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Review Article

Admissibility Analysis of a Sampled-Data Singular System Based on the Input Delay Approach

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This study investigates the sampled-data admissibility problem for a singular system. The objective of this paper is to design a sampled-data controller to ensure the admissibility of a singular system and to construct an appropriate Lyapunov–Krasovskii functional (LKF) to get less conservative results for the sampled-data singular system. To accomplish these objectives, the system is converted into a time-delay system by the input delay approach firstly, and both lower and upper bounds of the delay are considered. Secondly, by introducing a suitable LKF, the admissibility criteria are obtained. Then, when estimating the derivative of the LKF, a relaxation variable is introduced by the method of reciprocally convex inequality, and it is proved that the conservatism is reduced. Finally, two numerical examples are given to prove that the designed sampled-data controller can ensure the states of the systems under the influence of external interference.

1. Introduction

Singular systems have gained considerable attention recently, especially in the research of admissibility and other related theories. Compared with the state-space system, the stability, the regularity, and the impulsivity (or causality) have to be considered in singular systems. The singular system has extensive applications, which cover a variety of engineering fields and practical systems, such as the circuit system, mechanical engineering system, economic system, aerospace system, and robot. Besides, the singular system has more algebraic equations than the normal systems, so the singular system has better extension performance. In recent years, research on the admissibility problem of singular systems has become a hot subject [1–11]. In [7], the admissibility of singular systems is discussed in view of Lyapunov theory, and stability conditions and controller design methods are given. In [8], the robust admissibility issue for the uncertain discrete switched singular system is studied, and the arbitrary switching law and output feedback controller are designed to guarantee that the system is admissible. In [9], the delta operator model for a singular system is obtained by replacing the traditional discrete model with the generalized discrete model. And the relationship between discrete and delta operator models is established. In [10], the admissibility of the stochastic singular system with the T-S fuzzy model is studied, and a new quadratic Lyapunov function is proposed to obtain more relaxed admissibility conditions than the existing methods. The issue about robust admissibility for fuzzy singular systems is studied in [11]. By establishing the LMI, non-quadratic Lyapunov functions are constructed, which achieve less conservative result.

Recently, sampled-data systems have become a hot topic. The system's characteristic is that the discrete and continuous signals are both existing in one system, and it is difficult to be analyzed and designed. Recently, various viewpoints have been reported for sampled-data systems (see [12–20]). The main design methods for analyzing the sampled-data systems can be divided into three kinds. The first one is the lifting technique (see [21]), which converts a continuous system into a discrete system of equal finite dimension. The second one is jump system methods (see [22]), where the performance analysis of the sampling system is transformed into the solution of two Riccati equations with mutual jump. However, the two methods mentioned above cannot solve the uncertainty problems caused by the sampling time or system matrix. The third one is the input delay method (see [23]), and the system is transformed into a continuous time-varying delay system, and then the stability analysis can be accomplished using the method of time-delay system. Besides, the input delay approach can deal with the uncertainty of system parameters, and it has been widely used in the sampled-data system (see [24, 25]) and practical engineering field such as autonomous airship, high-speed train, and dynamic positioning ship (see [26–28]). In [24], based on the polytopic linear parameter-varying method, the issue of the sampled-data control is discussed for a nonlinear system. And Wirtinger's inequality is used to obtain less conservative exponential stability condition. In [25], based on the input delay approach, the exponential stability issue for the linear parameter-varying system with aperiodic sampled-data rates is studied. And the distance between actual parameters and measured parameters is considered. In [26], the robust sampled-data control problem of the high-speed train is studied. Through the input delay approach, the high-speed cruise sampled-data controller of the train is designed. It ensures that the train is robust and stable under the interference of external wind. In [27], the sampled-data control issue for an autonomous airship with polyhedral parameter uncertainty is discussed. And a sampled-data controller is designed to guarantee that the system is exponentially stable and meets the H_∞ performance index. In [28], a state-derivative control law is provided for a sampled-data dynamic positioning ship. By combining the delay-decomposition technique with Wirtinger's integral inequalities, less conservative results are obtained.

However, although the sampled-data control technology has been well developed in control theory, few results have been reported for the sampled-data singular systems, and the research process on this field is nearly blank. The technical challenges are listed as follows:

(1) The mathematical structure of the sampled-data singular system is more complex

(2) How to improve the LKF to get less conservative results for the sampled-data singular system

(3) How to design a sampled-data controller to guarantee the admissibility of the system and achieve the performance index

As mentioned above, compared with the state-space system, the singular system has richer connotation and wider range. Besides, many multidimensional and multilevel large-scale complex systems are suitable to be dealt with by singular systems. So, it is important and meaningful to analyze the sampled-data control problem for the singular system, which is the motivation of this paper.

In this paper, the admissibility of the sampled-data singular system is studied. Firstly, the system's model is established, and then it is transformed into a time-delay system by using the input delay method. Then, both lower and upper bounds of the sampling period are considered. LMI and Lyapunov functions are introduced for admissibility analysis. By introducing the convex reciprocal inequality, the less conservative results can be obtained. Then, the design method of the sampled-data controller is provided, which makes the system achieve good performance under external interference. Finally, two examples are given to prove the effectiveness of this method.

The main contributions of this article are summarized as follows:

(1) The admissibility condition is established for a sampled-data singular system to ensure the regularity, impulse free, and asymptotical stability. The system has wider application scopes than the existing sampled-data system.

(2) The upper and lower bounds of the variable sampling period are considered, which covers the previous research works as special cases.

(3) Reciprocally convex combination approach is used to estimate the integral terms of the LKF to obtain less conservative results.

Notations: the superscript "T" stands for the transpose. $\mathbb{R}^{m\times n}$ denotes the set of $m\times n$ real matrices. " $*$ "denotes the matrix entries implied by symmetry. $L_2[0,\infty)$ is the space of square integrable functions on $[0,\infty)$.

2. Problem Formulation

Consider the singular system as follows:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t), \\ y(t) = Cx(t) + Du(t), \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the control input, $y(t)\in\mathbb{R}^p$ is the control output, and $w(t)\in\mathbb{R}^s$ is the disturbance which belongs to $L_2[0,\infty)$ and satisfies that $\int_0^\infty w^T(t)w(t)\mathrm{d}t<\infty$, and it means that the energy of the disturbance is finite. Matrix $E\in\mathbb{R}^{n\times n}$ is assumed to be singular, and rank $(E)=r\le n$. A, B, C, D, and B_w are known constant matrices.

It is assumed that the control signals are obtained at each sampling time $0=t_0<t_1<\cdots<t_k<\cdots<\lim_{k\longrightarrow\infty} t_k=+\infty$, and they have

$$u(t) = u_d(t_k),\ t_k\le t<t_{k+1}, \tag{2}$$

where u_d represents the discrete-time control signal. The proposed sampled-data scheme is depicted in Figure 1.

The sampling period is assumed such that

$$d_1\le t_{k+1}-t_k\le d_2,\quad \forall k\ge 0,\ d_2>d_1\ge 0, \tag{3}$$

where d_1 and d_2 are the lower and upper bound of the sampling period. Consider the sampled-data controller as follows:

$$u(t) = Kx(t_k),\ t_k\le t<t_{k+1}, \tag{4}$$

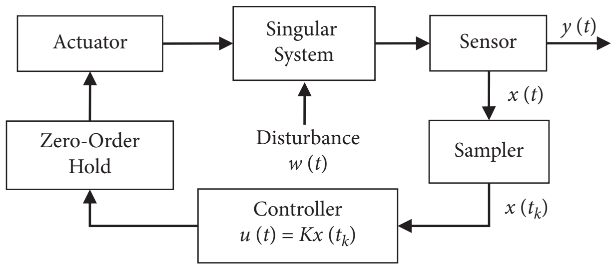

FIGURE 1: The schematic of the sampled-data singular system.

where K is a controller gain matrix. By substituting (4) into (1), then

$$\begin{cases} E\dot{x}(t) = Ax(t) + BKx(t_k) + B_w w(t),\ t > 0, \\ y(t) = Cx(t) + DK\ x(t_k). \end{cases} \tag{5}$$

Remark 1. It is noted that both continuous and discrete signals exist in a singular system (5). Therefore, compared with the existing methods for analyzing the continuous or discrete singular systems, it is more complex and meaningful to study the sampled-data control issue for singular systems.

To obtain the main results, we list the following definitions.

Definition 1 (see [29])

(1) If there exists a constant $s \in \mathbb{C}$ ($\mathbb{C}$ represents the complex field) satisfying $\det(sE - A) \neq 0$, then the matrix pair (E, A) is regular

(2) If $\deg(\det(sE - A)) = \text{rank}\,(E)$, then the matrix pair (E, A) is impulse free

Definition 2 (see [30])

(1) The singular system

$$E\dot{x}(t) = Ax(t) + Bx(t - \tau(t)) \tag{6}$$

is said to be regular and impulse free if the pair (E, A) is regular and impulse free.

(2) System (6) is said to be asymptotically admissible if it is regular, impulse free, and asymptotically stable.

The objective of this paper is designing a sampled-data controller to satisfy that

(1) System (5) with $w(t) = 0$ is asymptotically admissible

(2) Despite the external disturbances, the output signal $y(t)$ satisfies that $\|y(t)\|_2 \leq \gamma\|w(t)\|_2$ for all nonzero $w(t) \in L_2[0, \infty)$ under the zero condition, where $\gamma > 0$

Based on the input delay approach, define

$$\tau(t) = t - t_k,\ t_k \leq t < t_{k+1}, \tag{7}$$

where the time-varying delay $\tau(t)$ is piecewise linear satisfying $\tau(t) \in [d_1, d_2)$. And the derivative of $\tau(t)$ is

$$\dot{\tau}(t) = 1, \quad t \neq t_k. \tag{8}$$

Substituting the time delay $\tau(t)$ into the control input signal $u(t)$, sampled-data controller (4) can be converted such that

$$\begin{aligned} u(t) &= u_d(t_k) = u(t - (t - t_k)) \\ &= u(t - \tau(t)). \end{aligned} \tag{9}$$

Substituting (8) into (5), sampled-data system (5) is converted into a singular system with time-varying delay.

$$\begin{cases} E\dot{x}(t) = Ax(t) + BKx(t - \tau(t)) + B_w w(t),\ t > 0, \\ y(t) = Cx(t) + DK\ x(t - \tau(t)). \end{cases} \tag{10}$$

Remark 2. Compared with [24, 25, 31] which use the similar input delay approach to deal with the sampled-data problem, it can be seen that if $E = I$, then system (9) is simplified to the normal sampled-data system such as [24, 25, 31], which shows that the work covers these references as special issues. However, [31] did not consider the variable sampling period and only considered the constant sampling period. [24, 25] considered the upper bound of the sampling period, but the lower bound of the sampling period is omitted, which will limit its application scope, and it will also lead to conservatism to some extent. In this paper, the upper and lower bounds of the variable sampling period are considered. Let $d_1 = d_2 = d$; the sampling period will reduce to be a constant such as [24, 25, 31]. Thus, the proposed method has wider application scopes than the references.

The following lemma will be used for obtaining the main results.

Lemma 1 (see [32]). *For given matrix J, $R = R^T > 0$, scalars h_1 and h_2 satisfying $h_1 \leq d(t) \leq h_2$, and differentiable function x satisfying $\dot{x}$: $[-h_2, -h_1] \longrightarrow \mathbb{R}^n$, the following inequality holds:*

$$\begin{aligned} &(h_1 - h_2) \int_{t-h_2}^{t-h_1} \dot{x}^T(s) R\dot{x}(s) ds \leq \\ &\leq -\vartheta^T(t) \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix}^T W \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \vartheta(t), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \vartheta^T(t) &= \left[x^T(t - h_1)\ \ x^T(t - d(t))\ \ x^T(t - h_2) \right], \\ W &= \begin{bmatrix} -R & J \\ * & -R \end{bmatrix} \leq 0. \end{aligned} \tag{12}$$

3. Main Results

In this section, the sufficient admissibility criteria for sampled-data singular systems are exhibited by establishing the LKF and formulating in terms of LMI.

Theorem 1. *For constant delay d_1, d_2, system (9) is asymptotically admissible with H_∞ performance γ if there exist matrices $P, R, Q_i > 0, Z_i > 0, i = 1, 2, 3$, such that*

$$\begin{aligned} &E^T P = P^T E \geq 0, \\ &\begin{bmatrix} Z_3 & R \\ R^T & Z_3 \end{bmatrix} > 0, \\ &\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \Pi_{11} &= \begin{bmatrix} \Xi_{11} & E^T Z_1 E & P^T BK & E^T Z_2 E & P^T B_w \\ * & \Xi_{22} & \Xi_{23} & E^T R E & 0 \\ * & * & \Xi_{33} & \Xi_{34} & 0 \\ * & * & * & \Xi_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}, \\ \Pi_{12} &= \begin{bmatrix} C & 0 & DK & 0 & 0 \\ A & 0 & BK & 0 & B_w \end{bmatrix}^T, \\ \Pi_{22} &= diag\left\{-I, -Z^{-1}\right\}, \\ \Xi_{11} &= P^T A + A^T P + Q_1 + Q_2 + Q_3 - E^T Z_1 E - E^T Z_2 E, \\ \Xi_{22} &= -Q_1 - E^T Z_1 E - E^T Z_3 E, \\ \Xi_{23} &= E^T Z_3 E - E^T R E, \\ \Xi_{33} &= -2E^T Z_3 E + E^T R E + E^T R^T E, \\ \Xi_{34} &= E^T Z_3 E - E^T R E, \\ \Xi_{44} &= -Q_3 - E^T Z_2 E - E^T Z_3 E, \\ Z &= d_1^2 Z_1 + d_2^2 Z_2 + d_{21}^2 Z_3, \\ d_{21} &= d_2 - d_1. \end{aligned} \tag{14}$$

Proof. Firstly, we prove system (9) is regular and impulse free. From (13), it can be obtained that

$$Q_1 + Q_2 + Q_3 + A^T P + P^T A - E^T Z_1 E - E^T Z_2 E < 0. \tag{15}$$

As $Q_1 > 0, Q_2 > 0$, and $Q_3 > 0$, then

$$A^T P + P^T A - E^T Z_1 E - E^T Z_2 E < 0. \tag{16}$$

Since rank $(E) = r \leq n$, there exist nonsingular matrices $M \in \mathbb{R}^{n\times n}$ and $H \in \mathbb{R}^{n\times n}$ such that

$$\begin{aligned} \overline{E} &= MEH \\ &= \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{17}$$

Similar to (17), it can be defined that

$$\begin{aligned} \overline{A} &= MAH \\ &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\ \overline{P} &= M^{-T} P H \\ &= \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}. \end{aligned} \tag{18}$$

From (17) and (18),

$$\begin{aligned} \overline{P} &= \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}, \\ P_{11}^T &= P_{11} > 0. \end{aligned} \tag{19}$$

Premultiplying and postmultiplying H^T and H with $\Xi_{11} < 0$,

$$A_{22}^T P_{22} + P_{22}^T A_{22} < 0. \tag{20}$$

From (20), it means that A_{22} is nonsingular, and the pair (E, A) is regular and impulse free. According to Definition 1, system (9) is regular and impulse free.

Then, we will show that system (9) is asymptotically stable. The LKF is constructed as

$$\begin{aligned} V(t) &= \sum_{i=1}^{4} V_i(t), \quad t \in [t_k, t_{k+1}) \\ V_1(t) &= x(t)^T E^T P x(t), \\ V_2(t) &= \int_{t-d_1}^{t} x(s)^T Q_1 x(s) \mathrm{d}s + \int_{t-\tau(t)}^{t} x(s)^T Q_2 x(s) \mathrm{d}s \\ &\quad + \int_{t-d_2}^{t} x(s)^T Q_3 x(s) \mathrm{d}s, \\ V_3(t) &= d_1 \int_{-d_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) \mathrm{d}s \mathrm{d}\theta \\ &\quad + d_2 \int_{-d_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E^T Z_2 E \dot{x}(s) \mathrm{d}s \mathrm{d}\theta, \\ V_4(t) &= d_{21} \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} \dot{x}^T(s) E^T Z_3 E \dot{x}(s) \mathrm{d}s \mathrm{d}\theta. \end{aligned} \tag{21}$$

Calculating the derivative of $V(t)$, it can be obtained that

$$\begin{aligned} \dot{V}_1(t) &= 2x(t)^T E^T P \dot{x}(t), \\ \dot{V}_2(t) &= x(t)^T Q_1 x(t) - x(t-d_1)^T Q_1 x(t-d_1) \\ &\quad + x(t)^T Q_2 x(t) + x(t)^T Q_3 x(t) - x(t-d_2)^T Q_3 x(t-d_2), \\ \dot{V}_3(t) &= d_1^2 \dot{x}(t)^T E^T Z_1 E x(t) + d_2^2 \dot{x}(t)^T E^T Z_2 E \dot{x}(t) \\ &\quad - d_1 \int_{t-d_1}^{t} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) \mathrm{d}s \\ &\quad - d_2 \int_{t-d_2}^{t} \dot{x}^T(s) E^T Z_2 E \dot{x}(s) \mathrm{d}s, \\ \dot{V}_4(t) &= d_{21}^2 \dot{x}(t)^T E^T Z_3 E \dot{x}(t) - d_{21} \int_{t-d_2}^{t-d_1} \dot{x}^T(s) E^T Z_3 E \dot{x}(s) \mathrm{d}s. \end{aligned} \tag{22}$$

According to Jensen's inequality, it can be obtained that

$$\begin{aligned}&-d_1\int_{t-d_1}^{t}\dot{x}^T(s)E^TZ_1E\dot{x}(s)\mathrm{d}s\leq\\&-[x(t)-x(t-d_1)]^TE^TZ_1E[x(t)-x(t-d_1)],\\&-d_2\int_{t-d_2}^{t}\dot{x}^T(s)E^TZ_2E\dot{x}(s)\mathrm{d}s\leq\\&-[x(t)-x(t-d_2)]^TE^TZ_2E[x(t)-x(t-d_2)],\end{aligned}\tag{23}$$

and

$$\begin{aligned}&-d_{21}\int_{t-d_2}^{t-d_1}\dot{x}^T(s)E^TZ_3E\dot{x}(s)\mathrm{d}s\\&=-d_{21}\int_{t-d_2}^{t-\tau(t)}\dot{x}^T(s)E^TZ_3E\dot{x}(s)\mathrm{d}s\\&\quad-d_{21}\int_{t-\tau(t)}^{t-d_1}\dot{x}^T(s)E^TZ_3E\dot{x}(s)\mathrm{d}s\\&\leq-\frac{d_{21}}{d_2-\tau(t)}\left(x(t-\tau(t))-x(t-d_2)\right)^T\\&\quad\times E^TZ_3E\left(x(t-\tau(t))-x(t-d_2)\right)\\&\quad+\frac{d_{21}}{\tau(t)-d_1}\left(x(t-d_1)-x(t-\tau(t))\right)^T\\&\quad\times E^TZ_3E\left(x(t-d_1)-x(t-\tau(t))\right).\end{aligned}\tag{24}$$

It can be found from (12) that

$$\begin{bmatrix}\sqrt{\dfrac{d_2-d(t)}{\tau(t)-d_1}}\left(x(t-d_1)-x(t-\tau(t))\right)E\\-\sqrt{\dfrac{d(t)-d_1}{d_2-\tau(t)}}\left(x(t-\tau(t))-x(t-d_2)\right)E\end{bmatrix}^T\begin{bmatrix}Z_3 & R\\R^T & Z_3\end{bmatrix}\begin{bmatrix}\sqrt{\dfrac{d_2-\tau(t)}{\tau(t)-d_1}}\left(x(t-d_1)-x(t-\tau(t))\right)E\\-\sqrt{\dfrac{\tau(t)-d_1}{d_2-\tau(t)}}\left(x(t-\tau(t))-x(t-d_2)\right)E\end{bmatrix}\geq 0.\tag{25}$$

Thus, according to Lemma 1, we obtain from (24) and (25) that

$$\begin{aligned}&-d_{21}\int_{t-d_2}^{t-d_1}\dot{x}^T(s)E^TZ_3E\dot{x}(s)ds\\&\leq-\begin{bmatrix}x(t-d_1)-x(t-\tau(t))\\x(t-\tau(t))-x(t-d_2)\end{bmatrix}^T\begin{bmatrix}E^TZ_3E & E^TRE\\ * & E^TZ_3E\end{bmatrix}\begin{bmatrix}x(t-d_1)-x(t-\tau(t))\\x(t-\tau(t))-x(t-d_2)\end{bmatrix}\\&=-\vartheta^T(t)\begin{bmatrix}E^TZ_3E & -E^TZ_3E+E^TRE & -E^TRE\\ * & 2E^TZ_3E-E^T\left(R^T+R\right)E & -E^TZ_3E+E^TRE\\ * & * & E^TZ_3E\end{bmatrix}\vartheta(t),\end{aligned}\tag{26}$$

where

$$\vartheta(t)=\left[x^T(t-d_1)\ \ x^T(t-\tau(t))\ \ x^T(t-d_2)\right]^T.\tag{27}$$

Substituting (26) into (22),

$$\dot{V}(t)\leq\varsigma^T(t)\left(\Phi+[A\ \ 0\ \ BK\ \ 0]^T\left(d_1^2Z_1+d_2^2Z_2+d_{21}^2Z_3\right)[A\ \ 0\ \ BK\ \ 0]\right)\varsigma(t),\tag{28}$$

where

$$\Phi = \begin{bmatrix} \Xi_{11} & E^T Z_1 E & P^T BK & E^T Z_2 E \\ * & \Xi_{22} & \Xi_{23} & E^T RE \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * * & \Xi_{44} \end{bmatrix},$$

$$\zeta(t) = \left[x^T(t) \;\; x^T(t-d_1) \;\; x^T(t-\tau(t)) \;\; x^T(t-d_2) \right]^T. \tag{29}$$

By the Schur complement, (28) guarantees that

$$\Phi + [A \;\; 0 \;\; BK \;\; 0]^T \left(d_1^2 Z_1 + d_2^2 Z_2 + d_{21}^2 Z_3\right) [A \;\; 0 \;\; BK \;\; 0] < 0. \tag{30}$$

Therefore, it can be obtained that $\dot{V}(t) < -\sigma\|x(t)\|^2$ when $x(t) \neq 0$ and $\sigma > 0$. Then, system (9) is asymptotically stable.

Then, it will be proved that system (9) satisfies the H_∞ performance index γ. Consider the H_∞ performance for all nonzero $w(t) \in L_2[0,\infty)$ as follows:

$$J_{zw} \leq \int_0^\infty \left[y^T(s) y(s) - \gamma^2 w^T(s) w(s) \right] ds, \; \gamma > 0. \tag{31}$$

Then,

$$\begin{aligned} & y^T(t) y(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t) \\ & \leq \zeta^T(t) \left(\Theta + [C \;\; 0 \;\; DK \;\; 0 \;\; 0]^T [C \;\; 0 \;\; DK \;\; 0 \;\; 0] + [A \;\; 0 \;\; BK \;\; 0 \;\; B_w]^T \left(d_1^2 Z_1 + d_2^2 Z_2 + d_{21}^2 Z_3\right) [A \;\; 0 \;\; BK \;\; 0 \;\; B_w] \right) \zeta(t), \end{aligned} \tag{32}$$

where

$$\zeta(t) = \left[x^T(t) \;\; x^T(t-d_1) \;\; x^T(t-\tau(t)) \;\; x^T(t-d_2) \;\; w^T(t) \right]^T,$$

$$\Theta = \begin{bmatrix} \Xi_{11} & E^T Z_1 E & P^T BK & E^T Z_2 E & PB_w \\ * & \Xi_{22} & \Xi_{23} & E^T RE & 0 \\ * & * & \Xi_{33} & \Xi_{34} & 0 \\ * & * & * & \Xi_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}. \tag{33}$$

By the Schur complement, (16) guarantees that

$$\begin{aligned} & \Theta + [C \;\; 0 \;\; DK \;\; 0]^T [C \;\; 0 \;\; DK \;\; 0] \\ & + [A \;\; 0 \;\; BK \;\; B_w]^T \left(d_1^2 Z_1 + d_2^2 Z_2 + d_{21}^2 Z_3\right) [A \;\; 0 \;\; BK \;\; B_w] < 0. \end{aligned} \tag{34}$$

From (34), we can obtain that

$$y^T(t) y(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t) < 0. \tag{35}$$

Under zero initial conditions, we have $V(0) = 0$ and $V(\infty) \geq 0$. From (35), it can be obtained that $\|y(t)\|_2 \leq \gamma \|w(t)\|_2$ for all nonzero $w(t) \in L_2[0,\infty)$.

Then, according to the condition, system (9) is asymptotically stable and satisfies the H_∞ performance index γ. This completed the proof. □

Remark 3. If $E = I, Q_1 = Q_2 = 0$, and $R_2 = R_3 = 0$, then the LKF is similar with the one in [31]. Therefore, the LKF in [31] is a special case of (9), which means that the result in Theorem 1 has wider application scopes than [31].

Remark 4. To reduce the conservativeness, severable methods have been reported, such as the free-weighting matrix method and Wirtinger-based inequality. In the constructed LKF, we directly use tighter interactive convex combinatorial inequalities to estimate the integral terms $-d_{21} \int_{t-d_2}^{t_k} \dot{x}^T(s) E^T Z_3 E \dot{x}(s) ds$.

During this process, a matrix R is introduced to deal with the two terms

$$\begin{aligned} & \frac{d_{21}}{d_2 - \tau(t)} \left(x(t-\tau(t)) - x(t-d_2)\right)^T E^T Z_3 E \\ & \cdot \left(x(t-\tau(t)) - x(t-d_2)\right), \\ & \frac{d_{21}}{\tau(t) - d_1} \left(x(t-d_1) - x(t-\tau(t))\right)^T E^T Z_3 E \\ & \cdot \left(x(t-d_1) - x(t-\tau(t))\right). \end{aligned} \tag{36}$$

However, the above two terms are not considered in [33, 34], which will lead to conservatism to some extent. Besides, (25) is free as matrix R could be more than non-negative. Therefore, the conservatism has been greatly reduced.

Remark 5. It is noted that the number of decision variables is $4.5n^2 + 4.5n$, and it illustrates that the proposed methodology has lower computational complexity than the free-weighting matrix method or Jensen inequality lemma which increases the computational complexity with too much additional slack variables.

Furthermore, according to the following theorems, we will design the sampled-data controller to stabilize system (9).

Theorem 2. *For scales d_1, d_2, and γ, system (9) is asymptotically admissible with H_∞ performance γ if there exist matrices $P, R, Q_i > 0$, $Z_i > 0$, $i = 1, 2, 3$, satisfying*

$$\begin{gathered} E^T P = P^T E \geq 0, \\ \begin{bmatrix} \overline{Z}_3 & \overline{R} \\ \overline{R}^T & \overline{Z}_3 \end{bmatrix} > 0, \\ \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0, \end{gathered} \tag{37}$$

where

$$\begin{aligned} \Pi_{11} &= \begin{bmatrix} \overline{\Xi}_{11} & E^T\overline{Z}_1 E & B\overline{K} & E^T\overline{Z}_2 E & B_w \\ * & \overline{\Xi}_{22} & \overline{\Xi}_{23} & E^T\overline{R}E & 0 \\ * & * & \overline{\Xi}_{33} & \overline{\Xi}_{34} & 0 \\ * & * & * & \overline{\Xi}_{44} & 0 \\ * & * & * & * * & -\gamma^2 I \end{bmatrix}, \\ \Pi_{12} &= \begin{bmatrix} C\overline{P}^T & 0 & \overline{DK} & 0 & 0 \\ A\overline{P}^T & 0 & \overline{BK} & 0 & B_w\overline{P}^T \end{bmatrix}^T, \\ \Pi_{22} &= diag\left\{-I, -\overline{P}Z^{-1}\overline{P}\right\}, \\ \overline{\Xi}_{11} &= A\overline{P} + \overline{P}A^T + \overline{Q}_1 + \overline{Q}_2 + \overline{Q}_3 - E^T\overline{Z}_1 E - E^T\overline{Z}_2 E, \\ \overline{\Xi}_{22} &= -\overline{Q}_1 - E^T\overline{Z}_1 E - E^T\overline{Z}_3 E, \\ \overline{\Xi}_{23} &= E^T\overline{Z}_3 E - E^T\overline{R}E, \\ \Xi_{33} &= -2E^T\overline{Z}_3 E + E^T\overline{R}E + E^T\overline{R}^T E, \\ \overline{\Xi}_{34} &= E^T\overline{Z}_3 E - E^T\overline{R}E, \\ \overline{\Xi}_{44} &= -\overline{Q}_3 - E^T\overline{Z}_2 E - E^T\overline{Z}_3 E, \\ \overline{Z} &= d_1^2\overline{Z}_1 + d_2^2\overline{Z}_2 + d_{21}^2\overline{Z}_3, \\ d_{21} &= d_2 - d_1. \end{aligned} \tag{38}$$

Then, the controller gain matrix K can be obtained such that

$$K = \overline{KP}^{-1}. \tag{39}$$

Proof. By noticing that $-\overline{PZ}^{-1}\overline{P} \leq \overline{Z} - 2\overline{P}$, let $\eta = diag\{P^{-T}, P^{-T}, P^{-T}, P^{-T}, I, I, I\}$ and, $\kappa = diag\{P^{-T}, P^{-T}\}$, and define

$$\begin{aligned} \overline{P} &= P^{-1}, \\ \overline{K} &= KP^{-1}, \\ \overline{Z} &= P^{-T}ZP^{-1}, \\ \overline{R} &= P^{-T}RP^{-1}, \\ \overline{Z}_i &= P^{-T}Z_iP^{-1}, \\ \overline{Q}_i &= P^{-T}Q_iP^{-1},\ i = 1, 2, 3. \end{aligned} \tag{40}$$

Pre- and postmultiplying (12) by κ and κ^T, respectively, we can obtain (33). Pre- and postmultiplying (13) by η and η^T, respectively, (34) can be obtained by the Schur complement. The proof is completed. □

Remark 6. It can be seen that the control performance is affected by the design parameters d_1, d_2, and these parameters are chosen to solve the LMI problem. We can adjust the design parameters to achieve a better solution. The design and implementation step of the proposed method is provided as follows.

Step 1. We solve the LMI problem Ps to guarantee the system is admissible.

$$Ps: \begin{cases} \text{minimize}\, \gamma \\ \text{subject to}\, P, R, Q_i > 0,\ Z_i > 0,\ i = 1, 2, 3 \\ \text{LMIs}\, (11), (12), (13). \end{cases} \tag{41}$$

Step 2. If the above problem is feasible, then we solve the LMI problems Po to obtain the sampled-data controller. Otherwise, we adjust the design parameters d_1, d_2, and then go back to Step 1.

$$Po: \begin{cases} \text{minimize}\, \gamma, \\ \text{subject to}\, \overline{P}, \overline{R}, \overline{Q}_i > 0,\ \overline{Z}_i > 0, \quad i = 1, 2, 3, \\ \text{LMIs}\, (30), (31), (32). \end{cases} \tag{42}$$

Step 3. If the above problem is solvable, we can obtain the controller, that is, $K = \overline{KP}^{-1}$. Otherwise, we adjust the design parameters d_1, d_2, and then go back to Step 2.

The flowchart of the proposed method is provided in Figure 2.

Then, when $E = I$, the following Corollary 1 can be obtained based on Theorem 2. According to Corollary 1, we can use the proposed method to solve the sampled-data control problem of the practical systems, such as dynamic positioning ship and autonomous airship.

Corollary 1. *For scales d_1, d_2, and γ, system (9) is asymptotically stable with H_∞ performance γ if there exist matrices $P, R, Q_i > 0$, $Z_i > 0$, $i = 1, 2, 3$, such that*

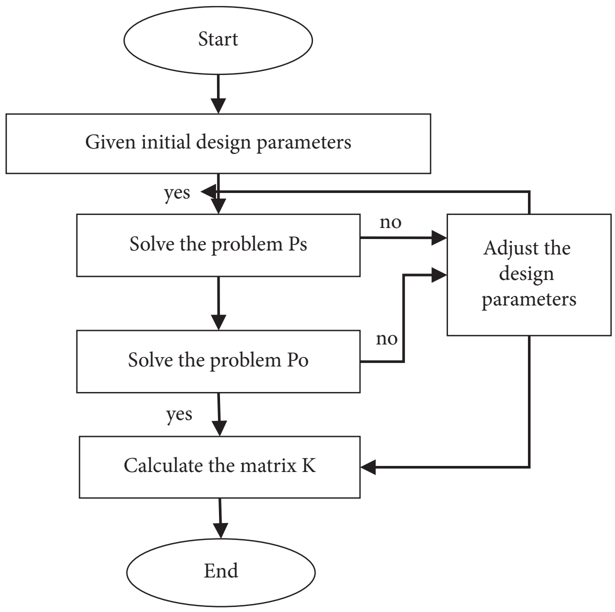

Figure 2: The flowchart of the proposed method.

$$\begin{bmatrix} \overline{Z}_3 & \overline{R} \\ \overline{R}^T & \overline{Z}_3 \end{bmatrix} > 0,$$
$$\begin{bmatrix} \overline{\Pi}_{11} & \overline{\Pi}_{12} \\ * & \overline{\Pi}_{22} \end{bmatrix} < 0, \tag{43}$$

where

$$\Pi_{11} = \begin{bmatrix} \overline{\Xi}_{11} & \overline{Z}_1 & B\overline{K} & \overline{Z}_2 & B_w \\ * & \overline{\Xi}_{22} & \overline{\Xi}_{23} & \overline{R} & 0 \\ * & * & \overline{\Xi}_{33} & \overline{\Xi}_{34} & 0 \\ * & * & * & \overline{\Xi}_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$
$$\Pi_{12} = \begin{bmatrix} C\overline{P}^T & 0 & \overline{DK} & 0 & 0 \\ A\overline{P}^T & 0 & \overline{BK} & 0 & B_w\overline{P}^T \end{bmatrix}^T,$$
$$\Pi_{22} = diag\left\{-I, -\overline{P}Z^{-1}\overline{P}\right\},$$
$$\overline{\Xi}_{11} = A\overline{P} + \overline{P}A^T + \overline{Q}_1 + \overline{Q}_2 + \overline{Q}_3 - \overline{Z}_1 - \overline{Z}_2, \tag{44}$$
$$\overline{\Xi}_{22} = -\overline{Q}_1 - \overline{Z}_1 - \overline{Z}_3,$$
$$\overline{\Xi}_{23} = \overline{Z}_3 - \overline{R},$$
$$\Xi_{33} = -2\overline{Z}_3 + \overline{R} + \overline{R}^T,$$
$$\overline{\Xi}_{34} = \overline{Z}_3 - \overline{R},$$
$$\Xi_{44} = -\overline{Q}_3 - \overline{Z}_2 - \overline{Z}_3,$$
$$\overline{Z} = d_1^2\overline{Z}_1 + d_2^2\overline{Z}_2 + d_{21}^2\overline{Z}_3,$$
$$d_{21} = d_2 - d_1.$$

Then, the controller gain matrix K can be obtained such that

$$K = \overline{KP}^{-1}. \tag{45}$$

Proof. The procedure of the proof is similar with Theorem 2, so it is omitted. □

4. Numerical Examples

In this section, to validate the effectiveness of the given method, two numerical examples are introduced.

Example 1. Consider the following parameters:

$$A = \begin{bmatrix} -13.1 & -13.7 \\ -15.4 & -23.8 \end{bmatrix},$$
$$B = \begin{bmatrix} -18.6 & -10.4 \\ -25.2 & -16.8 \end{bmatrix},$$
$$B_w = \begin{bmatrix} 1.9 \\ 1.8 \end{bmatrix}, \tag{46}$$
$$C = \begin{bmatrix} 0.4 & -0.8 \end{bmatrix},$$
$$E = \begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

When $w(t) = 0$, for different lower bounds of the time delay d_1, Table 1 lists the maximum time delay d_2 to guarantee the admissibility of the system by different methods. From Table 1, the upper bound of time delay d_2 obtained by Theorem 1 is larger than that in [33–36], which indicates that the proposed method in the paper can obtain lower conservative results than the other references.

Next, consider the situation about $w(t) \neq 0$. For different $d_2 > 0$, Table 2 lists the minimal H_∞ performance indicators γ that guarantee the admissibility of the system by Theorem 1 and [37] when $d_1 = 0$. From Table 2, it is easy to see that, for different upper bounds of time delay d_2, H_∞ performance γ obtained by Theorem 1 is smaller than that in [37]. Therefore, it is shown that Theorem 1 improves the results of [37].

Assuming the sampling interval $d_2 = 1.7$, the H_∞ performance is obtained such that $\gamma_{\min} = 0.8625$. Then, the controller gain can be computed such that

$$K = \begin{bmatrix} -0.1663 & -0.1109 \\ -0.0554 & -0.0370 \end{bmatrix}. \tag{47}$$

Consider the system's initial state $x_s(t) = \begin{bmatrix} -2 & -2 \end{bmatrix}$ and the external environment disturbance $w(t) = 0.5\cos(t)$ and random disturbance, respectively. Under different disturbances, the response curves of the system's state and the control input $u(t)$ are shown in Figures 3–6, respectively. From Figures 3 and 5, it is shown that the system's state $x_1(t)$ and $x_2(t)$ are stable in a short time under different disturbances, which further illustrates that the designed sampled-data controller can make the system's states stable and have acceptable control performance with external variable disturbances.

Table 1: Maximum values of the upper bound d_2.

d_1	1.4	1.6	1.8	2.0	2.2
[33]	2.1121	2.1450	2.2841	2.4328	2.5852
[34]	2.2314	2.2761	2.4041	2.5383	2.6777
[35]	2.3360	2.3704	2.4242	2.5425	2.7007
[36]	2.3372	2.3730	2.4923	2.6181	2.7494
Theorem 1	2.3418	2.3836	2.5013	2.6408	2.7826

Table 2: Minimum values of H_∞ performance γ ($d_1 = 0$).

d_2	0.9	1.1	1.3	1.5	1.7
[37]	0.2713	0.3548	0.4744	0.6378	0.8736
Theorem 1	0.2627	0.3468	0.4642	0.6174	0.8625

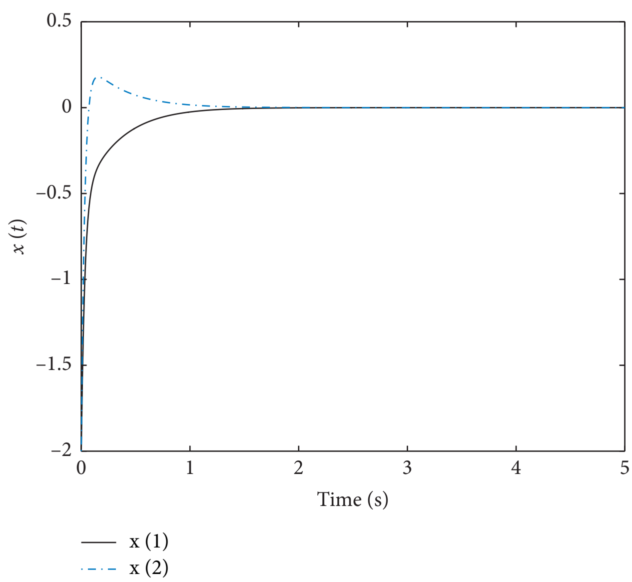

Figure 3: The state response curve of system (9) ($w(t) = 0.5\cos(t)$).

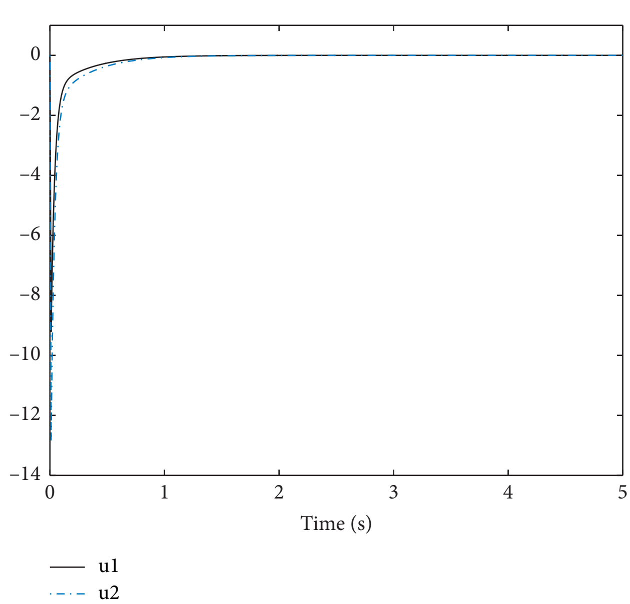

Figure 4: Control input for system (9) ($w(t) = 0.5\cos(t)$).

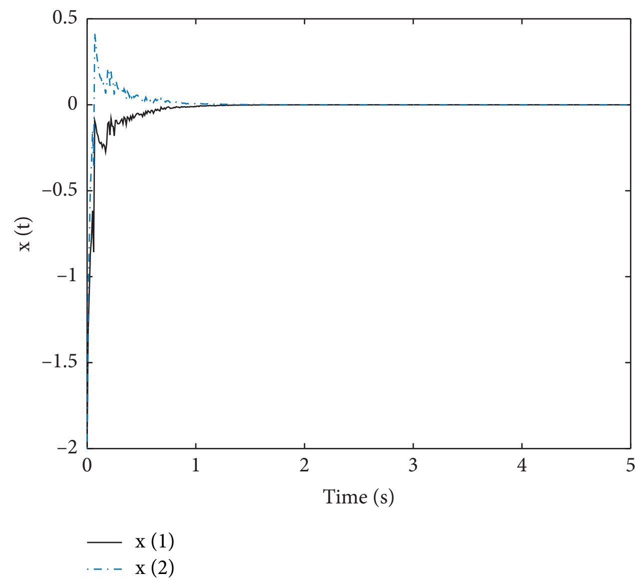

Figure 5: The state response curve of system (9) ($w(t)$ is the random disturbance).

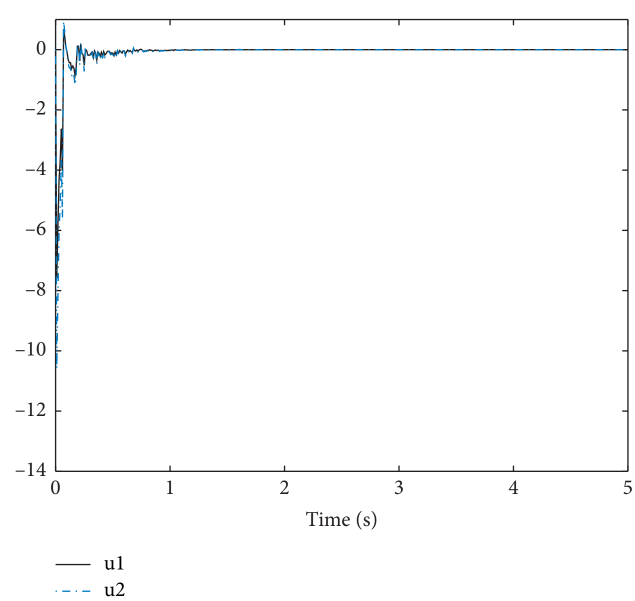

Figure 6: Control input for system (9) ($w(t)$ is the random disturbance).

Example 2. Similar with [38], the motion equations of a sampled-data dynamic positioning ship system are considered as follows:

$$M\dot{v} + Dv = \tau + w, \tag{48}$$

where τ denotes the control input, w denotes the external disturbance, M is the inertia matrix, D is the damping coefficient matrix, and $v = [p, v, r]^T$ includes the velocity of surge, sway, and yaw, respectively. Define the input matrix

$$x(t) = [x_a, y_a, \psi, p, v, r]^T, \tag{49}$$

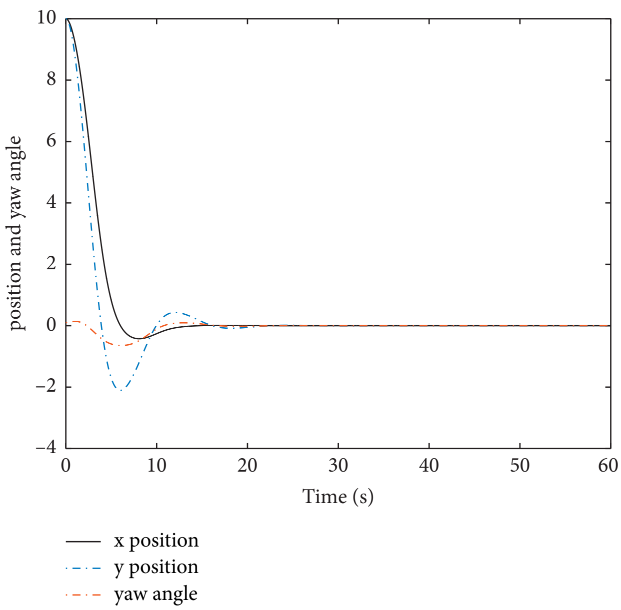

FIGURE 7: The responses of the position of x and y and yaw angle of DPS.

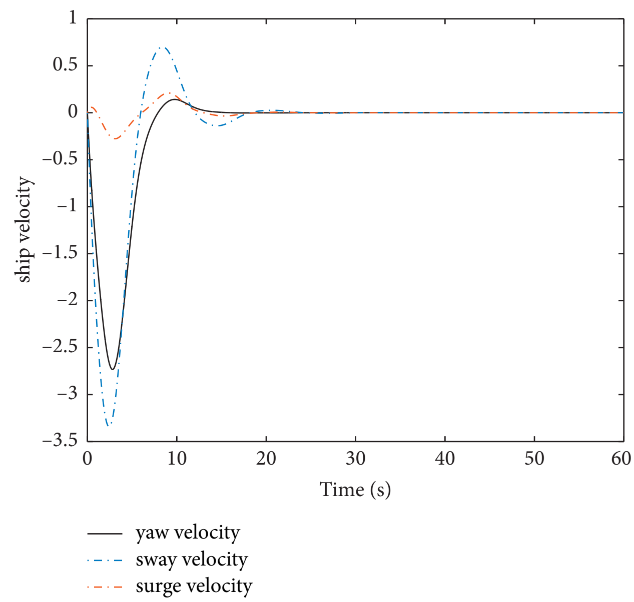

FIGURE 8: The response of velocities of DPS.

where x_a, y_a, and ψ represent the x-position, y-position, and yaw angle, respectively. The values of M and D are considered such that

$$M = \begin{bmatrix} 0.754 & 0 & 0 \\ 0 & 1.199 & 0.211 \\ 0 & 0.029 & 0.524 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.014 & 0 & 0 \\ 0 & 0.102 & -0.024 \\ 0 & 0.192 & 0.095 \end{bmatrix}. \quad (50)$$

Let $A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$, and $B_w = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$.

Then,

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & -0.0349 & 0 \\ 0 & 0 & 0 & 0.0349 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0186 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0208 & 0.0342 \\ 0 & 0 & 0 & 0 & -0.3653 & -0.1832 \end{bmatrix},$$
$$B = B_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.3263 & 0 & 0 \\ 0 & 0.8422 & -0.3391 \\ 0 & -0.0466 & 1.9272 \end{bmatrix}. \quad (51)$$

The initial states of the ship are assumed such that $x_s(t) = [\,10 \;\; 10 \;\; 0.1 \;\; 0 \;\; 0 \;\; 0\,]^T$. Let $w(t) = \sin(0.1t)$. And the values of the sampling period are $d_1 = 0.8$ and $d_2 = 1.6$; then, the H_∞ performance index is obtained according to Corollary 1 such that $\gamma_{\min} = 2.945$. Then, the controller gain is obtained as follows:

$$K = \begin{bmatrix} -0.2438 & -0.0109 & -0.0008 & -0.5459 & -0.0105 & 0.0002 \\ 0.0072 & -0.4429 & -0.1018 & 0.0186 & -0.5794 & -0.1722 \\ -0.0011 & 0.0088 & -0.2082 & -0.0022 & 0.2673 & -0.5638 \end{bmatrix}. \quad (52)$$

The responses of the DPS state are shown in Figures 7 and 8. Figure 7 shows the responses of the position of x and y and yaw angle of DPS. Figure 8 shows the response of velocities of DPS. It is shown that the DPS can be stabilized by the designed sampled-data controller even if external disturbances exist.

5. Conclusion

The sampled-data control issue for singular systems is discussed in this paper. Both lower and upper bounds of the variable period have been considered. Then, the criteria of asymptotical admissibility are given by constructing a suitable LKF. Then, the tighter reciprocally convex inequalities are used to estimate the derivative of the LKF, and less conservative results can be obtained. Two numerical examples are given to illustrate the effectiveness of the proposed method. In the future, we will improve the proposed methods and consider the actuator faults and then discuss the sampled-data fault-tolerant issue for the more complex nonlinear singular systems such as fuzzy singular systems and singular Markov jump systems.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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