Hindawi
Complexity
Volume 2022, Article ID 2006947, 18 pages
https://doi.org/10.1155/2022/2006947

Research Article

Nonuniform Sampled-Data Control for Synchronization of Semi-Markovian Jump Stochastic Complex Dynamical Networks with Time-Varying Delays

N. Sakthivel,[1] Yong-Ki Ma ,[2] M. Mounika Devi,[1] G. Manopriya,[1] V. Vijayakumar ,[3] and Mooyul Huh[4]

[1]*Department of Applied Mathematics, Bharathiar University, Coimbatore 641046, India*
[2]*Department of Applied Mathematics, Kongju National University, Chungcheongnam-do 32588, Republic of Korea*
[3]*Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632014, India*
[4]*Division of International Studies, Kongju National University, Chungcheongnam-do 32588, Republic of Korea*

Correspondence should be addressed to Yong-Ki Ma; ykma@kongju.ac.kr

Received 8 July 2022; Revised 17 September 2022; Accepted 22 September 2022; Published 17 November 2022

Academic Editor: Chittaranjan Hens

In this paper, the problem of exponential synchronization of semi-Markov jump stochastic complex dynamical networks using nonuniform sampled-data control with random delayed information exchanges among dynamical nodes are discussed. In particular, it is considered that random delayed information exchanges follow a Bernoulli distribution, in which stochastic variables are used to model randomness. To achieve exponential synchronization, we designed a nonuniform sampled-data control approach. By constructing an appropriate Lyapunov–Krasovskii functional and using the Wirtinger inequality, sufficient criteria were obtained in terms of linear matrix inequalities. Finally, numerical examples were implemented to demonstrate the effectiveness and superiority of the proposed design techniques.

1. Introduction

In the past few decades, complex dynamical networks (CDNs) have attracted increasing attention in many real-world applications such as power grids, the World Wide Web, cellular networks disease, transmission networks, traffic networks on roads, electricity distribution, neural networks, linguistic networks, and aviation networks. In general, a CDN consists of a vast set of associated nodes, which constitute the basic elementary units with definite dynamics. The complexity of these networks results in significant practical problems. Therefore, analyses of the topological properties and dynamical behaviors of the network nodes have been extensively conducted by many researchers [1–3]. Additionally, they have made significant efforts to examine and implement large-scale dynamical systems in fields of science and engineering. It is worth mentioning that CDNs can be expressed by differential equations demonstrating various dynamical behaviors such as self-organization, spatial-temporal chaos, synchronization, and spiral waves. Among these, synchronization is a key issue that has been extensively investigated [4–6]; it finds applications through synchronous communication and signals synchronization in geostationary satellites, synchronous motors, and databases. It is well known that the process of synchronization between two or more nodes aims to attain common trajectories by tuning a set of prescribed properties. In [7], global cluster synchronization via aperiodic intermittent control was proposed. Zhao et al. [8] studied the exponential synchronization of delayed CDNs under nonfragile sample data control, and Dai et al. [9] proposed an event-triggered mechanism with the objective of obtaining exponential synchronization for time-varying inner-coupling CDNs. In addition, in [10], a sampled data control scheme was adopted for exponential synchronization of CDNs with Markov jump, whereas in [11] a nonfragile sampled-data controller for exponential H_∞ synchronization of CDNs was examined. Therefore, it is more general and relevant to consider the exponential synchronization of CDNs.

Besides, it should be noted that CDNs are often subject to noisy environments and perturbations. In real-life scenarios, unpredicted modifications of the external environment or uncertainties lead to random fluctuations, which results in a noisy environment during signal transmission. Hence, stochastic modeling has become more significant for handling random fluctuations in realistic dynamical behaviors of complex networks. Recently, many researchers have focused on investigating synchronization problems in stochastic complex dynamical networks. However, time delays occurring in such systems can degrade the performance or damage the synchronization process. It is essential to consider time delays in the synchronization problem of CDNs. In practical situations, during information exchange between the nodes, a time delay may occur, and in some cases, randomness may also emerge in delayed information exchanges, which is a more tedious problem to address. These random delayed information exchanges are handled using a stochastic variable that satisfies the Bernoulli distribution. Therefore, it is necessary and practical to consider random delayed information exchanges between nodes in the study of CDNs [12].

Nowadays, more attention has been concentrated on the study of semi-Markov jumps in CDNs. In general, the abrupt variation occurring in communication topologies may lead to repairs and failures of components owing to sudden environmental changes, unstable systems, and interdependence among various points of a nonlinear plant. To overcome these practical problems, this class of systems is modeled using transitions according to the Markov chain methodology. Ye et al. [13] and Ma et al. [14] studied the synchronization of a class of semi-Markov jump CDNs. In Markov jump CDNs, the transition rates are constant, whereas, in semi-Markov jump CDNs, varying transition rates are exploited [15, 16]. Moreover, Markov-jump systems have limitations in their application because the time duration between two jumps is considered as a sojourn time that follows an exponential distribution in which the jump speed stochastic process is independent of the history, whereas in a semi-Markov jump process, the sojourn time obeys a nonexponential distribution, which not only depends on the present time but also on the sojourn time. Therefore, semi-Markov jump CDNs are more generic than Markov-jump CDNs and can deal with the highly complex factors occurring in systems [16, 17]. The authors, in [18], discussed about stochastic complex dynamical networks with the semi-Markov process. Also, the authors of [19, 20] and [21] have studied about semi-Markov jump systems. In addition, most real-time systems are affected by external noise factors and stochastic disturbances. Hence, it should be noted that stochastic disturbances are inevitable in the study of the synchronization of CDNs. Therefore, it is important to investigate stochastic CDNs subject to a semi-Markov jump topology and stochastic disturbances.

In general, all the dynamic behaviors of the nodes in CDNs are not synchronized. Several control schemes have been applied for the synchronization of CDNs. With the rapid development of high-speed computers, digital controllers are being used to control modern communication systems. Hence, only at the discrete-time, instants will the samples of the control input signals be employed; that is, only at sampling instants will the information be sent to the controller, which helps in reducing the amount of transmitted information. In addition, communication bandwidth is saved by implementing sampled-data control systems [22, 23]. However, for the sake of greater use of modern computer techniques, communication technology, and digital hardware systems, sampled-data feedback control is applied. This control plays a significant role in attaining low consumption, high reliability, and simple system deployment [24–27]. At certain instants, the introduction of suitable irregularities provides more benefits than classical sampling methods. In addition, the main aim of using different sampling schemes is to reduce the data size and data loss and ensure higher accuracy. Therefore, it is important to consider nonuniform sample techniques that take samples according to unequal time intervals [28]. These techniques are applied in chemical engineering, network control systems, nuclear magnetic resonance, automotive applications, and sensing devices. Based on this idea, many researchers focused on using nonuniform sampled-data controllers [29–32].

The problem of synchronization of stochastic CDNs constitutes a new challenge subjected to random time-varying delays and stochastic disturbances within a semi-Markov process. To handle stochastic disturbances, the Wiener process plays an important role, and to overcome these difficulties, a nonuniform sampled-data controller was investigated to achieve higher accuracy. To the best of our knowledge, no results have been reported concerning the exponential synchronization of semi-Markovian stochastic CDNs based on nonuniform sampled-data control subject to random delayed information exchanges and stochastic disturbance. Motivated by the above discussions, we derive a new set of criteria for exponential synchronization of semi-Markov stochastic CDNs with a Wiener process based on nonuniform sampled-data control in terms of linear matrix inequalities (LMIs). The main contributions of this study are summarized as follows:

(1) In our work, as the first attempt, studies on the synchronization problem for a class of stochastic CDNs subjected to random time delay under semi-Markov jump process and non-uniform sampled data controller have been considered, altogether, in which the challenging problem is that how to handle randomly occurring time delay that occurs during information exchange among the nodes and to attain exponential synchronization.

(2) To handle this randomness, Bernoulli distribution has been introduced, and also, a Wiener process technique is implemented for handling stochastic disturbances.

(3) Furthermore, a suitable Lyapunov–Krasovskii functional (LKF) is constructed to derive sufficient conditions in terms of LMIs. Finally, developed theoretical results are validated and achieved through two numerical simulations.

This paper is organized as follows. The problem formulation and preliminaries are presented in Section 2. The main results are presented in Section 3. In Section 4, numerical examples are presented to demonstrate the effectiveness of the proposed approach. Finally, the conclusions are presented in Section 5.

1.1. Notations. $\mathbb{R}^l$ denotes the l-dimensional Euclidean space and the set of all $p \times q$ real matrices is denoted as $\mathbb{R}^{p\times q}$. For any given matrix, $M = [M_{ij}]_{n\times n}$, M^T and M^{-1} denote the transpose and inverse of$M$, respectively. The elements presented below the main diagonal of the symmetric matrix are denoted as *. The notation $X > 0\,(X \geq 0)$ for $X \in \mathbb{R}^n \times \mathbb{R}^n$ implies that the matrix X is a real symmetric positive definite (positive semi-definite) matrix, and I is the identity matrix. diag$\{\ldots\}$ denotes a block diagonal matrix. For a given matrix, the largest eigenvalue is $\Theta_{\max}(.)$. Given any two matrices, $\mathcal{T} \in \mathbb{R}^{n\times m}$ and $\mathcal{S} \in \mathbb{R}^{p\times q}$, their Kronecker product is denoted as $\mathcal{T} \otimes \mathcal{S}$. The complete probability space $(\Xi, \mathcal{F}, P)$ in which filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfies that it contains all $P-$ null sets and is always right-continuous. $L^P_{\mathcal{F}_0}([-\zeta, 0])$ denotes the family of all $\mathcal{F}_0$ measurable $\mathscr{C}([-\zeta, 0]; \mathbb{R}^n)$ with random variables $\varphi = \{\varphi(s)\colon \zeta \leq s \leq 0\}$, such that $\sup_{-\zeta\leq s\leq 0} \mathbb{E}\|\varphi(s)\|^P < \infty$, where $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^n$, and $\mathbb{E}\{\cdot\}$ indicates the mathematical expectation operator.

2. Problem Formulation and Preliminaries

Consider a class of semi-Markov jump CDNs consisting of N identical coupled nodes defined over a probability space $(\Xi, \mathcal{F}, P)$ whose network model is designed as follows:

$$\begin{cases} dx_q(t) = \left[A_{\eta(t)}x_q(t) + f\big(t, x_q(t)\big) + (1-\varrho(t))\displaystyle\sum_{r=1}^{N} b_{qr}\Gamma_{\eta(t)}x_r(t) + \varrho(t)\sum_{r=1}^{N} b_{qr}\Gamma_{\eta(t)}x_r(t-\wp(t)) + u_q(t)\right]dt \\ \qquad +W_{\eta(t)}d\tau(t), \\ x_q(t) = \phi_q(t), \forall t \in [-\wp, 0], q = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_q(t) \in \mathbb{R}^n$ denotes the state vector, $u_q(t)$ represents the control input, $A_{\eta(t)}$ is the given matrix, $f(t, x_q(t))$ is a nonlinear vector-valued function, $\wp(t)$ denotes the time-varying delay satisfying $0 \leq \wp(t) \leq \wp$, with $\dot{\wp}(t) \leq \nu$, $\Gamma_{\eta(t)}$ indicates the inner coupling positive diagonal matrix, and $B = (b_{qr})_{N\times N} \in \mathbb{R}^{N\times N}$ represents the outer coupling matrix that provides the topological structure of the networks. If there is a connection between node q and node $r\,(q \neq r)$, then $b_{qr} \neq 0$; otherwise, $b_{qr} = 0$. The diagonal elements are considered to be $b_{qq} = -\sum_{r=1, r\neq q}^{N} b_{qr}$, where $r = 1, 2, \ldots, N$; $W_{\eta(t)}$ indicates the Wiener process matrix, which is considered to be an appropriate dimensional matrix, in which $\tau(t)$ is a standard one-dimensional Wiener process on the given probability space $(\Xi, \mathcal{F}, P)$ with $\mathbb{E}\{d\tau(t) = 0\}$, $\mathbb{E}\{d\tau^2(t) = dt\}$; and $\varrho(t)$ denotes the stochastic variable that follows the Bernoulli distributed sequence:

$$\varrho(t) = \begin{cases} 1, & \text{information exchange happen during delay,} \\ 0, & \text{information exchange does not happen during delay.} \end{cases} \tag{2}$$

Let $\{\eta(t), t \geq 0\}$, be a continuous-time homogeneous semi-Markov process that is defined on a probability space $(\Xi, \mathcal{F}, P)$ with right-continuous trajectories, and the values are taken in a finite set $\mathbb{S} = \{1, 2, \ldots, \mathcal{N}\}$ with the transition probability matrix $\Pi \triangleq \{\Pi_{ij}(\Delta)\}$ given by

$$\Pr\{\eta(t+\Delta) = j | \eta(t) = i\} = \left\{ \prod_{ij}(\Delta)\Delta + o(\Delta), i \neq j, 1 + \prod_{ii}(\Delta) + o(\Delta), i = j, \right. \tag{3}$$

where the small o - notation $o(\Delta)$ defined as $\lim_{\Delta\longrightarrow 0}(o(\Delta)/\Delta) = 0$, $\Delta > 0$ represents the sojourn time in the semi-Markov process, $\prod_{ij}(\Delta) \geq 0$ for $i \neq j$ is the transition rate from mode i at a time t to mode j at the time $(t+\Delta)$, and $\prod_{ii}(\Delta) = -\sum_{j\in s, j\neq i}\prod_{ij}(\Delta)$.

To derive the main results of this study, the following assumptions were made:

(A1) The vector-valued continuous function, $f\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$ satisfies the following sector-bound condition:

$$[f(t,\mathrm{x}(t)) - f(t,y(t)) - U(\mathrm{x}-y)]^T[f(t,\mathrm{x}(t)) - f(t,y(t)) - V(\mathrm{x}-y)] \le 0, \tag{4}$$

for every $x, y \in \mathbb{R}^n$, where U and V are suitable constant matrices.

(A2) If $\varrho(t)$ is the Bernoulli distributed sequence, then the given probability conditions satisfy

(i) $\mathrm{Prob}\{\varrho(t)=1\} = \mathbb{E}\{\varrho(t)\} = \varrho_0$,

(ii) $\mathrm{Prob}\{\varrho(t)=0\} = 1 - \mathbb{E}\{\varrho(t)=1\} - \varrho_0$, where $0 \le \varrho_0 \le 1$, is a constant.

(A3) The sampling period $h(t)$ is bounded by $h > 0$, such that $0 < t_{k+1} - t_k \le h, 0 < h(t) \le h$.

Based on the aforementioned conditions, for N identical nodes (1) synchronized to a general value, the synchronization error is constructed as $z_q(t) = \mathrm{x}_q(t) - s(t)$, where $s(t) \in \mathbb{R}^n$ denotes the state vector of the unforced isolated node satisfying $\mathrm{d}s(t) = [A_{\eta(t)}s(t) + f(t,s(t))]\mathrm{d}t$, which is assumed to be noise-free. Therefore, the error system is defined as follows:

$$\begin{aligned} dz_q(t) = {} & \left[A_{\eta(t)}z_q(t) + f\big(t,z_q(t)\big) + (1-\varrho(t))\sum_{r=1}^{N} b_{qr}\Gamma_{\eta(t)}z_r(t) + \varrho(t)\sum_{r=1}^{N} b_{qr}\Gamma_{\eta(t)}z_r(t-\wp(t)) + u_q(t)\right]dt \\ & + W_{\eta(t)}z_q(t)d\tau(t), \end{aligned} \tag{5}$$

where $g(t,z_q(t)) = f(t,\mathrm{x}_q(t)) - f(t,s(t))$. The controller is defined as follows:

$$u_q(t_k) = K_{\eta(t)}z_q(t_k), \quad t_k \le t < t_{k+1}, \tag{6}$$

where the control signals are represented by a sequence of sampling times satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots$ for $\lim_{k\longrightarrow\infty} t_k = +\infty$, such that only $u_q(t_k)$ is available for the interval $t_k \le t < t_{k+1}$, and $z_q(t_k)$ is the discrete measurement of $z_q(t)$ at sampling instant t_k. The sampling period T is defined as T: $t_{k+1} - t_k$, which is not constant. Therefore, the sampling period is considered to be $t_{k+1} - t_k = h(t) \le h$ for any integer $k \ge 0$, where $h > 0$ represents the largest sampling interval and $h(t) = t - t_k, \forall t \in [t_k, t_{k+1})$. Hence, the nonuniform sampled-data controller is constructed as follows:

$$u_q(t_k) = K_{\eta(t)}z(t-h(t)), \quad h(t) \le h,\ t \in [t_k, t_{k+1}). \tag{7}$$

Implementing the designed control input (7) into (5) yields

$$\begin{aligned} \mathrm{d}z_q(t) = {} & \left[A_{\eta(t)}z_q(t) + g\big(t,z_q(t)\big) + (1-\varrho(t))\sum_{r=1}^{N} b_{qr}\Gamma_{\eta(t)}z_r(t) + \varrho(t)\sum_{r=1}^{N} b_{qr}\Gamma_{\eta(t)}z_r(t-\wp(t)) \right. \\ & \left. + K_{\eta(t)}z(t-h(t))\right]\mathrm{d}t + W_{\eta(t)}z_q(t)\mathrm{d}\tau(t), \end{aligned} \tag{8}$$

where $K_{\eta(t)}$ indicates a set of nonuniform sampled-data feedback controller gain matrices that are to be determined. For convenience, matrices $A_{\eta(t)}, \Gamma_{\eta(t)}$, and $W_{\eta(t)}$ are represented by A_i, Γ_i, and W_i, respectively, for $i \in \mathbb{S}$. With the aid of Kronecker product properties, the compact form is obtained as follows:

$$\begin{aligned} \mathrm{d}z(t) = {} & \left[(I_N \otimes A_i)z(t) + G(t,z(t)) + (1-\varrho(t))(B\otimes\Gamma_i)z(t) + \varrho(t)(B\otimes\Gamma_i)z(t-\wp(t))\right] \\ & + (I_N\otimes K_i)z(t-h(t))]\mathrm{d}t + W_i z(t)\mathrm{d}\tau(t), \end{aligned} \tag{9}$$

where $z(t) = [z_1^T(t), z_2^T(t), \ldots, z_N^T(t)]^T$, $G(t,z(t)) = [g^T(t, z_1(t)), \ldots g^T(t, z_N(t))]^T$, $K_i = \mathrm{diag}\{K_1, K_2, \ldots, K_N\}$.

The following definitions and lemmas were used to derive the main results.

Lemma 1 (see [18]). *The noise intensity function $\hat{w}(t, z_i(t), z_i(t-\wp(t)))$, $\mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is uniformly Lipschitz continuous in terms of the following inequality of the trace inner product.*

$$\mathrm{trace}\left\{\hat{w}^T(t,z_i(t),z_i(t-\wp(t)))\hat{w}(t,z_i(t),z_i(t-\wp(t)))\right\} \le \eta_i z_i^T(t)z_i(t) + \zeta_i z_i^T(t-\wp(t))z_i(t-\wp(t)), \tag{10}$$

where $\eta_i, \zeta_i (i = 1, 2, \ldots)$ are non-negative real constants.

Lemma 2 (see [33]). *For any positive definite matrix Z, with scalars $0 < \mathscr{L} < \mathscr{U}$, the following integration holds:*

$$-(\mathscr{U} - \mathscr{L}) \int_{t-\mathscr{U}}^{t-\mathscr{L}} z^T(s) Z z(s) \mathrm{d}s \leq -\left(\int_{t-\mathscr{U}}^{t-\mathscr{L}} z(s)\mathrm{d}s\right)^T Z \left(\int_{t-\mathscr{U}}^{t-\mathscr{L}} z(s)\mathrm{d}s\right),$$
$$-\frac{\left(\mathscr{U}^2 - \mathscr{L}^2\right)}{2} \int_{t-\mathscr{U}}^{t-\mathscr{L}} \int_s^t z^T(u) Z z(u) \mathrm{d}u\mathrm{d}s \leq -\left(\int_{t-\mathscr{U}}^{t-\mathscr{L}} \int_s^t z(u)\mathrm{d}u\mathrm{d}s\right)^T Z \left(\int_{t-\mathscr{U}}^{t-\mathscr{L}} \int_s^t z(u)\mathrm{d}u\mathrm{d}s\right). \tag{11}$$

Lemma 3 (Ito formula) (see [34]). *Consider the time-varying stochastic system of the form*

$$dx = f\left(t, \hat{e}_i(t)\right)dt + h\left(t, \hat{e}_i(t)\right)d\tau(t), \tag{12}$$

where $\tau(t)$ is an independent r-dimensional standard Wiener process. The infinitesimal generator of the Markov process $\left\{t \geq 0, \hat{e}(t), \eta(t)\right\}$ is given by

$$\mathfrak{C}V = \mathscr{V}_t(t, \hat{e}(t)) + \mathscr{V}_{\hat{e}}(t, \hat{e}(t))\left\{f\left(t, \hat{e}_i(t)\right)\right\} + \frac{1}{2}\mathrm{trace}\left\{h^T(t, \hat{e}(t))\mathscr{V}_{\hat{e}\hat{e}}(t, \hat{e}(t))h(t, \hat{e}(t))\right\}, \tag{13}$$

where $\mathscr{V}_t(t, \hat{e}(t)) = (\partial\mathscr{V}(t, \hat{e}(t))/\partial t)$, $\mathscr{V}_{\hat{e}}(t, \hat{e}(t)) = (\partial\mathscr{V}(t, \hat{e}(t))/\partial\hat{e}_1, \partial\mathscr{V}(t, \hat{e}(t))/\partial\hat{e}_2 \ldots \partial\mathscr{V}(t, \hat{e}(t))/\partial\hat{e}_n)$, and $\mathscr{V}_{\hat{e}\hat{e}}(t, \hat{e}(t)) = (\partial^2\mathscr{V}(t, \hat{e}(t))/\partial\hat{e}_i\hat{e}_j)_{n\times n}$.

The expansion of the above equation is realized according to the following algebraic operations: $dt\ dt = 0, dt\ d\tau(t) = 0, d\tau(t)d\tau(t) = t$, which in turn follow the properties of stochastic effects.

Definition 1 (see [35]). CDNs are said to be exponentially synchronized, that is, the considered closed-loop error dynamics are exponentially stable if there exist positive constants δ, ϑ such that the following condition holds:

$$\mathbb{E}\left\{\|\mathscr{G}(t)\|^2\right\} \leq \vartheta e^{-\delta t} \sup_{-\max\{\gamma,\wp\}\leq\phi\leq 0} \mathbb{E}\left\{[\|\mathscr{G}(\phi)\|, \|\dot{\mathscr{G}}(\phi)\|]^2\right\}. \tag{14}$$

3. Main Results

This section presents the sufficient conditions for the considered CDN model (1) to guarantee exponential synchronization. Specifically, the exponential stability of the error system (5) is obtained, and turns up (1) is synchronized. Additionally, nonuniform sampled-data control was considered; it was derived in terms of LMIs.

Theorem 1. *For given positive scalars, $\varrho_0 \in [0, 1], \wp, h, \nu$, the error system (5) is exponentially mean-square stable with known controller gain K_i satisfying conditions (A1) - (A3); if there exist real symmetric matrices P_i, $B_1, B_2, B_3, C_1, C_2, D_1, D_2, F_1, F_2$, then the following LMIs hold:*

$$\mathfrak{J}_1 = \begin{bmatrix} [\mathfrak{J}_{ij}]_{8\times 8} & 0 \\ * & -\mathfrak{Z} \end{bmatrix} < 0, \tag{15}$$

where

$$\mathfrak{J}_{11} = \sum_{i=1}^N \Pi_{ii}P_i + B_1 + B_2 + B_3 - C_1 - C_2 - 2D_1 - 2F_1 + 2\beta P_i + 2P_i\left(I_N \otimes A_i\right) + 2\left(1 - \varrho_0\right)P_i\left(B \otimes \Gamma_i\right)$$
$$+ W_i^T P_i W_i - \gamma\overline{\mathfrak{U}},$$

$$\mathfrak{J}_{12} = \varrho_0 P_i\left(B \otimes \Gamma_i\right) + C_1, \mathfrak{J}_{14} = P_i K_i + C_2, \mathfrak{J}_{16} = \frac{2}{\wp}D_1, \mathfrak{J}_{17} = \frac{2}{h}F_1, \mathfrak{J}_{18} = -\gamma\overline{\mathfrak{V}} + P_i,$$

$$\mathfrak{J}_{22} = (1 - \nu)e^{-2\beta\wp}B_1 - C_1, \mathfrak{J}_{23} = C_1, \mathfrak{J}_{33} = -e^{-2\beta\wp}B_2 - C_1 - D_2, \mathfrak{J}_{36}$$

$$\mathfrak{J}_{45} = C_2, \mathfrak{J}_{55} = -e^{-2\beta h}B_3 - C_2 - 2F_2, \mathfrak{J}_{57} = \frac{2}{h}F_2, \mathfrak{J}_{66} = \frac{-2}{\wp^2}D_1 - \frac{2}{\wp^2}D_2, \mathfrak{J}_{77} = \frac{-2}{h^2}F_1 - \frac{2}{h^2}F_2,$$

$$\mathfrak{I}_{88} = -\gamma I, \mathfrak{Z} = \mathfrak{g}_1 C_1 + \mathfrak{g}_2 C_2 + \mathfrak{g}_3 D_1 + \mathfrak{g}_4 D_2 + \mathfrak{g}_5 F_1 + \mathfrak{g}_6 F_2, \mathfrak{g}_1 = \frac{\wp}{2\beta}\left(1 - e^{2\beta\wp}\right),$$
$$\mathfrak{g}_2 = \frac{h}{2\beta}\left(1 - e^{2\beta h}\right), \mathfrak{g}_3 = \frac{\wp}{2\beta} + \frac{1}{4\beta^2} - \frac{e^{2\beta\wp}}{4\beta^2}, \mathfrak{g}_4 = \frac{-1}{4\beta^2} + \frac{e^{2\beta\wp}}{4\beta^2} - \frac{\wp e^{2\beta\wp}}{2\beta}, \mathfrak{g}_5 = \frac{h}{2\beta} + \frac{1}{4\beta^2} - \frac{e^{2\beta h}}{4\beta^2},$$
$$\mathfrak{g}_6 = \frac{-1}{4\beta^2} + \frac{e^{2\beta h}}{4\beta^2} - \frac{h e^{2\beta h}}{2\beta}, \overline{\mathfrak{U}} = \frac{(I_N \otimes U)^T (I_N \otimes V)}{2} + \frac{(I_N \otimes V)^T (I_N \otimes U)}{2},$$
$$\overline{\mathfrak{V}} = \frac{(I_N \otimes U)^T + (I_N \otimes V)^T}{2}, U = \mathrm{diag}\underbrace{\{u, u, \ldots, u\}}_{N\text{ times}}, V = \mathrm{diag}\underbrace{\{v, v, \ldots v\}}_{N\text{ times}}. \tag{16}$$

The remaining parameters are set to zero.

Proof. Consider the following Lyapunov–Krasovskii functional:

$$V(z(t), t, i) = \sum_{m=1}^{5} V_m(z(t), t, i), \tag{17}$$

where

$$\begin{aligned}
V_1(z(t), t, i) &= e^{2\beta t} z^T(t) P_i z(t),\\
V_2(z(t), t, i) &= \int_{t-\wp(t)}^{t} e^{2\beta s} z^T(s) B_1 z(s) \mathrm{d}s + \int_{t-\wp}^{t} e^{2\beta s} z^T(s) B_2 z(s) \mathrm{d}s + \int_{t-h}^{t} e^{2\beta s} z^T(s) B_3 z(s) \mathrm{d}s,\\
V_3(z(t), t, i) &= \wp \int_{-\wp}^{0} \int_{t+\theta}^{t} e^{2\beta(s-\theta)} \dot{z}^T(s) C_1 \dot{z}(s) \mathrm{d}s\, d\theta + h \int_{-h}^{0} \int_{t+\theta}^{t} e^{2\beta(s-\theta)} \dot{z}^T(s) C_2 \dot{z}(s) \mathrm{d}s \mathrm{d}\theta,\\
V_4(z(t), t, i) &= \int_{-\wp}^{0} \int_{\lambda}^{0} \int_{t+\theta}^{t} e^{2\beta(s-\theta)} \dot{z}^T(s) D_1 \dot{z}(s) \mathrm{d}s \mathrm{d}\theta\, d\lambda + \int_{-\wp}^{0} \int_{-\wp}^{\lambda} \int_{t+\theta}^{t} e^{2\beta(s-\theta)} \dot{z}^T(s) D_2 \dot{z}(s) \mathrm{d}s \mathrm{d}\theta \mathrm{d}\lambda,\\
V_5(z(t), t, i) &= \int_{-h}^{0} \int_{\mu}^{0} \int_{t+\theta}^{t} e^{2\beta(s-\theta)} \dot{z}^T(s) F_1 \dot{z}(s) \mathrm{d}s \mathrm{d}\theta \mathrm{d}\mu + \int_{-h}^{0} \int_{-h}^{\mu} \int_{t+\theta}^{t} e^{2\beta(s-\theta)} \dot{z}^T(s) F_2 \dot{z}(s) \mathrm{d}s \mathrm{d}\theta \mathrm{d}\mu.
\end{aligned} \tag{18}$$

The time derivative of $V(t)$ is manipulated as follows:

$$\begin{aligned}
\mathbb{E}\{\mathfrak{C}(V_1(z(t), t, i))\} &= 2\beta e^{2\beta t} z^T(t) P_i z(t) + 2e^{2\beta t} z^T(t) P_i \left[(I_N \otimes A_i) z(t) + G(t, z(t)) + (1 - \varrho(t))(B \otimes \Gamma_i) z(t)\right.\\
&\quad \left.+ \varrho(t)(B \otimes \Gamma_i) z(t - \wp(t)) + K_i z(t - h(t))\right] \mathrm{d}t + e^{2\beta t} \mathrm{trace}\left[z^T(t) W_i^T P_i W_i z(t)\right]\\
&\quad + \sum_{j=1}^{N} \Pi_{ij} z^T(t) P_j z(t),
\end{aligned} \tag{19}$$

$$\begin{aligned}
\mathbb{E}\{\mathfrak{C}(V_2(z(t)), t, i)\} &= e^{2\beta t}\left[z^T(t) B_1 z(t) + z^T(t) B_2 z(t) + z^T(t) B_3 z(t)\right] + (1 - \nu) e^{2\beta\wp(t)} z^T(t - \wp(t))\\
&\quad B_1 z(t - \wp(t)) - e^{2\beta\wp} z^T(t - \wp) B_2 z(t - \wp) - e^{2\beta h} z^T(t - h) B_3 z(t - h),
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathbb{E}\{\mathfrak{C}(V_3(z(t), t, i)\} &= \wp \int_{-\wp}^{0} \left(e^{2\beta(t-\theta)} \dot{z}^T(t) C_1 \dot{z}(t) - e^{2\beta t} \dot{z}^T(t+\theta) C_1 \dot{z}(t+\theta)\right) \mathrm{d}\theta\\
&\quad + h \int_{-h}^{0} \left(e^{2\beta(t-\theta)} \dot{z}^T(t) C_2 \dot{z}(t) - e^{2\beta t} \dot{z}^T(t+\theta) C_2 \dot{z}(t+\theta)\right) \mathrm{d}\theta\\
&= e^{2\beta t}\left\{\mathfrak{g}_1 \dot{z}^T(t) C_1 \dot{z}(t) - \wp \int_{t-\wp}^{t} \dot{z}^T(s) C_1 \dot{z}(s) \mathrm{d}s + \mathfrak{g}_2 \dot{z}^T(t) C_2 \dot{z}(t) - h \int_{t-h}^{t} \dot{z}^T(s) C_2 \dot{z}(s) \mathrm{d}s\right\}.
\end{aligned} \tag{21}$$

The integral terms are also considered:

$$-\wp \int_{t-\wp}^{t} \dot{z}^T(s)C_1\dot{z}(s)\mathrm{d}s = -\wp \int_{t-\wp}^{t-\wp(t)} \dot{z}^T(s)C_1\dot{z}(s)\mathrm{d}s - \wp \int_{t-\wp(t)}^{t} \dot{z}^T(s)C_1\dot{z}(s)\mathrm{d}s,$$
$$-h \int_{t-h}^{t} \dot{z}^T(s)C_2\dot{z}(s)\mathrm{d}s = -h \int_{t-h}^{t-h(t)} \dot{z}^T(s)C_2\dot{z}(s)\mathrm{d}s - h \int_{t-h(t)}^{t} \dot{z}^T(s)C_2\dot{z}(s)\mathrm{d}s. \quad (22)$$

Applying Lemma 2, we obtain

$$\begin{aligned}\mathbb{E}\{\mathfrak{C}(V_3(z(t),t,i))\} \le e^{2\beta t}\Bigg\{ & \mathfrak{g}_1\dot{z}(t)C_1\dot{z}(t) - \begin{bmatrix} z(t) \\ z(t-\wp(t)) \end{bmatrix}^T \begin{bmatrix} -C_1 & C_1 \\ * & -C_1 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-\wp(t)) \end{bmatrix} \\ & - \begin{bmatrix} z(t-\wp(t)) \\ z(t-\wp) \end{bmatrix}^T \begin{bmatrix} -C_1 & C_1 \\ * & -C_1 \end{bmatrix} \begin{bmatrix} z(t-\wp(t)) \\ z(t-\wp) \end{bmatrix} + \mathfrak{g}_2\dot{z}^T(t)C_2\dot{z}(t) \\ & - \begin{bmatrix} z(t) \\ z(t-h(t)) \end{bmatrix}^T \begin{bmatrix} -C_2 & C_2 \\ * & -C_2 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h(t)) \end{bmatrix} \\ & - \begin{bmatrix} z(t-h(t)) \\ z(t-h) \end{bmatrix}^T \begin{bmatrix} -C_2 & C_2 \\ * & -C_2 \end{bmatrix} \begin{bmatrix} z(t-h(t)) \\ z(t-h) \end{bmatrix} \Bigg\} \end{aligned} \quad (23)$$

$$\begin{aligned}\mathbb{E}\{\mathfrak{C}(V_4(z(t),t,i))\} &= \int_{-\wp}^{0}\int_{\lambda}^{0}\left(e^{2\beta(t-\theta)}\dot{z}^T(s)D_1\dot{z}(s) - e^{2\beta t}\dot{z}^T(t+\theta)D_1\dot{z}(t+\theta)\right)\mathrm{d}\theta\mathrm{d}\lambda \\ &\quad + \int_{-\wp}^{0}\int_{-\wp}^{\lambda}\left(e^{2\beta(t-\theta)}\dot{z}^T(t)D_2\dot{z}(t) - e^{2\beta t}\dot{z}^T(t+\theta)D_2\dot{z}(t+\theta)\right)\mathrm{d}\theta\mathrm{d}\lambda \\ &= e^{2\beta t}\Bigg\{\mathfrak{g}_3\dot{z}(t)D_1\dot{z}(t) - \int_{-\wp}^{0}\int_{t+\lambda}^{t}\dot{z}^T(s)D_1\dot{z}(s)\mathrm{d}s\mathrm{d}\lambda + \mathfrak{g}_4\dot{z}(t)D_2\dot{z}(t) \\ &\quad - \int_{-\wp}^{0}\int_{t-\wp}^{t+\lambda}\dot{z}^T(s)D_2\dot{z}(s)\mathrm{d}s\mathrm{d}\lambda\Bigg\}. \end{aligned} \quad (24)$$

By applying Lemma 2 to the integral terms in (24), we obtain

$$\begin{aligned}\mathbb{E}\{\mathfrak{C}(V_4(z(t),t,i))\} \le e^{2\beta t}\Bigg\{ & \mathfrak{g}_3\dot{z}^T(t)D_1\dot{z}(t) - 2z^T(t)D_1z(t) + \frac{2}{\wp}z^T(t)D_1\left(\int_{t-\wp}^{t} z(s)\mathrm{d}s\right) \\ & + \frac{2}{\wp}\left(\int_{t-\wp}^{t} z(s)\mathrm{d}s\right)D_1z^T(t) - \frac{2}{\wp^2}\left(\int_{t-\wp}^{t} z^T(s)\mathrm{d}s\right)D_1\left(\int_{t-\wp}^{t} z(s)\mathrm{d}s\right) \\ & + \mathfrak{g}_4\dot{z}^T(t)D_2\dot{z}(t) - 2z^T(t-\wp)D_2z(t-\wp) + \frac{2}{\wp}z^T(t-\wp)D_2\left(\int_{t-\wp}^{t} z(s)\mathrm{d}s\right) \\ & + \frac{2}{\wp}\left(\int_{t-\wp}^{t} z(s)\mathrm{d}s\right)D_2z^T(t) - \frac{2}{\wp^2}\left(\int_{t-\wp}^{t} z^T(s)\mathrm{d}s\right)D_2\left(\int_{t-\wp}^{t} z(s)\mathrm{d}s\right)\Bigg\}, \end{aligned} \quad (25)$$

$$\begin{aligned}\mathbb{E}\{\mathfrak{C}(V_5(z(t)),t,i)\} &= \int_{-\wp}^{0}\int_{\lambda}^{0}\left[e^{2\beta(t-\theta)}\dot{z}^T(s)F_1\dot{z}(s) - e^{2\beta t}\dot{z}^T(t+\theta)F_1\dot{z}(t+\theta)\right]d\theta \\ &\quad + \int_{-h}^{0}\int_{-h}^{\mu}\left[e^{2\beta(t-\theta)}\dot{z}^T(t)F_2\dot{z}(t)d\mu - e^{2\beta t}\dot{z}^T(t+\theta)F_2\dot{z}(t+\theta)\right]d\theta\, d\mu \\ &= e^{2\beta t}\Bigg\{\mathfrak{g}_5\dot{z}(t)F_1\dot{z}(t) - \int_{-\wp}^{0}\int_{t+\lambda}^{t} e^{2\beta t}\dot{z}^T(s)F_1\dot{z}(s)ds\, d\mu + \mathfrak{g}_6\dot{z}(t)F_2\dot{z}(t) - \int_{-h}^{0}\int_{t-h}^{t+\mu} e^{2\beta t}\dot{z}^T(s)F_2\dot{z}(s)ds\, d\mu\Bigg\}. \end{aligned} \quad (26)$$

By applying Lemma 2 to the integral terms in (24), we obtain

$$\mathbb{E}\{\mathfrak{C}(V_5(z(t)),t,i)\} \le e^{2\beta t}\left\{\begin{array}{c} \mathfrak{g}_5\dot{z}^T(t)F_1\dot{z}(t) - 2z^T(t)F_1 z(t) + \frac{2}{h}z^T(t)F_1\left(\int_{t-h}^{t} z(s)\mathrm{d}s\right) \\ +\frac{2}{h}\left(\int_{t-h}^{t} z(s)\mathrm{d}s\right)F_1 z^T(t) - \frac{2}{h^2}\left(\int_{t-h}^{t} z^T(s)\mathrm{d}s\right)F_1\left(\int_{t-h}^{t} z(s)\mathrm{d}s\right) \\ +\mathfrak{g}_6\dot{z}^T(t)F_2\dot{z}(t) - 2z^T(t-h)F_2 z(t-h) + \frac{2}{h}z^T(t-h)F_2\left(\int_{t-h}^{t} z(s)\mathrm{d}s\right) \\ +\frac{2}{h}\left(\int_{t-h}^{t} z(s)ds\right)F_2 z^T(t) - \frac{2}{h^2}\left(\int_{t-h}^{t} z^T(s)\mathrm{d}s\right)F_2\left(\int_{t-h}^{t} z(s)\mathrm{d}s\right) \end{array}\right\}. \quad (27)$$

Based on (A2), for any scalar $\gamma > 0$,

$$-\gamma\begin{bmatrix} z(t) \\ G(t,z(t)) \end{bmatrix}^T\begin{bmatrix} \overline{\mathfrak{U}} & \overline{\mathfrak{B}} \\ * & I \end{bmatrix}\begin{bmatrix} z(t) \\ G(t,z(t)) \end{bmatrix} \le 0, \quad (28)$$

which can be written in the following simplified form:

$$-\gamma\left[z^T(t)\overline{\mathfrak{U}}z(t) + G^T(t,z(t))\overline{\mathfrak{B}}^T z(t) + z^T(t)\overline{\mathfrak{B}}G(t,z(t)) + G(t,z(t))I_N z(t)\right] \le 0. \quad (29)$$

Substituting expressions (19)–(27) into (17) and subtracting (29), we obtain

$$\mathbb{E}\{\mathfrak{C}(V(z(t),t,i)\} \le e^{-2\beta t}\mathbb{E}\left\{\zeta^T(t)\mathfrak{J}_1\zeta(t)\right\}, \quad (30)$$

where $\zeta^T(t) = [z(t)\, z(t-\wp(t))\, z(t-\wp)\, z(t-h(t))\, z(t-h)\, \int_{t-\wp}^{t} z(s)\mathrm{d}s \int_{t-h}^{t} z(s)\,\mathrm{d}s\, G(t,z(t))\, \dot{z}(t)]$

Therefore,

$$\mathbb{E}\{\mathfrak{C}V(z(t),t,i)\} \le 0, \quad t \in [t_{k-1}, t_k). \quad (31)$$

It follows from (31) and generalized Ito's formula that we have

$$\begin{aligned} &\mathbb{E}\{\mathfrak{C}(V(z(t),t,i))\} - \mathbb{E}\{\mathfrak{C}(V(z(0),0,i))\} = \int_0^t \mathbb{E}\{\mathfrak{C}(V(z(s),s,i))\}\mathrm{d}s < 0, \\ &\mathbb{E}\{\mathfrak{C}(V(z(0),0,\eta(0)))\} \le \begin{bmatrix} \Theta_{\max}(P_i) + \Theta_{\max}B_1\Lambda_1 + \Theta_{\max}B_2\Lambda_1 + \Theta_{\max}B_3\Lambda_2 + \Theta_{\max}C_1\Lambda_3 \\ +\Theta_{\max}C_2\Lambda_4 + \Theta_{\max}D_1\Lambda_5 + \Theta_{\max}D_2\Lambda_6 + \Theta_{\max}F_1\Lambda_7 + \Theta_{\max}F_2\Lambda_8 \end{bmatrix} \\ &\sup_{\max\{\wp,h\}\le s\le 0} \mathbb{E}\|z(s)\|^2, \end{aligned} \quad (32)$$

where

$$\begin{aligned} &\Lambda_1 = \left[\frac{1}{2\beta} - \frac{e^{-2\beta\wp}}{2\beta}\right], \Lambda_2 = \left[\frac{1}{2\beta} - \frac{e^{2\beta h}}{2\beta}\right], \Lambda_3 = \left[\frac{e^{2\beta\wp}}{(2\beta)^2} - \frac{\wp}{2\beta} - \frac{1}{(2\beta)^2}\right], \Lambda_4 = \left[\frac{e^{2\beta h}}{(2\beta)^2} - \frac{h}{2\beta} - \frac{1}{(2\beta)^2}\right], \\ &\Lambda_5 = \left[\frac{e^{2\beta\wp}}{(2\beta)^3} - \frac{1}{(2\beta)^3} - \frac{\wp}{(2\beta)^2} - \frac{\wp^2}{4\beta}\right], \Lambda_6 = \left[\frac{e^{2\beta h}}{(2\beta)^3} - \frac{1}{(2\beta)^3} - \frac{h}{(2\beta)^2} - \frac{h^2}{4\beta}\right], \\ &\Lambda_7 = \left[\frac{-1}{(2\beta)^3} - \frac{e^{4\beta\wp}}{(2\beta)^3} + \frac{\wp e^{2\beta\wp}}{(2\beta)^2} - \frac{\wp^2}{4\beta}\right], \Lambda_8 = \left[\frac{-1}{(2\beta)^3} - \frac{e^{2\beta h}}{(2\beta)^3} + \frac{de^{2\beta h}}{(2\beta)^2} - \frac{h^2}{4\beta}\right]. \end{aligned} \quad (33)$$

From Definition 1, we have

$$\mathbb{E}\left\{\|z(t)\|^2\right\} \le e^{-2\beta t}\Psi \sup_{\max\{\wp,h\}\le s\le 0} \mathbb{E}\|\|z(s)\|^2,$$
$$\Psi = \left\{ \begin{array}{c} \Theta_{\max}(P_i) + \Theta_{\max}B_1\Lambda_1 + \Theta_{\max}B_2\Lambda_1 + \Theta_{\max}B_3\Lambda_2 + \Theta_{\max}C_1\Lambda_3 \\ +\Theta_{\max}C_2\Lambda_4 + \Theta_{\max}D_1\Lambda_5 + \Theta_{\max}D_2\Lambda_6 + \Theta_{\max}F_1\Lambda_7 + \Theta_{\max}F_2\Lambda_8 \end{array} \right\}. \tag{34}$$

Therefore, we conclude that the error system considered in (5) is exponentially stable. □

Remark 1. Theorem 1 exclusively presents two noteworthy values based on the Lyapunov–Krasovskii stability theory: (a) a new synchronization criterion for stochastic CDNs (1) with random delayed information exchange among the nodes under semi-Markov jump; and (b) a design method of the nonuniform sample data controller (6) that ensures the desired synchronization of the plant (1). In order to achieve the desired synchronization, the LKF was specifically constructed using double integral terms in such a way utilizing the Wirtinger-type inequality. The primary benefit of the proposed method in this paper is that it can guarantee the necessary synchronization even in the presence of randomly occurring delays and nonlinearity under semi-Markov jump. The results that were generated in this paper have some potential benefits from an application viewpoint.

Theorem 2. *For given positive scalars, $\varrho_0 \in [0,1]$, $\wp$, h, ν, the error system (5) is exponentially mean-square stable with unknown controller gain K_i satisfying conditions (A1) - (A3); if there exist real symmetric matrices P_i, B_1, B_2, B_3, C_1, C_2, D_1, D_2, F_1, F_2, then the following LMIs hold:*

$$\left[\overline{\mathfrak{J}_{ij}}\right]_{9\times 9} < 0, \tag{35}$$

where

$$\begin{aligned}
\overline{\mathfrak{J}_{11}} &= \sum_{i=1}^{N} \Pi_{ii}P_i + B_1 + B_2 + B_3 - C_1 - C_2 - 2D_1 - 2F_1 + 2\beta P_i + 2P_i(I_N\otimes A_i) + 2(1-\varrho_0)P_i(B\otimes\Gamma_i) \\
&\quad + W_i^T P_i W_i - \gamma\mathfrak{U}, \\
\overline{\mathfrak{J}_{12}} &= \varrho_0 P_i(B\otimes\Gamma_i) + C_1, \overline{\mathfrak{J}_{14}} = X_i + C_2, \overline{\mathfrak{J}_{16}} = \frac{2}{\wp}D_1, \overline{\mathfrak{J}_{17}} = \frac{2}{h}F_1, \overline{\mathfrak{J}_{18}} = -\gamma\mathfrak{V} + P_i, \\
\overline{\mathfrak{J}_{22}} &= (1-\nu)e^{-2\beta\wp}B_1 - C_1, \overline{\mathfrak{J}_{23}} = C_1, \overline{\mathfrak{J}_{33}} = -e^{-2\beta\wp}B_2 - C_1 - 2D_2, \overline{\mathfrak{J}_{36}} = \frac{2}{\wp}D_2, \overline{\mathfrak{J}_{44}} = -C_2, \\
\overline{\mathfrak{J}_{45}} &= -e^{-2\beta h}B_2 + C_2, \overline{\mathfrak{J}_{55}} = -C_2 - 2F_2, \overline{\mathfrak{J}_{57}} = \frac{2}{h}F_2, \overline{\mathfrak{J}_{66}} = \frac{-2}{\wp^2}D_1 - \frac{2}{\wp^2}D_2, \overline{\mathfrak{J}_{77}} = \frac{-2}{h^2}F_1 - \frac{2}{h^2}F_2, \\
\overline{\mathfrak{J}_{88}} &= -\gamma I, \overline{\mathfrak{J}_{99}} = \mathfrak{g}_1C_1 + \mathfrak{g}_2C_2 + \mathfrak{g}_3D_1 + \mathfrak{g}_4D_2 + \mathfrak{g}_5F_1 + \mathfrak{g}_6F_2, \\
\mathfrak{g}_1 &= -\frac{\wp}{2\beta}\left(1-e^{2\beta\wp}\right), \mathfrak{g}_2 = \frac{-h}{2\beta}\left(1-e^{2\beta h}\right), \mathfrak{g}_3 = \frac{-\wp}{2\beta} - \frac{1}{4\beta^2} + \frac{e^{2\beta\wp}}{4\beta^2}, \mathfrak{g}_4 = \frac{1}{4\beta^2} - \frac{e^{2\beta\wp}}{4\beta^2} + \frac{\wp e^{2\beta\wp}}{2\beta}, \\
\mathfrak{g}_5 &= -\frac{h}{2\beta} - \frac{1}{4\beta^2} + \frac{e^{2\beta h}}{4\beta^2}, \mathfrak{g}_6 = \frac{1}{4\beta^2} - \frac{e^{2\beta h}}{4\beta^2} + \frac{he^{2\beta h}}{2\beta}.
\end{aligned} \tag{36}$$

The remaining parameters are set to zero. Moreover, the controller gain matrices are given by $K_i = P_i^{-1}X_i, i \in \mathbb{S}$.

Proof. Let us consider the same LKF as defined in Theorem 1 and follow the same procedure with an unknown gain matrix K_i. By setting $X_i = P_iK_i$, we obtain (35). This completes this proof. □

Remark 2. It is worth pointing out that a great number of research works regarding the issues of synchronization of CDNs have been reported in the recent literature, for instance see [13–15]. Also, it should be noted that all the aforementioned works do not consider stochastic behaviour in the system. Recently, some interesting results on the synchronization of stochastic CDNs behaviour have been discussed [18, 22, 34]. However, the issues of the synchronization of nonuniform samples data controller for stochastic CDNs subjected to semi-Markov jump and random delayed information exchanges has not been discussed in the literature. Thus, the main contribution of this paper is to fill such a gap through employing a nonuniform control law for achieving synchronization of stochastic CDNs in presence of semi-Markov jump and random delay occurred during information exchanges, which makes this work different from the existing works in stochastic CDNs.

Remark 3. In practical applications, randomness occurring in the delays during the information exchanges are usually inevitable owing to the influence of unexpectable changes. More precisely, the existence of randomness is apparent in a probabilistic manner. To handle this scenario, Bernoulli distributed parameter $\varrho(t)$ is introduced in which it should be noted that when $\varrho(t) = 0$ the considered stochastic CDNs (1) reduces to general stochastic CDNs without delays, which has been investigated in [12]. It should be noted that such a description is considered for nonuniform sampled data control law for synchronization of stochastic CDNs with semi-Markov jump for the first time.

Remark 4. It is generally well-known to research communities that implementing the LMI technique when considering a large number of decision variables in the designed synchronization criterion, the computational complexity certainly increases and also the required time to solve the criterion is being large value. As a result, there should be a trade-off between the aforementioned criteria and the quantity of decision variables. Additionally, the computation of the findings suggested in Section 4 is conveniently offline. Therefore, the defined LMI-based synchronization criterion may be easily solved by using the existing convex optimization tools.

Remark 5. Consider a class of semi-Markov jump CDNs (1) with stochastic noise consisting of N identical coupled nodes, defined over a probability space $(\Xi, \mathscr{F}, P)$, whose error system is described as follows:

$$dz_q(t) = \left[A_i z_q(t) + g\big(t, z_q(t)\big) + (1-\varrho(t))\sum_{r=1}^{N} b_{qr}\Gamma_i z_r(t) + \varrho(t)\sum_{r=1}^{N} b_{qr}\Gamma_i z_r(t-\wp(t)) + K_i z(t-h(t))\right]dt + W\big(t, z_q(t), z_q(t-\wp(t))\big)d\tau(t), \tag{37}$$

where $W(t, z_q(t), z_q(t-\wp(t)))$ denotes the noise intensity vector-valued function.

Corollary 1. *For given scalars $\varrho_0 \in [0,1], \wp > 0, h > 0, \nu > 0, \kappa_i > 0\ (i \in \mathbb{S})$, the exponential synchronization in the mean square of error system (37) can be achieved via unknown control (7) with controller gain K_i.*

If there exist positive definite matrices P_i, $B_1, B_2, B_3, C_1, C_2, D_1, D_2, F_1, F_2$ and diagonal matrices N_1, N_2 with the appropriate dimensions, the following inequality is satisfied:

$$\left[\mathfrak{Y}_{ij}\right]_{9\times 9} < 0, \tag{38}$$

where

$$\begin{aligned}
\mathfrak{Y}_{11} &= \sum_{i=1}^{N} \Pi_{ij}P_j + B_1 + B_2 + B_3 - C_1 - C_2 - 2D_1 - 2F_1 + 2\beta P_i + 2P_i(I_N \otimes A_i) + 2(1-\varrho_0)P_i(B\otimes\Gamma_i) - \nu\overline{\mathfrak{U}} + \kappa_i N_1,\\
\mathfrak{Y}_{12} &= \varrho_0 P_i(B\otimes\Gamma_i) + C_1, \mathfrak{Y}_{14} = X_i + C_2, \mathfrak{Y}_{16} = \frac{2}{\wp}D_1, \mathfrak{Y}_{17} = \frac{2}{h}F_1, \mathfrak{Y}_{18} = -\gamma\overline{\mathfrak{V}} + P_i,\\
\mathfrak{Y}_{22} &= (1-\nu)e^{-2\beta\wp}B_1 - C_1 + \kappa_i N_2, \mathfrak{Y}_{23} = C_1, \mathfrak{Y}_{33} = -e^{-2\beta\wp}B_2 - C_1 - 2D_2, \mathfrak{Y}_{36} = \frac{2}{\wp}D_2,\\
\mathfrak{Y}_{44} &= -C_2, \mathfrak{Y}_{45} = -e^{-2\beta h}B_2 + C_2, \mathfrak{Y}_{55} = -C_2 - 2F_2, \mathfrak{J}_{57} = \frac{2}{h}F_2, \mathfrak{Y}_{66} = \frac{-2}{\wp^2}D_1 - \frac{2}{\wp^2}D_2,\\
\mathfrak{Y}_{77} &= \frac{-2}{h^2}F_1 - \frac{2}{h^2}F_2, \mathfrak{Y}_{88} = -\gamma, \mathfrak{Y}_{99} = \mathfrak{g}_1 C_1 + \mathfrak{g}_2 C_2 + \mathfrak{g}_3 D_1 + \mathfrak{g}_4 D_2 + \mathfrak{g}_5 F_1 + \mathfrak{g}_6 F_2,\\
\mathfrak{g}_1 &= -\frac{\wp}{2\beta}\left(1-e^{2\beta\wp}\right), \mathfrak{g}_2 = \frac{-h}{2\beta}\left(1-e^{2\beta h}\right), \mathfrak{g}_3 = \frac{-\wp}{2\beta} - \frac{1}{4\beta^2} + \frac{e^{2\beta\wp}}{4\beta^2}, \mathfrak{g}_4 = \frac{1}{4\beta^2} - \frac{e^{2\beta\wp}}{4\beta^2} + \frac{\wp e^{2\beta\wp}}{2\beta},\\
\mathfrak{g}_5 &= -\frac{h}{2\beta} - \frac{1}{4\beta^2} + \frac{e^{2\beta h}}{4\beta^2}, \mathfrak{g}_6 = \frac{1}{4\beta^2} - \frac{e^{2\beta h}}{4\beta^2} + \frac{he^{2\beta h}}{2\beta}.
\end{aligned} \tag{39}$$

In this case, the control gain matrix is given by $K_i = P_i^{-1}X_i$.

Proof. Using Lemma 1, the inequality $\text{trace}\{W^T(t, z(t), z(t-\wp(t)))P_iW(t, z(t), z(t-\wp(t)))\} \le \kappa_i \{z(t)^T N_1 z(t) + z(t-\wp(t))^T N_2 z(t-\wp(t))\}$ is obtained. The remaining procedure follows from Theorem 2. □

Remark 6. Consider the CDNs (1) without a semi-Markov jump, with stochastic noise consisting of N identical coupled nodes, defined over a probability space $(\Xi, \mathcal{F}, P)$, whose error system is described as follows:

$$dz_q(t) = \left[Az_q(t) + g(t, z_q(t)) + (1-\varrho(t))\sum_{r=1}^{N} b_{qr}\Gamma z_r(t) + \varrho(t)\sum_{r=1}^{N} b_{qr}\Gamma z_r(t-\wp(t)) + Kz(t-h(t))\right]dt + Wz_q(t)d\tau(t), \quad (40)$$

where W is the Wiener process matrix, which is assumed to have appropriate dimensions.

Corollary 2. *For given scalars $\varrho_0 \in [0,1], \wp > 0, h > 0, \nu > 0$ the exponential synchronization in the mean square of the error system (40) can be achieved via control (7) with unknown controller gain K. If there exist positive definite matrices $B_1, B_2, B_3, C_1, C_2, D_1, D_2, F_1, F_2$, the following inequality is satisfied:*

$$\begin{bmatrix} [\mathfrak{X}_{ij}]_{8\times 8} & 0 \\ * & -\mathfrak{Z} \end{bmatrix} < 0, \quad (41)$$

where

$$\begin{aligned}
&\mathfrak{X}_{11} = B_1 + B_2 + B_3 - C_1 - C_2 - 2D_1 - 2F_1 + 2\beta P + 2P(I_N \otimes A) + 2(1-\varrho_0)P(B\otimes\Gamma) + W^T PW - \gamma\overline{\mathfrak{U}},\\
&\mathfrak{X}_{12} = \varrho_0 P(B\otimes\Gamma) + C_1, \mathfrak{X}_{14} = X + C_2, \mathfrak{X}_{16} = \frac{2}{\wp}D_1, \mathfrak{X}_{17} = \frac{2}{h}F_1, \mathfrak{X}_{18} = \gamma(I_N\otimes\overline{\mathfrak{B}}) + P,\\
&\mathfrak{X}_{22} = (1-\nu)e^{2\beta\wp}B_1 - C_1, \mathfrak{X}_{23} = C_1, \mathfrak{X}_{33} = -e^{-2\beta\wp}B_2 - C_1 - 2D_2, \mathfrak{X}_{36} = \frac{2}{\wp}D_2, \mathfrak{X}_{44} = -C_2,\\
&\mathfrak{X}_{45} = -e^{-2\beta h}B_2 + C_2, \mathfrak{X}_{55} = -C_2 - 2F_2, \mathfrak{J}_{57} = \frac{2}{h}F_2, \mathfrak{X}_{66} = \frac{-2}{\wp^2}D_1 - \frac{2}{\wp^2}F_2, \mathfrak{X}_{77} = \frac{-2}{h^2}F_1 - \frac{2}{h^2}F_2,\\
&\mathfrak{X}_{88} = -\gamma I.
\end{aligned} \quad (42)$$

The remaining parameters of (39) follow Theorem 1. Moreover, the controller gain matrix is given by $K = P^{-1}X$.

Proof. The proof follows from Theorem 2. □

4. Numerical Examples

In this section, two numerical examples are presented to evaluate the feasibility and effectiveness of the results obtained.

Example 1. Consider the error system (5) with following nodes. Furthermore, the outer and inner coupling matrices and the corresponding system matrices are set as follows:

$$B = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.6 \end{bmatrix}, A_1 = \begin{bmatrix} -0.5 & -1.5 \\ 1.3 & -0.5 \end{bmatrix}, A_2 = \begin{bmatrix} -0.8 & 0.1 \\ 0.8 & -0.15 \end{bmatrix}. \quad (43)$$

Besides, the nonlinear function is given by

$$g(z_q(t)) = [-2.3z(t) + \tanh(0.2z(t)) + 0.2z(t)0.95z(t) - \tanh(0.75z(t))], \tag{44}$$

satisfying the sector bound condition (4) with $U = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}$ and $V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Also, the Wiener process matrices are represented as

$$W_1^T = [\,0.2 \;\; 0.25 \;\; 0.27 \;\; 0.13 \;\; 0.27 \;\; 0.13\,], W_2^T = [\,0.8 \;\; 0.5 \;\; 0.84 \;\; 0.34 \;\; 0.57 \;\; 0.8\,]. \tag{45}$$

In addition, the semi-Markov jump topology exhibits transition rates of the model chosen with

$$\pi_{ij}(1) = \begin{bmatrix} -2.5 & 1.3512 \\ 1.6879 & -0.8090 \end{bmatrix}, \pi_{ij}(2) = \begin{bmatrix} -0.6399 & 0.3512 \\ 0.6879 & -0.8090 \end{bmatrix}, \text{Where} \quad i, j = 1, 2. \tag{46}$$

Without loss of generality, consider $\Pi_{11,1} = -2.5$, $\Pi_{12,1} = 1.3512$, $\Pi_{21,1} = 1.6879$, $\Pi_{22,1} = -0.8090$, $\Pi_{11,2} = -0.6399$, $\Pi_{12,2} = 0.3512$, $\Pi_{21,2} = 0.6879$, and $\Pi_{22,2} = -0.8090$. The time-varying delay is assumed to be $\wp(t) = 0.25 + 0.05\sin(10t), 0 \le \wp \le 0.3$ and the sampling interval is represented as $h(t) = 0.36 + 0.25\sin(5t), 0 \le h \le 0.7$. Moreover, to verify the feasibility of the attained results, the following parameter values are fixed as $\nu = 0.55$, $\varrho_0 = 0.25$, $\beta = 0.1$. The following gain matrices are obtained using the MATLAB LMI Control toolbox by solving the LMIs in Theorem 2:

$$\begin{aligned} K_1 &= \begin{bmatrix} -1.1805 & 0.6879 & 0 & 0 & 0 & 0 \\ 0.6529 & -1.5785 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.7084 & 0.2683 & 0 & 0 \\ 0 & 0 & 0.2554 & -1.8076 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.0587 & -0.0386 \\ 0 & 0 & 0 & 0 & -0.1150 & -2.3360 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -2.0261 & -0.7108 & 0 & 0 & 0 & 0 \\ -0.7242 & -2.6163 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.7165 & 0.5003 & 0 & 0 \\ 0 & 0 & 0.5501 & -3.0811 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.6637 & -0.6717 \\ 0 & 0 & 0 & 0 & -0.8431 & -4.6084 \end{bmatrix}. \end{aligned} \tag{47}$$

The initial values of the state nodes were set as $z_1(0) = [4; -1]^T$, $z_2(0) = [-5; 2]^T$, $z_3(0) = [-3; 4]^T$ and the isolated nodes were set as $s(0) = [4; -1]^T$. The error trajectories without control input are shown in Figure 1. Furthermore, from the obtained gain matrices and nonuniform sampled-data controller, we obtained the synchronization error trajectories shown in Figure 2 and also, note that these trajectories converge to zero within a satisfactory time interval. Figure 3 shows the responses of the control input. The external disturbances are depicted in Figure 4. In addition, Figure 5 shows the semi-Markov jumping mode process during the simulation, and Figure 6 presents the randomness occurred during information exchange. As a conclusion, based on the above discussions, it can be strongly mentioned that the system performance under nonuniform sampled data controller is effective and efficient to handle semi-Markov jump and the randomness behaviour. Therefore, it is of great significance to consider the nonuniform sampled data control strategy for stochastic CDNs.

Example 2. Consider a single Rossler oscillator for the error system (37) described by the following dimensionless form:

$$\begin{cases} \dot{x}_1(t) = -(x_2(t) + x_3(t)), \\ \dot{x}_2(t) = x_1(t) + c_1 x_2(t), \\ \dot{x}_3(t) = c_2 x_1(t) + x_3(t)(x_1(t) - c_3), \end{cases} \quad (48)$$

where $c_1 = 0.2$, $c_2 = 0.2$, $c_3 = 5.7$. The outer and inner coupling matrices are given by

$$B = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \quad (49)$$

Also, the nonlinear function is represented as $g(z_q(t)) = [-2.3z(t) + \tanh(0.2z(t)) + 0.2z(t)0.95z(t) - \tanh(0.75z(t))0]$ with sector bound condition satisfying (4), along with $U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $V = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}$. The noise intensity function is $W(t, z(t)) = 3.1 \sin(2.5z(t)) + 1.1 \sin(5.2z(t - \wp(t)))$. In addition, the transition rates of the model for the semi-Markov jump topology were chosen as

$$\pi_{ij}(1) = \begin{bmatrix} -0.2 & 0.2 \\ 0.3 & -0.3 \end{bmatrix}, \pi_{ij}(2) = \begin{bmatrix} -0.3 & 0.3 \\ 0.2 & -0.2 \end{bmatrix} (i, j = 1, 2). \quad (50)$$

Without loss of generality, consider $\Pi_{11,1} = -0.2, \Pi_{12,1} = 0.2, \Pi_{21,1} = 0.3, \Pi_{22,1} = -0.3, \Pi_{11,2} = -0.3, \Pi_{12,2} = 0.3, \Pi_{21,2} = 0.2, \Pi_{22,2} = -0.2$. The time-varying delay was set as $\wp(t) = 0.25 + 0.05 \sin(10t), 0 \leq \wp \leq 0.3$ and the nonuniform sampling interval was set as $h(t) = 0.36 + 0.25 \sin(5t), 0 \leq h \leq 0.7$. To demonstrate the effectiveness of the obtained results, the remaining parameter values were set as $\nu = 0.4, \kappa_i = 0.4, \varrho_0 = 0.25, \beta = 0.45$. Using the MATLAB LMI Control toolbox to solve the LMIs in Corollary 1, we obtain the following gain matrices: $K_1 = \text{diag}\{K_{11}, K_{12}, K_{13}\}$ and $K_2 = \text{diag}\{K_{21}, K_{22}, K_{23}\}$ where

$$\begin{aligned} K_{11} &= \begin{bmatrix} -3.1165 & 0.5989 & -0.2761 \\ 0.6938 & -3.0729 & -0.3872 \\ -0.2038 & -0.3273 & -1.2857 \end{bmatrix}, K_{21} = \begin{bmatrix} 0.0050 & -0.0088 & 0.0088 \\ -0.0158 & -0.0022 & 0.0116 \\ 0.0054 & 0.0066 & -0.0759 \end{bmatrix}, \\ K_{12} &= \begin{bmatrix} -4.0978 & 0.8297 & -0.3585 \\ 0.9565 & -4.3148 & -0.6072 \\ -0.2786 & -0.5296 & -1.3420 \end{bmatrix}, K_{22} = \begin{bmatrix} 0.0047 & -0.0091 & 0.0091 \\ -0.0155 & -0.0025 & 0.0112 \\ 0.0054 & 0.0069 & -0.0794 \end{bmatrix}, \\ K_{13} &= \begin{bmatrix} -4.0978 & 0.8297 & -0.3585 \\ 0.9565 & -4.3148 & -0.6072 \\ -0.2786 & -0.5296 & -1.3420 \end{bmatrix}, K_{23} = \begin{bmatrix} 0.0047 & -0.0091 & 0.0091 \\ -0.0155 & -0.0025 & 0.0112 \\ 0.0054 & 0.0069 & -0.0794 \end{bmatrix}. \end{aligned} \quad (51)$$

In the simulation, the initial values for the state and isolated nodes were represented as $z_1(0) = [4; -2; 3]$, $z_2(0) = [1; 3; -2]$, $z_3(0) = [-5; -1; 4]$, and $s(0) = [4; -2; 3]$. Then, according to the aforementioned controller gain, the synchronization of the error system is illustrated in Figure 7. In addition, the nonuniform sampling data controller is shown in Figure 8, and it can be seen from Figure 9 that the synchronization of the state responses converges to zero. The external disturbances are depicted in Figure 10. In addition, Figure 11 presents the randomness of the time-varying delay, and Figure 12 shows the chaotic attractor of the Rossler system. Eventually, based on the simulations carried out above, we conclude from these that the design of the nonuniform sampled data controller works effectively in the presence of a noise intensity function and random time-varying delay.

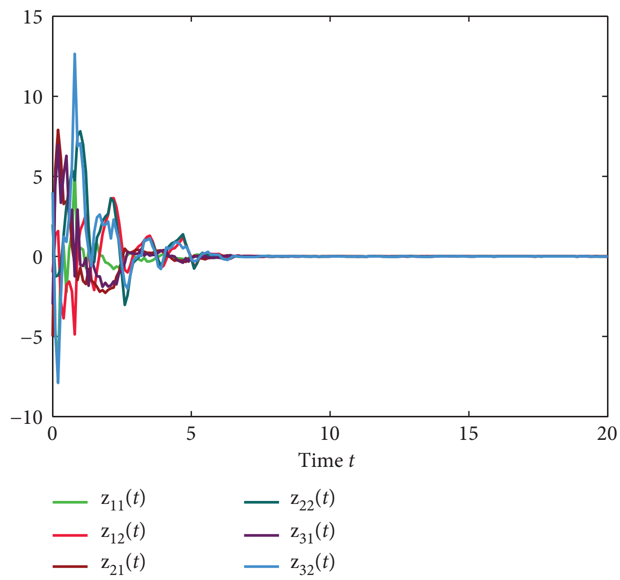

Figure 1: Trajectories of error system (5) with control inputs.

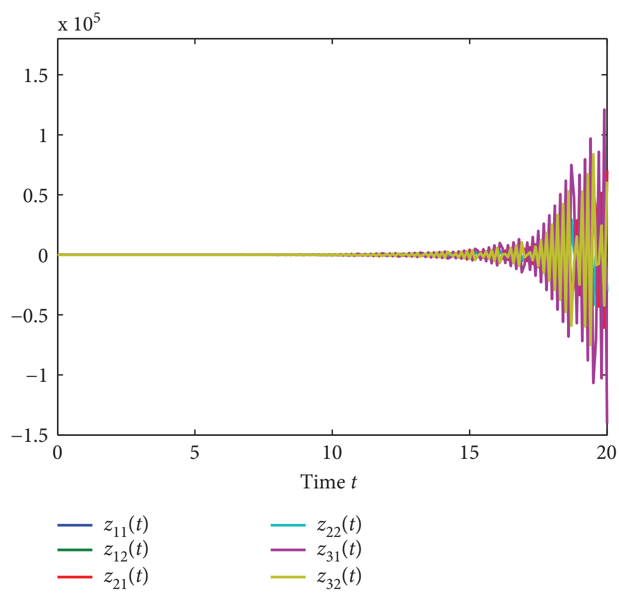

Figure 2: Trajectories of error system (5) without control inputs.

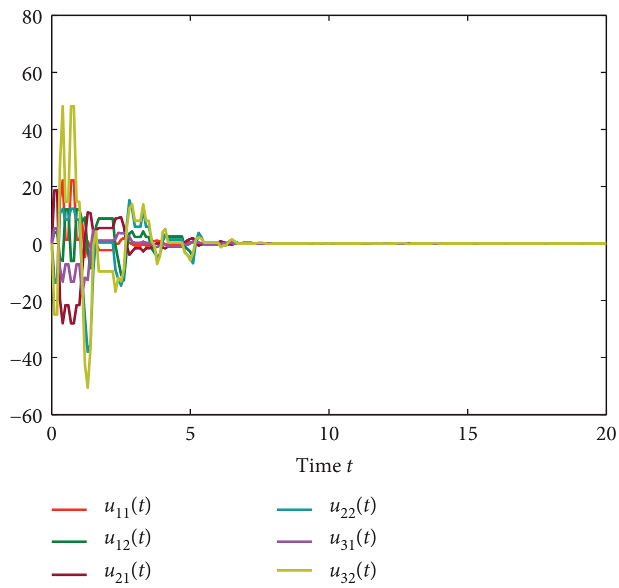

Figure 3: Trajectories of control inputs.

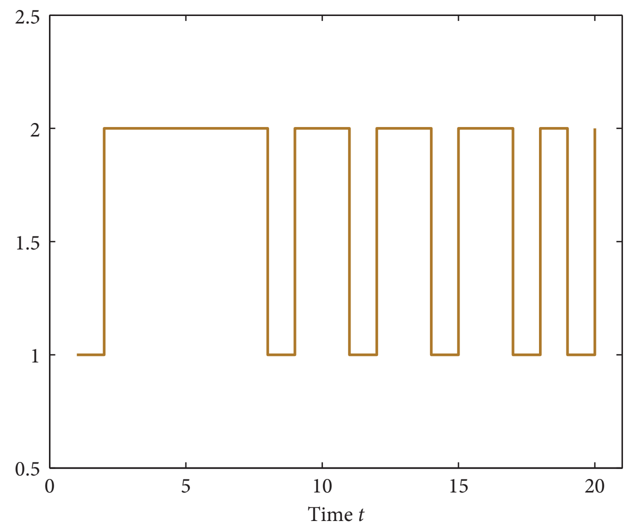

Figure 5: Mode transitions of semi-Markov jump CDNs.

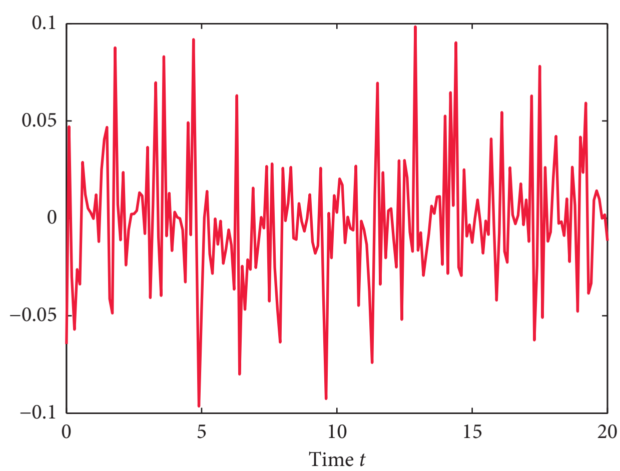

Figure 4: Response of disturbance.

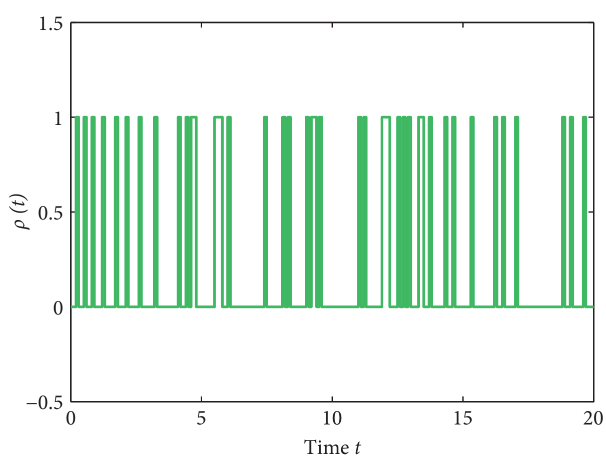

Figure 6: Bernoulli distribution.

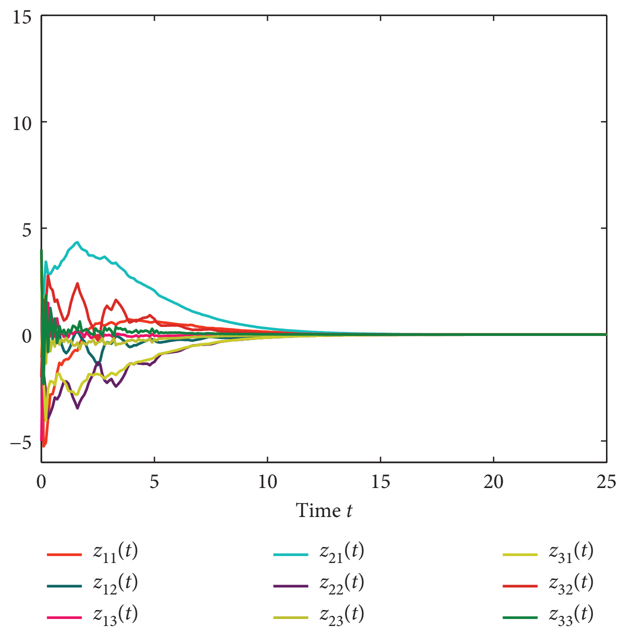

FIGURE 7: Trajectories of error system (37) with control inputs.

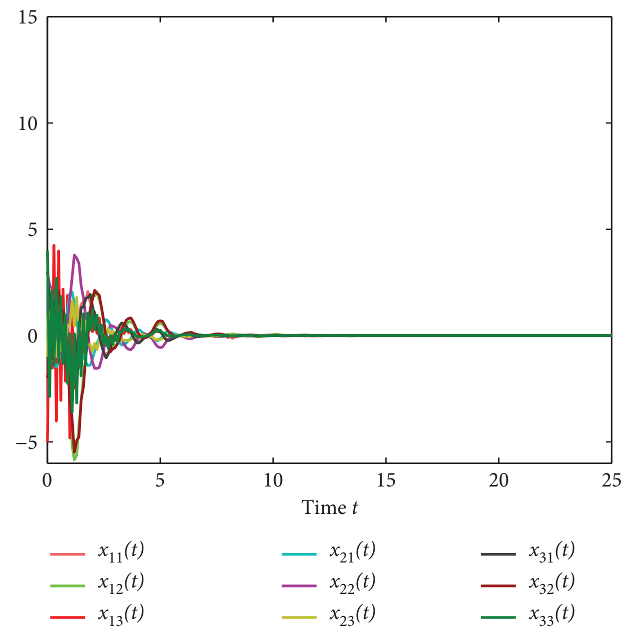

FIGURE 9: Trajectories of state system with control inputs.

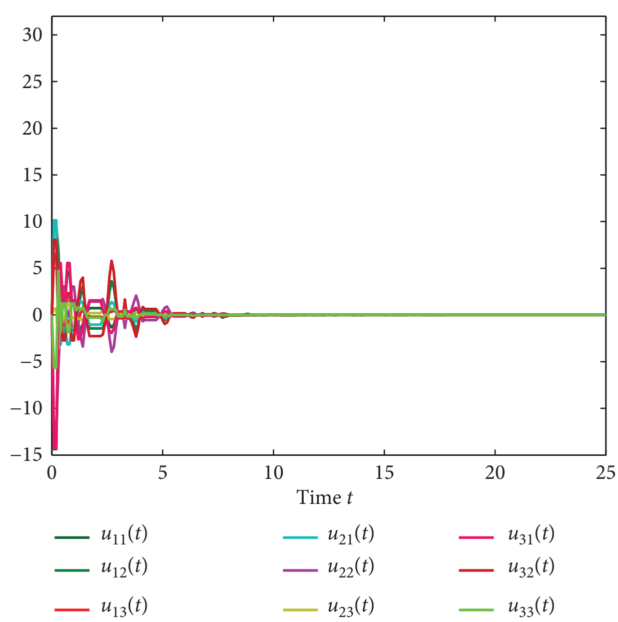

FIGURE 8: Trajectories of control inputs.

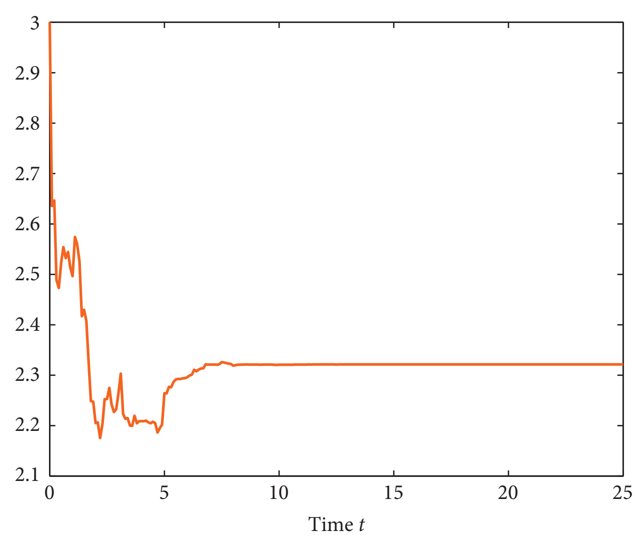

FIGURE 10: Response of disturbance.

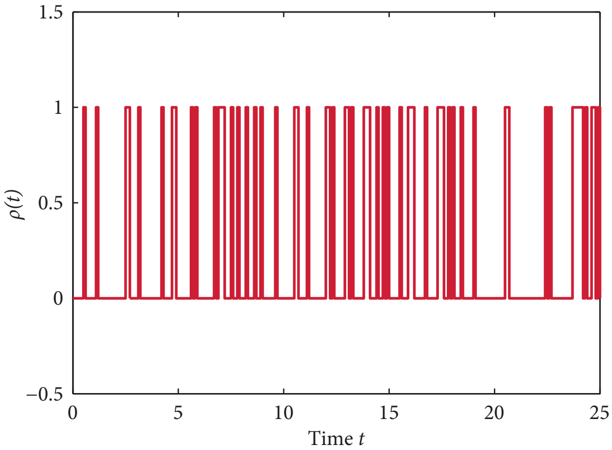

Figure 11: Bernoulli distribution.

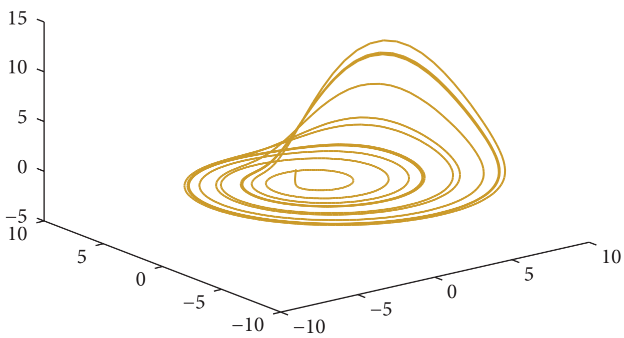

Figure 12: Chaotic attractor of the Rossler system.

5. Conclusions

In this study, the exponential synchronization of semi-Markov jump CDNs with random delayed information exchanges among dynamical nodes was investigated. In particular, to describe the information exchanges between nodes, a stochastic variable obeying the Bernoulli distribution was incorporated to model randomly occurring phenomena. The nonuniform sample data controller has been utilized to achieve the desired synchronization of considered CDNs. By constructing a suitable LKF using the Wirtinger-type inequality, sufficient criteria were obtained in terms of LMIs. Finally, numerical examples were presented to illustrate the effectiveness of the proposed method. Future work will focus on the framework of denial-of-service attacks and event-triggered control for a class of stochastic complex dynamical networks to obtain more industrial-oriented results.

Data Availability

This paper contains numerical experimental results; the values for these experiments are included in the paper. The data are freely available.

Consent

All the authors agree with this submission.

Conflicts of Interest

The authors declare that they have no conflict of interest.

Acknowledgments

This work was financially supported by a UGC-BSR Research Start-Up-Grant under the Ministry of Human Resource Development, Government of India (Grant No. F.30-410/2018(BSR)) awarded to the first author. This work was also supported by the National Research Foundation of Korea (NRF) grant funded by the Korean Government (MSIT) (No. 2021R1F1A1048937) to the second author.

References

[1] J. L. Wang, H. N. Wu, and T. Huang, “Passivity-based synchronization of a class of complex dynamical networks with time-varying delay,” *Automatica*, vol. 56, pp. 105–112, 2015.

[2] Y. Liu, B. Z. Guo, J. H. Park, and S. M. Lee, “Non-fragile exponential synchronization of delayed complex dynamical networks with memory sampled-data control,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 1, pp. 118–128, 2018.

[3] P. Chanthorn, G. Rajchakit, J. Thipcha et al., “Robust stability of complex-valued stochastic neural networks with time-varying delays and parameter uncertainties,” *Mathematics*, vol. 8, no. 5, p. 742, 2020.

[4] H. Shen, J. H. Park, Z. G. Wu, and Z. Zhang, “Finite-time H∞ synchronization for complex networks with semi-Markov jump topology,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 24, pp. 40–51, 2015.

[5] C. Aouiti and M. Bessifi, “Sliding mode control for finite-time and fixed-time synchronization of delayed complex-valued recurrent neural networks with discontinuous activation functions and nonidentical parameters,” *European Journal of Control*, vol. 59, pp. 109–122, 2021.

[6] X. Wang, J. Cao, J. Wang, and J. Qi, “A novel fixed-time stability strategy and its application to fixed-time synchronization control of semi-Markov jump delayed neural networks,” *Neurocomputing*, vol. 452, pp. 284–293, 2021.

[7] Q. Yang, H. Wu, and J. Cao, “Global cluster synchronization in finite time for complex dynamical networks with hybrid couplings via aperiodically intermittent control,” *Optimal Control Applications and Methods*, vol. 41, no. 4, pp. 1097–1117, 2020.

[8] C. Zhao, S. Zhong, X. Zhang, Q. Zhong, and K. Shi, “Novel results on nonfragile sampled-data exponential synchronization for delayed complex dynamical networks,” *International Journal of Robust and Nonlinear Control*, vol. 30, no. 10, pp. 4022–4042, 2020.

[9] H. Dai, W. Chen, J. Jia, J. Liu, and Z. Zhang, “Exponential synchronization of complex dynamical networks with time-varying inner coupling via event-triggered communication,” *Neurocomputing*, vol. 245, pp. 124–132, 2017.

[10] R. Rakkiyappan, N. Sakthivel, and S. Lakshmanan, “Exponential synchronization of complex dynamical networks with Markovian jumping parameters using sampled-data and mode-dependent probabilistic time-varying delays,” *Chinese Physics B*, vol. 23, no. 2, Article ID 020205, 2014.

[11] F. A. Gyamfi, Y. Cheng, C. Yin, and S. Zhong, “Exponential H∞ synchronization of non-fragile sampled-data controlled complex dynamical networks with random coupling and time

varying delay," *Systems Science & Control Engineering*, vol. 6, pp. 370–387, 2018.

[12] D. Li, Z. Wang, and G. Ma, "Controlled synchronization for complex dynamical networks with random delayed information exchanges: a non-fragile approach," *Neurocomputing*, vol. 171, pp. 1047–1052, 2016.

[13] D. Ye, X. Yang, and L. Su, "Fault-tolerant synchronization control for complex dynamical networks with semi-Markov jump topology," *Applied Mathematics and Computation*, vol. 312, pp. 36–48, 2017.

[14] C. Ma, W. Wu, and Y. Ji, "Dissipativity-based synchronization of mode-dependent complex dynamical networks with semi-Markov jump topology," *Complexity*, vol. 2018, pp. 1–10, Article ID 2348705, 2018.

[15] X. Song, R. Zhang, C. K. Ahn, and S. Song, "Dissipative synchronization of semi-markov jump complex dynamical networks via adaptive event-triggered sampling control scheme," *IEEE Systems Journal*, vol. 16, no. 3, pp. 4653–4663, 2022.

[16] J. Wang, X. Hu, Y. Wei, and Z. Wang, "Sampled-data synchronization of semi-Markov jump complex dynamical networks subject to generalized dissipativity property," *Applied Mathematics and Computation*, vol. 346, pp. 853–864, 2019.

[17] J. Liu, H. Wu, and J. Cao, "Event-triggered synchronization in fixed time for semi-Markov switching dynamical complex networks with multiple weights and discontinuous nonlinearity," *Communications in Nonlinear Science and Numerical Simulation*, vol. 90, Article ID 105400, 2020.

[18] R. Sakthivel, R. Sakthivel, B. Kaviarasan, C. Wang, and Y. K. Ma, "Finite-time nonfragile synchronization of stochastic complex dynamical networks with semi-Markov switching outer coupling," *Complexity*, vol. 201813 pages, 2018.

[19] L. Zhang, Y. Sun, Y. Pan, D. Hou, and S. Wang, "Network-based robust event-triggered control for continuous-time uncertain semi-Markov jump systems," *International Journal of Robust and Nonlinear Control*, vol. 31, no. 1, pp. 306–323, 2021.

[20] Y. Tian, H. Yan, W. Dai, S. Chen, and X. Zhan, "Observed-based asynchronous control of linear semi-markov jump systems with time-varying mode emission probabilities," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 12, pp. 3147–3151, 2020.

[21] H. Shen, F. Li, S. Xu, and V. Sreeram, "Slow state variables feedback stabilization for semi-markov jump systems with singular perturbations," *IEEE Transactions on Automatic Control*, vol. 63, no. 8, pp. 2709–2714, 2018.

[22] J. Wang, K. Shi, Q. Huang, S. Zhong, and D. Zhang, "Stochastic switched sampled-data control for synchronization of delayed chaotic neural networks with packet dropout," *Applied Mathematics and Computation*, vol. 335, pp. 211–230, 2018.

[23] G. Chen, J. Xia, J. H. Park, H. Shen, and G. Zhuang, "Robust sampled-data control for switched complex dynamical networks with actuators saturation," *IEEE Transactions on Cybernetics*, vol. 52, no. 10, pp. 10909–10923, 2022.

[24] N. Gunasekaran, M. S. Ali, S. Arik, H. A. Ghaffar, and A. A. Z. Diab, "Finite-time and sampled-data synchronization of complex dynamical networks subject to average dwell-time switching signal," *Neural Networks*, vol. 149, pp. 137–145, 2022.

[25] J. Li, Y. Ma, and L. Fu, "Fault-tolerant passive synchronization for complex dynamical networks with Markovian jump based on sampled-data control," *Neurocomputing*, vol. 350, pp. 20–32, 2019.

[26] A. Alsaedi, M. Usha, M. Syed Ali, and B. Ahmad, "Finite-time synchronization of sampled-data Markovian jump complex dynamical networks with additive time-varying delays based on dissipative theory," *Journal of Computational and Applied Mathematics*, vol. 368, Article ID 112578, 2020.

[27] T. Poongodi, P. P. Mishra, C. P. Lim et al., "TS fuzzy robust sampled-data control for nonlinear systems with bounded disturbances," *Computation*, vol. 9, no. 12, p. 132, 2021.

[28] S. Khan, R. M. Goodall, and R. Dixon, "Non-uniform sampling strategies for digital control," *International Journal of Systems Science*, vol. 44, no. 12, pp. 2234–2254, 2013.

[29] W. Kang, J. Cheng, B. Wang, J. H. Park, and H. M. Fardoun, "Event-triggered reliable control for Markovian jump systems subject to non-uniform sampled-data," *Journal of the Franklin Institute*, vol. 354, no. 14, pp. 5877–5894, 2017.

[30] J. Janczak and E. Pawluszewicz, "On stabilization of non-uniformly sampled control systems - a survey," *Solid State Phenomena*, vol. 260, pp. 156–174, 2017.

[31] L. Lu, Z. Xing, and B. He, "Non-uniform sampled-data control for stochastic passivity and passification of Markov-jump genetic regulatory networks with time-varying delays," *Neurocomputing*, vol. 171, pp. 434–443, 2016.

[32] N. Boonsatit, R. Sugumar, D. Ajay et al., "Mixed $\mathscr{H}$-infinity and passive synchronization of markovian jumping neutral-type complex dynamical networks with randomly occurring distributed coupling time-varying delays and actuator faults," *Complexity*, vol. 2021, Article ID 5553884, 2021.

[33] S. Rajavel, R. Samidurai, J. Cao, A. Alsaedi, and B. Ahmad, "Finite-time non-fragile passivity control for neural networks with time-varying delay," *Applied Mathematics and Computation*, vol. 297, pp. 145–158, 2017.

[34] H. Zhao, L. Li, H. Peng, J. Xiao, Y. Yang, and M. Zheng, "Finite-time topology identification and stochastic synchronization of complex network with multiple time delays," *Neurocomputing*, vol. 219, pp. 39–49, 2017.

[35] Y. Luo, Z. Ling, Z. Cheng, and B. Zhou, "New results of exponential synchronization of complex network with time-varying delays," *Advances in Difference Equations*, vol. 2019, no. 1, p. 10, 2019.