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Research Article

Technological Change and Market Conditions: Evidence from Bitcoin Fork

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This article examines the impact of technological changes to cryptocurrency—known as "forking" that triggers blockchain splits—on market conditions. Despite the explicit distinction in log return distributions between the two splitting blockchains, adopting new technology does not result in a disparity in market conditions: no significant difference exists in market efficiency and long-term market equilibrium between the two splitting blockchains. Technological changes accompanying market separation do not impede the underlying uniformity in market conditions. The findings suggest that mutual information flows linked to market liquidity explain the results between the new and old forks.

1. Introduction

> "The 10,000 altcoins tried all manner laughable physics and computer science failed things."—Adam Back (on Twitter), *Forbes*, May 16, 2021.

The cryptocurrency market experiences technological changes in the form of forking, which is classified into two types: soft and hard. Both forks refer to changes to (i) a protocol of blockchain networks and (ii) data structures. A soft fork indicates a change that is backward compatible, whereas a hard fork denotes a change that is not backward compatible and results in two versions of the same blockchain. In particular, the fragile nature of consensus in blockchain technology (e.g., the debate over security issues) provides the opportunity for technological change, resulting in blockchain splits caused by a hard fork. The controversy about the benefits of and challenges to adopting new technology triggered the first hard fork—splitting Bitcoin. Although hard forks—particularly splitting Bitcoin—were implemented several times, this article concentrates on Bitcoin Cash, ranked within the top five in terms of market capitalization among all cryptocurrencies. This offshoot of Bitcoin, called Bitcoin Cash, referred to as the "old" fork, whereas the change-implemented Bitcoin is called the "new" fork. A hard fork, generated from the absence of a consensus or a divergence of beliefs, changes market sentiments and leads to diverging price movements. By creating multiple versions of the same blockchain, hard forks contribute to an increase in technological diversity. The old and new forks manifest large price fluctuations after the split: in the first five days, Bitcoin rose by approximately 19.7% and Bitcoin Cash declined by approximately 43.9%.

As a result, market fundamentals are examined and an underlying factor causing changes in market conditions is investigated further. In particular, we estimated the Hurst exponent (HE) to test the weak-form efficient market hypothesis (EMH) [1] and calculated the entropy to capture uncertainty and the degree of long-run market equilibrium. Despite significant differences in descriptive statistics and probability density functions, technological divergence does not result in a disparity in market conditions in terms of market efficiency and long-term market equilibrium. The factor driving the main results is explained by mutual information flows linked to market liquidity via transfer entropy.

Since the seminal work of Fama [2], prior studies on market conditions—particularly in economics literature—have focused on market efficiency. However, recent studies on the Bitcoin market provided mixed evidence: some studies reported evidence of inefficiency [3–10], whereas others demonstrated that the market is efficient [11, 12], or at the very least, moves toward efficiency with the launch of Bitcoin futures and liquidity expansion [13–16]. Moreover, although many studies have been conducted on Bitcoin market's efficiency, studies on the impact of forking on the cryptocurrency market, particularly with its efficiency, have been extremely limited.

Another strand of literature on Bitcoin's market conditions studies the uncertainty and randomness of price series by employing the concept of long-term equilibrium in the economic context. In this article, the long-run equilibrium refers to the statistical equilibrium of the balance of each system [17–19]. The findings mostly suggested that the Bitcoin market was rife with randomness, unpredictability, and disorder [20, 21]. However, prior studies on long-run equilibrium are still limited for the cryptocurrency market in comparison to other financial markets, such as stock [22, 23], energy [19, 24, 25], and real estate [17, 26, 27]. In particular, to the best of our knowledge, no single study has been conducted to assess the long-run equilibrium in the new and old fork markets by using the concept of entropy.

The remainder of this paper is organized as follows. Section 2 describes the data and methodology. Section 3 presents the results and discussion. Finally, Section 4 provides the conclusion.

2. Data and Methodology

2.1. Data. The daily prices of two splitting blockchains, such as Bitcoin and Bitcoin Cash, are retrieved immediately following the hard fork: the first hard fork splitting Bitcoin occurred on August 1, 2017. All data are in US dollars and are obtained from CoinMarketCap, which provides trading data, including the exchange activities of 2,543 cryptocurrencies in 20,295 markets. Our dataset spans the period from splitting the two blockchains to June 5, 2019, with 674 observations. For further analysis, both price series are stationarized, converting the price series into log returns. Table 1 summarizes the descriptive statistics.

Table 1: Descriptive statistics of daily log returns.

	Mean	Min.	Max.	Std.	Skewness	Kurtosis
Bitcoin	0.00	−0.21	0.23	0.04	0.01	6.34***
Bitcoin Cash	0.00	−0.45	0.43	0.09	0.65***	9.29***

Note: *** denotes significance at the 1% level.

Over the sample period, the log returns of Bitcoin Cash vary more significantly than those of Bitcoin, implying greater market uncertainty. The return series of old and new forks are distributed with nonnegative skewness and positive excess kurtosis. Unlike Bitcoin, which has a skewness close to zero, Bitcoin Cash has positive skewness, indicating an asymmetric distribution. Therefore, investors favor a positive skew and love risk in the old fork market, complying with the classical expected utility theory [28] and indicating a smaller downside risk [29]. The old fork also showed larger excess kurtosis than the new fork, which denotes a fatter tail, resulting in a more leptokurtic distribution.

2.2. Hurst Exponent. The HE was used to test the weak-form EMH. Following Hurst [30, 31] and Mandelbrot and Wallis [32, 33], we defined the R/S statistic as follows:

$$(R/S)_n = c \times n^{HE}, \tag{1}$$

where n is the length of the subseries, c is a constant, and $(R/S)_n$ is the mean value of the rescaled range for all subseries of length n. The R/S statistics and the estimated standard deviation S_n are given by the following:

$$\begin{aligned}(R/S)_n &= \frac{1}{S_n}\left[\max_{1\le t\le n}\sum_{k=1}^{t}\left(r_k-\overline{r}_n\right)-\min_{1\le t\le n}\sum_{k=1}^{t}\left(r_k-\overline{r}_n\right)\right],\\ S_n &= \sqrt{\frac{1}{n}\sum_{k=1}^{t}\left(r_k-\overline{r}_n\right)^2},\end{aligned} \tag{2}$$

where t is the number of successive subintervals, r_k denotes the return at time k, and $\overline{r}_n$ indicates the mean value of the return series.

To confirm the robustness of the results, we estimate the corrected HE [34] and the classical HE. Specifically, by estimating the slope of $(R/S - AL)_n$ versus n in a log-log plot, we defined the corrected HE as follows:

$$(R/S - AL)_n = (R/S)_n - E(R/S)_n + \sqrt{\frac{n\pi}{2}}, \tag{3}$$

where $E(R/S)_n$ is approximated by the following:

$$E(R/S)_n = \begin{cases} \dfrac{(n-0.5)\Gamma((n-1)/2)}{\sqrt[n]{\pi}\,\Gamma(n/2)}\displaystyle\sum_{i=1}^{n-1}\sqrt{(n-i)/i}, & (\text{for } n\le 340),\\[2ex] \dfrac{(n-0.5)}{\sqrt[n]{n\pi/2}}\displaystyle\sum_{i=1}^{n-1}\sqrt{(n-i)/i}, & (\text{for } n>340),\end{cases} \tag{4}$$

where Γ denotes the Euler gamma function.

2.3. Entropy. Entropy is used to measure the long-run market equilibrium and to capture uncertainty with a small information loss [22, 35, 36]. In particular, entropy measures the dispersion of probability allocation to each state rather than that of realized outcomes; therefore, it is robust to extremes other than volatility [37]. Following Shannon [38], we define entropy (H) for the discrete random variable X as follows:

$$H(X) = -\sum_{i=1}^{M} p(x_i)\ln p(x_i), \tag{5}$$

where $p(x_i)$ and M are the probability mass function and the number of states, respectively.

This study uses two key approaches to calculate the Shannon entropy to ensure the robustness: (i) Shannon entropy through histogram, which has long been demonstrated to be a rigorous density estimator [39], is relatively simple to draw. However, the feasibility could be more dependent on the sample size and thus limited in use [22] and (ii) Shannon entropy via symbolic time series analysis (STSA), which has been extensively applied in various fields of study (i.e., physics, information theory, and finance), has been verified to be robust to noise [22, 35] and competitive in capturing uncertainty, particularly with time series data in finance [25]. Meanwhile, STSA may necessitate a better command of demanding calculations. First, the histogram-based entropy of the discrete random variable X is obtained by the following equation:

$$H(X) = -\sum_{i=1}^{N} \hat{f}(x_i)\ln \hat{f}(x_i), \tag{6}$$

where N refers to the number of intervals and $\hat{f}(x_i)$ denotes a histogram estimate of the underlying probability mass function when X equals x_i [40].

Second, the dispersion of probability allocation onto the dynamic rise–fall pattern of consecutive price series was detected using STSA [22, 41]. The symbolization of consecutive return series is conducted as 1s for the positive returns and 0s for the others [42]. Subsequently, we determined the size of the rolling window to quantify the subsequence bundles composed of S binary numbers. Each subsequence bundle is converted from a binary sequence to a new decimal number, that is, X^S [22]. Then, the entropy of the random variable X^S is derived as follows:

$$H\left(X^S\right) = -\sum_{i=1}^{M-(S-1)} p\left(x_i^S\right)\log_2 p\left(x_i^S\right), \tag{7}$$

where M is the number of outcomes in the entire series. In the end, the normalized Shannon entropy is given by the following:

$$h\left(X^S\right) = \frac{1}{S} H\left(X^S\right). \tag{8}$$

Hereafter, the mention of "Shannon entropy" or simply "entropy" refers to a normalized one, that is, $h(X^S)$.

2.3.1. Transfer Entropy. Transfer entropy detecting the information flow between the two markets is calculated as a proxy for the cause–effect relationship. By considering the attributes of the interactions, transfer entropy quantifies the amount of information transport in a nonsymmetric manner. In particular, finding nonzero rates of information transmission in both directions implies a dynamic correlation in producing and receiving information between the two time series [43–45]. Following Schreiber [43], we define the transfer entropy from system Y to X as follows:

$$\begin{aligned} TE_{Y\longrightarrow X} = \sum p\left(x_{n+1}, x_n^{(k)}, y_n^{(l)}\right)\Big(\log p\left(x_{n+1}|x_n^{(k)}, y_n^{(l)}\right) \\ - \log p\left(x_{n+1}|x_n^{(k)}\right)\Big), \end{aligned} \tag{9}$$

where $x_n^{(k)} = (x_n, \cdots, x_{n-k+1})$ and $y_n^{(l)} = (y_n, \cdots, y_{n-l+1})$ are the processes given by the k and l dimensional delay vectors, respectively. Therefore, $TE_{Y\longrightarrow X}$ reveals that asymmetry—the degree of dependence of X on Y—discerns the driving and responding sources [43].

We also consider the effective transfer entropy (ETE) to correct the noise caused by the finite size of the data. The ETE is derived as follows [46]:

$$ETE_{Y\longrightarrow X} = TE_{Y\longrightarrow X}(k, l) - \frac{1}{M}\sum_{i=1}^{M} TE_{Y_{(i)}\longrightarrow X}(k, l), \tag{10}$$

where $Y_{(i)}$ indicates the randomly shuffled variable Y. Accordingly, ETE is calculated by subtracting the arithmetic mean of the randomized transfer entropy values from the estimated transfer entropy value [47].

3. Results and Discussion

The Kolmogorov–Smirnov (KS) and Jarque–Bera (JB) tests are conducted to examine the difference between the two distributions: the new and old forks. As indicated in Table 2, the null hypothesis of the KS test can be rejected for both splitting blockchains, implying the significant difference between the two log return distributions. Additionally, the JB test indicates that neither Bitcoin nor Bitcoin Cash is normally distributed. Figure 1 further confirms these results: the new and old forks exhibit clearly different distributions, and both are close to the Laplace rather than the Gaussian distribution, in particular, near the center and the tail part.

However, two splitting blockchains reveal similar market conditions in terms of market efficiency and long-term equilibrium. First, HE is estimated to test the weak-form EMH, which examines price fairness, for the new and old fork markets. As shown in Table 3, both markets have relatively high values of classical and corrected HE ($HE > 0.5$), indicating a long-range dependence with no significant difference between the two, such as clustering tendency and delayed response to information flows [48]. Such persistence reinforces the predictability of the market and provides evidence of market inefficiency [49, 50]. However, two price series might move with a different trend, and the market could fluctuate according to different volatility clustering. Therefore, the long-term market equilibrium could differ between the new and old fork markets, unlike similar market efficiency.

Second, the Shannon entropy is estimated through two approaches, including histogram- and STSA-based, to determine the distance from the long-term market equilibrium

Table 2: Comparison between the two samples: Bitcoin and Bitcoin Cash.

	KS statistic	JB statistic
Bitcoin	0.15^{***}	$3.07 \times 10^{2***}$
Bitcoin Cash		$1.10 \times 10^{3***}$

The Kolmogorov–Smirnov (KS) statistic reports the outcome of a nonparametric equality test, the null hypothesis of which is that two samples are drawn from the same distribution. Meanwhile, the Jarque–Bera (JB) statistic documents the results of a normality test based on a Monte Carlo simulation, whose null hypothesis is that both skewness and excess kurtosis are all zero: the sample follows the normal distribution. *** denotes significance at the 1% level.

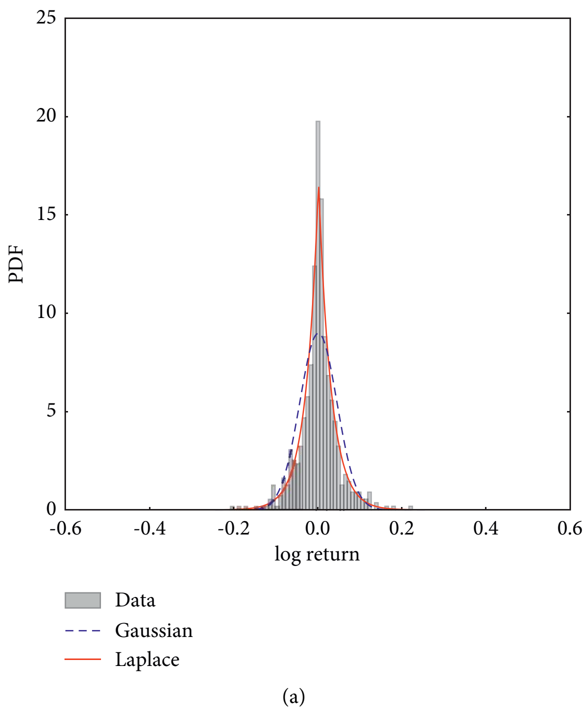

Figure 1: Probability density function of log returns in the new and old forks. Following the Freedman–Diaconis rule [39], we plotted the histograms of both datasets with the optimal bin width of 0.84×10^{-2} (Bitcoin) and 1.66×10^{-2} (Bitcoin Cash). There are two strands of rules for determining the optimal bin width: the Scott rule and the Freedman–Diaconis rule. The former is best suited for data that are close to being normally distributed, whereas the latter is less sensitive to outliers and thus better suited for data with heavy tails. As Figure 1 indicates, the data do not match well with the normal distribution; thus, the Freedman–Diaconis rule is used. (a) Bitcoin. (b) Bitcoin Cash.

Table 3: Hurst exponent.

	Classical	Corrected
Bitcoin	0.68 ± 0.00	0.54 ± 0.00
Bitcoin Cash	0.69 ± 0.01	0.55 ± 0.01

The estimated values of each measure with one standard error are presented. To validate the robustness, we estimated both the classical and corrected HE values with a minimum length of the subseries 2. The minimum subseries length (n) defines the length of the subseries when splitting a time series. This paper sets the minimum length of the subseries as $n = 2, \ldots, N/2$, where N indicates the length of the entire time series.

Table 4: Shannon entropy.

	Histogram	STSA
Bitcoin	0.75 ± 0.07	0.94 ± 0.02
Bitcoin Cash	0.76 ± 0.06	0.92 ± 0.03

The estimated values of each measure with one standard error are presented: the mean and standard deviation of entropy were calculated monthly basis (STSA: symbolic time series analysis).

on both new and old fork markets. As shown in Table 4, the Shannon entropy demonstrates the balance between the two markets, implying that the probability allocations onto (i) each state and (ii) dynamic rise–fall patterns are similar to high randomness in the two markets. This study considers the Shannon entropy as an optimal measure of long-run equilibrium, particularly in equilibrium systems [51] and determines whether each market is close to long-term market equilibrium. In summary, despite technological changes, the market conditions of the new and old forks are equivalent in terms of market efficiency and long-run market equilibrium. The term “long-term equilibrium” refers to statistical equilibrium, which is widely used in physics and information theory and is derived by maximizing the system’s entropy, indicating the system’s most likely state [18]. To enhance the robustness of the results on the market conditions, we further apply the power-law exponent (PLE), one of the most effective and powerful indicators for scaling behavior [52]. The results are summarized in the Appendix.

Table 5: Transfer entropy by quantile-based estimation.

Transfer entropy		Effective transfer entropy	
Bitcoin ⟶ Bitcoin Cash	0.03***	Bitcoin ⟶ Bitcoin Cash	0.02
Bitcoin Cash ⟶ Bitcoin	0.02**	Bitcoin Cash ⟶ Bitcoin	0.01

The directional link shown with the arrow represents the information flows between the new and old forks. For transfer entropy, ** and *** indicate significance at the 5% and 1% levels, respectively.

Table 6: Power-law exponent.

	Top 5%	Bottom 5%
Bitcoin	3.90 ± 0.98	3.25 ± 0.80
Bitcoin Cash	2.44 ± 0.61	2.53 ± 0.62

We estimate the power-law exponents (PLEs) based on the ordinary least square method in the positive and the negative tails of the normalized return distributions. Because the ranking process allows the residuals to be positively autocorrelated, the standard error (SE) could be incorrect. In this paper, the SE of the PLE is calculated from $\widehat{\xi}(n/2)^{-(1/2)}$, asymptotic SE of PLE $\widehat{\xi}$ [55].

The preceding results are explained using information flows related to market liquidity [53, 54]. In particular, transfer entropy distinguishes the bidirectional information flows between the new and old forks. As shown in Table 5, transfer entropy in both directions is statistically significant, implying that the new and old forks interact with each other. Accordingly, mutual information flows mitigate market uncertainty and alleviate investors' distrust. Moreover, the ETE supports the robustness of our findings: it is not due to random noise [46]. Therefore, significant mutual information flows linked to sufficient liquidity of Bitcoin Cash contribute to the two splits having similar market conditions. Since August 2017, Bitcoin Cash has been ranked in the top five cryptocurrencies in terms of market capitalization. Bitcoin splits other than Bitcoin Cash, such as Bitcoin Gold and Bitcoin Diamond, have far smaller market capitalizations. These are approximately 890–1,300 times smaller than Bitcoin in terms of market capitalization [54].

4. Conclusion

Using price series data, this study examines the market conditions for the two splitting blockchains, identifying commonalities and differences in the supporting technologies. The hard forks in cryptocurrency provide a novel setting for examining how technological advancements result in underlying market conditions that differ between the old and the new. The two splitting cryptocurrencies are clearly coupled in terms of market conditions, such as market efficiency and long-term equilibrium, despite the disparity in technology adoption. This study hypothesizes and finds supporting evidence that information flows linked to market liquidity can be attributed to similar market conditions of splitting blockchains.

As the cryptocurrency market is based on consensus-building systems, technological issues, such as hard forks, are likely to arise frequently. Our finding, significant bidirectional information flows between Bitcoin and Bitcoin Cash, suggests that the two cryptocurrencies could be used as hedging tools for one another. Moreover, monitoring information flows in conjunction with market liquidity could help policymakers and investors better understand and respond to future technological changes.

Appendix

As a signature for the collective phenomenon, PLE ($\widehat{\xi}$) for the tail of the normalized return distribution is estimated based on the ordinary least squares (OLS) method [55, 56]:

$$1 - F(x) = P(X > x) \sim x^{-\xi}, \tag{A.1}$$

where $F(x)$ denotes the cumulative density function. Taking the logarithm on both sides, we obtain the PLE as the slope of the estimated regression line from the following linear relationship [57, 58]:

$$\ln P(X > x) = c - \xi \ln x + \epsilon, \tag{A.2}$$

where c is a constant and ϵ is an identically and independently distributed error term following a normal distribution.

Table 6 summarizes the PLEs for Bitcoin and Bitcoin Cash. The authors choose the top and bottom 5% as critical values for estimating the PLEs of the positive and negative tails separately [59–61]. As shown in Table 6, the values of PLEs of Bitcoin and Bitcoin Cash exhibit the evidence of scaling behavior. All the PLEs summarized in Table 6 exhibit no significant difference between Bitcoin and Bitcoin Cash. In more detail, the two splitting blockchains follow the inverse cubic law, commonly found in the tails of stock returns [55, 60, 62], suggesting that both returns manifest a heavier tail as stocks do. These results further support the fact that market conditions between the two splitting blockchains are reached on par.

Data Availability

The daily prices of two splitting blockchains, such as Bitcoin and Bitcoin Cash, are retrieved immediately after the hard fork: the first hard fork splitting Bitcoin occurred on August 1, 2017. All data are in US dollars and are provided by CoinMarketCap, which provides trading data, including the exchange activities of 2,543 cyrptocurrencies in 20,295 markets.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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