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Research Article

Topological Models of Rough Sets and Decision Making of COVID-19

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The basic methodology of rough set theory depends on an equivalence relation induced from the generated partition by the classification of objects. However, the requirements of the equivalence relation restrict the field of applications of this philosophy. To begin, we describe two kinds of closure operators that are based on right and left adhesion neighbourhoods by any binary relation. Furthermore, we illustrate that the suggested techniques are an extension of previous methods that are already available in the literature. As a result of these topological techniques, we propose extended rough sets as an extension of Pawlak's models. We offer a novel topological strategy for making a topological reduction of an information system for COVID-19 based on these techniques. We provide this medical application to highlight the importance of the offered methodologies in the decision-making process to discover the important component for coronavirus (COVID-19) infection. Furthermore, the findings obtained are congruent with those of the World Health Organization. Finally, we create an algorithm to implement the recommended ways in decision-making.

1. Introduction

General topology [1] has recently been put as a topic in its own right as well as being a significant mathematical instrument with such varied topics as operational research approaches, biochemistry, genetics, and sociology. The construction of topology and its concepts via relations has become a remarkable and hot role in solving many problems such as [2–15]. Closure space has been described by Cech [10] as an extension of classical topology. Pal in [15] defined closure operators through binary relations. Relations represent a mathematical tool that has some real-life data and applications which can be resolved. Moreover, relations are applied in building some topological structures that are used in structure analysis, general view of space-time, biochemistry, biology, dynamics, fuzzy set model, and rough set idea. For more details, see [2–8, 12–14, 16–21]. In the viewpoint of topological structures, topological notions, especially, closure operators with relations, are useful in applications.

In covering-based rough sets, an adhesion-set is introduced [21]. Nawar et al. [22] utilized it to define the j-adhesion neighbourhood formed by any binary relation, and they proposed different types of covering-based rough sets as a result. Furthermore, Atef et al. [23] provided six rough approximations derived by j-adhesion neighbourhoods utilizing j-neighbourhood space [8] which was modified by El-Bably et al. in [5].

In this study, the idea of adhesion neighbourhood is utilized to construct various generalized closure operators via relations. These operators are a generalization of Galton's [17], Allam's et al. [18], and El-Bably's et al. operators [19, 20]. Furthermore, we demonstrated that the new operators are topological properties with no requirements, and we showed some of their features. Rough set theory was established by computer scientist Pawlak [24, 25] based on several difficulties in computer science to overcome this challenge by a modal approximation of a crisp set in the expressions of a pair of sets called the rough approximations of it. Many writers have focused on generalization rough sets [2, 4, 8, 11, 13, 20, 22, 23, 26–39].

The rough set model's basic subject is the notion of uncertainty areas. It seeks to find the boundary region by expanding the lower approximation and contracting the higher approximation. These approximations were induced by closure operators in this context. The comparison between our techniques with the Yao strategy is studied. Lellis Thivagar and Richard [40] proposed the concept of nano topology. It is mostly based on Pawlak's preliminary guesses (namely, the lower, upper, and boundary region of a rough set). Nano open sets are the characteristics of a nano topological space.

We construct novel nano topologies from generalized rough approximations using the proposed operators in this research, and we apply them in the decision-making of COVID-19 infection. We reduce the amount of data we collect in order to determine the risk variables for COVID-19 infection. As a result, we assert that remaining at home may reduce the danger. We create an algorithm for decision-making utilizing our ideas towards the conclusion of the article.

2. Basic Concepts and Properties

Some definitions and results from the sequel are provided.

Definition 1 (see [41]). Let R be a binary relation from a nonempty set X to a nonempty set Y. R can be from X to itself and called a relation on X. Consequently, if R is a binary relation from X to Y, we say that $a \in X$ is related to $b \in Y$ if $(a, b) \in R$, sometimes written aRb.

Definition 2 (see [41]). A binary relation R on X is called as

(i) Reflexive if aRa, $\forall a \in X$
(ii) Symmetric if $aRb \longrightarrow bRa$, $\forall a, b \in X$
(iii) Transitive if $aRb \wedge bRc \longrightarrow aRc$, $\forall a, b, c \in X$
(iv) Equivalence if conditions (i), (ii), and (iii) are satisfied.

Definition 3 (see [41]). Let R be a relation on X. After set and foreset of $x \in X$ are $xR = \{y \in X\colon xRy\}$ and $Rx = \{y \in X\colon yRx\}$, respectively.

Definition 4 (see [1]). A collection τ of subsets of U is a topology on U if it contains $U, \varnothing$, finite intersection (resp. arbitrary union) of its elements is closed. (U, τ) is a topological space, each set in τ is open, and its complement w. r. to U is closed. $int_\tau(X)$ (resp. $cl_\tau(X)$) denotes the interior (resp. closure) of X in (U, τ).

Definition 5 (see [27]). The operator $Cl\colon P(U) \longrightarrow P(U)$, where $P(U)$ is the power set of U, is called *the closure* of X, where $Cl(X) \subseteq U$, if satisfies

(i) $Cl(\varnothing) = \varnothing$
(ii) $X \subseteq Cl(X)$, $\forall X \subseteq U$
(iii) $Cl(X \cup Y) = cl(X) \cup cl(Y)$, $\forall\ X, Y \subseteq U$. The pair (U, Cl) is called a closure space.

Definition 6 (see [17]). Let (U, cl) be a closure space and let $X \subseteq U$. Then,

(i) $Int(X) = (Cl(X^c))^c$, where X^c is the complement of X w. r. to U
(ii) X is a neighbourhood of an element $x \in X$ if $x \in Int(X)$
(iii) If $B = Cl(B)$, then B is closed
(iv) if $A = Int(A)$, then A is open.

For each (U, Cl), $Int\colon P(U) \longrightarrow P(U)$ is called *the interior* of X [13] and satisfies.

(i) $Int(U) = U$
(ii) $Int(X) \subseteq X$, $\forall X \subseteq U$
(iii) $Int(X \cap Y) = Int(X) \cap Int(Y)$, $\forall X, Y \subseteq U$.

Remark 1. If $Cl(Cl(X)) = Cl(X)$, then the space is called a closure space.

Definition 7 (see [27]). Let R be a relation on X. Then, the closure operator on U by R is $cl_R(X) = X \cup \{y \in U\colon \exists x \in X\colon yRx\}$.

Lemma 1 (see [17]). *Let R be a transitive relation on a nonempty finite set U. Then, the operator $\mathbf{cl}_R$ is a topological closure.*

Definition 8 (see [24]). Let U be a universal set and R be an equivalence relation on U. $U/R = \{[x]_R\colon x \in U\}$ are equivalence classes of R. (U, R) is an approximation space. The lower and upper approximations of X are $\underline{R}(X) = \{x \in X\colon [x]_R \subseteq X\}$ and $\overline{R}(X) = \{x \in U\colon [x]_R \cap\ X \neq \varnothing\}$, respectively, for any $X \subseteq U$. X is rough set if $\underline{R}(X) \neq \overline{R}(X)$.

Proposition 1 (see [24]). *If X^c is a complement set of X in U, then $\varnothing$ represents an empty set. The properties of Pawlak's rough sets are as follows:*

$(L1)\,\underline{R}(X) \subseteq X$ $\quad (U1)\,X \subseteq \overline{R}(X)$
$(L2)\,\underline{R}(\varnothing) = \varnothing$ $\quad (U2)\,\overline{R}(\varnothing) = \varnothing$
$(L3)\,\underline{R}(U) = U$ $\quad (U3)\,\overline{R}(U) = U$
$(L4)\,\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$ $\quad (U4)\,\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$
$(L5)$ If $X \subseteq Y$, then $\underline{R}(X) \subseteq \underline{R}(Y)$ $\quad (U5)$ If $X \subseteq Y$, then $\overline{R}(X) \subseteq \overline{R}(Y)$
$(L6)\,\underline{R}(X) \cup \underline{R}(Y) \subseteq \underline{R}(X \cup Y)$ $\quad (U6)\,\overline{R}(X) \cap \overline{R}(Y) \supseteq \overline{R}(X \cap Y)$
$(L7)\,\underline{R}(X^c) = (\overline{R}(X))^c$ $\quad (U7)\,\overline{R}(X^c) = (\underline{R}(X))^c$
$(L8)\,\underline{R}(\underline{R}(X)) = \underline{R}(X)$ $\quad (U8)\,\overline{R}(\overline{R}(X)) = \overline{R}(X)$
$(L9)\,\underline{R}((\underline{R}(X))^c) = (\underline{R}(X))^c$ $\quad (U9)\,\overline{R}((\overline{R}(X))^c) = (\overline{R}(X))^c$
$(L10)\,\forall K \in U/R \Rightarrow \underline{R}(K) = K$ $\quad (U10)\,\forall K \in U/R \Rightarrow \overline{R}(K) = K$

Definition 9 (see [26]). For $A \subseteq U$, the lower and upper approximations of A w. r. to neighbourhood $n(x)$ of x are $\underline{apr}(A) = \{x \in U: n(x) \subseteq A\}$ and $\overline{apr}(A) = \{x \in U: \overline{n(x)} \cap A \neq \emptyset\}$, respectively. The properties (L3-L9) and (U1, U2, U4-U9) are satisfied, in general, while some Pawlak's properties are held in $n(x)$.

Definition 10 (see [40]). In (U, R), $\tau_R = \{U, \emptyset, \underline{R}(X), \overline{R}(X), B(X)\}$ is a nano topology on U w. r. to X with a base $\mathscr{B} = \{U, \underline{R}(X), B(X)\}$.

3. A New Closure Operator and Its Equivalences

Throughout this section, some different sorts of closure operators in terms of relations are introduced, and comparisons between them are discussed.

Definition 11 (see [5, 22, 23]). Let R be any relation on U. The right adhesion and left adhesion set of each $y \in U$ are defined, respectively, as follows:

(i) r-adhesion: $A_r(x) = \{y \in U: \ xR = yR\}$
(ii) l-adhesion: $A_l(x) = \{y \in U: \ Rx = Ry\}$

It is simple to demonstrate the following lemma, so we omit the proof.

Lemma 2. *Let R be any relation on U. Then, $\forall j \in \{r, l\}$.*

(i) $A_j(x) \neq \emptyset$
(ii) $x \in A_j(x)$
(iii) $A_j(U) = \{A_j(x): x \in U\}$ is a partition on U.

Lemma 3. *For all $j \in \{r, l\}$: $y \in A_j(x)$ if $A_j(y) = A_j(x)$, for any R on U.*

Proof. It is sufficient to prove this lemma for $j = r$ and the others similarly. Firstly, if $\in \in A_r(x) \Rightarrow yR = xR$. Now, it is sufficient to prove $A_r(y) \subseteq A_r(x)$. Let $z \in A_r(y)$, then $zR = yR$. Hence, $zR = xR$, and this implies $z \in A_r(x)$. Thus, $A_r(y) \subseteq A_r(x)$. Also, $A_r(y) \supseteq A_r(x)$ is proved similarly. □

Proposition 2. *For a reflexive relation R on U and $x \in U$, we get*

(i) $A_r(x) \subseteq xR$
(ii) $A_l(x) \subseteq Rx$.

Proof. (i) Let $y \in A_r(x)$, then $yR = xR$. But R is a reflexive relation on U, thus $x \in xR$, $\forall x \in U$. Hence, $y \in xR$, and this implies $A_r(x) \subseteq xR$, $\forall x \in U$. (ii) is proved similarly. □

Remark 2. The equality of Proposition 5 is not held, in general, as illustrated in Example 1.

Example 1. Let $U = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, b), (c, c), (c, a), (d, d)\}$. Then, we get the following: $aR = \{a, b\}$, $bR = \{b\}$, $cR = \{a, c\}$, and $dR = \{d\}$. It is clear that $A_r(a) = \{a\}$, $A_r(b) = \{b\}$, $A_r(c) = \{c\}$, and $A_r(d) = \{d\}$.

Definition 12. For any relation R on U and $X \subseteq U$, $Cl_j: P(U) \longrightarrow P(U)$ is $Cl_j(X) = \{x \in U: A_j(x) \cap X \neq \emptyset\}$, $\forall j \in \{r, l\}$.

The following proposition proves that Cl_j represents a closure operator.

Proposition 3. *If R is an arbitrary binary relation on U. Then, $\forall j \in \{r, l\}$, and the pair (U, Cl_j) is a closure space.*

Proof.

(i) Clearly, $Cl_j(\emptyset) = \emptyset$.
(ii) According to Lemma 3, Definition 10, and by contradiction, assume that $X \not\subset Cl_j(X)$, then there exists at least $x \in X$ such that $x \notin Cl_j(X)$. Thus, $x \in X$ such that $A_j(x) \cap X = \emptyset$. But $x \in A_j(x)$, $\forall x \in U$, thus, $A_j(x) \cap X \neq \emptyset$, and this is a contradiction to the assumption that $x \notin Cl_j(X)$. Hence, $X \subseteq Cl_j(X)$.

$$\begin{aligned} Cl_j(X \cup Y) &= \{x \in U: A_j(x) \cap (X \cup Y) \neq \Phi\} \\ &= \{x \in U: A_j(x) \cap X \neq \Phi\} \text{ or } \{x \in U: A_j(x) \cap Y \neq \Phi\} \\ &= \{x \in U: A_j(x) \cap X \neq \Phi\} \cup \{x \in U: A_j(x) \cap Y \neq \Phi\}. \end{aligned} \tag{1}$$

Thus, $Cl_j(X \cup Y) = Cl_j(X) \cup Cl_j(Y)$.

We call the pair (U, Cl_j) in Proposition 3, a *j-closure space*. □

Definition 13. Suppose that (U, Cl_j) is a *j-closure space*. Then, $\forall j \in \{r, l\}$.

(i) $X \subseteq U$ is j-closed if $X = Cl_j(X)$
(ii) $\Gamma_j = \{X \subseteq U: X = Cl_j(X)\}$ is the collection of all j-closed in U
(iii) The complement of j-closed is called j-open

Proposition 4. *Let (U, Cl_j) be a j-closure space. Then, $\forall j \in \{r, l\}$, and the j-closure Cl_j is a closure operator in a viewpoint of topology.*

Proof. It is sufficient to prove Cl_r is a topological closure, and Cl_l is so.

It is clear that $Cl_r(X) \subseteq Cl_r(Cl_r(X))$. Thus, we only prove that $Cl_r(Cl_r(X)) \subseteq Cl_r(X)$. Let $x \in Cl_r(Cl_r(X))$. Then, $A_r(x) \cap Cl_r(X) \neq \emptyset$ which implies $\exists y \in A_r(x)$ and $y \in Cl_r(X)$. Thus, $A_r(y) = A_r(x)$ and $A_r(y) \cap X \neq \emptyset$. Then, $A_r(x) \cap X \neq \emptyset$ which implies $x \in Cl_r(X)$. □

Theorem 1. *Every j-closure space (U, Cl_j), $\forall j \in \{r, l\}$, is topological space.*

Proof. Directly by Proposition 4. □

Remark 3. According to Theorem 1, we notice that the closure operator Cl_j, for each $j \in \{r, l\}$, represents a generalization of Galton [17] approach (Definition 7), Allam et al. [18], and El-Bably et al. [19, 20].

Proposition 5. *For a reflexive relation R on U and $X \subseteq U$, $Cl_j(X) \subseteq cl_R(X)$, $\forall j \in \{r, l\}$.*

Proof. From Lemma 4, we have $A_r(x) \subseteq xR$. Thus, if $x \in Cl_j(X)$. Then, $A_r(x) \cap X \neq \varnothing$, and this implies $xR \cap X \neq \varnothing$. Hence, by Definition 3, $x \in cl_R(X)$. □

Remark 4. The equality of Proposition 5 is not held, in general, as shown in Example 2.

Example 2. Using Example 1, we take $X = \{a, b, d\}$. Then, $Cl_r(X) = \{a, b, d\} = X$, but $cl_R(X) = U$.

Now, we also define the interior operation from the j-closure operation, which represents a topological interior of the topological space (U, Cl_j).

Definition 14. If (U, Cl_j) is a *j-closure space* and $\forall j \in \{r, l\}$, then for each $X \subseteq U$, the operator Int_j: $P(U) \longrightarrow P(U)$ is $Int_j(X) = \{x \in U\colon A_j(x) \subseteq X\}$.

In the following lemmas, we prove that the *j-interior operator (Abbriv. Int_j)* is an interior operator on U. Also, we prove that Cl_j and Int_j are dual.

Lemma 4. *Let (U, Cl_j) be a j-closure space and $\forall j \in \{r, l\}$. Then, $int_j(X) = [Cl_j(X^c)]^c$, where X^c denotes the complement of X.*

Proof. $[Cl_j(X^c)]^c = [\{x \in U\colon A_j(x) \cap X^c \neq \varnothing\}]^c = \{x \in U\colon A_j(x) \cap X^c \neq \varnothing\}^c. = \{x \in U\colon A_j(x) \cap X^c = \varnothing\} = \{x \in U\colon A_j(x) \cap X^c = \varnothing\}$ □

Lemma 5. *Let (U, Cl_j) be a j-closure space and $\forall j \in \{r, l\}$. Then, $Int_j(X)$ satisfies the following properties:*

(i) $Int_j(U) = U$

(ii) $Int_j(X) \subseteq X$, for each $X \subseteq U$

(iii) $Int_j(X \cap Y) = Int_j(X) \cap Int_j(Y)$

(iv) $Int_j(Int_j(X)) = Int_j(X)$.

Lemma 6. *Let (U, Cl_j) be a j-closure space, $\forall j \in \{r, l\}$ and $X \subseteq U$. Then, X is j-open if and only if $Int_j(X) = X$.*

4. Generalized Approximations in Terms of Closure Operators

In the current section, we aim to present some topological properties of operators Cl_j and Int_j for each $j \in \{r, l\}$ and apply its relationship with the rough set theory. This study is a generalization for Pawlak's approximations [24]. In addition, a comparison between them is discussed with different sorts of examples.

Definition 15. Let (U, Cl_j) be a *j-closure space*, $\forall j \in \{r, l\}$ and $X \subseteq U$. Then, j-lower and j-upper approximations of X are defined, respectively, by

$$\begin{aligned} \underline{R}_j(X) &= \{x \in U\colon A_j(x) \subseteq X\} = Int_j(X), \text{ and} \\ \overline{R}_j(X) &= \{x \in U\colon A_j(x) \cap X \neq \varnothing\} = Cl_j(X). \end{aligned} \tag{2}$$

Definition 16. Let (U, Cl_j) be a *j-closure space*, $\forall j \in \{r, l\}$ and $X \subseteq U$. The j-boundary, j-positive, and j-negative regions of X, for each $j \in \{r, l\}$, are defined, respectively, by

$$\begin{aligned} B_j(X) &= \overline{R}_j(X) - \underline{R}_j(X), \\ POS_j(X) &= \underline{R}_j(X), \textit{ and} \\ NEG_j(X) &= U - \overline{R}_j(X). \end{aligned} \tag{3}$$

Definition 17. Let (U, Cl_j) be a *j-closure space*, $\forall j \in \{r, l\}$ and $X \subseteq U$. Then, X is called j-exact set if $\underline{R}_j(X) = \overline{R}_j(X) = X$. Otherwise, it is called j-rough.

Definition 18. Let (U, Cl_j) be a *j-closure space*, $\forall j \in \{r, l\}$ and $X \subseteq U$. Thus, the j-accuracy of the approximations of X is defined as follows:

$\alpha_j(X) = |\underline{R}_j(A)|/|\overline{R}_j(A)|$, where $|\overline{R}_j(X)| \neq 0$ and $|X|$ refer to the cardinality of X.

Remark 5. According to Definitions 15–18, we have $\forall j \in \{r, l\}$.

(i) $0 \leq \alpha_j(X) \leq 1$, for every $X \subseteq U$, $\forall j \in \{r, l\}$

(ii) If $\alpha_j(X) = 1\, an\, d\, \mathscr{B}_j(X) = \varnothing$, then X is j-exact. Otherwise, it is j-rough.

Proposition 6. *Let (U, Cl_j) be a j-closure space, $\forall j \in \{r, l\}$ and $X, Y \subseteq U$. Then, we have*

$(L1)$ $\underline{R}_j(X) \subseteq X$ $(U1) X \subseteq \overline{R}_j(X)$

$(L2)$ $\underline{R}_j(\varnothing) = \varnothing$ $(U2) \overline{R}_j(\varnothing) = \varnothing$

$(L3)$ $\underline{R}_j(U) = U$ $(U3) \overline{R}_j(U) = U$

$(L4)$ $\underline{R}_j(X \cap Y) = \underline{R}_j(X) \cap \underline{R}_j(Y)$ $(U4) \overline{R}_j(X \cup Y) = \overline{R}_j(X) \cup \overline{R}_j(Y)$

$(L5)$ $If\ X \subseteq Y, then\ \underline{R}_j(X) \subseteq \underline{R}_j(Y)$ $(U5)$ If $X \subseteq Y$, then $\overline{R}(X) \subseteq \overline{R}(Y)$

$(L6)$ $\underline{R}_j(X) \cup \underline{R}_j(Y) \subseteq \underline{R}_j(X \cup Y)$ $(U6) \overline{R}_j(X) \cap \overline{R}_j(Y) \supseteq \overline{R}_j(X \cap Y)$

$(L7)$ $\underline{R}_j(X^c) = (\overline{R}_j(X))^c$ $(U7) \overline{R}_j(X^c) = (\underline{R}_j(X))^c$

$(L8)$ $\underline{R}_j(\underline{R}_j(X)) = \underline{R}_j(X)$ $(U8)\, \overline{R}_j(\overline{R}_j(X)) = \overline{R}_j(X)$

$(L9)\, \underline{R}_j((\underline{R}_j(X))^c) = (\underline{R}_j(X))^c$ $(U9)\, \overline{R}_j((\overline{R}_j(X))^c) = (\overline{R}_j(X))^c$

$(L10)\, \forall K \in \mathscr{A}_j(U) \Rightarrow \underline{R}_j(K) = K$ $(U10)\, \forall K \in \mathscr{A}_j(U) \Rightarrow \overline{R}(K) = K$

Proof. Directly, using the definitions and properties of Cl_j and Int_j, the proof is clear. □

Remark 6.

(1) If R represents an equivalence relation, then the j-adhesion set $A_j(x)$ of each $x \in U$ represents an

equivalence class of x; that is, $A_j(x) = [x]_R$. Thus, our approximations (j-approximations that given in Definitions 15 and 16) are conceded with Pawlak's approximations.

(2) Moreover, according to Proposition 4.6, the j-approximations satisfy all properties of the classical rough set model that was introduced by Pawlak [24], using any relation without any restrictions. Therefore, we say that the current methods represent an interesting generalization to the rough set theory.

(3) According to Definition 15, we have two different generalized rough approximation operators. Example 3 illustrates that these operators are independent.

Example 3. Let $U = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, b), (c, a), (d, b)\}$. Then, we get the following: $A_r(a) = \{a\}$, $A_r(b) = \{b, d\}$, $A_r(c) = \{c\}$, and $A_r(d) = \{b, d\}$. Furthermore, $A_l(a) = \{a\}$, $A_l(b) = \{b\}$, $A_l(c) = \{c, d\}$, and $A_l(d) = \{c, d\}$. Consider $X = \{a\}$, $Y = \{a, b\}$, and $Z = \{a, c\}$. Thus, we compute the approximations for some subsets of U as shown in Table 1.

From Table 1, it is clear that the subset X is r-exact and l-exact. However, Y is r-rough although it is l-exact, and also the subset Z is l-rough although it is r-exact.

Definition 19. Let (U, Cl_j) be a *j-closure space*, $\forall j \in \{r, l\}$ and $X \subseteq U$. The generalized lower and upper approximations, the boundary, positive and negative regions, and the accuracy of the approximations of X are defined by

$$\begin{aligned}
\underline{G}(X) &= \underline{R}_r(X) \cup \underline{R}_l(X),\\
\overline{G}(X) &= \overline{R}_r(X) \cap \overline{R}_l(X),\\
B(X) &= \overline{G}(X) - \underline{G}(X),\\
POS(X) &= \underline{G}(X),\\
NEG(X) &= U - \overline{G}(X)\ an\ d\\
\mu(X) &= \frac{|\underline{G}(X)|}{|\overline{G}(X)|}
\end{aligned} \tag{4}$$

where $|\overline{G}(X)| \neq 0$ and $|X|$ refer to the cardinality of X.

Definition 20. Let (U, Cl_j) be a *j-closure space*, $\forall j \in \{r, l\}$ and $X \subseteq U$. Then, X is a generalized exact (shortly, g-exact) set if $\underline{G}(X) = \overline{G}(X) = X$. Otherwise, it is called g-rough.

Remark 7

(1) $0 \leq \mu(X) \leq 1$, $\forall X \subseteq U$

(2) X is g-exact if $\mu(X) = 1$. Otherwise, it is a g-rough set.

Proposition 7. *If (U, Cl_j) is a j-closure space, $\forall j \in \{r, l\}$ and $X, Y \subseteq U$, then the generalized approximation operators $\underline{G}(X)$ and $\overline{G}(X)$) satisfy all Pawlak's properties (L1-L10) and (U1-U10).*

Proof. By using Proposition 6, the proof is obvious.

The main goal of the following results is to introduce the relationships between the g-approximations and j-approximations. Moreover, they show the best of these approximations. □

Theorem 2. *Let (U, Cl_j) be a j-closure space, $\forall j \in \{r, l\}$ and $X \subseteq U$, then*

(i) $\underline{R}_r(X) \subseteq \underline{G}(X) \subseteq X \subseteq \overline{G}(X) \subseteq \overline{R}_r(X)$

(ii) $\underline{R}_l(X) \subseteq \underline{G}(X) \subseteq X \subseteq \overline{G}(X) \subseteq \overline{R}_l(X)$.

Proof. It is sufficient to prove (i) and (ii) similarly. Let $x \in \underline{R}_r(X)$, then by Definition 19, $x \in \underline{G}(X)$, and thus, $x \in X$. Hence, $x \in \overline{G}(X)$ which means that, by Definition 19, $x \in \overline{R}_r(X)$. □

Corollary 1. *Let (U, Cl_j) be a j-closure space, $\forall j \in \{r, l\}$ and $Y \subseteq U$. So,*

(i) $B(Y) \subseteq B_j(Y)$

(ii) $\alpha_j(Y) \leq \mu(Y)$.

Corollary 2. *Let (U, Cl_j) be a j-closure space, $\forall j \in \{r, l\}$ and $X \subseteq U$. Then, X is g-exact if it is j-exact.*

The opposite of Corollary 2 is not correct in general.

Example 4. (Continuation of Example 3). The approximations of all subsets of U are calculated. Thus, Table 2 introduces comparisons between the approximations, boundary, and accurate measure of j-approximations and g-approximations.

Remark 8. From Table 2, it is noted that

(1) $\{a, b\}$ and $\{c, d\}$ are g-exact, but it is neither r-exact (r-rough) nor l-exact (*not* l-rough).

(2) $\{b, d\}$ and $\{a, b, d\}$ are g-exact, but it is neither r-exact (*not* l-rough) nor l-rough (*not* l-exact).

(3) $\{d\}$ and $\{b, c\}$ are g-rough. Also, it is r-rough and l-rough.

Example 5 (Continuation for Example 3). Table 3 represents a comparison between our method and Yao approach [26].

Remark 9. From Table 3, we notice that

(1) Our method in Definition 16 represents the best method for computing exactness and the roughness of sets, because the boundary regions are reduced or cancelled, and then, we obtain more accurate measures for approximating sets. On the other hand, in Yao approach, there are rough sets that are not defined, and then, we cannot be able, by Yao [26], to define it or remove the vagueness of it, but it is exact in our approaches.

(2) Moreover, in Yao approach, there are sets whose lower (resp., upper) approximation does not belong

Table 1: Comparison between r-approximations and l-approximations.

A	r-approximations				l-approximations			
	$\underline{R}_r(A)$	$\overline{R}_r(A)$	$B_r(X)$	$\alpha_r(A)$	$\underline{R}_l(A)$	$\overline{R}_l(A)$	$B_l(A)$	$\alpha_j(A)$
$\{a\}$	$\{a\}$	$\{a\}$	$\emptyset$	1	$\{a\}$	$\{a\}$	$\emptyset$	1
$\{a,b\}$	$\{a\}$	$\{a,b,d\}$	$\{b,d\}$	1/3	$\{a,b\}$	$\{a,b\}$	$\emptyset$	1
$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\emptyset$	1	$\{a\}$	$\{a,c,d\}$	$\{c,d\}$	1/3

Table 2: Comparison between j-approximations and g-approximations.

A	j-approximations								g-approximations			
	r-approximations				l-approximations							
	$\underline{R}_r(A)$	$\overline{R}_r(A)$	$B_r(X)$	$\alpha_r(A)$	$\underline{R}_l(A)$	$\overline{R}_l(A)$	$B_l(A)$	$\alpha_l(A)$	$\underline{G}(X)$	$\overline{G}(X)$	$\mathscr{B}(X)$	$\mu(X)$
$\{a\}$	$\{a\}$	$\{a\}$	$\emptyset$	1	$\{a\}$	$\{a\}$	$\emptyset$	1	$\{a\}$	$\{a\}$	$\emptyset$	1
$\{b\}$	$\emptyset$	$\{b,d\}$	$\{b,d\}$	0	$\{b\}$	$\{b\}$	$\emptyset$	1	$\{b\}$	$\{b\}$	$\emptyset$	1
$\{c\}$	$\{c\}$	$\{c\}$	$\emptyset$	1	$\emptyset$	$\{c,d\}$	$\{c,d\}$	0	$\{c\}$	$\{c\}$	$\emptyset$	1
$\{d\}$	$\emptyset$	$\{b,d\}$	$\{b,d\}$	0	$\emptyset$	$\{c,d\}$	$\{c,d\}$	0	$\emptyset$	$\{d\}$	$\{d\}$	0
$\{a,b\}$	$\{a\}$	$\{a,b,d\}$	$\{b,d\}$	1/3	$\{a,b\}$	$\{a,b\}$	$\emptyset$	1	$\{a,b\}$	$\{a,b\}$	$\emptyset$	1
$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\emptyset$	1	$\{a\}$	$\{a,c,d\}$	$\{c,d\}$	1/3	$\{a,c\}$	$\{a,c\}$	$\emptyset$	1
$\{a,d\}$	$\{a\}$	$\{a,b,d\}$	$\{b,d\}$	1/3	$\{a\}$	$\{a,c,d\}$	$\{c,d\}$	1/3	$\{a\}$	$\{a,d\}$	$\{d\}$	1/2
$\{b,c\}$	$\{c\}$	$\{b,c,d\}$	$\{b,d\}$	1/3	$\{b\}$	$\{b,c,d\}$	$\{c,d\}$	1/3	$\{b,c\}$	$\{b,c,d\}$	$\{d\}$	2/3
$\{b,d\}$	$\{b,d\}$	$\{b,d\}$	$\emptyset$	1	$\{b\}$	$\{b,c,d\}$	$\{c,d\}$	1/3	$\{b,d\}$	$\{b,d\}$	$\emptyset$	1
$\{c,d\}$	$\{c\}$	$\{b,c,d\}$	$\{b,d\}$	1/3	$\{c,d\}$	$\{c,d\}$	$\emptyset$	1	$\{c,d\}$	$\{c,d\}$	$\emptyset$	1
$\{a,b,c\}$	$\{a,c\}$	U	$\{b,d\}$	1/2	$\{a,b\}$	U	$\{c,d\}$	1/2	$\{a,b,c\}$	U	$\{d\}$	3/4
$\{a,b,d\}$	$\{a,b,d\}$	$\{a,b,d\}$	$\emptyset$	1	$\{a,b\}$	U	$\{c,d\}$	1/2	$\{a,b,d\}$	$\{a,b,d\}$	$\emptyset$	1
$\{a,c,d\}$	$\{a,c\}$	U	$\{b,d\}$	1/2	$\{a,c,d\}$	$\{a,c,d\}$	$\emptyset$	1	$\{a,c,d\}$	$\{a,c,d\}$	$\emptyset$	1
$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\emptyset$	1	$\{b,c,d\}$	$\{b,c,d\}$	$\emptyset$	1	$\{b,c,d\}$	$\{b,c,d\}$	$\emptyset$	1
U	U	U	$\emptyset$	1	U	U	$\emptyset$	1	U	U	$\emptyset$	1

Table 3: The boundary and accuracy of approximations for Yao Method 2.9 [26] and current method in Definition 4.2.

A	Yao Method				Method in Definition 16			
	$\underline{apr}(A)$	$\overline{apr}(A)$	$B(A)$	$\pi(A)$	$\underline{G}(A)$	$\overline{G}(A)$	$B(A)$	$\mu(A)$
$\{a\}$	$\{c\}$	$\{a,c,d\}$	$\{a,d\}$	1/3	$\{a\}$	$\{a\}$	$\emptyset$	1
$\{b\}$	$\{b,d\}$	$\{a,b,d\}$	$\{b\}$	2/3	$\{b\}$	$\{b\}$	$\emptyset$	1
$\{c\}$	$\emptyset$	$\emptyset$	$\emptyset$	Not defined	$\{c\}$	$\{c\}$	$\emptyset$	1
$\{d\}$	$\emptyset$	$\emptyset$	$\emptyset$	Not defined	$\emptyset$	$\{d\}$	$\{d\}$	0
$\{a,b\}$	U	U	$\emptyset$	1	$\{a,b\}$	$\{a,b\}$	$\emptyset$	1
$\{a,c\}$	$\{c\}$	$\{a,c\}$	$\{a\}$	1/2	$\{a,c\}$	$\{a,c\}$	$\emptyset$	1
$\{a,d\}$	$\{c\}$	$\{a,c\}$	$\{a\}$	1/2	$\{a\}$	$\{a,d\}$	$\{d\}$	1/2
$\{b,c\}$	$\{b,d\}$	$\{a,b,d\}$	$\{a\}$	2/3	$\{b,c\}$	$\{b,c,d\}$	$\{d\}$	2/3
$\{b,d\}$	$\{b,d\}$	$\{a,b,d\}$	$\{a\}$	2/3	$\{b,d\}$	$\{b,d\}$	$\emptyset$	1
$\{c,d\}$	$\emptyset$	$\emptyset$	$\emptyset$	Not defined	$\{c,d\}$	$\{c,d\}$	$\emptyset$	1
$\{a,b,c\}$	U	U	$\emptyset$	1	$\{a,b,c\}$	U	$\{d\}$	3/4
$\{a,b,d\}$	U	U	$\emptyset$	1	$\{a,b,d\}$	$\{a,b,d\}$	$\emptyset$	1
$\{a,c,d\}$	$\{c\}$	$\{a,c\}$	$\{a\}$	1/2	$\{a,c,d\}$	$\{a,c,d\}$	$\emptyset$	1
$\{b,c,d\}$	$\{b,d\}$	$\{a,b,d\}$	$\{a\}$	2/3	$\{b,c,d\}$	$\{b,c,d\}$	$\emptyset$	1
U	U	U	$\emptyset$	1	U	U	$\emptyset$	1

to the set which represents a contradiction to the original rough set theory given by Pawlak (e.g.,, $\{a,\}$, $\{b\}$, $\{b,c\}$, $\{a,b,c\}$, $\{a,b,d\}$).

5. Decision Making of COVID–19 as Medical Application

In this part, we provide a realistic example of how our methodologies might be used to make decisions for an information system concerning coronavirus infections (COVID-19). In fact, we have identified the risk factors for COVID-19 infection in people. In this model, the only decisive criteria for infection transmission are gathering, interaction with wounded individuals, and employment in hospitals. We conclude that remaining at home and avoiding contact with people protect against coronavirus infection. The authors of [42] state that human-to-human transmissions have been described with incubation times ranging from 2 to 10 days, allowing the virus to spread through

Persistence of coronaviruses on different types of inanimate surfaces					
Type of surface	Virus	Strain/isolate	Inoculum (viral titer)	Temperature	Persistence
Steel	MERS-CoV	Isolate HCoV-EMC/2012	10^5	20°C	48 h
				30°C	8-24 h
	TGEV	Unknown	10^6	4°C	≥ 28 d
				20°C	3-28 d
				40°C	4-96 d
	MHV	Unknown	10^6	4°C	≥ 28 d
				20°C	3-28 d
				40°C	4-96 h
	HCoV	Strain 229 E	10^3	21°C	5 d
Aluminium	HCoV	Strains 229 E and OC43	5×10^3	21°C	2-8 h
Metal	SARS-CoV	Strain P9	10^5	RT	5 d
Wood	SARS-CoV	Strain P9	10^5	RT	4 d
Paper	SARS-CoV	Strain P9	10^5	RT	4-5 d
	SARS-CoV	Strain GVU6109	6	RT	24 h
			10^5		3 h
			10^4		< 5 min
Glass	SARS-CoV	Strain P9	10^5	RT	4 d
	SARS-CoV	Strain 229 E	10^3	21°C	5 d
Plastic	SARS-CoV	Strain HKU39849	10^5	22-25°C	≤ 5 d
	MERS-CoV	Isolate HCoV-EMC/2012	10^5	20°C	4-24 d
				30°C	48 d
	SARS-CoV	Strain P9	10^5	RT	4 d
	SARS-CoV	Strain FFM1	10^7	RT	6-9d
	HCoV	Strain 229 E	10^7	RT	2-6 d
PVC	HCoV	Strain 229 E	10^3	21°C	5 d
Silicon rubber	HCoV	Strain 229 E	10^3	21°C	5 d
Surgicalg love (latex)		Strains 229 E and OC43	5×10^3	21°C	≤ 8 h
Disposable gown	SARS-CoV	Strain GVU6109	10^6	RT	2 d
			10^5		24 h
			10^4		1 h
Ceramic	HCoV	Strain 229 E	10^3	21°C	5 d
Teflon	HCoV	Strain 229 E	10^3	21°C	5 d

MERS = Middle East Respiratory Syndrome; HCoV = human coronavirus; TGEV = transmissible gastroenteritis virus; MHV = mouse hepatitis virus; SARS = Severe Acute Respiratory Syndrome; RT = room temperature.

Figure 1: Perseverance of coronaviruses on different kinds of stationary sides [42].

droplets, contaminated hands, or surfaces. The persistence of coronaviruses on diverse inanimate surfaces is seen in Figure 1. We utilized real-life data from an experiment involving six patients in our application, as well as a generic binary relation established by a multi-information system. As a result, Pawlak and some of the other approaches are inapplicable here, and we can conclude that our method broadens the application range of rough sets.

Example 6. Here, we use the notion of nano topology, as defined in Definition 10, to discover the essential elements of "COVID-19" infection via topological reduction of characteristics in a multi-information system. It should be noted that Pawlak's technique cannot be applied in this case since the used connection is a reflexive relation (not an equivalence relation). Table 4 gives information about six persons of patients $U = \{p_1, p_2, p_3, \dots, p_6\}$ and the set of attributes $AT = \{a_1, a_2, a_3\}$ where $a_1 =$ handled surfaces $= \{Metal, Woo\ d, Paper\}$, $a_2 =$ protection tools $= \{Muzzle, Glove\}$, and $a_3 =$ person status $= \{Stay\ at\ home, Go\ out\ home\}$, and decision set of the infected persons with COVID-19 is $D = \{Yes, No\}$.

Now, suppose the reflexive relation is $p_i R p_j \Leftrightarrow V(p_i) \subseteq V(p_j)$. This relation can be applied on Table 4 to get $= \{(p_1, p_1), (p_1, p_3), (p_2, p_2), (p_3, p_1), (p_3, p_3),$

Table 4: Multi-information table of causing factors for COVID-19 infection.

	$\mathbf{a}_1$	$\mathbf{a}_2$	$\mathbf{a}_3$	D
$\mathbf{p}_1$	$\{M, W\}$	$\{M, G\}$	$\{G\}$	Yes
$\mathbf{p}_2$	$\{W\}$	$\{G\}$	$\{S\}$	No
$\mathbf{p}_3$	$\{M, W\}$	$\{M, G\}$	$\{G\}$	Yes
$\mathbf{p}_3$	$\{P\}$	$\{M\}$	$\{S\}$	No
$\mathbf{p}_3$	$\{P\}$	$\{M\}$	$\{G\}$	Yes
$\mathbf{p}_6$	$\{M\}$	$\{G\}$	$\{G\}$	No

$(p_4, p_4), (p_5, p_5), (p_6, p_1), (p_6, p_3), (p_6, p_6)\}$. Thus, the after set of each element in U of this relation is $p_1 R = p_3 R = \{p_1, p_3\}$, $p_2 R = \{p_2\}$, $p_4 R = \{p_4\}$, $p_5 R = \{p_5\}$, and $p_6 R = \{p_1, p_3, p_6\}$. Accordingly, the r-adhesion neighbourhoods are $A_r(p_1) = A_r(p_3) = \{p_1, p_3\}$, $A_r(p_2) = \{p_2\}$, $A_r(p_4) = \{p_4\}$, $A_r(p_5) = \{p_5\}$, and $A_r(p_6) = \{p_6\}$.

Now, from Table 4, there are two types of patients according to infections of COVID-19.

Case 1 **(Infection).** Using the set $X = \{p_1, p_3, p_5\}$, we get $\underline{R}_r(X) = \overline{R}_r(X) = X$ and $B_r(X) = \emptyset$. Hence, by Definition 8, the nano topology of X is $\tau_{N_r} = \{U, \emptyset, \{p_1, p_3, p_5\}\}$ with a base $\beta_{N_r} = \{U, \emptyset, \{p_1, p_3, p_5\}\}$. The reduction proceeds as follows:

Step 1: Write the finite set U and the set of attributes represent the data as an information table, with rows labelled by attributes (C), columns labelled by objects, and table entries representing attribute values.
Step 2: Compute the j-upper approximation, j-lower approximation, and j-boundary for each $X \subseteq U$. As $j = r$, say $\overline{R}_r(X)$, $\underline{R}_r(Y)$, and $B_r(Y)$, respectively, as defined in Definition 11.
Step 3: Establish τ_{N_r} on U with a base β_{N_r} by Definition 10.
Step 4: Remove a_i from (C). Accordingly, find the r-upper approximation, r-lower approximation, and r-boundary $X \subseteq U$ on $C-(a_i)$.
Step 5: Generate the nano topology $\tau_{N_r-a_i}$ on U and its base $\beta_{N_r-a_i}$ by Definition 10.
Step 6: Repeat Steps 4 and 5, $\forall a_i$ in C.
Step 7: a_i which satisfies $\beta_{N_r-a_i} \neq \beta_{N_r}$ forms the *CORE* (U).

ALGORITHM 1: A decision making via **j**-rough approximations.

Step 1. a_1 is removed. Then, r-adhesion neighbourhoods are $A_{r-a_1}(p_1) = A_{r-a_1}(p_3) = \{p_1, p_3\}$, $A_{r-a_1}(p_2) = \{p_2\}$, $A_{r-a_1}(p_4) = \{p_4\}$, $A_{r-a_1}(p_5) = \{p_5\}$, and $A_{r-a_1}(p_6) = \{p_6\}$. Accordingly, $\underline{R}_{r-a_1}(X) = \overline{R}_{r-a_1}(X) = X$ and $B_{r-a_1}(X) = \varnothing$. Therefore, by Definition 10, $\tau_{N_r-a_1} = \{U, \varnothing, \{p_1, p_3, p_5\}\} = \tau_{N_r}$. Also, $\beta_{N_r-a_1} = \{U, \varnothing, \{p_1, p_3, p_5\}\} = \beta_{N_r}$.

Step 2. a_2 is removed. So, r-adhesion neighbourhoods are $A_{r-a_2}(p_1) = A_{r-a_2}(p_3) = \{p_1, p_3\}$, $A_{r-a_2}(p_2) = \{p_2\}$, $A_{r-a_2}(p_4) = \{p_4\}$, $A_{r-a_2}(p_5) = \{p_5\}$, and $A_{r-a_2}(p_6) = \{p_6\}$. Accordingly, $\underline{R}_{r-a_2}(X) = \overline{R}_{r-a_2}(X) = X$ and $B_{r-a_2}(X) = \varnothing$. Therefore, by Definition 10, $\tau_{N_r-a_2} = \{U, \varnothing, \{p_1, p_3, p_5\}\} = \tau_{N_r}$. Also, $\beta_{N_r-a_2} = \{U, \varnothing, \{p_1, p_3, p_5\}\} = \beta_{N_r}$.

Step 3. a_3 is removed. Then, r-adhesion neighbourhoods are $A_{r-a_3}(p_1) = A_{r-a_3}(p_3) = \{p_1, p_3\}$, $A_{r-a_3}(p_2) = \{p_2\}$, $A_{r-a_3}(p_4) = A_{r-a_3}(p_5) = \{p_4, p_5\}$, and $A_{r-a_3}(p_6) = \{p_6\}$. Accordingly, $\underline{R}_{r-a_3}(X) = \{p_1, p_3\}$, $\overline{R}_{r-a_3}(X) = \{p_1, p_3, p_4, p_5\}$ and $B_{r-a_3}(X) = \{p_4, p_5\}$. Therefore, by Definition 10, $\tau_{N_r-a_3} = \{U, \varnothing, \{p_1, p_3\}, \{p_4, p_5\}, \{p_1, p_3, p_5\}\} \neq \tau_{N_r}$. In this case, $\beta_{N_r-a_3} = \{U, \{p_1, p_3\}, \{p_4, p_5\}\} \neq \beta_{N_r}$. So, the CORE is the attribute a_3. This means that "Person status" is the effective factor for COVID-19 infection.

Case 2 **(Noninfection).** Using the set $Y = \{p_2, p_6\}$, we get $\underline{R}_r(Y) = \overline{R}_r(Y) = Y$ and $B_r(Y) = \varnothing$. Hence, by Definition 8, $\tau_{N_r} = \{U, \varnothing, \{p_2, p_6\}\}$ with a base $\beta_{N_r} = \{U, \varnothing, \{p_2, p_6\}\}$. Here, the reduction proceeds as follows:

Step 4. a_1 is removed. Hence, like procedures of Case 1, $\tau_{N_r-a_1} = \{U, \varnothing, \{p_2, p_6\}\} = \tau_{N_r}$ with a base $\beta_{N_r-a_1} = \{U, \varnothing, \{p_2, p_6\}\} = \beta_{N_r}$.

Step 5. a_2 is removed. Hence, like procedures of Case 1, $\tau_{N_r-a_2} = \{U, \varnothing, \{p_2, p_6\}\} = \tau_{N_r}$ with a base $\beta_{N_r-a_2} = \{U, \varnothing, \{p_2, p_6\}\} = \beta_{N_r}$.

Step 6. a_3 is removed. Then, like procedures of Case 1, $\tau_{N_r-a_3} = \{U, \varnothing, \{p_2, p_6\}, \{p_4, p_5\}, \{p_2, p_4, p_5, p_6\}\} \neq \tau_{N_r}$ with a base $\beta_{N_r-a_3} = \{U, \{p_2, p_6\}, \{p_4, p_5\}\} \neq \beta_{N_r}$. Therefore, the CORE is a_3. This means that "Person status" is the effective factor for COVID-19 infection.

Observation: According to the CORE, "Person status" (whether you remain at home or not) is the most important factor in COVID-19 infection. Proper medical care for those who remain at home may reduce the danger.

We provide an algorithm for decision-making based on our ideas towards the conclusion of the study.

6. Conclusion and Discussion

The current paper can be divided into three main parts besides the introduction and basic concept sections. The first section examines two kinds of closure operators based on right and left adhesion neighbourhoods formed by a broad binary relation and their features. These operators are extensions of Galton [17], Allam et al. [18], and El-Bably and Fleifel [19, 20], as shown by Theorem 1 with a counterexample. The second part is devoted to the application of the closure operators, proposed in the current paper, in the notion of rough sets. In fact, we have presented three models to approximate the rough sets, which are generalizations of previously presented methods (such as [2, 4, 8, 11, 13, 20, 22, 23, 26–39]). We studied the properties of these approximations, and we were able to demonstrate all of Pawlak's properties, which were not fulfilled in some other generalizations such as Yao [26] without adding any conditions to the relation. Several comparisons between our methods and previous methods have been presented. Theorem 2 and its results demonstrate that our methods are more accurate and general than other methods of approximations. As a result, we can state that our technique will be beneficial in decision-making for real-world challenges, contributing to the extraction of knowledge from concealed data. Furthermore, the closure operators pave the door for further topological contributions to the rough set theory and applications. Part three of the article offered a medical decision-making application for identifying the effect elements affecting the transmission of the coronavirus (Covid-19) infection. We built a nano topology in this application using a general relation (rather than Pawlak's approximations, which need an equivalence relation). It is worth mentioning that we have offered algorithms for our decision-making technique, which may be used for any real-world problem.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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