactions, involving combining sensing, perception, response, selection, and programming delays, must be considered [2].

Over the last few decades, time-delayed systems have been studied by researchers in recognition of their theoretical and practical importance. Many control problems have been extensively studied, such as population dynamics [3, 4], biological systems [5, 6], analysis of human respiratory stability [7], analysis of a model of HIV pathogenesis [8], teleoperation [9], spinal cord injury sitting stability [10], accuracy increase of discrete sensors [11], and satellite image encryption in OFDM communication systems [12].

Input delays can happen generally for two reasons. First, consider the physical nature of the plant, i.e., fluid transportation and biological systems. Second, the controller may introduce delays in computation or communication. Nowadays, with the fast development of remote-controlled systems such as networked control systems and teleoperation, the investigation of input delay systems has been most interesting [13]. Regarding DC motors, input delay is observed in different ways, some of which have been mentioned in the literature. Time delays can happen in networked DC motors or when the DC motor is controlled through a network. In the following, some examples will be reviewed. For example, in a networked DC motor, to transmit the control signal from the central controller to the remote controller, a time delay may occur. For another example, transmitting the measured signal from the remote controller to the central controller in network-induced time delays are unavoidable. It is worth mentioning that DC motor control systems are stable systems in general, when time delays are not considered. However, inevitable time delays may destabilize the closed-loop system. Therefore, time delays must be considered in the process of controller design in practical and physical systems [14, 15].

DC motors and their control drives due to their high reliability, low cost, simple control of speed and position, low energy consumption, and compatibility with digital systems have been widely used in various industrial processes and home applications [16], such as electric wheelchairs, rolling mills, machine tools, robotic arms, lathes, drills, elevators, and cranes. These applications require very precise control of speed. Furthermore, the simple modeling of DC motors can be utilized as a benchmark system for the evaluation of new control laws [13–17]. Numerous techniques have been applied for driving DC motors. For example, sliding mode control [18], optimal control [19], digital control technique [20], adaptive variable structure control [21], PID speed control using the LQR approach [22], online self-tuning ANN-based speed control [23], switched LQ controllers [24], robust speed control using adaptive gain low [25], robust speed control based on the Lyapunov direct method [26], DC motor control based on multisensor information [27], robust flatness tracking control [28], linear parameter varying control approach [29], UM Shaper command inputs [30], and control with fuzzy reasoning [31]. However, they do not consider delays. In [32], the tuning problem of PID parameters for a DC motor controlled via the (CAN) was investigated. In this paper, time delays are considered stochastic, but the delay is not an input delay.

Some research can be listed among a few studies for the control of DC motors in the presence of input delay. For instance, the stability of a DC motor with input delay using a PI controller [33] and the use of an adaptive controller to follow the network's "Quality of service" (QOS) variations on the network [34]. However, they do not consider external disturbance, and the input delay is very small, less than the sampling period.

A PID controller based on a predictive approach is one of the techniques that have been investigated for DC motor control with input delay and disturbance [13]. In that study, the gain (K) of the controller was obtained by trial and error in the Simulink environment. The most important result of that research was controlling the speed of a DC motor in the presence of constant disturbance.

In this paper, we have used a delay-dependent memory state-feedback predictor-based H_∞ controller for a DC motor with input delay and disturbance that will be able to stabilize the speed of a DC motor. Therefore, the gain of the mentioned controller, K is obtained with linear matrix inequalities (LMIs), and the stabilization of this method is guaranteed by Lyapunov theory analysis and has theoretical analysis. While in the study of PID controllers [13], there is no method to acquire the gain of the controller.

In this study, the predictor-based H_∞ controller is investigated to control the speed of a DC motor with three different modes of external disturbance. These are constant, sinusoidal, and stochastic disturbances, while in the real world, signals are random in nature. The predictor-based H_∞ controller will be able to stabilize the speed of the DC motor in the presence of sinusoidal and stochastic disturbances with higher performance and more efficiency than the PID controller [13].

2. Presentation and Formulation of the Method

The H_∞ control method based on the predictor that has been presented in [35] for an LTI system in the presence of input delay and external disturbance is briefly recalled in this section.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-\tau) + Dw(t), \\ y(t) &= C_1 x(t), \\ Z(t) &= Cx(t) + Eu(t-\tau), \\ u(t) &= \phi, \\ &\quad t \in [-\tau, 0], \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the input vector, $y(t) \in R^p$ is the measured output, $Z(t) \in R^d$ is the controlled output, τ is the constant single delay, $\phi(t)$ is a real-valued initial function on $t \in [-\tau.0]$, $w(t) \in R^q$ is the external disturbance vector, matrices A, B ,D, C_1,C, and E are all constant real matrices with appropriate dimensions. Also, it is assumed that the exogenous disturbance signal $w(t)$ is square-integrable, i.e., $w(t)^2_{\mathscr{L}_2} = \int_0^\infty w(s)^2 ds < M_1$ and $M_1 \geq 0$.

Lemma 1. *The prediction vector for system (1) in the input time-delay horizon is given by*

$$\overline{P}(t) = x(t+\tau) = e^{A\tau}x(t) + \int_{t-\tau}^{t} e^{A(t-s)}[Bu(s) + Dw(s+\tau)]ds. \tag{2}$$

Lemma 2. *In the absence of external disturbance ($w = 0$), the prediction vector for system (1) in the input time-delay horizon is specified.*

$$P(t) = x(t+\tau) - \int_{t-\tau}^{t} e^{A(t-s)}[Dw(s+\tau)]ds = e^{A\tau}x(t) + \int_{t-\tau}^{t} e^{A(t-s)}[Bu(s)]ds. \quad (3)$$

Theorem 1. *Let positive constants γ, λ, L_R, and L_S are given. The linear delayed system (1) with prediction-based controller in the absence of external disturbance is asymptotically stable and in the presence of external disturbance satisfies $Z(t)^2_{\mathscr{L}_2} < \gamma^2 \overline{w}(t)^2_{\mathscr{L}_2}$ for $\overline{w}(t) \in \mathscr{L}_2[0\infty)$, if there exist symmetric positive definite matrix $X > 0$ and matrix Y with appropriate dimensions, such that the following LMIs hold:*

$$\begin{bmatrix} \psi_{11} & 0 & D & B & XC^T + Y^T E^T \\ 0 & \psi_{22} & \lambda M\ D & 0 & 0 \\ D^T & \lambda D^T M^T & -\gamma^2 I & 0 & 0 \\ B^T & 0 & 0 & -\gamma^2 I & E^T \\ CX + EY & 0 & 0 & E & -I \end{bmatrix}, \quad (4)$$

$$\begin{bmatrix} L_R I & Y^T \\ Y & I \end{bmatrix} > 0,$$

$$\begin{bmatrix} L_S I & I \\ I & X \end{bmatrix} > 0,$$

where $\Psi_{11} = XA^T + Y^T I^T + AX + BY$ *and* $\Psi_{22} = \lambda(XA^T + Y^T I^T + AX + BY)$.

The appropriate gain for the prediction-based controller (9) is given by $K = YX^{-1}$.

If the initial function is assumed, the predictive vector can be estimated with the recursive formula obtained in Lemmas 1 and 2 for subsequent periods. From (2) and (3), Ii is possible to write

$$\overline{P}(t) = P(t) + e_p(t), \quad (5)$$

where guarantee

$$e_p(t) = \int_{t-\tau}^{t} e^{A(t-s)}[Dw(s+\tau)]ds. \quad (6)$$

Finally, the predictive-based controller is established with the following structure:

$$u(t) = KP(t) = K\overline{P}(t) - Ke_p(t). \quad (7)$$

Therefore, the design of the controller is performed as follows:

$$u = KP = Ke^{A\tau}x(t) + K\int_{t-\tau}^{t} e^{A(t-s)}[Bu(s)]ds. \quad (8)$$

The prediction vector $P(t)$ is obtained by Lemmas 1 and 2. In order to provide sufficient conditions for the existence of a delay-dependent state-feedback H_∞ controller, the gain of this controller (K) is obtained by solving the LMIs in theorem (1).

Remark 1. the proof of Theorem 1, Lemmas 1 and 2, and more details about the method are referred to [33].

3. Application to a DC Motor

In this section, the delay-dependent memory state-feedback predictor-based H_∞ controller is applied to control the angular velocity of a DC motor and the result is compared with the PID controller.

3.1. First Order DC Motor. The simplified transfer function and its state-space representation of the DC motor with delayed input u is [13]

$$\frac{\Omega(s)}{U(s)} = \frac{k}{1+sT}e^{-\tau s}, \quad (9)$$

where Ω and U are the Laplace transforms of the angular velocity (ω) and the input voltage (u) respectively. The steady-state gain k, the time constant T and the input delay τ are known. The term $e^{-\tau s}$ accounts for the delayed input. This delay is not intrinsic to the motor model. It is due to the delay in input that can be introduced, for example, by remote control over a network or the time needed to compute the control law.

$$\dot{\omega} = a\omega + bu(t-\tau) + d. \quad (10)$$

with $a = -1/T$, $b = k/T$, and d is the disturbance. Table 1 shows the system parameter values of the first-order DC motor.

The memory state-feedback predictor-based H_∞ controller for the first-order DC motor is designed in this way, $u(t) = KP$. $P(t)$ is the predictive vector and is obtained by Lemmas 1 and 2. K is the state-feedback gain matrix to be designed since the closed-loop system without external disturbance is asymptotically stable. Under zero initial conditions, the $\mathscr{L}_2$ gain (i.e., H_∞ norm) of the closed-loop system guarantees the following criterion for all nonzero $\overline{w}(t) \in \mathscr{L}_2[0\infty)$, and some scalar $\gamma > 0$ [36].

$$Z(t)_{\mathscr{L}_2} < \gamma \overline{w}(t)_{\mathscr{L}_2}. \quad (11)$$

3.2. Simulation Results. In this section, the results of the first-order DC motor simulation are shown in the following figures. The performance of the predictor-based H_∞ controller is compared with the performance of the PID controller. In order to show the behavior of controllers, the system in the presence of delay will be examined in three moods. First of all, the disturbance is constant ($d = 24$ rad/s). Second, the disturbance is sinusoidal ($w = 5 + 10\sin(t)$). Finally, a stochastic disturbance is used. In each figure, the behavior of controllers is compared with the case of open-loop system, too.

In physical and practical systems, signals to control systems are not known but are random and stochastic in nature [35]. Therefore, it seems that the behavior of the

Table 1: System parameter values of a first-order DC motor [13].

Parameter	Value
k	177.75
T	1.14 s
ω_{ref}	150 rad/s

controller in the presence of a stochastic disturbance is more like the real world than the constant disturbance.

For predictor-based H_∞ controller, by sitting γ, λ, L_R, L_s and by solving the LMIs in theorem (1) using the YALMIP toolbox [36], the optimum gain of a prediction-based controller is obtained. If the LMIs are feasible, the predictor-based H_∞ controller guarantees robust asymptotic stability of the system. However, there is no defined technique to obtain K for the PID controller in [13].

3.2.1. Simulation Results of a First-Order DC Motor in MATLAB. Both of the controllers are able to eliminate the effect of input delay. In this case, the simulation results are shown in three moods, as follows:

(1) Constant Disturbance. In this subsection, the external disturbance is constant ($d = 24$ rad/s), which affects the system between 10 s and 30 s.

By sitting parameters in Table 2, when the value of input delay is 0.1 s, the optimum gain of a predictor-based H_∞ controller is obtained, $K = -22.7842$.

By sitting parameters in Table 3, when the value of input delay is 1 s, the optimum gain of the predictor-based H_∞ controller is obtained, $K = -67.1073$.

In Figures 1 and 2, for predictor-based H_∞ controller in each step, the optimum gain of the controller is obtained by solving the LMIs. While in the PID controller, there is no specific strategy to acquire K. The PID controller has a big overshoot, while the predictor-based H_∞ controller has approximately no overshoot. The PID controller has more transient state error than the predictor-based H_∞ controller. The speed of the predictor-based H_∞ controller is more than the PID controller. The predictor-based H_∞ controller fluctuations are less than the PID controller.

(2) Sinusoidal Disturbance. In this subsection, the external disturbance is sinusoidal ($W = 5 + 10\sin\ (0.3t)$). By sitting parameters similar to Table 2, when the value of input delay is 0.1 s, the optimum gain of the predictor-based H_∞ controller is obtained, $K = -22.7842$.

By sitting parameters in Table 4, when the value of input delay is 1 s, the optimum gain of the predictor-based H_∞ controller is $K = -50.5419$.

In Figures 3 and 4, in the presence of sinusoidal disturbance, the speed and performance of the predictor-based H_∞ controller are definitely better than the PID controller. The PID controller has a big overshoot and the fluctuation of the system is more than the predictor-based H_∞ controller. The disturbance attenuation in the predictor-based H_∞ controller is more than the PID controller.

(3) Stochastic Disturbance. In this subsection, the behaviors of controllers are investigated when the system is in the presence of a stochastic disturbance. The stochastic disturbance that has been used for this part is shown in Figure 5.

By sitting parameters in Table 5, when the value of input delay is 0.1 s, the optimum gain of the predictor-based H_∞ controller is $K = -72.4111$.

By sitting parameters in Table 6, when the value of input delay is 1 s, the optimum gain of the predictor-based H_∞ controller is $K = -41.4002$.

In Figures 6 and 7, the best performance of the predictor-based H_∞ controller is in the presence of stochastic disturbance. The PID controller has a big overshoot, while the predictor-based H_∞ controller has approximately no overshoot. The transient state error of the PID controller is much more than that of the predictor-based H_∞ controller. The disturbance attenuation of the predictor-based H_∞ controller is more than the PID controller.

3.2.2. Simulation Results of a Second-Order DC Motor. Consider the state-space model of a second-order DC motor as follows [37]:

$$\frac{d}{dt}\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix} = \begin{bmatrix}-\frac{b}{j} & \frac{K}{j}\\ -\frac{K}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}u(t-\tau) + D\begin{bmatrix}v\\ 0\end{bmatrix},$$

$$\text{where } \dot{z} = \frac{d}{dt}\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix},$$

$$\mathrm{A} = \begin{bmatrix}-\frac{b}{j} & \frac{K}{j}\\ -\frac{K}{L} & -\frac{R}{L}\end{bmatrix}, \mathrm{B} = \begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}, z = \begin{bmatrix}\dot{\theta}\\ i\end{bmatrix}, \mathrm{D} = \mathrm{I}\,w = \begin{bmatrix}v\\ 0\end{bmatrix}. \tag{12}$$

Table 7 shows the system parameter values of the second-order DC motor.

In this section, the simulation results are developed into a second-order DC motor. The behavior of controllers, while the system is in the presence of delay will be investigated. External disturbances are considered in two cases. The first is a sinusoidal disturbance ($w = 5 + 10\sin(10t)$) and the second is a stochastic disturbance. Both of the controllers are able to eliminate the influence of input delay.

(1) Sinusoidal Disturbance. In this subsection, the behaviors of controllers will be investigated when the second-order DC motor is in the presence of a time delay for 1 s and 5 s. The external disturbance is sinusoidal ($w = 5 + 10\sin(10t)$).

By sitting parameters in Table 8, when the value of input delay is 1 s and 5 s, the optimum gain of the predictor-based H_∞ controller is $K = -93.385\ \ -1.1953$.

Table 2: Parameters for obtaining K and simulation for $\tau = 0.1$ s and constant disturbance.

Parameters	Value
γ	0.1
λ	0.01
L_R	10^6
L_s	10^6
c	2
dt	0.002

Table 3: Parameters for obtaining K and simulation for $\tau = 1$ s and constant disturbance.

Parameters	Value
γ	0.1
λ	0.01
L_R	10^6
L_s	10^6
c	6
dt	0.002

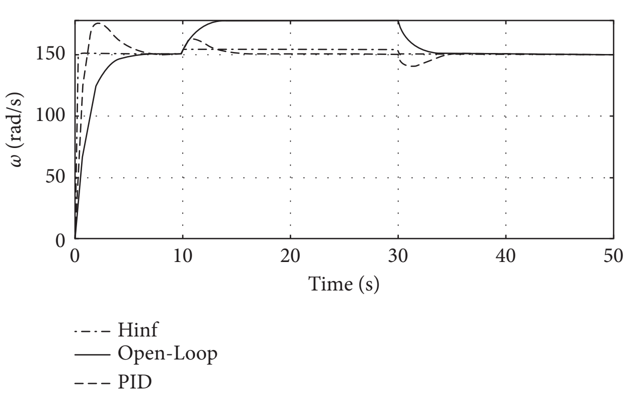

Figure 1: The angular velocity of the first order system for $\tau = 0.1$ s for constant disturbance.

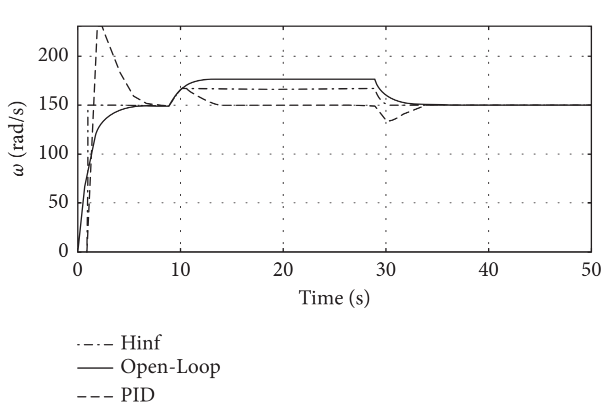

Figure 2: The angular velocity of the first order system for $\tau = 1$ s for constant disturbance.

Table 4: Parameters for obtaining K and simulation for $\tau = 1s$ and sinusoidal disturbance.

Parameters	Value
γ	0.1
λ	0.01
L_R	10^8
L_s	10^8
c	5
dt	0.002

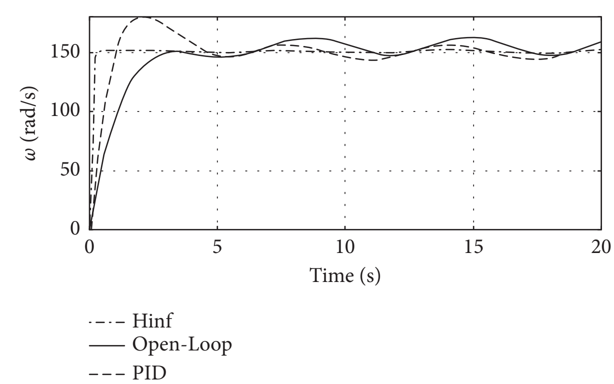

Figure 3: The angular velocity of the first order system for $\tau = 0.1$ s for sinusoidal disturbance.

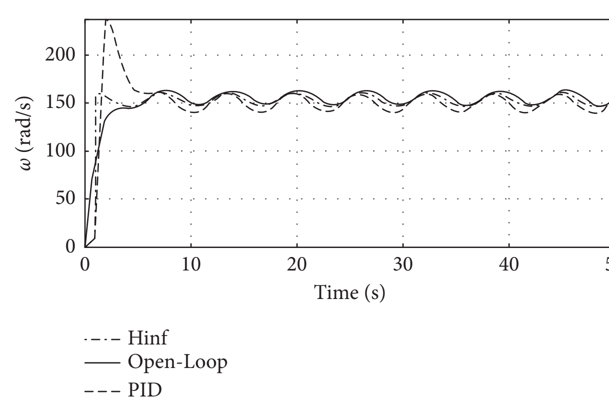

Figure 4: The angular velocity of the first order system for $\tau = 1$ s for sinusoidal disturbance.

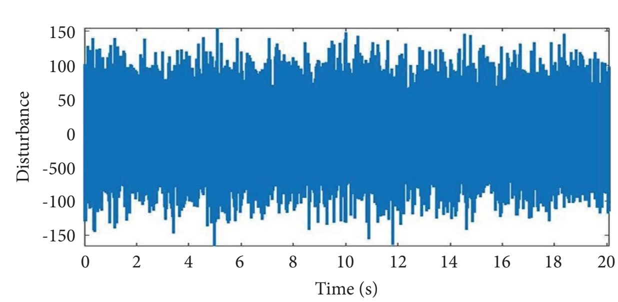

Figure 5: The stochastic disturbance.

Table 5: Parameters for obtaining K and simulation for $\tau = 0.1$ s and stochastic disturbance.

Parameters	Value
γ	0.1
λ	0.01
L_R	10^7
L_s	10^7
c	7
dt	0.002

Table 6: Parameters for obtaining K and simulation for $\tau = 1$ s and stochastic disturbance.

Parameters	Value
γ	0.1
λ	0.01
L_R	10^7
L_s	10^7
c	4
dt	0.002

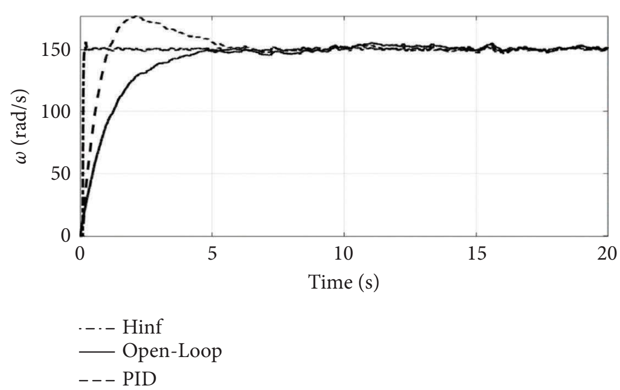

Figure 6: The angular velocity of the first-order system for $\tau = 0.1$ s and stochastic disturbance.

In Figures 8 and 9, for the second-order DC motor in the presence of sinusoidal disturbance, the speed and the performance of the predictor-based H_∞ controller are definitely better than the PID controller, similar to the first-order. The PID controller has a big overshoot. The fluctuation of the PID controller is more than the predictor-based H_∞ controller. The disturbance attenuation in the predictor-based H_∞ controller is more than the PID controller.

(2) Stochastic Disturbance. In this subsection, the external disturbance is stochastic that as shown in Figure 5. Time delays are 1 s and 5 s.

By sitting parameters in Table 9, when the value of input delay is 1 s and 5 s, the optimum gain of the predictor-based H_∞ controller is $K = -93.385\ -1.1953$.

In Figures 10 and 11, the best performance of the predictor-based H_∞ controller for second-order DC motors is in the presence of stochastic disturbance. The PID controller has a bigger overshoot than the predictor-based H_∞ controller. The transient state error of the PID controller is more than H_∞ controller. The disturbance attenuation of the predictor-based H_∞ controller is more than the PID controller.

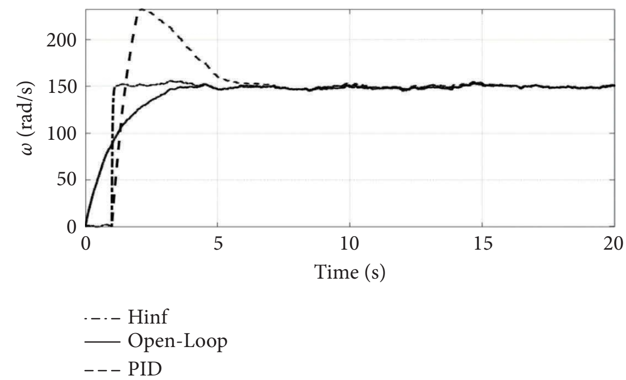

Figure 7: The angular velocity of the first order system for $\tau = 1$ s and stochastic disturbance.

3.3. Comparison of Predictor-Based Robust H_∞ Controller with the Common H_∞ Controller. In this section, a common H_∞ controller is applied to a DC motor and the performance of the predictor-based robust H_∞ controller is compared with that of a common H_∞ controller. System parameter values for the DC motor are $k = 177.75$, $T = 1.14$ s and $\omega_{\text{ref}} = 150$ rad/s.

The common H_∞ controller is a state-feedback controller $u(t) = Kx(t)$. The gain of this controller is obtained by solving the LMIs in Theorem 1 by minimizing the H_∞ norm of the system. The parameters for simulations are similar to Table 10 and the obtained gain with this parameter is $K = -2.4128$.

3.3.1. Comparing in the Presence of Sinusoidal Disturbance. In this subsection, both of H_∞ controllers (predictor-based H_∞ controller and a common H_∞ controller) are applied to DC motors with an input delay of 0.8 ss, 1 s, and 1.1 s in the presence of sinusoidal disturbance ($w = 5 + 10\sin(t)$).

3.3.2. Comparing in the Presence of Stochastic Disturbance. In this subsection, both of H_∞ controllers are applied to a DC motor, where the external disturbance is stochastic that as shown in Figure 5. Time delays are considered at 0.8 s, 1 s, and 1.1 s.

The predictor-based H_∞ controller is a memory or dynamic state-feedback H_∞ controller that can compensate for the effect of input time-delay using predictor feedback. To compensate for input time-delay, the prediction vector in Lemma 1 is used and the control input is constructed with this. However, the common H_∞ controller is a memoryless or static state-feedback H_∞ controller. In Figure 12–17, the predictor-based H_∞ controller is compared with the common H_∞ controller. The mentioned figures show the excellent performance of the predictor-based H_∞ controller than the common H_∞ controller. It is observed that the predictor-based H_∞ provided acceptable results and

Table 7: System parameter values of a second-order DC motor [37].

Parameter	Value
J	$0.01\ \mathrm{kg{\cdot}m^2}$
b	0.1 N·M S
k	0.01 N·M/Am
R	1 ohm
L	0.5 H
ω_{ref}	150 rad/s

Table 8: Parameters for obtaining K and simulation for $\tau = 1$ s, 5 s and sinusoidal disturbance.

Parameters	Value
γ	0.2
λ	1
L_R	10^4
L_s	10^4
c	[10 0]
dt	0.002

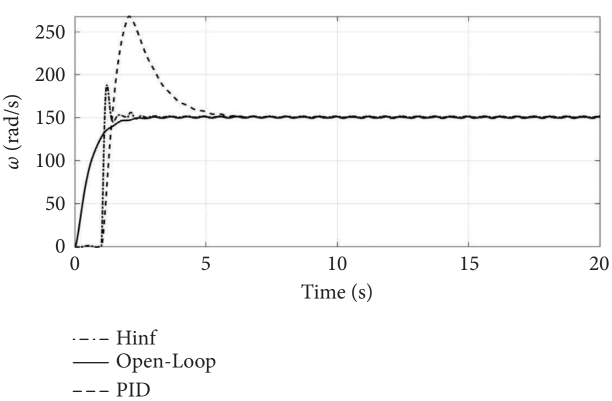

Figure 8: The angular velocity of the second-order system for $\tau =$ 1 s and sinusoidal disturbance.

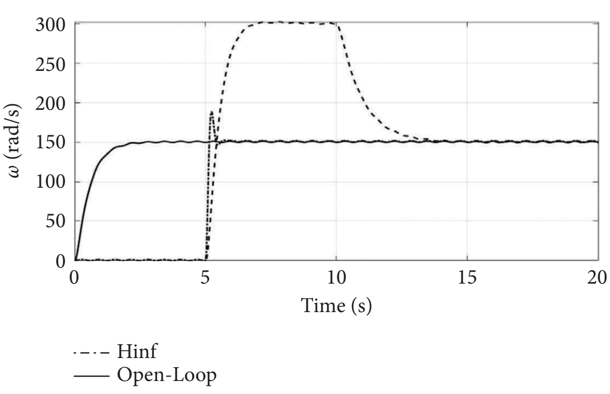

Figure 9: The angular velocity of the second-order system for $\tau =$ 5 s and sinusoidal disturbance.

Table 9: Parameters for obtaining K and simulation for $\tau = 1$ s, 5 s and stochastic disturbance.

Parameters	Value
γ	0.2
λ	1
L_R	10^4
L_s	10^4
c	[10 0]
dt	0.002

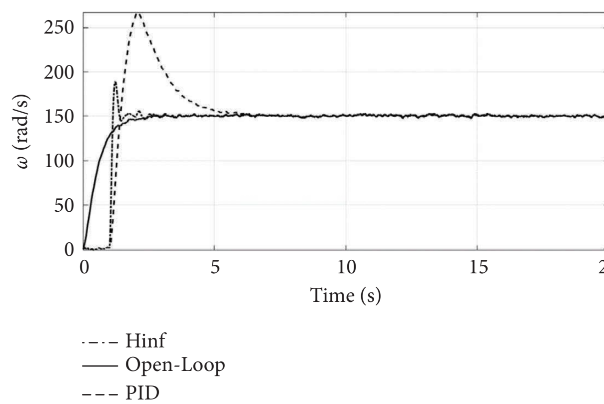

Figure 10: The angular velocity of the second-order system for $\tau = 1$ s and stochastic disturbance.

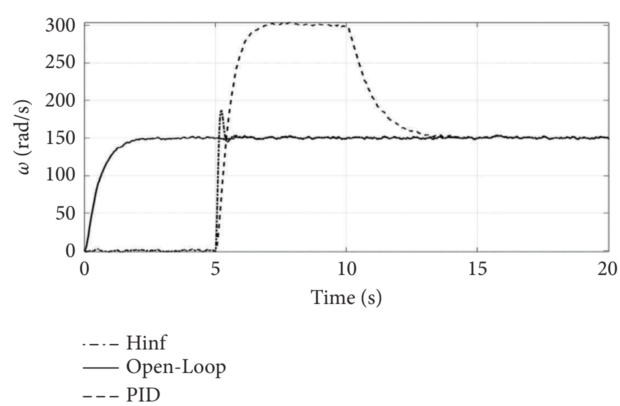

Figure 11: The angular velocity of the second-order system for $\tau = 5$ s and stochastic disturbance.

Table 10: Parameters for obtaining K.

Parameters	Value
γ	0.3
λ	0.01
L_R	10^3
L_s	10^3
c	0.7
dt	0.01

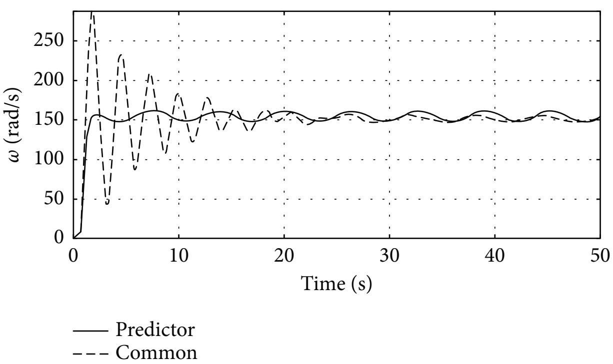

FIGURE 12: Comparing a predictive-based H_∞ controller with a common H_∞ controller for $\tau = 0.8$ s.

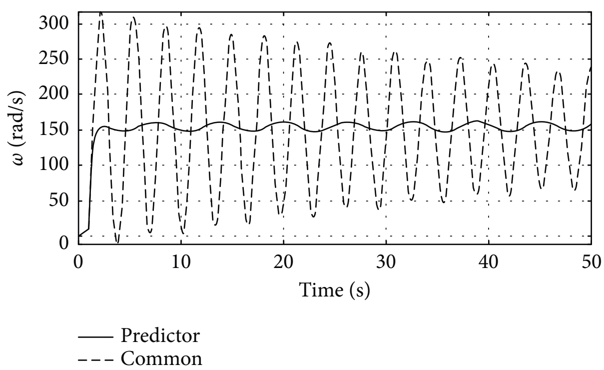

FIGURE 13: Comparing a predictive-based H_∞ controller with a common H_∞ controller for $\tau = 1$ s.

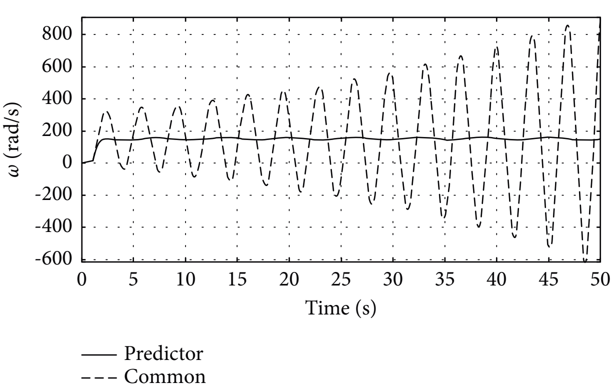

FIGURE 14: Comparing a predictive-based H_∞ controller with a common H_∞ controller for $\tau = 1.1$ s.

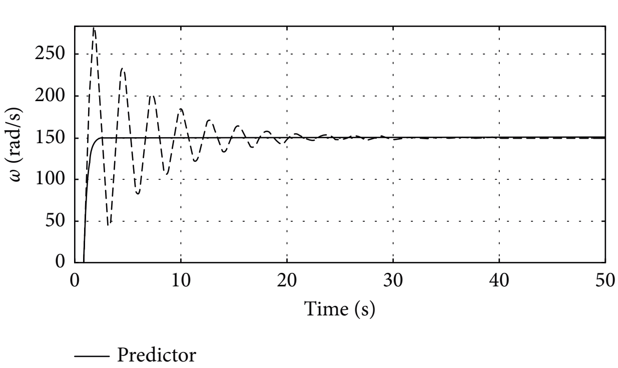

FIGURE 15: Comparing a predictive-based H_∞ controller with a common H_∞ controller for $\tau = 0.8$ s.

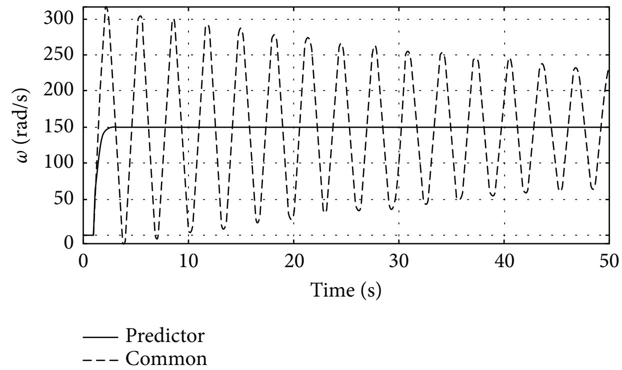

FIGURE 16: Comparing a predictive-based H_∞ controller with a common H_∞ controller for $\tau = 1$ s.

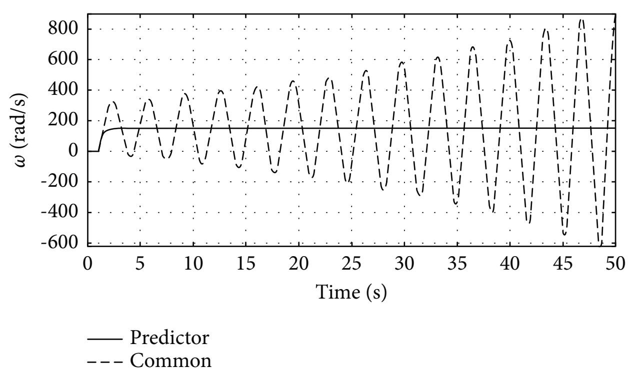

FIGURE 17: Comparing a predictive-based H_∞ controller with a common H_∞ controller for $\tau = 1.1$ s.

eliminated the disturbance and the effect of time delays with very high accuracy. As shown in Figures 12–17, when time-delay increases, the common H_∞ controller can cause unsuitable system performance and even instability.

4. Conclusion

In this paper, a predictive-based robust H_∞ controller is applied to a DC motor in the presence of input time delay and external disturbances. Previously, the first order of this application was studied by designing a PID controller. In this study, the predictor-based H_∞ controller is utilized for first-order and second-order DC motors, and the behavior of the predictor-based H_∞ controller has been compared with the PID controller that has been used in [13] and a common H_∞ controller.

The best performance of the PID controller was when the system was in the presence of constant disturbance. Although the predictor-based H_∞ controller has better speed and performance than the PID controller for sinusoidal and stochastic disturbances, signals in control systems are not known to be stochastic in nature [38].

In the previous research, there was no strategy to obtain the gain of the PID controller, and it seems that the gain has been acquired by tuning in the Simulink environment. But, in this study, the gains of H_∞ controllers are obtained by a sufficient condition in the form of delay-dependent LMIs in each step of the investigation. Using the gains acquired from LMIs, H_∞ controllers guarantee robust asymptotic stability of the system, and because of delay-dependent LMIs, conservatism is decreased.

For the first-order DC motor, the predictor-based H_∞ controller and the PID controller could eliminate the effect of input delay. The speed and performance of the predictor-based H_∞ controller for sinusoidal and stochastic disturbances are definitely better than the PID controller. The PID controller has a big overshoot, and the transient state error of the PID controller is much more than the predictor-based H_∞ controller. The disturbance attenuation of the predictor-based H_∞ controller is more than the PID controller. Only in constant disturbance, the disturbance attenuation of the PID controller is better than the predictor-based H_∞ controller. But, in this case, the speed of the predictor-based H_∞ controller is better than the PID controller. For the second-order DC motor, the predictor-based H_∞ controller and the PID controller are able to eliminate the influence of input delay. In this case, the speed and performance of the predictor-based H_∞ controller are better than those of the PID controller. The other results of the second-order DC motor are similar to the first-order one.

The results of the comparison between the predictor-based H_∞ controller and the common H_∞ controller are as follows: the performance of a predictor-based H_∞ controller is definitely better than a common H_∞ controller. The predictor-based H_∞ controller could reject the disturbance and the time delay effects with high precision. While the time delay increases, the common H_∞ controller is going into instability.

Data Availability

All data that support the results of this paper are available in table 1–10. By simulaation the systems in matlab, the results can be achieved. The gain of controllers are obtained by solving LMIs with YALMIP solvers.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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