

## Research Article

# Exponential Synchronization of Complex Dynamical Networks via a Novel Sampled-Data Control

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This paper investigates the exponential synchronization of complex dynamical networks based on the sampled-data control method. The sampled-data control means that the control input remains unchanged for a long time after each sampling, which can reduce the sampling number. By using the stability theory of the dynamical systems, this paper provides a novel sampling controller and estimates the bound of the sampling interval. Finally, a numerical example is given to demonstrate the effectiveness of the proposed design technique.

## 1. Introduction

Recently, complex networks have received a great deal of attention from researchers in several fields such as industrial networks, financial networks, biology, and power systems [1–4]. Complex networks are usually composed of lots of interconnected nodes, in which each node can adjust its behavior according to the received information from its neighbor nodes, so complex networks can be used to describe some complex phenomena in science and engineering fields. As a result, there are some existing papers [5–8] to study the complex networks from diverse facets.

Synchronization is one of the important issues concerning complex networks, which means that the collective behaviour that all the states of the network converge towards a predefined target trajectory along with the time evolution. In fact, synchronization is a difficult problem because of the different initial values and various dynamical structures. Thus, there are many papers [9–15] to investigate the synchronization of complex networks by using different theories and methods. Furthermore, many kinds of synchronization of complex networks have been investigated, which include asymptotic synchronization [16], finite-time synchronization [17], and fixed-time synchronization [18].

With the rapid development of computer hardware technology, sampled-data control has been applied widely for its convenience and good control efficiency. Sampled-data control only requires the system states information at the sampling instants, which can greatly reduce the computation and the number of transmitted information. Therefore, there are many results reported in the literature [19–25]. For example, for the periodic sampling case, the authors presented some sufficient conditions of sampled-data synchronization criteria for the complex dynamical networks with time-varying coupling delay by constructing a suitable augmented Lyapunov function, and with the help of introduced integral inequalities, and employing the convex combination technique in [22]. By employing a time-dependent Lyapunov functional and the sampled-data control, the authors of [23] investigated the synchronization control problem for chaotic neural networks subject to actuator saturation and provided a sampled-data controller to regionally synchronize the drive neural networks and response neural networks. For the stochastic sampling case, the authors of [24] studied the stabilization problem for a class of sampled-data systems under noisy sampling interval by introducing a Vandermonde matrix and Kronecker product operation and provided the corresponding stabilization controller. By using the reciprocally convex matrix

inequality, proper integral inequalities, and the linear convex combination method, the authors of [25] investigated the dissipative analysis and quantized sampled-data control design issues for T-S fuzzy networked control system under stochastic cyberattacks and obtained a new quantitative sample data controller to ensure that the system is asymptotically stable and dissipative.

Motivated by the above discussion, we will consider the exponential synchronization control problem for complex dynamical networks in this paper. The main contributions of our paper are as follows: (1) we consider the exponential synchronization of complex networks by using the sampled-data control; (2) the presented controller is novel; (3) the upper bound of the sampling interval is estimated.

The rest of this paper is organized as follows: in Section 2, the model description and preliminary results are presented. In Section 3, the sampled-data controller guaranteed the complex network to be exponential synchronization is derived. In Section 4, a numerical example is provided to illustrate the electiveness of our obtained results. Finally, this paper is ended with a conclusion in Section 5.

Throughout this paper, the following notations are used.  $R^+$  =  $(0, +\infty)$  denotes the set of the positive real number.  $R^n$  and  $R^{n \times m}$ , respectively, denote the  $n$ -dimensional Euclidean space and the set of  $n \times m$  real matrices. For a given vector  $x = (x_1, x_2, \dots, x_n)^T \in R^n$ ,  $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$  denotes its norm. The notation  $X \geq Y$  (respectively,  $X > Y$ ), where  $X, Y$  are symmetric matrices, means that  $X - Y$  is a symmetric semidefinite matrix (respectively, positive definite matrix). For a given matrix  $A$ ,  $A^T$  denotes its transpose, and  $\|A\|$  denotes its norm defined as  $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$ .

## 2. Problem Formulation and Preliminaries

In this paper, we consider the following complex networks composed of  $N$  identical nodes with linear couplings. The  $i$ th node is an  $n$ -dimensional dynamical system, whose state equation is

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^N c_{ij}\Gamma x_j(t) + u_i(t), \\ &i = 1, 2, \dots, N, \end{aligned} \quad (1)$$

where  $N$  is the number of coupled nodes.  $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in R^n$  denotes the state vector,  $A \in R^{n \times n}$  and  $B \in R^{n \times n}$  are known constant real matrices,  $f(\cdot) \in R^n$  is a continuous differential vector function with  $f(0) = 0$ .  $C = [c_{ij}] \in R^{N \times N}$  is the outer-coupling matrix, where  $c_{ij}$  is defined as follows: if there exists a connection between node  $i$  with node  $j$ , then  $c_{ij} > 0$ ; otherwise,  $c_{ij} = 0$ . In addition, the elements of matrix  $C$  satisfy

$$c_{ii} = - \sum_{j=1, j \neq i}^N c_{ij}, \quad i = 1, 2, \dots, N, \quad (2)$$

$\Gamma \in R^{n \times n}$  is the innercoupling matrix, and  $u_i(t) \in R^n$  is the control input to be designed in the sequel.

Assume that the target node is

$$\dot{s}(t) = As(t) + Bf(s(t)), \quad (3)$$

where  $s(t) = (s_1(t), s_2(t), \dots, s_n(t)) \in R^n$  may be an equilibrium point, a periodic orbit, or a chaotic orbit of dynamical systems.

*Definition 1.* Complex network (1) is said to be exponential synchronization with target node (3) if there exist scalars  $\kappa \geq 0$  and  $\theta > 0$  such that

$$\|x_i(t) - s(t)\| \leq \kappa e^{-\theta t}, \quad i = 1, 2, \dots, N, \quad (4)$$

for  $t \geq 0$  and any initial conditions.

Next, for the node  $i$ , we let the sequence of sampling data instants be  $t_0, t_1, t_2, \dots$  and correspond a sequence of control updates  $u_i(t_0), u_i(t_1), u_i(t_2), \dots$ . Between the adjacent control updates, the value of input  $u_i(t)$  is held constant in a zero-order hold fashion. That is,

$$u_i(t) = u_i(t_k), \quad \forall t \in [t_k, t_{k+1}]. \quad (5)$$

Thus, the controllers are piecewise constant between any sampling interval  $[t_k, t_{k+1}]$  for  $k = 0, 1, 2, \dots$

In order to ensure that complex network (1) exponentially synchronizes with target node (3), we intend to use the following feedback controller:

$$u_i(t) = -\mu e^{-\mu(t-t_k)} [x_i(t_k) - s(t_k)], \quad t \in [t_k, t_{k+1}], \quad (6)$$

to control the  $i$  th node, where  $\mu > 0$  is a scalar to be determined.

Letting  $y_i(t) = x_i(t) - s(t)$ , while  $t \in [t_k, t_{k+1}]$ , we obtain the closed-loop system

$$\begin{aligned} \dot{y}_i(t) &= Ay_i(t) + B[f(x_i(t)) - f(s(t))] \\ &+ \sum_{j=1}^N c_{ij}\Gamma y_j(t) - \mu e^{-\mu(t-t_k)} y_i(t_k), \quad i = 1, 2, \dots, N. \end{aligned} \quad (7)$$

Define  $e_i(t) = y_i(t) - e^{-\mu(t-t_k)} y_i(t_k)$ , then the above system can be rewritten as

$$\begin{aligned} \dot{y}_i(t) &= Ay_i(t) + B[f(x_i(t)) - f(s(t))] + \sum_{j=1}^N c_{ij}\Gamma y_j(t) \\ &- \mu y_i(t) + \mu e_i(t), \quad i = 1, 2, \dots, N, \quad t \in [t_k, t_{k+1}]. \end{aligned} \quad (8)$$

Denote by  $y(t) = (y_1^T(t), y_2^T(t), \dots, y_N^T(t))^T, e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T, \phi_i(t) = f(x_i(t)) - f(s(t)), \phi(t) = (\phi_1^T(t), \phi_2^T(t), \dots, \phi_N^T(t))^T$ . Following the above notations, system (8) can be written as the compact form

$$\begin{aligned} \dot{y}(t) &= (I_N \otimes A - \mu I_{nN} + C \otimes \Gamma)y(t) + (I_N \otimes B)\phi(t) + \mu e(t) \\ &= \tilde{A}y(t) + (I_N \otimes B)\phi(t) + \mu e(t), \end{aligned} \quad (9)$$

for  $t \in [t_k, t_{k+1}]$ , where  $\tilde{A} = I_N \otimes (A - \mu I_n) + C \otimes \Gamma$

Before presenting our main results, we give the following assumptions and lemmas.

*Assumption 1.* Assume that there exists a constant  $L \geq 0$  such that

$$\|f(\xi_1(t)) - f(\xi_2(t))\| \leq L \|\xi_1(t) - \xi_2(t)\|, \quad (10)$$

for any  $\xi_1(t), \xi_2(t) \in R$ , and  $t \geq t_0$ .

*Assumption 2.* The complex network is connection. That is, there does not exist isolated node. Intuitively, it is a necessary condition for the synchronization of complex network.

**Lemma 1.** [26]. *The Kronecker product  $\otimes$  has the following properties:*

- (1)  $(A + B) \otimes C = A \otimes C + B \otimes C, C \otimes (A + B) = C \otimes A + C \otimes B$
- (2)  $(A \otimes B)^T = A^T \otimes B^T$
- (3)  $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$
- (4)  $(A \otimes C)(B \otimes D) = AB \otimes CD$
- (5)  $\|A \otimes B\| = \|A\| \cdot \|B\|$

where  $A, B, C, D$  are matrices with appropriate dimensions.

**Lemma 2.** [27]. *Consider a differentiable function  $v(t): R^+ \rightarrow R^+$  satisfying the following inequality:*

$$\dot{v}(t) \leq -av(t) + b \int_{t_k}^t v(\vartheta) d\vartheta, \quad \forall t \in [t_k, t_{k+1}], \quad (11)$$

where  $0 < t_{k+1} - t_k \leq \tau_M < +\infty$ ,  $a$  and  $b$  are positive reals satisfying  $b\tau_M < a$ . Then, the function  $v(t)$  exponentially converges to zero; that is,

$$v(t) \leq v(t_0) e^{-\theta(t-t_0)}, \quad (12)$$

with  $\theta = (a - b\tau_M) e^{-a\tau_M}$ .

### 3. Main Results

In this section, we will present several updated conditions of sampling data such that complex network (1) synchronizes with the target node (3).

**Theorem 1.** *Suppose that Assumptions 1 and 2 hold. If there exist a positive definite symmetric matrix  $P \in R^{n \times n}$ , scalars  $\alpha > 0, \beta > 0$  and  $\mu > 0$  such that*

$$PB + B^T P - \beta I_n < 0, \quad (13)$$

$$I_N \otimes (PA + A^T P - 2\mu P + \alpha I_n) + C \otimes (P\Gamma + \Gamma^T P) < 0, \quad (14)$$

and

$$\alpha > \beta L, \quad (15)$$

then complex network (1) exponentially synchronizes with target node (3) under the action of the controller (6).

*Proof.* Choose the following Lyapunov functional as

$$V(t) = y^T(t) (I_N \otimes P) y(t). \quad (16)$$

For  $t \in [t_k, t_{k+1}]$ , the derivative of  $V(t)$  with respect to the trajectories of system (9) is

$$\begin{aligned} \dot{V}(t) &= 2y^T(t) (I_N \otimes P) [\tilde{A}y(t) + (I_N \otimes B)\phi(t) + \mu e(t)] \\ &= y^T(t) \tilde{A}y(t) + y^T(t) [I_N \otimes (PB + B^T P)] \phi(t) \\ &\quad + 2\mu y^T(t) (I_N \otimes P) e(t), \end{aligned} \quad (17)$$

where  $\tilde{A} = I_N \otimes (PA + A^T P - 2\mu P) + C \otimes (P\Gamma + \Gamma^T P)$ . It follows from Assumption 1 and inequality (13) that

$$\begin{aligned} y^T(t) [I_N \otimes (PB + B^T P)] \phi(t) &\leq \|y(t)\| \cdot \|PB + B^T P\| \cdot \|\phi(t)\| \\ &\leq \beta \|y(t)\| \cdot \|\phi(t)\| \leq \beta L \|y(t)\|^2. \end{aligned} \quad (18)$$

In view of (14) and (15), we have

$$\begin{aligned} \dot{V}(t) &\leq y^T(t) (\tilde{A} + \beta L \cdot I_n) y(t) + 2\mu y^T(t) (I_N \otimes P) e(t) \\ &\leq (\beta L - \alpha) y^T(t) y(t) + 2\mu \|y(t)\| \cdot \|P\| \cdot \|e(t)\| \\ &\leq \frac{\beta L - \alpha}{\lambda_{\max}(P)} V(t) + 2\mu \|y(t)\| \cdot \|P\| \cdot \|e(t)\|. \end{aligned} \quad (19)$$

Since

$$\begin{aligned} \dot{e}(t) &= \dot{y}(t) + \mu e^{-\mu(t-t_k)} y(t_k) \\ &= \tilde{A}y(t) + (I_N \otimes B)\phi(t) + \mu e(t) + \mu e^{-\mu(t-t_k)} y(t_k) \\ &= \tilde{A}y(t) + (I_N \otimes B)\phi(t) + \mu y(t). \end{aligned} \quad (20)$$

Integrating from  $t_k$  to  $t$  on the above equation and using  $e(t_k) = 0$ , one gets

$$e(t) = \int_{t_k}^t [\tilde{A}y(\vartheta) + (I_N \otimes B)\phi(\vartheta) + \mu y(\vartheta)] d\vartheta. \quad (21)$$

Thus, we obtain

$$\begin{aligned} \|e(t)\| &\leq \int_{t_k}^t \|\tilde{A}y(\vartheta) + (I_N \otimes B)\phi(\vartheta) + \mu y(\vartheta)\| d\vartheta \\ &\leq \int_{t_k}^t [\|\tilde{A}\| \cdot \|y(\vartheta)\| + \|B\| \cdot \|\phi(\vartheta)\| + \mu \cdot \|y(\vartheta)\|] d\vartheta \\ &\leq (\|\tilde{A}\| + L\|B\| + \mu) \int_{t_k}^t \|y(\vartheta)\| d\vartheta. \end{aligned} \quad (22)$$

Substituting (22) into (19), then

$$\begin{aligned}
\dot{V}(t) &\leq \frac{\beta L - \alpha}{\lambda_{\max}(P)} V(t) + 2\mu \|y(t)\| \cdot \|P\| \\
&\quad \cdot (\|\tilde{A}\| + L\|B\| + \mu) \int_{t_k}^t \|y(\vartheta)\| d\vartheta \\
&\leq \frac{\beta L - \alpha}{\lambda_{\max}(P)} V(t) + \frac{2\mu}{\lambda_{\min}(P)} \sqrt{V(t)} \cdot \|P\| \\
&\quad \cdot (\|\tilde{A}\| + L\|B\| + \mu) \int_{t_k}^t \|V(\vartheta)\| d\vartheta \\
&\leq -2g_1 V(t) + 2g_2 \sqrt{V(t)} \int_{t_k}^t \|V(\vartheta)\| d\vartheta,
\end{aligned} \tag{23}$$

where  $g_1 = \alpha - \beta L / 2\lambda_{\max}(P)$  and  $g_2 = 2\mu / \lambda_{\min}(P) \cdot \|P\| \cdot (\|\tilde{A}\| + L\|B\| + \mu)$  which leads to

$$\frac{d\sqrt{V(t)}}{dt} \leq -g_1 \sqrt{V(t)} + g_2 \int_{t_k}^t \sqrt{V(\vartheta)} d\vartheta. \tag{24}$$

It follows from Lemma 2 that

$$V(t) \leq V(t_0) e^{-2\rho(t-t_0)}, \tag{25}$$

with  $\rho = (g_1 - g_2 \tau_M) e^{-g_1 \tau_M} > 0$ . By the fact  $\lambda_{\min}(P) \|y(t)\|^2 \leq V(t)$ , we get

$$\|y(t)\| \leq \sqrt{\frac{V(t_0)}{\lambda_{\min}(P)}} e^{-\rho(t-t_0)}, \tag{26}$$

which shows that complex network (1) exponentially synchronizes with target node (3) under the action of the controller (6). This completes the proof.  $\square$

*Remark 1.* It follows from  $\rho = (g_1 - g_2 \tau_M) e^{-g_1 \tau_M} > 0$  that

$$\begin{aligned}
\tau_M &< \frac{g_1}{g_2} = \frac{\alpha - \beta L}{2\lambda_{\max}(P)} \cdot \frac{\lambda_{\min}(P)}{\mu \|P\| (\|\tilde{A}\| + L\|B\| + \mu)} \\
&\leq \frac{\alpha - \beta L}{2\mu \|P\| (\|\tilde{A}\| + L\|B\| + \mu)},
\end{aligned} \tag{27}$$

which gives the upper bound of the sampling data interval. That is, only if the sampling period is not bigger than the bound  $\tau_M$ , then complex network (1) can exponentially synchronize with the target node (3) under controller (6). Moreover, if we take the sampling period  $T \in (0, \tau_M)$  as a constant, then it becomes a zero-order hold control with a fixed sampling period.

*Remark 2.* Although inequalities (13) ~ (15) are nonlinear on variables, if we take  $\mu$  beforehand, then these inequalities are linear matrix inequalities. Furthermore, by solving

$$\begin{aligned}
&\min \mu, \\
&\text{subject to (9), (10), (11),}
\end{aligned} \tag{28}$$

one can obtain the most optical  $\mu$ .

In this paper, the graph of the considered complex network is undirect, so the out-coupling matrix  $C$  is real symmetric. According to the properties of a real symmetric matrix, there must exist an orthogonal matrix  $U$  such that  $U^T C U = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\}$ , where  $\lambda_i (1 \leq i \leq N)$  are the eigenvalues of matrix  $C$ . By this fact, we have the following results.

**Theorem 2.** Suppose that Assumptions 1 and 2 hold and the graph is undirect. If there exist a positive definite symmetric matrix  $P \in R^{n \times n}$ , scalars  $\alpha > 0, \beta > 0$ , and  $\mu > 0$  such that

$$\begin{aligned}
PB + B^T P - \beta I_n &< 0, \\
PA + A^T P - 2\mu P + \alpha I_n + \lambda_i (\Gamma \Gamma + \Gamma^T P) &< 0, \quad i = 1, 2, \dots, N,
\end{aligned} \tag{29}$$

and

$$\alpha > \beta L, \tag{30}$$

then complex network (1) exponentially synchronizes with target node (3) under the action of controller (6).

*Proof.* Lift multiply  $U^T \otimes I_n$  and right multiply  $U^T \otimes I_n$  on both sides of inequality (14), one can obtain inequality (29). The proof is completed.

It is noticed that we also can use controller (6) to control a part of nodes in complex network (1), which is called the pinning control and has been usually used to investigate the synchronization of complex networks in the literature such as [28–30]. Without loss of generality, we only control the former  $l (1 \leq l \leq N)$  nodes with index  $i = 1, 2, \dots, l$ . Otherwise, we can rearrange the index of nodes.  $\square$

**Theorem 3.** Suppose that Assumptions 1 and 2 hold. If there exist a positive definite symmetric matrix  $P \in R^{n \times n}$ , scalars  $\alpha > 0, \beta > 0$  and  $\mu > 0$  such that

$$\begin{aligned}
PB + B^T P - \beta I_n &< 0, \\
I_N \otimes (PA + A^T P + \alpha I_n) - 2\bar{\mu} \otimes P + C \otimes (\Gamma \Gamma + \Gamma^T P) &< 0,
\end{aligned} \tag{31}$$

and

$$\alpha > \beta L, \tag{32}$$

then complex network (1) exponentially synchronizes with target node (3) under the pinning controller

$$\begin{aligned}
u_i(t) &= -\mu e^{-\mu(t-t_k)} [x_i(t_k) - s(t_k)], \\
t &\in [t_k, t_{k+1}], \quad 1 \leq i \leq l \leq N,
\end{aligned} \tag{33}$$

where  $\bar{\mu} = \text{diag} \left\{ \underbrace{\mu, \dots, \mu}_l, \underbrace{0, \dots, 0}_{N-l} \right\}$ .

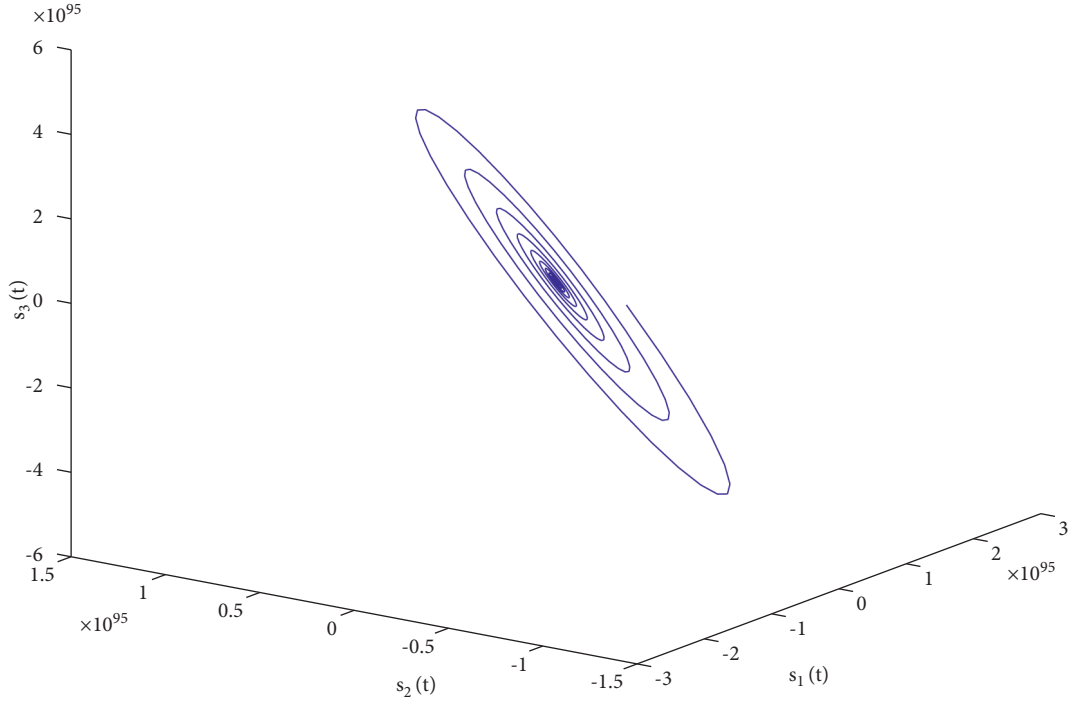


FIGURE 1: State trajectories of the Chua's system with the given initial condition  $s(0)$ .

*Proof.* Substituting (33) into (1), we yield

$$\begin{aligned} \dot{y}(t) &= (I_N \otimes A - \bar{\mu} \otimes I_n + C \otimes \Gamma)y(t) \\ &\quad + (I_n \otimes B)\phi(t) + \bar{\mu} \otimes I_n e(t) \\ &= \bar{A}y(t) + (I_n \otimes B)\phi(t) + \bar{\mu} \otimes I_n e(t), \end{aligned} \quad (34)$$

where  $\bar{A} = I_N \otimes A - \bar{\mu} \otimes I_n + C \otimes \Gamma$ . Similar to the proof of Theorem 1, we can obtain the above results. This completes the proof.  $\square$

*Remark 3.* It is similar to Remark 1, we obtain the length of the sampling interval

$$\bar{\tau}_M \leq \frac{\alpha - \beta L}{2\mu \|P\| (\|\bar{A}\| + L\|B\| + \mu)}. \quad (35)$$

In addition, (14) holds if (32) holds because of  $\hat{\mu} \otimes P > 0$ ,

where  $\hat{\mu} = \text{diag} \left\{ \underbrace{0, \dots, 0}_l, \underbrace{\mu, \dots, \mu}_{N-l} \right\}$ . But the converse does

not hold, which shows that the complex network can achieve synchronization by using the pinning control, then it can achieve synchronization by controlling all the nodes.

#### 4. A Numerical Example

In this section, we provide the following example to show the effectiveness of our proposed method.

It is well known that Chua's dynamical system is described by

$$\begin{cases} \dot{s}_1(t) = 10[s_2(t) - s_1(t) - f_1(s_1(t))], \\ \dot{s}_2(t) = s_1(t) - s_2(t) + s_3(t), \\ \dot{s}_3(t) = -15s_2(t) - 0.0385s_3(t), \end{cases} \quad (36)$$

where

$f_1(s_1(t)) = bs_1(t) + 0.5(a - b)(|s_1(t) + 1| - |s_1(t) - 1|)$ ,  $a$  and  $b$  are two constants. Furthermore, it has been shown that system (36) possesses a chaotic behavior if  $a$  and  $b$  are appropriately chosen. In this example, we choose  $a = -1.14$  and  $b = -0.714$ , then the phase diagram of this system is shown in Figure 1 with the initial condition  $s(0) = (-500, 400, -300)^T$ .

On the other hand, this system can be rewritten as

$$\dot{s}(t) = As(t) + Bf(s(t)), \quad (37)$$

where

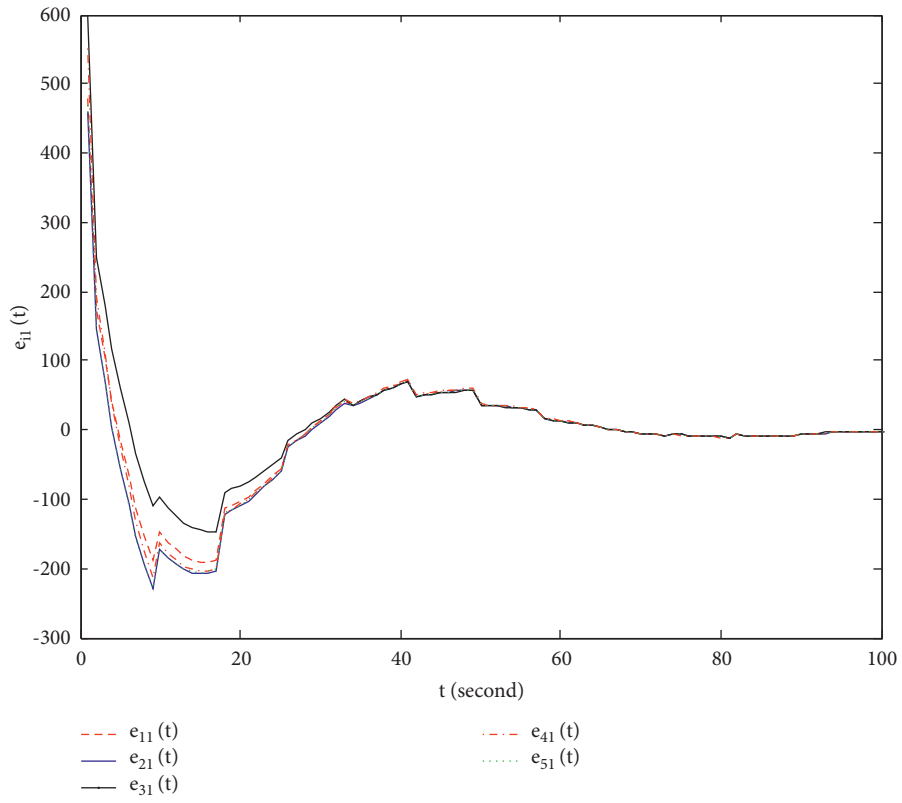
$$s(t) = (s_1(t), s_2(t), s_3(t))^T, A = \begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -15 & -0.0385 \end{bmatrix}, B =$$

$$I_3, f(s(t)) = (-10f_1(s_1(t)), 0, 0)^T.$$

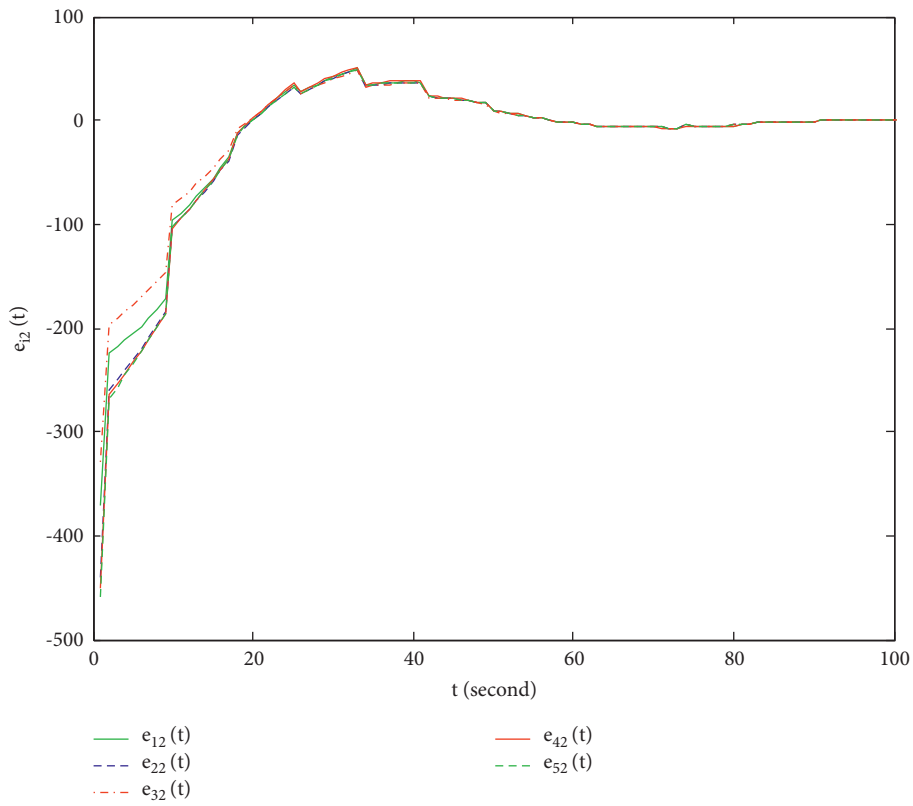
Since

$$\begin{aligned} \|f(s(t)) - f(\hat{s}(t))\| &= 10|f_1(s_1(t)) - f_1(\hat{s}(t))| \\ &\leq 10(|b| + |a - b|) \cdot |s_1(t) - \hat{s}(t)| \\ &\leq 10(|b| + |a - b|) \cdot |s(t) - \hat{s}(t)|, \end{aligned} \quad (38)$$

for any  $s(t), \hat{s}(t) \in \mathbb{R}^3$ ,  $f(s(t))$  satisfies Assumption 1 as  $L = 11.4$ .



(a)



(b)

FIGURE 2: Continued.

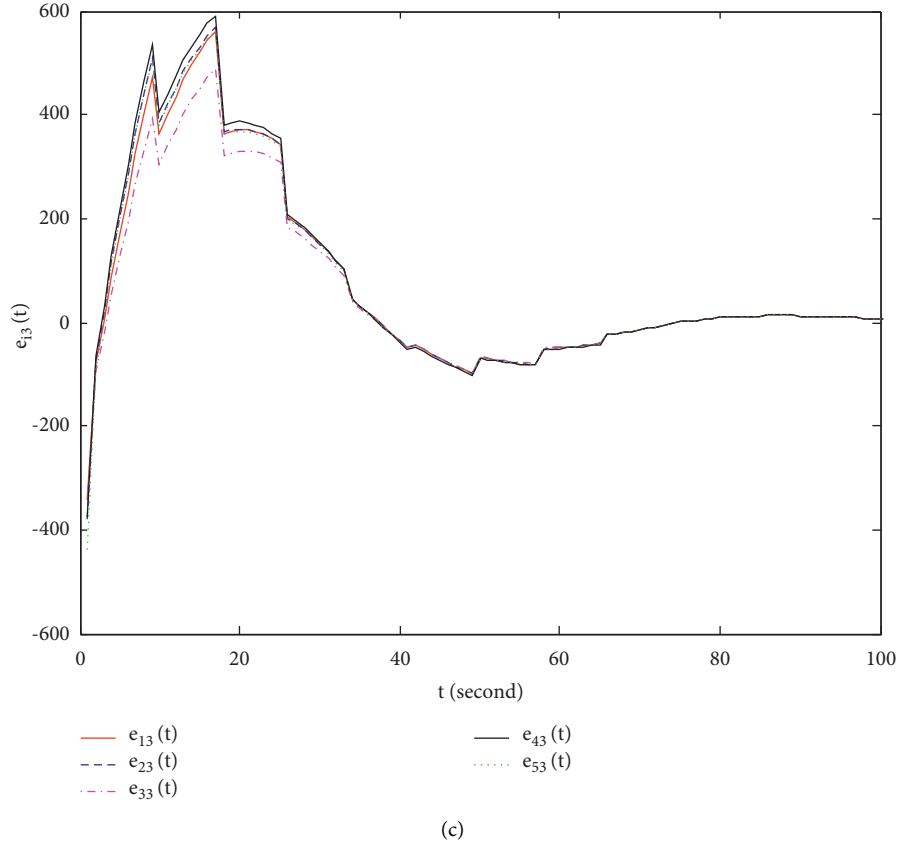


FIGURE 2: The error trajectories of system (9) with each node being the Chua's system ( $i = 1, 2, \dots, 5$ ).

Consider complex network (1) with five nodes ( $N = 5$ ) and take Chua's dynamical system (37) as the target node. Assume that the outer-coupling matrix is

$$C = \begin{bmatrix} -5 & 2 & 0 & 1 & 2 \\ 2 & -6 & 1 & 0 & 3 \\ 0 & 1 & -3 & 0 & 2 \\ 1 & 0 & 0 & -2 & 1 \\ 2 & 3 & 2 & 1 & -8 \end{bmatrix}, \quad (39)$$

and the inner-coupling matrix is  $\Gamma = \text{diag}\{0.2, 0.5, 0.7\}$ , respectively. By the LMI toolbox in Matlab, we obtain the feasible solutions of inequalities (13)–(15) as follows:

$$\begin{aligned} \alpha &= 5.0296, \\ \beta &= 0.4101, \\ P &= \begin{bmatrix} 0.1055 & 0.0252 & -0.0143 \\ 0.0252 & 0.1284 & -0.0386 \\ -0.0143 & -0.0386 & 0.1574 \end{bmatrix}. \end{aligned} \quad (40)$$

According to Theorem 1, we know that complex network (1) can synchronize with target node  $\dot{s}(t) = As(t) + f(s(t))$  under the action of controller (6). Taking the initial condition

$$y(0) = (-20, 30, -40, -40, -40, -80, 100, 70, -40, 50, -50, -75, -30, -60, -140)^T, \quad (41)$$

the state trajectories of the error system (9) with the given parameters are shown in Figure 2, which shows that all the nodes synchronize well with each other, and our proposed method is effective.

## 5. Conclusions

This paper has investigated the exponential synchronization of complex network based on sampled-data control. By constructing a novel sampling-data controller, the complex networks can exponentially synchronize with the target node. Moreover, we have estimated the bound of the sampling intervals. A numerical example has shown that our method is effective. In the future, we will continue to study the synchronization of complex networks with multiple influencing factors, find out the quantity relationship of the synchronization with the complex network parameters, and provide some more effective synchronization controllers. Specially, we will consider how to apply the existing results and methods of complex networks to solve practical problems.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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