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Research Article

Application of RQMC for CDO Pricing with Stochastic Correlations under Nonhomogeneous Assumptions

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In consideration of that the correlation between any two assets of the asset pool is always stochastic in the actual market and that collateralized debt obligation (CDO) pricing models under nonhomogeneous assumptions have no semianalytic solutions, we designed a numerical algorithm based on randomized quasi-Monte Carlo (RQMC) simulation method for CDO pricing with stochastic correlations under nonhomogeneous assumptions and took Gaussian factor copula model as an example to conduct experiments. The simulation results of RQMC and Monte Carlo (MC) method were compared from the perspective of variance changes. The results showed that this numerical algorithm was feasible, efficient, and stable for CDO pricing with stochastic correlation under nonhomogeneous assumptions. This numerical algorithm is expected to be extended to other factor Copula models for CDO pricing with stochastic correlations under nonhomogeneous assumptions.

1. Introduction

Collateralized debt obligation (CDO) is one of the most typical securitized assets in credit derivative markets, and its pricing has always been the focus of scholars. In the pricing process of CDO, the reduced model is widely used for establishing default distribution of a single reference entity, factor Copula methods are the mainstream of joint default distribution of asset portfolio [1, 2], and the spread of each tranche of CDO can be calculated according to the principle of arbitrage-free pricing [3].

The Gaussian factor Copula model is usually considered to be the standard model for CDO pricing, but its correlation coefficient is a deterministic parameter. In fact, the stochastic correlation copula is closer to the market [3]. Therefore, some scholars extended the constant correlation coefficient to a stochastic one. Burtschell et al. [4] provided a thorough analysis of the three-state stochastic correlation model with homogeneity and obtained the semianalytical pricing formula. Yang et al. [5] proposed a semianalytical method for the credit spreads of each CDO tranche under the conditions of homogeneity and stochastic correlations based on single-factor Copula model with mixed distribution.

As we can see, the above literature is all based on the assumption of homogeneity. However, it is difficult to meet the requirements of homogeneity in actual markets and the arbitrage-free pricing models under nonhomogeneous assumptions have no semianalytical solutions [6]. Therefore, it might be a good idea to resort to numerical methods.

Monte Carlo (MC) is one of the most common numerical methods [7, 8]. However, this method has a certain dimension disaster problem. Quasi-Monte Carlo (QMC) [9, 10] is an extension of MC, and it is usually superior to MC in dealing with high-dimensional problems. Furthermore, by introducing a randomized component into the QMC method, the randomized quasi-Monte Carlo (RQMC) method can effectively improve the cycle problem [11], which usually has higher convergence order than MC and QMC. Some scholars applied RQMC to option pricing [12–14]. Johansson et al. [12] studied the pricing of American options

using RQMC and found that RQMC could reduce both the variance and the bias of the option. Amal et al. [13] applied array-RQMC for option pricing under a stochastic volatility process, and the empirical results showed that it could bring very significant variance reductions compared with MC. He [14] found that RQMC had a better rate of convergence in financial option pricing.

Qu et al. [15] introduced RQMC into the CDO pricing based on a single-factor Copula framework and nonhomogeneous hypothesis and achieved good results. However, only the constant correlation coefficient was considered in the model. To our knowledge, there have been no further reports of RQMC being used to solve the CDO pricing issues. In view of this, we intend to design a numerical algorithm for the CDO pricing with stochastic correlations based on RQMC and Gaussian factor Copula model under the conditions of nonhomogeneous assumptions and conduct empirical research. We try to provide some ideas for the pricing problem with stochastic correlations under the nonhomogeneous assumptions in the credit derivative markets.

The rest of this paper is organized as follows: Some preliminary knowledge is introduced in Section 2. In the following section, the concrete algorithm process is given based on RQMC. The empirical study is carried out in Section 4. Finally, the conclusion and the discussion are presented in Section 5.

2. Methodology

2.1. Arbitrage-Free Pricing Model. First, we give some symbols used in this paper and their meanings are shown in Table 1.

According to the principle of arbitrage-free pricing, for the m-th tranche, the discounted expectation of premium leg (PL) and default leg (DL) should be equal, i.e.,

$$E\left(\mathrm{PL}\left[a_{m-1}, a_m\right]\right) = E\left(\mathrm{DL}\left[a_{m-1}, a_m\right]\right), \quad m = 1, 2, \ldots, M. \tag{1}$$

In the continuous case, they can be expressed as follows:

$$E\left(\mathrm{PL}\left[a_{m-1}, a_m\right]\right) = E\left[\int_0^T s_m E_m(t) e^{-r_f t} \mathrm{d}t\right] \approx s_m\left(A_m + B_m\right), \tag{2}$$

$$E\left(\mathrm{DL}\left[a_{m-1}, a_m\right]\right) = E\left[\int_0^T e^{-r_f t} \mathrm{d}L_m(t)\right] \approx C_m. \tag{3}$$

From equations (1)–(3), we have

$$s_m = \frac{C_m}{A_m + B_m} \quad m = 1, 2, \ldots, M. \tag{4}$$

2.2. Gaussian Factor Copula Model with Stochastic Correlations. In Gaussian factor Copula model with stochastic correlations, the yield rate $X_i\,(i = 1, 2, \ldots, n)$ of the i-th asset is determined by a common factor F and a special factor $Z_i\,(i = 1, 2, \ldots, n)$, namely,

$$X_i = \sqrt{\widetilde{\rho}_i} F + \sqrt{1 - \widetilde{\rho}_i} Z_i, \quad i = 1, 2, \ldots, n, \tag{5}$$

where F and $Z_i\,(i = 1, 2, \ldots, n)$ are independent of each other and they are all subject to $N(0, 1)$; $\widetilde{\rho}_i\,(i = 1, 2, \ldots, n)$ are stochastic correlation coefficients between $X_i\,(i = 1, 2, \ldots, n)$ and F, and it is independent of F and $Z_i\,(i = 1, 2, \ldots, n)$; $X_i\,(i = 1, 2, \ldots, n)$ is also independent of each other when F and $\widetilde{\rho}_i\,(i = 1, 2, \ldots, n)$ are conditions. The distribution functions of F, Z_i, and X_i are denoted as G_F, G_{Z_i}, and G_{X_i}, respectively, where $i = 1, 2, \ldots, n$.

Now, we consider the case that the random variable $\widetilde{\rho}_i$ of equation (5) is in two states.

State 1. When $\widetilde{\rho}_i = \rho_i\,(i = 1, 2, \ldots, n)$, equation (5) can be written as

$$X_i = \sqrt{\rho_i} F + \sqrt{1 - \rho_i} Z_i \quad i = 1, 2, \ldots, n. \tag{6}$$

The probability of this state is p.

State 2. When $\widetilde{\rho}_i = \eta_i\,(i = 1, 2, \ldots, n)$, we have

$$X_i = \sqrt{\eta_i} F + \sqrt{1 - \eta_i} Z_i \quad i = 1, 2, \ldots, n. \tag{7}$$

In this case, the probability is $1 - p$.

We point out that in this paper, we use corresponding superscript symbols (1) and (2) to indicate States 1 and 2, respectively.

For the m-th tranche, according to equation (4), we have

$$s_m^{(1)} = \frac{C_m^{(1)}}{A_m^{(1)} + B_m^{(1)}}, \quad m = 1, 2, \ldots, M, \tag{8}$$

$$s_m^{(2)} = \frac{C_m^{(2)}}{A_m^{(2)} + B_m^{(2)}}, \quad m = 1, 2, \ldots, M. \tag{9}$$

Now, let us take State 1 as an example to illustrate.

In State 1, according to the reduced model, the accumulative probability of the i-th asset defaulting before $t_i^{(1)}$ can be written as

$$Q_{\mathcal{T}_i^{(1)}}\left(t_i^{(1)}\right) = 1 - e^{-\lambda_i^{(1)} t_i^{(1)}}, \tag{10}$$

where $\lambda_i^{(1)}$ is the default intensity of the i-th asset and $\mathcal{T}_i^{(1)}$ is default time. For the i-th asset, we know that the default correlation of yield rate $X_i^{(1)}$ can be reflected by the correlation of default time $\mathcal{T}_i^{(1)}$, so we can assume that the relationship between $\mathcal{T}_i^{(1)}$ and $X_i^{(1)}$ can be described as

$$P\left\{X_i^{(1)} \le x_i^{(1)}\right\} = P\left\{\mathcal{T}_i^{(1)} \le t_i^{(1)}\right\}, \tag{11}$$

namely,

$$G_{X_i^{(1)}}\left(x_i^{(1)}\right) = Q_{\mathcal{T}_i^{(1)}}\left(t_i^{(1)}\right). \tag{12}$$

Then, according to equations (10)–(12), $t_i^{(1)}$ can be obtained:

$$t_i^{(1)} = Q_{\mathcal{T}_i^{(1)}}^{-1}\left(G_{X_i^{(1)}}\left(x_i^{(1)}\right)\right) = -\frac{\ln\left(1 - G_{X_i^{(1)}}\left(x_i^{(1)}\right)\right)}{\lambda_i^{(1)}}. \tag{13}$$

Table 1: Symbols and their meanings.

Symbol	Meaning	Note
n	Number of reference entities in the asset pool of CDO	
N_i	Nominal value of the i-th reference entity	$i = 1, 2, \ldots, n$
N	Total nominal value	$N = \sum_{i=1}^{n} N_i$
R_i	Recovery rate of the i-th reference entity	$i = 1, 2, \ldots, n$
l_i	Loss of the i-th reference entity	$l_i = N_i(1 - R_i)$, $i = 1, 2, \ldots, n$
$L(t)$	Accumulative default loss at time t	$L(t) = \sum_{i=1}^{n} l_i 1_{\{\mathcal{T}_i \le t\}}, 1_{\{\mathcal{T}_i \le t\}} = \begin{cases} 1, & \text{if } \mathcal{T}_i \le t \\ 0, & \text{if } \mathcal{T}_i > t \end{cases}$
r_f	Risk-free interest rates	
T	Term of CDO	Unit: year
τ_j	Payment of time nodes	$j = 1, 2, \ldots, J, \tau_J = T$
$\Delta\tau_j$	Payment interval	$\Delta\tau_j = \tau_j - \tau_{j-1}, j = 1, 2, \ldots, J, \tau_0 = 0$
M	Total number of tranches	
$[a_{m-1}, a_m]$	The m-th tranche	$m = 1, 2, \ldots, M$
$L_m(t)$	Loss suffered by the m-th tranche at time t	$L_m(t) = \max\{L(t) - Na_{m-1}, 0\} - \max\{L(t) - Na_m, 0\}$, $m = 1, 2, \ldots, M$
$E_m(t)$	Residual value of the m-th tranche at time t	$E_m(t) = Na_m - Na_{m-1} - L_m(t)$, $m = 1, 2, \ldots, M$
s_m	Spread of the m-th tranche	
$s_m A_m$	Discounted value of normal payment for promotion of the m-th tranche	
$s_m B_m$	Accrual payment of the m-th tranche when default occurs	$m = 1, 2, \ldots, M$
C_m	Discounted value of compensation of the m-th tranche	

By substituting $t_i^{(1)}$ into equations (2) and (3), $s_m^{(1)}$ $(m = 1, \ldots, M)$ can be obtained by equation (8).

Similarly, the corresponding default time $t_i^{(2)}$ in State 2 can be obtained as follows:

$$t_i^{(2)} = Q_{\mathcal{T}_i^{(2)}}^{-1}\left(G_{X_i^{(2)}}\left(x_i^{(2)}\right)\right) = -\frac{\ln\left(1 - G_{X_i^{(2)}}\left(x_i^{(2)}\right)\right)}{\lambda_i^{(2)}}, \quad (14)$$

where $\lambda_i^{(2)}$ is default intensity of the i-th asset in State 2. Then, $s_m^{(2)}$ $(m = 1, ..., M)$ can be obtained.

Finally, we have

$$s_m = ps_m^{(1)} + (1 - p)s_m^{(2)}, \quad m = 1, \ldots, M. \quad (15)$$

2.3. Random Sobol Sequences. Figure 1 shows the scatter diagrams of pseudo-random sequences used in MC, Sobol sequences used in QMC, and randomized Sobol (Sobol scramble) sequences used in RQMC in high-dimensional cases (125-th dimension and 126-th dimension), respectively, where the number of points are all 1100.

It can be seen from Figure 1 that randomized Sobol sequences not only maintain good uniformity but also improve circulation problems of Sobol sequences in high dimension. In this paper, RQMC method based on randomized Sobol sequences is adopted.

In this paper, we call command equation (16) in MATLAB to generate $(n+1)$-dimensional Sobol sequences and $(n+1)$-dimensional randomized Sobol sequences.

$$P = \text{sobolset}(n+1); P = \text{scramble} \cdot (P, \text{'Matousek Affine Owen'}). \quad (16)$$

3. Algorithm

In this section, based on the Gaussian factor Copula model, we design the RQMC simulation algorithm for CDO pricing with stochastic correlations of the two states under non-homogeneous assumptions.

First, we draw the algorithm flow chart (concise format), as shown in Figure 2. The meanings of ρ_i, η_i, p, and ST are shown in Step 1 of the algorithm.

Next, we give the concrete steps of the algorithm.

Step 1. Determine and input the relevant data.

Assume that there are only two values ρ_i and η_i for stochastic correlation coefficients $\widetilde{\rho}_i$ and corresponding probabilities are p and $1 - p$, respectively, where $i = 1, \ldots, n$. In addition, we preset the total number of simulations, denoted as ST.

Step 2. Generate randomized Sobol sequences.

$(n+1)$-dimensional randomized sequences $(\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_n)$ are generated by command equation (16); then, we can get corresponding sequences $(G_F^{-1}(\varepsilon_0), G_{Z_1}^{-1}(\varepsilon_1), \ldots, G_{Z_n}^{-1}(\varepsilon_n))$, which is a set of values $(F, Z_1, \ldots, Z_n)$, denoted as $(y, z_1, \ldots, z_n)$.

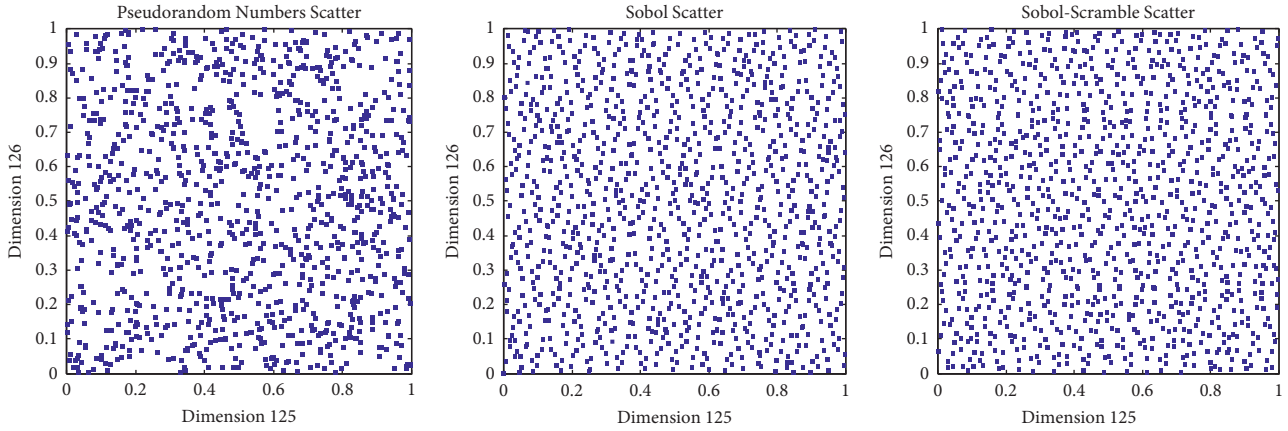

Figure 1: Scatter plots of different sequences.

Step 3. Simulate a path of yields in the two states, respectively.

Substitute $y, z_i\,(i = 1, \ldots, n)$ obtained in Step 2 and $\rho_i\,(i = 1, \ldots, n)$ determined in Step 1 into equation (6) to get the corresponding values $x_i^{(1)}\,(i = 1, \ldots, n)$ of yield rates $X_i^{(1)}\,(i = 1, \ldots, n)$ in State 1, and substitute $y, z_i\,(i = 1, \ldots, n)$ and $\eta_i\,(i = 1, ..., n)$ into equation (7) to get the corresponding values $x_i^{(2)}\,(i = 1, ..., n)$ of yield rates $X_i^{(2)}\,(i = 1, \ldots, n)$ in State 2.

Step 4. Generate default times in the two states, respectively.

Substitute $x_i^{(1)}$ and $x_i^{(2)}\,(i = 1, \ldots, n)$ obtained in Step 3 into equations (13) and (14); then, we can obtain $t_i^{(1)}, t_i^{(2)}\,(i = 1, ..., n)$.

Step 5. Find out the actual default time in the two states.

Find out the actual default time for the two states: $\left\{\widetilde{t}_k^{(1)} | \widetilde{t}_k^{(1)} \in \left\{t_i^{(1)} | t_i^{(1)} \le T,\ i = 1, \ldots, n\right\},\ k = 1, 2, \ldots, K_1\right\}$ and $\left\{\widetilde{t}_k^{(2)} | \widetilde{t}_k^{(2)} \in \left\{t_i^{(2)} | t_i^{(2)} \le T,\ i = 1, \ldots, n\right\},\ k = 1, \ldots, K_2\right\}$, where $K_1\,(K_1 \le n)$ and $K_2\,(K_2 \le n)$ are the total number of real default assets in the two states, respectively. Here, we agree that $\widetilde{t}_k^{(1)}\,(k = 1, 2, \ldots, K_1)$ and $\widetilde{t}_k^{(2)}\,(k = 1, 2, \ldots, K_2)$ have been arranged in an order from the smallest to the largest, i.e.,

$$\begin{aligned} \widetilde{t}_k^{(1)} &\le \widetilde{t}_{k+1}^{(1)}\,(k = 1, 2, \ldots K_1 - 1), \\ \widetilde{t}_k^{(2)} &\le \widetilde{t}_{k+1}^{(2)}\,(k = 1, 2, \ldots K_2 - 1). \end{aligned} \tag{17}$$

We introduce default matrices $L^{(1)}$ and $L^{(2)}$:

$$\begin{aligned} L^{(1)} &= \begin{pmatrix} \widetilde{t}_1^{(1)} & \widetilde{t}_2^{(1)} & \widetilde{t}_3^{(1)} & \ldots & \widetilde{t}_{K_1}^{(1)} \\ h_1^{(1)} & h_2^{(1)} & h_3^{(1)} & \ldots & h_{K_1}^{(1)} \\ H_1^{(1)} & H_2^{(1)} & H_3^{(1)} & \ldots & H_{K_1}^{(1)} \end{pmatrix}, \\ L^{(2)} &= \begin{pmatrix} \widetilde{t}_1^{(2)} & \widetilde{t}_2^{(2)} & \widetilde{t}_3^{(2)} & \ldots & \widetilde{t}_{K_2}^{(2)} \\ h_1^{(2)} & h_2^{(2)} & h_3^{(2)} & \ldots & h_{K_2}^{(2)} \\ H_1^{(2)} & H_2^{(2)} & H_3^{(2)} & \ldots & H_{K_2}^{(2)} \end{pmatrix}. \end{aligned} \tag{18}$$

where $h_k^{(1)}\,(k = 1, 2, \ldots K_1)$ and $h_k^{(2)}\,(k = 1, 2, \ldots K_2)$ are the corresponding nominal values of the k-th real default in the two states, respectively, and $H_k^{(1)}\,(k = 1, 2, \ldots K_1)$ and $H_k^{(2)}\,(k = 1, 2, \ldots K_2)$ are the corresponding cumulative nominal values of the k-th real default by time $\widetilde{t}_k^{(1)}$ and $\widetilde{t}_k^{(2)}$, respectively.

Step 6. Allocate default losses for the two states.

Let $b_m^{(1)}$ and $b_m^{(2)}$ be the corresponding positions of the maximum loss that the m-th tranche can bear in $L^{(1)}$ and $L^{(2)}$, respectively, which means that $H_{b_m^{(1)}}^{(1)}(1 - R) = a_m, H_{b_m^{(2)}}^{(2)}(1 - R) = a_m$, where a_m is a separation point (maximum loss should be taken by the m-th tranche); then, the default information allocated to the m-th tranche in the two states can be represented by the following default matrices:

$$\begin{aligned} L_m^{(1)} &= \begin{pmatrix} \widetilde{t}_{b_{m-1}+1}^{(1)} & \widetilde{t}_{b_{m-1}+2}^{(1)} & \ldots & \widetilde{t}_{b_m}^{(1)} \\ h_{b_{m-1}+1}^{(1)} & h_{b_{m-1}+2}^{(1)} & \ldots & h_{b_m}^{(1)} \\ H_{b_{m-1}+1}^{(1)} & H_{b_{m-1}+2}^{(1)} & \ldots & H_{b_m}^{(1)} \end{pmatrix}, \\ L_m^{(2)} &= \begin{pmatrix} \widetilde{t}_{b_{m-1}+1}^{(2)} & \widetilde{t}_{b_{m-1}+2}^{(2)} & \ldots & \widetilde{t}_{b_m}^{(2)} \\ h_{b_{m-1}+1}^{(2)} & h_{b_{m-1}+2}^{(2)} & \ldots & h_{b_m}^{(2)} \\ H_{b_{m-1}+1}^{(2)} & H_{b_{m-1}+2}^{(2)} & \ldots & H_{b_m}^{(2)} \end{pmatrix}. \end{aligned} \tag{19}$$

Step 7. Calculate $A_m^{(1)}, B_m^{(1)}, C_m^{(1)}$ and $A_m^{(2)}, B_m^{(2)}, C_m^{(2)}$.

For the m-th tranche $[a_{m-1}, a_m]\,(m = 1, \ldots, M)$, the initial nominal values are $E_0^{(m)^{(1)}} = E_0^{(m)^{(1)}} = N\mathrm{a}_m - N\mathrm{a}_{m-1}$ and we can calculate the residual values of States 1 and 2 at τ_j:

$$\begin{aligned} E_j^{(m)^{(1)}} &= E_{j-1}^{(m)^{(1)}} - \sum_{k \in \Omega_1} (1 - R) h_k^{(1)}, \quad j = 1, \ldots, J_1 \\ E_j^{(m)^{(2)}} &= E_{j-1}^{(m)^{(2)}} - \sum_{k \in \Omega_2} (1 - R) h_k^{(2)}, \quad j = 1, \ldots, J_2. \end{aligned} \tag{20}$$

Here, $\Omega_1 = \left\{k|\tau_{j-1} \le \widetilde{t}_k^{(1)} \le \tau_j, b_{m-1}^{(1)} + 1 \le k \le b_m^{(1)}\right\}$ and $\Omega_2 = \left\{k|\tau_{j-1} \le \widetilde{t}_k^{(2)} \le \tau_j, b_{m-1}^{(2)} + 1 \le k \le b_m^{(2)}\right\}$. Then, we have

$$\begin{aligned}
A_m^{(1)} &= \sum_{j=1}^{J_1} \Delta\tau \times E_j^{(m)^{(1)}} \times e^{-r_f\tau_j},\\
B_m^{(1)} &= \sum_{k=b_{m-1}^{(1)}+1}^{b_m^{(1)}} \left(\widetilde{t}_k^{(1)} - t_*^{(1)}\right) \times h_k^{(1)} \times (1-R) \times e^{-r_f\widetilde{t}_k^{(1)}},\\
C_m^{(1)} &= \sum_{k=b_{m-1}^{(1)}+1}^{b_m^{(1)}} h_k^{(1)} \times (1-R) \times e^{-r_f\widetilde{t}_k^{(1)}},\\
A_m^{(2)} &= \sum_{j=1}^{J_2} \Delta\tau \times E_j^{(m)^{(2)}} \times e^{-r_f\tau_j},\\
B_m^{(2)} &= \sum_{k=b_{m-1}^{(2)}+1}^{b_m^{(2)}} \left(\widetilde{t}_k^{(2)} - t_*^{(2)}\right) \times h_k^{(2)} \times (1-R) \times e^{-r_f\widetilde{t}_k^{(2)}},\\
C_m^{(2)} &= \sum_{k=b_{m-1}^{(2)}+1}^{b_m^{(2)}} h_k^{(2)} \times (1-R) \times e^{-r_f\widetilde{t}_k^{(2)}},
\end{aligned} \tag{21}$$

where $t_*^{(1)}$ and $t_*^{(2)}$ are the last coupon payment time periods nearest to $\widetilde{t}_k^{(1)}$ and $\widetilde{t}_k^{(2)}$ in the two states, respectively.

Note: at this point, the values of $A_m^{(1)}, B_m^{(1)}, C_m^{(1)}$ and $A_m^{(2)}, B_m^{(2)}, C_m^{(2)}$ of the m-th coupon on a path have been calculated, where $m = 1, \ldots, M$.

Repeat Steps 2–7 until the total number of simulations reaches ST.

Step 8. Calculate $s_m\,(m = 1, \ldots, M)$.

For $m = 1, 2, \ldots, M$, we calculate the average values of $A_m^{(1)}, B_m^{(1)}, C_m^{(1)}$ and $A_m^{(2)}, B_m^{(2)}, C_m^{(2)}$, respectively, and denote them as $\overline{A_m^{(1)}}, \overline{B_m^{(1)}}, \overline{C_m^{(1)}}$ and $\overline{A_m^{(2)}}, \overline{B_m^{(2)}}, \overline{C_m^{(2)}}$ in sequence. Then, we substitute $\overline{A_m^{(1)}}, \overline{B_m^{(1)}}, \overline{C_m^{(1)}}$ into equation (8) to obtain $s_m^{(1)}$ and substitute $\overline{A_m^{(2)}}, \overline{B_m^{(2)}}, \overline{C_m^{(2)}}$ into equation (9) to obtain $s_m^{(2)}$. Finally, we plug $s_m^{(1)}\,(m = 1, \ldots, M)$ and $s_m^{(2)}\,(m = 1, \ldots, M)$, and the value of p into equation (15) to get $s_m\,(m = 1, \ldots, M)$.

This is the end of the algorithm.

4. Empirical Study

4.1. Parameter Values. In this paper, the values of each parameter are as follows (refer to [3]). Let $n = 125$, $N = 1$, the nominal value and return rate of each reference entity be all equal, respectively; that is, $N_i = 1/n$, $R_i = R = 0.4$, and payment intervals are all $\Delta\tau = 0.25$. Let $r_f = 0.035$ and duration $T = 5$ (year). CDO tranches are [0, 3%], [3%, 6%], [6%, 9%], [9%, 12%], [12%, 22%], and [22%, 100%]. We assume that the default intensity is equal, that is, $\lambda_i^{(1)} = \lambda_i^{(2)} = \lambda = 0.0083$, and the Copula correlation coefficients of the two states are all equal, respectively; let $\rho_i = \rho = 0.12$, $\eta_i = \eta = 0.21$, and the probability of state 1 be $p = 0.3$. (Note: in the practical application, corresponding values of ρ_i and η_i are just substituted into equations (6) and (7), respectively; the default intensities $\lambda_i^{(1)}$ and $\lambda_i^{(2)}$ are just substituted into equations (13) and (14), respectively; and other steps in Section 3 are all the same.)

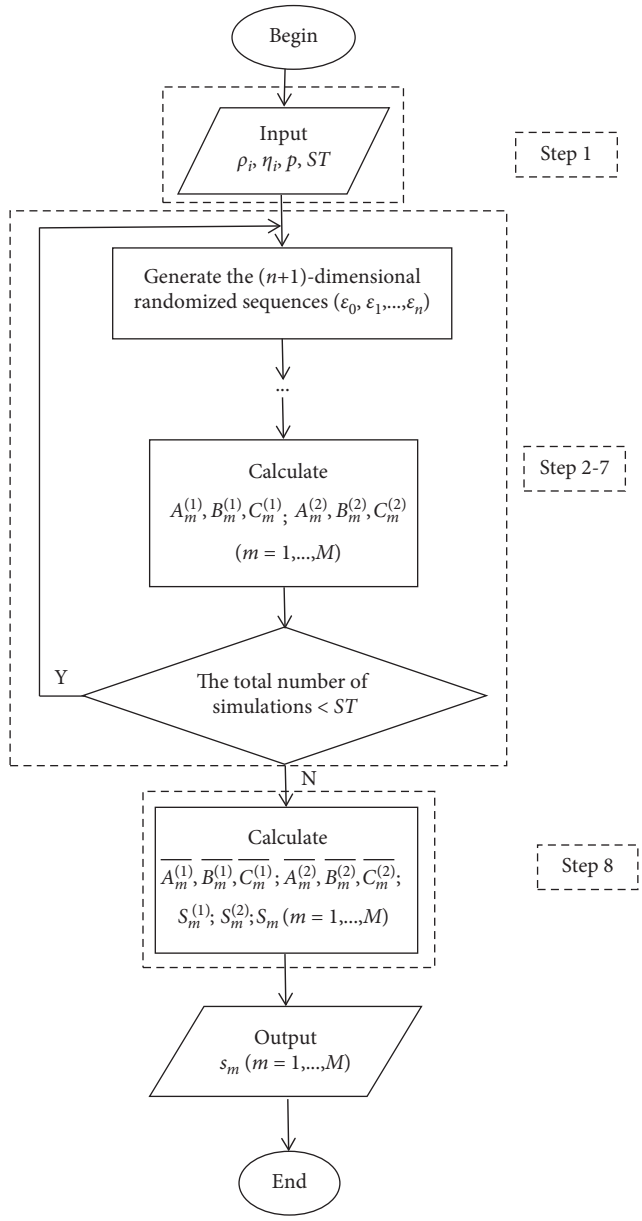

Figure 2: Algorithm flow chart (concise format).

4.2. Result Analysis. In order to investigate the stability of simulation results of the RQMC method and MC method, we calculate the variances of the two simulation methods under different simulation times and observe their changes for each CDO tranche, respectively. We set the variation range of simulation times from 5000 to 50000, with the step size of 5000. For each different simulation times, we all simulate 40 times and then calculate the corresponding variances of MC and RQMC of each CDO tranche. The variance changes of the simulation results of the two

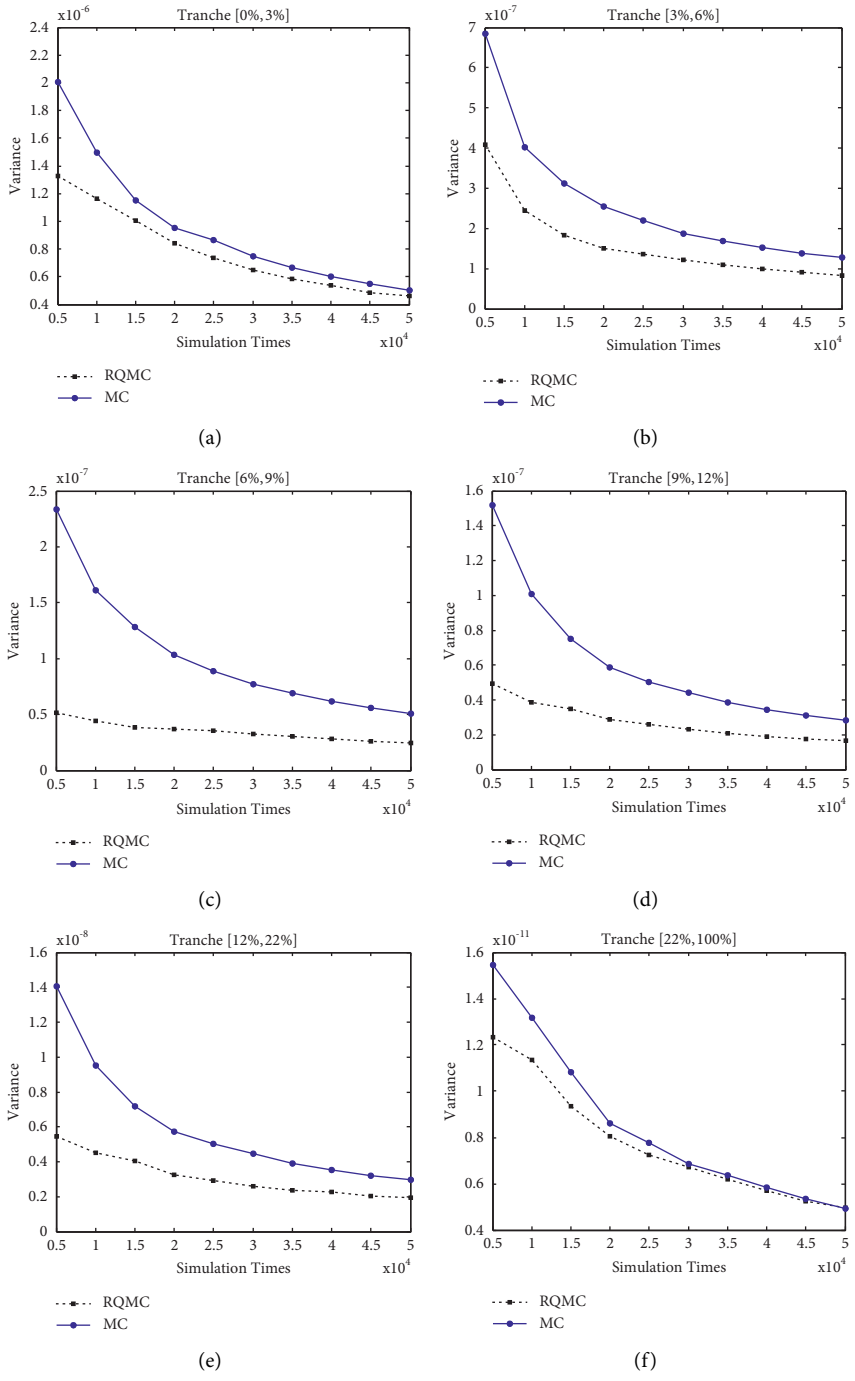

Figure 3: Variance changes of CDO pricing in each tranche with the increase in simulation times.

simulation methods for the pricing of each CDO tranche with the increase in simulation times are shown in Figure 3.

On the whole, we can see from Figure 3 that the variances of the two simulation methods in six different tranches are all decreased with the increase in simulation times, and in each subgraph, the gaps between the two curves all reduce gradually with the increase in simulation times. These show that the results of the two simulation methods are all gradually stable with the increase in simulation times. It can also be seen from Figure 3 that, compared with the MC method, the variance changes of each CDO tranche of the RQMC method are more gentle with the increase in simulation times. Especially, when the simulation time is relatively smaller, with the increase in simulation times, the variances of MC all decrease faster than those of RQMC in each CDO tranche. This indicates that the simulation results of the RQMC method are more stable than those of MC even when the simulation time is not too large. In other words, the RQMC method can obtain more stable results with fewer simulation times.

Table 2: Comparison of simulation times required of MC and RQMC at the same variance (taking 50000 times of simulation of MC as an example).

Tranches	Variance	Simulation times required (RQMC)	Simulation times required (MC)
[0%–3%]	$5.042e-07$	43860	50000
[3%–6%]	$1.273e-07$	28030	
[6%–9%]	$5.11e-08$	6054	
[9%–12%]	$2.833e-8$	21400	
[12%–22%]	$2.95e-9$	25170	
[22%–100%]	$4.923e-12$	49998	

From the local point of view on each tranche diagram, when the simulation times are the same, the variances of the RQMC method are much smaller than those of the MC method, which indicates that the simulation results of the RQMC method are more stable than those of the MC method when the simulation times are the same.

Particularly, in order to compare the simulation times required under the same variance of two simulation methods, we carry out spline interpolations for the variances data of the RQMC method for each subgraph in Figure 3. Without loss of generality, in each subgraph of Figure 3, we take the variances corresponding to 50000 simulation times of the MC method as an example to illustrate. At this time, the simulation times required by the RQMC method are shown in Table 2.

We can see from Table 2, for the same variances, when the total number of simulations of the MC method required is 50000, in addition to the tranche [22%–100%], simulation times of the other five tranches of the RQMC method are all far lower than 50000. Furthermore, the RQMC method can save about 25000 times on average and reduce the cost of the program running effectively.

5. Conclusion and Discussion

In this paper, a numerical algorithm based on the RQMC simulation method was designed and an empirical study was carried out to solve CDO pricing with the stochastic correlation problem of the two states under nonhomogeneous assumptions. The variances of RQMC simulation results were compared with those of the MC method from the perspective of variance changes. The results showed that the algorithm designed in this paper is more stable and reliable whether from the local view or from the global view, and it is an efficient and stable algorithm to solve CDO pricing with stochastic correlation problems under nonhomogeneous conditions, which would provide theoretical and empirical support for solving similar problems.

The algorithm in this paper is only empirically analyzed based on the Gaussian Copula model with stochastic correlations under nonhomogeneous assumptions. We point out that the algorithm can be generalized. For example, under the framework of the algorithm, the Gaussian Copula model in the algorithm can be replaced with other single-factor Copula models and the algorithm still works. Of course, for different single-factor Copula models, the implementation of the algorithm may be more complex.

In addition, the algorithm can also be extended to CDO pricing with three-state stochastic correlations under nonhomogeneous assumptions.

In the following study, we will further optimize the algorithm to improve its versatility. It is also considered to apply the numerical algorithm to the pricing of other assets in credit derivative markets under nonhomogeneity assumptions.

Data Availability

The data for the empirical study are randomly generated by MATALB code according to the algorithm in this paper. If readers need it, please contact the corresponding author for discussion.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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