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Research Article

Investigation of Blockchain Technology by Using the Innovative Concepts of Complex Pythagorean Fuzzy Soft Information

Naeem Jan,[1] Jeonghwan Gwak ,[1,2,3,4] Younghoon Jeon,[1] and Bushra Akram[5]

[1]*Department of Biomedical Engineering, Korea National University of Transportation, Chungju 27469, Republic of Korea*
[2]*Department of Software, Korea National University of Transportation, Chungju 27469, Republic of Korea*
[3]*Department of AI Robotics Engineering, Korea National University of Transportation, Chungju 27469, Republic of Korea*
[4]*Department of IT & Energy Convergence (BK21 Four), Korea National University of Transportation, Chungju 27469, Republic of Korea*
[5]*Department of Mathematics, Institute of Numerical Sciences, Gomal University, KPK, Dera Ismail Khan 29050, Pakistan*

Correspondence should be addressed to Jeonghwan Gwak; jgwak@ut.ac.kr

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A blockchain is a valuable and proficient type of digital ledger technology that involves of expanding list of records, called blocks, that are strongly connected simultaneously using cryptography. Further, complex Pythagorean fuzzy sets (CPFSs) are the generalized form of the intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PyFSs), and complex intuitionistic fuzzy sets (CIFSs), used for evaluating the awkward and unreliable information in genuine life problems. In this analysis, we aim to diagnose the innovative idea of complex Pythagorean fuzzy soft relations (CPyFSRs) by using the Cartesian product (CP) of two complex Pythagorean fuzzy soft sets (CPyFSSs), which are computed with the help of two different ideas, called complex Pythagorean fuzzy relation and soft sets. Additionally, using the presented approaches, we examined different kinds of relations and also justified them with the help of some suitable examples. The CPyFSRs has a comprehensive structure because it is discussing both degrees of membership and non-membership with multidimensional variable. Further, includes the CPyFSR-based modeling techniques that use the score function to choose the best blockchain technology (BCT) to enhance the worth of the evaluated information. Using a good BCT, the transaction may be simply transferred record between users. Finally, the benefit of this proposed framework is demonstrated by comparing it to other frameworks to show the supremacy and feasibility of the diagnosed approaches.

1. Introduction

The uncertainty involved in any problem-solving situation is a result of some information inadequacy. Uncertainty is a natural part of our life. Many everyday decisions are highly unpredictable. It frequently happens when there is not enough information available regarding the results, the future environment is unpredictable, and everything is unstable. A novel mathematical innovation called fuzzy set (FS) was presented by Zadeh [1] in 1965 for detecting and resolving ambiguity. Each element in this collection is given a membership degree between 0 and 1, which represents the element's quality or effectiveness. The FS is more important in human decision-making. Zimmermann [2] proposed the FS theory and its applications. Maiers and Sherif [3] use of FSs theory apply to a wide range of issues and fuzzy control techniques. Roberts [4] explained ordination based on FS theory. Kahraman [5] used FS in industrial engineering. Mendel [6] proposed the concept of fuzzy relationships (FRs). FRs used the membership degree of every element to indicate the quality of the relationship. If membership is closer to 1 then it indicates a good relationship and if the degree closer to 0 shows poor relationships. The FRs are an extended structure than classical relations. Nemitz [7] goes into much detail about FRs and fuzzy functions. Yang and Shih [8] designated the cluster analysis based on FRs. After

FS, Ramot et al. [9] nominated the new set called complex fuzzy set (CFS) that explains membership ranging between unit circles. It defines membership using two terms: amplitude which describes effectiveness, and phase which describes the duration of effectiveness. It lowers the likelihood of errors and ambiguity. Hu et al. [10] developed the orthogonality relation of CFSs. Li and Tu [11] examined CFSs and their applications in multi-class prediction. Zhang et al. [12] explored the various operating features and δ-equalities of CFSs. Moreover, he also defines complex fuzzy relations (CFRs). Khan et al. [13] established the CFRs in the future commission market.

After all of these advancements in human decision-making, people can become confused while deciding on the best alternative. There are numerous doubts and ambiguities in this situation. Molodtsov [14] examined the idea of the soft set (SS) in 1999, which helps people make better decisions in difficult situations. SS chooses the items based on some parameters. Ali et al. [15] developed some novel SSs operations. Kostek [16] used an SS approach to analyze sound quality. Mushrif et al. [17] suggested a new SS theory-based technique for evaluating natural textures. Maji et al. [18] employed an SS theory to resolve a decision-making difficulty. Babitha and Sunil [19] introduced the soft relations (SRs) between the CP of SSs. Park et al. [20] studied some features of equivalence SRs. Maji et al. [21] created the fuzzy soft set (FSS) by merging the FS and the SS. It helps humans make better decisions by reducing uncertainty in daily life decisions. Kong et al. [22] employed FSS in decision-making issues. Gogoi et al. [23] looked into how FSS theory may be used to solve various difficulties. Borah et al. [24] established the innovative idea of fuzzy soft relations (FSRs) by examining the CP of FSSs. Mockor and Hurtik [25] used image processing to approximate FSSs using FSRs. Thirunavukarasu et al. [26] looked into the novel idea of complex fuzzy soft sets (CFSSs) in which the degree of membership is expressed in complex numbers and sorted out all the problems by using multi-variables. TAmir et al. [27] analyzed an outline of CFS and complex fuzzy logic theory and applications.

Atanassov [28] established the idea of an intuitionistic fuzzy set (IFSs), which is broader than the FSs. An IFSs examined both degrees of membership and non-membership, whereas FSs only discussed the membership degree. Both of these values between the unit interval [0, 1] and sum also lie within this interval. Szmidt and Kacprzyk [29] resolved the distances among IFSs; Gerstenkorn and Manko [30] determined the IFS correlation. Alkouri [31] defined the notion of the complex intuitionistic fuzzy set (CIFS). The CIFS uses a complex number to define both membership and non-membership degrees. It consists of both amplitude term and phase term. Ngan et al. [32] used quaternion numbers to represent CIFS and applied them in decision-making. Xu et al. [33] nominate the intuitionistic fuzzy soft set (IFSS), which combines the SS and IFS. The IFSS is the expansion form of the FSS. Agarwal et al. [34] invented the modified IFSS with applications in decision-making. Dinda and Samanta [35] used the CP of IFSS to recommend the intuitionistic fuzzy soft relation (IFSR). Kumar and Bajaj [36] evaluated the concept of complex intuitionistic fuzzy soft sets (CIFSSs), which are parametric. The CIFSSs are used to apply parametrization tools to explain multicriteria decision-making issues. Yager [37] proposed Pythagorean fuzzy sets (PyFS), which increased the space by imposing new constraints. The constraint of PyFS is that the total of the squares of membership and non-membership degrees must be in the range [0, 1]. Garg [38] applied PyFS in the form of new logarithmic operational laws. Ullah et al. [39] suggested the thought of a complex Pythagorean fuzzy set (CPyFS) with application in pattern recognition. The CPyFS provides membership and non-membership values as a complex number. Dick et al. [40] described the CPyFS operations. Nasir et al. [41] used economic relationships to define the concept of a complex Pythagorean fuzzy relation (CPyFR). Peng et al. [42] presented the Pythagorean fuzzy soft set (PyFSS), by merging the SS with the PyFS and interpreted this notion through various possible applications. Akram et al. [43] introduced the complex Pythagorean fuzzy soft set (CPyFSS) with the application. Gillpatrick et al. [44] evaluated the blockchain contribute to developing country economies.

To expose the significance and proficiency of the evaluated theories by comparing them with other prevailing theories, for this, we demonstrated it with the help of some genuine life examples. Assume an enterprise N decided to purchase some new cars from a carmaker, for this the owner of the enterprise N provided two types of information regarding each car: (a) model of cars; (b) making the date of cars. Very carmaker produced the same model of car with some improvements or upgrading based on some parameters (like improving the quality of the fuel consumption, tire quality, comfort zone, etc.) in every new year. Where the model of the car expressed the amplitude term, and the production date of the car shows the phase term which changes time by time continuously. Traditionally PFS or Pythagorean fuzzy soft sets are not able to deal with it. For this, the theory of CPyFSR is much better than the prevailing theories. Because the theory of CPyFSR deals with two-dimension information at a time and the because of this reason, the IFS, PyFS, and CIFS are special cases of the proposed work. The concept of CPyFSS is a convenient tool in CIFSS theory for dealing with ambiguity and uncertainty. However, the concept of relations has not yet been defined for the CPyFSS. Based on our observation, the main analyses of this analysis are listed below:

(1) To propose the concept of CPFSRs by studying the CP of two CPyFSS.

(2) To describe different types of CPyFSR as well as the CPyFS-reflexive relation, CPyFS-symmetric relation, CPyFS-transitive relation, CPyFS-equivalence relation, CPyFS-partial order relation, CPyFS-linear order relation, CPyFS-strict order relation, CPyFS-converse relation, CPyFS-composite relation and many more. Each CPyFSR definition has been illustrated with examples.

(3) To illustrate numerous results for the type of CPyFSRs.

(4) To derive the innovative idea of CPyFSR is superior to pre-defined structures of SS, FSS, CFSS, IFSS, CIFSS, and PyFSS. The CPyFSS discussed both membership and non-membership degrees with increased space. They can also solve problems with multi-variables due to complex-valued mappings. Additionally, offered an application for selecting the best BCT by using CPyFSRs. The score function has been utilized to choose the best BCT. Experts have recommended a variety of parameters and selected the finest BCT based on those criteria.

(5) To compare the presented work with some prevailing work is to show the reliability of the evaluated work.

The rest of this article is arranged as follows: Section 2 contains all pre-existing structures of fuzzy algebra. Section 3 introduced the newly defined notion of CPyFSRs and CP of two CPyFSSs for example. Section 4 proposed an application of BCT by using the study of CPyFSRs. Section 5 compares the proposed structure with pre-existing structure. Section 6 concludes the results.

2. Preliminaries

The theory of CFS, SS, SR, FSS, CFSS, CIFS, IFSS, CIFSS, CPyFS, and CPyFSS are the part of this section which are very useful for evaluating the proposed ideas in next section.

Definitioni 1 (see [9]). Let $\dot{Y}$ be a universal set, then a CFS $\mp$ on $\dot{Y}$ can be defined as:

$$\mp = \{(\dot{s}, m_c(\dot{s})): \dot{s} \in \dot{Y}\}, \quad (1)$$

Where, $m(\dot{s}) = r_{m}(\dot{s})e^{(q_{m}(\dot{s}))2\pi i}$ represented the membership grade with $r_m, q_m: \dot{Y} \longrightarrow [0,1]$. Further, the mathematical terms r_m and q_m are represented the amplitude and phase terms of the membership degree individually.

Definition 2 (see [14].) Let $\dot{Y}$ be a universal set and $\breve{\xi}$ be the set of parameters, $\not{p}(\dot{Y})$ denote the power set of $\dot{Y}$. Then, a pair $(\mp, k)$ is called SS on $\dot{Y}$ with mapping $\mp: k \longrightarrow \not{p}(\dot{Y})$ is defined as:

$$\mp = \{\acute{\bar{o}}, \mp(\acute{\bar{o}}), \acute{\bar{o}} \in k, \mp(\acute{\bar{o}}) \in \not{p}(\dot{Y})\}. \quad (2)$$

Example 1. Suppose $\dot{Y}$ is a universal set consisting of the set of five watches $= \{\dot{\alpha}_1, \dot{\alpha}_2, \dot{\alpha}_3, \dot{\alpha}_4, \dot{\alpha}_5\}\dot{Y}$ under consideration, and $\breve{\xi}$ is the set of parameters $\breve{\xi} = \{\acute{\bar{o}}_1, \acute{\bar{o}}_2, \acute{\bar{o}}_3, \acute{\bar{o}}_4\}$ for universal set $\dot{Y}$, where each parameter stands for beautiful, expensive, very beautiful, and cheap individually. Suppose a SS $(\mp, k)$ shows the attractiveness of the watches, such that

$$\mp(\acute{\bar{o}}_1) = \{\dot{\alpha}_1, \dot{\alpha}_3, \dot{\alpha}_5\}\mp(\acute{\bar{o}}_2) = \{\dot{\alpha}_2, \dot{\alpha}_5\}\mp(\acute{\bar{o}}_3) = \{\dot{\alpha}_1, \dot{\alpha}_4\}\mp(\acute{\bar{o}}_4) = \{\dot{\alpha}_3, \dot{\alpha}_4, \dot{\alpha}_5\}. \quad (3)$$

Then, the SS $(\mp, k)$ is a parameterized family, and $\{\mp(\acute{\bar{o}}_i), i = 1,2,3,4\}$.

Definition 3 (see [19]). Let $(\mp, \tilde{A})$ and $(\mathscr{G}, \underline{B})$ be two SSs on $\dot{Y}$ and $\tilde{A}, \underline{B} \subseteq \breve{\xi}$. Then their CP of $(\mp, \tilde{A}) \times (\mathscr{G}, \underline{B}) = (\mathbb{H}, \mathscr{D})$ with a mapping $\mathbb{H}: \mathscr{D} \longrightarrow \not{p}(\dot{Y})$ is defined as:

$$\mathbb{H}(\hat{u}, \dot{u}) = \{(\acute{\bar{o}}_{\hat{u}}, t_{\dot{u}}): \acute{\bar{o}}_{\hat{u}} \in (\mp, \tilde{A}), t_{\dot{u}} \in (\mathscr{J}, \underline{B})\}. \quad (4)$$

Any subset of the CP of two SSs is called SR.

Definition 4 (see [21]). Let $\dot{Y}$ be a universal set and $\breve{\xi}$ be the set of parameters, $\not{p}^{\dot{Y}}$ represents the set of fuzzy subsets of $\dot{Y}$. Then FSS $(\mp, k)$ with mapping $\mp: k \longrightarrow \not{p}^{\dot{Y}}$ is defined as:

$$\mp = \{(\acute{\bar{o}}, m(\acute{\bar{o}})): \acute{\bar{o}} \in k, m(\acute{\bar{o}}) \in \not{p}^{\dot{Y}}\}. \quad (5)$$

Where $m(\acute{\bar{o}})$ is called the membership degree.

Example 2. Let $\dot{Y}$ is the set of LED companies and $\breve{\xi}$ be the set of parameters. The FSS $(\mp, k)$ express the LED characteristics concerning some parameters and each membership degree assigned by the experts. $\dot{Y} = \{\dot{\alpha}_1, \dot{\alpha}_2, \dot{\alpha}_3, \dot{\alpha}_4\}$ i.e., $\dot{\alpha}_1 =$ Orient, $\dot{\alpha}_2 =$ Samsung, $\dot{\alpha}_3 =$Haier, and $\dot{\alpha}_4 =$ Sony.
$\breve{\xi} = \{\acute{\bar{o}}_1, \acute{\bar{o}}_2, \acute{\bar{o}}_3\}$ i.e., $\acute{\bar{o}}_1$ = no electromagnetic radiation, $\acute{\bar{o}}_2$ =Price, and $\acute{\bar{o}}_3$ =higher resolution.

$$\begin{aligned}\mp(\acute{\bar{o}}_1) &= \{\dot{\alpha}_1 = 0.7, \dot{\alpha}_2 = 0.2, \dot{\alpha}_3 = 0.4, \dot{\alpha}_4 = 0.1\}\mp(\acute{\bar{o}}_2)\\ &= \{\dot{\alpha}_1 = 0.9, \dot{\alpha}_2 = 0.8, \dot{\alpha}_3 = 0.3, \dot{\alpha}_4 = 0.6\}\mp(\acute{\bar{o}}_3)\\ &= \{\dot{\alpha}_1 = 0.1, \dot{\alpha}_2 = 0.5, \dot{\alpha}_3 = 0.9, \dot{\alpha}_4 = 0.7\}.\end{aligned} \quad (6)$$

Then $(\mp, k)$ is a parameterized family $\{\mp(\acute{\bar{o}}_i), i = 1,2,3\}$.

Definition 5 (see [26]). Let $\dot{Y}$ be a universal set and $\breve{\xi}$ be the set of parameters, $C(\not{p}^{\dot{Y}})$ express the set of all complex fuzzy subsets of $\dot{Y}$. Then CFSS $(\mp, k)$ with mapping $\mp: k \longrightarrow C(\not{p}^{\dot{Y}})$ is defined as,

$$\mp = \{(\acute{\bar{o}}, m_c(\acute{\bar{o}})): \acute{\bar{o}} \in k, m(\acute{\bar{o}}) \in C(\not{p}^{\dot{Y}})\}. \quad (7)$$

And $m_c(\acute{\bar{o}}) = r_m(\acute{\bar{o}})e^{(q_m(\acute{\bar{o}}))2\pi i}$ Since $r_m, q_m: \dot{Y} \longrightarrow [0,1]$.
Where r_m and q_m are called amplitude terms and phase terms of the membership degree individually.

Definition 6 (see [31]). Let $\dot{Y}$ be a universal set. Then CIFS $\mp$ on $\dot{Y}$ with a mapping $m_c, m_c: \dot{Y} \longrightarrow [0,1]$ is defined as,

$$\mp = \{(\acute{\bar{o}}, m_c(\acute{\bar{o}}), m_c(\acute{\bar{o}})): \acute{\bar{o}} \in \dot{Y}\}. \quad (8)$$

Since $m_c(\acute{\bar{o}}) = r_m(\acute{\bar{o}})e^{q_m(\acute{\bar{o}})2\pi i}$ and $m_c(\acute{\bar{o}}) = r_m(\acute{\bar{o}})e^{q_m(\acute{\bar{o}})2\pi i}$, on condition that, $r_m(\acute{\bar{o}}) + r_m(\acute{\bar{o}}) \in [0,1]$ and $q_m(\acute{\bar{o}}) + q_m(\acute{\bar{o}}) \in [0,1]$.
Where r_m, r_m are known as amplitude terms of membership and non-membership degree individually. q_m, q_m are known as the phase terms of membership and non-membership degree, individually.

Definition 7 (see [33]). Let $\dot{Y}$ be a universal set and $\breve{\xi}$ be the set of parameters, $\not{p}\mp^{\dot{Y}}$ denotes the set of all intuitionistic fuzzy subsets of $\dot{Y}$. Then an IFSS $(\mp, k)$ with mapping $\mp: k \longrightarrow \not{p}\mp^{\dot{Y}}$ is defined as:

$$Ŧ = \left\{ (ő, ɱ_{ᵲ}(ő), ɱ_{ᵲ}(ő)) \colon ő \in ķ, ɱ_{ᵲ}(ő) ɱ_{ᵲ}(ő) \in ṗŦ^{Ẏ} \right\}, \tag{9}$$

Where $ɱ_{ᵲ}(ő), ɱ_{ᵲ}(ő)$ are called membership and non-membership degrees, individually.

Example 3. From example 2, Assume an IFSS $(Ŧ, ķ)$ describe the characteristic of the LED concerning some parameters and each membership and non-membership degree given by experts.

$$\begin{aligned} Ŧ(ő_1) &= \{ ą̇_1 = (0.2, 0.4), ą̇_2 = (0.5, 0.3), ą̇_3 = (0.1, 0.9), ą̇_4 = (0.3, 0.5) \} \\ Ŧ(ő_2) &= \{ ą̇_1 = (0.1, 0.3), ą̇_2 = (0.2, 0.6), ą̇_3 = (0.5, 0.3), ą̇_4 = (0.3, 0.2) \} \\ Ŧ(ő_3) &= \{ ą̇_1 = (0, 0.6), ą̇_2 = (0.4, 0), ą̇_3 = (0.7, 0.1), ą̇_4 = (0.4, 0.5) \}. \end{aligned} \tag{10}$$

Then the IFSS $(Ŧ, ķ)$ is a parameterized family $\{Ŧ(ő_i), i = 1, 2, 3\}$.

Definition 8 (see [36]). Let $Ẏ$ be a universal set and $Ȅ$ be the set of parameters, $C(ṗŦ^{Ẏ})$ denotes the set of all complex intuitionistic fuzzy subsets of $Ẏ$. Then CIFSS $(Ŧ, ķ)$ with mapping $Ŧ\colon ķ \longrightarrow C(ṗŦ^{Ẏ})$ is defined as,

$$Ŧ = \left\{ \left(ő, ɱ_{ᵲc}(ő), ɱ_{ᵲc}(ő) \right) \colon ő \in ķ, ɱ_{ᵲ}(ő), ɱ_{ᵲ}(ő) \in C\left(ṗŦ^{Ẏ} \right) \right\}. \tag{11}$$

Since $ɱ_{ᵲc}(ő) = r_{ɱ_{ᵲ}}(ő) e^{q_{ɱ_{ᵲ}}(ő) 2\pi i}, ɱ_{ᵲc}(ő) = r_{ɱ_{ᵲ}}(ő) e^{q_{ɱ_{ᵲ}}(ő) 2\pi i}$

Definition 9 (see [39]). Let $Ẏ$ be a universal set. Then a CPyFS $Ŧ$ on $Ẏ$ with mapping $ɱ_{ᵲc}, ɱ_{ᵲc}\colon Ẏ \longrightarrow [0, 1]$ is defined as,

$$Ŧ = \left\{ \left(ő, ɱ_{ᵲc}(ő), ɱ_{ᵲc}(ő) \right) \colon ő \in Ẏ \right\}. \tag{12}$$

Since $ɱ_{ᵲc}(ő) = r_{ɱ_{ᵲ}}(ő) e^{q_{ɱ_{ᵲ}}(ő) 2\pi i}$ and $ɱ_{ᵲc}(ő) = r_{ɱ_{ᵲ}}(ő) e^{q_{ɱ_{ᵲ}}(ő) 2\pi i}$ on condition that, $(r_{ɱ_{ᵲ}}(ő))^2 + (r_{ɱ_{ᵲ}}(ő))^2 \in [0, 1]$ and $(q_{ɱ_{ᵲ}}(ő))^2 + (q_{ɱ_{ᵲ}}(ő))^2 \in [0, 1]$.

Where $r_{ɱ_{ᵲ}}, r_{ɱ_{ᵲ}}$ are known as amplitude terms of membership and non-membership degree, individually. $q_{ɱ_{ᵲ}}, q_{ɱ_{ᵲ}}$ is called the phase terms of membership and non-membership degree, individually.

Definition 10 (see [43]). Let $Ẏ$ be a universal set and $Ȅ$ be the set of parameters, $C(Pyṗ^{Ẏ})$ denotes the set of all complex Pythagorean fuzzy subsets of $Ẏ$. Then CPyFSS $(Ŧ, ķ)$ with mapping $Ŧ\colon ķ \longrightarrow C(Pyṗ^{Ẏ})$ is defined as:

$$Ŧ = \left\{ \left(ő, ɱ_{ᵲc}(ő), ɱ_{ᵲc}(ő) \right) \colon ő \in ķ, ɱ_{ᵲ}(ő), ɱ_{ᵲ}(ő) \in C\left(Pyṗ^{Ẏ} \right) \right\}. \tag{13}$$

Since $ɱ_{ᵲc}(ő) = r_{ɱ_{ᵲ}}(ő) e^{q_{ɱ_{ᵲ}}(ő) 2\pi i}$ and $ɱ_{ᵲc}(ő) = r_{ɱ_{ᵲ}}(ő) e^{q_{ɱ_{ᵲ}}(ő) 2\pi i}$.

3. Main Result

In this section, we aim to diagnose the innovative idea of CPyFSRs by using the CP of two CPyFSSs, which are computed with the help of two different ideas, called CPF relation and soft sets. Additionally, using the presented approaches, we examined different kinds of relations and also justified them with the help of some suitable examples. The CPyFSRs has a comprehensive structure because it is discussing both degrees of membership and non-membership with multidimensional variable.

Definition 11. Suppose $(Ŧ, \tilde{A})$ and $(\mathscr{G}, \underline{B})$ be two complex Pythagorean fuzzy soft sets (CPyFSSs) on $Ẏ$, $Ȅ$ be the set of parameters. Let $(Ŧ, \tilde{A}) \times (\mathscr{G}, \underline{B}) = (\mathbb{H}, \mathscr{D})$ and $\tilde{A}, \underline{B} \subseteq Ȅ$ with a mapping $\mathbb{H}\colon \mathscr{D} \longrightarrow C(PyŦ^{Ẏ})$ then the CP of CPyFSSs

$$\tilde{A} = \left\{ \begin{pmatrix} ő, r_{ɱ_{ᵲ}}^{\tilde{A}}(ő) e^{q_{ɱ_{ᵲ}}^{\tilde{A}}(ő) 2\pi i}, \\ r_{ɱ_{ᵲ}}^{\tilde{A}}(ő) e^{q_{ɱ_{ᵲ}}^{\tilde{A}}(ő) 2\pi i} \end{pmatrix} \colon ő \in \tilde{A} \right\} \quad \text{and}$$

$$\underline{B} = \left\{ \begin{pmatrix} ᶖ, r_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) e^{q_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) 2\pi i}, \\ r_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) e^{q_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) 2\pi i} \end{pmatrix} \colon ᶖ \in \underline{B} \right\}$$

Is denoted and defined as,

$$(\mathbb{H}, \mathscr{D}) = \tilde{A} \times \underline{B} = \left\{ \begin{pmatrix} (ő, ᶖ), r_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) e^{q_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) 2\pi i}, \\ r_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) e^{q_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) 2\pi i} \end{pmatrix} \colon ő \in \tilde{A}, ᶖ \in \underline{B} \right\}, \tag{14}$$

Where $\left\{ r_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) = \min\left\{ \mathbf{r}_{ɱ_{ᵲ}}^{\tilde{A}}(ő), r_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) \right\} r_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) = \max\left\{ r_{ɱ_{ᵲ}}^{\tilde{A}}(ő), r_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) \right\} \right\}$

$$\left\{ q_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) = \min\left\{ q_{ɱ_{ᵲ}}^{\tilde{A}}(ő), q_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) \right\} q_{ɱ_{ᵲ}}^{\tilde{A}\times\underline{B}}(ő, ᶖ) = \max\left\{ q_{ɱ_{ᵲ}}^{\tilde{A}}(ő), q_{ɱ_{ᵲ}}^{\underline{B}}(ᶖ) \right\} \right\}. \tag{15}$$

Example 4. Let the universal set $\dot{Y} = \{\dot{q}_1, \dot{q}_2, \dot{q}_3\}$ consist of three types of shoe brands i.e., $\dot{q}_1$ =Bata, $\dot{q}_2$ =Servis, and $\dot{q}_3$ =Metro and there are three parameters $\ddot{E} = \{\acute{o}_1, \acute{o}_2, \acute{o}_3\}$ i.e., $\acute{o}_1$ =Good condition, $\acute{o}_2$ =attrictive appearance, and $\acute{o}_3$ =Stable. Then $(₮, \tilde{A})$ and $(\mathscr{G}, \underline{B})$ be two CPyFSSs on$\dot{Y}$ individually, Their corresponding membership and non-membership are as follows; for $n = 2$.

$$= \left\{ \begin{array}{l} \left(\begin{array}{c} \acute{o}_1, \left(0.4e^{0.6\pi i}, 0.6e^{0.3\pi i}\right), \left(0.5e^{0.2\pi i}, 0.6e^{0.9\pi i}\right), \\ \left(0.4e^{0.1\pi i}, 0.8e^{0.4\pi i}\right), \left(0.8e^{0.8\pi i}, 0.3e^{0.6\pi i}\right) \end{array} \right), \\ \left(\begin{array}{c} \acute{o}_2, \left(0.7e^{0.8\pi i}, 0.2e^{0.5\pi i}\right), \left(0.9e^{0.7\pi i}, 0.2e^{0.1\pi i}\right), \\ \left(0.3e^{0.7\pi i}, 0.6e^{0.4\pi i}\right), \left(0.6e^{0.7\pi i}, 0.3e^{0.5\pi i}\right) \end{array} \right), \\ \left(\begin{array}{c} \acute{o}_3, \left(0.8e^{0.7\pi i}, 0.4e^{0.6\pi i}\right), \left(0.1e^{0.5\pi i}, 0.8e^{0.3\pi i}\right), \\ \left(0.7e^{0.4\pi i}, 0.5e^{0.1\pi i}\right), \left(0.5e^{0.2\pi i}, 0.5e^{0.8\pi i}\right) \end{array} \right), \end{array} \right\} \tag{16}$$

In the above observations, the first three values characterize the membership and non-membership degree of each brand and the fourth value shows the general belongingness of each parameter to the company. Each row represents the parametric observations. Now the CP of $(₮, \tilde{A})$ and $(\mathscr{G}, \underline{B})$ in Table 1 is:

Definition 12. The complex Pythagorean fuzzy soft relations (CPyFSRs) $\bar{R}$ is a subset of the CP of two CPyFSSs.

Example 5. From Table 1, take a subset of the CP. Then the CPyFSR $\bar{R}$ are as:

$$\bar{R} = \left\{ \begin{array}{l} \left((\acute{o}_1, \acute{o}_2), \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.2e^{0.5\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.6e^{0.2\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left((\acute{o}_2, \acute{o}_3), \begin{pmatrix} 0.8e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.3\pi i}, \\ 0.5e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.4\pi i}, \\ 0.7e^{0.8\pi i} \end{pmatrix} \right), \\ \left((\acute{o}_3, \acute{o}_1), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right), \\ \left((\acute{o}_3, \acute{o}_3), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}. \tag{17}$$

Definition 13. Suppose that CPyFSR $\bar{R}$ on $(₮, ķ)$ is said to be CPyFS-inverse relation if

$$\begin{aligned} &\forall\left(\left((\acute{o}, \dot{i})\left[r_{m}(\acute{o}, \dot{i})\right]e^{q_{m}(\acute{o},\dot{i})\pi i}, (\acute{o}, \dot{i})\left[r_{m}(\acute{o}, \dot{i})\right]e^{q_{m}(\acute{o},\dot{i})\pi i}\right)\right) \\ &\in \bar{R} \Leftrightarrow \forall\left(\left((\dot{i}, \acute{o})\left[r_{m}(\dot{i}, \acute{o})\right]e^{q_{m}(\dot{i},\acute{o})\pi i}, (\dot{i}, \acute{o})\left[r_{m}(\dot{i}, \acute{o})\right]e^{q_{m}(\dot{i},\acute{o})\pi i}\right)\right) \in \bar{R}^{-1}. \end{aligned} \tag{18}$$

Example 6. Take a relation from Table 1 as:

Table 1: Cartesian product.

Ordered pair	$\dot{q}_1$	$\dot{q}_2$	$\dot{q}_3$	λ
$(ő_1, ő_1)$	$\begin{pmatrix} 0.4e^{0.6\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{0.2\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.2e^{0.1\pi i}, \\ 0.8e^{0.4\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix}$
$(ő_1, ő_2)$	$\begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.2e^{0.5\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.6e^{0.2\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix}$
$(ő_1, ő_3)$	$\begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.4e^{0.6\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.5e^{0.3\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}$
$(ő_2, ő_1)$	$\begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.6e^{0.8\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.3e^{0.2\pi i}, \\ 0.8e^{0.9\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.8e^{0.9\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.6\pi i} \end{pmatrix}$
$(ő_2, ő_2)$	$\begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix}$
$(ő_2, ő_3)$	$\begin{pmatrix} 0.8e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.1e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{0.3\pi i}, \\ 0.5e^{0.9\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{0.4\pi i}, \\ 0.7e^{0.8\pi i} \end{pmatrix}$
$(ő_3, ő_1)$	$\begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}$
$(ő_3, ő_2)$	$\begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.3e^{0.6\pi i}, \\ 0.3e^{0.3\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.3e^{0.5\pi i}, \\ 0.9e^{0.4\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.7\pi i}, \\ 0.3e^{0.5\pi i} \end{pmatrix}$
$(ő_3, ő_3)$	$\begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}$

$$\bar{\underset{.}{R}} = \left\{ \begin{array}{l} \left((ő_1, ő_3), \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.4e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.5e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right), \\ \left((ő_2, ő_2), \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left((ő_3, ő_2), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.6\pi i}, \\ 0.3e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.5\pi i}, \\ 0.9e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.7\pi i}, \\ 0.3e^{0.5\pi i} \end{pmatrix} \right), \\ \left((ő_3, ő_3), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}. \tag{19}$$

Then inverse relation $\bar{\underset{.}{R}}^{-1}$ of $\bar{\underset{.}{R}}$ is

$$\bar{\underset{.}{R}}^{-1} = \left\{ \begin{array}{l} \left((ő_3, ő_1), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right), \\ \left((ő_2, ő_2), \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left((ő_2, ő_3), \begin{pmatrix} 0.8e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.3\pi i}, \\ 0.5e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.4\pi i}, \\ 0.7e^{0.8\pi i} \end{pmatrix} \right), \\ \left((ő_3, ő_3), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}. \tag{20}$$

Definition 14. Suppose that CPyFSR $\bar{\underset{.}{R}}$ on $(\mp, ķ)$ is known as CPyFS-reflexive relation if

$$\Leftrightarrow \forall \left(\begin{pmatrix} (ő, ő)\left[r_{\mathfrak{m}}(ő, ő) \right] e^{q_{\mathfrak{m}}(ő, ő)\pi i}, \\ \left[r_{\mathfrak{m}}(ő, ő) \right] e^{q_{\mathfrak{m}}(ő, ő)\pi i} \end{pmatrix} \right) \in \bar{\underset{.}{R}}. \tag{21}$$

Example 7. Take a relation from Table 1 as:

$$\bar{\text{Ṛ}} = \left\{ \begin{array}{l} \left((\text{ő}_1, \text{ő}_1), \begin{pmatrix} 0.4e^{0.6\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.2\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1\pi i}, \\ 0.8e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left((\text{ő}_2, \text{ő}_2), \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left((\text{ő}_3, \text{ő}_3), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}, \tag{22}$$

is a CPyFS-reflexive relation $\bar{\text{Ṛ}}$.

Definition 15. Suppose that CPyFSR $\bar{\text{Ṛ}}$ on $(\text{₮}, \text{ķ})$ is known as CPyFS-irreflexive relation if

$$\Leftrightarrow \forall \left(\begin{pmatrix} (\text{ő}, \text{ő}) \left[r_{\text{m}} (\text{ő}, \text{ő}) \right] e^{q_{\text{m}} (\text{ő},\text{ő}) \pi i}, \\ \left[r_{\text{m}} (\text{ő}, \text{ő}) \right] e^{q_{\text{m}} (\text{ő},\text{ő}) \pi i} \end{pmatrix} \right) \notin \bar{\text{Ṛ}}. \tag{23}$$

Definition 16. Suppose that CPyFSR $\bar{\text{Ṛ}}$ on $(\text{₮}, \text{ķ})$ is known as CPyFS-symmetric relation if

$$\Leftrightarrow \forall \left(\begin{pmatrix} (\text{i̧}, \text{ő}) \left[r_{\text{m}} (\text{i̧}, \text{ő}) \right] e^{q_{\text{m}} (\text{i̧},\text{ő}) \pi i}, \\ (\text{i̧}, \text{ő}) \left[r_{\text{m}} (\text{i̧}, \text{ő}) \right] e^{q_{\text{m}} (\text{i̧},\text{ő}) \pi i} \end{pmatrix} \right) \in \bar{\text{Ṛ}}. \tag{24}$$

Definition 17. Suppose that CPyFSR $\bar{\text{Ṛ}}$ on $(\text{₮}, \text{ķ})$ is known as CPyFS-antisymmetric relation if

$$\forall \left((\text{ő}, \text{i̧}), \begin{pmatrix} \left[r_{\text{m}} (\text{ő}, \text{i̧}) \right] e^{q_{\text{m}} (\text{ő},\text{i̧}) \pi i}, \\ \left[r_{\text{m}} (\text{ő}, \text{i̧}) \right] e^{q_{\text{m}} (\text{ő},\text{i̧}) \pi i} \end{pmatrix} \right) \in \bar{\text{Ṛ}}, \tag{25}$$

and $((\text{i̧}, \text{ő}), ([r_{\text{m}} (\text{i̧}, \text{ő})] e^{q_{\text{m}} (\text{i̧},\text{ő}) \pi i}, [r_{\text{m}} (\text{i̧}, \text{ő})]\ e^{q_{\text{m}} (\text{i̧},\text{ő}) \pi i})) \in \bar{\text{Ṛ}}$

$$\Leftrightarrow \left((\text{ő}, \text{i̧}), \begin{pmatrix} \left[r_{\text{m}} (\text{ő}, \text{i̧}) \right] e^{q_{\text{m}} (\text{ő},\text{i̧}) \pi i}, \\ \left[r_{\text{m}} (\text{ő}, \text{i̧}) \right] e^{q_{\text{m}} (\text{ő},\text{i̧}) \pi i} \end{pmatrix} \right) = \left((\text{i̧}, \text{ő}), \begin{pmatrix} \left[r_{\text{m}} (\text{i̧}, \text{ő}) \right] e^{q_{\text{m}} (\text{i̧},\text{ő}) \pi i}, \\ \left[r_{\text{m}} (\text{i̧}, \text{ő}) \right] e^{q_{\text{m}} (\text{i̧},\text{ő}) \pi i} \end{pmatrix} \right). \tag{26}$$

Definition 18. Suppose that CPyFSR $\bar{\text{Ṛ}}$ on $(\text{₮}, \text{ķ})$ is known as CPyFS-transitive relation if

$$\forall \left((\text{ő}, \text{i̧}), \begin{pmatrix} \left[\mathrm{r}_{\text{m}} (\text{ő}, \text{i̧}) \right] \mathrm{e}^{q_{\text{m}} (\text{ő},\text{i̧}) \pi \mathrm{i}}, \\ \left[\mathrm{r}_{\text{m}} (\text{ő}, \text{i̧}) \right] \mathrm{e}^{q_{\text{m}} (\text{ő},\text{i̧}) \pi \mathrm{i}} \end{pmatrix} \right) \in \bar{\text{Ṛ}}, \tag{27}$$

and $((\text{i̧}, \text{ŧ}), ([\mathrm{r}_{\text{m}} (\text{i̧}, \text{ŧ})] \mathrm{e}^{q_{\text{m}} (\text{i̧},\text{ŧ}) \pi \mathrm{i}}, [\mathrm{r}_{\text{m}} (\text{i̧}, \text{ŧ})] \mathrm{e}^{q_{\text{m}} (\text{i̧},\text{ŧ}) \pi \mathrm{i}})) \in \bar{\text{Ṛ}}$

$$\Rightarrow \left((\text{ő}, \text{ŧ}), \begin{pmatrix} \left[\mathrm{r}_{\text{m}} (\text{ő}, \text{ŧ}) \right] \mathrm{e}^{q_{\text{m}} (\text{ő},\text{ŧ}) \pi \mathrm{i}}, \\ \left[\mathrm{r}_{\text{m}} (\text{ő}, \text{ŧ}) \right] \mathrm{e}^{q_{\text{m}} (\text{ő},\text{ŧ}) \pi \mathrm{i}} \end{pmatrix} \right) \in \bar{\text{Ṛ}}. \tag{28}$$

Definition 19. Suppose that CPyFSR $\bar{\text{Ṛ}}$ on $(\text{₮}, \text{ķ})$ is known as CPyFS-equivalence relation if,

(i) Reflexive
(ii) Symmetric
(iii) Transitive

Example 8. Take a relation from Table 1 as:

$$\bar{\underset{\cdot}{R}} = \left\{ \begin{array}{l} \left(\left(\acute{\bar{o}}_1, \acute{\bar{o}}_1 \right), \begin{pmatrix} 0.4e^{0.6\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.2\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1\pi i}, \\ 0.8e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_1, \acute{\bar{o}}_2 \right), \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.2e^{0.5\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.6e^{0.2\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_2, \acute{\bar{o}}_1 \right), \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.6e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2\pi i}, \\ 0.8e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.8e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.6\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_2, \acute{\bar{o}}_2 \right), \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_2, \acute{\bar{o}}_3 \right), \begin{pmatrix} 0.8e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.3\pi i}, \\ 0.5e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.4\pi i}, \\ 0.7e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_3, \acute{\bar{o}}_2 \right), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.6\pi i}, \\ 0.3e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.5\pi i}, \\ 0.9e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.7\pi i}, \\ 0.3e^{0.5\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_3, \acute{\bar{o}}_3 \right), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}, \tag{29}$$

is a CPyFS-equivalence relation $\bar{\underset{\cdot}{R}}$.

Definition 20. Suppose that CPyFSR $\bar{\underset{\cdot}{R}}$ on $(\mp, \underset{,}{k})$ is known as CPyFS-partial order relation if;

(i) Reflexive

(ii) Antisymmetric

(iii) Transitive

Example 9. Take a relation from Table 1 as:

$$\bar{\underset{\cdot}{R}} = \left\{ \begin{array}{l} \left(\left(\acute{\bar{o}}_1, \acute{\bar{o}}_1 \right), \begin{pmatrix} 0.4e^{0.6\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.2\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1\pi i}, \\ 0.8e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_1, \acute{\bar{o}}_2 \right), \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.2e^{0.5\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.6e^{0.2\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_2, \acute{\bar{o}}_2 \right), \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_2, \acute{\bar{o}}_3 \right), \begin{pmatrix} 0.8e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.3\pi i}, \\ 0.5e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.4\pi i}, \\ 0.7e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_1, \acute{\bar{o}}_3 \right), \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.4e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.5e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\bar{o}}_3, \acute{\bar{o}}_3 \right), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}, \tag{30}$$

is a CPyFS-partial order relation $\bar{\underset{\cdot}{R}}$.

Definition 21. Suppose that CPyFSR Ṝ on (Ŧ, ķ) is known as CPyFS-pre order relation if;

(i) Reflexive

(ii) Transitive

Definition 22. Suppose that CPyFSR Ṝ on (Ŧ, ķ) is known as CPyFS-complete relation if;

$$\forall\left(\left(ő\right),\begin{pmatrix}\left[r_{m}\left(ő\right)\right]e^{q_{m}\left(ő\right)\pi i},\\ \left[r_{m}\left(ő\right)\right]e^{q_{m}\left(ő\right)\pi i}\end{pmatrix}\right),\left(\left(i̦\right),\begin{pmatrix}\left[r_{m}\left(i̦\right)\right]e^{q_{m}\left(i̦\right)\pi i},\\ \left[r_{m}\left(i̦\right)\right]e^{q_{m}\left(i̦\right)\pi i}\end{pmatrix}\right)\in \bar{Ṛ},$$
$$\Rightarrow\left(\left(ő,i̦\right),\begin{pmatrix}\left[r_{m}\left(ő,i̦\right)\right]e^{q_{m}\left(ő,i̦\right)\pi i},\\ \left[r_{m}\left(ő,i̦\right)\right]e^{q_{m}\left(ő,i̦\right)\pi i}\end{pmatrix}\right)\in \bar{Ṛ}\text{ or }\left(\left(i̦,ő\right),\begin{pmatrix}\left[r_{m}\left(i̦,ő\right)\right]e^{q_{m}\left(i̦,ő\right)\pi i},\\ \left[r_{m}\left(i̦,ő\right)\right]e^{q_{m}\left(i̦,ő\right)\pi i}\end{pmatrix}\right)\in \bar{Ṛ}. \tag{31}$$

Definition 23. Suppose that CPyFSR Ṝ on (Ŧ, ķ) is known as CPyFS-linear order relation if;

(i) Reflexive

(ii) Antisymmetric

(iii) Transitive

(iv) Complete

Definition 24. Suppose that CPyFSR Ṝ on (Ŧ, ķ) is known as CPyFS-strict order relation if;

(i) Irreflexive

(ii) Transitive

Example 10. Take a relation from Table 1 as:

$$\bar{Ṛ}=\left\{\begin{array}{c}\left(\left(ő_{1},ő_{2}\right),\begin{pmatrix}0.6e^{0.7\pi i},\\ 0.2e^{0.5\pi i}\end{pmatrix},\begin{pmatrix}0.6e^{0.7\pi i},\\ 0.6e^{0.2\pi i}\end{pmatrix},\begin{pmatrix}0.2e^{0.3\pi i},\\ 0.6e^{0.4\pi i}\end{pmatrix},\begin{pmatrix}0.4e^{0.1\pi i},\\ 0.3e^{0.8\pi i}\end{pmatrix}\right),\\ \left(\left(ő_{1},ő_{3}\right),\begin{pmatrix}0.6e^{0.7\pi i},\\ 0.4e^{0.6\pi i}\end{pmatrix},\begin{pmatrix}0.1e^{0.5\pi i},\\ 0.8e^{0.3\pi i}\end{pmatrix},\begin{pmatrix}0.2e^{0.3\pi i},\\ 0.5e^{0.3\pi i}\end{pmatrix},\begin{pmatrix}0.4e^{0.1\pi i},\\ 0.5e^{0.8\pi i}\end{pmatrix}\right),\\ \left(\left(ő_{2},ő_{3}\right),\begin{pmatrix}0.8e^{0.4\pi i},\\ 0.4e^{0.8\pi i}\end{pmatrix},\begin{pmatrix}0.1e^{0.3\pi i},\\ 0.8e^{0.6\pi i}\end{pmatrix},\begin{pmatrix}0.5e^{0.3\pi i},\\ 0.5e^{0.9\pi i}\end{pmatrix},\begin{pmatrix}0.5e^{0.4\pi i},\\ 0.7e^{0.8\pi i}\end{pmatrix}\right)\end{array}\right\}, \tag{32}$$

is a CPyFS-strict order relation Ṝ.

Definition 25. Let Ṝ be a CPyFSR on (Ŧ, ķ) is known as CPyFS equivalence class of ő mod Ṝ is defined as:

$$\bar{Ṛ}\left[ő\right]=\left\{\begin{array}{c}\left(ő,\left(\left[r_{m}\left(ő\right)\right]e^{q_{m}\left(ő\right)\pi i},\left[r_{m}\left(ő\right)\right]e^{q_{m}\left(ő\right)\pi i}\right)\right):\\ \left(\left(i̦,ő\right),\left(\left[r_{m}\left(i̦,ő\right)\right]e^{q_{m}\left(i̦,ő\right)\pi i},\left[r_{m}\left(i̦,ő\right)\right]e^{q_{m}\left(i̦,ő\right)\pi i}\right)\right)\in \bar{Ṛ}\end{array}\right\}. \tag{33}$$

Example 11. Take an equivalence relation from example 8 as:

$$\underset{\cdot}{\bar{R}} = \left\{ \begin{array}{l} \left(\left(\acute{\dot{o}}_1, \acute{\dot{o}}_1\right), \begin{pmatrix} 0.4e^{0.6\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.2\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1\pi i}, \\ 0.8e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\dot{o}}_1, \acute{\dot{o}}_2\right), \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.2e^{0.5\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.7\pi i}, \\ 0.6e^{0.2\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\dot{o}}_2, \acute{\dot{o}}_1\right), \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.6e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2\pi i}, \\ 0.8e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.8e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.6\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\dot{o}}_2, \acute{\dot{o}}_2\right), \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\dot{o}}_2, \acute{\dot{o}}_3\right), \begin{pmatrix} 0.8e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.3\pi i}, \\ 0.5e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.4\pi i}, \\ 0.7e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\dot{o}}_3, \acute{\dot{o}}_2\right), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.6\pi i}, \\ 0.3e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.5\pi i}, \\ 0.9e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.7\pi i}, \\ 0.3e^{0.5\pi i} \end{pmatrix} \right), \\ \left(\left(\acute{\dot{o}}_3, \acute{\dot{o}}_3\right), \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}. \tag{34}$$

Now, the equivalence class of.

(i) $\acute{\dot{o}}_1$ mod $\underset{\cdot}{\bar{R}}$

$$\underset{\cdot}{\bar{R}}\left[\acute{\dot{o}}_1\right] = \left\{ \begin{array}{l} \left(\acute{\dot{o}}_1, \begin{pmatrix} 0.4e^{0.6\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.2\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1\pi i}, \\ 0.8e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\acute{\dot{o}}_2, \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right) \end{array} \right\}. \tag{35}$$

(ii) $\acute{\dot{o}}_2$ mod $\underset{\cdot}{\bar{R}}$

$$\underset{\cdot}{\bar{R}}\left[\acute{\dot{o}}_2\right] = \left\{ \begin{array}{l} \left(\acute{\dot{o}}_1, \begin{pmatrix} 0.4e^{0.6\pi i}, \\ 0.6e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.2\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1\pi i}, \\ 0.8e^{0.4\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1\pi i}, \\ 0.3e^{0.8\pi i} \end{pmatrix} \right), \\ \left(\acute{\dot{o}}_2, \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left(\acute{\dot{o}}_3, \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}. \tag{36}$$

(iii) $\acute{\dot{o}}_3$ mod $\underset{\cdot}{\bar{R}}$

$$\underset{\cdot}{\bar{R}}\left[\acute{\dot{o}}_3\right] = \left\{ \begin{array}{l} \left(\acute{\dot{o}}_2, \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.8e^{0.6\pi i} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3\pi i}, \\ 0.6e^{0.9\pi i} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.7e^{0.5\pi i} \end{pmatrix} \right), \\ \left(\acute{\dot{o}}_3, \begin{pmatrix} 0.7e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.5\pi i}, \\ 0.8e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.4\pi i}, \\ 0.9e^{0.3\pi i} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2\pi i}, \\ 0.5e^{0.8\pi i} \end{pmatrix} \right) \end{array} \right\}. \tag{37}$$

Definition 26. Let $\bar{R}$ be a CPyFSR on $(\text{₮}, k)$ is said to be CPyFS composite relation $\bar{R}_1 \circ \bar{R}_2$ is defined as:

$$\left((\acute{\bar{o}}, i), \left(\begin{array}{c}\left[r_{m_r}(\acute{\bar{o}}, i)\right]e^{q_{m_r}(\acute{\bar{o}}, i)\pi i}, \\ \left[r_{m_r}(\acute{\bar{o}}, i)\right]e^{q_{m_r}(\acute{\bar{o}}, i)\pi i}\end{array}\right)\right) \in \bar{R}_1, \tag{38}$$

and $((i, \text{ŧ}), ([r_{m_r}(i, \text{ŧ})]e^{q_{m_r}(i, \text{ŧ})\pi i}, [r_{m_r}(i, \text{ŧ})]e^{q_{m_r}(i, \text{ŧ})\pi i})) \in \bar{R}_2$

$$\Rightarrow \left((\acute{\bar{o}}, \text{ŧ}), \left(\begin{array}{c}\left[r_{m_r}(\acute{\bar{o}}, \text{ŧ})\right]e^{q_{m_r}(\acute{\bar{o}}, \text{ŧ})\pi i}, \\ \left[r_{m_r}(\acute{\bar{o}}, \text{ŧ})\right]e^{q_{m_r}(\acute{\bar{o}}, \text{ŧ})\pi i}\end{array}\right)\right) \bar{R}_1 \circ \bar{R}_2. \tag{39}$$

Theorem 1. *A CPyFSR $\bar{R}$ is CPyFS symmetric relation on a CPyFSS ₮ iff $\bar{R} = \bar{R}^c$.*

Proof 1. Suppose that $\bar{R} = \bar{R}^c$, then $((\acute{\bar{o}}, i), (m_{r_c}(\acute{\bar{o}}, i)), (m_{r_c}(\acute{\bar{o}}, i))) \in \bar{R}$

$$\begin{aligned} &\Longrightarrow \left((i, \acute{\bar{o}}), \left(m_{r_c}(i, \acute{\bar{o}})\right), \left(m_{r_c}(i, \acute{\bar{o}})\right)\right) \in \bar{R}^c \\ &\Longrightarrow \left((i, \acute{\bar{o}}), \left(m_{r_c}(i, \acute{\bar{o}})\right), \left(m_{r_c}(i, \acute{\bar{o}})\right)\right) \in \bar{R}. \end{aligned} \tag{40}$$

Thus, $\bar{R}$ is CPyFS symmetric relation on a CPyFSS₮.

Conversely, assume that $\bar{R}$ is CPyFS symmetric relation on CPyFSS₮ then

$$\begin{aligned} &\left((\acute{\bar{o}}, i), \left(m_{r_c}(\acute{\bar{o}}, i)\right), \left(m_{r_c}(\acute{\bar{o}}, i)\right)\right) \in \bar{R} \\ &\quad \Longrightarrow \left((i, \acute{\bar{o}}), \left(m_{r_c}(i, \acute{\bar{o}})\right), \left(m_{r_c}(i, \acute{\bar{o}})\right)\right) \in \bar{R}. \end{aligned} \tag{41}$$

However, $((i, \acute{\bar{o}}), (m_{r_c}(i, \acute{\bar{o}})), (m_{r_c}(i, \acute{\bar{o}}))) \in \bar{R}^c$

$$\Longrightarrow \bar{R} = \bar{R}^c. \tag{42}$$

□

Theorem 2. *A CPyFSR $\bar{R}$ is CPyFS transitive relation on CPyFSS ₮ iff $\bar{R} \circ \bar{R} \subseteq \bar{R}^c$.*

Proof 2. Suppose that $\bar{R}$ is CPyFS transitive relation on CPyFSS₮.

Let $((\acute{\bar{o}}, \text{ŧ}), (m_{r_c}(\acute{\bar{o}}, \text{ŧ}), (m_{r_c}(\acute{\bar{o}}, \text{ŧ}))) \in \bar{R} \circ \bar{R}$,

Then by the definition of CPyFS transitive relation, $((\acute{\bar{o}}, i), (m_{r_c}(\acute{\bar{o}}, i)), (m_{r_c}(\acute{\bar{o}}, i))) \in \bar{R}$ And $((i, \text{ŧ}), (m_{r_c}(i, \text{ŧ})), (m_{r_c}(i, \text{ŧ}))) \in \bar{R}$

$$\Longrightarrow \bar{R} \circ \bar{R} \subseteq \bar{R}. \tag{43}$$

Conversely suppose that $\bar{R} \circ \bar{R} \subseteq \bar{R}$, then.

For $((\acute{\bar{o}}, i), (m_{r_c}(\acute{\bar{o}}, i), (m_{r_c}(\acute{\bar{o}}, i))) \in \bar{R}$ and $((i, \text{ŧ}), (m_{r_c}(i, \text{ŧ})), (m_{r_c}(i, \text{ŧ}))) \in \bar{R}$

$$\left((\acute{\bar{o}}, \text{ŧ}), \left(m_{r_c}(\acute{\bar{o}}, \text{ŧ})\right), \left(m_{r_c}(\acute{\bar{o}}, \text{ŧ})\right)\right) \in \bar{R}. \tag{44}$$

Thus $\bar{R}$ is a CPyFS transitive relation on CPyFSS₮. □

Theorem 3. *A CPyFSR $\bar{R}$ is CPyFS equivalence relation on a CPyFSS ₮ iff $\bar{R} \circ \bar{R} = \bar{R}$.*

Proof 3. Suppose that $((\acute{\bar{o}}, i), (m_{r_c}(\acute{\bar{o}}, i), (m_{r_c}(\acute{\bar{o}}, i))) \in \bar{R}$,

Then by the definition of CPyFS symmetric relation,

$$\left((i, \acute{\bar{o}}), \left(m_{r_c}(i, \acute{\bar{o}})\right), \left(m_{r_c}(i, \acute{\bar{o}})\right)\right) \in \bar{R}. \tag{45}$$

Now by the definition of CPyFS transitive relation,

$$\left((\acute{\bar{o}}, \acute{\bar{o}})\left[r_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\right]e^{q_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\pi i}, \left[r_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\right]e^{q_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\pi i}\right) \in \bar{R}. \tag{46}$$

However, by the definition of CPyFS composite relation,

$$\left((\acute{\bar{o}}, \acute{\bar{o}})\left[r_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\right]e^{q_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\pi i}, \left[r_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\right]e^{q_{m_r}(\acute{\bar{o}}, \acute{\bar{o}})\pi i}\right) \in \bar{R} \circ \bar{R}. \tag{47}$$

Hence,

$$\bar{R} \subseteq \bar{R} \circ \bar{R}. \tag{48}$$

Conversely, assume that

$$\left((\acute{\bar{o}}, i), \left(m_{r_c}(\acute{\bar{o}}, i), \left(m_{r_c}(\acute{\bar{o}}, i)\right)\right)\right) \in \bar{R} \circ \bar{R}. \tag{49}$$

Then there exist $\text{ŧ} \in \text{₮} \ni ((\acute{\bar{o}}, \text{ŧ}), ([r_{m_r}(\acute{\bar{o}}, \text{ŧ})] e^{q_{m_r}(\acute{\bar{o}}, \text{ŧ})\pi i}, [r_{m_r}(\acute{\bar{o}}, \text{ŧ})]e^{q_{m_r}(\acute{\bar{o}}, \text{ŧ})\pi i})) \in \bar{R}$ and

$$\left((\text{ŧ}, i), \left(\left[r_{m_r}(\text{ŧ}, i)\right]e^{q_{m_r}(\text{ŧ}, i)\pi i}, \left[r_{m_r}(\text{ŧ}, i)\right]e^{q_{m_r}(\text{ŧ}, i)\pi i}\right)\right) \in \bar{R}. \tag{50}$$

However, □ is a CPyFS equivalence relation on CPyFSS ₮, so $\bar{R}$ is also CPyFS transitive relation. Therefore,

$$\Rightarrow \bar{R} \circ \bar{R} \subseteq \bar{R}. \tag{51}$$

Henceforth, (48) and (51),

$$\bar{R} \circ \bar{R} = \bar{R}. \tag{52}$$

□

4. Applications

Here, is an application of the proposed concepts to select the best blockchain technology (BCT) by applying the idea of complex Pythagorean fuzzy soft relation (CPyFSR) and their types.

4.1. Blockchain Technology (BCT). Blockchain is one of the most talked-about technologies in business right now. BCT has the potential to drive major changes and create new opportunities across industries from banking and cybersecurity to intellectual property and healthcare. BCT is a database system that preserves and stores data in a way that enables various organizations and people to reliably share access to the same data in real time while alleviating concerns about security, privacy, and control. The technology provides a reliable technique for individuals to contract directly with each other, without an intermediary like a government, bank, or other third parties. Figure 1 shows the algorithm of the application.

Firstly, express the universal set that consists of three types of BCT. The universal set consists of three types of BCT i.e., $\dot{\alpha}_1$ = Public blockchains, $\dot{\alpha}_2$ = Private blockchain and $\dot{\alpha}_3$ = Consortium blockchains. Figure 2 discusses the types of BCT.

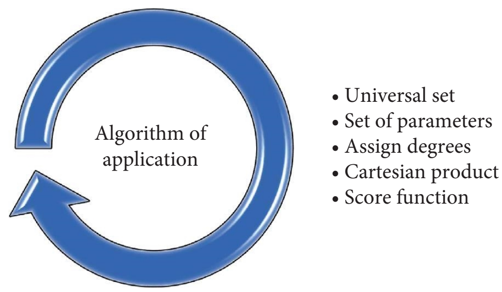

Figure 1: Algorithm of application.

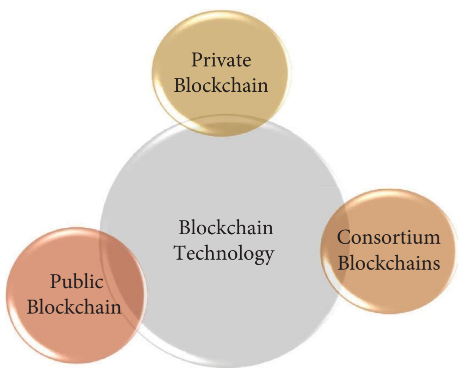

Figure 2: Summary of BCT.

4.1.1. Public Blockchains. Public blockchains are non-restrictive, permissionless distributed ledger systems. They are open networks that allow anyone to participate in the network. Public blockchains are generally secure if the users rigidly follow security guidelines and procedures.

4.1.2. Private (or Managed) Blockchains. Private blockchains are frequently used within an association or enterprise where only nominated members are applicants of a blockchain network. The level of security, authorizations, permissions, and availability is in the hands of the monitoring organizations.

4.1.3. Consortium Blockchains. Consortium blockchains are a semi-decentralized form where more than one association accomplishes a blockchain network. More than one association can act as a node in this nature of blockchains and exchange information. Consortium blockchains are classically used by banks, government organizations, etc.

Secondly, describe the parameter of the BCT. Figure 3 shows the summary of BCT parameters.

(1) Increased Capacity. The most amazing thing about this BCT is that it increases the capacity of the entire network. The average number of transactions successfully recorded per second in the BCT.

(2) Better Security. BCT produces a structure of data with inherent security qualities. BCT is considered more secure than its contemporaries because of the lack of a single point of failure.

(3) Faster Settlement. It can explain this problem by employing blockchains as they can allow money transfer at certainly fast speeds. This eventually saves a lot of time and money from these establishments and provides accessibility to the consumer also.

(4) Decentralized System. Decentralized technology gives the power to store resources in a network without the oversight and control of a single person organization or entity. BCT shows to be an operative tool for decentralizing the web so it is a small revolt in the Internet's world.

(5) Distributed Ledger. The distributed ledger allows anyone with the required access to view the ledger and makes the process transparent and reliable. This distributed computational power across the computer to ensure a better outcome.

The expert examines the BCT in which characteristic of all parameters. Let observations $(\mathscr{F}, \mathscr{A})$ by experts individually. They give the value of membership and nonmembership on the base of parameters.

Assume that their corresponding membership and nonmembership matrices are as follows.

$$(\mathscr{F}, \mathscr{A}) = \begin{pmatrix} \begin{pmatrix} 0.6e^{0.5\pi i}, \\ 0.5e^{0.6\pi i} \end{pmatrix} & \begin{pmatrix} 0.5e^{0.3\pi i}, \\ 0.4e^{0.8\pi i} \end{pmatrix} & \begin{pmatrix} 0.4e^{0.7\pi i}, \\ 0.2e^{0.5\pi i} \end{pmatrix} & \begin{pmatrix} 0.7e^{0.7\pi i}, \\ 0.2e^{0.3\pi i} \end{pmatrix} \\ \begin{pmatrix} 0.7e^{0.7\pi i}, \\ 0.2e^{0.3\pi i} \end{pmatrix} & \begin{pmatrix} 0.3e^{0.4\pi i}, \\ 0.6e^{0.7\pi i} \end{pmatrix} & \begin{pmatrix} 0.5e^{0.9\pi i}, \\ 0.3e^{0.1\pi i} \end{pmatrix} & \begin{pmatrix} 0.9e^{0.4\pi i}, \\ 0.3e^{0.2\pi i} \end{pmatrix} \\ \begin{pmatrix} 0.8e^{0.4\pi i}, \\ 0.3e^{0.3\pi i} \end{pmatrix} & \begin{pmatrix} 0.6e^{0.9\pi i}, \\ 0.1e^{0.2\pi i} \end{pmatrix} & \begin{pmatrix} 0.8e^{0.6\pi i}, \\ 0.3e^{0.5\pi i} \end{pmatrix} & \begin{pmatrix} 0.5e^{0.8\pi i}, \\ 0.6e^{0.3\pi i} \end{pmatrix} \\ \begin{pmatrix} 0.9e^{0.6\pi i}, \\ 0.4e^{0.2\pi i} \end{pmatrix} & \begin{pmatrix} 0.7e^{0.6\pi i}, \\ 0.4e^{0.4\pi i} \end{pmatrix} & \begin{pmatrix} 0.6e^{0.4\pi i}, \\ 0.3e^{0.3\pi i} \end{pmatrix} & \begin{pmatrix} 0.9e^{0.6\pi i}, \\ 0.1e^{0.4\pi i} \end{pmatrix} \\ \begin{pmatrix} 0.5e^{0.5\pi i}, \\ 0.2e^{0.7\pi i} \end{pmatrix} & \begin{pmatrix} 0.8e^{0.6\pi i}, \\ 0.3e^{0.4\pi i} \end{pmatrix} & \begin{pmatrix} 0.7e^{0.8\pi i}, \\ 0e^{0.2\pi i} \end{pmatrix} & \begin{pmatrix} 0.7e^{0.4\pi i}, \\ 0.6e^{0.7\pi i} \end{pmatrix} \end{pmatrix}. \tag{53}$$

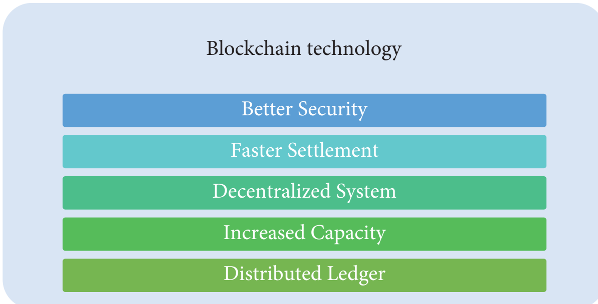

Figure 3: Summary of Blockchain technology parameters.

The first value of each parameter indicates the degrees of membership and non-membership assigned by experts to the $\dot{\alpha}_1$, the second value of each parameter indicates the degree of membership and non-membership assigned by experts to the $\dot{\alpha}_2$, the third value of each parameter shows the degree of membership and non-membership assigned by experts to the $\dot{\alpha}_3$ and the last value of each parameter shows the general belongingness of each parameter to the BCT and is denoted by λ.

Then, the CP of $(\mathscr{F}, \mathscr{A})$ shown in Table 2.

Now to calculate the score function, convert the complex values into real values to calculate the score values. Firstly, given values in complex polar form, i.e., $e^{\pi\vartheta i} = \cos\pi\vartheta + i\sin\pi\vartheta$. To convert the all-exponential values to the form of $a + ib$. i.e., $a + ib = re^{\pi i\theta}$, as $r = \sqrt{a^2 + b^2}$ and $e^{\pi i\theta} = \cos\pi(\theta) + i \quad \sin\pi(\theta)$. Then $a = r\cos\pi(\theta), b = \sin\pi(\theta)$. Here, π is just a notion and shows the cycle of the circle. Make polar form into standard form and then take modulus. After all, this process, apply to membership and non-membership score formula to $1/2(m_{+}^{2} + q_{m_{+}}^{2} - m_{+}^{2} - q_{m_{+}}^{2})$, as shown in Table 3.

Now, to determine the best BCT, take the largest value from each row and ignore the last column. The last column is the general belongingness of each BCT parameter. Now every BCT score is calculated by adding the product of these numerical degrees with the corresponding value of λ. The best BCT preferred by any user is the one that gets a greater numerical value than others. We do not study the numerical degree of the same parametric ordered pair's BCT because it is not a unique work to compare with itself. Now, estimate the score function in Table 4.

$$\begin{aligned} S(\dot{\alpha}_3) &= (0.155 \times 0.235) + (0.09 \times 0.145) + (0.18 \times -0.1) + (0.155 \times 0.235) + (0.38 \times -0.1) + (0.09 \times 0.145) \\ &\quad + (0.255 \times -0.22) + (0.18 \times -0.1) + (0.38 \times -0.1) + (0.255 \times -0.22) = -0.125. \end{aligned} \tag{54}$$

Thus, public blockchains are the best BCT as compared to other blockchain technology.

5. Comparative Analysis

In this section, the innovative framework of CPyFSRs is compared to the numerous pre-existing structures in FSS theory, such as FSRs, CFSRs, IFSRs, CIFSRs, and PyFSRs.

5.1. Comparison of FSRs, CFSRs with CPyFSRs. The structure of FSS and CFSSs is explained by a membership degree which is a fuzzy number, and the associated relations are known as FSRs and CFSRs. FSRs and CFSRs are defined only by the membership degree. The FSRs in an ordered pair show only the effectiveness of the first parameter over the second. The FSRs have only one dimension and provide limited information. The CFSRs analyzed only the membership degree with the complex number. The CFSRs are basic two components, i.e., amplitude terms and phase terms. An amplitude term describes the strength of the different BCT, and the phase term is used to define the period over specified situations. Therefore, the CPyFSRs defined both the membership and non-membership degrees with complex numbers.

Assume that their corresponding membership of CFSRs matrices is as follows.

$$(\mathscr{F}, \mathscr{A}) = \begin{pmatrix} (0.6e^{0.5\pi i}) & (0.5e^{0.3\pi i}) & (0.4e^{0.7\pi i}) & (0.7e^{0.7\pi i}) \\ (0.7e^{0.7\pi i}) & (0.3e^{0.4\pi i}) & (0.5e^{0.9\pi i}) & (0.9e^{0.4\pi i}) \\ (0.8e^{0.4\pi i}) & (0.6e^{0.9\pi i}) & (0.8e^{0.6\pi i}) & (0.5e^{0.8\pi i}) \end{pmatrix}. \tag{55}$$

The first three values of each parameter indicate the degrees of membership assigned by experts, and the last value of each parameter shows the general belongingness of each parameter to the BCT and is denoted by λ. Then, its self CP is shown in Table 5:

5.2. IFSR and Cifsr with CPyFSRS. The IFSSs and CIFSSs are defined as membership and non-membership degrees. The related relations are known as IFSRs and CIFSRs. The IFSRs indicate the effectiveness and ineffectiveness of the first parameter over the second in an ordered pair. An IFSRs

Table 2: Cartesian product.

Ordered pair	$\dot{q}_1$	$\dot{q}_2$	$\dot{q}_3$	λ
$(\H{o}_1, \H{o}_1)$	$(0.6e^{0.5\pi i}, 0.5e^{0.6\pi i})$	$(0.5e^{0.3\pi i}, 0.4e^{0.8\pi i})$	$(0.4e^{0.7\pi i}, 0.2e^{0.5\pi i})$	$(0.7e^{0.7\pi i}, 0.2e^{0.3\pi i})$
$(\H{o}_1, \H{o}_2)$	$(0.6e^{0.5\pi i}, 0.5e^{0.6\pi i})$	$(0.3e^{0.3\pi i}, 0.6e^{0.8\pi i})$	$(0.4e^{0.7\pi i}, 0.3e^{0.5\pi i})$	$(0.7e^{0.4\pi i}, 0.3e^{0.3\pi i})$
$(\H{o}_1, \H{o}_3)$	$(0.6e^{0.4\pi i}, 0.5e^{0.6\pi i})$	$(0.5e^{0.3\pi i}, 0.4e^{0.8\pi i})$	$(0.4e^{0.6\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.7\pi i}, 0.6e^{0.3\pi i})$
$(\H{o}_1, \H{o}_4)$	$(0.6e^{0.5\pi i}, 0.5e^{0.6\pi i})$	$(0.5e^{0.3\pi i}, 0.4e^{0.8\pi i})$	$(0.4e^{0.4\pi i}, 0.3e^{0.5\pi i})$	$(0.7e^{0.6\pi i}, 0.2e^{0.4\pi i})$
$(\H{o}_1, \H{o}_5)$	$(0.5e^{0.5\pi i}, 0.5e^{0.7\pi i})$	$(0.5e^{0.3\pi i}, 0.4e^{0.8\pi i})$	$(0.4e^{0.7\pi i}, 0.2e^{0.5\pi i})$	$(0.7e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_2, \H{o}_1)$	$(0.6e^{0.5\pi i}, 0.5e^{0.6\pi i})$	$(0.3e^{0.3\pi i}, 0.6e^{0.8\pi i})$	$(0.4e^{0.7\pi i}, 0.3e^{0.5\pi i})$	$(0.7e^{0.4\pi i}, 0.3e^{0.3\pi i})$
$(\H{o}_2, \H{o}_2)$	$(0.7e^{0.7\pi i}, 0.2e^{0.3\pi i})$	$(0.3e^{0.4\pi i}, 0.6e^{0.7\pi i})$	$(0.5e^{0.9\pi i}, 0.3e^{0.1\pi i})$	$(0.9e^{0.4\pi i}, 0.3e^{0.2\pi i})$
$(\H{o}_2, \H{o}_3)$	$(0.7e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.3e^{0.4\pi i}, 0.6e^{0.7\pi i})$	$(0.5e^{0.6\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.4\pi i}, 0.6e^{0.3\pi i})$
$(\H{o}_2, \H{o}_4)$	$(0.7e^{0.6\pi i}, 0.4e^{0.3\pi i})$	$(0.3e^{0.4\pi i}, 0.6e^{0.7\pi i})$	$(0.5e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.9e^{0.4\pi i}, 0.3e^{0.4\pi i})$
$(\H{o}_2, \H{o}_5)$	$(0.5e^{0.5\pi i}, 0.2e^{0.7\pi i})$	$(0.3e^{0.4\pi i}, 0.6e^{0.7\pi i})$	$(0.5e^{0.8\pi i}, 0.3e^{0.2\pi i})$	$(0.7e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_3, \H{o}_1)$	$(0.6e^{0.4\pi i}, 0.5e^{0.6\pi i})$	$(0.5e^{0.3\pi i}, 0.4e^{0.8\pi i})$	$(0.4e^{0.6\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.7\pi i}, 0.6e^{0.3\pi i})$
$(\H{o}_3, \H{o}_2)$	$(0.7e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.3e^{0.4\pi i}, 0.6e^{0.7\pi i})$	$(0.5e^{0.6\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.4\pi i}, 0.6e^{0.3\pi i})$
$(\H{o}_3, \H{o}_3)$	$(0.8e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.6e^{0.9\pi i}, 0.1e^{0.2\pi i})$	$(0.8e^{0.6\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.8\pi i}, 0.6e^{0.3\pi i})$
$(\H{o}_3, \H{o}_4)$	$(0.8e^{0.4\pi i}, 0.4e^{0.3\pi i})$	$(0.6e^{0.6\pi i}, 0.4e^{0.4\pi i})$	$(0.6e^{0.4\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.6\pi i}, 0.6e^{0.4\pi i})$
$(\H{o}_3, \H{o}_5)$	$(0.5e^{0.4\pi i}, 0.3e^{0.7\pi i})$	$(0.6e^{0.6\pi i}, 0.3e^{0.4\pi i})$	$(0.7e^{0.6\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_4, \H{o}_1)$	$(0.6e^{0.5\pi i}, 0.5e^{0.6\pi i})$	$(0.5e^{0.3\pi i}, 0.4e^{0.8\pi i})$	$(0.4e^{0.4\pi i}, 0.3e^{0.5\pi i})$	$(0.7e^{0.6\pi i}, 0.2e^{0.4\pi i})$
$(\H{o}_4, \H{o}_2)$	$(0.7e^{0.6\pi i}, 0.4e^{0.3\pi i})$	$(0.3e^{0.4\pi i}, 0.6e^{0.7\pi i})$	$(0.5e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.9e^{0.4\pi i}, 0.3e^{0.4\pi i})$
$(\H{o}_4, \H{o}_3)$	$(0.8e^{0.4\pi i}, 0.4e^{0.3\pi i})$	$(0.6e^{0.6\pi i}, 0.4e^{0.4\pi i})$	$(0.6e^{0.4\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.6\pi i}, 0.6e^{0.4\pi i})$
$(\H{o}_4, \H{o}_4)$	$(0.9e^{0.6\pi i}, 0.4e^{0.2\pi i})$	$(0.7e^{0.6\pi i}, 0.4e^{0.4\pi i})$	$(0.6e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.9e^{0.6\pi i}, 0.1e^{0.4\pi i})$
$(\H{o}_4, \H{o}_5)$	$(0.5e^{0.5\pi i}, 0.4e^{0.7\pi i})$	$(0.7e^{0.6\pi i}, 0.4e^{0.4\pi i})$	$(0.6e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.7e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_5, \H{o}_1)$	$(0.5e^{0.5\pi i}, 0.5e^{0.7\pi i})$	$(0.5e^{0.3\pi i}, 0.4e^{0.8\pi i})$	$(0.4e^{0.7\pi i}, 0.2e^{0.5\pi i})$	$(0.7e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_5, \H{o}_2)$	$(0.5e^{0.5\pi i}, 0.2e^{0.7\pi i})$	$(0.3e^{0.4\pi i}, 0.6e^{0.7\pi i})$	$(0.5e^{0.8\pi i}, 0.3e^{0.2\pi i})$	$(0.7e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_5, \H{o}_3)$	$(0.5e^{0.4\pi i}, 0.3e^{0.7\pi i})$	$(0.6e^{0.6\pi i}, 0.3e^{0.4\pi i})$	$(0.7e^{0.6\pi i}, 0.3e^{0.5\pi i})$	$(0.5e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_5, \H{o}_4)$	$(0.5e^{0.5\pi i}, 0.4e^{0.7\pi i})$	$(0.7e^{0.6\pi i}, 0.4e^{0.4\pi i})$	$(0.6e^{0.4\pi i}, 0.3e^{0.3\pi i})$	$(0.7e^{0.4\pi i}, 0.6e^{0.7\pi i})$
$(\H{o}_5, \H{o}_5)$	$(0.5e^{0.5\pi i}, 0.2e^{0.7\pi i})$	$(0.8e^{0.6\pi i}, 0.3e^{0.4\pi i})$	$(0.7e^{0.8\pi i}, 0e^{0.2\pi i})$	$(0.7e^{0.4\pi i}, 0.6e^{0.7\pi i})$

Table 3: Modulus of each complex number.

Ordered pair	$\dot{q}_1$	$\dot{q}_2$	$\dot{q}_3$	λ
$(\acute{o}_1,\acute{o}_1)$	0	−0.23	0.18	0.425
$(\acute{o}_1,\acute{o}_2)$	0	−0.41	0.155	0.235
$(\acute{o}_1,\acute{o}_3)$	−0.045	−0.23	0.09	0.145
$(\acute{o}_1,\acute{o}_4)$	0	−0.23	−0.01	0.325
$(\acute{o}_1,\acute{o}_5)$	−0.12	−0.23	0.18	−0.1
$(\acute{o}_2,\acute{o}_1)$	0	−0.41	0.155	0.235
$(\acute{o}_2,\acute{o}_2)$	0.425	−0.3	0.48	0.42
$(\acute{o}_2,\acute{o}_3)$	0.235	−0.3	0.135	−0.02
$(\acute{o}_2,\acute{o}_4)$	0.3	−0.3	0.115	0.36
$(\acute{o}_2,\acute{o}_5)$	−0.015	−0.3	0.38	−0.1
$(\acute{o}_3,\acute{o}_1)$	−0.045	−0.23	0.09	0.145
$(\acute{o}_3,\acute{o}_2)$	0.235	−0.3	0.135	−0.02
$(\acute{o}_3,\acute{o}_3)$	0.31	0.56	0.33	0.22
$(\acute{o}_3,\acute{o}_4)$	0.275	0.2	0.09	0.045
$(\acute{o}_3,\acute{o}_5)$	−0.085	0.235	0.255	−0.22
$(\acute{o}_4,\acute{o}_1)$	0	−0.23	−0.01	0.325
$(\acute{o}_4,\acute{o}_2)$	0.3	−0.3	0.115	0.36
$(\acute{o}_4,\acute{o}_3)$	0.275	0.2	0.09	0.045
$(\acute{o}_4,\acute{o}_4)$	0.485	0.265	0.17	0.5
$(\acute{o}_4,\acute{o}_5)$	−0.075	0.265	0.17	−0.1
$(\acute{o}_5,\acute{o}_1)$	−0.12	−0.23	0.18	−0.1
$(\acute{o}_5,\acute{o}_2)$	−0.015	−0.3	0.38	−0.1
$(\acute{o}_5,\acute{o}_3)$	−0.085	0.235	0.255	−0.22
$(\acute{o}_5,\acute{o}_4)$	−0.075	0.265	0.17	−0.1
$(\acute{o}_5,\acute{o}_5)$	−0.015	0.375	0.545	−0.1

Table 4: Score function.

$\bar{R}$	$(\acute{o}_1,\acute{o}_1)$	$(\acute{o}_1,\acute{o}_2)$	$(\acute{o}_1,\acute{o}_3)$	$(\acute{o}_1,\acute{o}_4)$	$(\acute{o}_1,\acute{o}_5)$	$(\acute{o}_2,\acute{o}_1)$	$(\acute{o}_2,\acute{o}_2)$	$(\acute{o}_2,\acute{o}_3)$	$(\acute{o}_2,\acute{o}_4)$
$\dot{q}_i$	$\dot{q}_3$	$\dot{q}_3$	$\dot{q}_3$	$\dot{q}_1$	$\dot{q}_3$	$\dot{q}_3$	$\dot{q}_3$	$\dot{q}_1$	$\dot{q}_1$
Highest degree	×	0.155	0.09	0	0.18	0.155	×	0.235	0.3
λ	×	0.235	0.145	0.325	−0.1	0.235	×	−0.02	0.36
$\bar{R}$	$(\acute{o}_2,\acute{o}_5)$	$(\acute{o}_3,\acute{o}_1)$	$(\acute{o}_3,\acute{o}_2)$	$(\acute{o}_3,\acute{o}_3)$	$(\acute{o}_3,\acute{o}_4)$	$(\acute{o}_3,\acute{o}_5)$	$(\acute{o}_4,\acute{o}_1)$	$(\acute{o}_4,\acute{o}_2)$	$(\acute{o}_4,\acute{o}_3)$
$\dot{q}_i$	$\dot{q}_3$	$\dot{q}_3$	$\dot{q}_1$	$\dot{q}_2$	$\dot{q}_1$	$\dot{q}_3$	$\dot{q}_1$	$\dot{q}_1$	$\dot{q}_1$
Highest degree	0.38	0.09	0.235	×	0.275	0.255	0	0.3	0.275
λ	−0.1	0.145	−0.02	×	0.045	−0.22	0.325	0.36	0.045
$\bar{R}$	$(\acute{o}_4,\acute{o}_4)$	$(\acute{o}_4,\acute{o}_5)$	$(\acute{o}_5,\acute{o}_1)$	$(\acute{o}_5,\acute{o}_2)$	$(\acute{o}_5,\acute{o}_3)$	$(\acute{o}_5,\acute{o}_4)$	$(\acute{o}_5,\acute{o}_5)$		
$\dot{q}_i$	$\dot{q}_1$	$\dot{q}_2$	$\dot{q}_3$	$\dot{q}_3$	$\dot{q}_3$	$\dot{q}_2$	$\dot{q}_3$		
Highest degree	×	0.265	0.18	0.38	0.255	0.265	×		
λ	×	−0.1	−0.1	−0.1	−0.22	−0.1	×		

cannot characterize such problems that include time, so they are provided incomplete information. The CIFSRs are examined the degrees of membership and non-membership with complex numbers. They explain both the amplitude term and phase term. The innovative structure of CPyFSRs discusses the degree of membership and non-membership with complex numbers and increases the space of CIFSRs. So, it provides comprehensive information on any problem.

Assume that their corresponding membership and non-membership of CIFSRs matrices are as follows.

$$(\mathscr{F},\mathscr{A})=\begin{pmatrix} \begin{pmatrix}0.3e^{0.5\pi i},\\ 0.5e^{0.4\pi i}\end{pmatrix} & \begin{pmatrix}0.5e^{0.1\pi i},\\ 0.4e^{0.8\pi i}\end{pmatrix} & \begin{pmatrix}0.4e^{0.3\pi i},\\ 0.2e^{0.5\pi i}\end{pmatrix} & \begin{pmatrix}0.7e^{0.7\pi i},\\ 0.2e^{0.3\pi i}\end{pmatrix} \\ \begin{pmatrix}0.7e^{0.7\pi i},\\ 0.2e^{0.3\pi i}\end{pmatrix} & \begin{pmatrix}0.3e^{0.4\pi i},\\ 0.6e^{0.5\pi i}\end{pmatrix} & \begin{pmatrix}0.5e^{0.9\pi i},\\ 0.3e^{0.1\pi i}\end{pmatrix} & \begin{pmatrix}0.6e^{0.4\pi i},\\ 0.3e^{0.2\pi i}\end{pmatrix} \\ \begin{pmatrix}0.5e^{0.4\pi i},\\ 0.3e^{0.3\pi i}\end{pmatrix} & \begin{pmatrix}0.6e^{0.6\pi i},\\ 0.1e^{0.2\pi i}\end{pmatrix} & \begin{pmatrix}0.5e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix} & \begin{pmatrix}0.5e^{0.5\pi i},\\ 0.4e^{0.3\pi i}\end{pmatrix} \end{pmatrix} \tag{56}$$

Table 5: Cartesian product.

Ordered pair	$\dot{q}_1$	$\dot{q}_2$	$\dot{q}_3$	λ
$(\acute{\acute{o}}_1, \acute{\acute{o}}_1)$	$(0.6e^{0.5\pi i})$	$(0.5e^{0.3\pi i})$	$(0.4e^{0.7\pi i})$	$(0.7e^{0.7\pi i})$
$(\acute{\acute{o}}_1, \acute{\acute{o}}_2)$	$(0.6e^{0.5\pi i})$	$(0.3e^{0.3\pi i})$	$(0.4e^{0.7\pi i})$	$(0.7e^{0.4\pi i})$
$(\acute{\acute{o}}_1, \acute{\acute{o}}_3)$	$(0.6e^{0.4\pi i})$	$(0.5e^{0.3\pi i})$	$(0.4e^{0.6\pi i})$	$(0.5e^{0.7\pi i})$
$(\acute{\acute{o}}_2, \acute{\acute{o}}_1)$	$(0.6e^{0.5\pi i})$	$(0.3e^{0.3\pi i})$	$(0.4e^{0.7\pi i})$	$(0.7e^{0.4\pi i})$
$(\acute{\acute{o}}_2, \acute{\acute{o}}_2)$	$(0.7e^{0.7\pi i})$	$(0.3e^{0.4\pi i})$	$(0.5e^{0.9\pi i})$	$(0.9e^{0.4\pi i})$
$(\acute{\acute{o}}_2, \acute{\acute{o}}_3)$	$(0.7e^{0.4\pi i})$	$(0.3e^{0.4\pi i})$	$(0.5e^{0.6\pi i})$	$(0.5e^{0.4\pi i})$
$(\acute{\acute{o}}_3, \acute{\acute{o}}_1)$	$(0.6e^{0.4\pi i})$	$(0.5e^{0.3\pi i})$	$(0.4e^{0.6\pi i})$	$(0.5e^{0.7\pi i})$
$(\acute{\acute{o}}_3, \acute{\acute{o}}_2)$	$(0.7e^{0.4\pi i})$	$(0.3e^{0.4\pi i})$	$(0.5e^{0.6\pi i})$	$(0.5e^{0.4\pi i})$
$(\acute{\acute{o}}_3, \acute{\acute{o}}_3)$	$(0.8e^{0.4\pi i})$	$(0.6e^{0.9\pi i})$	$(0.8e^{0.6\pi i})$	$(0.5e^{0.8\pi i})$

Table 6: Cartesian product.

Ordered pair	$\dot{q}_1$	$\dot{q}_2$	$\dot{q}_3$	λ
$(\acute{\acute{o}}_1, \acute{\acute{o}}_1)$	$\begin{pmatrix}0.3e^{0.5\pi i},\\ 0.5e^{0.4\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.1\pi i},\\ 0.4e^{0.8\pi i}\end{pmatrix}$	$\begin{pmatrix}0.4e^{0.3\pi i},\\ 0.2e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.7e^{0.7\pi i},\\ 0.2e^{0.3\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_1, \acute{\acute{o}}_2)$	$\begin{pmatrix}0.3e^{0.5\pi i},\\ 0.5e^{0.4\pi i}\end{pmatrix}$	$\begin{pmatrix}0.3e^{0.1\pi i},\\ 0.6e^{0.8\pi i}\end{pmatrix}$	$\begin{pmatrix}0.4e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.6e^{0.4\pi i},\\ 0.3e^{0.3\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_1, \acute{\acute{o}}_3)$	$\begin{pmatrix}0.3e^{0.4\pi i},\\ 0.5e^{0.4\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.1\pi i},\\ 0.4e^{0.8\pi i}\end{pmatrix}$	$\begin{pmatrix}0.4e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.5\pi i},\\ 0.4e^{0.3\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_2, \acute{\acute{o}}_1)$	$\begin{pmatrix}0.3e^{0.5\pi i},\\ 0.5e^{0.4\pi i}\end{pmatrix}$	$\begin{pmatrix}0.3e^{0.1\pi i},\\ 0.6e^{0.8\pi i}\end{pmatrix}$	$\begin{pmatrix}0.4e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.6e^{0.4\pi i},\\ 0.3e^{0.3\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_2, \acute{\acute{o}}_2)$	$\begin{pmatrix}0.7e^{0.7\pi i},\\ 0.2e^{0.3\pi i}\end{pmatrix}$	$\begin{pmatrix}0.3e^{0.4\pi i},\\ 0.6e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.9\pi i},\\ 0.3e^{0.1\pi i}\end{pmatrix}$	$\begin{pmatrix}0.6e^{0.4\pi i},\\ 0.3e^{0.2\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_2, \acute{\acute{o}}_3)$	$\begin{pmatrix}0.5e^{0.4\pi i},\\ 0.3e^{0.3\pi i}\end{pmatrix}$	$\begin{pmatrix}0.3e^{0.4\pi i},\\ 0.6e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.4\pi i},\\ 0.4e^{0.3\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_3, \acute{\acute{o}}_1)$	$\begin{pmatrix}0.3e^{0.4\pi i},\\ 0.5e^{0.4\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.1\pi i},\\ 0.4e^{0.8\pi i}\end{pmatrix}$	$\begin{pmatrix}0.4e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.5\pi i},\\ 0.4e^{0.3\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_3, \acute{\acute{o}}_2)$	$\begin{pmatrix}0.5e^{0.4\pi i},\\ 0.3e^{0.3\pi i}\end{pmatrix}$	$\begin{pmatrix}0.3e^{0.4\pi i},\\ 0.6e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.4\pi i},\\ 0.4e^{0.3\pi i}\end{pmatrix}$
$(\acute{\acute{o}}_3, \acute{\acute{o}}_3)$	$\begin{pmatrix}0.5e^{0.4\pi i},\\ 0.3e^{0.3\pi i}\end{pmatrix}$	$\begin{pmatrix}0.6e^{0.6\pi i},\\ 0.1e^{0.2\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.3\pi i},\\ 0.3e^{0.5\pi i}\end{pmatrix}$	$\begin{pmatrix}0.5e^{0.5\pi i},\\ 0.4e^{0.3\pi i}\end{pmatrix}$

The first three value of each parameter indicates the degrees of membership and non-membership assigned by experts, and the last value of each parameter shows the general belongingness of each parameter to the BCT and is denoted by λ. Then, its CP is shown in Table 6:

5.3. PyFSRs with CPyFSRS. The structure of PyFSS is explained by membership and non-membership degrees and the associated relations are known as PyFSRs. The structure of PyFSRs is improved by the limitation of FSRs and IFSRs, but they do not provide the time duration. The innovative structure of CPyFSRs discusses the degree of membership and non-membership with complex numbers and increases the space of CIFSRs. So, it provides comprehensive information on any problem. Table 7 summarizes the comparative study of CPyFSRs with a pre-defined structure.

Table 7: Summary of comparative analysis based on the structure.

Structure	Membership	Non-membership	Multi-dimension	Space
SR	No	No	No	No
FSR	Yes	No	No	n = 1
CFSR	Yes	No	Yes	n = 1
IFSR	Yes	Yes	No	n = 1
CIFSR	Yes	Yes	Yes	n = 1
PyFSR	Yes	Yes	No	n = 1
CPyFSR	Yes	Yes	Yes	n = 2

6. Conclusion

This paper defined the novel concept of CPyFSRs by using the CP of two CPyFSS. Furthermore, various types of CPyFSRs are also discussed, such as CPyFS-converse relation, CPyFS-reflexive relation, CPyFS-irreflexive relation, CPyFS symmetric relation, CPyFS anti-symmetric relation, CPyFS asymmetric relation, CPyFS-complete relation, CPyFS transitive relation, CPyFS equivalence relation, CPyFS-partial order relation, CPyFS-strict order relation, CPyFS preorder relation, and CPyFS equivalence classes. Some outcomes were proved with appropriate examples. Moreover, this novel concept of CPyFSRs has used the application of BCT. The goal of this application is to find the most effective BCT. The BCT represents the different parameters. The expert gives the membership and non-membership values of each BCT parameter. Then, using the score function, they choose the best BCT based on a set of parameters. The score function is used in this article to choose the best object, or anything based on some parameters. Finally, the innovative framework of the CPyFSRs is the more generalization form of all the pre-determined structures. Because it discusses both the degrees of membership and non-membership with complex numbers. The CPyFSRs can solve periodicity.The proposed work is more generalized then the bundle of exiting ideas, for instance, fuzzy relations, soft relations, complex fuzzy relations, fuzzy soft relations, complex fuzzy soft relations, intuitionistic fuzzy relations, intuitionistic fuzzy soft relations, complex intuitionistic fuzzy relations, complex intuitionistic fuzzy soft relations, Pythagorean fuzzy soft relations, and complex Pythagorean fuzzy soft relations, they all are the special cases of the pioneered relations.For better approach in the future, we aim to utilize different types of aggregation operators, hybrid aggregation operators, similarity measures, distance measures, TOPSIS technique, VIKOR technique, AHP technique, and MARCOS technique in the environment of CPyFSR and justified their application with the help of medical diagnosis, pattern recognition, network signals, artificial intelligence, risk analysis, and game theory are to enhance the quality of the presented information.

Data Availability

All the data used is given in the paper.

Conflicts of Interest

The authors do not have any conflicts.

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