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Research Article

Nonlinear dynamics of the media addiction model using the fractal-fractional derivative technique

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Excessive use of social media is a developing concern in the twenty-first century. This issue needs to be addressed before it has any more significant consequences than what we are currently experiencing. As a preventive technique, advertisements and awareness-raising campaigns about the detrimental impact of digital technologies are used. The application of novel mathematical techniques and terminologies in this field of study will have significant potential to enhance healthy living by preventing certain ailments. This is the most compelling justification for conducting a new study with the most up-to-date techniques at our disposal. This study investigates clear and concise transmission in order to generate a deterministic mathematical model of social media addiction $\mathbb{SMA}$ using the fractal-fractional (FF) derivative operator. Also, the analysis of the $\mathbb{SMA}$ model in terms of the invariant domain, the existence of a positive invariant solution, and equilibria assumptions are stated in a detailed manner. Besides that, the basic reproduction number $\mathbb{R}_0 < 1$ is computed, demonstrating that the proposed methodology is more efficacious. The Atangana-Baleanu FF differential operators are recently defined in FF differential operators that are applied to characterize the $\mathbb{SMA}$ model's mathematical algorithm. We investigated the numerical behaviour of the $\mathbb{SMA}$ in three ways: (i) changing the fractional-order (α) as well as the fractal-dimension ($\wp$); (ii) changing α while keeping $\wp$ constant; and (iii) changing $\wp$ while keeping α constant. Our examined visualizations and simulation studies using MATLAB for the numerical modelling of the aforementioned system showed that the novel developed Atangana-Baleanu FF differential operators produce remarkable results when compared to the classical frame.

1. Introduction

With the popularity of networking communication comes an increase in the psychoactive properties of this technological innovation. Numerous investigations have linked compulsive media platform use to undesirable outcomes, including decreased efficiency, adverse social interactions, and decreased life contentment. The misuse of social networking sites has increased at an alarming rate. For example, Facebook has 68 percent and 73 percent of the adolescent community in the United States, respectively [1]. Over prescription of social media addiction is connected with impaired effectiveness appraisal, problematic interpersonal interactions, insomnia, reduced levels of happiness, and sentiments of hostility, stress, and melancholy, see [2–7].To be consistent with the preponderance of the research, we included the terms "social media addiction" ($\mathbb{SMA}$) or "addicted social networking usage" (in a non-clinical meaning) in the entirety of that kind of analysis, while acknowledging the concerns surrounding the terminology [8]. Whenever it becomes more appropriate to employ certain concepts, exclusions are created. For the purposes of this evaluation, we characterize excessive social media adoption as becoming extremely apprehensive regarding social media, intrinsically incentivized, and dedicating a significant portion of time and vitality to incorporating social media to the juncture where an individual's group opportunities, intimate communication, surveys, and/or general well-being and well-being are jeopardized [9].There is now a discrepancy in theorizing $\mathbb{SMA}$, particularly how it progresses. There have been multiple evaluations of conceptualizations to elucidate $\mathbb{SMA}$, but limited research has addressed the mathematical frameworks implemented in actual research on $\mathbb{SMA}$. One evaluation, in particular, barely discussed three extensively acknowledged internet dependency scenarios. One questionnaire range of interventions includes broad scientific viewpoints (for example, neurological aspects and behavioural insights) without addressing relevant paradigms. Although some alternative analyses reviewed various relevant conceptual models, analyzed their flaws, and proposed hypothetical modifications, they did not describe how scientific findings about $\mathbb{SMA}$ implement these conceptualizations and the various attributes in these structures. Furthermore, new scientific findings on $\mathbb{SMA}$ are appearing at an astonishing rate; there is a need for a comprehensive overview of essential mathematical approaches. Alemneh and Alemu [10] constructed and examined a mathematical formulation for the dissemination and prevention of $\mathbb{SMA}$ in global society in 2021, as continues to follow:

$$\begin{cases} \dot{S}(\zeta) = \varsigma + \gamma R(\zeta) - \lambda\theta A(\zeta)S(\zeta) - (\kappa + \nu)S(\zeta), \\ \dot{E}(\zeta) = \lambda\theta A(\zeta)S(\zeta) - (\phi + \nu)E(\zeta), \\ \dot{A}(\zeta) = \beta\phi E(\zeta) - (\nu + \varepsilon + \psi)A(\zeta), \\ \dot{R}(\zeta) = (1-\beta)\phi E(\zeta) + \varepsilon A - (\nu +)R(\zeta), \\ \dot{Q}(\zeta) = \kappa S(\zeta) + (1-\gamma)R(\zeta) - \nu Q(\zeta), \end{cases} \tag{1}$$

The current community is categorized into five compartments depending on addiction level in the scheme (1). Compartment 1: individuals that are likely to be exposed are those that are not obsessed but are accessible to $\mathbb{SMA}$ is represented by $\mathbf{S}(\zeta)$. Compartment 2: individuals who utilize SM relatively regularly yet do not become obsessed are referred to as exposed individuals $\mathbf{E}(\zeta)$. Compartment 3: compulsive populace are persons who are obsessed to SM and spend a significant amount of time doing it $\mathbf{A}(\zeta)$. Compartment 4: retrieved individuals are persons who have progressed from $\mathbb{SMA}$, $\mathbf{R}(\zeta)$ is the number of individuals who have benefited from $\mathbb{SMA}$. Compartment 5: $\mathbf{Q}(\zeta)$ represents anyone that doesn't use or stop using SM perpetually. The number of individuals in the community is: $\mathbf{N} = \mathbf{S} + \mathbf{E} + \mathbf{A} + \mathbf{R} + \mathbf{Q}$. The model's suppositions are as follows: the dissemination of the $\mathbb{SMA}$ issue occurs in a confined area and is not reliant on intimate relations, racial group, or sentient socialist system. Representatives combining uniformly, and social networking dependent individuals will convey to non-addictive people when they are in contact with the compression of habit forming. Furthermore, the mathematical expressions of this scheme are amalgamated by employing the social networking addiction process, which begins with the introduction of vulnerable people into the community at a rate of ς. They are prompted to transfer to the revealed condition by addicted humans having a stress interaction incidence of and a probabilistic transference ratio of θ. At a proportion of κ, certain vulnerable persons migrate to a set of individuals who never access digital platforms. The challenged people are divided into two classes: one that develops obsession and advances to the addictive class at a pace $\beta\phi$, and another that recovers using therapy at a pace $(1-\beta)\phi$. Many dependent individuals transfer to the recovery class at a pace of or die as a result of the overuse of dependence on social networks at a ratio of ψ. People who have overcome become vulnerable again at a ratio of or eventually discontinue access to digital platforms at a ratio of $(1-\gamma)\varrho$. Eventually, all of the inhabitants in each group have a spontaneous mortality rate of ν. Alemneh and Alemu [10] further examined the robustness of the equilibria and applied Pontryagin's maximal principle to design the best monitoring mechanism.In past times, fractional calculus has been consistently proven to be an outstanding approach for illustrating the heredity properties of diverse formations. Furthermore, fractional differential operators have been used in a plethora of distinctive manifestations, such as hydroelectricity, fluid dynamics, chromatography, commerce, and viruses, utilising numerous fractional strategies such as Caputo, Hadamard, Riemann-Liouville, Katugumpola, Caputo–Fabrizio, Atangana-Baleanu and fractal–fractional, see [11–13]. This combo has suddenly accumulated a wealth of traction, partly because fractional differential equations have been shown to be superb tools for displaying a few incredibly challenging phenomena in a wide range of various and endless academic disciplines; assessors are encouraged to [14–20]. In 2019, the African mathematician Abdon-Atangana [21] proposed novel differential operators with fractal derivatives based on the combination of index law, modified Mittag-Leffler rule, and exponential decay law. Novel formulations, according to [20], offer additional non-local physical challenges with fractal characteristics simultaneously. Chen et al. [21] investigated the classification of anomalous propagation by deriving the underlying component of the numerical scheme using fractal derivative. For the purpose of evaluating processing performance and dispersion velocity, they compared fractional and fractal derivatives. Wei et al. [22] employed a scalability transformation technique to offer fractal modelling of elastic deformation. Sania et al. [24] introduced differential operators with fractional order and fractal dimension to describe additional chaotic complexity, wherein various kinds of orientations were mentioned: King Cobra, Shilnikov, Thomas cyclically symmetric, Langford, and Rössler. Rashid et al. [25] discussed the complex oscillatory behaviour of a human liver model via FF derivative operator technique. According to current evidence analysis of the data collected for this study, none of the works have addressed the mathematical framework of $\mathbb{SMA}$ involving fractal-fractional derivatives. The novelty of this article is the addition of the FF fractional derivative to the $\mathbb{SMA}$ model. The improved $\mathbb{SMA}$ transmitting system incorporating the fractal-fractional derivative in the Atangana-Baleanu sense is proposed in the following framework:

$$\begin{cases} {}^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\alpha,\wp}\mathbf{S}(\zeta) = \varsigma + \gamma\varrho\mathbf{R}(\zeta) - \lambda\theta\mathbf{A}(\zeta)\mathbf{S}(\zeta) - (\kappa + \nu)\mathbf{S}(\zeta), \\ {}^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\alpha,\wp}\mathbf{E}(\zeta) = \lambda\theta\mathbf{A}(\zeta)\mathbf{S}(\zeta) - (\phi + \nu)\mathbf{E}(\zeta), \\ {}^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\alpha,\wp}\mathbf{A}(\zeta) = \beta\phi\mathbf{E}(\zeta) - (\nu + \epsilon + \psi)\mathbf{A}(\zeta), \\ {}^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\alpha,\wp}\mathbf{R}(\zeta) = (1 - \beta)\phi\mathbf{E}(\zeta) + \epsilon\mathbf{A} - (\nu + \varrho)\mathbf{R}(\zeta), \\ {}^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\alpha,\wp}\mathbf{Q}(\zeta) = \kappa\mathbf{S}(\zeta) + (1 - \gamma)\varrho\mathbf{R}(\zeta) - \nu\mathbf{Q}(\zeta). \end{cases} \tag{2}$$

having initial conditions $(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}) = (\mathbf{S}_0, \mathbf{E}_0, \mathbf{A}_0, \mathbf{R}_0, \mathbf{Q}_0)$, where ${}^{\mathbf{FF}}\mathbf{D}_{\zeta}^{\alpha,\wp}(.)$ denotes the FF operator in terms of Atangana-Baleanu derivative operator, whilst α presents the fractional-order and $\wp$ denotes the fractal-dimension, respectively. Furthermore, Table 1 contains explanations of all attributes. Recently, Alemneh and Alemu [10] demonstrated the mathematical modeling with optimal control analysis of $\mathbb{SMA}$. Kongson et al. [27] expounded the estimates for $\mathbb{SMA}$ model pertaining to the Atnagana-Baleanu fractional derivative operator in the Caputo context. Leveraging the aforementioned proclivity, we employ a newly formed arbitrary order derivative in the $\mathbb{SMA}$ model. The main purpose of this research is to investigate the $\mathbb{SMA}$ model via a pioneering FF derivative operator in the Atangana-Baleanu sense and to characterize the complexities of the uniqueness and existence of the aforementioned framework response by utilizing the Picard-Lindlöf and contraction mapping techniques. Furthermore, to the best of our knowledge, there is no previous paper related to $\mathbb{SMA}$ qualitative aspects of fractal–fractional derivative based on the Atangana-Baleanu context. As a result, the main objective of this paper is to bridge that gap. Graphical results demonstrate that while fixing and varying fractional-order and fractal-dimension how they affect the model's characterizations.This article's entire tasks are organized into five portions, which are provided as they continue to be implemented: Section 2 summarizes and presents the core notions and relevant formulations of FF derivatives in the Atangana-Baleanu interpretations. Section 3 describes the analysis of the model that corresponds to the positivity, bounded domain, invariant region, and equilibria points via the FF derivative operator technique. Section 4 illustrates and performs a convergence and uniqueness investigation for the abovementioned $\mathbb{SMA}$ with the aid of the FF derivative operator and the numerical formulation of the Newton polynomial approach. Section 5 deals with the numerical outcomes and description of the suggested $\mathbb{SMA}$ model's solution. The prime focus of the planned study is on which we will exhibit simulation consequences. Ultimately, Section 6 summarizes the remarks and discusses the promising possibilities of the fractal-fractional $\mathbb{SMA}$ model.

2. Preliminaries

In what follows, it is vital to investigate some fundamental FF operator notions prior to continuing on to the formal model. Consider there be a function $\mathbf{y}(\zeta)$, which is continuous and fractal differentiable on $[c, d]$ and has fractal-dimension $\wp$ and fractional-order α, in addition to the specifications in [21, 24].

Definition 2.1 (see [21, 24]). Suppose there be a FF operator of $\mathbf{y}(\zeta)$ having power law kernel in terms of Riemann–Liouville (RL) can be expressed in the form:

$${}^{FFP}\mathbf{D}_{0,\zeta}^{\alpha,\mathfrak{p}}(\mathbf{y}(\zeta)) = \frac{1}{\Gamma(\mathbf{r} - \alpha)} \frac{d}{d\zeta^{\wp}} \int_0^{\zeta} (\zeta - \varpi)^{\mathbf{r} - \alpha - 1}\mathbf{y}(\varpi)d\varpi, \tag{3}$$

where $(d\mathbf{y}(\varpi)/d\varpi^{\wp}) = \lim_{\zeta \mapsto \varpi} (\mathbf{y}(\zeta) - \mathbf{y}(\varpi)/\zeta^{\wp} - \varpi^{\wp})$ and $\mathbf{r} - 1 < \alpha, \wp \le \mathbf{r} \in \mathbb{N}$.

Definition 2.2 (see [21, 24]). Suppose there be a FF operator of $\mathbf{y}(\zeta)$ having exponential kernel in terms of RL can be expressed in the form:

$${}^{FFE}\mathbf{D}_{0,\zeta}^{\alpha,\wp}(\mathbf{y}(\zeta)) = \frac{\mathbf{M}(\alpha)}{1 - \alpha} \frac{d}{d\zeta^{\wp}} \int_0^{\zeta} \exp\Big(-\frac{\alpha}{1 - \alpha}(\zeta - \varpi)\Big)\mathbf{y}(\varpi)d\varpi, \tag{4}$$

such that $\mathbf{M}(0) = \mathbf{M}(1) = 1$ with $\alpha > 0, \wp \le \mathbf{r} \in \mathbb{N}$.

Definition 2.3 (see [21, 24]). Suppose there be a FF operator of $\mathbf{y}(\zeta)$ with Mittag-Leffler kernel in terms of RL can be expressed in the form:

$${}^{FFE}\mathbf{D}_{0,\zeta}^{\alpha,\wp}(\mathbf{y}(\zeta)) = \frac{\mathbf{ABC}(\alpha)}{1 - \alpha} \frac{d}{d\zeta^{\wp}} \int_0^{\zeta} E_{\alpha}\Big(-\frac{\alpha}{1 - \alpha}(\zeta - \varpi)\Big)\mathbf{y}(\varpi)d\varpi, \tag{5}$$

such that $\mathbf{ABC}(\alpha) = 1 - \alpha + (\alpha/\Gamma(\alpha))$ with $\alpha > 0, \wp \le 1 \in \mathbb{N}$.

Definition 2.4 (see [21, 24]). The corresponding FF integral form of (3) is described as:

$${}^{FFP}\mathbb{J}_{0,\zeta}^{\alpha}(\mathbf{y}(\zeta)) = \frac{\wp}{\Gamma(\alpha)} \int_0^{\zeta} (\zeta - \varpi)^{\alpha - 1}\varpi^{\wp - 1}\mathbf{y}(\varpi)d\varpi. \tag{6}$$

Definition 2.5 (see [21, 24]). The corresponding FF integral version of (4) is described as:

$${}^{FFP}\mathbb{J}_{0,\zeta}^{\alpha}(\mathbf{y}(\zeta)) = \frac{\alpha\wp}{\mathbf{M}(\alpha)} \int_0^{\zeta} \varpi^{\wp - 1}\mathbf{y}(\varpi)d\varpi + \frac{\wp(1 - \alpha)\zeta^{\wp - 1}\mathbf{y}(\zeta)}{\mathbf{M}(\alpha)}. \tag{7}$$

Definition 2.6 (see [21, 24]). The corresponding FF integral form of (5) is described as:

$$\begin{aligned} {}^{FFP}\mathbb{J}_{0,\zeta}^{\alpha}(\mathbf{y}(\zeta)) = {} & \frac{\alpha\wp}{\mathbf{ABC}(\alpha)} \int_0^{\zeta} \varpi^{\wp - 1}(\zeta - \varpi)^{\alpha - 1}\mathbf{y}(\varpi)d\varpi \\ & + \frac{\wp(1 - \alpha)\zeta^{\wp - 1}\mathbf{y}(\zeta)}{\mathbf{ABC}(\alpha)}. \end{aligned} \tag{8}$$

Definition 2.7 (see [14]). Let $\mathbf{y} \in H^1(\delta, \gamma), \delta < \gamma$ and the Atangana-Baleanu-Caputo derivative operator is defined as:

Table 1: Table of specified variables and their descriptions

Parameters	Explanation	Data estimated	References
ς	Acquisition of susceptible people	0.5	Supposed
ν	natural death rate	0.05	[39]
λ	Addiction transfer percentage to vulnerable persons	0.1-0.8	[39]
θ	Interaction proportion of vulnerable people involving addicted people	0.2	[39]
β	Percentage of uncovered persons who get obsessed	0.7	[39]
ψ	induce mortality rate	0.01	Supposed.
ϕ	People who quit the exposed group	0.25	[39].
ϵ	Addicts who enter the rehabilitated group as a result of the therapy	0.7	[40]
κ	Susceptible persons who do not take and/or stop utilizing SM	0.01	Supposed.
γ	The percentage of rehabilitated people susceptible to SMA	0.35	[41]
ϱ	People who depart the rehabilitated category	0.4	[40]

$$
{}_{\mathbf{c}}^{ABC}\mathbf{D}_{\zeta}^{\alpha}(\mathbf{y}(\zeta)) = \frac{\mathbf{ABC}(\alpha)}{1-\alpha}\int_{\mathbf{c}}^{\zeta} y'(\varpi) \cdot E_{\alpha}\left(-\frac{\alpha(\zeta-\varpi)^{\alpha}}{1-\alpha}\right)\mathrm{d}\varpi, \alpha \in [0,1], \tag{9}
$$

where $\mathbf{ABC}(\alpha)$ represents the normalization function.

3. Qualitative analysis of $\mathbb{SMA}$

In this section, we demonstrate that $\mathbb{SMA}$ presented in (1) is epidemiologically viable by ensuring that the system's corresponding model parameters are non-negative for every time-step ζ. This is based on the more straightforward argument that the $\mathbb{SMA}$ model with non-negative ICs becomes non-negative for all $\zeta > 0$. The preceding is a lemma.

Lemma 3.1. *Suppose there be the initial data $\mathscr{G} \geq 0$, where $\mathscr{G}(\zeta) = (\mathbf{S}(\zeta), \mathbf{E}(\zeta), \mathbf{A}(\zeta), \mathbf{R}(\zeta), \mathbf{Q}(\zeta))$. Thus the $\mathbb{SMA}$ system (1) are positive for all $\zeta > 0$. Also, $\lim_{\zeta\mapsto\infty} \mathbf{N}(\zeta) \leq (\varsigma/\nu)$ having $\mathbf{N}(\zeta) = \mathbf{S}(\zeta) + \mathbf{E}(\zeta) + \mathbf{A}(\zeta) + \mathbf{R}(\zeta) + \mathbf{Q}(\zeta)$.*

Proof. Assume that $\Xi = \sup\{\zeta > 0: \mathscr{G}(\zeta) > 0 \in [0,\zeta]\}$. Therefore, $\Xi > 0$, the first equation of the framework (1) consists of following

$$
\frac{\mathbf{S}(\zeta)}{d\zeta} = \varsigma + \gamma\varrho\mathbf{R}(\zeta) - \lambda\theta\mathbf{A}(\zeta)\mathbf{S}(\zeta) - (\kappa+\nu)\mathbf{S}(\zeta), \tag{10}
$$

with $\eta = \lambda\theta\mathbf{A}(\zeta)$, then (10) diminishes to

$$
\frac{\mathbf{S}(\zeta)}{d\zeta} = \varsigma - \eta\mathbf{S}(\zeta) - (\kappa+\nu)\mathbf{S}(\zeta). \tag{11}
$$

It follows that

$$
\frac{d}{\mathrm{d}\zeta}\left\{\mathbf{S}(\zeta)\exp\left((\kappa+\mu)\zeta + \int_0^{\Xi}\eta(\phi_1)\mathrm{d}\phi_1\right)\right\} = \varsigma\left((\kappa+\mu)\zeta + \int_0^{\Xi}\eta(\phi_1)\mathrm{d}\phi_1\right). \tag{12}
$$

Consequently, we have

$$
\mathbf{S}(\Xi)\exp\left((\kappa+)\mu\Xi + \int_0^{\Xi}\eta(\phi_1)\mathrm{d}\phi_1\right) - \mathbf{S}(0) = \varsigma\,\exp\left((\kappa+\mu)y_1 + \int_0^{\Xi}\eta(\phi_2)\mathrm{d}\phi_2\right)\mathrm{d}y_1. \tag{13}
$$

Note that

$$
\begin{aligned}
\mathbf{S}(\Xi) = \mathbf{S}(0)\exp\left(-(\kappa+\mu)\Xi - \int_0^{\Xi}\eta(\phi_1)\mathrm{d}\phi_1\right) &+ \exp\left(-(\kappa+\mu)\Xi - \int_0^{\Xi}\eta(\phi_1)\mathrm{d}\phi_1\right) \\
&\times \int_0^{\Xi}\varsigma\,\exp\left((\kappa+\mu)y_1 + \int_0^{\Xi}\eta(\phi_2)d\phi_2\right)dy_1 > 0.
\end{aligned} \tag{14}
$$

Thus, we can obtain $\mathscr{G}(\zeta) > 0$ for any $\zeta > 0$ by repeating the previous methods for the leftover equations of model (1).

Now adding the $\mathbb{SMA}$ cohorts lead to the subsequent

$$
\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}\zeta} \leq \varsigma - \nu\mathbf{N} - \psi\mathbf{A}. \tag{15}
$$

If there is no death to the $\mathbb{SMA}$, then

$$
\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}\zeta} \leq \varsigma - \nu\mathbf{N}. \tag{16}
$$

Therefore, we have

$$
\lim_{\zeta\mapsto\infty}\mathbf{N}(\zeta) \leq \frac{\varsigma}{\nu}, \tag{17}
$$

which is the desired proof.

Further, in order to show the invariant region for the proposed $\mathbb{SMA}$ system (1), suppose

$$
\nabla = \left\{(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q}) \in \mathbb{R}_+^5 : 0 < \mathbf{N}(\zeta) \leq \frac{\varsigma}{\nu}\right\}. \tag{18}
$$

□

Lemma 3.2. *The domain represented by ∇ is positively invariant for the $\mathbb{SMA}$ system (1) along with non-negative ICs $\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q} > 0$ for all $\zeta \geq 0$.*

Proof. In view of (12), then we have

$$\frac{d\mathbf{N}}{d\zeta} = \varsigma - \nu\mathbf{N}. \quad (19)$$

Therefore, we have $(d\mathbf{N}/d\zeta) \le 0$, if $\mathbf{N}(0) \ge (\varsigma/\nu)$. Thus, we have

$$\mathbf{N}(\zeta) \le \mathbf{N}(0)\exp(-\nu\zeta) + \frac{\varsigma}{\nu}(1 - \exp(-\nu\zeta)), \quad (20)$$

which shows that, the domain presented by ∇ is positively invariant. Moreover, if $\mathbf{N}(0) > (\varsigma/\nu)$ or $\mathbf{N}(\zeta)$ approaches to ς/ν asymptotically. Hence, the domains presented by ∇ capture all of the possibilities in $\mathbb{R}_+^5$. □

3.1. Existence and nonnegativity of the solution. Further, we investigate the existence and nonnegativity of the $\mathbb{SMA}$ system (2).

Theorem 3.3. *If there be a unique solution of the system (2) and capture the solution in*$\mathbb{R}_+^5$.

Proof. In order to prove the solution of the system (2) is positive, we have

$$\begin{cases} {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{S}(\zeta)|_{\mathbf{S}=0} = \varsigma \ge 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{E}(\zeta)|_{\mathbf{E}=0} = 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{A}(\zeta)|_{\mathbf{A}=0} = \beta\phi\mathbf{E}(\zeta) \ge 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{R}(\zeta)|_{\mathbf{R}=0} = (1-\beta)\phi\mathbf{E} + \epsilon\mathbf{A}(\zeta) \ge 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{Q}(\zeta)|_{\mathbf{Q}=0} = \kappa\mathbf{S}(\zeta) + (1-\gamma)\varrho\mathbf{R}(\zeta) \ge 0, \end{cases} \quad (21)$$

which indicates that the system (2) solution exist in $\mathbb{R}_+^5 \forall\zeta > 0$. Summing up all cohorts in (2), we have

$${}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{N}(\zeta) = \varsigma - \nu\mathbf{N}. \quad (22)$$

Moreover, we have

$$\lim_{\mathfrak{n}\mapsto\infty} \sup\mathbf{N}(\zeta) \le \frac{\varsigma}{\nu} \quad (23)$$

and hence, the biologically viable domain for the system (2) can be represented by

$$\nabla^1 = \left\{(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}) \in \mathbb{R}_+^5 \colon 0 < \mathbf{N}(\zeta) \le \frac{\varsigma}{\nu}\right\}. \quad (24)$$

□

The framework for $\mathbb{SMA}$ mentioned-above (2) in the FF operator in the Atangana-Baleanu sense is implemented to provide the outcomes in the next subsection.

3.2. Stability result for disease free case. The stability outcomes for the $\mathbb{SMA}$ framework introduced by at disease free equilibrium (DFE) E_0 are explored in this section. We can get the respective formulas by changing the right side terms of the $\mathbb{SMA}$ (2) to zero, as

$$\begin{cases} {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{S}(\zeta) = 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{E}(\zeta) = 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{A}(\zeta) = 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{R}(\zeta) = 0, \\ {}^{\mathbf{FF}}\mathbf{D}_{0,\zeta}^{\alpha,\wp}\mathbf{Q}(\zeta) = 0, \end{cases} \quad (25)$$

we have the following *DF E* as follows

$$E_0 = \left(\frac{\varsigma}{\kappa+\nu}, 0, 0, 0, \frac{\kappa\varsigma}{\nu(\nu+\kappa)}\right). \quad (26)$$

Furthermore, the fundamental reproduction number $\mathbb{R}_0$, which may be determined by applying the next generation methodology for the scheme, can be used to examine the robustness of DFE at E_0. The infectious cohorts in the $\mathbb{SMA}$ system (2) are $\mathbf{E}, \mathbf{A}$, and the matrices $\mathbf{F}$ and $\mathbf{V}$ are obtained as follows:

$$\mathbf{F} = \begin{bmatrix} \lambda\theta\mathbf{AS} \\ 0 \\ 0 \end{bmatrix} \text{and} \mathbf{V} = ((\nu+\phi)\mathbf{E} - \phi\beta\mathbf{E} + (\nu+\epsilon+\psi)\mathbf{A} \\ - (1-\beta)\phi\mathbf{E} + (\nu+\varrho)\mathbf{R}). \quad (27)$$

Hence, the fundamental reproductive number can be calculated as

$$\mathbb{R}_0 = \tilde{\rho}\left(\mathcal{F}\mathcal{V}^{-1}\right) = \frac{\lambda\varsigma\beta\phi\theta}{(\kappa+\nu)(\phi+\nu)(\nu+\epsilon+\psi)}. \quad (28)$$

3.3. Strength number. Following the work [28], we will present the strength number.In recent years, the concept of reproduction in a specific infectious problem has been extensively used in epidemiology modelling. As predicted by the concept, two components, $\mathbf{F}$ and $\mathbf{V}$, will be identified in (27)

$$\left(\mathbf{F}\mathbf{V}^{-1} - \mu\mathbf{I}\right) = 0 \quad (29)$$

will be employed to generate the reproductive number [29]. The nonlinear portion of the classes that are infected is how the component $\mathbf{F}$, which is quite intriguing, gets derived

$$\frac{\partial}{\partial\mathbf{A}}\left(\frac{\mathbf{A}}{\mathbf{N}}\right) = \frac{[\mathbf{N}-\mathbf{A}]}{\mathbf{N}^2}. \quad (30)$$

Again, we have

$$\begin{aligned} &\frac{\partial^2}{\partial\mathbf{A}^2}\left(\frac{(\mathbf{N}-1)}{\mathbf{N}^3}\right) \\ &= \frac{-2[\mathbf{N}-\mathbf{A}]}{\mathbf{N}^3} \\ &= \frac{-2(\varsigma - \nu(\mathbf{S}(\zeta)+\mathbf{E}(\zeta)+\mathbf{R}(\zeta)+\mathbf{Q}(\zeta))) - \beta\phi\mathbf{E}(\zeta) + \epsilon\mathbf{A}(\zeta)}{(\mathbf{S}(\zeta)+\mathbf{E}(\zeta)+\mathbf{A}(\zeta)+\mathbf{R}(\zeta)+\mathbf{Q}(\zeta))^3}. \end{aligned} \quad (31)$$

At DFE $E_0 = (\varsigma/\kappa+\nu, 0, 0, 0, \kappa\varsigma/\nu(\nu+\kappa))$, we have

$$\frac{\partial^2}{\partial \mathbf{A}^2}\left(\frac{(\mathbf{N}-1)}{\mathbf{N}^3}\right) = \frac{-2(\varsigma - \nu(\mathbf{S}_0+\mathbf{E}_0+\mathbf{R}_0+\mathbf{Q}_0)) - \beta\phi\mathbf{E}_0 + \epsilon\mathbf{A}_0}{(\mathbf{S}_0+\mathbf{E}_0+\mathbf{A}_0+\mathbf{R}_0+\mathbf{Q}_0)^3} = \frac{-2(\varsigma - \nu(\mathbf{S}_0+\mathbf{Q}_0))}{(\mathbf{S}_0+\mathbf{Q}_0)^3}. \tag{32}$$

In this case, we have the following

$$\mathbf{F}_{\mathbb{A}} = \begin{bmatrix} \dfrac{-2\lambda(\varsigma-\nu(\mathbf{S}_0+\mathbf{Q}_0))}{(\mathbf{S}_0+\mathbf{Q}_0)^3} \\ 0 \\ 0 \end{bmatrix}. \tag{33}$$

Then $\det(\mathbf{F}_{\mathbb{A}}\mathbf{V}^{-1} - \mu\mathbf{I}) = 0$ leads to

$$\mathbb{A}_0 = \frac{-2\lambda\nu^3(\kappa+\nu-1)}{\varsigma^2} < 0. \tag{34}$$

Therefore, the dispersion will have a single magnitude and fade out if there is no regeneration mechanism, which is indicated by a value of $\mathbb{A}_0 = 0$. Furthermore, $\mathbb{A}_0 > 0$ denotes a strength that will trigger a renewing mechanism, indicating that the expansion will contain multiple waves. However, biologists will give a precise explanation of the aforementioned number.

Theorem 3.4. *The DFE at E_0 for $\mathbb{SMA}$ model (2) is locally asymptotically stable when$\mathbb{R}_0 < 1$satisfying the assumption*

$$\left|\arg(\mu_j)\right| > \frac{\varphi\pi}{2}. \tag{35}$$

Proof. To illustrate the provided hypothesis, we must first acquire the Jacobian matrix by evaluating $\mathbb{SMA}$ system (2) at the DFE E_0, we have

$$\mathbb{J}_{E_0} = \begin{bmatrix} -(\kappa+\nu) & 0 & -\dfrac{\lambda\varsigma\theta}{\kappa+\nu} & \gamma\varrho & 0 \\ 0 & -(\phi+\nu) & \dfrac{\lambda\varsigma\theta}{\kappa+\nu} & 0 & 0 \\ 0 & \beta\phi & -(\epsilon+\psi+\nu) & 0 & 0 \\ 0 & (1-\beta)\phi & \epsilon & -(\varrho+\nu) & 0 \\ \kappa & 0 & 0 & (1-\gamma)\varrho & -\nu \end{bmatrix}. \tag{36}$$

The negative eigenvalues are $-\nu, -(\nu+\kappa), -(\nu+\varrho), -(\nu+\phi)$ and remaining eigenvalues can be achieved from the following expression $\mu^2 + (\epsilon+\psi+\phi+2\nu)\mu + (\phi+\nu)(\epsilon+\psi+\nu) - (\lambda\varsigma\beta\phi\theta/\kappa+\nu)$.the coefficients $(\epsilon+\psi+\phi+2\nu)$ and $(\phi+\nu)(\epsilon+\psi+\nu) - (\lambda\varsigma\beta\phi\theta/\kappa+\nu)$ are positive, for the DFE case, the value of $\mathbb{R}_0$ should be less than 1. So that the Rough-Hurtwiz condition is satisfied for the assumptions presented if and only if $(\epsilon+\psi+\phi+2\nu) > 0$, $(\phi+\nu)(\epsilon+\psi+\nu) - (\lambda\varsigma\beta\phi\theta/\kappa+\nu) > 0$ and $(\epsilon+\psi+\phi+2\nu)(\phi+\nu)(\epsilon+\psi+\nu) - (\lambda\varsigma\beta\phi\theta/\kappa+\nu) > 0$. Thus, the Rough-Hurtwiz condition promise the local asymptotic stability of the $\mathbb{SMA}$ system presented (2) at DFE E_0. The aforesaid results were achieved utilizing the FF framework as described and utilized in [30]. □

3.4. Endemic equilibrium and their stability. The endemic equilibria of the $\mathbb{SMA}$ (2) designated by $E_1 = (\mathbf{S}^*, \mathbf{E}^*, \mathbf{A}^*, \mathbf{R}^*, \mathbf{Q}^*)$ and the outcome indicated in (2) are presented in this subsection as

$$\begin{aligned}
\mathbf{S}^* &= \frac{(\nu+\epsilon+\psi)(\nu+\phi)}{\lambda\beta\phi\theta}, \\
\mathbf{E}^* &= \frac{(\nu+\epsilon+\psi)\left(\varsigma\beta\phi\lambda\theta(\varrho+\nu) - (\kappa+\nu)(\epsilon\phi(\varrho+\nu)+\epsilon\varrho\nu) - (\psi+\nu)(\kappa+\nu)(\nu+\phi)(\nu+\varrho) - \epsilon\nu^2(\nu+\kappa)\right)}{\lambda\beta\phi\theta(\beta\phi\gamma\varrho(\psi+\nu) - \phi\gamma\varrho(\epsilon+\varrho+\nu) + (\epsilon+\psi+\nu)(\nu+\phi)(\varrho+\nu))}, \\
\mathbf{A}^* &= \frac{\left(\varsigma\beta\phi\lambda\theta(\varrho+\nu) - (\kappa+\nu)(\epsilon\phi(\varrho+\nu)+\epsilon\varrho\nu) - (\psi+\nu)(\kappa+\nu)(\nu+\phi)(\nu+\varrho) - \epsilon\nu^2(\nu+\kappa)\right)}{\lambda\theta(\beta\phi\gamma\varrho(\psi+\nu) - \phi\gamma\varrho(\epsilon+\varrho+\nu) + (\epsilon+\psi+\nu)(\nu+\phi)(\varrho+\nu))}, \\
\mathbf{R}^* &= \frac{((\alpha-1)(\nu+\psi)-\epsilon)(\varsigma\beta\lambda\phi\theta - (\nu+\phi)(\kappa+\nu)(\nu+\epsilon+\psi))}{\lambda\beta\theta(\beta\phi\gamma\varrho(\psi+\nu) - \phi\gamma\varrho(\epsilon+\varrho+\nu) + (\epsilon+\psi+\nu)(\nu+\phi)(\varrho+\nu))}, \\
\mathbf{Q}^* &= \frac{\kappa}{\nu}\left(\frac{(\nu+\epsilon+\psi)(\nu+\phi)}{\lambda\beta\phi\theta}\right) + \frac{\varrho(1-\gamma)}{\nu}\,\frac{((\alpha-1)(\nu+\psi)-\epsilon)(\varsigma\beta\lambda\phi\theta - (\nu+\phi)(\kappa+\nu)(\nu+\epsilon+\psi))}{\lambda\beta\theta(\beta\phi\gamma\varrho(\psi+\nu) - \phi\gamma\varrho(\epsilon+\varrho+\nu) + (\epsilon+\psi+\nu)(\nu+\phi)(\varrho+\nu))}.
\end{aligned} \tag{37}$$

Theorem 3.5. *The $\mathbb{SMA}$ presented by (2) has the following assertions:*

(a) *if $\mathbb{R}_0 > 1$, then system (2) exhibits unique endemic equilibrium.*
(b) *if $\mathbb{R}_0 = 1$, then system (2) has forward bifurcation.*
(c) *if $\mathbb{R}_0 < 1$, then system (2) does not contain the endemic equilibrium point or has backward bifurcation.*

4. The fractal-fractional $\mathbb{SMA}$ model

Here, we used the novel FF methodology in this section to reassemble the classical integer-order $\mathbb{SMA}$ system with a non-singular and nonlocal kernel (2). The $\mathbb{SMA}$ framework that ensues when the FF operator is taken into account is (2).

4.1. Existence-uniqueness outcomes of FF-$\mathbb{SMA}$. Now, the existence-uniqueness of the $\mathbb{SMA}$ obtained in the FF operator are succinctly discussed in (2). To do so, we shall use a FF derivative to generate the generic Cauchy problem:

$$\begin{cases} {}^{FF}\mathbf{D}^{\alpha,\wp}_{0,\zeta}\Lambda(\zeta) = \Upsilon(\zeta,\Lambda(\zeta)), \\ \Lambda(0) = \Lambda_0. \end{cases} \tag{38}$$

In view of Definition (12), the right hand side of (38) yields:

$$\begin{aligned} &\frac{\mathbf{ABC}(\alpha)}{1-\alpha}\frac{d}{d\zeta}\int_0^\zeta \Upsilon(\mathbf{u},\Lambda(\mathbf{u}))\overline{E}_\alpha\left(-\frac{\alpha}{1-\alpha}(\zeta-\mathbf{u})^\alpha\right)d\mathbf{u} \\ &\quad = \wp\zeta^{\wp-1}\Upsilon(\zeta,\Lambda(\zeta)). \end{aligned} \tag{39}$$

Considering the implementation of the appropriate integral, the following conclusions are drawn as:

$$\begin{aligned} \Lambda(\zeta) = {}& \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\Upsilon(\zeta,\Lambda(\zeta)) + \frac{\wp\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \\ & \int_0^\zeta (\zeta-\mathbf{u})^{\alpha-1}\Upsilon(\mathbf{u},\Lambda(\mathbf{u}))\mathbf{u}^{\wp-1}d\mathbf{u} + \Lambda(0). \end{aligned} \tag{40}$$

Employing the Picard-Lindelöf method, we have

$$\prod_{\eta_1}^{\eta_2} = \mathfrak{I}_{\mathbf{p}}(\zeta_{\mathbf{p}}) \times \overline{\mathscr{M}_0(\Lambda_0)}, \tag{41}$$

where $\overline{\mathfrak{I}_{\mathbf{p}}(\zeta_{\mathbf{p}})} = [\zeta_{\mathbf{p}-\mu_1}, \zeta_{\mathbf{p}+\mu_1}]$, $\overline{\mathscr{M}_0(\Lambda_0)} = [\zeta_0 - \nu_1, \zeta_0 + \nu_1]$.

Accordingly, surmise that

$$\hbar = \sup_{\zeta\in\prod_{\eta_1}^{\eta_2}} \|\Upsilon\| \tag{42}$$

Furthermore, the norm is written as follows:

$$\|\mathfrak{I}\|_\infty = \sup_{\zeta\in\prod_{\eta_1}^{\eta_2}} \|\mathfrak{I}\|, \tag{43}$$

and consider the operations

$$\Theta\left[\mathscr{C}\left[\mathfrak{I}_{\mathbf{p}}(\zeta_{\mathbf{p}}), \mathscr{M}_{\mathfrak{b}}(\zeta_{\mathbf{p}})\right]\right] \longrightarrow \mathscr{C}\left(\mathfrak{I}_{\mathbf{p}}(\mathfrak{b}), \mathscr{M}_{\mathfrak{b}}(\zeta_{\mathbf{p}})\right), \tag{44}$$

described as

$$\begin{aligned} \Theta\Upsilon(\zeta) = {}& \Upsilon_0 + \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\Upsilon(\zeta,\Lambda(\zeta)) + \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \\ & \int_0^\zeta (\zeta-\mathbf{u})^{\alpha-1}\Upsilon(\mathbf{u},\Lambda(\mathbf{u}))\mathbf{u}^{\wp-1}d\mathbf{u}. \end{aligned} \tag{45}$$

The essential objective is to illustrate that the aforementioned operator can convert a completely empty metric space onto itself. We also aim to illustrate that it has the potential to map contractions. First and foremost, we show that

$$\begin{aligned} &\left\|\Theta\Lambda(\zeta) - \Lambda_0\right\| \le \mathfrak{b}, \\ &\left\|\Theta\Lambda(\zeta) - \Lambda_0\right\| \le \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\|\Upsilon(\zeta,\Lambda(\zeta))\|_\infty + \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \\ &\quad \int_0^\zeta (\zeta-\mathbf{u})^{\alpha-1}\|\Upsilon(\mathbf{u},\Lambda(\mathbf{u}))\|\mathbf{u}^{\wp-1}d\mathbf{u} \\ &\le \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\mathfrak{I} + \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\mathfrak{I}\int_0^\zeta (\zeta-\mathbf{u})^{\alpha-1}\mathbf{u}^{\wp-1}d\mathbf{u}. \end{aligned} \tag{46}$$

Inserting $\mathbf{u} = \zeta\mathbf{x}$, then produces the foregoing

$$\left\|\Theta\Lambda(\zeta) - \Lambda_0\right\| \le \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\mathfrak{I} + \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\mathfrak{I}\zeta^{\alpha+\wp-1}\mathfrak{B}_1(\wp,\alpha). \tag{47}$$

Therefore,

$$\begin{aligned} &\left\|\Theta\Lambda(\zeta) - \Lambda_0\right\| \le \mathfrak{b} \mapsto \mathfrak{I} < \\ &\frac{\mathfrak{b}\mathfrak{B}_1(\wp,\alpha)}{(1-\alpha/\mathbf{ABC}(\alpha))\wp\zeta^{\wp-1} + (\alpha\wp/\mathbf{ABC}(\alpha)\Gamma(\alpha))\zeta^{\alpha+\wp-1}}. \end{aligned} \tag{48}$$

Then, surmising $\Lambda_1, \Lambda_2 \in \mathscr{C}[\mathfrak{I}_{\mathbf{p}}(\zeta_{\mathbf{p}}), \mathscr{M}_{\mathfrak{b}}(\zeta_{\mathbf{p}})]$. To obtain at the following result, apply the Banach fixed point theorem:

$$\left\|\Theta\Lambda_1 - \Theta\Lambda_2\right\| \le \mathbb{L}_\Omega\left\|\Lambda - \Lambda_2\right\|_\infty, \tag{49}$$

where $\mathbb{L}_\Omega < 1$.

$$\begin{aligned} &\left\|\Theta\Lambda_1 - \Theta\Lambda_2\right\| \le \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\left\|\Upsilon(\zeta,\Lambda_1) - \Upsilon(\zeta,\Lambda_2)\right\| \\ &+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^\zeta (\zeta-\mathbf{u})^{\alpha-1}\mathbf{u}^{\wp-1}\left\|\Upsilon(\zeta,\mathbf{u}_1) - \Upsilon(\zeta,s_2)\right\|d\mathbf{u}, \end{aligned} \tag{50}$$

Owing to the contraction mapping Υ, we have

$$\left\|\Theta\Lambda_1-\Theta\Lambda_2\right\|\leq\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\mathbb{L}_\Lambda\left\|\Lambda_1-\Lambda_2\right\|_\infty+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\mathbb{L}_\Lambda\left\|\Lambda_1-\Lambda_2\right\|_\infty\int_0^\zeta(\zeta-\mathbf{u})^{\alpha-1}\mathbf{u}^{\wp-1}d\mathbf{u}$$
$$\leq\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\mathbb{L}_\Lambda\left\|\Lambda_1-\Lambda_2\right\|_\infty+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\mathbb{L}_\Lambda\left\|\Lambda_1-\Lambda_2\right\|_\infty\zeta^{\alpha+\wp-3}\mathfrak{B}_1(\wp,\alpha). \tag{51}$$

Consequently, we have

$$\left\|\Theta\Lambda_1-\Theta\Lambda_2\right\|\leq\left(\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\mathbb{L}_\Lambda+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\mathbb{L}_\Lambda\zeta^{\alpha+\wp-3}\mathfrak{B}_1(\wp,\alpha)\right)\left\|\Lambda_1-\Lambda_2\right\|_\infty$$
$$<\left(\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\mathbf{a}^{\wp-1}\mathbb{L}_\Lambda+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\mathbb{L}_\Lambda\mathbf{a}^{\alpha+\wp-3}\mathfrak{B}_1(\wp,\alpha)\right)\left\|\Lambda_1-\Lambda_2\right\|_\infty. \tag{52}$$

If the supposition made is correct, then

$$\mathbb{L}_\Lambda<\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\mathbf{a}^{\wp-1}\mathbb{L}_\Lambda+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\mathbb{L}_\Lambda\mathbf{a}^{\alpha+\wp-3}\mathfrak{B}_1(\wp,\alpha), \tag{53}$$

then the contraction criterion is achieved, i.e.,

$$\left\|\Theta\Lambda_1-\Theta\Lambda_2\right\|\leq\left\|\Lambda_1-\Lambda_2\right\|_\infty. \tag{54}$$

In a nutshell, the proof is completed by demonstrating that there is only one solution.

In the next, we describes the numerical solutions for the proposed $\mathbb{SMA}$ system.

4.2. Newton polynomial approach. Here, we configure a comprehensive analysis of the numerical approach, which relies on an efficient Newton polynomial method. This methodology, which was also originally envisioned in [34], is more effective than some of the previous methods available in the analysis. To continue further with the approach, we apply the equation:

$${}^{\mathbf{FF}}\mathbf{D}_\zeta^{\alpha,\wp}\Lambda(\zeta)=\Upsilon(\zeta,\Lambda(\zeta)). \tag{55}$$

Integrating (55) with respect to ζ, produces

$$\Lambda(\zeta)-\Lambda(0)=\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta^{\wp-1}\Upsilon(\zeta,\Lambda(\zeta))+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^\zeta(\zeta-\mathbf{u})^{\alpha-1}\mathbf{u}^{\wp-1}\Upsilon(\zeta,\Lambda(\zeta))d\mathbf{u}. \tag{56}$$

Taking $\mathscr{W}(\zeta,\Lambda(\zeta))=\wp\zeta^{\wp-1}\Upsilon(\zeta,\Lambda(\zeta))$, then (56) reduces to

$$\Lambda(\zeta)-\Lambda(0)=\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\mathscr{W}(\zeta,\Lambda(\zeta))+\frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^\zeta(\zeta-\mathbf{u})^{\alpha-1}\mathscr{W}(\mathbf{u},\Lambda(\mathbf{u}))d\mathbf{u}. \tag{57}$$

At $\zeta_{\mathbf{p}+1}=(n+1)\Delta\zeta$, we have

$$\Lambda\left(\zeta_{\mathbf{p}+1}\right)-\Lambda(0)=\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\mathscr{W}\left(\zeta_\mathbf{p},\Lambda\left(\zeta_\mathbf{p}\right)\right)+\frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta_{\mathbf{p}+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}\mathscr{W}(\mathbf{u},\Lambda(\mathbf{u}))d\mathbf{u}. \tag{58}$$

Therefore, we have

$$\Lambda\left(\zeta_{\mathbf{p}+1}\right)=\Lambda(0)+\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\mathscr{W}\left(\zeta_\mathbf{p},\Lambda\left(\zeta_\mathbf{p}\right)\right)+\frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\sum_{\iota=2}^{\mathbf{p}}\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}\mathscr{W}(\mathbf{u},\Lambda(\mathbf{u}))d\mathbf{u}. \tag{59}$$

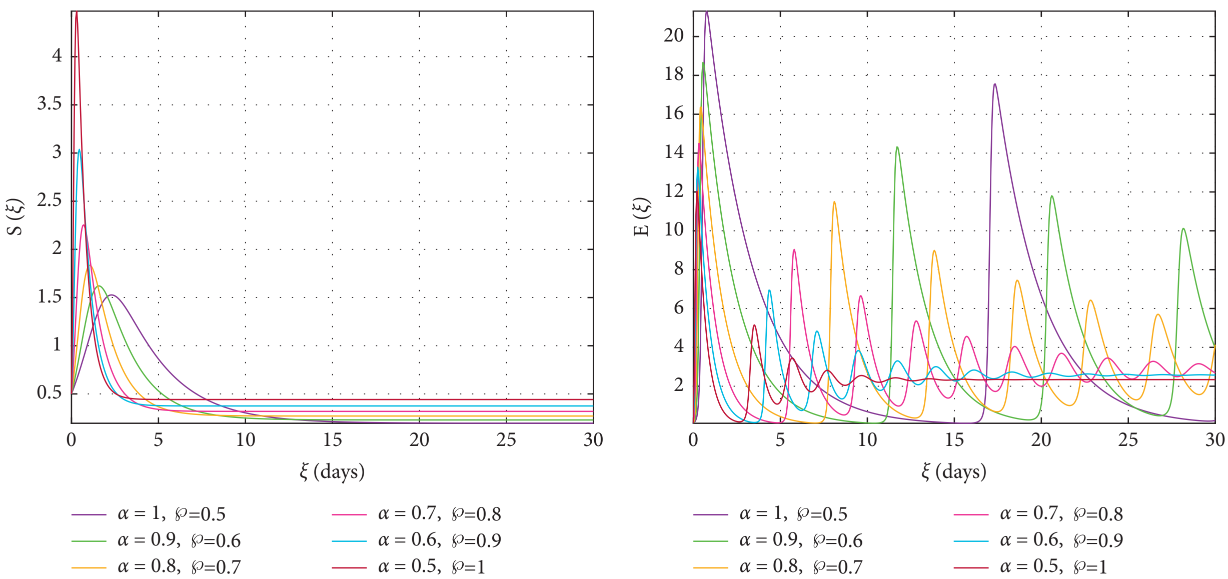
To estimate the mapping, employ the Newton polynomial $\mathscr{W}(\zeta,\Lambda(\zeta))$, we have

$$\mathscr{U}_\mathbf{p}(\mathbf{u})=\mathscr{W}\left(\zeta_{\mathbf{p}-2},\Lambda\left(\zeta_{\mathbf{p}-2}\right)\right)+\frac{\mathscr{W}\left(\zeta_{\mathbf{p}-1},\Lambda\left(\zeta_{\mathbf{p}-1}\right)\right)-\mathscr{W}\left(\zeta_{\mathbf{p}-2},\Lambda\left(\zeta_{\mathbf{p}-2}\right)\right)}{\Delta\zeta}\left(\mathbf{u}-\zeta_{\mathbf{p}-2}\right)$$
$$+\frac{\mathscr{W}\left(\zeta_\mathbf{p},\Lambda\left(\zeta_\mathbf{p}\right)\right)-2\mathscr{W}\left(\zeta_{\mathbf{p}-1},\Lambda\left(\zeta_{\mathbf{p}-1}\right)\right)+\mathscr{W}\left(\zeta_{\mathbf{p}-2},\Lambda\left(\zeta_{\mathbf{p}-2}\right)\right)}{2(\Delta\zeta)^2}\left(\mathbf{u}-\zeta_{\mathbf{p}-2}\right)\left(\mathbf{u}-\zeta_{\mathbf{p}-1}\right). \tag{60}$$

Substituting (60) into (57), yields

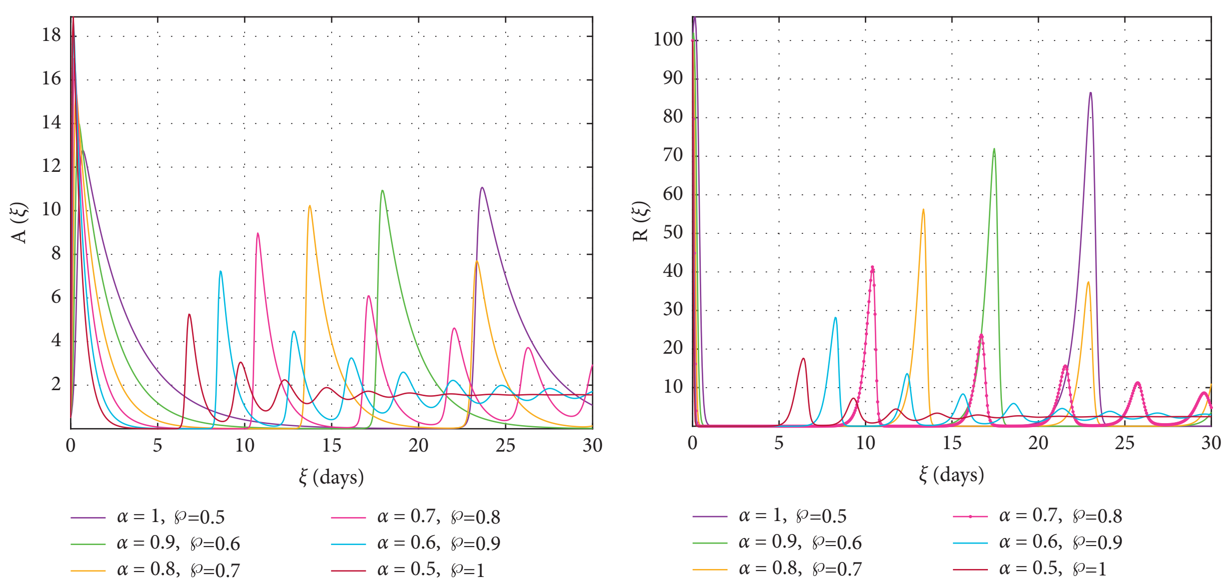
$$\Lambda^{\mathbf{p}+1}=\Lambda^0+\frac{1-\alpha}{\mathbf{ABC}(\alpha)}\mathscr{W}\left(\zeta_\mathbf{p},\Lambda\left(\zeta_\mathbf{p}\right)\right)+\frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\sum_{\iota=2}^{\mathbf{p}}\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}\left(\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)\right.$$
$$+\frac{\mathscr{W}\left(\zeta_{\ell-1},\Lambda^{\ell-1}\right)-\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)}{\Delta\zeta}\left(\mathbf{u}-\zeta_{\ell-2}\right)$$
$$\left.+\frac{\mathscr{W}\left(\zeta_\ell,\Lambda^\ell\right)-2\mathscr{W}\left(\zeta_{\ell-1},\Lambda^{\ell-1}\right)+\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)}{2(\Delta\zeta)^2}\left(\mathbf{u}-\zeta_{\ell-2}\right)\left(\mathbf{u}-\zeta_{\ell-1}\right)\right)d\mathbf{u}. \tag{61}$$

Simple computations yield



(a) (b)

FIGURE 1: (a) Displays of susceptible individuals $\mathbf{S}(\zeta)$ (b) Displays of exposed individuals $\mathbf{E}(\zeta)$ using a Newton polynomial approach for decreasing fractional-order α and increasing fractal-dimension $\wp$.



(a) (b)

FIGURE 2: (a) Displays of addicted individuals $\mathbf{A}(\zeta)$ (b) Displays of recovered individuals $\mathbf{R}(\zeta)$ using a Newton polynomial approach for decreasing fractional-order α and increasing fractal-dimension $\wp$.

$$
\begin{aligned}
\Lambda^{\mathbf{p}+1} &= \Lambda^{0} + \frac{1-\alpha}{\mathbf{ABC}(\alpha)} \mathscr{W}\left(\zeta_{\mathbf{p}}, \Lambda\left(\zeta_{\mathbf{p}}\right)\right) \\
&+ \frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \sum_{\iota=2}^{\mathbf{p}} \left\{ \int_{\zeta_\iota}^{\zeta_{\iota+1}} \left(\zeta_{\mathbf{p}+1} - \mathbf{u}\right)^{\alpha-1} \mathscr{W}\left(\zeta_{\ell-2}, \Lambda^{\ell-2}\right) d\mathbf{u} + \int_{\zeta_\iota}^{\zeta_{\iota+1}} \left(\zeta_{\mathbf{p}+1} - \mathbf{u}\right)^{\alpha-1} \frac{\mathscr{W}\left(\zeta_{\ell-1}, \Lambda^{\ell-1}\right) - \mathscr{W}\left(\zeta_{\ell-2}, \Lambda^{\ell-2}\right)}{\Delta\zeta} \left(\mathbf{u} - \zeta_{\ell-2}\right) d\mathbf{u} \right. \\
&\left. + \int_{\zeta_\iota}^{\zeta_{\iota+1}} \left(\zeta_{\mathbf{p}+1} - \mathbf{u}\right)^{\alpha-1} \frac{\mathscr{W}\left(\zeta_{\ell}, \Lambda^{\ell}\right) - 2\mathscr{W}\left(\zeta_{\ell-1}, \Lambda^{\ell-1}\right) + \mathscr{W}\left(\zeta_{\ell-2}, \Lambda^{\ell-2}\right)}{2(\Delta\zeta)^2} \left(\mathbf{u} - \zeta_{\ell-2}\right)\left(\mathbf{u} - \zeta_{\ell-1}\right) d\mathbf{u} \right\}.
\end{aligned}
\tag{62}
$$

Note that

$$\begin{aligned}\Lambda^{\mathbf{p}+1} &= \Lambda^{0} + \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\mathscr{W}\left(\zeta_{\mathbf{p}},\Lambda\left(\zeta_{\mathbf{p}}\right)\right)\\ &+ \frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\sum_{\iota=2}^{\mathbf{p}}\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}\mathbf{du}\\ &+ \frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\sum_{\iota=2}^{\mathbf{p}}\frac{\mathscr{W}\left(\zeta_{\ell-1},\Lambda^{\ell-1}\right)-\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)}{\Delta\zeta}\\ &\quad\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}\left(\mathbf{u}-\zeta_{\ell-2}\right)\mathbf{du}\\ &+ \frac{\alpha}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\sum_{\iota=2}^{\mathbf{p}}\frac{\mathscr{W}\left(\zeta_{\ell},\Lambda^{\ell}\right)-2\mathscr{W}\left(\zeta_{\ell-1},\Lambda^{\ell-1}\right)+\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)}{2(\Delta\zeta)^2}\\ &\quad\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}\left(\mathbf{u}-\zeta_{\ell-2}\right)\left(\mathbf{u}-\zeta_{\ell-1}\right)\mathbf{du}.\end{aligned} \tag{63}$$

Using the fact that

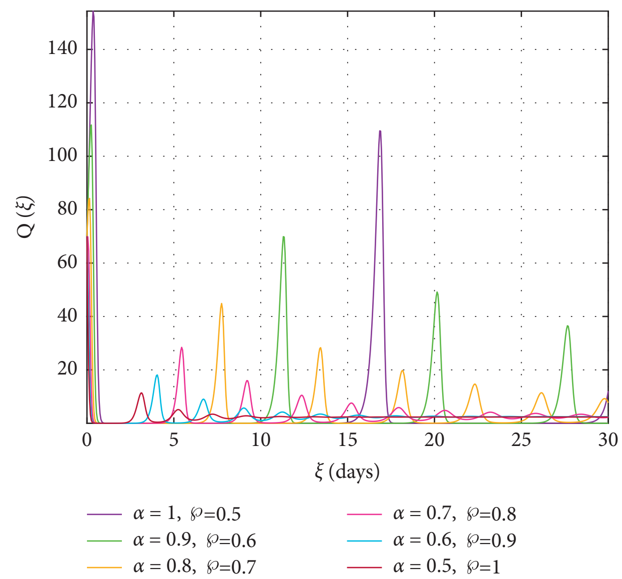


FIGURE 3: Displays of individuals who quit or not using $\mathbf{Q}(\zeta)$ social media using a Newton polynomial approach for decreasing fractional-order α and increasing fractal-dimension $\wp$.

$$\begin{aligned}&\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}d\mathbf{u} = \frac{(\Delta\zeta)^{\alpha}\left\{(\mathbf{p}-\iota+1)^{\alpha}-(\mathbf{p}-\iota)^{\alpha}\right\}}{\alpha},\\ &\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\mathbf{u}-\zeta_{\iota-2}\right)\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}d\mathbf{u} = \frac{(\Delta\zeta)^{\alpha+1}\left\{(\mathbf{p}-\iota+1)^{\alpha}(\mathbf{p}-\iota+2\alpha+3)-(\mathbf{p}-\iota+1)^{\alpha}(\mathbf{p}-\iota+3\alpha+3)\right\}}{\alpha(\alpha+1)},\\ &\int_{\zeta_\iota}^{\zeta_{\iota+1}}\left(\zeta_{\mathbf{p}+1}-\mathbf{u}\right)^{\alpha-1}\left(\mathbf{u}-\zeta_{\ell-2}\right)\left(\mathbf{u}-\zeta_{\ell-1}\right)d\mathbf{u} = \frac{(\Delta\zeta)^{\alpha+2}}{\alpha(\alpha+1)(\alpha+2)}\\ &\times\left\{(\mathbf{p}-\iota+1)^{\alpha}\left[2(\mathbf{p}-\iota)^2+(3\alpha+10)(\mathbf{p}-\iota)+2\alpha^2+9\alpha+12\right]-(\mathbf{p}-\iota)^{\alpha}\left[2(\mathbf{p}-\iota)^2+(5\alpha+10)(\mathbf{p}-\iota)+6\alpha^2+18\alpha+12\right]\right\}.\end{aligned} \tag{64}$$

Furthermore, we have

$$\begin{aligned}\Lambda_{\mathbf{p}+1} &= \Lambda_0 + \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\mathscr{W}\left(\zeta_{\mathbf{p}},\Lambda\left(\zeta_{\mathbf{p}}\right)\right)\\ &+ \frac{\alpha(\Delta\zeta)^{\alpha}}{\mathbf{ABC}(\alpha)\Gamma(\alpha+1)}\sum_{\iota=2}^{\mathbf{p}}\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)\left\{(\mathbf{p}-\iota+1)^{\alpha}-(\mathbf{p}-\iota)^{\alpha}\right\}\\ &+ \frac{\alpha(\Delta\zeta)^{\alpha}}{\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\iota=2}^{\mathbf{p}}\left\{\mathscr{W}\left(\zeta_{\ell-1},\Lambda^{\ell-1}\right)-\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)\right\}\\ &\times\left\{(\mathbf{p}-\iota+1)^{\alpha}(\mathbf{p}-\iota+2\alpha+3)-(\mathbf{p}-\iota+1)^{\alpha}(\mathbf{p}-\iota+3\alpha+3)\right\}\\ &+ \frac{\alpha(\Delta\zeta)^{\alpha}}{2\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\iota=2}^{\mathbf{p}}\left\{\mathscr{W}\left(\zeta_{\ell},\Lambda^{\ell}\right)-2\mathscr{W}\left(\zeta_{\ell-1},\Lambda^{\ell-1}\right)+\mathscr{W}\left(\zeta_{\ell-2},\Lambda^{\ell-2}\right)\right\}\\ &\times\left\{(\mathbf{p}-\iota+1)^{\alpha}\left[2(\mathbf{p}-\iota)^2+(3\alpha+10)(\mathbf{p}-\iota)+2\alpha^2+9\alpha+12\right]\right.\\ &\quad\left.-(\mathbf{p}-\iota)^{\alpha}\left[2(\mathbf{p}-\iota)^2+(5\alpha+10)(\mathbf{p}-\iota)+6\alpha^2+18\alpha+12\right]\right\}.\end{aligned} \tag{65}$$

Therefore, a general approximate solution of the $\mathbb{SMA}$ is as follows:

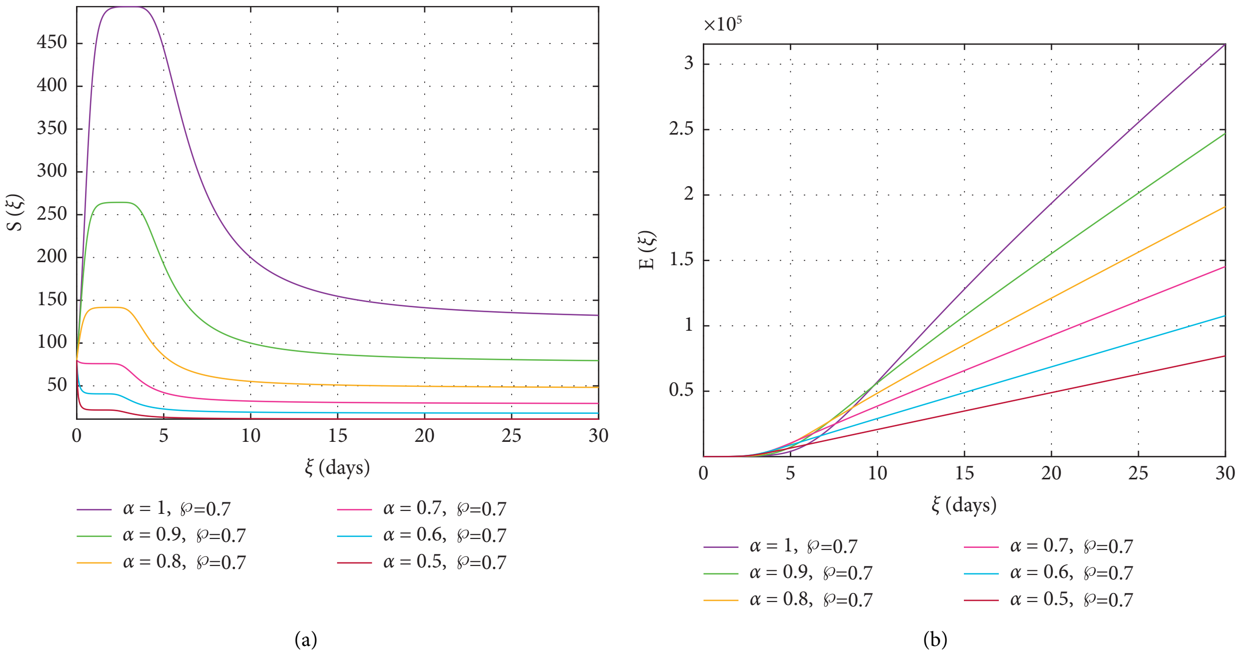


FIGURE 4: (a) Displays of susceptible individuals $\mathbf{S}(\zeta)$ (b) Displays of exposed individuals $\mathbf{E}(\zeta)$ using a Newton polynomial approach for decreasing fractional-order α and fractal-dimension $\wp = 0.7$.

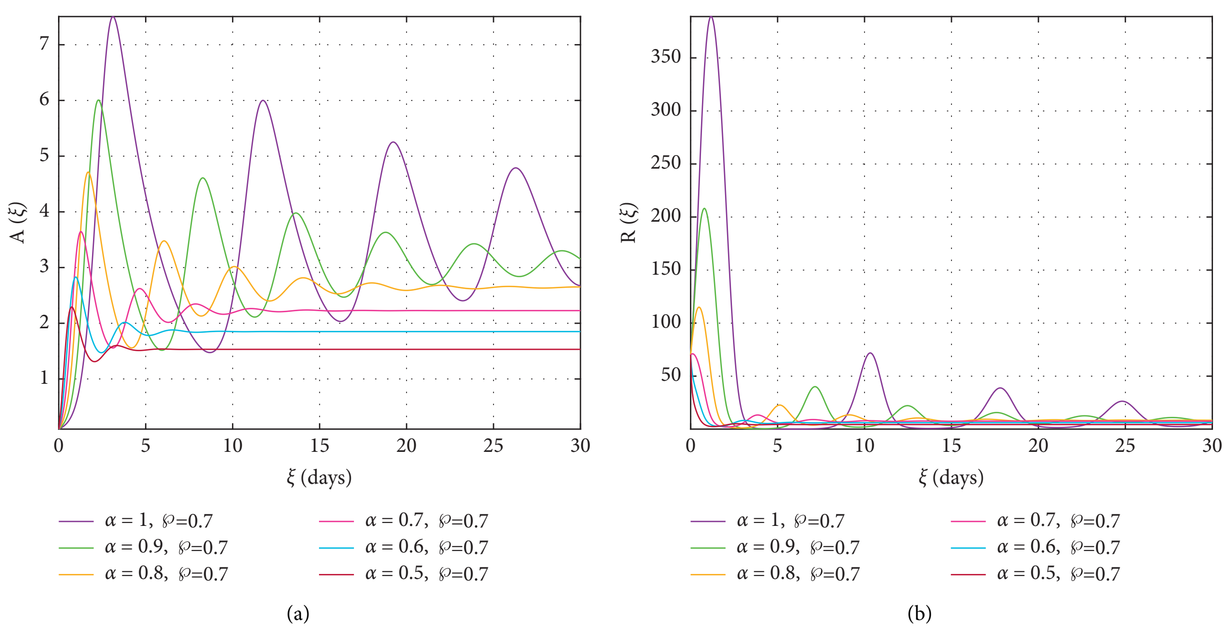


FIGURE 5: (a) Displays of addicted individuals $\mathbf{A}(\zeta)$ (b) Displays of recovered individuals $\mathbf{R}(\zeta)$ using a Newton polynomial approach for decreasing fractional-order α and fractal-dimension $\wp = 0.7$.

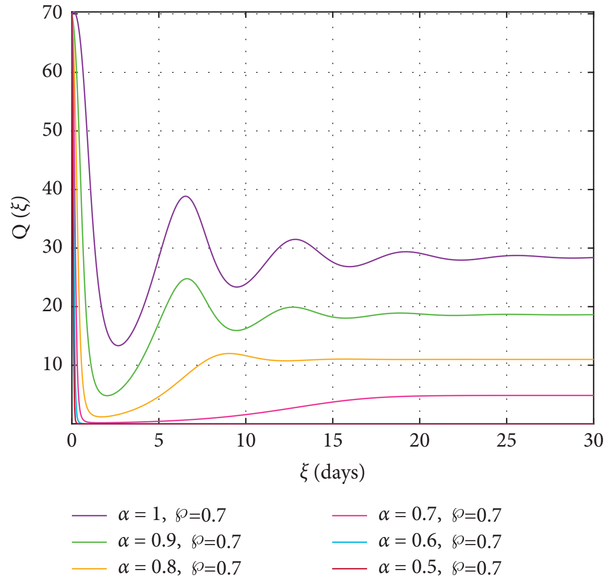


FIGURE 6: Displays of individuals who quit or not using $\mathbf{Q}(\zeta)$ using a Newton polynomial approach for decreasing fractional-order α and fractal-dimension $\wp = 0.7$.

$$\begin{aligned}
\Lambda_{\mathbf{p}+1} &= \Lambda_0 + \frac{1-\alpha}{\mathbf{ABC}(\alpha)}\wp\zeta_{\mathbf{p}}^{\wp-1}\mathscr{W}\left(\zeta_{\mathbf{p}}, \Lambda\left(\zeta_{\mathbf{p}}\right)\right.\\
&+ \frac{\alpha(\Delta\zeta)^{\alpha}}{\mathbf{ABC}(\alpha)\Gamma(\alpha+1)}\sum_{\iota=2}^{\mathbf{p}}\wp\zeta_{\iota-2}^{\wp-1}\mathscr{W}\left(\zeta_{\ell-2}, \Lambda^{\ell-2}\right)\left\{(\mathbf{p}-\iota+1)^{\alpha}-(\mathbf{p}-\iota)^{\alpha}\right\}\\
&+ \frac{\wp\alpha(\Delta\zeta)^{\alpha}}{\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\iota=2}^{\mathbf{p}}\left\{\zeta_{\iota-1}^{\wp-1}\mathscr{W}\left(\zeta_{\ell-1}, \Lambda^{\ell-1}\right)-\zeta_{\iota-2}^{\wp-1}\mathscr{W}\left(\zeta_{\ell-2}, \Lambda^{\ell-2}\right)\right\}\\
&\times\left\{(\mathbf{p}-\iota+1)^{\alpha}(\mathbf{p}-\iota+2\alpha+3)-(\mathbf{p}-\iota+1)^{\alpha}(\mathbf{p}-\iota+3\alpha+3)\right\}\\
&+ \frac{\wp\alpha(\Delta\zeta)^{\alpha}}{2\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\iota=2}^{\mathbf{p}}\left\{\zeta_{\iota}^{\wp-1}\mathscr{W}\left(\zeta_{\ell}, \Lambda^{\ell}\right)-2t_{\iota-1}^{\wp-1}\mathscr{W}\left(\zeta_{\ell-1}, \Lambda^{\ell-1}\right)+\zeta_{\iota-2}^{\wp-1}\mathscr{W}\left(\zeta_{\ell-2}, \Lambda^{\ell-2}\right)\right\}\\
&\times\left\{(\mathbf{p}-\iota+1)^{\alpha}\left[2(\mathbf{p}-\iota)^2+(3\alpha+10)(\mathbf{p}-\iota)+2\alpha^2+9\alpha+12\right]-(\mathbf{p}-\iota)^{\alpha}\left[2(\mathbf{p}-\iota)^2+(5\alpha+10)(\mathbf{p}-\iota)+6\alpha^2+18\alpha+12\right]\right\}.
\end{aligned} \tag{66}$$

4.3. New numerical technique for $\mathbb{SMA}$ *FF-AB derivative model.* The objective of this task is to provide a structured approach technique for interacting with the (1) social media framework, using the FF operator in the Atangana-Baleanu context. converting the (2) system to the FF-Atangana-Baleanu derivative configuration as follows:

$$\begin{aligned}
&{}^{ABR}\mathbf{D}_{0,\zeta}^{\alpha,\wp}(\mathbf{S}(\zeta)) = \wp\zeta^{\wp-1}\mathscr{F}_1(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta), {}^{ABR}\mathbf{D}_{0,\zeta}^{\alpha,\wp}(\mathbf{E}(\zeta)) = \wp\zeta^{\wp-1}\mathscr{F}_2(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta), {}^{ABR}\mathbf{D}_{0,\zeta}^{\alpha,\wp}(\mathbf{A}(\zeta)) = \wp\zeta^{\wp-1}\mathscr{F}_3(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta), {}^{ABR}\mathbf{D}_{0,\zeta}^{\alpha,\wp}(\mathbf{R}(\zeta)) = \wp\zeta^{\wp-1}\mathscr{F}_4(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta),\\
&{}^{ABR}\mathbf{D}_{0,\zeta}^{\alpha,\wp}(\mathbf{Q}(\zeta)) = \wp\zeta^{\wp-1}\mathscr{F}_5(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta).
\end{aligned} \tag{67}$$

Employing the AB fractional integral operator, the preceding conclusions were made as

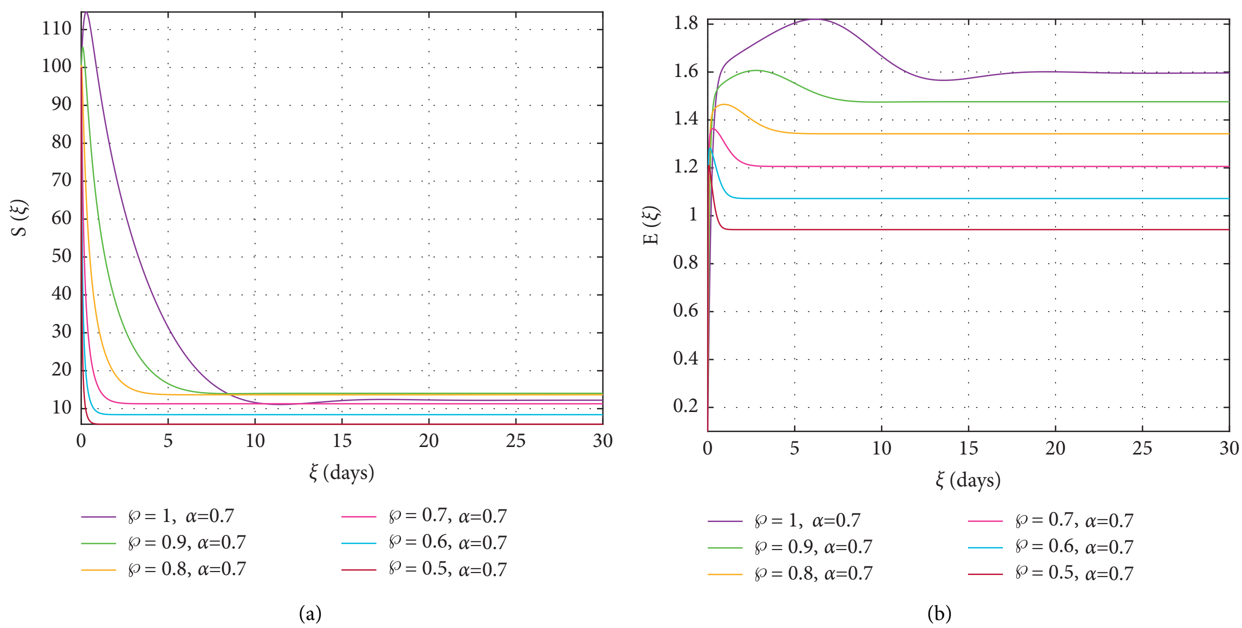


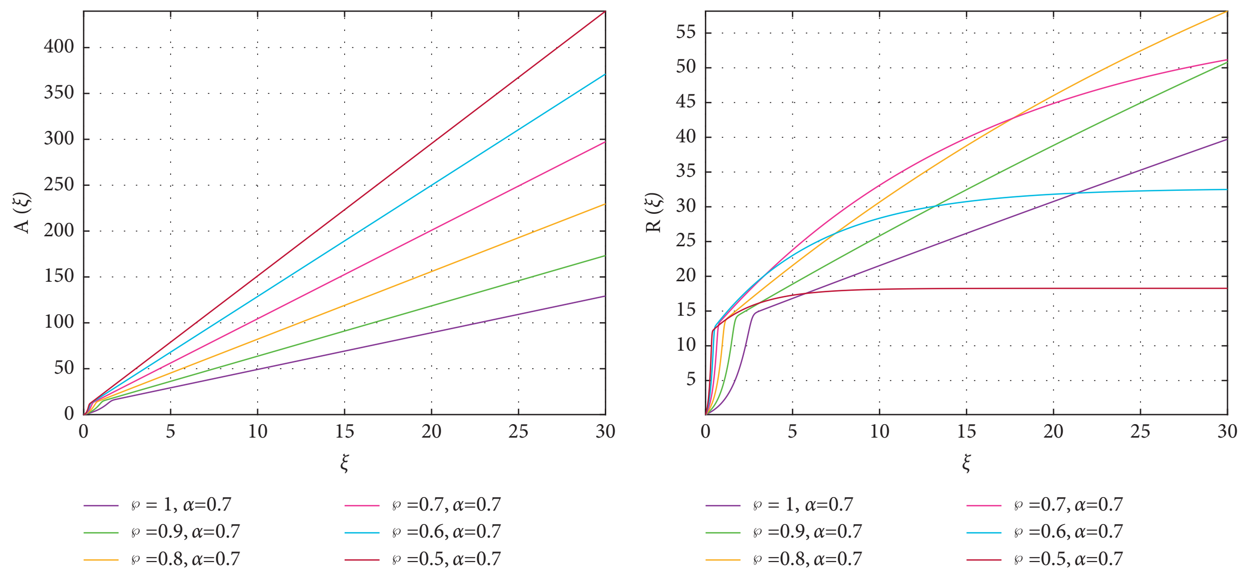
FIGURE 7: (a) Displays of susceptible individuals $\mathbf{S}(\zeta)$ (b) Displays of exposed individuals $\mathbf{E}(\zeta)$ using a Newton polynomial approach for decreasing fractal-dimension $\wp$ and fixed $\alpha = 0.7$.

$$
\begin{aligned}
\mathbf{S}(\zeta) &= \mathbf{S}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_1(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta}\xi^{\wp-1}(\zeta-\xi)^{\alpha-1}\mathscr{F}_1(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)d\xi,\\
\mathbf{E}(\zeta) &= \mathbf{E}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_2(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta}\xi^{\wp-1}(\zeta-\xi)^{\alpha-1}\mathscr{F}_2(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)d\xi,\\
\mathbf{A}(\zeta) &= \mathbf{A}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_3(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta}\xi^{\wp-1}(\zeta-\xi)^{\alpha-1}\mathscr{F}_3(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)d\xi,\\
\mathbf{R}(\zeta) &= \mathbf{R}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_4(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta}\xi^{\wp-1}(\zeta-\xi)^{\alpha-1}\mathscr{F}_4(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)d\xi,\\
\mathbf{Q}(\zeta) &= \mathbf{Q}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_5(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta}\xi^{\wp-1}(\zeta-\xi)^{\alpha-1}\mathscr{F}_5(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\zeta)d\xi.
\end{aligned}
\tag{68}
$$

At ζ_{n+1}, We obtain the below

$$
\begin{aligned}
\mathbf{S}^{n+1}(\zeta) &= \mathbf{S}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_1(\mathbf{S}^n,\mathbf{E}^n,\mathbf{A}^n,\mathbf{R}^n,\mathbf{Q}^n,\zeta_n)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta_{n+1}}\xi^{\wp-1}(\zeta_{n+1}-\xi)^{\alpha-1}\mathscr{F}_1(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\xi)d\xi,\\
\mathbf{E}^{n+1}(\zeta) &= \mathbf{E}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_2(\mathbf{S}^n,\mathbf{E}^n,\mathbf{A}^n,\mathbf{R}^n,\mathbf{Q}^n,\zeta_n)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta_{n+1}}\xi^{\wp-1}(\zeta_{n+1}-\xi)^{\alpha-1}\mathscr{F}_2(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\xi)d\xi,\\
\mathbf{A}^{n+1}(\zeta) &= \mathbf{A}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_3(\mathbf{S}^n,\mathbf{E}^n,\mathbf{A}^n,\mathbf{R}^n,\mathbf{Q}^n,\zeta_n)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta_{n+1}}\xi^{\wp-1}(\zeta_{n+1}-\xi)^{\alpha-1}\mathscr{F}_3(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\xi)d\xi,\\
\mathbf{R}^{n+1}(\zeta) &= \mathbf{R}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_4(\mathbf{S}^n,\mathbf{E}^n,\mathbf{A}^n,\mathbf{R}^n,\mathbf{Q}^n,\zeta_n)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta_{n+1}}\xi^{\wp-1}(\zeta_{n+1}-\xi)^{\alpha-1}\mathscr{F}_4(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\xi)d\xi,\\
\mathbf{Q}^{n+1}(\zeta) &= \mathbf{Q}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_5(\mathbf{S}^n,\mathbf{E}^n,\mathbf{A}^n,\mathbf{R}^n,\mathbf{Q}^n,\zeta_n)\\
&+\frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)}\int_0^{\zeta_{n+1}}\xi^{\wp-1}(\zeta_{n+1}-\xi)^{\alpha-1}\mathscr{F}_5(\mathbf{S},\mathbf{E},\mathbf{A},\mathbf{R},\mathbf{Q},\xi)d\xi.
\end{aligned}
\tag{69}
$$

(69) has been modified significantly, yielding the following results:



(a) (b)

Figure 8: (a) Displays of addicted individuals $\mathbf{S}(\zeta)$ (b) Displays of recovered individuals $\mathbf{E}(\zeta)$ using a Newton polynomial approach for decreasing fractal-dimension $\wp$ and fixed $\alpha = 0.7$.

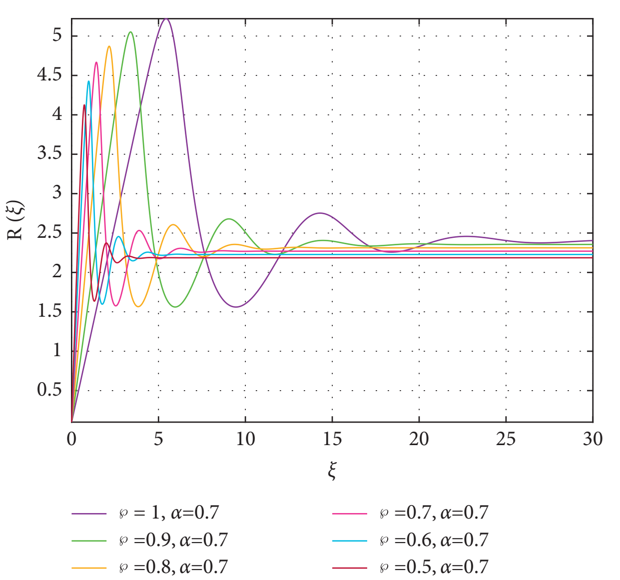


Figure 9: Displays of individuals who quit or not using $\mathbf{Q}(\zeta)$ employing the Newton polynomial approach for decreasing fractal-dimension $\wp$ and fixed $\alpha = 0.7$.

$$
\begin{aligned}
\mathbf{S}^{n+1}(\zeta) &= \mathbf{S}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathcal{F}_1\left(\mathbf{S}^n, \mathbf{E}^n, \mathbf{A}^n, \mathbf{R}^n, \mathbf{Q}^n, \zeta_n\right) \\
&+ \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \sum_{\ell=0}^{n} \int_{\zeta_\ell}^{\zeta_{\ell+1}} \xi^{\wp-1}\left(\zeta_{n+1}-\xi\right)^{\alpha-1}\mathcal{F}_1(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}, \xi)d\xi, \\
\mathbf{E}^{n+1}(\zeta) &= \mathbf{E}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathcal{F}_2\left(\mathbf{S}^n, \mathbf{E}^n, \mathbf{A}^n, \mathbf{R}^n, \mathbf{Q}^n, \zeta_n\right) \\
&+ \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \sum_{\ell=0}^{n} \int_{\zeta_\ell}^{\zeta_{\ell+1}} \xi^{\wp-1}\left(\zeta_{n+1}-\xi\right)^{\alpha-1}\mathcal{F}_2(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}, \xi)d\xi, \\
\mathbf{A}^{n+1}(\zeta) &= \mathbf{A}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathcal{F}_3\left(\mathbf{S}^n, \mathbf{E}^n, \mathbf{A}^n, \mathbf{R}^n, \mathbf{Q}^n, \zeta_n\right) \\
&+ \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \sum_{\ell=0}^{n} \int_{\zeta_\ell}^{\zeta_{\ell+1}} \xi^{\wp-1}\left(\zeta_{n+1}-\xi\right)^{\alpha-1}\mathcal{F}_3(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}, \xi)d\xi, \\
\mathbf{R}^{n+1}(\zeta) &= \mathbf{R}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathcal{F}_4\left(\mathbf{S}^n, \mathbf{E}^n, \mathbf{A}^n, \mathbf{R}^n, \mathbf{Q}^n, \zeta_n\right) \\
&+ \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \sum_{\ell=0}^{n} \int_{\zeta_\ell}^{\zeta_{\ell+1}} \xi^{\wp-1}\left(\zeta_{n+1}-\xi\right)^{\alpha-1}\mathcal{F}_4(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}, \xi)d\xi, \\
\mathbf{Q}^{n+1}(\zeta) &= \mathbf{Q}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathcal{F}_5\left(\mathbf{S}^n, \mathbf{E}^n, \mathbf{A}^n, \mathbf{R}^n, \mathbf{Q}^n, \zeta_n\right) \\
&+ \frac{\alpha\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha)} \sum_{\ell=0}^{n} \int_{\zeta_\ell}^{\zeta_{\ell+1}} \xi^{\wp-1}\left(\zeta_{n+1}-\xi\right)^{\alpha-1}\mathcal{F}_5(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}, \xi)d\xi.
\end{aligned}
\tag{70}
$$

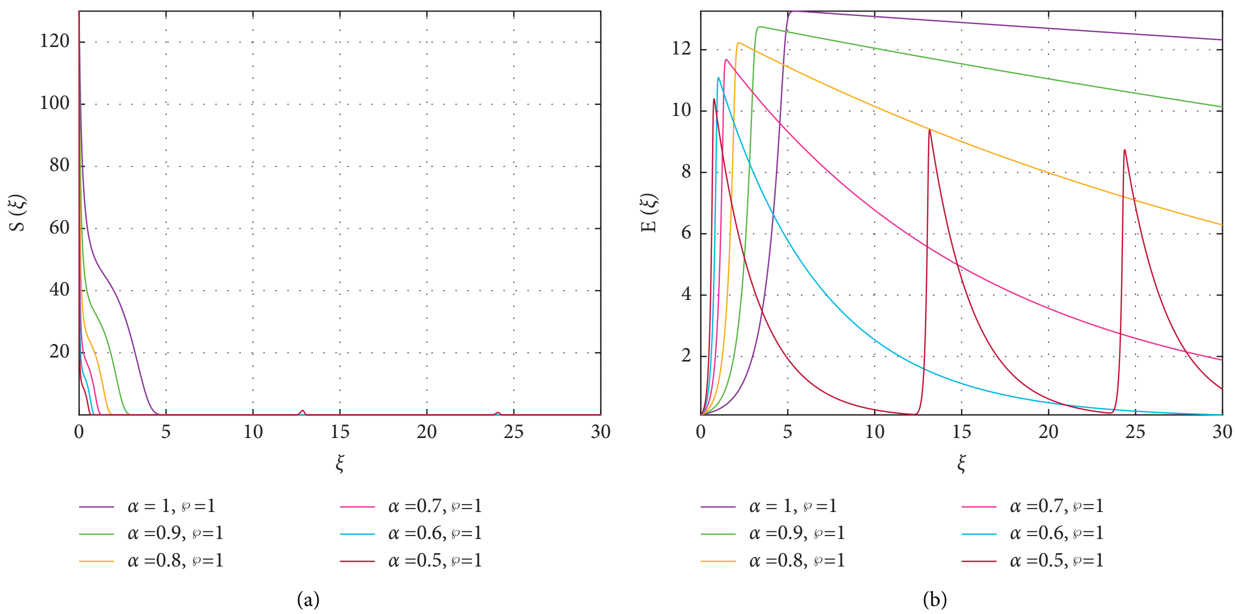


FIGURE 10: (a) Displays of susceptible individuals $\mathbf{S}(\zeta)$ (b) Displays of exposed individuals $\mathbf{E}(\zeta)$ using an another numerical approach for decreasing fractional-order α and fixed fractal-dimension $\wp = 1$.

Moreover, implementing $\xi^{\alpha-1}\mathscr{F}_{\iota}(\mathbf{S}, \mathbf{E}, \mathbf{A}, \mathbf{R}, \mathbf{Q}, \xi)for\iota = 1, 2, \ldots, 5$ to describe the expressions in (70), in the defined interval $[\zeta_{\ell}, \zeta_{\ell+1}]$, the relevant numerical strategy is constructed as

$$
\begin{aligned}
\mathbf{S}^{\mathfrak{n}+1}(\zeta) &= \mathbf{S}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_1(\mathbf{S}^{\mathfrak{n}}, \mathbf{E}^{\mathfrak{n}}, \mathbf{A}^{\mathfrak{n}}, \mathbf{R}^{\mathfrak{n}}, \mathbf{Q}^{\mathfrak{n}}, \zeta_{\mathfrak{n}}) \\
&+ \frac{(\Delta\zeta)^{\alpha}\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\ell=0}^{\mathfrak{n}}\Big\{\Big\{\zeta_{\ell}^{\alpha-1}\mathscr{F}_1\big(\mathbf{S}^{\ell}, \mathbf{E}^{\ell}, \mathbf{A}^{\ell}, \mathbf{R}^{\ell}, \mathbf{Q}^{\ell}, \zeta_{\ell}\big)\Big\} \times ((\mathfrak{n}+1-\ell)^{\alpha}(\mathfrak{n}-\ell+2\alpha) - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+2+2\alpha)) - \Big\{\zeta_{\ell-1}^{\wp-1}\mathscr{F}_1\big(\mathbf{S}^{\ell-1}, \mathbf{E}^{\ell-1}, \mathbf{A}^{\ell-1}, \mathbf{R}^{\ell-1}, \mathbf{Q}^{\ell-1}, \zeta_{\ell-1}\big)\Big\} \times \big((\mathfrak{n}+1-\ell)^{\alpha+1} - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+1+\alpha)\big)\Big\}, \\
\mathbf{E}^{\mathfrak{n}+1}(\zeta) &= \mathbf{E}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_2(\mathbf{S}^{\mathfrak{n}}, \mathbf{E}^{\mathfrak{n}}, \mathbf{A}^{\mathfrak{n}}, \mathbf{R}^{\mathfrak{n}}, \mathbf{Q}^{\mathfrak{n}}, \zeta_{\mathfrak{n}}) \\
&+ \frac{(\Delta\zeta)^{\alpha}\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\ell=0}^{\mathfrak{n}}\Big\{\Big\{\zeta_{\ell}^{\alpha-1}\mathscr{F}_2\big(\mathbf{S}^{\ell}, \mathbf{E}^{\ell}, \mathbf{A}^{\ell}, \mathbf{R}^{\ell}, \mathbf{Q}^{\ell}, \zeta_{\ell}\big)\Big\} \times ((\mathfrak{n}+1-\ell)^{\alpha}(\mathfrak{n}-\ell+2\alpha) - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+2+2\alpha)) - \Big\{\zeta_{\ell-1}^{\wp-1}\mathscr{F}_2\big(\mathbf{S}^{\ell-1}, \mathbf{E}^{\ell-1}, \mathbf{A}^{\ell-1}, \mathbf{R}^{\ell-1}, \mathbf{Q}^{\ell-1}, \zeta_{\ell-1}\big)\Big\} \times \big((\mathfrak{n}-\ell+1)^{\alpha+1} - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+1+\alpha)\big)\Big\}, \\
\mathbf{A}^{\mathfrak{n}+1}(\zeta) &= \mathbf{A}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_3(\mathbf{S}^{\mathfrak{n}}, \mathbf{E}^{\mathfrak{n}}, \mathbf{A}^{\mathfrak{n}}, \mathbf{R}^{\mathfrak{n}}, \mathbf{Q}^{\mathfrak{n}}, \zeta_{\mathfrak{n}}) \\
&+ \frac{(\Delta\zeta)^{\alpha}\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\ell=0}^{\mathfrak{n}}\Big\{\Big\{\zeta_{\ell}^{\alpha-1}\mathscr{F}_3\big(\mathbf{S}^{\ell}, \mathbf{E}^{\ell}, \mathbf{A}^{\ell}, \mathbf{R}^{\ell}, \mathbf{Q}^{\ell}, \zeta_{\ell}\big)\Big\} \times ((\mathfrak{n}+1-\ell)^{\alpha}(\mathfrak{n}-\ell+2\alpha) - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+2+2\alpha)) - \Big\{\zeta_{\ell-1}^{\wp-1}\mathscr{F}_3\big(\mathbf{S}^{\ell-1}, \mathbf{E}^{\ell-1}, \mathbf{A}^{\ell-1}, \mathbf{R}^{\ell-1}, \mathbf{Q}^{\ell-1}, \zeta_{\ell-1}\big)\Big\} \times \big((\mathfrak{n}-\ell+1)^{\alpha+1} - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+1+\alpha)\big)\Big\}, \\
\mathbf{R}^{\mathfrak{n}+1}(\zeta) &= \mathbf{R}(0) + \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_4(\mathbf{S}^{\mathfrak{n}}, \mathbf{E}^{\mathfrak{n}}, \mathbf{A}^{\mathfrak{n}}, \mathbf{R}^{\mathfrak{n}}, \mathbf{Q}^{\mathfrak{n}}, \zeta_{\mathfrak{n}}) \\
&+ \frac{(\Delta\zeta)^{\alpha}\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\ell=0}^{\mathfrak{n}}\Big\{\Big\{\zeta_{\ell}^{\alpha-1}\mathscr{F}_4\big(\mathbf{S}^{\ell}, \mathbf{E}^{\ell}, \mathbf{A}^{\ell}, \mathbf{R}^{\ell}, \mathbf{Q}^{\ell}, \zeta_{\ell}\big)\Big\} \times ((\mathfrak{n}+1-\ell)^{\alpha}(\mathfrak{n}-\ell+2\alpha) - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+2+2\alpha)) - \Big\{\zeta_{\ell-1}^{\wp-1}\mathscr{F}_4\big(\mathbf{S}^{\ell-1}, \mathbf{E}^{\ell-1}, \mathbf{A}^{\ell-1}, \mathbf{R}^{\ell-1}, \mathbf{Q}^{\ell-1}, \zeta_{\ell-1}\big)\Big\} \times \big((\mathfrak{n}-\ell+1)^{\alpha+1} - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+1+\alpha)\big)\Big\},
\end{aligned}
\tag{71}
$$

$$
\begin{aligned}
\mathbf{Q}^{\mathfrak{n}+1}(\zeta) = \mathbf{Q}(0) &+ \frac{\wp\zeta^{\wp-1}(1-\alpha)}{\mathbf{ABC}(\alpha)}\mathscr{F}_5(\mathbf{S}^{\mathfrak{n}}, \mathbf{E}^{\mathfrak{n}}, \mathbf{A}^{\mathfrak{n}}, \mathbf{R}^{\mathfrak{n}}, \mathbf{Q}^{\mathfrak{n}}, \zeta_{\mathfrak{n}}) + \frac{(\Delta\zeta)^{\alpha}\wp}{\mathbf{ABC}(\alpha)\Gamma(\alpha+2)}\sum_{\ell=0}^{\mathfrak{n}}\Big\{\Big\{\zeta_{\ell}^{\alpha-1}\mathscr{F}_5\big(\mathbf{S}^{\ell}, \mathbf{E}^{\ell}, \mathbf{A}^{\ell}, \mathbf{R}^{\ell}, \mathbf{Q}^{\ell}, \zeta_{\ell}\big)\Big\} \times ((\mathfrak{n}+1-\ell)^{\alpha}(\mathfrak{n}-\ell+2\alpha) - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+2+2\alpha)) \\
&- \Big\{\zeta_{\ell-1}^{\wp-1}\mathscr{F}_5\big(\mathbf{S}^{\ell-1}, \mathbf{E}^{\ell-1}, \mathbf{A}^{\ell-1}, \mathbf{R}^{\ell-1}, \mathbf{Q}^{\ell-1}, \zeta_{\ell-1}\big)\Big\} \times \big((\mathfrak{n}-\ell+1)^{\alpha+1} - (\mathfrak{n}-\ell)^{\alpha}(\mathfrak{n}-\ell+1+\alpha)\big)\Big\}.
\end{aligned}
\tag{72}
$$

5. Numerical results and description

In this part, we exhibit simulation results for the FF derivative operator for the $\mathbb{SMA}$ model (2) assuming the numerical methods proposed by [34], as mentioned previously. Such numeric findings are obtained using nonnegative factors, as indicated in Table 1. We examine a FF $\mathbb{SMA}$ model, including the Atangana-Baleanu approach in the Caputo context, to demonstrate the reliability and usefulness of the new efficient and Newton polynomial approach. We can effortlessly acquire approximate methods

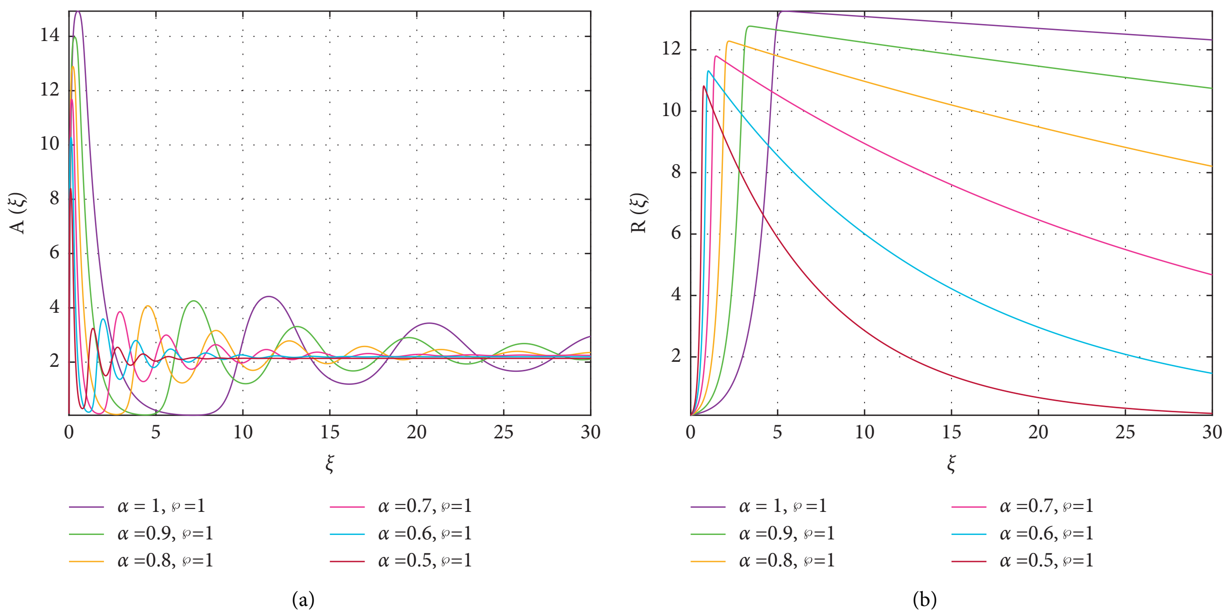


Figure 11: (a) Displays of addicted individuals $\mathbf{S}(\zeta)$ (b) Displays of recovered individuals $\mathbf{E}(\zeta)$ using a another numerical approach for decreasing fractional-order α and fixed fractal-dimension $\wp = 1$.

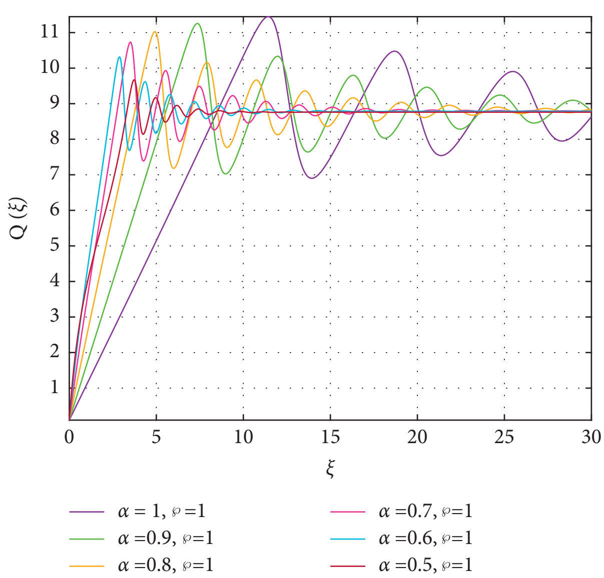


Figure 12: Displays of individuals who quit or not using $\mathbf{Q}(\zeta)$ applying an another numerical approach for decreasing fractional-order α and fixed fractal-dimension $\wp = 1$.

for it using new efficient and Newton polynomial approach. The primary goal of this review is to identify individuals who are vulnerable and exposed to various fractional Brownian patterns as well as classical motion.The simulation results of the scheme (2) for various Brownian motions are $\alpha = 1.0, 0.9, 0.8, 0.7, 0.6, 0.5$ are presented in Figure11(a)-1(b), Figure 2(a)-2(b) and Figure 3(a)-3(b). As seen in Figure 1(a), the susceptible group diminishes as fractional orders increase and reach 1 while increasing the fractal-dimension $\wp$, and finally remains consistent including all Brownian motion at $\mathbf{S} = 8.1212$. Furthermore, when $\mathbf{E} = 0.0450$ and $\mathbf{A} = 0.0104$, Figure 1(b) and Figure 2(a) illustrate that the unprotected and addictive groups rise dramatically and decline, respectively, exhibiting behaviour of different fractional orders eventually reaching 1. When $\mathbf{R} = 0.0236$ and $\mathbf{Q} = 1.7517$, Figure 2(b) and Figure 3 indicate that the community of individuals who are rehabilitated and consistently do not utilize and discontinue using SM significantly improves and drops, respectively, using multiple fractional-orders changes, reaching 1.

In a similar way, we can discuss the behaviour of Figure 4(a)-4(b), Figure 5(a)-5(b) and Figure 6(a)-6(b), by incorporating the values of $\lambda = 0.5$, the induced mortality rate of 0.05 and $\kappa = 0.06$ via the FF operator in the Atangana-Baleanu sense. In this case, the graphical representation has a lower fractional-order α while keeping the fractal-dimension $\wp = 0.7$ constant. The fundamental target of this article is that subtle improvements in the fractional derivative order have no impact on the general behaviour of the consequent structures; simply simulation studies are altered.

Figure 7(a)-7(b), Figure 8(a)-8(b) and Figure 9(a)-9(b) demonstrates that the percentage of unprotected people appears to have decreased in the first two years, but affected people revert to utilizing social platforms owing to the scheme's ineffectiveness via FF derivative operator in the Atangana-Baleanu sense. In this case, the graphical representation has a lower fractal-dimension $\wp$ while retaining a constant fractional-order $\alpha = 0.7$. As a result, fighting $\mathbb{SMA}$ in the community is ineffective.

As shown in Figure 10(a)-10(b), Figure 11(a)-11(b) and Figure 12(a)-12(b), the proportion of affected and intoxicated people was diminished when the method was used against no approach within the FF derivative operator in the Atangana-Baleanu sense. The graphical illustration in this approach has a lower fractional-order α while maintaining a constant fractal-dimension $\wp = 1$. The proposed technique appears to be efficacious in diminishing dependency load during the implementation, and hence can be considered an ideal contender for managing the stress of $\mathbb{SMA}$.

6. Conclusion

In this investigation, we presented a mathematical framework for the prevalence and distribution of the $\mathbb{SMA}$ model, including the fractal-fractional derivative operator in the Atangana-Baleanu sense. According to the analysis, the system's disease-free equilibrium is locally asymptotically stable when the $\mathbb{R}_0 < 1$, but generally unstable. The stability of equilibria points was explored utilizing $\mathbb{R}_0$. Bifurcation investigation shows that the model demonstrates forward bifurcation at $\mathbb{R}_0 = 1$. Furthermore, this study proposes two effective mathematical approaches for numerically solving a fractional SMA model in the fractal-fractional derivatives perspective. One initial approach relies on product integration formulation, while the other is focused on the numeric Newton polynomial approach. When the obtained simulations from the two methodologies are compared, it is clear that their respective behaviour in fixing the challenges is remarkably analogous. The findings demonstrate that our generated results are in close harmony with the precise outcomes. In a simulation study, the responses from the two different techniques exhibit the same behaviour for fractional order and fractal-dimension. Such mathematical approaches can also be leveraged to generate analytical results for other complex scientific systems of any complexity. The numerical behaviours produced by the aforesaid approaches are perfectly compatible with the model's projected rational behaviour. [30–34].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

S. Rashid provided the main ideas of the article, constructed the main algorithm and proved the convergence, also submitted the article. R. Ashraf drafted the manuscript and provided qualitative analysis. E. Bonyah provided the solution and illustrations of the final revision. All authors read and approved the final manuscript.

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