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*Research Article*

# Dynamical Behavior of a Predator-Prey System Incorporating a Prey Refuge with Impulse Effect

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A predator-prey model with Holling II functional response incorporating a prey refuge with impulse effect is considered in this paper. With the help of the Floquet theory of impulsive differential equations, local stability and global attractivity of the boundary periodic solution of the system are derived, and then sufficient conditions for global asymptotic stability of the boundary periodic solution are obtained. Next, the permanence of the system is proved by constructing a Lyapunov function. Further, by applying the bifurcation theory of impulsive differential equations, conditions under which the system has a positive periodic solution are obtained. Finally, numerical simulations are presented to illustrate the analytical results.

## 1. Introduction

The study of population dynamic system has always been the focus of scholars. Population dynamics behavior is always affected by many factors. Smith [1] pointed out that scholars often study the effects on the behaviors such as territorial behavior and migration. The challenge of reducing the negative environmental impacts of land use and maintaining economic and social benefits is considered [2]. Wildlife species are also affected by agricultural activities, such as farming, intercropping, drainage, rotation, grazing, and the widespread use of pesticides and fertilizers [3]. One of the more relevant behavioral traits that affect the dynamics of predator-prey systems is the use of spatial refuges by the prey. Some fractions of the prey population are partially protected against predators by using refuges. González-Olivares et al. [4] proposed a predator-prey system with Holling type II functional response incorporating a constant prey refuge:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = \alpha x(t)\left(1 - \dfrac{x(t)}{K}\right) - \dfrac{\beta(x(t)-m)y(t)}{1 + a(x(t)-m)}, \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = -\mathrm{d}y(t) + \dfrac{c\beta(x(t)-m)y(t)}{1 + a(x(t)-m)}. \end{cases} \tag{1}$$

They investigated the local stability of equilibria and the existence of limit cycle of the system. Chen et al. [5] discussed the instability and global stability properties of the equilibria and the existence and uniqueness of limit cycle of the above system. The predator-prey systems with prey refuge have been focused by several authors. Tripathi et al. studied the model with Beddington-DeAngelis type function response incorporating a prey refuge [6, 7], the model with time delay and prey refuge [8], and the model for prey with variable rates in protected areas [9]. Mondal and Samanta [10] investigated the model with prey refuge dependent on both species and constant harvest in predator [10] and the model with nonlinear prey refuge [11, 12]. Many models with prey refuge have been studied; one can also refer to [13–21] and the references cited therein.

It has been noticed that many dynamical systems may be disturbed by impulsive factors. With profound understanding of human nature, the theory of impulsive differential equations [22–24] becomes more perfect. The impulsive differential equation has become a widely concerned subject in recent years, and it is more appropriate to apply to biological systems for the actual reality, for example [25–35].

Shea and Amarasekare [33] pointed out that "conservation, harvesting, and pest control are three aspects of the same general problem: population management." People seek to maintain exploited populations at productive levels by harvesting.

The results show that the population model with refuge is practical. Many periodic factors, such as seasonal periodic change, migration, and harvest, have a great impact on population dynamics. The model with refuge and impulsive effect has also been studied by many scholars, one can refer to [13, 34, 35]. Driven by the above reasons, we consider system (1) with prey subject to periodic impulse harvesting, the model is as follows:

$$\begin{cases} \left.\begin{aligned} \frac{\mathrm{d}x(t)}{\mathrm{d}t} &= \alpha x(t)\left(1-\frac{x(t)}{K}\right)-\frac{\beta(x(t)-m)y(t)}{1+a(x(t)-m)}, \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= -\mathrm{d}y(t)+\frac{c\beta(x(t)-m)y(t)}{1+a(x(t)-m)}, \end{aligned}\right\} t\neq nT, \\ \left.\begin{aligned} \triangle x(t) &= -px(t), \\ \triangle y(t) &= 0, \end{aligned}\right\} t=nT. \end{cases} \tag{2}$$

where $x(t)$ and $y(t)$ are the population density of prey and predator, respectively, $\alpha$ represents the intrinsic growth rate, $K$ is the environment carrying capacity, $\beta$ is the capture rate of predator, $d$ is the death rate of the predator, $c$ is the conversion factor denoting the number of newly born predators for each captured prey, $m$ is a constant number of prey using refuges, which protect prey from predation, and $0<p<1$ is impulse capture rate of prey. We assume that the population density of prey is more than the constant $m(m<x(t))$, $t$, $d$, $K$, $\alpha$, $\beta$, $c$, $a$, and $m$ are all positive constants, $\triangle x(t)=x(t^+)-x(t)$, $\triangle y(t)=y(t^+)-y(t)$, and $T$ is the period of the impulse effect.

The rest of this paper is organized as follows. Some definitions and preliminary lemmas are introduced in Section 2. Existence and global attractivity of boundary periodic solution of system (2) are discussed in Section 3. Section 4 contributes to permanence of system (2), and Section 5 aims at discussion of positive periodic solution. Section 6 contains some numerical simulations and illustrates our analytical results. Some simple discussions are given in Section 7.

## 2. Definitions and Preliminary Lemmas

Let $R_+=[0,\infty)$ and $R_+^2=\left\{\|z\in R^2z\geq 0\right\}$. Denote the map $f=(f_1,f_2)$ defined by the right hand of system (2). Let $V$: $R_+\times R_+^2\longrightarrow R_+$, then $V$ is said to belong to class $V_0$, if

(1) $V$ is continuous in $(nT,(n+1)T]$ and $\lim_{(t,u)\longrightarrow(nT,z),t>nT} V(t,u)=V(nT^+,z)$

(2) $V$ is locally Lipschitzian in $x$

*Definition 1.* If $V\in V_0$, then for $(t,z)\in(nT,(n+1)T]\times R_+^2$, the upper right derivative of $V(t,x)$ of system (2) is defined as $D^+V\quad(t,z)=\lim_{h\longrightarrow 0}\sup 1/h[V(t+h,z\quad+hf(t,z))-V(t,z)]$.

It follows from the second equation of (2) that $y(t)\geq 0$ for $y(0)\geq 0$, so

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}\geq\alpha x(t)\left(1-\frac{x(t)}{K}\right)-\frac{\beta x(t)y(t)}{1+a(x(t)-m)}. \tag{3}$$

Therefore, the following Lemma 1 is obvious.

**Lemma 1.** *If $x(t)$ is a solution of system (2) with $x(0^+)\geq 0$, then $x(t)\geq 0$ for all $t\geq 0$.*

*Definition 2.* System (2) is said to be permanent if there are constants $m,M>0$ (independent of initial value) and a finite time $T_0$, such that $m\leq x(t)\leq M$ and $m\leq y(t)\leq M$ when $t\geq T_0$ for all solutions $(x(t),y(t))$, with all initial values $x(0^+)>0$ and $y(0^+)>0$.

**Lemma 2** (see [22]). *Let $V$: $R_+\times R^n\longrightarrow R_+$ and $V\in V_0$. Assume that*

$$\begin{cases} D^+V(t,z)\leq h(t,V(t,z)),\ t\neq nT, \\ V(t,z(t^+))\leq\psi_n(V(t,z)),\ t=nT, \end{cases} \tag{4}$$

*where $h$: $R_+\times R_+\longrightarrow R$ satisfies $\psi_n$: $R_+\longrightarrow R_+$ is nondecreasing; let $r(t)$ be the maximal solution of the following scalar impulsive differential equation:*

$$\begin{cases} \frac{\mathrm{d}u(t)}{\mathrm{d}t}=h(t,u(t)),t\neq nT, \\ u(t^+)=\psi_n(u(t)),t=nT, \\ u(0^+)=u_0\geq 0, \end{cases} \tag{5}$$

*existing on $[t_0,+\infty)$. Then, $V(t_0^+,x_0)\leq u_0$ implies that*

$$V(t,z(t))\leq r(t),t\geq t_0, \tag{6}$$

*where $z(t)=z(t,t_0,z_0)$ is any solution of system (2) existing on $[t_0,+\infty)$.*

Now we first consider some basic properties of the following subsystem of system (2):

$$\begin{cases} \frac{dx(t)}{dt} = \alpha x(t)\left(1 - \frac{x(t)}{K}\right), t \neq nT, \\ \triangle x(t) = -px(t), t = nT. \end{cases} \tag{7}$$

**Lemma 3** (see [36]). *System (7) has a positive periodic solution $x^*(t)$. Any solution $x(t)$ of system (7) satisfies $x(t) \longrightarrow x^*(t)$ as $t \longrightarrow \infty$. It is easy to see that*

$$x^*(t) = \frac{K(1 - p - \exp(-\alpha T))}{p \exp(-\alpha(t - nT)) + (1 - p - \exp(-\alpha T))}, t \in (nT, (n+1)T]. \tag{8}$$

Therefore, $x^*(t)$ is the positive periodic solution of system (7).

## 3. Global Attractivity of Boundary Periodic Solution

First of all, we give the extinction of predator.

**Theorem 1.** *Let $(x(t), y(t))$ be any solution of system (2), if*

$$\begin{aligned} &T > -\frac{\ln(1-p)}{\alpha}, \\ &\left(-d - \frac{c\beta m}{q}\right)T + \frac{c\beta K}{\alpha(q^2 - Kaq)} \ln \frac{qp + \exp(\alpha T)(qR + KRa)}{qp + qR + KRa} < 0, \end{aligned} \tag{9}$$

*then the stability of boundary periodic solution $(x^*(t), 0)$ is globally asymptotically stable, where $q = 1 - am$ and $R = 1 - p - \exp(-\alpha T) > 0$.*

*Proof.* Firstly, we will prove the local stability of boundary periodic solution. Let $x(t) = u(t) + x^*(t)$ and $y(t) = v(t)$, then the solution of variation equation of system (2) is

$$\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \end{pmatrix}, \tag{10}$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} \alpha - \frac{2\alpha x^*(t)}{K} & -\frac{\beta(x^*(t) - m)}{1 + a(x^*(t) - m)} \\ 0 & -d + \frac{c\beta(x^*(t) - m)}{1 + a(x^*(t) - m)} \end{pmatrix} \Phi(t), \tag{11}$$

and $\Phi(0) = I$, in which $I$ is the identity matrix. Hence, the fundamental solution matrix is

$$\Phi(T) = \begin{pmatrix} \exp\left(\int_0^T \left(\alpha - \frac{2\alpha x^*(t)}{K}\right)\right) dt & * \\ 0 & \int_0^T \left(-d + \frac{c\beta(x^*(t) - m))}{1 + a(x^*(t) - m)}\right) dt \end{pmatrix}. \tag{12}$$

Then, the third equation and the forth equation of system (2) are linearized to be

$$\begin{pmatrix} u(nT^+) \\ v(nT^+) \end{pmatrix} = \begin{pmatrix} 1 - p & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \end{pmatrix}. \tag{13}$$

Stability of boundary periodic solution depends on the eigenvalue of $J$ by

$$J = \begin{pmatrix} 1-p & 0 \\ 0 & 1 \end{pmatrix} \Phi(T), \tag{14}$$

and the eigenvalues of $J$ are

$$\mu_1 = (1-p)\exp\left(\int_0^T \left(\alpha - \frac{2\alpha x^*(t)}{K}\right) dt\right),$$
$$\mu_2 = \exp\left(\int_0^T \left(-d + \frac{c\beta(x^*(t)-m)}{1+a(x^*(t)-m)}\right) dt\right). \tag{15}$$

According to the Floquet theory of impulsive equation, the boundary periodic solution $(x^*(t), 0)$ is locally asymptotically stable if $|\mu_1| < 1$ and $|\mu_2| < 1$; that is,

$$T > -\frac{\ln(1-p)}{\alpha},$$
$$\left(-d - \frac{c\beta m}{q}\right)T + \frac{c\beta K}{\alpha(q^2 - Kaq)} \ln\frac{qp + \exp(\alpha T)(qR + KRa)}{qp + qR + KRa} < 0. \tag{16}$$

Secondly, we will prove the global attractivity of boundary periodic solution $(x^*(t), 0)$. Let $\epsilon > 0$, so as to $\delta = \int_0^T (-d + c\beta(x^*(t) + \epsilon - m)/1 + a(x^*(t) + \epsilon - m))dt$; that is.

$$\left(-d - \frac{c\beta(m-\epsilon)}{1-a(m-\epsilon)}\right)T + \frac{c\beta K}{\alpha(q^2 - Kaq)} \ln\frac{qp + \exp(\alpha T)(qR + KRa)}{qp + qR + KRa} < 0, \tag{17}$$

where $q = 1 - am$, $R = 1 - p - \exp(-\alpha T)$.

From the first equation of system (2), it is easy to obtain

$$\frac{dx(t)}{dt} < \alpha x(t)\left(1 - \frac{x(t)}{K}\right). \tag{18}$$

Consider the following comparison system:

$$\begin{cases} \dfrac{dz(t)}{dt} = \alpha z(t)\left(1 - \dfrac{z(t)}{K}\right), t \neq nT, \\ \triangle z(t) = -pz, t = nT, \\ z(0^+) = x(0^+) > 0. \end{cases} \tag{19}$$

When $t$ is large enough, by Lemmas 2 and 3, we can obtain that

$$x(t) \le z(t) < x^*(t) + \epsilon. \tag{20}$$

For $t \ge 0$, we assume that $x(t) \le z(t) < x^*(t) + \epsilon$, and then we have

$$\frac{dy(t)}{dt} \le -dy + \frac{c\beta(x^*(t) - m + \epsilon)y}{1 + a(x^*(t) - m + \epsilon)}. \tag{21}$$

Integrating and solving the above system on $(nT, (n+1)T]$, we can derive that

$$y((n+1)T) \le y(nT)\exp\left(\int_{nT}^{(n+)T} \left(-d + \frac{c\beta(x^*(t) - m + \epsilon)}{1 + a(x^*(t) - m + \epsilon)}\right) dt\right). \tag{22}$$

Therefore, $y(nT) \le y(0^+)(1-p)^n\delta$, then $y(nT) \longrightarrow 0$ as $n \longrightarrow \infty$. When $t \in (nT, (n+1)T]$, we can easily know that $0 < y(nT) \le y(nT)H$, so $y(t) \longrightarrow 0$ as $t \longrightarrow \infty$, where $H = \sup_u \in (0, T] \exp(\int_0^u (-dc\beta(x^*(t) - m \quad +\epsilon)/1 + a(x^*(t) - m + \epsilon))dt)$.

Next, we prove $x(t) \longrightarrow x^*(t)$ as $t \longrightarrow \infty$. By $dx(t)/dt < \alpha x(t)(1 - x(t)/K)$, we can obtain $x(t) < x^*(t)$. By Lemma 3, we can get

$$x^*(t) = \frac{K(1-p-\exp(-\alpha T))}{p\exp(-\alpha(t-nT)) + (1-p-\exp(-\alpha T))} \le \frac{K(1-p-\exp(-\alpha T))}{(1-p)(1-\exp(-\alpha T))}, t \in (nT, (n+1)T]. \tag{23}$$

Let $m_2 = K(1-p-\exp(-\alpha T))/(1-p)(1-\exp(-\alpha T))$, we have proved that $x(t) < x^*(t) + \epsilon$ for any $\epsilon > 0$ when $t$ is large enough. Suppose that $x(t) \le m_2$ as $t > 0$. There exist $T_0 > 0$ and $m_1 > 0$ for $t \ge T_0$ such that $y(t) \le m_1$. Assume that $y(t) \le m_1$ as $t > 0$, it follows by system (2)

$$\frac{dx(t)}{dt} \ge \alpha x(t)\left(1 - \frac{x(t)}{K}\right) - \frac{\beta m_1(m_2 - m)}{1 + a(m_2 - m)}. \tag{24}$$

Consider the comparison system

$$\begin{cases} \dfrac{dw(t)}{dt} = \alpha w(t)\left(1 - \dfrac{w(t)}{K}\right) - \dfrac{\beta m_1(m_2-m)}{1+a(m_2-m)}, t \ne nT, \\ \triangle w(t) = -pw(t), t = nT, \\ w(0^+) = x(0^+) > 0. \end{cases} \tag{25}$$

By equation (25), we can obtain

$$w^*(t) = \exp(M(t-nT))\left(\frac{(1-p)Q_1(1-\exp(MT))}{1-(1-p)(1-\exp(MT))} - \frac{Q_1}{M}\right) + \frac{Q_1}{M}, \tag{26}$$

where $M = \alpha(1 - m_2/K) = \alpha p \exp(-\alpha T)/(1-p)(1-\exp(-\alpha T))$ and $Q_1 = \beta m_1(m_2 - m)/1 + a(m_2 - m)$.

By Lemmas 2 and 3, one can obtain that $w(t) \longrightarrow x^*(t)$ and $w(t) \longrightarrow w^*(t)$, as $t \longrightarrow \infty$. So, for any $\epsilon_2$, there exists a $T_1$, such that $x^*(t) - \epsilon_2 < x(t) < x^*(t) - \epsilon_2$ as $t > T_1$. Let $\epsilon_2 \longrightarrow 0$, then $x(t) \longrightarrow x^*(t)$ as $t \longrightarrow \infty$. The theorem is completely proved. □

## 4. Permanence of System (2)

**Theorem 2.** *There exists a constant $S > 0$ such that $x(t) < S$ and $y(t) < S$ for each solution $(x(t), y(t))$ of system (2) when $t$ is large enough.*

*Proof.* Suppose that $(x(t), y(t))$ is any solution of system (2). Let $V = cx(t) + y(t)$ and $0 < \lambda < d$; we calculate the upper right derivative of $V(t)$ along system (2) and get the following inequality:

$$\begin{aligned} D^+V(t) + \lambda V(t) &= c\alpha x(t)\left(1 - \frac{x(t)}{K}\right) - dy(t) \\ &\quad + c\lambda x(t) + \lambda y(t) \\ &= -\frac{c\alpha x^2(t)}{K} + c(\alpha+\lambda)x(t) + (\lambda - d)y(t) \\ &\le \frac{Kc(\alpha+\lambda)^2}{4\alpha}, t \ne nT. \end{aligned} \tag{27}$$

Therefore, it is bounded. We choose an $M_0$ such that

$$D^+V(t) \le -\lambda V(t) + M_0. \tag{28}$$

It is easy to know that

$$V(nT^+) = y(nT) + (1-p)x(nT) \le V(nT). \tag{29}$$

According to Lemma 2, we have

$$V(t) \le \left(V(0^+) - \frac{M_0}{\lambda}\right)\exp(-\lambda t) + \frac{M_0}{\lambda}, t \in (nT, (n+1)T]. \tag{30}$$

Therefore, $V(t)$ is ultimately bounded. There exists a constant $S > 0$ such that any solution $(x(t), y(t))$ of system (2) satisfies $x(t) \le S$ and $y(t) \le S$ for $t$ large enough. □

**Theorem 3.** *System (2) is permanent if*

$$\left(-d - \frac{c\beta m}{1-am}\right)T + \frac{c\beta K}{\alpha(q^2 - Kaq)}\ln\frac{qp + \exp(\alpha T)(qR + KRa)}{qp + qR + KRa} > 0, \tag{31}$$

*where $q = 1 - am$ and $R = 1 - p - \exp(-\alpha T)$.*

*Proof.* Suppose $(x(t), y(t))$ is any solution of system (2); we have proved there exists an $M$ such that $x(t) \le M$ and $y(t) \le M$. Let $h = K(1-p-\exp(-\alpha T))/1 - \exp(-\alpha T) - \epsilon$; we can know $x(t) > h$ when $t$ is large enough. We will find a constant $m_4$ such that $y(t) \ge m_4$ when $t$ is large enough, where $m_4 = m_3\exp((1+n_2+n_3)T\delta_2)$ and $m_3$ is a constant. Next, we consider the following two cases. Firstly, there exists a sufficient small $\epsilon_1$ such that

$$\delta_1 = \int_0^T\left(-d + \frac{c(v^*(t) - m - \epsilon_1)}{1 + a(x^*(t) - m - \epsilon_1)}\right)dt. \tag{32}$$

We can prove $y(t) < m_3$ is not satisfied for any $t$; otherwise,

$$\frac{dx(t)}{dt} > \alpha x(t)\left(1 - \frac{m_2}{K}\right) - \frac{\beta(m_2 - m)m_3}{1 + a(m_2 - m)}. \tag{33}$$

Consider the comparison system

$$\begin{cases} \frac{dv(t)}{dt} = \alpha v(t)\left(1 - \frac{v(t)}{K}\right) - \frac{\beta m_3(m_2 - m)}{1 + a(m_2 - m)}, t \neq nT, \\ \triangle v(t) = -pv(t), t = nT, \\ v(0^+) = x(0^+) > 0. \end{cases} \quad (34)$$

It is not hard to get $v^*(t) = \exp(M(t - nT))((1 - p)Q_2(1 - \exp(MT))/1 - (1 - p)(1 - \exp(MT)) - Q_2/M) + Q_2/M$,where

$$M = \alpha\left(1 - \frac{m_2}{K}\right) = \frac{\alpha p \exp(-\alpha T)}{(1 - p)(1 - \exp(-\alpha T))},$$
$$Q_2 = \frac{\beta m_3(m_2 - m)}{1 + a(m_2 - m)}. \quad (35)$$

Therefore, there exists a $t_2 > 0$ such that $v^*(t) - \epsilon_1 < v^*(t) \leq x(t)$ for $t > t_2$. Let $n_1 \in \mathbb{N}$ and $n_1 T \geq t_1$ as $n > n_1$. It is easy to know that

$$\frac{dy(t)}{dt} \geq y(t)\left(-d + \frac{c\beta(v^*(t) - \epsilon_1 - m)}{1 + a(v^*(t) - \epsilon_1 - m)}\right). \quad (36)$$

Integrating and solving the above system on $(nT, (n + 1)T]$, we can derive that

$$y((n + 1)T) \geq y(nT)\exp\left(\int_{nT}^{(n+1)T}\left(-d + \frac{c\beta(v^*(t) - \epsilon_1 - m)}{1 + a(v^*(t) - \epsilon_1 - m)}\right)dt\right) = y(nT)\exp(\delta_1). \quad (37)$$

Then, $j \longrightarrow \infty, y((n + j)T) \geq y(nT)\exp(j\delta_1) \longrightarrow \infty$; it contradicts with $y(t) < m_3$. Thus, there is a $t_0 > 0$ such that $y(t_0) \geq m_3$.

Secondly, if $t > t_0$, then $y(t) \geq m_3$. Otherwise, let $t^* = \inf_{t > t_0}\{t | y(t) < m_3\}$, we can easily know $y(t) \geq m_3$ when $t \in [t_1, t^*)$, choose $n_2, n_3 \in \mathbb{N}$ such that $\exp((n_2 + q)\delta_2 T)\exp(n_3\delta_1 T) > 1$, where $\delta_2 = -d + c\beta(h - m)/1 + a(h - m)$. Let $\overline{T} = (n_2 + n_3)T$, then there is a $t_2 \in [n_1 T, (n_1 + 1)T + \overline{T}]$ such that $y(t_2) \geq m_3$. Otherwise, $y(t) < m_3$ as $t \in [n_1 T, (n_1 + 1)T + \overline{T}]$. From system (25), we can obtain

$$y((n_1 + 1 + n_2 + n - 3)T) \geq y(n_1 + 1 + n_2)\exp(n_3\delta_1). \quad (38)$$

as $dy(t)/dt = -dy(t) + c\beta(x(t) - m)y(t)/1 + a(x(t) - m)$, so $dy/dt \geq -dy(t) + c\beta(h - m)y(t)/1 + a(h - m)$.

Integrating and solving the above system on $(t^*, (n_1 + n_2 + 1)T]$, we can derive that $y((n_1 + n_2 + 1)T) \geq m_3 \exp((n_2 + 1)\delta_2)T)$, so $y((n_1 + 1 + n_2 + n - 3)T) \geq m_3 \exp((n_2 + 1)\delta_2 T)\exp(n_3\delta_1 T) > m_3$, it contradicts with the assumption $y(t) < m_3$. Let $\overline{t} = \inf\{t | y(t) \geq m_3\}, t > t^*$, we have $x(t) = m_3 \exp((1 + n_2 + n_3)T\delta_2) \triangleq m_4$. For $t > \overline{t}$, as $x(\overline{t}) \geq m_3$, we discuss it in the same way, at last we obtain $y(t) \geq m_4$, for any $t \geq t_1$. □

## 5. Existence of Positive Periodic Solution

In this section, we will study the existence of positive periodic solution by bifurcation theory of impulsive differential equations. In order to be consistent with [37], we denote $x(t)$, $y(t)$ as $x_1(t)$, $x_2(t)$, respectively. System (2) is

$$\begin{cases} \left.\begin{aligned} \frac{dx_1(t)}{dt} &= \alpha x_1(t)\left(1 - \frac{x_1(t)}{K}\right) - \frac{\beta(x_1(t) - m)x_2(t)}{1 + a(x_1(t) - m)} \triangleq F_1(x_1(t), x_2(t)), \\ \frac{dx_2(t)}{dt} &= -dx_2(t) + \frac{c\beta(x_1(t) - m)x_2(t)}{1 + a(x_1(t) - m)} \triangleq F_2(x_1(t), x_2(t)), \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} x_1(nT^+) &= (1 - p)x_1(nT) \triangleq \theta_1(x_1(nT), x_1(nT)), \\ x_2(nT^+) &= x_2(nT) \triangleq \theta_2(x_1(nT), x_1(nT)), \end{aligned}\right\} t = nT. \end{cases} \quad (39)$$

By formally deriving the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\Phi\left(t,x\left(0\right)\right)\right)=F\left(\Phi\left(t,x\left(0\right)\right)\right), \tag{40}$$

which characterized the dynamics of the unperturbed flow associated to the first two equations in (2), we obtain that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[D_x\Phi\left(t,x\left(0\right)\right)\right]=D_xF\left(\Phi\left(t,x\left(0\right)\right)\right)D_x\Phi\left(t,x\left(0\right)\right). \tag{41}$$

This relation will be integrated in what follows in order to compute the components of $D_x\Phi\left(t,x^{(0)}\right)$ explicitly. Firstly, it is clear that

$$\Phi\left(t,x_0\right)=\left(\phi_1\left(t,x_0\right),\phi_2\left(t,x_0\right)\right). \tag{42}$$

Then, we derive that

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\dfrac{\partial\phi_1}{\partial x_1} & \dfrac{\partial\phi_1}{\partial x_2}\\[2ex] \dfrac{\partial\phi_2}{\partial x_1} & \dfrac{\partial\phi_2}{\partial x_2}\end{pmatrix}=\begin{pmatrix}\alpha-\dfrac{2\alpha x^*\left(t\right)}{K} & -\dfrac{\beta\left(x^*\left(t\right)-m\right)}{1+a\left(x^*\left(t\right)-m\right)}\\[2ex] 0 & -d+\dfrac{c\beta\left(x^*\left(t\right)-m\right)}{1+a\left(x^*\left(t\right)-m\right)}\end{pmatrix}\begin{pmatrix}\dfrac{\partial\phi_1}{\partial x_1} & \dfrac{\partial\phi_1}{\partial x_2}\\[2ex] \dfrac{\partial\phi_2}{\partial x_1} & \dfrac{\partial\phi_2}{\partial x_2}\end{pmatrix}\left(t,x_0\right). \tag{43}$$

The initial condition at $t=0$ being $D_x\Phi\left(0,x_0\right)=I$. Here $I$ is the identity matrix. It follows that

$$\frac{\partial\phi_2\left(t,x_0\right)}{\partial x_1}=\exp\left(\int_0^t\left(-d+\frac{c\beta\left(x^*\left(s\right)-m\right)}{1+a\left(x^*\left(s\right)-m\right)}\right)\mathrm{d}s\right)\frac{\partial\phi_2\left(0,x_0\right)}{\partial x_1}. \tag{44}$$

As $D_x\Phi\left(0,x_0\right)=I$, we can get $\partial\phi_2\left(t,x_0\right)/\partial x_1=0$ for $t\geq 0$. We can obtain from (43) that

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\phi_1\left(t,x_0\right)}{\partial x_1}&=\left(\alpha-\frac{2\alpha x^*\left(t\right)}{K}\right)\frac{\partial\phi_1\left(t,x_0\right)}{\partial x_1},\\
\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\phi_1\left(t,x_0\right)}{\partial x_2}&=\left(\alpha-\frac{2\alpha x^*\left(t\right)}{K}\right)\frac{\partial\phi_1\left(t,x_0\right)}{\partial x_2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\phi_1\left(t,x_0\right)}{\partial x_2}-\frac{\beta\left(x^*\left(t\right)-m\right)}{1+a\left(x^*\left(t\right)-m\right)}\frac{\partial\phi_2\left(t,x_0\right)}{\partial x_2},\\
\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\phi_2\left(t,x_0\right)}{\partial x_2}&=\left(-d+\frac{\beta\left(x^*\left(t\right)-m\right)}{1+a\left(x^*\left(t\right)-m\right)}\right)\frac{\partial\phi_2\left(t,x_0\right)}{\partial x_2}.
\end{aligned} \tag{45}$$

According to the initial condition, we obtain that

$$\begin{aligned}
\frac{\partial\phi_1\left(t,x_0\right)}{\partial x_1}&=\exp\left(\int_0^t\left(\alpha-\frac{2\alpha x^*\left(s\right)}{K}\right)\mathrm{d}s\right),\\
\frac{\partial\phi_1\left(t,x_0\right)}{\partial x_2}&=\int_0^t\left[\exp\int_u^t\left(\alpha-\frac{2\alpha x^*\left(r\right)}{K}\right)\mathrm{d}r\left(-\frac{\beta\left(x^*\left(u\right)-m\right)}{1+a\left(x^*\left(u\right)-m\right)}\right)\exp\left(\int_0^u\left(-d+\frac{c\beta\left(x^*\left(r\right)-m\right)}{1+a\left(x^*\left(r\right)-m\right)}\right)\mathrm{d}r\right)\right]\mathrm{d}u,\\
\frac{\partial\phi_2\left(t,x_0\right)}{\partial x_2}&=\exp\left(\int_0^t\left(-d+\frac{\beta\left(x^*\left(r\right)-m\right)}{1+a\beta\left(x^*\left(r\right)-m\right)}\right)\mathrm{d}r\right).
\end{aligned} \tag{46}$$

From (4), we obtain that $D_xN\left(0,\ \left(0,0\right)\right)=I-D_x\Phi\left(T_0,x_0\right)$,which implies

$$D_xN(0,(0,0)) = \begin{pmatrix} a_0' & b_0' \\ 0 & d_0' \end{pmatrix}, \tag{47}$$

where $a_0'$, $b_0'$, and $d_0'$ are

$$\begin{aligned} a_0' &= 1-(1-p)\exp\left(\alpha T_0 - 2\ln\frac{(1-p-\exp(-\alpha T))\exp(\alpha T_0)+p}{1-\exp(-\alpha T)}\right) > 0, \\ b_0' &= (p-1)\int_0^{T_0}\left(\exp\left(\int_u^{T_0}\left(\alpha-\frac{2\alpha x^*(r)}{K}\right)\mathrm{d}r\right)\left(-\frac{\beta(x^*(u)-m)}{1+a(x^*(u)-m)}\right)\right. \\ &\quad \left.\exp\left(\int_0^u\int_0^{T_0}\left(-d+\frac{c\beta(x^*(r)-m)}{1+a(x^*(r)-m)}\mathrm{d}r\right)\mathrm{d}u\right) > 0, \\ d_0' &= 1-\exp\left(\int_0^{T_0}\left(-d+\frac{c\beta(x^*(r)-m)}{1+a(x^*(r)-m)}\right)\mathrm{d}r\right), \end{aligned} \tag{48}$$

where $T_0$ is the root of $d_0'=0$.

Firstly, we make sure the sign of $C$, where

$$C = 2\frac{\partial\theta_2}{\partial x_2}\frac{b_0'}{a_0'}\frac{\partial^2\phi_2(T_0,x_0)}{\partial x_1\partial x_2} - \frac{\partial\theta_2}{\partial x_2}\frac{\partial^2\phi_2(T_0,x_0)}{\partial x_2^2}. \tag{49}$$

For determining $C$, we must calculate $\partial^2\phi_2(T_0,x_0)/\partial x_2^2$ and $\partial^2\phi_2(T_0,x_0)/\partial x_1\partial x_2$.

We have

$$\frac{\partial^2\phi_2(T_0,x_0)}{\partial x_2^2} = \int_0^{T_0}\left\{\begin{array}{c}\left(\exp\left(\int_u^{T_0}\left(-d+\frac{c\beta(x^*(r)-m)}{1+a(x^*(r)-m)}\right)\mathrm{d}r\right)\right)\left(\frac{c\beta}{(1+a(x^*(u)-m))^2}\right) \\ \left[\exp\left(\left(\int_u^{T_0}\left(\alpha-\frac{2\alpha x^*(r)}{K}\right)\mathrm{d}r\right)\left(-\frac{\beta(x^*(u)-m)}{1+a^*(u)-m}\right)\exp\left(\int_0^p\left(\int_u^{T_0}\left(-d+\frac{c\beta(x^*(r)-m)}{1+a(x^*(r)-m)}\right)\mathrm{d}r\right)\mathrm{d}p\right)\right)\right]\end{array}\right\}\mathrm{d}u < 0. \tag{50}$$

Then, we have

$$\frac{\partial^2\phi_2(T_0,x_0)}{\partial x_1\partial x_2} = \int_0^{T_0}\left(\frac{c\beta}{(1+a(x^*(u)-m))^2}\right)\mathrm{d}u\ \exp\left(\int_0^{T_0}\left(-d+\frac{c\beta(x^*(r)-m)}{1+a(x^*(r)-m)}\right)\mathrm{d}r\right) > 0. \tag{51}$$

Therefore, $C>0$.

Secondly, we make sure the sign of $B$, where

$$\begin{aligned} B = &-\frac{\partial^2\theta_2}{\partial x_1\partial x_2}\left(\frac{\partial\phi_1(T_0,x_0)}{\partial\overline{T}} + \frac{\partial\phi_1(T_0,x_0)}{\partial x_1}\frac{1}{a_0'}\frac{\partial\phi_1(T_0,x_0)}{\partial\overline{T}}\right)\frac{\partial\phi_2(T_0,x_0)}{\partial x_2} \\ &-\frac{\partial\theta_2}{\partial x_2}\left(\frac{\partial^2\phi_2(T_0,x_0)}{\partial\overline{T}\partial x_2} + \frac{\partial^2\phi_2(T_0,x_0)}{\partial x_1\partial x_2}\frac{1}{a_0'}\frac{\theta_1}{x_1}\frac{\partial\phi_1(T_0,x_0)}{\partial\overline{T}}\right). \end{aligned} \tag{52}$$

Therefore, we have to calculate $\partial^2\phi_2(T_0,x_0)/\partial\overline{T}\partial x_2$,

$$\frac{\partial^2 \phi_2 (T_0, x_0)}{\partial \overline{T} \partial x_2} = \left( -d + \frac{c\beta (x^* (T_0) - m)}{1 + a(x^* (T_0) - m)} \right) \exp \left( \int_0^{T_0} \left( -d + \frac{c\beta (x^* (r) - m)}{1 + a(x^* (r) - m)} \right) \mathrm{d}r \right). \tag{53}$$

Then, we have

$$B = - \left( -d + \frac{c\beta (x^* (T_0) - m)}{1 + a(x^* (T_0) - m)} \right) \exp \left( \int_0^{T_0} \left( -d + \frac{c\beta (x^* (r) - m)}{1 + a(x^* (r) - m)} \right) \mathrm{d}r \right). \tag{54}$$

In order to make sure the sign of $B$, let

$$f(t) = -d + \frac{c\beta (x^* (t) - m)}{1 + a(x^* (t) - m)}. \tag{55}$$

Then,

$$f'(t) = \frac{c\beta}{(1 + a(x^* (u) - m))^2} \frac{K(1 - p - \exp(-\alpha T)) p\alpha \exp(-\alpha T)}{(1 - p - \exp(-\alpha T)) + p \exp(-\alpha t))^2} > 0. \tag{56}$$

As $d_0' = 0$, we can obtain $\int_0^{T_0} f(t) \mathrm{d}t = 0$. Because $f(t)$ is an increasing function, so $f(T_0) > 0$. According to equation, we can know $B < 0$, thus $BC < 0$. According to [37], the following conclusion can be derived.

**Theorem 4.** *System (2) has a critical bifurcation; there exists a positive periodic solution of system (2) if $t > T_0$, near $T_0$.*

## 6. Numerical Simulations

In this section, we investigate the effects of impulsive perturbations on system (2) by using numerical method to illustrate our theoretical results. For convenience, we assume keeping some parametric values of system (2) as $\alpha = 0.5$, $K = 4$, $c = 0.5$, $\beta = 0.4$, $m = 2$, $a = 0.2$, and $d = 0.2$. By simple calculation, we can get $T_0 \approx 2.4023$ for

$$d_0' = 1 - \exp \left( \int_0^{T_0} \left( -d + \frac{c\beta (x^* (r) - m)}{1 + a(x^* (r) - m)} \right) \mathrm{d}r \right) = 0. \tag{57}$$

Now, we choose different parameter $T$ to illustrate our main results by numerical simulation. Let $T = 1.5$, $x(0) = 3.3$, and $y(0) = 0.05$; bounded periodic solution $(x^* (t), 0)$ is globally asymptotically stable (see Figure 1); By Theorem 2, consider the comparison system, let $T = 3$, $x(0) = 3.3$, and $y(0) = 0.14$, and one can get system (2) is permanent (see Figure 2). By Theorem 4, let $T = 2.42$ nearly to $T_0$, $x(0) = 3.3$, and $y(0) = 0.005$; system (2) has a critical bifurcation (Figure 3). Let $T = 2.38$, $x(0) = 3.3$, and $y(0) = 0.005$; the number of predators population $y(t)$ of system (2) decreases significantly (Figure 4). Further, take the smaller $y(0) = 0.00001$, $T = 2.38$, and $x(0) = 3.3$, and we can obtain that the predator will be extinct (Figure 5).

We simulate the influence of the refuge constant $m$ on the system (see Figures 6 and 7) and the influence of pulse time $T$ on the system (see Figure 8).

## 7. Discussion

We have studied a predator-prey model with Holling II functional response incorporating a prey refuge and impulsive effect. Bounded periodic solution $(x^* (t), 0)$ is globally asymptotically stable if $T < T_0$. System (2) is permanent if $T > T_0$. By Figures 1 and 2, we show the facts above. System (2) has a critical bifurcation; there exists a positive periodic solution of system (2) if $t > T_0$, nearby $T_0$. The facts above are shown in Figure 3. By Theorem 1, one can know that the predator is extinct when $T$ is little smaller than $T_0$, as shown in Figure 5. It is easily known that $T_0$ is a threshold. From the view point of biology, if the harvest period $T$ of the prey is too short ($T < T_0$), that is, the harvest frequency is relatively frequent, the number of the prey is bound to be small, which cannot guarantee the food supply of the predator, resulting in the final extinction of the predator. On the contrary, if the harvest period exceeds the threshold ($T > T_0$), the food supply of the predator is sufficient, and the system is persistent. This provides a good theoretical basis for how to select appropriate parameters to maintain ecological balance.

The influence of refuge constant $m$ on system (2) can be seen in Figures 6 and 7. This shows that the predator becomes extinct and the number of prey remains unchanged when the prey refuge constant reaches a certain value. This shows that when the number of prey entering the shelter reaches a certain amount, the prey that the predator can catch is less and insufficient, and the supply of prey is in short supply, resulting in the extinction of the predator. It is consistent with the reality. The impact of the change of pulse

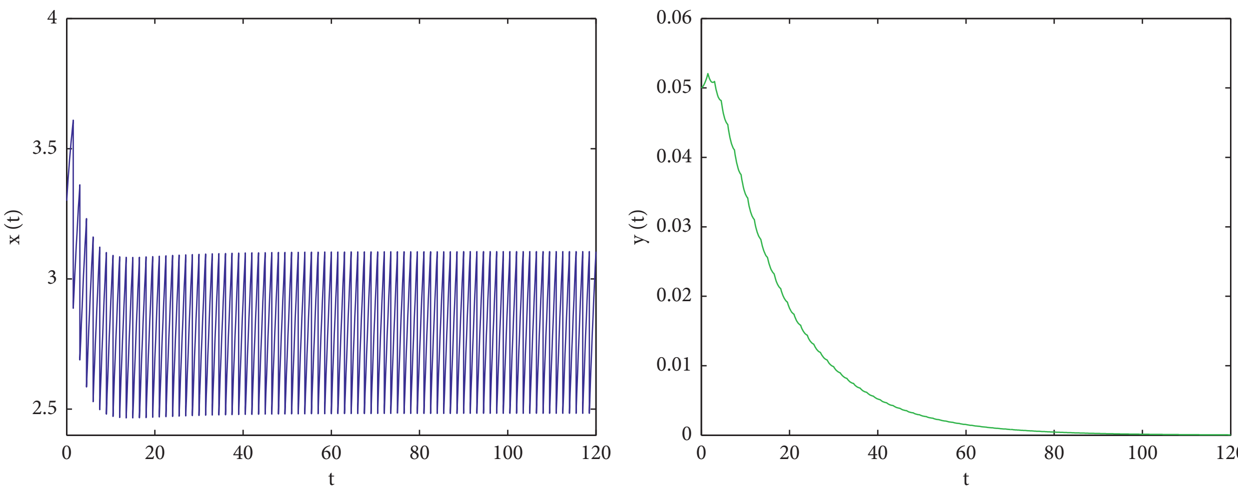


Figure 1: The $x(t)$ time series and $y(t)$ time series are simulated by numerical integration of system (2) with $T = 1.5$, $x(0) = 3.3$, and $y(0) = 0.05$. Bounded periodic solution $(x^*(t), 0)$ is globally asymptotically stable.

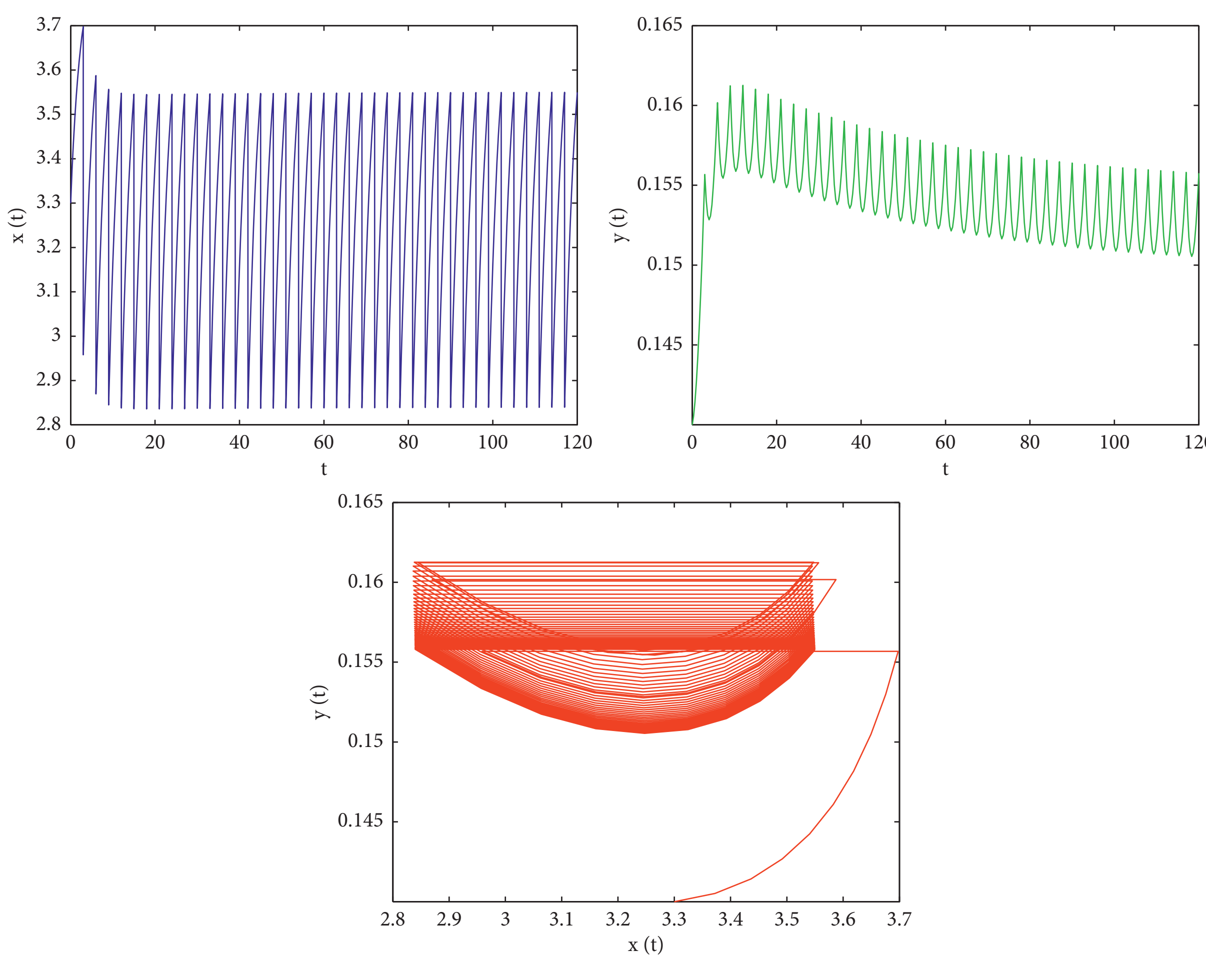


Figure 2: The $x(t)$ time series and $y(t)$ time series are simulated by numerical integration of system (2) with $T = 3$ and $x(0) = 3.3$ $y(0) = 0.14$. System (2) is permanent.

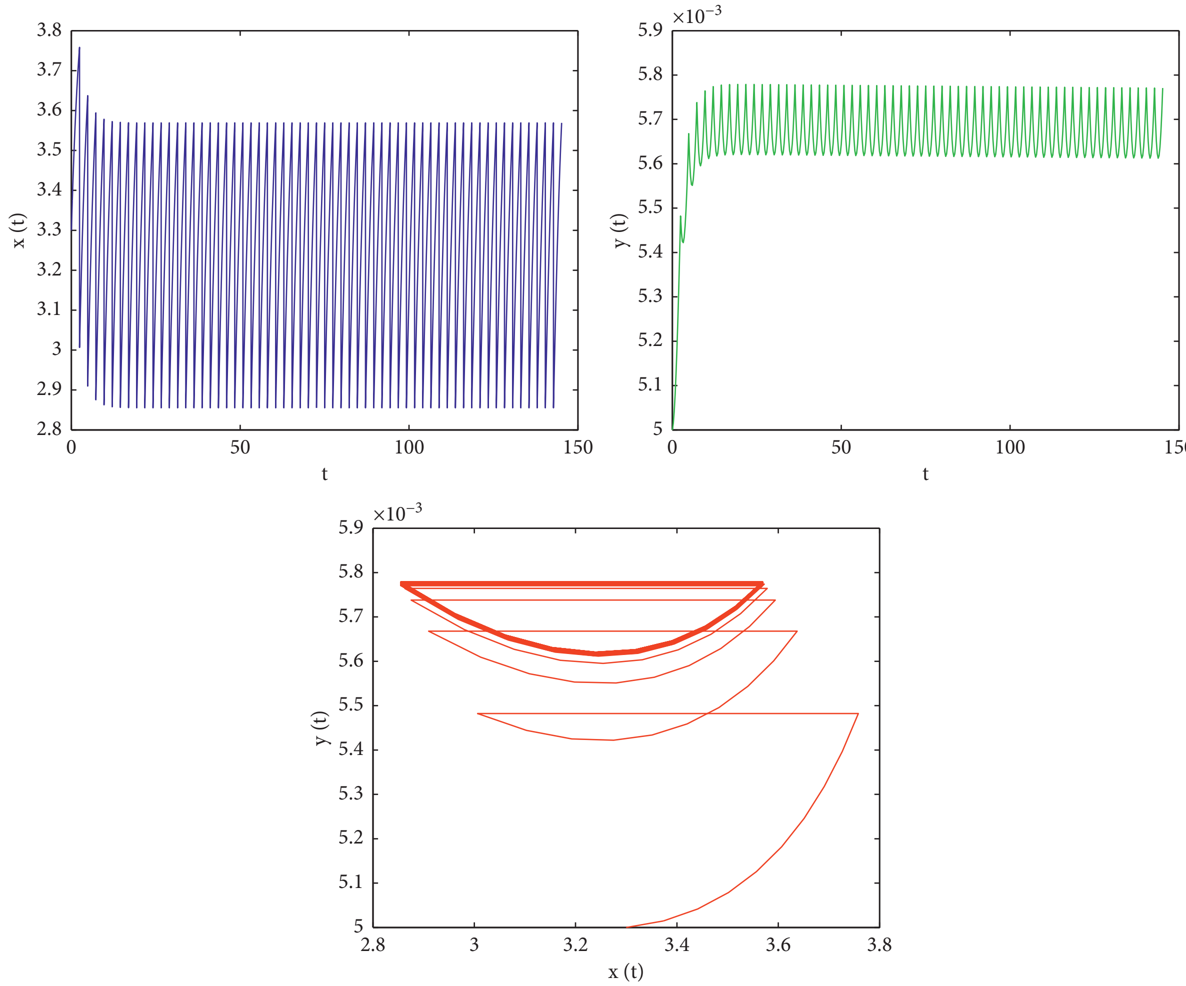


FIGURE 3: The $x(t)$ time series and $y(t)$ time series are simulated by numerical integration of system (2) with $T = 2.42$, nearly to $T_0$, $x(0) = 3.3$, and $y(0) = 0.005$. System (2) has a critical bifurcation.

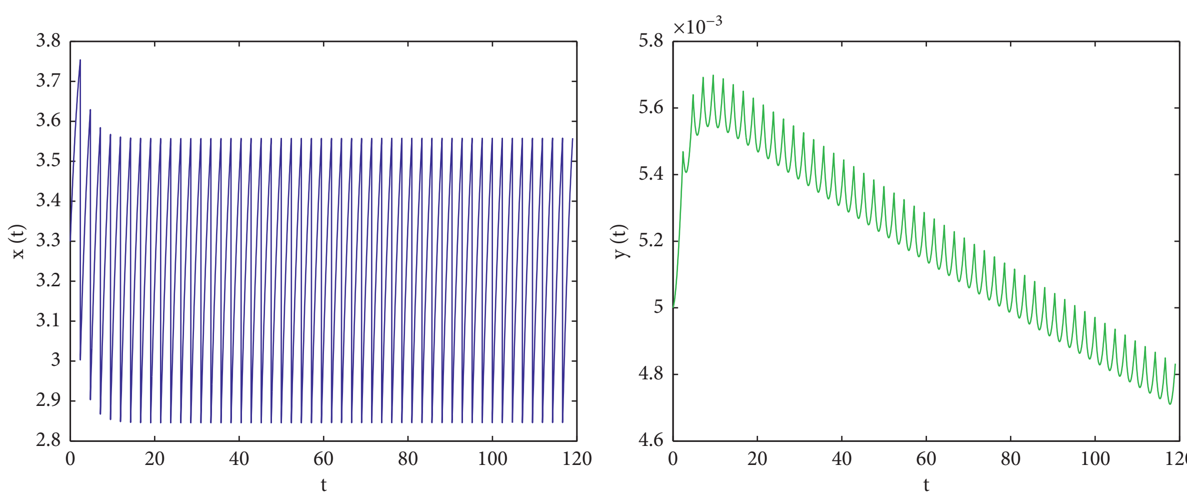


FIGURE 4: The $x(t)$ time series and $y(t)$ time series of system (2) with $T = 2.38$, $x(0) = 3.3$, and $y(0) = 0.005$.

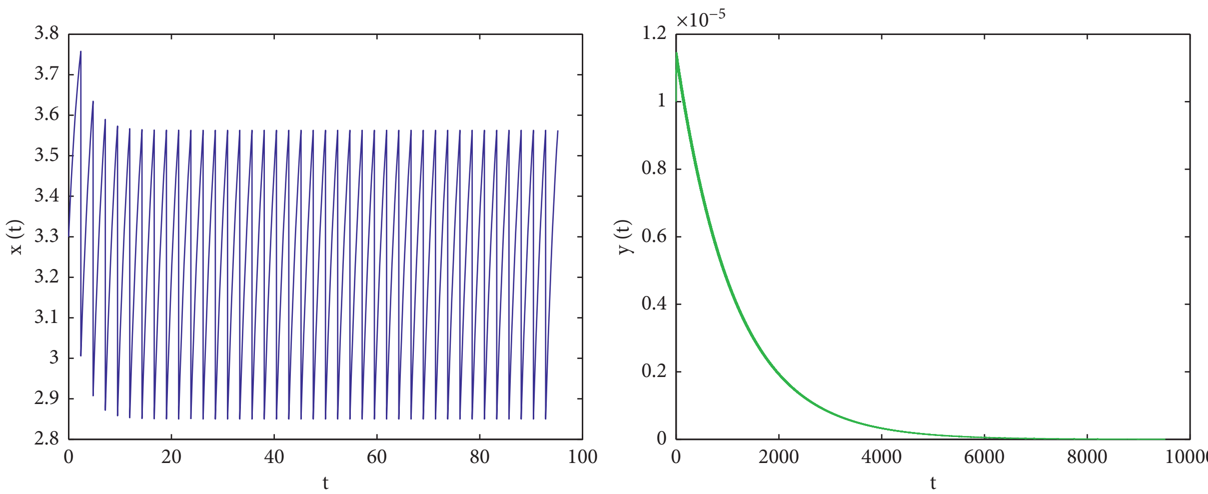


Figure 5: The $x(t)$ time series and $y(t)$ time series of system (2) with $T = 2.38$, $x(0) = 3.3$, and $y(0) = 0.00001$.

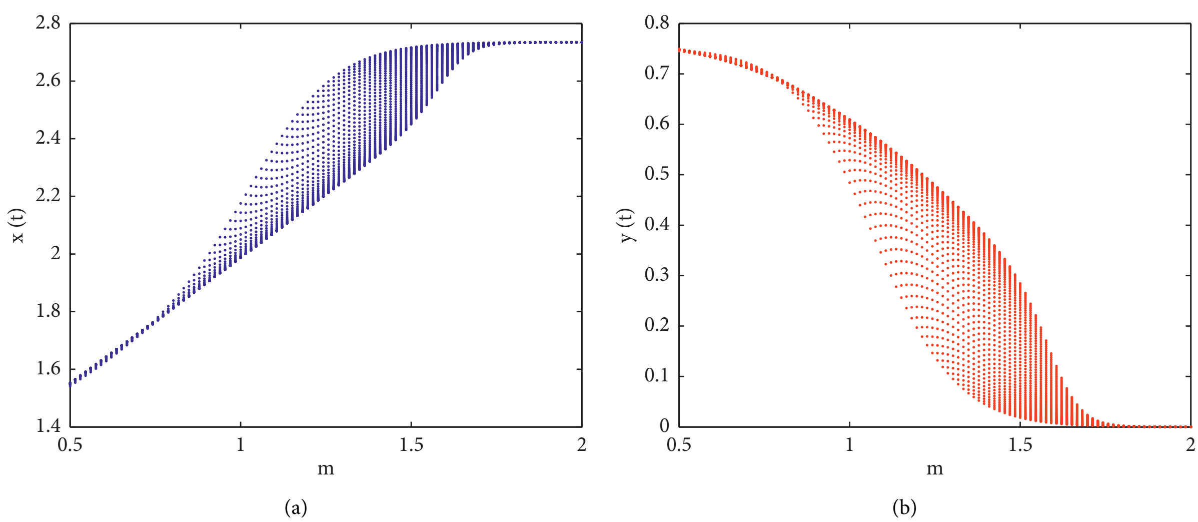


Figure 6: Bifurcation diagram of system (2) affected by prey refuge constant number $m$ with $T = 2$.

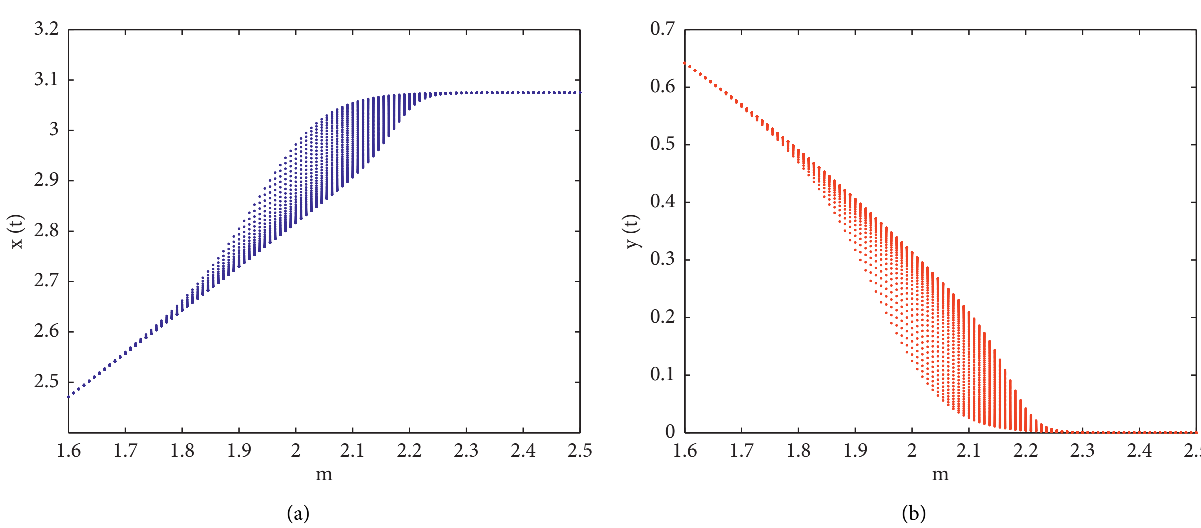


Figure 7: Bifurcation diagram of system (2) affected by prey refuge constant number $m$ with $T = 4$.

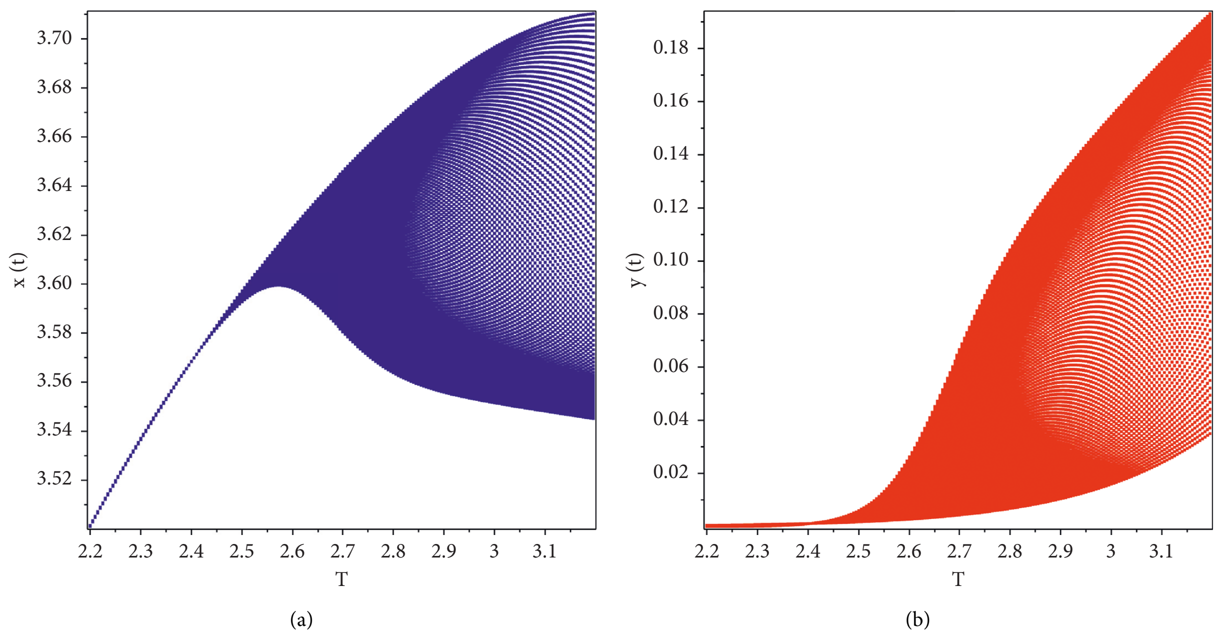


FIGURE 8: Bifurcation diagram of system (2) affected by impulsive periodic $T$.

time $T$ on system (2) is shown in Figure 8, this shows that the selection of pulse period $T$ also has an obvious impact on the system. From $(b)$ of Figure 8, we can see that the predator $y(t)$ is persistent when $T > T_0 \approx 2.4023$.

The effects of different pulse forms on the dynamic behavior of the system are also very different, so it is necessary for us to study the effects of these different pulse forms on the dynamic behavior of the system. The state feedback control of the system is also close to the real problem, we will study it in the following work.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

ZL and XY designed the study and carried out the analysis. ZL, XY, and SF wrote the paper. XY and SF performed numerical simulations. All authors read and approved the final manuscript.

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