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Research Article

Consensus of Second-Order Heterogeneous Hybrid Multiagent Systems via Event-Triggered Protocols

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This paper investigates the event-based consensus problem for the heterogeneous hybrid multiagent system (MAS). First, the heterogeneous hybrid MAS is proposed which contains continuous and discrete-time subsystems with second-order and first-order heterogeneous dynamics. Second, the event-triggered protocols are proposed, which mainly include the event-based control laws and event-triggered conditions for different kinds of agents. Then, the consensus conclusions of fixed topology and switching topologies are obtained based on graph theory and nonnegative matrix theory, which include the constraints on control parameters, coupling gains, and sampling interval to guarantee consensus. Finally, a simulation example is given to verify the efficiency of the proposed protocols.

1. Introduction

With the popularity of distributed artificial intelligence, multiagent system (MAS) has been widely researched and applied to engineering, military, and other fields. It can accomplish huge and complex tasks in the real world through the mutual communication and coordination among individuals. It can also explain some complex phenomena in nature and human society, such as fish schools, bird flocks [1, 2], and the dynamics of opinion forming in human society [3]. At present, the research on multiagent system is mainly about consensus [4–8], flocking [9–11], formation [12–15], and so on.

Consensus, as one of the most fundamental cooperative behaviors of MASs, has attracted extensive interest. It means that the agents can reach the same states from any initial states by a suitable consensus algorithm or control law. Up to now, researchers have proposed many consensus algorithms for MASs through different analysis methods, such as the analysis based on nonnegative matrix theory [16–19], Lyapunov function analysis [20, 21], and frequency domain analysis [22–24]. In 2006, Xiao et al. studied the consensus problem for discrete-time first-order MASs with fixed topology and considered the structural decomposition of the leader-follower model [25]. Then, Xie et al. and Ren et al. gave some sufficient conditions for solving the consensus problem for second-order MASs with fixed and switching topologies [26, 27]. Shi et al. further considered the weighted-average consensus problem for second-order MASs and obtained necessary and sufficient conditions [28]. In recent years, the multiagent networks studied have become more and more complicated with the wide application of MASs. The heterogeneous MASs composed of agents with different dynamics are more suitable for real systems. Taking the multirobots systems into account, heterogeneous systems composed of robots with different perceptual capabilities can complete tasks faster [29]. A number of results about the consensus of heterogeneous MASs have been obtained, including low-order linear systems [30–32] and high-order linear systems [33–35]. In addition, the hybrid MASs including continuous and discrete-time subsystems have also attracted attention. Examples include the refrigeration and heating system. The heat-loss dynamics and the control of air conditioners belong to continuous-time systems, whereas the thermostat is controlled by a discrete-time system [36]. Since 2018, Zheng et al. have studied the consensus problems of first-order and second-order hybrid systems and have proposed a game-theoretic approach to analyze the hybrid systems [37–39]. Su et al. designed an event-triggered consensus strategy for the second-order hybrid system [40] in 2019. Other researchers obtained more results about the hybrid MAS [41, 42].

In addition, interest in the event-based consensus problem for MASs has grown in the past decade. Compared with traditional methods, the event-triggered method has certain advantages in studying practical problems, such as minimizing the number of control actions and saving energy by updating the controller only at the trigger time and avoiding continuous communications. However, it also brings new theoretical and practical problems. The main task is the design of distributed event-triggered protocols, including event-triggered control laws and trigger conditions. In 2012, Dimarogonas et al. proposed effective event-triggered consensus algorithms for the first-order agents under undirected connected communication topologies [43]. Then, Fan et al. designed a fully distributed event-triggered strategy for solving the consensus problem of general linear MASs [44]. In recent years, several event-triggered consensus problems based on state feedback, output feedback, and leader-follower models were considered in [45–50].

In this paper, we consider the heterogeneous multi-agent systems with continuous-time and discrete-time individuals (the heterogeneous hybrid MASs). For example, in the complex system of nature and human interaction, the biological signals are continuous signals, whereas the automated instruments with different functions are mostly discrete-time systems. Compared with the MASs in [42], the consensus problem for the heterogeneous hybrid MASs with a more general form is investigated, and the event-triggered protocols are proposed. First-order and second-order dynamic agents coexist in a system. Some of them belong to the continuous-time system, whereas the other agents are controlled by the discrete-time system. The main contributions of this paper are as follows. First, for the different dynamic characteristics of first-order and second-order agents, two kinds of event-triggered control laws are proposed. The event-triggered conditions are designed, which contain the position of all agents and the velocity of only second-order agents. Second, the sampled-data approach is used to solve the consensus problem for the heterogeneous hybrid MASs. On the one hand, the continuous-time subsystem and discrete-time subsystem can be better analyzed by the overall analysis method. On the other hand, this approach shows that the trigger time interval exists in a lower bound. Hence, the Zeno behavior is avoided. Finally, under the assumption that the fixed topology or the union of switching topologies contains a spanning tree, several results for solving the consensus problem are obtained by using graph theory and nonnegative matrix theory. Some selection conditions of control parameters, as well as the constraints of coupling gains and sampling interval, are given to guarantee consensus.

Throughout this paper, assume $0 < M < N$, $I_M = \{1, 2, \ldots, M\}$ and $I_N/I_M = \{M+1, M+2, \ldots, N\}$. $\mathcal{N}^+$ represents the set of positive integers. Consider a vector or a matrix A, $A \in R^n$ means A is a real column vector of length N. Similarly, A is an $n \times p$-dimensional real matrix, which is defined by $A \in R^{n\times p}$. The symbol A^T and $\|A\|$ represent the transpose and Euclidean norm of A, respectively. The calculational symbols $\otimes$ represent the Kronecker product of matrices.

2. Preliminaries and Problem Formulation

2.1. Preliminaries. The communication topology is described by a weighted directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ represents the nodes set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of edges between nodes. $\mathcal{A} = [a_{ij}] \in R^{N\times N}$ denotes the related adjacency matrix. If there is a directed path $\epsilon_{ji} \in \mathcal{E}$, indicating that the j th agent can transmit data to the i th agent, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$. A diagonal matrix $\mathcal{D} = \mathrm{diag}(d_1, d_2, \ldots, d_N)$ is defined as the degree matrix of the directed graph $\mathcal{G}$, where $d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix is expressed as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. For a weighted directed graph $\mathcal{G}$, it is said to contain a directed spanning tree, indicating that there is a node, which has directed paths that can lead to all other nodes.

A nonnegative matrix $\mathcal{C} \in R^{n\times r}$ is also called a row stochastic matrix if the sum of its each row is equal to 1. Furthermore, if it also satisfies $\lim_{k\longrightarrow\infty} \mathcal{C}^k = 1_n \times v^T$, where $v \in R^n$, then it is indecomposable and aperiodic (SIA).

2.2. Problem Formulation. Considering the position and velocity states of the second-order agents and the position states of the first-order agents, the dynamic models are proposed, respectively. Suppose the number of agents in the entire system is N, where the first $M\,(M < N)$ agents are second-order agents, the remaining $(N - M)$ agents are first-order. The second-order dynamic agents are expressed as follows:

$$\begin{cases} \dot{x}_i^c(t) = v_i^c(t), & i \in I_M, \\ \dot{v}_i^c(t) = u_i^c(t), & i \in I_M, \\ x_i^d(kh+h) = x_i^d(kh) + hv_i^d(kh) + \dfrac{h^2}{2}u_i^d(kh), & i \in I_M, \\ v_i^d(kh+h) = v_i^d(kh) + hu_i^d(kh), & i \in I_M, \end{cases} \tag{1}$$

where $x_i \in R$, $v_i \in R$, and $u_i \in R$ represent the position, velocity, and control input of the second-order agent i, respectively. The superscripts c and d denote that the agent belongs to continuous and discrete-time subsystems. $h > 0$ is the sampling interval of the discrete-time subsystem. The dynamic model of first-order agents is given by

$$\begin{cases} \dot{x}_l^c(t) = u_l^c(t)\, l \in \dfrac{I_N}{I_M}, \\ \\ x_l^d(kh+h) = x_l^d(kh) + hu_l^d(kh),\ l \in \dfrac{I_N}{I_M}. \end{cases} \tag{2}$$

Definition 1. The heterogeneous hybrid MAS is said to reach consensus if the position of all agents satisfies the following conditions from any initial state.

$$\begin{aligned} \lim_{t\longrightarrow\infty} x_i^c(t) &= \lim_{kh\longrightarrow\infty} x_i^d(kh) \\ &= C,\ i \in I_N, \end{aligned} \tag{3}$$

where C is a constant. To ensure that all agents can keep the same position state, (3) also indicates that the velocity of the second-order agents will tend to zero when $t \longrightarrow \infty$ or $kh \longrightarrow \infty$.

In order to propose the event-triggered protocols to solve this consensus problem, we first design the following event-triggered control laws for second-order agents:

$$\begin{cases} u_i^c(t) = -\beta v_i^c\left(t_\xi^i\right) + \alpha \sum_{j=1}^{N} a_{ij}\left[x_j^{c,d}\left(t_\xi^i\right) - x_i^c\left(t_\xi^i\right)\right],\ i \in I_N,\ t \in \left(t_\xi^i, t_{\xi+1}^i\right], \\ u_i^d(kh) = -\beta v_i^d\left(k_\xi^i h\right) + \alpha \sum_{j=1}^{N} a_{ij}\left[x_j^{c,d}\left(k_\xi^i h\right) - x_i^d\left(k_\xi^i h\right)\right],\ i \in I_N,\ kh \in \left(k_\xi^i h, k_{\xi+1}^i h\right], \end{cases} \tag{4}$$

where $\alpha, \beta > 0$ are control parameters, and $(t_0^i, \ldots, t_\xi^i, \cdots)$ and $(k_0^i h, \ldots, k_\xi^i h, \cdots)$ are the trigger time series. The event-triggered control laws of the first-order agents are designed as follows:

$$\begin{cases} u_i^c(t) = \alpha \sum_{j=1}^{N} a_{ij}\left[x_j^{c,d}\left(t_\xi^i\right) - x_i^c\left(t_\xi^i\right)\right],\ i \in \dfrac{I_N}{I_M},\ t \in \left(t_\xi^i, t_{\xi+1}^i\right], \\ \\ u_i^d(kh) = \alpha \sum_{j=1}^{N} a_{ij}\left[x_j^{c,d}\left(k_\xi^i h\right) - x_i^d\left(k_\xi^i h\right)\right],\ i \in \dfrac{I_N}{I_M},\ kh \in \left(k_\xi^i h, k_{\xi+1}^i h\right]. \end{cases} \tag{5}$$

Before proposing the event-triggered conditions, the following definitions of the combined measurement and combined measurement error are given. The combined measurement is defined as

$$\begin{cases} q_i^c(t) &= \sum_{j=1}^{N} a_{ij}\left[x_j^{c,d}(t) - x_i^c(t)\right],\ i \in I_N, \\ \\ q_i^d(kh) &= \sum_{j=1}^{N} a_{ij}\left[x_j^{c,d}(kh) - x_i^d(kh)\right],\ i \in I_N. \end{cases} \tag{6}$$

The combined measurement error is defined as

$$\begin{cases} e_{xi}^c(t) &= q_i^c\left(t_\xi^i\right) - q_i^c(t), i \in I_N, \\ e_{xi}^d(kh) &= q_i^d\left(k_\xi^i h\right) - q_i^d(kh),\ i \in I_N, \\ e_{vi}^c(t) &= v_i^c\left(t_\xi^i\right) - v_i^c(t),\ i \in I_M, \\ e_{vi}^d(kh) &= v_i^d\left(k_\xi^i h\right) - v_i^d(kh),\ i \in I_M. \end{cases} \tag{7}$$

In this paper, the event-triggered conditions are presented as follows:

$$\begin{aligned} t_{\xi+1}^i &= t_\xi^i + \min\{l_i h\},\ i \in I_N,\ l_i \in \mathcal{N}^+, \\ k_{\xi+1}^i h &= k_\xi^i h + \min\{l_i h\},\ i \in I_N,\ l_i \in \mathcal{N}^+, \end{aligned} \tag{8}$$

where $\min\{l_i h\}$ is the time interval between two adjacent trigger times, which $l_i h$ satisfies the following inequalities for the continuous and discrete-time agents:

$$\begin{aligned} &\begin{cases} \left\|e_{xi}^c\left(t_\xi^i + l_i h\right)\right\| \geq \gamma_1 \left\|q_i^c\left(t_\xi^i\right)\right\| + \gamma_2 \delta^{t_\xi^i + l_i h}, & i \in I_N, \\ \left\|e_{vi}^c\left(t_\xi^i + l_i h\right)\right\| \geq \gamma_1 \left\|z_i^c\left(t_\xi^i\right)\right\| + \gamma_2 \delta^{t_\xi^i + l_i h}, & i \in I_M, \end{cases} \\ &\begin{cases} \left\|e_{xi}^d\left(k_\xi^i ht + nl_i qh\right)\right\| \geq \gamma_1 \left\|q_i^d\left(k_\xi^i h\right)\right\| + \gamma_2 \delta^{k_\xi^i h + l_i h}, & i \in I_N, \\ \left\|e_{vi}^d\left(k_\xi^i h + l_i h\right)\right\| \geq \gamma_1 \left\|z_i^d\left(k_\xi^i h\right)\right\| + \gamma_2 \delta^{k_\xi^i h + l_i h}, & i \in I_M, \end{cases} \end{aligned} \tag{9}$$

where $\gamma_1, \gamma_2 > 0$ are the threshold parameters, $\delta > 0$ is the exponential component. And z_i is a new definition of hybrid state quantity for the second-order agent, which is defined as

$$z_i = x_i + hv_i,\ i = 1, 2, \ldots, M \tag{10}$$

According to the event-triggered conditions proposed above, the Zeno behavior is avoided because there is a lower bound $h > 0$ for the event-trigger interval $l_i h$, which ensures that the agent does not trigger infinitely for a limited time. Then, based on the definitions of (6) and (7), the event-triggered control laws (4) and (5) can be written as follows:

$$\begin{cases} u_i^c(t) = -\beta\left[v_i^c(t)t + ne_{vi}^c q(t)\right] \\ \quad + \alpha\left[q_i^c(t)t + ne_{xi}^c q(t)\right], & i \in I_M, t \in \left(t_\xi^i, t_{\xi+1}^i\right], \\ u_i^d(kh) = -\beta\left[v_i^d(kh)t + ne_{vi}^d q(kh)\right] \\ \quad + \alpha\left[q_i^d(kh)t + ne_{xi}^d q(kh)\right], & i \in I_M, kh \in \left(k_\xi^i h, k_{\xi+1}^i h\right], \\ u_l^c(t) = \alpha\left[q_l^c(t)t + ne_{xl}^c q(t)\right], & l \in I_N/I_M, t \in \left(t_\xi^l, t_{\xi+1}^l\right], \\ u_l^d(kh) = \alpha\left[q_l^d(kh)t + ne_{xl}^d q(kh)\right], & l \in I_N/I_M, kh \in \left(k_\xi^l h, k_{\xi+1}^l h\right]. \end{cases} \tag{11}$$

Each controller updates only at its own trigger time, so the energy can be saved. For any $t \in (t_\xi^i, t_{\xi+1}^i]$, $u_i(t) = u_i(t_\xi^i)$ does not change, and the continuous-time dynamics in (1) and (2) can be described as

$$\begin{cases} x_i^c(t) = x_i^c\left(t_\xi^i\right) + \int_{t_\xi^i}^t v_i^c(\tau)d\tau \\ \quad = x_i^c\left(t_\xi^i\right) + \left(t - t_\xi^i\right)v_i^c\left(t_\xi^i\right) + \dfrac{\left(t - t_\xi^i\right)^2}{2}u_i^c\left(t_\xi^i\right), \ i \in I_M, \\ v_i^c(t) = v_i^c\left(t_\xi^i\right) + \int_{t_\xi^i}^t u_i^c(\tau)d\tau = v_i^c\left(t_\xi^i\right) + \left(t - t_\xi^i\right)u_i^c\left(t_\xi^i\right), i \in I_M, \\ x_l^c(t) = x_l^c\left(t_\xi^l\right) + \int_{t_\xi^l}^t u_l^c(\tau)d\tau = x_l^c\left(t_\xi^l\right) + \left(t - t_\xi^l\right)u_l^c\left(t_\xi^l\right), \ l \in I_N/I_M. \end{cases} \tag{12}$$

To analyze the entire MAS with continuous and discrete-time subsystems by the overall analysis method, the sampled-data method is applied in this paper. Considering the continuous subsystem in the discrete-time scale, a new discrete-time scale $\kappa h = l_i^l h$ is defined to describe the entire MASs. $\kappa_\xi^i h$ represents the trigger time t_ξ^i and $k_\xi^i h$. The unified form of the entire MASs can be described as the following expression:

$$\begin{cases} x_i^{c,d}(\kappa h + h) = x_i^{c,d}(\kappa h) + h v_i^{c,d}(\kappa h) + \dfrac{h^2}{2}\left[-\beta v_i^{c,d}\left(\kappa_\xi^i h\right) + \alpha q_i^{c,d}\left(\kappa_\xi^i h\right)\right], \ i \in I_M, \\ v_i^{c,d}(\kappa h + h) = v_i^{c,d}(\kappa h) + h\left[-\beta v_i^{c,d}\left(\kappa_\xi^i h\right) + \alpha q_i^{c,d}\left(\kappa_\xi^i h\right)\right], \ i \in I_M, \\ x_l^{c,d}(\kappa h + h) = x_l^{c,d}(\kappa h) + h\alpha q_l^{c,d}\left(\kappa_\xi^i h\right), \ l \in I_N/I_M. \end{cases} \tag{13}$$

Substituting (7) into (13), we get the following expression:

$$\begin{cases} x_i^{c,d}(\kappa h + h) = x_i^{c,d}(\kappa h) + hv_i^{c,d}(\kappa h) - \dfrac{h^2\beta}{2}\left[v_i^{c,d}(\kappa h) + e_{vi}^{c,d}(\kappa h)\right] \\ +\dfrac{h^2\alpha}{2}\left[q_i^{c,d}(\kappa h) + e_{xi}^{c,d}(\kappa h)\right],\ i \in I_M, \\ v_i^{c,d}(\kappa h + h) = v_i^{c,d}(\kappa h)\left[-h\beta v_i^{c,d}(\kappa h) + e_{vi}^{c,d}(\kappa h)\right] \\ +h\alpha\left[q_i^{c,d}(\kappa h) + e_{xi}^{c,d}(\kappa h)\right],\ i \in I_M, \\ x_l^{c,d}(\kappa h + h) = x_l^{c,d}(\kappa h) + h\alpha\left[q_l^{c,d}(\kappa h) + e_{xl}^{c,d}(\kappa h)\right],\ l \in I_N/I_M. \end{cases} \tag{14}$$

In order to obtain the relationship between consensus achievement and system parameters, the following definitions are given:

$$\begin{aligned} x_s(kh) &= \left[x_1^T(kh), x_2^T(kh), \ldots, x_M^T(kh)\right]^T, \\ x_f(kh) &= \left[x_{M+1}^T(kh), x_{M+2}^T(kh), \ldots, x_N^T(kh)\right]^T, \\ v_s(kh) &= \left[v_1^T(kh), v_2^T(kh), \ldots, v_M^T(kh)\right]^T, \\ z_s(kh) &= \left[z_1^T(kh), z_2^T(kh), \ldots, z_M^T(kh)\right]^T, \\ Q_s(kh) &= \left[q_1^T(kh), q_2^T(kh), \ldots, q_M^T(kh)\right]^T, \\ Q_f(kh) &= \left[q_{M+1}^T(kh), q_{M+2}^T(kh), \ldots, q_N^T(kh)\right]^T, \\ E_{xs}(kh) &= \left[e_{x,1}^T(kh), e_{x,2}^T(kh), \ldots, e_{x,M}^T(kh)\right]^T, \\ E_{xf}(kh) &= \left[e_{x,M+1}^T(kh), e_{x,M+2}^T(kh), \ldots, e_{x,N}^T(kh)\right]^T, \\ E_v(kh) &= \left[e_{v,1}^T(kh), e_{v,2}^T(kh), \ldots, e_{v,M}^T(kh)\right]^T. \end{aligned} \tag{15}$$

Then, (14) can be written as

$$\begin{cases} x_s(\kappa h + h) = \dfrac{h\beta}{2}x_s(\kappa h) + \dfrac{2-h\beta}{2}z_s(\kappa h) + \dfrac{h^2\alpha}{2}Q_s(\kappa h) \\ -\dfrac{h^2\beta}{2}E_v(\kappa h) + \dfrac{h^2\alpha}{2}E_{xs}(\kappa h), \\ z_s(\kappa h + h) = \dfrac{3h\beta - 2}{2}x_s(\kappa h) + \dfrac{4-3h\beta}{2}z_s(\kappa h) + \dfrac{3h^2\alpha}{2}Q_s(\kappa h) \\ -\dfrac{3h^2\beta}{2}E_v(\kappa h) + \dfrac{3h^2\alpha}{2}E_{xs}(\kappa h), \\ x_f(\kappa h + h) = x_f(\kappa h) + h\alpha Q_f(\kappa h) + h\alpha E_{xf}(\kappa h). \end{cases} \tag{16}$$

Considering the communication between different dynamics, the Laplacian matrix representing the communication topology is divided into four parts.

$$\mathscr{L}(kh) = \begin{bmatrix} \mathscr{L}_1(kh)\mathscr{L}_2(kh) \\ \mathscr{L}_3(kh)\mathscr{L}_4(kh) \end{bmatrix}, \tag{17}$$

where $\mathscr{L}_1$ represents the directed communication from second-order agents to second-order agents; $\mathscr{L}_2$, $\mathscr{L}_3$, and $\mathscr{L}_4$ represent the directed communication from first-order agents to second-order agents, from first-order agents to first-order agents, and from second-order agents to first-order agents, respectively. (16) can be converted into the following expression:

$$\begin{cases} x_s(\kappa h + h) = \left[\dfrac{h\beta}{2} I_M - \dfrac{h^2\alpha}{2}\mathscr{L}_1(\kappa h)\right] x_s(\kappa h) + \dfrac{2 - h\beta}{2} z_s(\kappa h) \\ -\dfrac{h^2\alpha}{2}\mathscr{L}_2(\kappa h) x_f(\kappa h) + \dfrac{h^2\alpha}{2} E_{xs}(\kappa h) \dfrac{h^2\beta}{2} E_v(\kappa h), \\ z_s(\kappa h + h) = \left[\dfrac{3h\beta - 2}{2} I_M - \dfrac{3h^2\alpha}{2}\mathscr{L}_1(\kappa h)\right] x_s(\kappa h) + \dfrac{4 - 3h\beta}{2} z_s(\kappa h) \\ -\dfrac{3h^2\alpha}{2}\mathscr{L}_2(\kappa h) x_f(\kappa h) + \dfrac{3h^2\alpha}{2} E_{xs}(\kappa h) - \dfrac{3h^2\beta}{2} E_v(\kappa h), \\ x_f(\kappa h + h) = -h\alpha\mathscr{L}_3(\kappa h) x_s(\kappa h) + \left[I_{N-M} - h\alpha\mathscr{L}_4(\kappa h)\right] x_f(\kappa h) \\ +h\alpha E_{xf}(\kappa h). \end{cases} \tag{18}$$

Then, two large matrices

$$\begin{aligned} Y(kh) &= \left[x_s^T(kh), z_s^T(kh), x_f^T(kh)\right]^T, \\ E(kh) &= \left[E_{xs}^T(kh), E_v^T(kh), E_{xf}^T(kh)\right]^T. \end{aligned} \tag{19}$$

are defined. We can get

$$Y(\kappa h + h) = H(\kappa h) Y(\kappa h) + PE(\kappa h), \tag{20}$$

where

$$\begin{aligned} H(\kappa h) &= \begin{bmatrix} \dfrac{h\beta}{2} I_M - \dfrac{h^2\alpha}{2}\mathscr{L}_1(\kappa h) & \dfrac{2 - h\beta}{2} I_M & -\dfrac{h^2\alpha}{2}\mathscr{L}_2(\kappa h) \\ \dfrac{3h\beta - 2}{2} I_M - \dfrac{3h^2\alpha}{2}\mathscr{L}_1(\kappa h) & \dfrac{4 - 3h\beta}{2} I_M & -\dfrac{3h^2\alpha}{2}\mathscr{L}_2(\kappa h) \\ -h\alpha\mathscr{L}_3(\kappa h) & 0 & I_{N-M} - h\alpha\mathscr{L}_4(\kappa h) \end{bmatrix} \otimes I_n, \\ P &= \begin{bmatrix} \dfrac{h^2\alpha}{2} I_M & -\dfrac{h^2\beta}{2} I_M & 0 \\ \dfrac{3h^2\alpha}{2} I_M & -\dfrac{3h^2\beta}{2} I_M & 0 \\ 0 & 0 & h\alpha I_{N-M} \end{bmatrix} \otimes I_n. \end{aligned} \tag{21}$$

Based on the analysis method in [40], the error system is further constructed. Define

$$\begin{aligned}
\phi_s(kh) &= \left[x_2^T(kh) - x_1^T(kh), \ldots, x_M^T(kh) - x_1^T(kh)\right]^T, \\
\psi_s(kh) &= \left[z_2^T(kh) - z_1^T(kh), \ldots, z_M^T(kh) - z_1^T(kh)\right]^T, \\
\phi_f(kh) &= \left[x_{M+1}^T(kh) - x_1^T(kh), \ldots, x_N^T(kh) - x_1^T(kh)\right]^T, \\
\mathscr{L} &= \begin{bmatrix} l_{2,2} - l_{1,2} & l_{2,3} - l_{1,3} & \cdots & l_{2,N} - l_{1,N} \\ l_{3,2} - l_{1,2} & l_{3,3} - l_{1,3} & \cdots & l_{3,N} - l_{1,N} \\ \vdots & \vdots & \ddots & \vdots \\ l_{N,2} - l_{1,2} & l_{N,3} - l_{1,3} & \cdots & l_{N,N} - l_{1,N} \end{bmatrix} \\
&= \begin{bmatrix} \overline{\mathscr{L}}_1 \overline{\mathscr{L}}_2 \\ \overline{\mathscr{L}}_3 \overline{\mathscr{L}}_4 \end{bmatrix}, \\
\overline{Y}(kh) &= \left[\phi_s^T(kh)\psi_s^T(kh)\phi_f^T(kh)\right]^T.
\end{aligned} \tag{22}$$

Then, (18) can be rewritten as

$$\overline{Y}(\kappa h + h) = \overline{H}(\kappa h)\overline{Y}(\kappa h) + \overline{P}E(\kappa h), \tag{23}$$

where

$$\overline{H}(\kappa h) = \begin{bmatrix} \frac{h\beta}{2} I_M - \frac{h^2\alpha}{2}\overline{\mathscr{L}}_1(\kappa h) & \frac{2 - h\beta}{2} I_M & -\frac{h^2\alpha}{2}\overline{\mathscr{L}}_2(\kappa h) \\ \frac{3h\beta - 2}{2} I_M - \frac{3h^2\alpha}{2}\overline{\mathscr{L}}_1(\kappa h) & \frac{4 - 3h\beta}{2} I_M & -\frac{3h^2\alpha}{2}\overline{\mathscr{L}}_2(\kappa h) \\ -h\alpha\overline{\mathscr{L}}_3(\kappa h) & 0 & I_{N-M} - h\alpha\overline{\mathscr{L}}_4(\kappa h) \end{bmatrix} \otimes I_n,$$

$$\overline{P} = \begin{bmatrix} \frac{h^2\alpha}{2} I_M & -\frac{h^2\beta}{2} I_M & 0 \\ \frac{3h^2\alpha}{2} I_M & -\frac{3h^2\beta}{2} I_M & 0 \\ 0 & 0 & h\alpha I_{N-M} \end{bmatrix} \times \begin{bmatrix} -1_{M-1} & I_{M-1} & 0 & 0 & 0 \\ 0 & 0 & -1_{M-1} & I_{M-1} & 0 \\ -1_{N-M} & 0 & 0 & 0 & I_{N-M} \end{bmatrix} \otimes I_n. \tag{24}$$

3. Main Result

3.1. Fixed Communication Topology. Consider the consensus problem of fixed communication topology, which means the directed graph $\mathscr{G}(\mathscr{V}, \mathscr{E}, \mathscr{A})$ does not change over time and $a_{ij}(t) = a_{ij}(kh) = a_{ij}$.

Lemma 1. *[8] If the sum of each row of nonnegative matrix $H = [h_{ij}] \in R^{(M+N)\times(M+N)}$ is a positive constant $\mu > 0$, then μ is an eigenvalue of H corresponding to the eigenvector 1_{N+M}. Furthermore, if the algebraic multiplicity of the eigenvalue μ of H is 1 and $h_{ii} > 0, i = 1, 2, \ldots, M + N$, then, for each eigenvalue $\lambda \neq \mu$, $|\lambda| < \mu$ is satisfied.*

Remark 1. In practical applications, the movement of the agents should be multidimensional. This article only considers motion in a single dimension; in other words, $I_n = 1$, because motion in a certain direction can be decomposed into motion in several independent directions. Therefore, the conclusion under a single dimension can also be extended to a multidimensional scale.

Theorem 1. *The matrix H defined in (18) contains eigenvalue 1, and the remaining eigenvalues satisfy $|\lambda| < 1$, and all the eigenvalues of $\overline{H}$ defined in (20) satisfy $|\lambda| < 1$, if and only if the fixed communication topology contains a directed spanning tree and coupling gains, and sampling interval and control parameters satisfy as follows:*

$$\begin{cases} \frac{4}{3} > h\beta > \frac{2}{3} + h^2\alpha \max_{i \in I_M}\left(\sum_{j=1}^{N} a_{ij}\right), \\ 1 > h\alpha \max_{i \in I_N/I_M}\left(\sum_{j=1}^{N} a_{ij}\right). \end{cases} \tag{25}$$

Proof. Under condition (21), H is a row stochastic matrix with positive diagonal elements, and $\lambda = 1$ is one of the eigenvalues of H corresponding to eigenvector 1_{N+M}. The column and row transformations of $H - I_{N+M}$ are performed as follows:

$$H - I_{N+M} = \begin{bmatrix} \frac{h\beta - 2}{2} I_M - \frac{h^2\alpha}{2}\mathscr{L}_1 & \frac{2 - h\beta}{2} I_M & -\frac{h^2\alpha}{2}\mathscr{L}_2 \\ \frac{3h\beta - 2}{2} I_M - \frac{3h^2\alpha}{2}\mathscr{L}_1 & \frac{2 - 3h\beta}{2} I_M & -\frac{3h^2\alpha}{2}\mathscr{L}_2 \\ -h\alpha\mathscr{L}_3 & 0 & -h\alpha\mathscr{L}_4 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathscr{L}_1 & 0 & \mathscr{L}_2 \\ I_M & -I_M & 0 \\ \mathscr{L}_3 & 0 & \mathscr{L}_4 \end{bmatrix} \longrightarrow \begin{bmatrix} I_M & 0 & 0 \\ 0 & \mathscr{L}_1 & \mathscr{L}_2 \\ 0 & \mathscr{L}_3 & \mathscr{L}_4 \end{bmatrix}. \tag{26}$$

We obtain $\operatorname{rank}(H - I_{M+N}) = M + \operatorname{rank}(\mathscr{L})$. H has one eigenvalue $\lambda = 1$ with algebraic multiplicity 1, if and only if $\operatorname{rank}(L) = N - 1$, which is equivalent to the topology containing a directed spanning tree [8]. Based on Lemma 1, we can obtain that all the other eigenvalues of H satisfy $|\lambda| < 1$.

Next, it can be proved that all the nonzero eigenvalues of $\mathscr{L}$ are also the eigenvalues of $\overline{\mathscr{L}}$. By observing (18) and (20), the mapping of the eigenvalues of H and $\mathscr{L}$ is the same as the mapping of the eigenvalues of $\overline{H}$ and $\overline{\mathscr{L}}$. Thus, the eigenvalues of H and $\overline{H}$ are equal if $\mathscr{L}$ and $\overline{\mathscr{L}}$ have the same eigenvalues. Additionally, the eigenvalue of H corresponding to the zero eigenvalue of $\mathscr{L}$ is one. Therefore, all eigenvalues of H are also eigenvalues of $\overline{H}$ except for eigenvalue 1. In other words, all the eigenvalues of $\overline{H}$ satisfy $|\lambda| < 1$. □

Lemma 2. *[40] If a matrix $\overline{H}$ satisfies that all its eigenvalues are inside the unit circle, then the following inequality is satisfied:*

$$\|\overline{H}\|^k \leq a \cdot b^k, \tag{27}$$

where a and b are positive constants that satisfy $a \geq 1$ and $0 < b < 1$.

Theorem 2. *Consider the consensus of the heterogeneous hybrid MASs (1) and (2) with fixed communication topology under the event-triggered control laws (4) and (5) and event-triggered conditions (8), (9), and (10) with $\gamma_1 \in (0, 1)$, $\gamma_2 \in (0, \infty)$, and $\delta \in (b, 1)$. If the fixed communication topology $\mathscr{G}$ has a directed spanning tree and conditions (21) in Theorem 1 are satisfied, the heterogeneous hybrid MASs can reach the consensus condition (3).*

Proof. Firstly, (20) is written in the following form by iteration.

$$\overline{Y}(\kappa h) = \overline{H}^{\kappa}\overline{Y}(0) + \overline{P}\sum_{s=0}^{\kappa-1}\overline{H}^{\kappa-1-s}E(sh). \tag{28}$$

By Theorem 1 and Lemma 2, we have

$$\|\overline{Y}(\kappa h)\| \leq ab^{\kappa}\|\overline{Y}(0)\| + \|\overline{P}\|\left\|\sum_{s=0}^{\kappa-1} ab^{\kappa-1-s}\|E(sh)\|\right\|, \tag{29}$$

where $a \geq 1$ and $0 < b < 1$. According to the designed event-trigger conditions (9) and (10), $\|E(\kappa h)\|$ can be expressed as

$$\|E(\kappa h)\| \leq \gamma_1 \left\| \begin{bmatrix} Q_s(\kappa h) \\ Q_f(\kappa h) \\ z_s(\kappa h) \end{bmatrix} \right\| + \gamma_2\sqrt{N + M}\delta^{\kappa}. \tag{30}$$

Two positive constants C_1 and C_2 are defined, and then

$$\left\| \begin{bmatrix} Q_s(\kappa h) \\ Q_f(\kappa h) \end{bmatrix} \right\| \leq \|\mathscr{L}\| \cdot \left\| \begin{bmatrix} \phi_s(\kappa h) \\ \phi_f(\kappa h) \end{bmatrix} \right\| = C_1 \left\| \begin{bmatrix} \phi_s(\kappa h) \\ \phi_f(\kappa h) \end{bmatrix} \right\|,$$

$$\begin{aligned}\left\|z_s(\kappa h)\right\| &\le \left\|\begin{bmatrix} 1 & 0_{1\times(M-1)} \\ -1_{(M-1)\times 1} & I_{M-1}\end{bmatrix}\right\|\cdot\left\|\begin{bmatrix} z_1(\kappa h) \\ \psi_s(\kappa h)\end{bmatrix}\right\| \\ &\le \left\|\begin{bmatrix} 1 & 0_{1\times(M-1)} \\ -1_{(M-1)\times 1} & I_{M-1}\end{bmatrix}\right\|\left\{\left\|z_s(\kappa h)\right\| + \left\|\psi_s(\kappa h)\right\|\right\}.\end{aligned} \tag{31}$$

Through (28), we further obtain

$$\left\|z_s(\kappa h)\right\| \le \frac{\left\|\begin{bmatrix} 1 & 0_{1\times(M-1)} \\ -1_{(M-1)\times 1} & I_{M-1}\end{bmatrix}\right\|}{1-\left\|\begin{bmatrix} 1 & 0_{1\times(M-1)} \\ -1_{(M-1)\times 1} & I_{M-1}\end{bmatrix}\right\|}\cdot\left\|\psi_s(\kappa h)\right\| = C_2\cdot\left\|\psi_s(\kappa h)\right\|. \tag{32}$$

From (27) and (29), (30) can be converted as follows:

$$\|E(\kappa h)\| \le \gamma_1\|C\overline{Y}(\kappa h)\| + \gamma_2\sqrt{N+M}\delta^{\kappa}, \tag{33}$$

where C is a bounded positive constant only related to communication topology. Letting $\rho = \|\overline{P}\|$ and $\mathscr{L} = \max\{\rho a\gamma_2\sqrt{N+M}/(\delta-b-\rho a\gamma_1 C), a\|\overline{Y}(0)\|\}$, ρ and $\mathscr{L}$ are also bounded positive constants related to communication topology. Then, the following inequality will be proved by contradiction:

$$\|\overline{Y}(\kappa h)\| \le \mathscr{L}\delta^{\kappa}. \tag{34}$$

Consider that there is a constant $k^* > \kappa > 0$ that makes inequality (31) invalid. Thus,

$$\left\|\overline{Y}(k^* h)\right\| > \mathscr{L}\delta^{k^*}. \tag{35}$$

According to (25) and (30), one has

$$\begin{aligned}\mathscr{L}\delta^{k^*} &< \left\|\overline{Y}(k^* h)\right\| \\ &\le ab^{k^*}\|\overline{Y}(0)\| + \rho\sum_{s=0}^{k^*-1} ab^{k^*-1-s}\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)\delta^s \\ &\le ab^{k^*}\|\overline{Y}(0)\| + \rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)\frac{b^{k^*}-\delta^{k^*}}{b-\delta} \\ &= \left[a\|\overline{Y}(0)\| + \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{b-\delta}\right]b^{k^*} \\ &\quad + \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{\delta-b}\delta^{k^*}.\end{aligned} \tag{36}$$

Next, (31) can be proved in three cases. □

Case 1.

$$\mathscr{L} = \rho a\gamma_2\frac{\sqrt{N+M}}{(\delta-b-\rho a\gamma_1 C)}, \tag{37}$$

which indicates that

$$\frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{(\delta-b)} = \mathscr{L} \tag{38}$$

$$\frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{(\delta-b)} > a\|\overline{Y}(0)\|. \tag{39}$$

According to (33), we have

$$\begin{aligned}\mathscr{L}\delta^{k^*} &< \left[a\|\overline{Y}(0)\| + \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{b-\delta}\right]b^{k^*} \\ &\quad + \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{\delta-b}\delta^{k^*} \\ &< \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{\delta-b}\delta^{k^*} = \mathscr{L}\delta^{k^*}.\end{aligned} \tag{40}$$

Case 2. $\mathscr{L} = a\|\overline{Y}(0)\|$ and $\rho a\gamma_2\sqrt{N+M}/(\delta-b-\rho a\gamma_1 C) > 0$, which indicates $\delta > b$. According to (33), we can obtain that

$$\begin{aligned}\mathscr{L}\delta^{k^*} &< \left[a\|\overline{Y}(0)\| + \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{b-\delta}\right]\delta^{k^*} \\ &\quad + \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{\delta-b}\delta^{k^*} \\ &= a\|\overline{Y}(0)\|\delta^{k^*} = \mathscr{L}\delta^{k^*}.\end{aligned} \tag{41}$$

Case 3. $\mathscr{L} = a\|\overline{Y}(0)\|$ and $\rho a\gamma_2\sqrt{N+M}/(\delta-b-\rho a\gamma_1 C) > 0$, which indicates $a\|\overline{Y}(0)\| < \rho a(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M})/(\delta-b)$. The proof is similar to case 1 that

$$\mathscr{L}\delta^{k^*} < \frac{\rho a\left(\gamma_1 C\mathscr{L} + \gamma_2\sqrt{N+M}\right)}{\delta-b}\delta^{k^*} < \mathscr{L}\delta^{k^*}. \tag{42}$$

Combining the three cases, the inequality (31) holds, from which we can obtain that

$$\lim_{\kappa h\longrightarrow\infty}\left\|\begin{bmatrix}\phi_s(kh)\\ \psi_s(kh)\\ \phi_f(kh)\end{bmatrix}\right\| = 0. \tag{43}$$

Then, we further consider the consensus of the continuous-time agents in real time. The following inequality is established:

$$\begin{aligned}\left\|v_i^c(t)-v_j^c(t)\right\| &\le \left\|v_i^c(t)-v_i^c(kh)\right\| + \left\|v_i^c(kh)-v_j^c(kh)\right\|\\ &\quad + \left\|v_j^c(kh)-v_j^c(t)\right\|, i\in I_M,\\ \left\|x_i^c(t)-x_j^c(t)\right\| &\le \left\|x_i^c(t)-x_i^c(kh)\right\| + \left\|x_i^c(kh)-x_j^c(kh)\right\|\\ &\quad + \left\|x_i^c(kh)-x_j^c(t)\right\|, i\in I_N.\end{aligned} \tag{44}$$

When $\kappa h\longrightarrow\infty$, we can get $v_i^{c,d}(k_\xi^i h)\longrightarrow 0, q_i^{c,d}(k_\xi^i h)\longrightarrow 0$. Combined with (4), (5), (6), and (12), when $t\in(kh, kh+h]$, the following inequalities are obtained:

$$\begin{aligned}&\lim_{t,kh\longrightarrow\infty}\left\|v_i^c(t)-v_i^c(kh)\right\| \le \lim_{t,kh\longrightarrow\infty} h\left\{-\beta v_i^{c,d}\left(k_\xi^i h\right)+\alpha q_i^{c,d}\left(k_\xi^i h\right)\right\},\ i\in I_M,\\ &\lim_{t,kh\longrightarrow\infty} = \left\|x_i^c(t)-x_i^c(kh)\right\| \le \lim_{t,kh\longrightarrow\infty} h v_i^{c,d}\left(k_\xi^i h\right)\\ &\quad + \frac{\left(t-k_\xi^i h\right)^2-\left(kh-k_\xi^i h\right)^2}{2}\left\{-\beta v_i^{c,d}\left(k_\xi^i h\right)+\alpha q_i^{c,d}\left(k_\xi^i h\right)\right\},\ i\in I_M,\\ &\lim_{t,kh\longrightarrow\infty}\|\theta\| \le \lim_{t,kh\longrightarrow\infty} h\alpha q_l^{c,d}\left(k_\xi^l h\right),\ l\in I_N/I_M.\end{aligned} \tag{45}$$

Then,

$$\begin{aligned}&\lim_{t,kh\longrightarrow\infty}\left\|x_i^c(t)-x_i^c(kh)\right\| = 0,\ i\in I_M,\\ &\lim_{t,kh\longrightarrow\infty}\left\|x_i^c(t)-x_i^c(kh)\right\| = 0,\ l\in \frac{I_N}{I_M}.\end{aligned} \tag{46}$$

From (44) and (46), continuous-time individuals can also achieve consensus on continuous-time scales. Combined with (43), the consensus conditions (3) are satisfied.

3.2. Switching Communication Topologies. According to the above conclusions with the fixed topology, the consensus of the heterogeneous hybrid MASs with switching communication topologies is considered. The system description (18) is rewritten as

$$Y\left(k_{p+1}h\right) = \Omega\left(k_p\right)\left(k_p h\right) + Q_E\left(k_p\right),\ p = 1, 2, \ldots, \infty, \tag{47}$$

where $k_p\in\kappa$ is the p th event-triggering instant of an agent from its initial state, $\Omega(k_p) = \prod_{s=k_p}^{k_{p+1}-1} H(sh)$ is the matrix product of H, corresponding to all switching communication topologies during each interval $[k_p h, k_{p+1}h)$, and

$$Q_E\left(k_p\right) = \mathrm{PE}\left[\left(k_{p+1}-1\right)h\right] + P\sum_{s=k_p}^{k_{p+1}-2} E(sh)\cdot\prod_{d=s+1}^{k_{p+1}-1} H(dh) \tag{48}$$

is the accumulation of errors. Similarly, the error system description (20) can also be rewritten as

$$\overline{Y}\left(k_{p+1}h\right) = \overline{\Omega}\left(k_p\right)\overline{Y}\left(k_p h\right) + \overline{Q}_E\left(k_p\right),\ p = 1, 2, \ldots, \infty, \tag{49}$$

where

$$\overline{\Omega}\left(k_p\right) = \prod_{s=k_p}^{k_{p+1}-1}\overline{H}(sh), \tag{50}$$

$$\overline{Q}_E\left(k_p\right) = \overline{P}E\left[\left(k_{p+1}-1\right)h\right] + \overline{P}\sum_{s=k_p}^{k_{p+1}-2} E(sh)\cdot\prod_{d=s+1}^{k_{p+1}-1}\overline{H}(dh). \tag{51}$$

Remark 2. To simplify subsequent proofs, it is assumed that $k_{p+1}-k_p > 1$ in the subsection on switching communication topologies, which means there are at least two different communication topologies in the period $[k_p, k_{p+1})$. Otherwise, it can be regarded as a fixed communication topology in a short period of time, and the convergence will not be destroyed according to the existing conclusions.

Theorem 3. *The matrix product $\Omega(k_p)$ is SIA, and all the eigenvalues of $\overline{\Omega}(k_p)$ satisfy $|\lambda_\Omega| < 1$ if the union of communication topologies $\{G[k_p h], G[(k_p+1)h], \ldots, G[(k_{p+1}-1)h]\}$ of each interval $[k_p h, k_{p+1}h)$ contains a spanning tree and the coupling gains, and sampling interval and control parameters satisfy conditions (21) in Theorem 1.*

Proof. Define $\widetilde{\mathscr{L}} = \sum_{s=k_p}^{k_{p+1}-1} \mathscr{L}(sh)/(k_{p+1}-k_p)$ as the Laplacian matrix of the union of directed graphs during time interval $[k_p h, k_{p+1}h)$, and $\overline{\widetilde{\mathscr{L}}}$ is the matrix obtained by $\widetilde{\mathscr{L}}$ through the same transformation as $\overline{\mathscr{L}}$ in (19).

$$\widetilde{H} = \sum_{s=k_p}^{k_{p+1}-1} \frac{H(sh)}{\left(k_{p+1}-k_p\right)} \tag{52}$$

still satisfies the sum of each row is 1. Then, we can get $\operatorname{rank}(\widetilde{H} - I_{N+M}) = M + \operatorname{rank}(\widetilde{\mathscr{L}})$ by taking the elements column and row transforms as follows:

$$\widetilde{H} - I_{N+M} = \begin{bmatrix} \frac{h\beta-2}{2}I_M - \frac{h^2\alpha}{2}\widetilde{\mathscr{L}}_1 & \frac{2-h\beta}{2}I_M & -\frac{h^2\alpha}{2}\widetilde{\mathscr{L}}_2 \\ \frac{3h\beta-2}{2}I_M - \frac{3h^2\alpha}{2}\widetilde{\mathscr{L}}_1 & \frac{2-3h\beta}{2}I_M & -\frac{3h^2\alpha}{2}\widetilde{\mathscr{L}}_2 \\ -h\alpha\widetilde{\mathscr{L}}_3 & 0 & -h\alpha\widetilde{\mathscr{L}}_4 \end{bmatrix} \longrightarrow \begin{bmatrix} I_M & 0 \\ 0 & \widetilde{\mathscr{L}} \end{bmatrix}. \tag{53}$$

Thus, $\widetilde{H}$ has eigenvalue $\widetilde{\lambda} = 1$ with algebraic multiplicity 1, if and only if $\operatorname{rank}(\widetilde{\mathscr{L}}) = N-1$, which means that the union of topologies contains a spanning tree. According to Lemma 3.1 in [18], we can get that

$$\prod_{s=k_p}^{k_{p+1}-1} H(sh) > \sum_{s=k_p}^{k_{p+1}-1} H(sh)/\left(k_{p+1}-k_p\right), \tag{54}$$

which indicates that the graph of the matrix product $\Omega(k_p)$ also contains a spanning tree. Besides, $\Omega(k_p)$ is a stochastic matrix with positive diagonal elements if conditions (21) are satisfied, because the matrix multiplication among stochastic matrixes with positive diagonal elements is closed. In other words, $\Omega(k_p)$ is SIA.

Similar to the proof of Theorem 1, for the error system, it can be proved that all eigenvalues of $\Omega(k_p)$ except eigenvalue 1 are also eigenvalues of $\overline{\Omega}(k_p)$. □

Theorem 4. *Consider the consensus of the heterogeneous hybrid MASs (1) and (2) with switching communication topologies under the event-triggered control laws (4) and (5) and event-triggered conditions (8), (9), and (10) with $\gamma_1 \in (0,1)$, $\gamma_2 \in (0,\infty)$, and $\delta \in (b,1)$. If the union of switching topologies $\{G[k_p h], G[(k_p+1)h], \ldots, G[(k_{p+1}-1)h]\}$ of each interval $[k_p h, k_{p+1}h)$ has a directed spanning tree and conditions (21) in Theorem 1 are satisfied, the heterogeneous hybrid MASs can reach the consensus condition (3).*

Proof. Through iteration, the error system description (39) with switching communication topologies can be expressed as

$$\begin{aligned} \overline{Y}\left(k_p h\right) &= \overline{\Omega}(k_0)\overline{\Omega}(k_1)\cdots\overline{\Omega}\left(k_{p-1}\right)\overline{Y}(0) \\ &\quad + \overline{\Omega}(k_1)\Omega(k_2)\cdots\overline{\Omega}\left(k_{p-1}\right)\overline{Q}_E(k_0) \\ &\quad + \cdots \\ &\quad + \overline{\Omega}\left(k_{p-1}\right)\overline{Q}_E\left(k_{p-2}\right) \\ &\quad + \overline{Q}_E\left(k_{p-1}\right). \end{aligned} \tag{55}$$

Let

$$\Omega^* = \max\left\{\left\|\overline{\Omega}(k_0)\right\|, \left\|\overline{\Omega}(k_1)\right\|, \ldots, \left\|\overline{\Omega}\left(k_{p-1}\right)\right\|\right\}. \tag{56}$$

Combined with Lemma 2, we have

$$\begin{aligned} \left\|\overline{Y}\left(k_p h\right)\right\| &\le \Omega^{*p}\|\overline{Y}(0)\| + \sum_{s=0}^{p-1} \Omega^{*p-1-s}\left\|\overline{Q}_E(k_s)\right\| \\ &\le a' B^p \|\overline{Y}(0)\| + \sum_{s=0}^{p-1} a' B^{p-1-s}\left\|\overline{Q}_E(k_s)\right\|, \end{aligned} \tag{57}$$

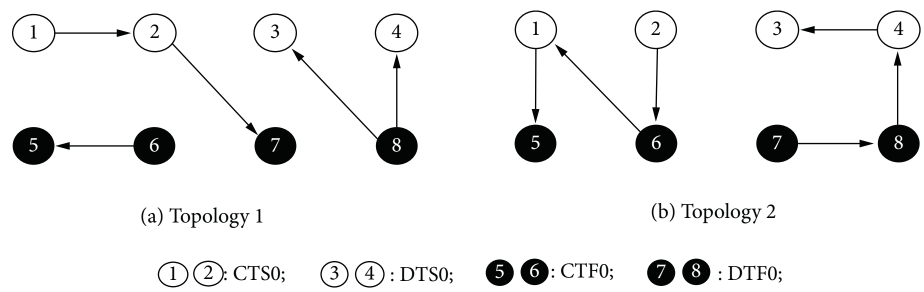

Figure 1: Switching topologies.

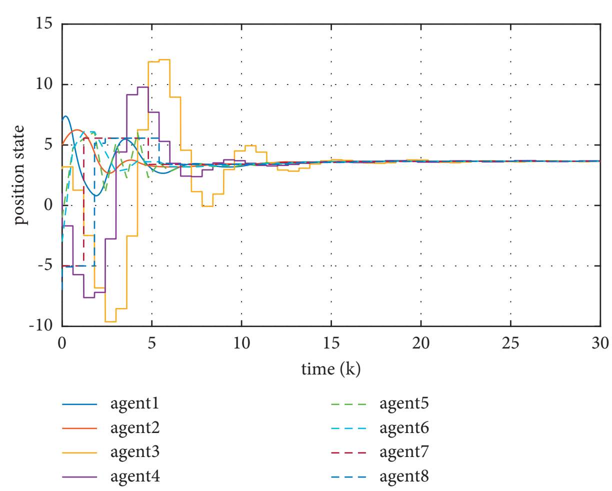

Figure 2: Position of the dynamic agents.

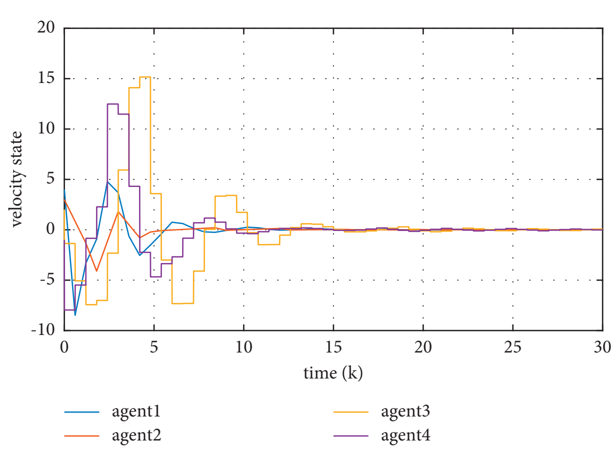

Figure 3: Velocity of the second-order dynamic agents.

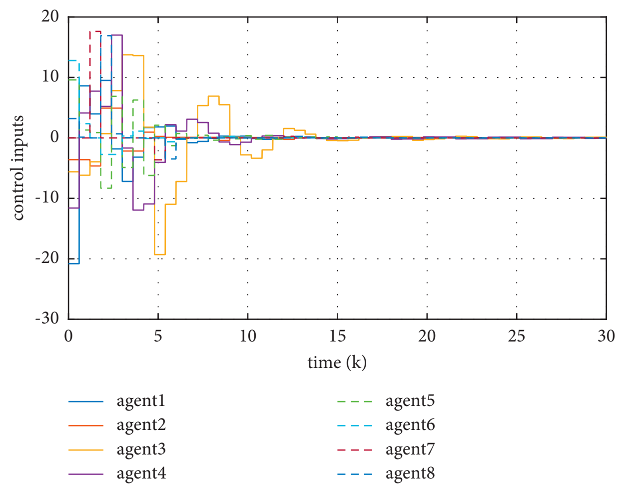

FIGURE 4: Control inputs of the dynamic agents.

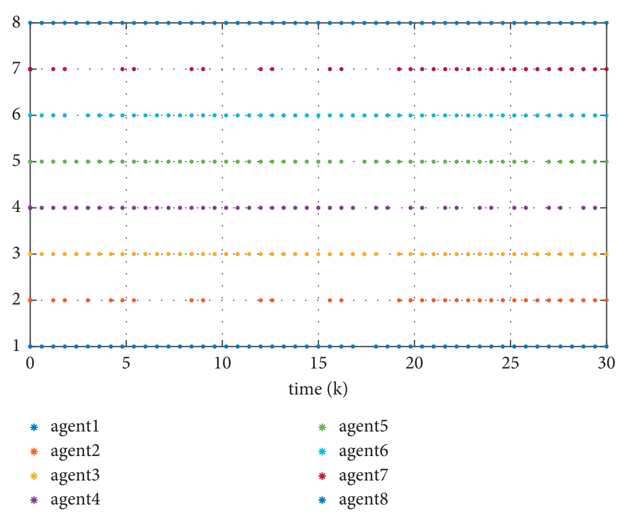

FIGURE 5: Event-triggered instants of the agents.

where $0 < B < 1$ and $a' > 1$ are positive constants. Next, the norm of $\overline{Q}_E(k_s)$ is considered as follows:

$$\begin{aligned}\left\|\overline{Q}_E(k_s)\right\| &\leq \rho\left\|\overline{H}[(k_s+1)h]\right\|\left\|\overline{H}[(k_s+2)h]\right\|\cdots\left\|\overline{H}[(k_{s+1}-1)h]\right\|\left\|E(k_sh)\right\| \\ &\quad+\cdots \\ &\quad+\rho\left\|\overline{H}[(k_{s+1}-1)h]\right\|\left\|\overline{H}[(k_{s+1}-2)h]\right\| \\ &\quad+\rho\left\|\overline{H}[(k_{s+1}-1)h]\right\| \\ &<\rho\eta_s\sum_{\tau=k_s}^{k_{s+1}-1}\|E(\tau h)\|,\end{aligned} \tag{58}$$

where

$$\eta_s = \max\left\{\overline{H}[(k_s+1)h]\cdots\|H[(k_s+1)h]\|,\ldots,\right.\\ \left.\|\overline{H}[(k_s+1)h]\|,1\right\}, \tag{59}$$

is a positive constant. In addition, there are b_p and b_p' that satisfy $b_p^{k_{p+1}-k_p} = B$ and $b_p^{k_{p+1}-k_p-1} = B$ for each interval $[k_p h, k_{p+1}h)$. Letting $\eta = \max\{\eta_s\}$, $b' = \max\{b_p, b_p'\}$, we can get $b' < 1$ and (42) can be converted into

$$\begin{aligned} B\left\|\overline{Y}(k_p h)\right\| &< a' b'^{kp} B\|\overline{Y}(0)\| \\ &\quad + a'\rho\eta b'^{k_p-1}\left[\|E(k_0)\| + \|E(k_0+1)\| + \cdots + \|E(k_1-1)\|\right] + \cdots \\ &\quad + a'\rho\eta b'^{k_0-1}\left[\left\|E(k_{p-1})\right\| + \left\|E(k_{p-1}+1)\right\| + \cdots + \left\|E(k_p-1)\right\|\right] \\ &< a' b'^{kp} B\|\overline{Y}(0)\| + a'\rho\eta\sum_{s=0}^{k_p-1} b'^{k_p-1-s}\|E(sh)\|. \end{aligned} \tag{60}$$

Thus, let

$$\rho' = \frac{a' p\eta}{B}, \tag{61}$$

$$\left\|\overline{Y}(k_p h)\right\| < a' b'^{k_p}\|\overline{Y}(0)\| + \rho'\sum_{s=0}^{k_p-1} b'^{k_p-1-s}\|E(sh)\|, \tag{62}$$

which has a similar form to (25). The subsequent proof of this theorem is the same as Theorem 2. □

Remark 3. The parameters of switching communication topologies, such as the switching rate and the dwell time, have certain impacts on the convergence rate. It mainly depends on the structure of each switching communication topology. In a period of time, more different agents communicating can improve the convergence efficiency. If there are different edges in the switching topologies, increasing the switching rate can reduce the time for the system to reach consensus.

4. Simulation Examples

A heterogeneous hybrid MAS is assumed to consist of four second-order (SO) agents and four first-order (FO) agents. They both contain two continuous-time (CT) individuals and two discrete-time (DT) individuals, respectively. Let $[x_s(0), x_f(0)]^T = [7, 5, 3, 1, -1, -3, -5, -7]^T$ and $v(0)^T = [4, 3, 2, -1]^T$.

Consider the consensus of heterogeneous hybrid MASs with switching topologies. The communication topology can be switched between topology 1 and topology 2 in Figure 1 every step. The union of topology 1 and topology 2 has a spanning tree. Suppose the coupling gains of each edge in topology 1 and topology 2 are 1 and $\alpha = 1.6$, $\beta = 1.2$. By calculating, we choose $h = 0.6 < 0.625$ to satisfy conditions (21) in Theorem 1. Then, choosing $\gamma_1 = 0.2$ and $\gamma_2 = 0.01$, we can obtain the simulation graphics as follows. The position and velocity are shown in Figures 2 and 3. The control inputs are shown in Figure 4. And the triggered instants of each agent are shown in Figure 5.

All agents can achieve consensus, and the controllers are triggered a limited number of times within a finite time, which indicates that the event-triggered protocols algorithm is effective.

5. Conclusion

In this paper, the event-triggered consensus was studied for the heterogeneous hybrid MASs, consisting of continuous and discrete-time subsystems with second-order and first-order heterogeneous dynamics. We designed the effective event-triggered protocols, including the event-triggered control laws for the first-order and second-order agents, respectively, and the event-triggered conditions, which can make the controllers only update at their own trigger time and ensure all agents meet consensus. Some criteria were obtained for solving the consensus problems of the heterogeneous hybrid MASs with fixed topology and switching topologies. The main results showed that the MASs can reach consensus if the control parameter, coupling gains, and sampling interval meet certain conditions, and the fixed topology or the union of switching topologies contain a directed spanning tree. Future work may consider the consensus of the heterogeneous hybrid MASs with time delay or communication noise.

Data Availability

No other data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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