

Research Article

Forecasting Renminbi Exchange Rate Volatility Using CARR-MIDAS Model

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Received 22 February 2022; Accepted 13 April 2022; Published 26 April 2022

Academic Editor: Gang Jin Wang

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In this study, we propose to employ the conditional autoregressive range-mixed-data sampling (CARR-MIDAS) model to model and forecast the renminbi exchange rate volatility. The CARR-MIDAS model exploits intraday information from the intraday high and low prices, which has the capacity to capture the high persistence of conditional range (volatility). The empirical results show that the range-based CARR-MIDAS model provides more accurate out-of-sample forecasts of the renminbi exchange rate volatility compared to the return-based GARCH and GARCH-MIDAS models and the range-based CARR model for forecast horizons of 1 day up to 3 months. In addition, the superior predictive ability of the CARR-MIDAS model is robust to different forecast windows. Hence, our CARR-MIDAS model provides a promising tool for forecasting the renminbi exchange rate volatility.

1. Introduction

Modeling and forecasting financial market volatility have attracted a great deal of attention in the financial econometric literature due to its important role in many financial applications, such as portfolio allocation, risk management, and option pricing. In the past decades, numerous volatility models have been developed to model and forecast the dynamics of the volatility process. The generalized autoregressive conditional heteroskedasticity (GARCH) model by Bollerslev [1] is among the most popular volatility models. However, the GARCH model is a return-based model that uses only closing prices to estimate volatility and fails to exploit the intraday information.

An alternative approach for estimating volatility is to use the daily intraday range from the intraday high and low prices. It is clear that the range contains more information about intraday price movements than the traditional return-based volatility estimator that is based on a single measurement of the closing price. It has been documented in the literature that the range is a more efficient volatility estimator than the return-based one (see, e.g., [2–5] and Chou [6] propose a range-based volatility model: the conditional

autoregressive range (CARR) model, and show that the model provides more accurate volatility estimates than the traditional return-based GARCH model. Since then, the CARR model has received considerable attention in the literature (see, e.g., [7–16]).

Despite the empirical success of the CARR model, the model with a constant long-run trend of the range is still not adequate to account for the high persistence (long memory) of the conditional range (volatility). To address this issue, an extension of the CARR model, namely, the CARR-mixed-data sampling (CARR-MIDAS) model has been proposed by Wu et al. [17]. The CARR-MIDAS model inherits the strength of the range-based CARR model and its capacity to exploit intraday information for estimating volatility. Most importantly, the CARR-MIDAS model features a multiplicative decomposition of the conditional range into a short-run and a long-run component, where the short-run component is governed by a CARR(1,1) process, while the long-run component is modeled using a MIDAS approach. The multiplicative component structures have been recently proposed by Engle and Rangel [18]; Engle et al. [19], and Amado and Teräsvirta [20, 21] in the context of the return-based GARCH framework. It is claimed that this structure is

useful to capture complex volatility dynamics such as the high persistence of volatility and to well handle the structural changes or nonstationarities in volatility [22, 23]. Our proposed CARR-MIDAS model is motivated by the multiplicative component GARCH-MIDAS model of Engle et al. [19], which allows to capture time-varying long-run trends in volatility through a parsimonious and flexible MIDAS structure.

While Wu et al. [17] employ the CARR-MIDAS model to investigate the impact and predictive power of EPU on the Chinese stock market volatility, in this study we apply the model to forecast the renminbi exchange rate volatility. To the best of our knowledge, the usefulness of the CARR-MIDAS model for forecasting the renminbi exchange rate volatility has not been investigated in the literature. Since the implementation of the renminbi exchange rate regime reform in 2005, the renminbi exchange rate has experienced significant fluctuations. Forecasting the renminbi exchange rate volatility is crucial as it has an important impact on international trade and economic growth. We examine and compare the out-of-sample forecast performance of the range-based CARR-MIDAS model with that of the two popular return-based volatility models: the GARCH model of Bollerslev [1] and the GARCH-MIDAS model of Engle et al. [19], and the range-based CARR model of Chou [6]. Our results show that the CARR-MIDAS model provides more accurate out-of-sample forecasts of the renminbi exchange rate volatility compared to the return-based GARCH and GARCH-MIDAS models and the range-based CARR model for forecast horizons of 1 day up to 3 months. We also find that the superior forecast ability of the CARR-MIDAS model is robust to different forecasting windows. These results highlight the value of incorporating the intraday range and a MIDAS component (long-run component) into the volatility model for forecasting the renminbi exchange rate volatility.

The remainder of this study is organized as follows. In Section 2, we introduce the CARR-MIDAS model. In Section 3, we illustrate the forecast evaluation method. Section 4 presents the empirical results, while Section 5 concludes the study.

2. The Model

In this study, we utilize the intraday range to model and forecast the dynamic behavior of the renminbi exchange rate volatility. It has been theoretically shown that the intraday range is a more accurate volatility estimator compared to the realized volatility estimator, which is based on five, or less, equidistance points in time (Degiannakis and Livada, 2013). The intraday range of Parkinson [2] is defined as follows:

$$R_i = \frac{\log(H_i) - \log(L_i)}{\sqrt{4\log(2)}}, \quad (1)$$

where H_i and L_i are the highest and lowest prices observed at day i , respectively. Parkinson [2] shows that the range given by equation (1) is an effective estimator of the volatility and demonstrates the efficiency of this range-based estimator

versus traditional volatility estimator based on the close-to-close returns.

2.1. The CARR Model. To describe the dynamics of the range, Chou [6] introduces the CARR model, which can be written as follows:

$$\begin{aligned} R_i &= \lambda_i \varepsilon_i, \varepsilon_i | \mathcal{F}_{i-1} \sim \exp(1), \\ \lambda_i &= \omega + \alpha R_{i-1} + \beta \lambda_{i-1}, \end{aligned} \quad (2)$$

where λ_i is the conditional mean of the range based on the information set, \mathcal{F}_{i-1} , up to day $i-1$, and $\exp(1)$ is an exponential distribution with unit mean. The coefficients, ω , α , and β , in the conditional mean equation are all assumed to be nonnegative to ensure positivity of the range. Furthermore, the stationary condition for the process is $\alpha + \beta < 1$, where $\alpha + \beta$ determines the persistence of range shocks, and the unconditional (long run) mean of the range is $\omega / (1 - (\alpha + \beta))$.

2.2. The CARR-MIDAS Model. The CARR model is a range-based analog to the traditional return-based GARCH model, which is capable of capturing the well-known phenomenon of volatility clustering. Chou [6] shows that the range-based CARR model outperforms the return-based GARCH model in terms of out-of-sample volatility forecasts. However, the CARR model with a constant long-run trend of the range is still very restrictive and does not account for high persistence (long memory) of conditional volatility. Motivated by the return-based GARCH-MIDAS model of Engle et al. [19], in this study we introduce an extension of the CARR model, namely, the CARR-MIDAS model, which can be written as follows:

$$\begin{aligned} R_{i,t} &= \lambda_{i,t} \varepsilon_{i,t}, \varepsilon_{i,t} | \mathcal{F}_{i-1,t} \sim \exp(1), \\ \lambda_{i,t} &= \tau_t g_{i,t}, \\ g_{i,t} &= (1 - \alpha - \beta) + \alpha \frac{R_{i-1,t}}{\tau_t} + \beta g_{i-1,t}, \end{aligned} \quad (3)$$

$$\log(\tau_t) = m + \theta \sum_{k=1}^K \varphi_k(\gamma) \log(RRV_{t-k}),$$

$$RRV_t = \sum_{i=1}^{N_t} R_{i,t}^2,$$

where $R_{i,t}$ is the range on day i in month t , and $\lambda_{i,t}$ is the conditional mean of the range based on the information set, $\mathcal{F}_{i-1,t}$, up to day $i-1$ of month t , which is multiplicatively decomposed into two components, a short-run component, $g_{i,t}$, and a long-run component, τ_t . The short-run component, $g_{i,t}$, is specified as a CARR(1,1) process, while the long-run component, τ_t , is modeled in the spirit of the MIDAS regression, which is driven by the smoothing monthly realized range volatility (RRV) with the weighting scheme φ_k . To ensure nonnegativity and stationarity for the short-run component $g_{i,t}$, we assume that $\alpha > 0$, $\beta > 0$, and $\alpha + \beta < 1$.

One-parameter beta polynomial is employed as the weighting scheme φ_k due to its parsimony and flexibility:

$$\varphi_k(\gamma) = \frac{(1 - k/K)^{\gamma-1}}{\sum_{j=1}^K (1 - j/K)^{\gamma-1}}, \quad (4)$$

where K is the number of MIDAS lags with $\sum_{k=1}^K \varphi_k(\gamma) = 1$.

It is clear that the CARR-MIDAS model is more flexible relative to the CARR model. It is straightforward to show that

$$\lambda_{i,t} = \omega_t + \alpha R_{i-1,t} + \beta \lambda_{i-1,t}, \quad (5)$$

where $\omega_t = (1 - \alpha - \beta)\tau_t$ implies a time-varying parameter, which allows to capture structural changes in conditional volatility. Lamoureux and Lastrapes [24] show that structural changes should be taken into account when modeling volatility; otherwise, it may induce spurious apparent persistence (long memory features) in the volatility process. By assuming a constant long-run component, the CARR-MIDAS model reduces to the original CARR model.

2.3. Maximum Likelihood Estimation. The CARR-MIDAS model is easy to estimate. We estimate the CARR-MIDAS model using the quasi-maximum likelihood method. The log-likelihood function of the CARR-MIDAS model can be written as follows:

$$\ell(\Theta) = - \sum_{t=1}^T \sum_{i=1}^{N_t} \left[\log(\lambda_{i,t}) + \frac{R_{i,t}}{\lambda_{i,t}} \right], \quad (6)$$

where $\Theta = (m, \theta, \gamma, \alpha, \beta)'$ is the vector of all model parameters. The maximum likelihood estimators, $\hat{\Theta}$, can be obtained by maximizing the log-likelihood function in equation (6).

3. Forecast Evaluation

To evaluate the forecast performance of the CARR-MIDAS model, we use two robust loss functions, the mean squared error (MSE), and the quasi-likelihood (QLIKE), which are given as follows:

$$\begin{aligned} \text{MSE} : \text{Loss}_{i,t} &= (MV_{i,t} - FV_{i,t})^2, \\ \text{QLIKE} : \text{Loss}_{i,t} &= \log(FV_{i,t}) + \frac{MV_{i,t}}{FV_{i,t}}, \end{aligned} \quad (7)$$

where $MV_{i,t}$ is a measure of the ex-postvolatility, and $FV_{i,t}$ is the forecasted volatility. We use the range given in equation (1) as the ex-postvolatility. Patton [25] shows that the MSE and QLIKE loss functions are robust to imperfect proxy of actual volatility and provide a consistent ranking of forecasts.

We test the significant differences between competing models by employing the model confidence set (MCS) approach of Hansen et al. [26]. Let \mathcal{M}^0 be a set of competing models. We identify the set of the best-performing models with a given confidence level α , namely, the MCS $\widehat{\mathcal{M}}_{1-\alpha}^*$. MCS approach tests the null hypothesis of equal forecasting accuracy:

$$H_{0,\mathcal{M}}: E(d_{uv,i,t}) = 0, \quad \forall u, v \in \mathcal{M}, \mathcal{M} \subset \mathcal{M}^0, \quad (8)$$

where $d_{uv,i,t} \equiv \text{Loss}_{i,t}(u) - \text{Loss}_{i,t}(v)$ denotes the difference in the MSE or QLIKE loss of models u and v . If the null hypothesis $H_{0,\mathcal{M}}$ is rejected, the worst-performing model from the set \mathcal{M} is eliminated. The procedure is iteratively performed, until no further model can be eliminated. The final set of surviving models is denoted by $\widehat{\mathcal{M}}_{1-\alpha}^*$. Following Hansen et al. [26], we implement the MCS procedure using a block bootstrap of 10^5 replications and a significance level of $\alpha = 10\%$.

Moreover, we examine the significance of the difference between the competing models for volatility forecasting using the Diebold and Mariano [27] test. We test the superiority of model u over model v using a t -test for the coefficient $d_{u,v}$ in the following:

$$(MV_{i,t} - FV_{i,t}(u))^2 - (MV_{i,t} - FV_{i,t}(v))^2 = d_{u,v} + \eta_{i,t}. \quad (9)$$

A significantly positive $d_{u,v}$ indicates that the model v dominates the model u and vice versa.

4. Empirical Results

4.1. Data. The data used in the study consist of daily open, high, low, and close prices for the renminbi exchange rate of the Chinese Yuan (CNY) against the US Dollar (USD). The exchange rate is measured as CNY of one unit of USD. The data are obtained from the Wind Database of China for the period January 2, 2006, to December 31, 2020, for a total of 3,716 trading days. Daily intraday ranges are calculated using the equation (1). For comparison, we also compute the daily log returns as $r_i = \log(P_i/P_{i-1})$, where P_i is the closing price on day i . Figure 1 plots the daily returns, ranges for the USD/CNY exchange rate, and shows that the well-known behaviors of volatility clustering in the USD/CNY exchange rate are apparent. It is also worth noting that the USD/CNY exchange rate experienced significant fluctuations, particularly in recent years.

Table 1 presents descriptive statistics for the USD/CNY daily return and range series and the series of the absolute return. The three series exhibit positive skewness and leptokurtosis, and the Jarque-Bera statistics show that all the three series fail the normality assumption. The Ljung-Box Q statistics up to 12 lags for the absolute return and range series show the existence of high persistence (serial correlation) of the USD/CNY exchange rate volatility. In particular, the obviously larger Ljung-Box Q statistic for the range series than for the absolute return series suggests a much higher persistence in the USD/CNY volatility for the range than for the absolute return series. Our proposed CARR-MIDAS model aims to capture this high degree of persistence by assuming a MIDAS component (long-run component) for the conditional range of the USD/CNY exchange rate.

4.2. Estimation Results. Table 2 reports the estimation results for the CARR-MIDAS model. In addition, estimates for the CARR model of Chou [6] are presented for the purpose of

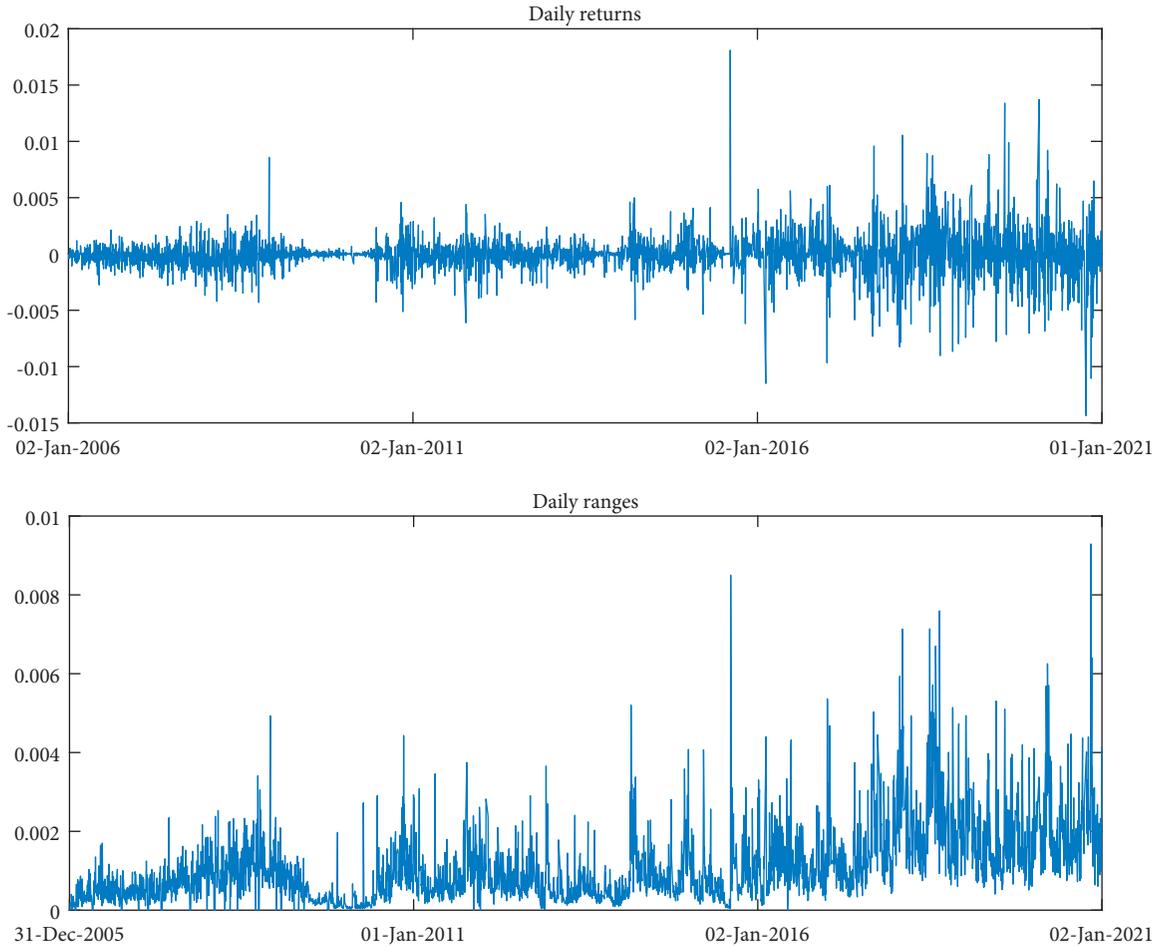


FIGURE 1: USD/CNY daily returns and ranges for the period January 2, 2006, to December 31, 2020.

TABLE 1: Descriptive statistics for the USD/CNY daily returns and ranges for the period January 2, 2006, to December 31, 2020.

	Return	Absolute return	Range
Mean	-0.0001	0.0010	0.0011
Min.	-0.0144	0.0000	0.0000
Max.	0.0181	0.0181	0.0093
Std. Dev.	0.0017	0.0014	0.0009
Skewness	0.4067	3.5882	2.1745
Kurtosis	15.9693	24.8488	10.7412
Jarque-Bera	26138.8708	81865.2329	12207.1151
Q(12)	36.9645	2015.4260	11143.1941

Note: Q(12) is the Ljung-Box statistic for autocorrelation up to 12 lags.

comparison. For the CARR-MIDAS specification, we employ three MIDAS lag years, i.e., we choose $K = 36$. Conrad and Kleen [23] show that the data will identify the optimal weighting scheme as long as K is chosen reasonably large.

It can be seen from Table 2 that the estimate of the persistence coefficient $\alpha + \beta$ in the CARR model is close to one, showing high persistence in the conditional range process. Note also that in the CARR-MIDAS estimation results, the estimate of the persistence coefficient of the short-run component, $\alpha + \beta$, is less than one, with its magnitude obviously smaller than that of the CARR (0.7716

TABLE 2: Estimation results.

Parameter	CARR	CARR-MIDAS
$m(\omega)$	0.0000 (0.0000)	-2.3302 (0.0255)
θ	—	0.4256 (0.0032)
γ	—	15.3240 (0.2479)
α	0.2925 (0.0108)	0.4000 (0.0134)
β	0.6693 (0.0104)	0.3717 (0.0160)
$\alpha + \beta$	0.9618	0.7716
Log-lik	22397.4605	22434.3209
AIC	-44788.9210	-44858.6417
BIC	-44770.2598	-44827.5397

Note: Log-lik is the log-likelihood, AIC is the Akaike information criterion, and BIC is the Bayesian information criterion. The numbers in parentheses are asymptotic standard errors for the model parameters.

vs. 0.9618), indicating that accounting for the long-run component reduces persistence in the short-run component. Additionally, the estimate of the parameter θ is significant positive, which suggests the presence of the MIDAS component (long-run component), and the monthly RRV is positively related to the long-run component. Figure 2 plots the conditional range ($\lambda_{i,t}$) along with the long-run component (τ_t) and the short-run component ($g_{i,t}$) from the CARR-MIDAS model. The long-term component appears smooth and tracks secular volatility trends over the sample

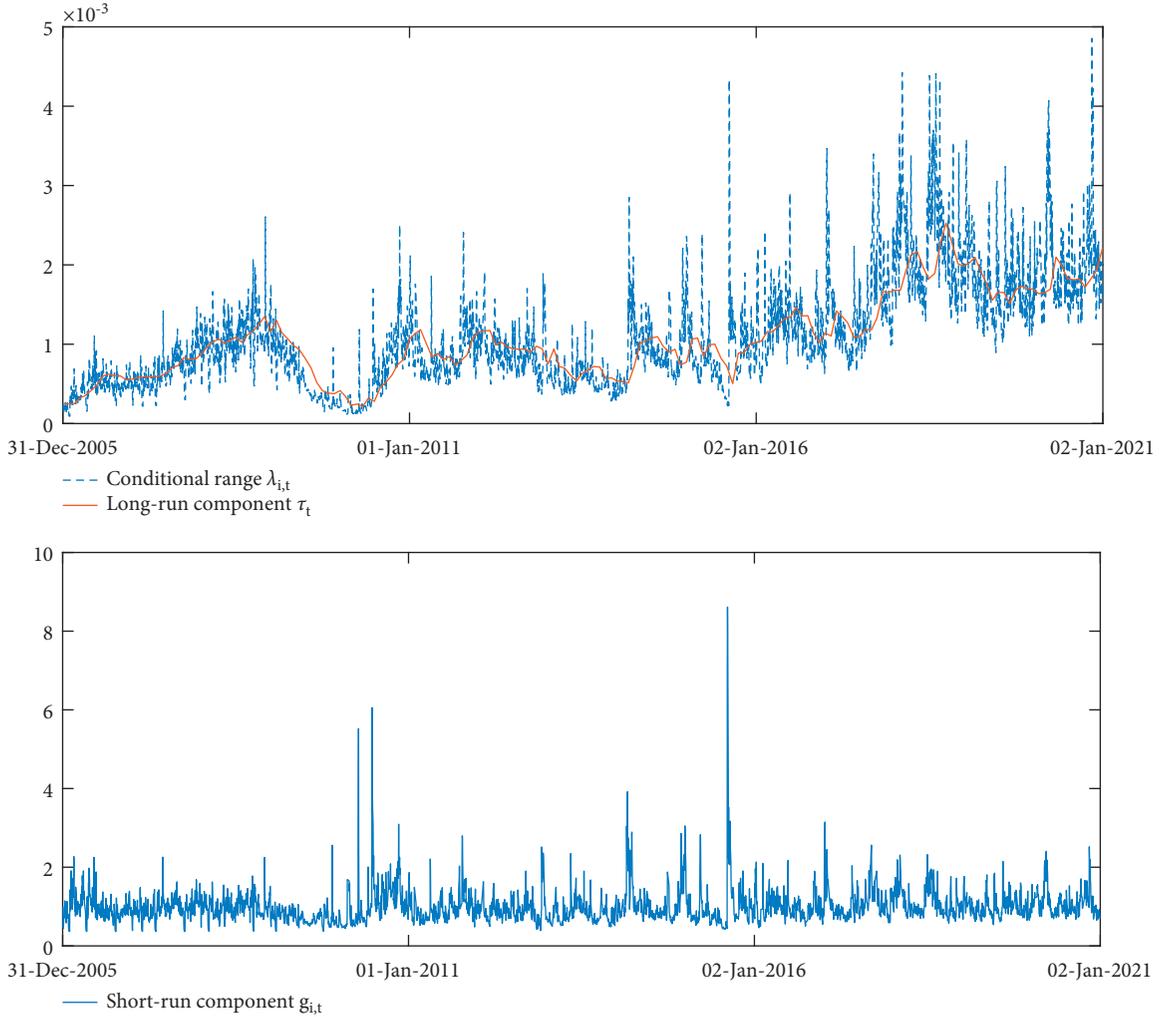


FIGURE 2: Conditional range ($\lambda_{i,t}$) and its long-term component (τ_t) and short-run component ($g_{i,t}$) from the CARR-MIDAS model.

period, while the short-run component exhibits the mean reverting property (reverts to a long-run mean of one).

According to the values of the log-likelihood, the Akaike and Bayesian information criteria shown in Table 2, the CARR-MIDAS model fits the data better compared to the CARR model. This result highlights the importance of incorporating the MIDAS component (long-run component) for modeling the renminbi exchange rate volatility.

4.3. Out-of-Sample Results. In this section, we investigate the out-of-sample forecast performance of the CARR-MIDAS model in forecasting the renminbi exchange rate volatility. We compare the performance of the range-based CARR-MIDAS model with that of the two popular return-based volatility models: the GARCH model of Bollerslev [1] and the GARCH-MIDAS model of Engle et al. [19], and the range-based CARR model of Chou [6]. We employ a rolling window scheme to perform the out-of-sample forecasts. In particular, we estimate model parameters on a rolling basis with 3,000 observations and leave the remaining (716) observations for out-of-sample evaluation. The forecast horizon is set to one day (1d), two days (2d), three days (3d), four

days (4d), one week (1w), two weeks (2w), one month (1m), two months (2m), and three months (3m), i.e., 1 day, 2 days, 3 days, 4 days, 5 days, 10 days, 22 days, 44 days, and 66 days ahead forecasts.

Table 3 reports the out-of-sample forecast evaluation results. It can be seen from the table that the range-based CARR (CARR-MIDAS) model generally outperforms the return-based GARCH (GARCH-MIDAS) model for the nine forecast horizons in terms of the MSE and QLIKE loss functions, which highlights the value of employing the intraday range for forecasting the renminbi exchange rate volatility. Moreover, we find that the GARCH-MIDAS (CARR-MIDAS) model improves upon the forecasting performance of the original GARCH (CARR) model for all forecast horizons. As the forecast horizon increases, the improvements appear to grow. These findings illustrate that incorporating the MIDAS component (long-run component) is important for improving the volatility forecasts, particularly for longer forecast horizons. In summary, the CARR-MIDAS model gives the lowest loss values for all forecast horizons and is clearly the preferred and best model for forecasting the renminbi exchange rate volatility.

TABLE 3: Out-of-sample forecast evaluation results.

Horizon	GARCH	GARCH-MIDAS	CARR	CARR-MIDAS
<i>Panel A: MSE loss function</i>				
1d	1.8812 E - 06	1.6093 E - 06	1.0439 E - 06	1.0100 E - 06
2d	2.0191 E - 06	1.6917 E - 06	1.2202 E - 06	1.1935 E - 06
3d	2.1171 E - 06	1.7454 E - 06	1.2792 E - 06	1.2428 E - 06
4d	2.1787 E - 06	1.7707 E - 06	1.3163 E - 06	1.2705 E - 06
1w	2.2375 E - 06	1.7879 E - 06	1.3601 E - 06	1.2956 E - 06
2w	2.4510 E - 06	1.8444 E - 06	1.6051 E - 06	1.4181 E - 06
1m	2.9237 E - 06	1.8923 E - 06	2.0200 E - 06	1.4109 E - 06
2m	4.3222 E - 06	2.2110 E - 06	2.4252 E - 06	1.3921 E - 06
3m	5.5743 E - 06	2.1918 E - 06	2.5807 E - 06	1.3845 E - 06
<i>Panel B: QLIKE loss function</i>				
1d	-5.1629	-5.1733	-5.2063	-5.2171
2d	-5.1539	-5.1672	-5.1782	-5.1961
3d	-5.1485	-5.1637	-5.1687	-5.1920
4d	-5.1457	-5.1628	-5.1617	-5.1909
1w	-5.1440	-5.1629	-5.1510	-5.1880
2w	-5.1362	-5.1611	-5.1023	-5.1728
1m	-5.1270	-5.1677	-4.9900	-5.1788
2m	-5.0712	-5.1431	-4.8115	-5.1863
3m	-5.0353	-5.1448	-4.7389	-5.1908

Note: MSE is the mean squared error, and QLIKE is the quasi-likelihood. Bold entries indicate the model with the lowest loss value per horizon (in each row). Shaded entries indicate the model includes the MCS at a 10% significance level. For the forecast horizons, d = day, w = week, and m = month.

TABLE 4: Diebold-Mariano statistics.

Horizon		CARR	GARCH-MIDAS	GARCH
1d	CARR-MIDAS	1.8760	8.3026	9.7510
	CARR		7.6976	9.3290
2d	GARCH-MIDAS			10.4591
	CARR-MIDAS	1.1952	6.1360	8.0666
	CARR		5.7277	7.8760
3d	GARCH-MIDAS			11.3301
	CARR-MIDAS	1.3302	5.7830	7.8312
	CARR		5.3741	7.6804
4d	GARCH-MIDAS			11.5681
	CARR-MIDAS	1.4811	5.4140	7.5467
	CARR		4.9367	7.3325
1w	GARCH-MIDAS			11.7271
	CARR-MIDAS	1.9078	5.0917	7.4106
	CARR		4.4220	7.0158
2w	GARCH-MIDAS			12.1817
	CARR-MIDAS	4.9525	3.8211	6.8328
	CARR		1.9808	5.3129
1m	GARCH-MIDAS			13.1066
	CARR-MIDAS	13.0779	4.3357	8.8228
	CARR		-0.8880	4.4641
2m	GARCH-MIDAS			15.1071
	CARR-MIDAS	15.0205	9.3679	14.9260
	CARR		-1.4974	7.6723
3m	GARCH-MIDAS			18.1445
	CARR-MIDAS	15.5806	8.9919	15.9034
	CARR		-2.5555	9.3778
	GARCH-MIDAS			18.7436

Note: A positive statistic indicates that the model in the row dominates the model in the column, and a negative statistic indicates that the model in the column dominates the model in the row. For the forecast horizons, d = day, w = week, and m = month.

The shaded entries in Table 3 identify the model included in the MCS at the significance level of 10%. The results show that the CARR-MIDAS model is included in the MCS for all forecast horizons, and in most cases, it is the only model that

is included in the MCS, suggesting that the CARR-MIDAS model significantly outperforms all other models.

Table 4 reports Diebold-Mariano test statistics for all pairs of the four volatility models over the nine different

TABLE 5: Out-of-sample forecast evaluation results for forecast window of 500.

Horizon	GARCH	GARCH-MIDAS	CARR	CARR-MIDAS
<i>Panel A: MSE loss function</i>				
1d	2.0068 E - 06	1.6990 E - 06	9.0186 E - 07	8.6854 E - 07
2d	2.1629 E - 06	1.7962 E - 06	1.0384 E - 06	9.9931 E - 07
3d	2.2868 E - 06	1.8680 E - 06	1.0871 E - 06	1.0256 E - 06
4d	2.3687 E - 06	1.9056 E - 06	1.1331 E - 06	1.0539 E - 06
1w	2.4308 E - 06	1.9265 E - 06	1.1882 E - 06	1.0895 E - 06
2w	2.6363 E - 06	1.9515 E - 06	1.3307 E - 06	1.1545 E - 06
1m	3.0896 E - 06	1.9259 E - 06	1.5823 E - 06	1.1650 E - 06
2m	4.3618 E - 06	2.1107 E - 06	1.8549 E - 06	1.1911 E - 06
3m	5.5636 E - 06	2.0560 E - 06	2.0134 E - 06	1.2128 E - 06
<i>Panel B: QLIKE loss function</i>				
1d	-5.2147	-5.2273	-5.2764	-5.2867
2d	-5.2055	-5.2205	-5.2535	-5.2707
3d	-5.1996	-5.2167	-5.2467	-5.2695
4d	-5.1950	-5.2143	-5.2391	-5.2680
1w	-5.1906	-5.2117	-5.2279	-5.2625
2w	-5.1804	-5.2092	-5.1952	-5.2534
1m	-5.1659	-5.2144	-5.1322	-5.2532
2m	-5.1183	-5.2024	-5.0378	-5.2602
3m	-5.0770	-5.1996	-4.9712	-5.2538

Note: MSE is the mean squared error, and QLIKE is the quasi-likelihood. Bold entries indicate the model with the lowest loss value per horizon (in each row). Shaded entries indicate the model includes the MCS at a 10% significance level. For the forecast horizons, d = day, w = week, and m = month.

TABLE 6: Out-of-sample forecast evaluation results for forecast window of 1,000.

Horizon	GARCH	GARCH-MIDAS	CARR	CARR-MIDAS
<i>Panel A: MSE loss function</i>				
1d	1.6488 E - 06	1.4435 E - 06	9.3247 E - 07	8.9878 E - 07
2d	1.7634 E - 06	1.5265 E - 06	1.1026 E - 06	1.0841 E - 06
3d	1.8302 E - 06	1.5694 E - 06	1.1623 E - 06	1.1379 E - 06
4d	1.8754 E - 06	1.5939 E - 06	1.2060 E - 06	1.1746 E - 06
1w	1.9301 E - 06	1.6259 E - 06	1.2743 E - 06	1.2281 E - 06
2w	2.0993 E - 06	1.6973 E - 06	1.5302 E - 06	1.3638 E - 06
1m	2.4848 E - 06	1.8318 E - 06	2.0408 E - 06	1.4434 E - 06
2m	3.3213 E - 06	2.0724 E - 06	2.3934 E - 06	1.4053 E - 06
3m	3.8112 E - 06	1.9729 E - 06	2.5319 E - 06	1.4155 E - 06
<i>Panel B: QLIKE loss function</i>				
1d	-5.2591	-5.2696	-5.3043	-5.3150
2d	-5.2495	-5.2615	-5.2714	-5.2880
3d	-5.2431	-5.2564	-5.2571	-5.2801
4d	-5.2391	-5.2535	-5.2470	-5.2754
1w	-5.2345	-5.2499	-5.2314	-5.2660
2w	-5.2238	-5.2431	-5.1694	-5.2402
1m	-5.1942	-5.2218	-5.0010	-5.2156
2m	-5.1519	-5.1964	-4.8021	-5.2212
3m	-5.1284	-5.1926	-4.7188	-5.2122

Note: MSE is the mean squared error, and QLIKE is the quasi-likelihood. Bold entries indicate the model with the lowest loss value per horizon (in each row). Shaded entries indicate the model includes the MCS at a 10% significance level. For the forecast horizons, d = day, w = week, and m = month.

forecast horizons. It can be seen from the table that the differences in forecast accuracy among the four models are significant in most cases, and the significance tends to increase as the forecast horizon increases. In particular, the Diebold-Mariano statistics for the CARR-MIDAS model are unanimously reported to be positive and significant in most cases, which indicates that the CARR-MIDAS model significantly dominates the other models.

4.4. Robustness Check. For the robustness check, the out-of-sample forecast is also performed over different forecast windows (out-of-sample periods). We consider three different forecast windows, 500, 1,000, and 1,500. The out-of-sample forecast evaluation results are presented in Tables 5–7 for the three forecast windows, respectively. As is consistent with the results in Tables 3 and 4, the CARR-MIDAS model significantly outperforms the other models.

TABLE 7: Out-of-sample forecast evaluation results for forecast window of 1,500.

Horizon	GARCH	GARCH-MIDAS	CARR	CARR-MIDAS
<i>Panel A: MSE loss function</i>				
1d	1.5384 E - 06	1.3723 E - 06	8.2349 E - 07	7.8460 E - 07
2d	1.6969 E - 06	1.4975 E - 06	9.5137 E - 07	9.6109 E - 07
3d	1.7951 E - 06	1.5689 E - 06	9.9754 E - 07	1.0141 E - 06
4d	1.8409 E - 06	1.5924 E - 06	1.0223 E - 06	1.0430 E - 06
1w	1.8853 E - 06	1.6136 E - 06	1.0535 E - 06	1.0800 E - 06
2w	2.0442 E - 06	1.6625 E - 06	1.1651 E - 06	1.1862 E - 06
1m	2.4191 E - 06	1.7385 E - 06	1.3695 E - 06	1.2373 E - 06
2m	3.1551 E - 06	1.8347 E - 06	1.6240 E - 06	1.2379 E - 06
3m	3.7905 E - 06	1.7622 E - 06	1.8127 E - 06	1.2149 E - 06
<i>Panel B: QLIKE loss function</i>				
1d	-5.4350	-5.4477	-5.4963	-5.5060
2d	-5.4027	-5.4197	-5.4507	-5.4637
3d	-5.3801	-5.4004	-5.4282	-5.4471
4d	-5.3742	-5.3955	-5.4241	-5.4434
1w	-5.3721	-5.3938	-5.4168	-5.4353
2w	-5.3579	-5.3873	-5.3856	-5.4156
1m	-5.3368	-5.3762	-5.3529	-5.4052
2m	-5.3158	-5.3780	-5.2787	-5.4077
3m	-5.2903	-5.3790	-5.1855	-5.4140

Note: MSE is the mean squared error, and QLIKE is the quasi-likelihood. Bold entries indicate the model with the lowest loss value per horizon (in each row). Shaded entries indicate the model includes the MCS at a 10% significance level. For the forecast horizons, d = day, w = week, and m = month.

5. Conclusions

In this study, we propose to use the range-based CARR-MIDAS model to modeling and forecasting the renminbi exchange rate volatility. The CARR-MIDAS model exploits intraday information from the intraday high and low prices, and features a multiplicative decomposition of the conditional range into a short-run and a long-run component, where the short-run component is governed by a CARR(1,1) process and the long-run component is modeled by a MIDAS structure, which is capable of capturing the high persistence of conditional range (volatility). To the best of our knowledge, the usefulness of the CARR-MIDAS model for forecasting the renminbi exchange rate volatility has not been investigated in the literature. Empirical results show that the range-based CARR-MIDAS model provides more accurate out-of-sample volatility forecasts compared to the return-based GARCH and GARCH-MIDAS models and the range-based CARR model for forecast horizons ranging from 1 day to 3 months ahead. Moreover, according to the robustness check, the superior predictive ability of the CARR-MIDAS model is robust to different forecast windows. These results highlight the importance of incorporating the intraday range and the MIDAS component (long-run component) for forecasting the renminbi exchange rate volatility. Against the backdrop of repeated shocks to the global economic environment and widespread global epidemics, the risks of capital outflows and financial assets have increased. This study focuses on the issue of renminbi exchange rate volatility forecasting based on the CARR-MIDAS model, which has important implications for all researchers, investors, policy-makers, and regulators that focus on financial applications in risk measurement, portfolio allocation, and option pricing.

The CARR-MIDAS model is flexible, which allows additional macroeconomic variables such as economic policy uncertainty to be easily incorporated. Thus, future research could be extended to investigate whether macroeconomic information has predictive power for the renminbi exchange rate volatility relying on our CARR-MIDAS approach. [28].

Data Availability

The data on the renminbi exchange rate of the Chinese Yuan (CNY) against the US Dollar (USD) are obtained from the Wind Database of China. All the data are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (no. 71971001), the University Natural Science Research Project of Anhui Province (no. KJ2019A0659), the Southern Jiangsu Capital Markets Research Center (no. 2017ZSJD020), and the Innovative Research Project for Graduates of Anhui University of Finance and Economics (no. ACYC2021302).

References

- [1] T. Bollerslev, "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, vol. 31, no. 3, pp. 307–327, 1986.

- [2] M. Parkinson, "The extreme value method for estimating the variance of the rate of return," *Journal of Business*, vol. 53, no. 1, pp. 61–65, 1980.
- [3] S. Alizadeh, M. W. Brandt, and F. X. Diebold, "Range-based estimation of stochastic volatility models," *The Journal of Finance*, vol. 57, no. 3, pp. 1047–1091, 2002.
- [4] M. W. Brandt and C. S. Jones, "Volatility forecasting with range-based EGARCH models," *Journal of Business & Economic Statistics*, vol. 24, no. 4, pp. 470–486, 2006.
- [5] R. Y. Chou, H. Chou, and N. Liu, "Range volatility: a review of models and empirical studies," in *Handbook of Financial Econometrics and Statistics*, CF. Lee and J. Lee, Eds., Springer, New York, NY, USA, pp. 2029–2050, 2015.
- [6] R. Y.-T. Chou, "Forecasting financial volatilities with extreme values: the conditional autoregressive range (CARR) model," *Journal of Money, Credit, and Banking*, vol. 37, no. 3, pp. 561–582, 2005.
- [7] C. W. S. Chen, R. Gerlach, and E. M. H. Lin, "Volatility forecasting using threshold heteroskedastic models of the intra-day range," *Computational Statistics & Data Analysis*, vol. 52, no. 6, pp. 2990–3010, 2008.
- [8] R. Y. Chou and N. Liu, "The economic value of volatility timing using a range-based volatility model," *Journal of Economic Dynamics and Control*, vol. 34, no. 11, pp. 2288–2301, 2010.
- [9] M.-H. Chiang and L.-M. Wang, "Volatility contagion: a range-based volatility approach," *Journal of Econometrics*, vol. 165, no. 2, pp. 175–189, 2011.
- [10] E. M. H. Lin, C. W. S. Chen, and R. Gerlach, "Forecasting volatility with asymmetric smooth transition dynamic range models," *International Journal of Forecasting*, vol. 28, no. 2, pp. 384–399, 2012.
- [11] C.-Y. Sin, "Using CARRX models to study factors affecting the volatilities of Asian equity markets," *The North American Journal of Economics and Finance*, vol. 26, pp. 552–564, 2013.
- [12] R. I. Anderson, Y.-C. Chen, and L.-M. Wang, "A range-based volatility approach to measuring volatility contagion in securitized real estate markets," *Economic Modelling*, vol. 45, pp. 223–235, 2015.
- [13] K. H. Ng, S. Peiris, J. S.-K. Chan, D. Allen, and K. H. Ng, "Efficient modelling and forecasting with range based volatility models and its application," *The North American Journal of Economics and Finance*, vol. 42, pp. 448–460, 2017.
- [14] H. Xie and X. Wu, "A conditional autoregressive range model with gamma distribution for financial volatility modelling," *Economic Modelling*, vol. 64, pp. 349–356, 2017.
- [15] H. Xie and X. Wu, "Range-based volatility forecasting: an extended conditional autoregressive range model," *Journal of Risk*, vol. 21, no. 3, pp. 55–80, 2019.
- [16] J. S.-K. Chan, K.-H. Ng, and R. Ragell, "Bayesian return forecasts using realised range and asymmetric CARR model with various distribution assumptions," *International Review of Economics & Finance*, vol. 61, pp. 188–212, 2019.
- [17] X. Y. Wu, T. Y. Liu, and H. B. Xie, "Economic policy uncertainty and Chinese stock market volatility: a CARR-MIDAS approach," *Complexity*, vol. 2021, Article ID 4527314, 10 pages, 2021.
- [18] R. F. Engle and J. G. Rangel, "The spline-GARCH model for low-frequency volatility and its global macroeconomic causes," *Review of Financial Studies*, vol. 21, no. 3, pp. 1187–1222, 2008.
- [19] R. F. Engle, E. Ghysels, and B. Sohn, "Stock market volatility and macroeconomic fundamentals," *The Review of Economics and Statistics*, vol. 95, no. 3, pp. 776–797, 2013.
- [20] C. Amado and T. Teräsvirta, "Modelling volatility by variance decomposition," *Journal of Econometrics*, vol. 175, no. 2, pp. 142–153, 2013.
- [21] C. Amado and T. Teräsvirta, "Specification and testing of multiplicative time-varying GARCH models with applications," *Econometric Reviews*, vol. 36, no. 4, pp. 421–446, 2017.
- [22] F. Wang and E. Ghysels, "Econometric analysis of volatility component models," *Econometric Theory*, vol. 31, no. 2, pp. 362–393, 2015.
- [23] C. Conrad and O. Kleen, "Two are better than one: volatility forecasting using multiplicative component GARCH-MIDAS models," *Journal of Applied Econometrics*, vol. 35, no. 1, pp. 19–45, 2020.
- [24] C. G. Lamoureux and W. D. Lastrapes, "Persistence in variance, structural change, and the GARCH model," *Journal of Business & Economic Statistics*, vol. 8, no. 2, pp. 225–234, 1990.
- [25] A. J. Patton, "Volatility forecast comparison using imperfect volatility proxies," *Journal of Econometrics*, vol. 160, no. 1, pp. 246–256, 2011.
- [26] P. R. Hansen, A. Lunde, and J. M. Nason, "The model confidence set," *Econometrica*, vol. 79, no. 2, pp. 453–497, 2011.
- [27] F. X. Diebold and R. S. Mariano, "Comparing predictive accuracy," *Journal of Business & Economic Statistics*, vol. 13, no. 3, pp. 253–263, 1995.
- [28] S. Degiannakis and A. Livada, "Realized volatility or price range: evidence from a discrete simulation of the continuous time diffusion process," *Economic Modelling*, vol. 30, pp. 212–216, 2013.