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Research Article

Uncertainty Measure for Multisource Intuitionistic Fuzzy Information System

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Multisource information systems and multigranulation intuitionistic fuzzy rough sets are important extended types of Pawlak's classical rough set model. Multigranulation intuitionistic fuzzy rough sets have been investigated in depth in recent years. However, few studies have considered this combination of multisource information systems and intuitionistic fuzzy rough sets. In this paper, we give the uncertainty measure for multisource intuitionistic fuzzy information system. Against the background of multisource intuitionistic fuzzy information system, each information source is regarded as a granularity level. Considering the different importance of information sources, we assign different weights to them. Firstly, the paper proposes an optimal source selection method. Secondly, we study the weighted generalized, weighted optimistic, and weighted pessimistic multigranularity intuitionistic fuzzy rough set models and uncertainty measurement methods in the multisource intuitionistic fuzzy information system, and we further study the relationship between the three models and related properties. Finally, an example is given to verify the validity of the models and methods.

1. Introduction

Rough set theory [1] was proposed by Polish mathematician Pawlak in 1982. It is an effective mathematical tool to analyze and process inaccurate data and uncertain information. Rough set theory has received more and more attention in recent years. It is widely used in many fields such as natural science, social science, and engineering technology [2–11]. Uncertainty measurement [12, 13] is one of the important research contents in rough set theory. It can measure the dependency and similarity between attributes and provide an effective measurement tool for attribute reduction and cluster analysis [14]. Traditional uncertainty measurement considers single-source information system. With the advent of the era of big data, it is necessary to study uncertainty measurement methods in multisource intuitionistic fuzzy information system [15].

In current study, by extending the equivalence relationship to a general binary relationship, Qian et al. extended the single-granularity rough set model to multigranularity structure [16, 17]. By combining fuzzy set theory and rough set theory, a fuzzy rough set model [18] and rough fuzzy set model [19, 20] are obtained. Literature [21] extends the fuzzy set theory to the intuitionistic fuzzy set theory and extends the relationship between elements and sets from membership degree to nonmembership degree and hesitation degree. Therefore, the intuitionistic fuzzy rough set theory is a very effective mathematical tool when analyzing and processing inaccurate, incomplete, and other rough information, and the result accuracy is significantly improved. Literature [22, 23] combined rough set theory with intuitionistic fuzzy set theory and established an intuitionistic fuzzy rough set model. Literature [24] investigated the upper approximation reduction problem of intuitionistic fuzzy information system based on dominant relationship. Literature [25, 26] extended the single-granularity intuitionistic fuzzy rough set model to the multigranularity intuitionistic fuzzy rough set model and presented the optimistic multigranularity and pessimistic multigranularity intuitionistic fuzzy rough set models under the dominant relationship. Literature [27] proposes the intuitionistic fuzzy soft set, which is an effective tool for solving multiple attribute decision-making with intuitionistic fuzzy information. Literature [28] proposes the hesitant fuzzy sets and studies their relationship with intuitionistic fuzzy sets.

In the research of multigranularity intuitionistic fuzzy rough set model, if all granularity levels are considered equally important, optimistic multigranularity only requires one granularity knowledge and target concept to meet inclusion relation, while pessimistic multigranularity requires all granularity knowledge and target cocept to meet inclusion relation, ultimately leading to inaccurate decision-making results. The importance of granulation is usually different for decision-making in real life. Taking various factors into account, some granularity is very important, so we give it a larger weight, but some granularity is not important, so we give it a smaller weight. Considering the importance of different granularity levels, literature [29] proposed a weighted multigranularity intuitionistic fuzzy rough set model.

Now, in our real life, we no longer face a single-source information system, but a multisource information system. Multisource information systems [30] are used to represent information that comes from multiple sources. Literature [31] proposed a fuzzy multigranulation decision-theoretic rough set model in multisource fuzzy information systems. Literature [32] investigated the attribute reduction in multisource decision systems. Literature [33] combined rough set model and multisource decision systems and established a decision-theoretic rough set model of multisource decision systems. Literature [34] built the information source selection criteria and proposed some principles of information fusion. Literature [35–42] proposed information fusion methods of multisource information system. In this paper, we treat each single information system as a granular structure. As the number of information sources increases, a large amount of data is unreliable. Therefore, we propose a corresponding algorithm to select reliable information sources and one that can greatly improve the efficiency of information processing. Up to now, few scholars have combined the rough set model with the multisource intuitionistic fuzzy information system. Different weights are given to the granularity; the weighted multigranularity intuitionistic fuzzy rough set models and the uncertainty measurement methods for multisource intuitionistic fuzzy information system are proposed. This is the purpose of this article.

The rest of this paper is organized as follows. In Section 2, we mainly review the relevant concepts and properties of intuitionistic fuzzy rough sets, multigranulation intuitionistic fuzzy rough sets, and multisource intuitionistic fuzzy information system. In Section 3, we first study the optimal source selection of multisource intuitionistic fuzzy information system. Further, the weighted multigranulation intuitionistic fuzzy rough set model and its related properties of multisource intuitionistic fuzzy information system are researched. In Section 4, we give the uncertainty measurement methods for the weighted multigranulation intuitionistic fuzzy rough set model of multisource intuitionistic fuzzy information system. At the same time, we verify the effectiveness of the proposed models and methods through a specific example. Section 5 uses a numerical experiment to verify the effectiveness of the proposed methods. In Section 6, we present the conclusion and the future work.

2. Preliminaries

This section mainly reviews related concepts and properties of intuitionistic fuzzy rough set models, multigranulation intuitionistic fuzzy rough set models, and multisource intuitionistic fuzzy information system.

2.1. Intuitionistic Fuzzy Rough Set Model

Definition 1 (see [21]): Given the universe of discourse U, an intuitionistic fuzzy rough set A on U is defined by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in U\}$, where the functions $\mu_A(x)\colon \mu \longrightarrow [0,1]$ and $\nu_A(x)\colon \nu \longrightarrow [0,1]$ satisfy $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in U$. $\mu_A(x)$ and $\nu_A(x)$ are called the degrees of membership and nonmembership of element $x \in U$ to A, respectively. The family of all intuitionistic fuzzy sets in U is denoted by $IF(U)$. When $\mu_A(x) + \nu_A(x) = 1$, A degenerates into a fuzzy set.

Definition 2 (see [21]): Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in U\} \in IF(U)$, $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in U\} \in IF(U)$; then:

(1) The supplementary set of A $\sim A = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in U\}$.
(2) $A = B \Leftrightarrow \mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \forall x \in U$.
(3) $A \subseteq B \Leftrightarrow \mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \forall x \in U$.
(4) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle | x \in U\}$.
(5) $A \cup B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle | x \in U\}$.
(6) $A - B = A \cap (\sim B)$.
(7) $A \triangle B = (A \cap B^c) \cup (B \cap A^c)$.

Definition 3 (see [21]): Let $\mathrm{IFIS} = \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, where $U = \{x_1, x_2 \ldots x_n\}$ is a nonempty finite set of objects called the universe of discourse; $AT = \{a_1, a_2 \ldots a_n\}$ is a nonempty finite attribute set; $V = \cup_{a \in AT} V_a$ is the attribute value set; $f\colon U \times AT \longrightarrow V$ is the information function, $A \subseteq AT$; a binary dominant relation R_A is defined as follows:

$$\begin{aligned} R_A &= \left\{\left(x_i, x_j\right) \in U \times U | f\left(x_i, a\right) \le f\left(x_j, a\right), \forall a \in A\right\} \\ &= \left\{\left(x_i, x_j\right) \in U \times U | \mu_a\left(x_i\right) \le \mu_a\left(x_j\right) \wedge \nu_a\left(x_i\right) \ge \nu_a\left(x_j\right), \forall a \in A\right\}. \end{aligned} \tag{1}$$

Obviously, this binary dominant relation satisfies reflexivity, antisymmetry, and transitivity, and $R_A = \cap R_{a \in A}$. Based on the above dominant relation, the dominant class $R_A(x_i)$ definition of object x_i can be obtained as follows:

$$R_A(x_i) = \{x_j \in U:\ f(x_i, a) \le f(x_j, a), \forall a \in A\}. \quad (2)$$

Definition 4 (see [22]): Let $\mathrm{IFIS} = \{U, AT, V, f\}$ be an intuitionistic fuzzy information system; for any $X \subseteq U, A \subseteq AT$, the lower approximation and upper approximation of X for R_A are defined as follows:

$$\begin{aligned} \underline{R_A}(X) &= \{x \in U \mid R_A(x_i) \subseteq X\}; \\ \overline{R_A}(X) &= \{x \in U \mid R_A(x_i) \cap X \ne \varnothing\}. \end{aligned} \quad (3)$$

By definition, $\underline{R_A}(X)$ consists of all objects that are definitely contained in the set X. $\overline{R_A}(X)$ consists of all objects that are possibly contained in the set X. If $\underline{R_A}(X) = \overline{R_A}(X)$, then X is the exact set of R_A; otherwise, it is the rough set about R_A.The positive, negative, and boundary regions of X can be defined as follows:

$$\begin{aligned} \mathrm{POS}_{R_A}(X) &= \underline{R_A}(X), \\ NEG_{R_A}(X) &= u - \overline{R_A}(X), \\ BND_{R_A}(X) &= \overline{R_A}(X) - \underline{R_A}(X). \end{aligned} \quad (4)$$

Definition 5 (see [22]): Let $\mathrm{IFIS} = \{U, AT, V, f\}$ be an intuitionistic fuzzy information system; for any $X \subseteq U, A \subseteq AT$, the approximate accuracy and the roughness of the target set X are defined as follows:

$$\begin{aligned} \alpha(R_A, X) &= \frac{|\underline{R_A}(X)|}{|\overline{R_A}(X)|}; \\ \rho(R_A, X) &= 1 - = \frac{|\underline{R_A}(X)|}{|\overline{R_A}(X)|}, \end{aligned} \quad (5)$$

when $|\underline{R_A}(X)| = 0$, $\alpha(R_A, X) = 0, \rho(R_A, X) = 1$.

Definition 6 (see [23]): Let $A = \{\langle x, \mu_A(x)\rangle | x \in U\}$ be a fuzzy sets on universe U; the nonmembership degree of A is defined as follows:

$$\nu_A(x) = \begin{cases} 0, & \mu_A(x) > 0.5, \\ 0.5, & \mu_A(x) \le 0.5. \end{cases} \quad (6)$$

Then, $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in U\}$ is the correspondence intuitionistic fuzzy sets of the fuzzy set A.

2.2. Multigranulation Intuitionistic Fuzzy Rough Set Model. Multigranulation rough set model was first proposed by Qian et al. It is an extension of the classical rough set theory. In this subsection, the single-granulation intuitionistic fuzzy rough set is extended to the multigranulation intuitionistic fuzzy rough set model under multiple dominant relations. Generalized multigranulation intuitionistic fuzzy rough set is the generalization of optimistic multigranulation intuitionistic fuzzy rough set and pessimistic multigranulation intuitionistic fuzzy rough set.

Given the definition of the support characteristic function, we use this function to complete the object selection.

Definition 7 (see [26]): Let $\mathrm{IFIS} = \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, $A_i \subseteq AT$, $i = 1, 2, \ldots m\,(m \le 2^{|AT|})$; for any $X \subseteq U$, the support characteristic function of x for X is denoted as

$$S_X^{A_i}(x) = \begin{cases} 1, & R_{A_i}(x) \subseteq X, \\ 0, & \text{else}. \end{cases} \quad (7)$$

The support characteristic function is used to describe the inclusion relation between dominance class $R_{A_i}(x)$ and concept X, Which indicates whether object x accurately supports X by A_i, or whether object x has a positive description of X by A_i.

Definition 8 (see [26]): Let $\mathrm{IFIS} = \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, $A_i \subseteq AT$, $i = 1, 2, \ldots m$ $(m \le 2^{|AT|})$; $S_X^{A_i}(x)$ is the characteristic function of x for X. Given information level $\beta \in (0.5, 1]$, for any $X \subseteq U$, the generalized lower and upper approximation of X are defined as follows:

$$\begin{aligned} \underline{GM}_{\sum_{i=1}^m A_i}(X)_\beta &= \left\{x \in U | \frac{\sum_{i=1}^m S_X^{A_i}(x)}{m} \ge \beta\right\} \\ \underline{GM}_{\sum_{i=1}^m A_i}(X)_\beta &= \left\{x \in U | \frac{\sum_{i=1}^m 1 - S_{X^c}^{A_i}(x)}{m} > 1 - \beta\right\}. \end{aligned} \quad (8)$$

If $\underline{GM}_{\sum_{I=1}^m A_i}(X)_\beta = \overline{GM}_{\sum_{I=1}^m A_i}(X)_\beta$, then the target set X is generalized and definable; otherwise, it is generalized and rough. $(\underline{GM}_{\sum_{I=1}^m A_i}(X)_\beta = \overline{GM}_{\sum_{I=1}^m A_i}(X)_\beta)$ is called the generalized multigranulation intuitionistic fuzzy rough set model (GMIFRS).

Definition 9 (see [26]): Let $\mathrm{IFIS} = \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, $A_i \subseteq AT$, $i = 1, 2, \ldots m$ $(m \le 2^{|AT|})$; $S_X^{A_i}(x)$ is the support characteristic function of x for X; for any $X \subseteq U$, the pessimistic lower and upper approximation of X are defined as follows:

$$\underline{PM}_{\sum_{i=1}^{m} A_i}(X) = \left\{x \in U | \wedge_{i=1}^{m}\left(R_{A_i}(x) \subseteq X\right)\right\} = \left\{x \in U | \frac{\sum_{i=1}^{m} S_X^{A_i}(x)}{m} \geq 1\right\},$$
$$\underline{PM}_{\sum_{i=1}^{m} A_i}(X) == \left\{x \in U | \vee_{i=1}^{m}\left(R_{A_i}(x) \cap X \neq \varnothing\right)\right\} = \left\{x \in U | \frac{\sum_{i=1}^{m}\left(1 - S_{X^c}^{A_i}(x)\right)}{m} > 0\right\}, \quad (9)$$

where "∨" denotes "or" and "∧" denotes "and."

If $\underline{PM}_{\sum_{I=1}^{M} A_i}(X) = \overline{PM}_{\sum_{I=1}^{M} A_i}(X)$, then the target set X is pessimistic and definable; otherwise, X is pessimistic and rough. ($\underline{PM}_{\sum_{I=1}^{M} A_i}(X) = \overline{PM}_{\sum_{I=1}^{M} A_i}(X)$) is called the pessimistic multigranulation intuitionistic fuzzy rough set model (PMIFRS).

Definition 10 (see [26]): Let IFIS $= \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, $A_i \subseteq AT$, $i = 1, 2, \ldots m$ $(m \leq 2^{|AT|})$; $S_X^{A_i}(x)$ is the support characteristic function of x for X; for any $X \subseteq U$, the optimistic lower and upper approximation of X are defined as follows:

$$\underline{OM}_{\sum_{i=1}^{m} A_i}(X) = \left\{x \in U | \wedge_{i=1}^{m}\left(R_{A_i}(x) \subseteq X\right)\right\} = \left\{x \in U | \frac{\sum_{i=1}^{m} S_X^{A_i}(x)}{m} > 0\right\},$$
$$\overline{OM}_{\sum_{i=1}^{m} A_i}(X) == \left\{x \in U | \vee_{i=1}^{m}\left(R_{A_i}(x) \cap X \neq \varnothing\right)\right\} = \left\{x \in U | \frac{\sum_{i=1}^{m}\left(1 - S_{X^c}^{A_i}(x)\right)}{m} > 1\right\}, \quad (10)$$

where "∨" denotes "or" and "∧" denotes "and."

If $\underline{OM}_{\sum_{I=1}^{M} A_i}(X) = \overline{OM}_{\sum_{I=1}^{M} A_i}(X)$, then the target set X is optimistic and definable; otherwise, X is optimistic and rough. ($\underline{OM}_{\sum_{I=1}^{M} A_i}(X) = \overline{OM}_{\sum_{I=1}^{M} A_i}(X)$) is called the optimistic multigranulation intuitionistic fuzzy rough set model (OMIFRS).

2.3. Attribute Reduction of Intuitionistic Fuzzy Information System Based on Dominant Relation. In this subsection, we define the attribute importance of intuitionistic fuzzy information system based on the dominant relation. We also give the attribute reduction algorithm of intuitionistic fuzzy information system based on knowledge rough entropy.

Definition 11 (see [9]): Let IFIS $= \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, $A \subseteq AT$, $U/R_A = \{R_A(x_i) | x_i \in U\}$; R_A is a binary dominant relation; the rough entropy of A is defined as

$$Er(A) = \sum_{i=1}^{|U|} \frac{\left|R_A(x_i)\right|}{|U|} \log_2\left|R_A(x_i)\right|. \quad (11)$$

It is obvious that there are minimum and maximum values of the rough entropy. When R_A is the finest classification, the rough entropy of A has a minimum value of 0; if R_A is the coarsest classification, the rough entropy of A has a maximum value of $|U| \log_2 |U|$.

Each information system has many attributes, but some of these are redundant. To measure the significance of a single attribute, [3, 4] give the concepts of relative importance and absolute importance of attribute.

Definition 12 Let IFIS $= \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, $A \subseteq AT$; for any $a \in A$, for any $b \in (AT - A)$, the relative importance and absolute importance of attribute a in attribute set A is defined as

$$\text{sig}_{in}(a, A) = Er(A - \{a\}) - Er(A);$$
$$\text{sig}_{in}(b, A) = Er(A) - Er(A \cup \{b\}). \quad (12)$$

According to this definition, the following properties are established:

(1) $0 \leq \text{sig}_{\text{in}}(a, A) \leq \log_2 |U|$.

(2) Attribute a is necessary $\Leftrightarrow \text{sig}_{in}(a, A) > 0$.

(3) The attribute core of A is $\text{core}(A) = \{a \in A | \text{sig}_{\text{in}}(a, A) > 0\}$.

Definition 13 Let IFIS $= \{U, AT, V, f\}$ be an intuitionistic fuzzy information system, $A \subseteq AT$; if $Er(A) = Er(AT)$, for any $a \in A$ and $Er_{(A-\{a\})} \neq Er_{AT}$, A is the attribute reduction of intuitionistic fuzzy information system based on dominance relation.

Since the attribute core is a subset of attribute reduction, in the heuristic reduction process starting from the attribute core, we often add attributes to attribute core through a measurement method to obtain attribute reduction. Next, an attribute reduction algorithm for intuitionistic fuzzy information system based on knowledge rough entropy is given.

Example 1. Table 1 shows an intuitionistic fuzzy information system IFIS $= \{U, AT, V, f\}$, where the universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and attribute set $AT = \{a_1, a_2, a_3, a_4\}$. We use Algorithm 1 to calculate the minimum attribute

Input: IFIS = $\{U, AT, V, f\}$, $AT = \{a_1, a_2, \ldots, a_n\}$.
Output: The minimum attribute reduction $\mathrm{Red}(AT)$.
begin
  set $\mathrm{core}(AT) \leftarrow \emptyset$; $\mathrm{Red}(AT) \leftarrow \emptyset$; $A_0 \leftarrow \emptyset$;
  for $a \in AT$ do
    $\mathrm{sig}_{\mathrm{in}}(a, A) = Er(AT - \{a\}) - Er(AT)$;
      if $\mathrm{sig}_{\mathrm{in}}(a, A) > 0$ then
        $\mathrm{core}(AT) = \mathrm{core}(AT) \cup \{a\}$;
      end
  end
  set $A_0 \leftarrow \mathrm{core}(AT)$
  while $Er(A_0) \neq Er(A_T)$ do
    for $a* \in AT/A_0$
      $\mathrm{sig}_{\mathrm{out}}(a^*, A_0) = Er(A_0) - Er(A_0 \cup a^*)$;
      $a^{**} = \max\{\mathrm{sig}_{\mathrm{out}}(a^*, A_0)\}$;
    set $A_1 \leftarrow a^{**}$
    end
  set $A_0 \leftarrow A_0 \cup A_1$
  end
return: $\mathrm{Red}(AT) = A_0$
end

Algorithm 1: The minimum attribute reduction of intuitionistic fuzzy information system.

Table 1: Intuitionistic fuzzy information system.

U	a_1	a_2	a_3	a_4
x_1	(0.3, 0.5)	(0.6, 0.4)	(0.5, 0.2)	(0.7, 0.1)
x_2	(0.2, 0.7)	(0.1, 0.8)	(0.4, 0.5)	(0.1, 0.8)
x_3	(0.2, 0.7)	(0.1, 0.8)	(0.4, 0.5)	(0.7, 0.1)
x_4	(0.1, 0.8)	(0.1, 0.8)	(0.2, 0.7)	(0.1, 0.8)
x_5	(0.9, 0.1)	(0.8, 0.1)	(0.8, 0.1)	(0.9, 0.0)
x_6	(0.4, 0.6)	(0.7, 0.3)	(0.6, 0.3)	(0.7, 0.1)

reduction of the intuitionistic fuzzy information system given in Table 1.

From this table, it is easy to calculate the granular structure:

$U/AT = \{\{x_1, x_5\}, \{x_1, x_2, x_3, x_5, x_6\}, \{x_1, x_3, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_5\}, \{x_5, x_6\}\}$.

According to Definition 11, the rough entropy of AT is as follows:

$Er(AT) = 2/6\log_2 2 + 5/6\log_2 5 + 4/6\log_2 4 + 6/6\log_2 6 + 1/6\log_2 1 + 2/6\log_2 2 \approx 6.520$.

After calculating the core of attribute set AT,

$U/(AT - \{a_1\}) = U/(AT - \{a_2\}) = U/(AT - \{a_3\}) = U/AT$, $U/(AT - \{a_4\}) = \{\{x_1, x_5\}, \{x_1, x_2, x_3, x_5, x_6\}, \{x_1, x_3, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_5\}, \{x_5, x_6\}\}$.

Hence, $Er(AT - \{a_1\}) = Er(AT - \{a_2\}) = Er(AT - \{a_3\}) = Er(AT) \approx 6.520$, $Er(AT - a_4) = 2/6\log_2 2 + 5/6\log_2 5 + 4/6\log_2 4 + 6/6\log_2 6 + 1/6\log_2 1 + 2/6\log_2 2 \approx 7.122$.

$\mathrm{sig}_{\mathrm{in}}(a_1, AT) = \mathrm{sig}_{\mathrm{in}}(a_2, AT) = \mathrm{sig}_{\mathrm{in}}(a_3, AT) = 0$, $\mathrm{sig}_{\mathrm{in}}(a_4, AT) = 0.602 > 0$.

Accordingly, $\mathrm{core}(AT) = \{a_4\}$. Let $A_0 = \{a_4\}$.

$U/(\{a_4\}) = \{\{x_1, x_5\}, \{x_1, x_2, x_3, x_5, x_6\}, \{x_1, x_3, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}\}, \{\{x_5\}, \{x_1, x_3, x_5, x_6\}\}$.

Therefore, $Er(\{a_4\}) \approx 9.170$, $Er(\{a_4\}) \approx Er(AT)$.

$A/A_0 = \{a_1, a_2, a_3\}$; for any $a_i \in A/A_0\,(i = 1, 2, 3)$, we calculate $Er(A_0 \cup \{a_i\})$ as follows:

$Er(A_0 \cup \{a_1\}) = Er(\{a_1, a_4\}) \approx 6.520$,

$Er(A_0 \cup \{a_2\}) = Er(\{a_2, a_4\}) \approx 7.629$,

$Er(A_0 \cup \{a_3\}) = Er(\{a_3, a_4\}) \approx 6.520$.

Thus, $Er(A_0 \cup \{a_1\}) = Er(A_0 \cup \{a_3\}) < Er(A_0 \cup \{a_2\})$.

Then, let $A_0 = \{a_1, a_4\}$ or $A_0 = \{a_3, a_4\}$.

Similarly, by calculation, we know that $Er(\{a_1, a_4\}) = Er(AT)$, $Er(\{a_3, a_4\}) = Er(AT)$.

Therefore, the minimal attribute reduction of AT is $\mathrm{Red}(AT) = \{a_1, a_4\}$ or $\{a_3, a_4\}$.

2.4. Multisource Intuitionistic Fuzzy Information System (MSIFIS)

2.4. Multisource Intuitionistic Fuzzy Information System (MSIFIS). In this subsection, we introduce multisource intuitionistic fuzzy information systems. When a person obtains information about a group of objects from different sources, each source can be regarded as a classical information system, which is an attribute with some intuitionistic fuzzy attribute values. The information system is called multisource intuitionistic fuzzy information system (MSIFIS). A MSIFIS is shown in Figure 1.

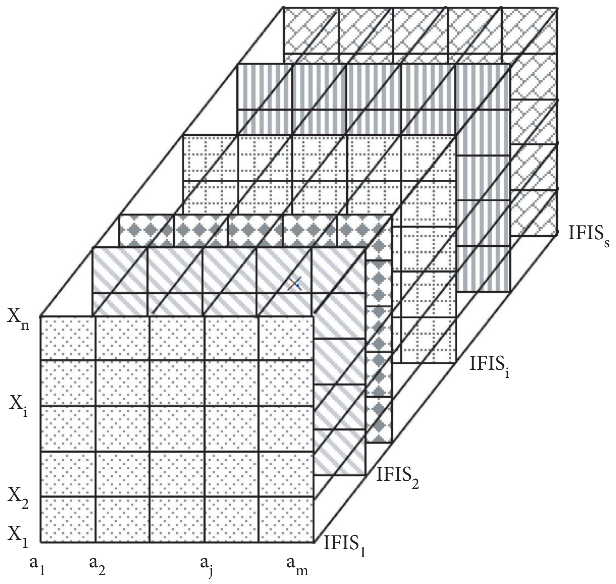

FIGURE 1: A multisource intuitionistic fuzzy information system.

This paper mainly discusses the case which shares the same structure. That is, the same object, attributes, and the value of the object's attributes have the same digital characteristics under different information sources.

Definition 14 (see [30]): Let $\text{MSIFIS} = \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system, where:

(1) U is a set of nonempty finite objects called the universe.

(2) A_i is a finite nonempty set of the attributes of each subsystem.

(3) $V_i = \{(V_a)_{a\in A_i}\}$, V_a is the value range of attribute a.

(4) f_i represents the corresponding relation between the object and the feature under the information source i.

Let $\text{MSIFDIS} = \{IFDIS_1, IFDIS_2, \ldots, IFDIS_n\}$ be a multisource intuitionistic fuzzy decision information system, where $IFDIS_i = \{U, A_i \cup D, V_i, f_i\}$ $(i \leq n)$ and D is the decision attribute.

3. Weighted Multigranulation Intuitionistic Fuzzy Rough Set Model for MSIFIS

In the theory of multigranulation intuitionistic fuzzy rough set, optimistic multigranulation only requires the existence of the granular knowledge and the target concept to satisfy the inclusion relation; the requirements are too loose for approximate characterization. However, the pessimistic multigranularity requires that the knowledge granule and the target concept satisfy the inclusion relation at all granularity spaces; the requirements are too strict for approximate characterization. These multigranularity models treat each granularity space equally.

However, in practical applications, considering the application background and user preferences, the granularity spaces are not equally important. Therefore, in this section, we study multigranularity intuitionistic fuzzy rough set with weights in multisource intuitionistic fuzzy information system.

3.1. Optimal Source Selection for MSIFIS. MSIFISs are used to express information from multiple sources. As the number of information sources increases, the selection of reliable information sources is a key issue in the field of information technology research. To characterize the effectiveness of an information source, we define the two source quality metrics of the internal-confidence degree and external-confidence degree.

Definition 15. Let $\text{MSIFIS} = \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system. For each single information source $IFIS_i \in \text{MSIFIS}$, let $\text{Red}(AT_i)$ be the reduction of $IFIS_i$. The internal-confidence degree of $IFIS_i$ can be defined as follows:

$$IC(IFIS_i) = \frac{|\text{Red}(A_i)|}{|A_i|}. \tag{13}$$

From the above definition, the following is obvious:

(1) $IC(IFIS_i)$ is the ratio of the cardinalities of $|\text{Red}(AT_i)|$ and $|AT_i|$, and $0 \leq IC(IFIS_i) \leq 1$.

(2) If $IC(IFIS_i) > 0.5$, the majority of the attributes are useful and the source is reliable. In practical applications, different thresholds $IC(IFIS_i) > \alpha$ can be defined in different fields according to the specific requirements.

Definition 16. Let $\text{MSIFIS} = \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system; $\forall IFIS_i, IFIS_j \in \text{MSIFIS}$, the difference between them can be defined as follows:

$$D\left(IFIS_i, IFIS_j\right) = \sum_{k=1}^{|U|}\left(\left|R_{A_i}(x_k) \cup R_{A_j}(x_k)\right| - \left|R_{A_i}(x_k) \cap R_{A_j}(x_k)\right|\right), \tag{14}$$

where $R_{AT_i}(x_k)$ is the dominant class of x_k with respect to R_{A_i}.

According to this definition, the following is obvious:

(1) $\forall IFIS_i, IFIS_j \in \text{MSIFIS}, D(IFIS_i, IFIS_j) \geq 0$.

(2) $\forall IFIS_i, IFIS_j \in \text{MSIFIS}$, $D(IFIS_i, IFIS_j) = D(IFIS_j, IFIS_i)$.

(3) When the single information sources $IFIS_i$ and $IFIS_j$ have the same granular structure, the difference between $IFIS_i$ and $IFIS_j$ has a minimum of 0. If the single information source $IFIS_i$ has the finest granular structure and $IFIS_j$ has the coarsest structure, then the difference between $IFIS_i$ and $IFIS_j$ reaches a maximum of $|U|(|U| - 1) = |U|^2 - |U|$.

Definition 17. Let MSIFIS $= \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system; $\forall IFIS_i, IFIS_j \in$ MSIFIS, the external correlation between $IFIS_i$ and $IFIS_j$ can be defined as

$$ec\left(IFIS_i, IFIS_j\right) = 1 - \frac{D\left(IFIS_i, IFIS_j\right)}{|U|^2 - |U|}, \quad (15)$$

where $D(IFIS_i, IFIS_j)$ is the difference between $IFIS_i$ and $IFIS_j$.

From this definition, the following is obvious:

To clarify the relationship between the external-confidence degrees of $IFIS_i$ and $IFIS_j$, we can construct an external-confidence degree matrix M_{EC} as follows:

$$M_{EC} = \begin{pmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & e_{nn} \end{pmatrix}, \quad (16)$$

where $e_{ij} = ec(IFIS_i, IFIS_j)$.

Definition 18. Let MSIFIS $= \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system; $\forall IFIS_i, IFIS_j \in$ MSIFIS, the external-confidence degree between $IFIS_i$ and $IFIS_j$ can be defined as

$$EC(IFIS_i) = \frac{1}{n}\sum_{j=1}^{n}\left(1 - \frac{D\left(IFIS_i, IFIS_j\right)}{|U|^2 - |U|}\right) = \frac{1}{n}\sum_{j=1}^{n} ec\left(IFIS_i, IFIS_j\right). \quad (17)$$

From this definition, the following is obvious:

(1) For any $IFIS_i \in MS$, $0 \le EC(IFIS_i) \le 1$.

(2) Similar to the internal-confidence degree, different thresholds $EC(IFIS_i) > \beta$ may be applicable in different fields according to specific requirements.

Definition 19. Let MSIFIS $= \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system; $\forall IFIS_i \in$ MSIFIS, we define a total score for information source IFIS$_i$ as follows:

$$TC(IFIS_i) = IC(IFIS_i) + EC(IFIS_i). \quad (18)$$

where $IC(IFIS_i)$ is the internal-confidence degree of a source $IFIS_i$, and $EC(IFIS_i)$ is the external-confidence degree of a source $IFIS_i$.

Example 2. Table 2 shows a multisource intuitionistic fuzzy information system, which consists of three intuitionistic fuzzy information tables; $x_i \in U(i = 1, 2, 3, 4, 5, 6)$ represents six evaluated objects, and $a_i \in A(i = 1, 2, 3, 4|)$ represents conditional attribute set.

From this table, it is easy to calculate the granular structure for each information source:

$U/AT_1 = \{\{x_1, x_5\}, \{x_1, \quad x_2, x_3, x_5, x_6\}, \{x_1, x_3, x_5, x_6\}, \{x_1, x_2, \quad x_3, x_4, x_5, x_6\}, \{x_5\}, \{x_5, x_6\}\}$, $U/AT_2 = \{\{x_1, x_2\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_2, \quad x_3, x_4, x_5\}, \{x_2, x_3, x_4, x_6\}\}$, $U/AT_3 = \{\{x_1, x_3\}, \{x_2\}, \{x_3\}, \{x_2, x_3, x_4\}, \{x_3, x_5\}, \{x_2, x_3, x_6\}\}$.

From the result of example 1, $\text{Red}(AT_1) = \{a_1, a_4\}$ or $\{a_3, a_4\}$.

Similarly, $\text{Red}(AT_2) = \{a_2, a_3, a_4\}$, and $\text{Red}(AT_3) = \{a_1, a_2, a_4\}$ or $\{a_1, a_3, a_4\}$.The calculation results of all sources $IFIS_i (i = 1, 2, 3)$ are presented in Table 3.

Then, the internal-confidence degree of source $IFIS_i (i = 1, 2, 3)$ is as follows:

$IC(IFIS_1) = (|\text{Red}(AT_1)|/|AT_1|) = (2/4) = 0.5$,
$IC(IFIS_2) = (|\text{Red}(AT_2)|/|AT_2|) = (3/4) = 0.75$,
$IC(IFIS_3) = (|\text{Red}(AT_3)|/|AT_3|) = (3/4) = 0.75$.

Next, the external-confidence degree between $IFIS_i$ and $IFIS_j$ can be calculated as follows:

$D(IFIS_1, IFIS_1) = D(IFIS_2, IFIS_2) = D(IFIS_3, IFIS_3) = 0$,
$D(IFIS_1, IFIS_2) = 21$, $D(IFIS_1, IFIS_3) = 16$, $D(IFIS_2, IFIS_3) = 7$.

By Definition 18, the external correlation between $IFIS_i$ and $IFIS_j$ can be computed. The result is as follows:

$ec(IFIS_1, IFIS_1) = ec(IFIS_2, IFIS_2) = ec(IFIS_3, IFIS_3) = 1$,
$ec(IFIS_1, IFIS_2) = 1 - (21/36 - 1) = 0.3$,
$ec(IFIS_1, IFIS_3) = 1 - (16/36 - 1) \approx 0.467$,
$ec(IFIS_2, IFIS_3) = 1 - (7/36 - 1) = 0.767$.Thus, we can get the external-confidence degree matrix M_{EC} as follows:

$$M_{EC} = \begin{pmatrix} 1 & 0.3 & 0.467 \\ 0.3 & 1 & 0.767 \\ 0.467 & 0.767 & 1 \end{pmatrix}. \quad (19)$$

Therefore, the external-confidence degrees for each information source $IFIS_i (i = 1, 2, 3)$ is $EC(IFIS_i) = \{0.589, 0.689, 0.745\}$.

Furthermore, the total score for each information source $IFIS_i (i = 1, 2, 3)$ can be calculated using Definition 19:

$TC(IFIS_1) = 0.5 + 0.589 = 1.089$,
$TC(IFIS_2) = 0.5 + 0.589 = 1.439$,
$TC(IFIS_3) = 0.5 + 0.589 = 1.495$.

Therefore, the quality ranking of information sources is $IFIS_3 > IFIS_2 > IFIS_1$, and $IFIS_3$ is the optimal source of the multisource intuitionistic fuzzy information system.

3.2. Weighted Generalized Multigranularity Intuitionistic Fuzzy Rough Set Model for MSIFIS

Definition 20. Let MSIFIS $= \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system; $S_X^{A_i}(x)$ is the support characteristic function of x for X. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1 \, (0 \le \omega_i \le 1)$, parameter $\min \omega_i \le \beta \le 1 \, (i = 1, 2, \ldots, n)$ denotes the information level with respect to $\sum_{i=1}^{n} A_i$; for any $X \subseteq U$, the lower approximation and upper approximation of X weighted generalized multigranularity intuitionistic fuzzy rough set are defined as follows:

Table 2: Multisource intuitionistic fuzzy information system.

	$IFIS_1$				$IFIS_2$				$IFIS_3$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	(0.3, 0.5)	(0.6, 0.4)	(0.5, 0.2)	(0.7, 0.1)	(0.3, 0.5)	(0.7, 0.3)	(0.5, 0.1)	(0.7, 0.1)	(0.2, 0.5)	(0.6, 0.2)	(0.4, 0.6)	(0.3, 0.5)
x_2	(0.2, 0.7)	(0.1, 0.8)	(0.4, 0.5)	(0.1, 0.8)	(0.8, 0.2)	(0.8, 0.1)	(0.7, 0.1)	(1.0, 0.0)	(0.4, 0.5)	(0.5, 0.4)	(0.3, 0.5)	(0.6, 0.4)
x_3	(0.2, 0.7)	(0.1, 0.8)	(0.4, 0.5)	(0.7, 0.1)	(0.8, 0.2)	(0.9, 0.0)	(0.7, 0.1)	(0.8, 0.2)	(0.6, 0.2)	(0.7, 0.1)	(0.4, 0.3)	(0.5, 0.2)
x_4	(0.1, 0.8)	(0.1, 0.8)	(0.2, 0.7)	(0.1, 0.8)	(0.9, 0.1)	(0.9, 0.0)	(0.8, 0.1)	(0.6, 0.3)	(0.3, 0.7)	(0.2, 0.8)	(0.2, 0.7)	(0.4, 0.6)
x_5	(0.9, 0.1)	(0.8, 0.1)	(0.8, 0.1)	(0.9, 0.0)	(0.7, 0.2)	(0.6, 0.4)	(0.6, 0.2)	(0.5, 0.3)	(0.6, 0.2)	(0.2, 0.7)	(0.1, 0.7)	(0.1, 0.8)
x_6	(0.4, 0.6)	(0.7, 0.3)	(0.6, 0.3)	(0.7, 0.1)	(0.6, 0.3)	(0.2, 0.5)	(0.7, 0.2)	(0.4, 0.5)	(0.2, 0.7)	(0.1, 0.8)	(0.1, 0.6)	(0.5, 0.4)

Table 3: The calculation results for each IFIS$_i$.

	$Er(A_i)$	$\mathrm{sig}_{\mathrm{in}}(a_1, A_i)$	$\mathrm{sig}_{\mathrm{in}}(a_2, A_i)$	$\mathrm{sig}_{\mathrm{in}}(a_3, A_i)$	$\mathrm{sig}_{\mathrm{in}}(a_4, A_i)$	$\mathrm{Red}(A_i)$
$IFIS_1$	6.52	0	0	0	0.602	$\{a_1, a_4\}$ or $\{a_3, a_4\}$
$IFIS_2$	3	0	0.333	0.602	2.126	$\{a_2, a_3, a_4\}$
$IFIS_3$	2.252	1	0	0	0.874	$\{a_1, a_2, a_4\}$ or $\{a_1, a_3, a_4\}$

$$\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = \left\{ x \in U \mid \sum_{i=1}^{n} \omega_i S_X^{A_i}(x) \geq \beta \right\};$$
$$\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = \sim \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X)_\beta = \left\{ x \in U \mid \sum_{i=1}^{n} \omega_i \left(1 - S_{X^c}^{A_i}(x)\right) > 1 - \beta \right\}. \tag{20}$$

If $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$, then the target set X is definable for $\sum_{i=1}^{n} A_i$; otherwise, X is rough for $\sum_{i=1}^{n} A_i$. $(\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta, \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)$ is called the weighted generalized multigranularity intuitionistic fuzzy rough set model for MSIFIS.

In the weighted generalized multigranularity intuitionistic fuzzy rough set model, the positive, negative, and boundary regions of X can be defined as follows:

$$POS^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta;$$
$$NEG^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = U - \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta;$$
$$BND^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta - \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta. \tag{21}$$

The basic properties of the weighted generalized multigranularity intuitionistic fuzzy rough set for MSIFIS are given by the following theorem.

Theorem 1. *Let $MSIFIS = \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ be a multisource intuitionistic fuzzy information system. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1\,(0 \leq \omega_i \leq 1)$, parameter $\min \omega_i \leq \beta \leq 1\,(i = 1, 2, \ldots, n)$ denotes the information level with respect to $\sum_{i=1}^{n} A_i$; $\forall X, Y \subseteq U$, the following conclusions hold:*

(1) $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X)_\beta = \sim \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$, $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X)_\beta = \sim \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

(2) $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq X \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

(3) $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing)_\beta = \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing)_\beta = \varnothing$.

(4) $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U)_\beta = \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U)_\beta = U$.

(5) $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta \supseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \underline{MISF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$, $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta \supseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

(6) $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cap \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$, $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y)_\beta \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cap \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

(7) $X \subseteq Y \Rightarrow \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$, $X \subseteq Y \Rightarrow \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

(8) $\beta_1 \leq \beta_2 \Rightarrow \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_2} \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_1}$, $\beta_1 \leq \beta_2 \Rightarrow \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_2} \supseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_1}$.

(9) $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)_\beta = \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$, $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\overline{MSIF}_{\sum_{i=}} 1^n A_i{}^{\omega}(X)_\beta)_\beta = \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

Proof. (1) Because

$$\sim \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = \left\{x \in U | \sum_{i=1}^{n} \omega_i\left(1 - S^{A_i}_{X^c}(x)\right) \le 1-\beta\right\}$$
$$= \left\{x \in U | \sum_{i=1}^{n} \omega_i - \sum_{i=1}^{n} \omega_i S^{A_i}_{X^c}(x) \le 1-\beta\right\}$$
$$= \left\{x \in U | 1 - \sum_{i=1}^{n} \omega_i S^{A_i}_{X^c}(x) \le 1-\beta\right\}$$
$$= \left\{x \in U | \sum_{i=1}^{n} \omega_i S^{A_i}_{X^c}(x) \ge \beta\right\}$$
$$= \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X)_\beta, \tag{22}$$

$\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X)_\beta = \sim \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

Similarly,
$\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X)_\beta = \sim \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

Therefore, the property is clearly established.

(2) For any $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$, we can know $\sum_{i=1}^{n} \omega_i S^{A_i}_X(x) \ge \beta$. In addition, $\min \omega_i \le \beta \le 1$, so $\exists A_i$, s.t. $S^{A_i}_X(x) = 1$. Namely, $[x]_{A_i} \subseteq X$. We have $x \in X$; thus, $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq X$.

$\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X)_\beta = \sim \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta = \sim \subseteq X$.
Hence, $X \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

Therefore, property (2) has been proved.

(3) For any $x \in U$, we can easily get $S^{A_i}_{\varnothing}(x) = 0, S^{A_i}_U(x) = 1$. Therefore, $\sum_{i=1}^{n} \omega_i S^{A_i}_{\varnothing}(x) = 0, \sum_{i=1}^{n} \omega_i S^{A_i}_U(x) = 1$.

Thus, $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing)_\beta = \{x \in U | \sum_{i=1}^{n} \omega_i \; S^{A_i}_{\varnothing}(x) \ge \beta\} = \{x \in U | 0 \ge \beta\} = \varnothing$,

$\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing)_\beta = \{x \in U | \sum_{i=1}^{n} \omega_i \; (1 - S^{A_i}_U(x)) > 1-\beta\} = \{x \in U | 0 > 1-\beta\} = \varnothing$.

Therefore, property (3) has been proved.

(4) According to (3), $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U)_\beta = \{x \in U | \sum_{i=1}^{n} \omega_i S^{A_i}_U(x) \ge \beta\} = \{x \in U | 1 \ge \beta\} = U$,

$\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U)_\beta = \{x \in U | \sum_{i=1}^{n} \omega_i (1 - S^{A_i}_{\varnothing}(x)) > 1-\beta\} = \{x \in U | 1 > 1-\beta\} = U$.

Therefore, property (4) has been proved.

(5) For any $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta$

$\Leftrightarrow x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$ or $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$

$\Leftrightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_X(x) \ge \beta$ or $\sum_{i=1}^{n} \omega_i S^{A_i}_Y(x) \ge \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_{X \cup Y}(x) \ge \sum_{i=1}^{n} \omega_i (S^{A_i}_X(x) \vee S^{A_i}_Y(x)) \ge \sum_{i=1}^{n} \omega_i S^{A_i}_X(x) \wedge \sum_{i=1}^{n} \omega_i S^{A_i}_Y(x) \ge \beta$.

Therefore, $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta$.

Then, we can get that $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta \supseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Meanwhile, for any $x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$,

$\Leftrightarrow x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$ or $x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$,

$\Leftrightarrow \sum_{i=1}^{n} \omega_i (1 - S^{A_i}_{X^c}(x)) > 1-\beta$ or $\sum_{i=1}^{n} \omega_i (1 - S^{A_i}_{Y^c}(x)) > 1-\beta$

$\Leftrightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_{X^c}(x) < \beta$ or $\sum_{i=1}^{n} \omega_i S^{A_i}_{Y^c}(x) < \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_{(X \cup Y)^c}(x) = \sum_{i=1}^{n} \omega_i S^{A_i}_{X^c \cap Y^c}(x) = \sum_{i=1}^{n} \omega_i (S^{A_i}_{X^c}(x) \wedge S^{A_i}_{Y^c}(x)) \le \sum_{i=1}^{n} \omega_i S^{A_i}_{X^c}(x) \wedge \sum_{i=1}^{n} \omega_i S^{A_i}_{Y^c}(x) < \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i (1 - S^{A_i}_{(X \cup Y)^c}(x)) > 1-\beta$

$\Rightarrow x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta$.

Then, we obtain that $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta \supseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Therefore, the property is clearly established.

(6) $\forall x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y)_\beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_{(X \cap Y)}(x) = \sum_{i=1}^{n} \omega_i \; (S^{A_i}_X(x) \wedge S^{A_i}_Y(x)) \ge \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_X(x) \wedge \sum_{i=1}^{n} \omega_i S^{A_i}_Y(x) \ge \sum_{i=1}^{n} \omega_i (S^{A_i}_X(x) \wedge S^{A_i}_Y(x)) \ge \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_X(x) \ge \beta$ and $\sum_{i=1}^{n} \omega_i S^{A_i}_Y(x) \ge \beta$

$\Rightarrow x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$ and $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$

$\Rightarrow x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cap MSIF^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Thus, $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cap \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Meanwhile, for any $x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y)_\beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i (1 - S^{A_i}_{(X \cap Y)^c}(x)) > 1-\beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S^{A_i}_{(X \cap Y)^c}(x) < \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i \; S^{A_i}_{X^c \cup Y^c}(x) < \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S_{X^c}^{A_i}(x) \vee \sum_{i=1}^{n} \omega_i S_{Y^c}^{A_i}(x) \leq \sum_{i=1}^{n} \omega_i (S_{X^c}^{A_i}(x) \vee S_{Y^c}^{A_i}(x)) \leq \sum_{i=1}^{n} \omega_i S_{X^c \cup Y^c}^{A_i}(x) < \beta$

$\Rightarrow \sum_{i=1}^{n} \omega_i S_{X^c}^{A_i}(x) < \beta$ and $\sum_{i=1}^{n} \omega_i S_{Y^c}^{A_i}(x) < \beta$

$\Rightarrow 1 - \sum_{i=1}^{n} \omega_i S_{X^c}^{A_i}(x) > 1 - \beta$ and $1 - \sum_{i=1}^{n} \omega_i S_{Y^c}^{A_i}(x) > 1 - \beta$

$\Rightarrow x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$ and $x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$

$\Rightarrow x \in \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cap \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Therefore, the property is clearly established.

(7) Because $X \subseteq Y$, we can know $X \cup Y = Y$.

According to property (5), $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta$.

Therefore, $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Thus, $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Meanwhile, according to property (5), $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y)_\beta$.

Therefore, $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \cup \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Thus, $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)_\beta$.

Therefore, the property is clearly established.

(8) For any $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_2}$, we have $\sum_{i=1}^{n} \omega_i S_X^{A_i}(x) \geq \beta_2 \geq \beta_1$.

Therefore, $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_1}$.

Then, $\beta_1 \leq \beta_2$ implies that $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_2} \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_1}$.

Similarly, we can prove that $\beta_1 \leq \beta_2 \Rightarrow \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_2} \supseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_{\beta_1}$.

Therefore, property (8) has been proved.

(9) According to properties (2) and (7), $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq X \Rightarrow \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

The following is the proof: $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)_\beta$.

For any $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$, we can know $\sum_{i=1}^{n} \omega_i S_X^{A_i}(x) \geq \beta$.

$\min \omega_i \leq \beta \leq 1$; thus, $\exists A_i, \text{s.t.}\, S_X^{A_i}(x) = 1$; namely, $[x]_{A_i} \subseteq X$.

Then, according to property (7), we have $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}([x]_{A_i})_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

Therefore, $[x]_{A_i} \subseteq \underline{MSFI}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$; we can get $\sum_{i=1}^{n} \omega_i S^{A_i}_{\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta}(x) \geq \beta$, and $x \in \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)_\beta$.

Therefore, $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)_\beta$, from which one can get that $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)_\beta = \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

Meanwhile, we can prove that $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta)_\beta = \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta$.

Property (1) shows that in multisource intuitionistic fuzzy information system, the lower approximation operator, and the upper approximation operator of weighted generalized multigranularity intuitionistic fuzzy rough sets satisfy duality. Property (2) illustrates the inclusion relationship between the lower approximation, the upper approximation, and the target concept. Properties (3) and (4) show the approximation of two special sets. The lower and upper approximation of the empty set $\varnothing$ and the universe U are themselves. Properties (5) – (8) state the monotonicity of $\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}$ and $\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}$. Moreover, property (9) expresses the idempotency between the lower approximation operator and the upper approximation operator. □

3.3. Weighted Optimistic Multigranularity Intuitionistic Fuzzy Rough Set Model for MSIFIS

Definition 21. Let $\text{MSIFIS} = \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system; $S_X^{A_i}(x)$ is the support characteristic function of x for X. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1\, (0 \leq \omega_i \leq 1)$, $\forall X \subseteq U$, the lower approximation and upper approximation of X weighted optimistic multigranularity intuitionistic fuzzy rough set are defined as follows:

$$\begin{aligned} \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) &= \left\{x \in U | \sum_{i=1}^{n} \omega_i S_X^{A_i}(x) > 0\right\}; \\ \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) &= \sim \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X) \\ &= \left\{x \in U | \sum_{i=1}^{n} \omega_i \left(1 - S_{X^c}^{A_i}(x)\right) \geq 1\right\}. \end{aligned} \tag{23}$$

If $\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) = \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$, then the target set X is optimistic and definable for $\sum_{i=1}^{n} A_i$; otherwise, X is optimistic and rough for $\sum_{i=1}^{n} A_i$. $(\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X), \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X))$ is called the weighted optimistic multigranularity intuitionistic fuzzy rough set model for MSIFIS.

In the weighted optimistic multigranularity intuitionistic fuzzy rough set model, the positive, negative, and boundary regions of X can be defined as follows:

$$POS^{\omega}_{\sum_{i=1}^{n} A_i}(X)_O = \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X);$$
$$NEG^{\omega}_{\sum_{i=1}^{n} A_i}(X)_O = U - \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X);$$
$$BND^{\omega}_{\sum_{i=1}^{n} A_i}(X)_O = \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) - \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X). \quad (24)$$

The basic properties of the weighted optimistic multigranularity intuitionistic fuzzy rough set for MSIFIS are given by the following theorem.

Theorem 2. *Let $MSIFIS = \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ be a multisource intuitionistic fuzzy information system. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1\, (0 \le \omega_i \le 1)$, $\forall X, Y \subseteq U$, the following conclusions hold:*

(1) $\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X) = \sim \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$, $\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X) = \sim \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$.

(2) $\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq X \subseteq \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$.

(3) $\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing) = \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing) = \varnothing$.

(4) $\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U) = \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U) = U$.

(5) $\underline{OMSF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y) \supseteq \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cup \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$, $\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y) \supseteq \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cup \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$.

(6) $\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y) \subseteq \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cap \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$, $\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y) \subseteq \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cup \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$.

(7) $X \subseteq Y \Rightarrow \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$, $X \subseteq Y \Rightarrow \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$.

(8) $\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)) = \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$,

$$\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)) = \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X).$$

Proof. It is similar to that of Theorem 1. □

3.4. Weighted Pessimistic Multigranularity Intuitionistic Fuzzy Rough Set Model for MSIFIS

Definition 22. Let MSIFIS $= \{IFIS_i | IFIS_i = (U, A_i, V_i, f_i)\}$ $(i = 1, 2, \ldots, n)$ be a multisource intuitionistic fuzzy information system; $S_X^{A_i}(x)$ is the support characteristic function of x for X. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1$, $(0 \le \omega_i \le 1)$, $\forall X \subseteq U$, the lower approximation and upper approximation of X weighted pessimistic multigranularity intuitionistic fuzzy rough set are defined as follows:

$$\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) = \left\{x \in U \mid \sum_{i=1}^{n} \omega_i S_X^{A_i}(x) \ge 1\right\};$$
$$\overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) = \sim \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X) = \left\{x \in U \mid \sum_{i=1}^{n} \omega_i \left(1 - S_{X^c}^{A_i}(x)\right) > 0\right\}. \quad (25)$$

If $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) = \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$, then the target set X is pessimistic and definable for $\sum_{i=1}^{n} A_i$; otherwise, X is pessimistic and rough for $\sum_{i=1}^{n} A_i$. $(\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X), \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X))$ is called the weighted pessimistic multigranularity intuitionistic fuzzy rough set model for MSIFIS.

In the weighted pessimistic multigranularity intuitionistic fuzzy rough set model, the positive, negative, and boundary regions of X can be defined as follows:

$$POS^{\omega}_{\sum_{i=1}^{n} A_i}(X)_P = \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X);$$
$$NEG^{\omega}_{\sum_{i=1}^{n} A_i}(X)_P = U - \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X);$$
$$BND^{\omega}_{\sum_{i=1}^{n} A_i}(X)_P = \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) - \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X). \quad (26)$$

The basic properties of the weighted pessimistic multigranularity intuitionistic fuzzy rough set for MSIFIS are given by the following theorem.

Theorem 3. *Let $MSIFIS = \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ be a multisource intuitionistic fuzzy information system. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1$, $(0 \le \omega_i \le 1)$, $\forall X, Y \subseteq U$, the following conclusions hold:*

(1) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X) = \sim \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$, $\overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\sim X) = \sim \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$.

(2) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq X \subseteq \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$.

(3) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing) = \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\varnothing) = \varnothing$.

(4) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U) = \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(U) = U$.

(5) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y) \supseteq \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cup \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$, $\overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cup Y) = \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cup \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$.

(6) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y) = \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cap \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$,

$\overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X \cap Y) \subseteq \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \cup \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$.

(7) $X \subseteq Y \Rightarrow \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$, $X \subseteq Y \Rightarrow \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(Y)$.

(8) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)) = \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$, $\overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(\overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)) = \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$.

Proof. It is similar to that of Theorem 1. □

3.5. The Relationship between the Three Models of MSIFIS. In this subsection, we investigate the relationship between the three models of MSIFIS.

Theorem 4. *Let MSIFIS* $= \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ *be a multisource intuitionistic fuzzy information system. If the weight corresponding to each granularity space is derived from each dominance relation as* $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1$, $(0 \le \omega_i \le 1)$, $\forall X \subseteq U$, *the following equations can be obtained:*

(1) *If* $\beta = 1$, *then*

$$\begin{aligned} \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta &= \underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X), \\ \overline{MISF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta &= \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X). \end{aligned} \tag{27}$$

(2) *If* $\beta = \min \omega_i \, (i = 1, 2, \ldots, n)$, *then*

$$\begin{aligned} \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta &= \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X), \\ \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta &= \overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X). \end{aligned} \tag{28}$$

Proof. It is easily obtained from Definition 15, Definition 20, and Definition 21.

From the above theorems, if $\beta = 1$, weighted generalized multigranularity intuitionistic fuzzy rough set degenerates into weighted pessimistic multigranularity intuitionistic fuzzy rough set. If $\beta = \min \omega_{A_i}$, weighted generalized multigranularity intuitionistic fuzzy rough set degenerates into weighted optimistic multigranularity intuitionistic fuzzy rough set. Hence, weighted generalized multigranularity intuitionistic fuzzy rough set is a generalization of weighted pessimistic multigranularity intuitionistic fuzzy rough set and weighted optimistic multigranularity intuitionistic fuzzy rough set. On the other hand, weighted pessimistic multigranularity intuitionistic fuzzy rough set and weighted optimistic multigranularity intuitionistic fuzzy rough set are the special cases of weighted generalized multigranularity intuitionistic fuzzy rough set. □

Theorem 5. *Let MSIFIS* $= \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ *be a multisource intuitionistic fuzzy information system. If the weight corresponding to each granularity space is derived from each dominance relation as* $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1$, $(0 \le \omega_i \le 1)$, $\forall X \subseteq U$, *the following properties are established:*

(1) $\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$.

(2) $\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta \subseteq \overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)$.

Proof. From Definition 20–Definition 22, the theorem clearly holds.

The theorem shows that the lower and upper approximation of the weighted generalized multigranularity intuitionistic fuzzy rough set are between weighted pessimistic multigranularity intuitionistic fuzzy rough set and weighted optimistic multigranularity intuitionistic fuzzy rough set. □

4. Uncertainty Measurement of $MSIFIS$

Like Pawlak rough sets, the uncertainty of knowledge is caused by the boundary region. The larger the boundary area, the lower the accuracy and the higher roughness. For the weighted multigranularity intuitionistic fuzzy rough set model in the MSIFIS, this section gives definition of rough accuracy, roughness, and attribute dependence.

Definition 23. Let MSIFIS $= \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ be a multisource intuitionistic fuzzy information system. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\sum_{i=1}^{n} \omega_i = 1 \, (0 \le \omega_i \le 1)$, parameter $\min \omega_i \le \beta \le 1 \, (i = 1, 2, \ldots, n)$ denotes the information level with respect to $\sum_{i=1}^{n} A_i$; $\forall X \subseteq U$, the approximation accuracy and roughness of the weighted generalized multigranularity intuitionistic fuzzy rough set model of the set X with respect to $\sum_{i=1}^{n} A_i$ are defined as follows:

$$\begin{aligned} \alpha^{\omega}_{MSIF}(X)_\beta &= \frac{\left|\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta\right|}{\left|\overline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta\right|}; \\ \rho^{\omega}_{MSIF}(X)_\beta &= 1 - \alpha^{\omega}_{MSIF}(X)_\beta. \end{aligned} \tag{29}$$

Similarly, the approximation accuracy and roughness of the weighted optimistic multigranularity intuitionistic fuzzy rough set model of the set X with respect to $\sum_{i=1}^{n} A_i$ are defined as follows:

$$\begin{aligned} \alpha^{\omega}_{OMSIF}(X) &= \frac{\left|\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)\right|}{\left|\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)\right|}; \\ \rho^{\omega}_{OMSIF}(X) &= 1 - \alpha^{\omega}_{OMSIF}(X). \end{aligned} \tag{30}$$

The approximation accuracy and roughness of the weighted pessimistic multigranularity intuitionistic fuzzy rough set model of the set X with respect to $\sum_{i=1}^{n} A_i$ are defined as follows:

$$\alpha^{\omega}_{PMSIF}(X) = \frac{\left|\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)\right|}{\left|\overline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)\right|};$$
$$\rho^{\omega}_{PMSIF}(X) = 1 - \alpha^{\omega}_{PMSIF}(X). \quad (31)$$

Definition 24. Let MSIFIS $= \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ be a multisource intuitionistic fuzzy information system. If the weight corresponding to each granularity space is derived from each dominance relation as $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}, \sum_{i=1}^{n} \omega_i = 1 \quad (0 \le \omega_i \le 1)$, parameter $\min \omega_i \le \beta \le 1\,(i = 1, 2, \ldots, n)$ denotes the information level with respect to $\sum_{i=1}^{n} A_i$; $\forall X \subseteq U$, the approximation accuracy and roughness of the weighted generalized multigranularity intuitionistic fuzzy rough set model of the set X with respect to $\sum_{i=1}^{n} A_i$ are defined as follows:

$$\gamma^{\omega}_{MSF}(X)_\beta = \frac{\left|\underline{MSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta\right|}{|U|} = \frac{\left|POS^{\omega}_{\sum_{i=1}^{n} A_i}(X)_\beta\right|}{|U|}. \quad (32)$$

Similarly, the dependence of the weighted optimistic multigranularity intuitionistic fuzzy rough set model of the set X with respect to $\sum_{i=1}^{n} A_i$ is defined as follows:

$$\gamma^{\omega}_{OMSF}(X) = \frac{\left|\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)\right|}{|U|} = \frac{\left|POS^{\omega}_{\sum_{i=1}^{n} A_i}(X)_O\right|}{|U|}. \quad (33)$$

The dependence of the weighted pessimistic multigranularity intuitionistic fuzzy rough set model of the set X with respect to $\sum_{i=1}^{n} A_i$ is defined as follows:

$$\gamma^{\omega}_{PMSF}(X) = \frac{\left|\underline{PMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X)\right|}{|U|} = \frac{\left|POS^{\omega}_{\sum_{i=1}^{n} A_i}(X)_P\right|}{|U|}. \quad (34)$$

Theorem 6. *Let MSIFIS* $= \{IFIS_1, IFIS_2, \ldots, IFIS_n\}$ *be a multisource intuitionistic fuzzy information system, where* $IFIS_i = \{U, A_i, V_i, IF_i\}\,(i \le n)$; $S^{A_i}_X(x)$ *is the support characteristic function of x for X. If the weight corresponding to each granularity space is derived from each dominance relation as* $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}, \sum_{i=1}^{n} \omega_i = 1$, *for any* $X \subseteq U$, *under the three rough set models, the relationships between the approximate accuracy, roughness, and dependence of the set X with respect to* $\sum_{i=1}^{n} A_i$ *are as follows:*

(1) $\alpha^{\omega}_{PMSIF}(X) \le \alpha^{\omega}_{MSIF}(X)_\beta \le \alpha^{\omega}_{OMSIF}(X)_\beta$.

(2) $\rho^{\omega}_{OMSIF}(X) \le \rho^{\omega}_{MSIF}(X)_\beta \le \rho^{\omega}_{PMSIF}(X)_\beta$.

(3) $\gamma^{\omega}_{PMSIF}(X) \le \gamma^{\omega}_{MSIF}(X)_\beta \le \gamma^{\omega}_{OMSIF}(X)$.

Proof. It is easy to obtain by Definition 23 and Definition 24.

The theorem shows that the accuracy, roughness, and dependence of the weighted generalized multigranularity intuitionistic fuzzy rough set model are between weighted pessimistic multigranularity intuitionistic fuzzy rough set and weighted optimistic multigranularity intuitionistic fuzzy rough set model. □

Example 3. In example 1, the quality ranking of the three information sources in the multisource intuitionistic fuzzy information system is calculated. Let us suppose that the granularity weights corresponding to the three information systems are assigned as $\omega\{0.2, 0.3, 0.5\}$; the distribution of granularity weights in specific applications can be given subjectively based on the experience of domain experts, with threshold $\beta = 0.6$.

According to Definition 2, we calculate the dominant classes of object under each source.

Dominant classes under source 1 are as follows:

$R_{A_1}(x_1) = \{x_1, x_5\}$, $R_{A_1}(x_2) = \{x_1, x_2, x_3, x_5, x_6\}$,
$R_{A_1}(x_3) = \{x_1, x_3, x_5, x_6\}$,
$R_{A_1}(x_4) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$,
$R_{A_1}(x_5) = \{x_5\}$, $R_{A_1}(x_6) = \{x_5, x_6\}$.

Dominant classes under source 2 are as follows:

$R_{A_2}(x_1) = \{x_1, x_2\}$, $R_{A_2}(x_2) = \{x_2\}$,
$R_{A_2}(x_3) = \{x_3\}$, $R_{A_2}(x_4) = \{x_4\}$,
$R_{A_2}(x_5) = \{x_2, x_3, x_4, x_5\}$, $R_{A_2}(x_6) = \{x_2, x_3, x_4, x_6\}$.

Dominant classes under source 3 are as follows:

$R_{A_3}(x_1) = \{x_1, x_3\}$, $R_{A_3}(x_2) = \{x_2\}$,
$R_{A_3}(x_3) = \{x_3\}$, $R_{A_3}(x_4) = \{x_2, x_3, x_4\}$,
$R_{A_3}(x_5) = \{x_3, x_5\}$, $R_{A_3}(x_6) = \{x_2, x_3, x_6\}$.

Given a concept set $X = \{x_1, x_2, x_3, x_5\}$, the support characteristic function of X and support characteristic function of X^c under each source are computed.

$S^{A_1}_X(x_1) = 1$, $S^{A_2}_X(x_1) = 1$, $S^{A_3}_X(x_1) = 1$,
$S^{A_1}_X(x_2) = 0$, $S^{A_2}_X(x_2) = 1$, $S^{A_3}_X(x_2) = 1$,
$S^{A_1}_X(x_3) = 0$, $S^{A_2}_X(x_3) = 1$, $S^{A_3}_X(x_3) = 1$,
$S^{A_1}_X(x_4) = 0$, $S^{A_2}_X(x_4) = 0$, $S^{A_3}_X(x_4) = 0$,
$S^{A_1}_X(x_5) = 1$, $S^{A_2}_X(x_5) = 0$, $S^{A_3}_X(x_5) = 1$,
$S^{A_1}_X(x_6) = 0$, $S^{A_2}_X(x_6) = 0$, $S^{A_3}_X(x_6) = 0$.
$X^c = \{x_4, x_6\}$,
$S^{A_1}_{X^c}(x_1) = 0$, $S^{A_2}_{X^c}(x_1) = 0$, $S^{A_3}_{X^c}(x_1) = 0$,
$S^{A_1}_{X^c}(x_2) = 0$, $S^{A_2}_{X^c}(x_2) = 0$, $S^{A_3}_{X^c}(x_2) = 0$,
$S^{A_1}_{X^c}(x_3) = 0$, $S^{A_2}_{X^c}(x_3) = 0$, $S^{A_3}_{X^c}(x_3) = 0$,
$S^{A_1}_{X^c}(x_4) = 0$, $S^{A_2}_{X^c}s(x_4) = 1$, $S^{A_3}_{X^c}(x_4) = 0$,
$S^{A_1}_{X^c}(x_5) = 0$, $S^{A_2}_{X^c}(x_5) = 0$, $S^{A_3}_{X^c}s(x_5) = 0$,
$S^{A_1}_{X^c}s(x_6) = 0$, $S^{A_2}_{X^c}(x_6) = 0$, $S^{A_3}_{X^c}(x_6) = 0$.

According to Definition 20, the lower approximation and upper approximation of the weighted generalized multigranularity intuitionistic fuzzy rough set X are obtained as follows:

$$\underline{MSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X)_\beta = \{x_1, x_2, x_3, x_5\},$$
$$\overline{MSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X)_\beta = \{x_1, x_2, x_3, x_4, x_5, x_6\}.$$

The positive region, boundary region, and negative region of the weighted generalized multigranularity intuitionistic fuzzy rough set X are as follows:

$POS^{\omega}_{\sum_{i=1}^{3} A_i}(X)_{\beta} = \{x_1, x_2, x_3, x_5\}$,

$BND^{\omega}_{\sum_{i=1}^{3} A_i}(X)_{\beta} = \{x_4, x_6\}$,

$NEG^{\omega}_{\sum_{i=1}^{3} A_i}(X)_{\beta} = \varnothing$.

The approximation accuracy, roughness, and dependence of the weighted generalized multigranularity intuitionistic fuzzy rough set X are as follows:

$\alpha^{\omega}_{MSIF}(X)_{\beta} = (4/6) = (2/3)$,
$\rho^{\omega}_{MSIF}(X)_{\beta} = 1 - (2/3) = (1/3)$,
$\gamma^{\omega}_{MSIF}(X)_{\beta} = (4/6) = (2/3)$.

According to Definition 21, the lower approximation and upper approximation of the weighted optimistic multigranularity intuitionistic fuzzy rough set X are obtained as follows:

$\underline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) = \{x_1, x_2, x_3, x_5\}$,
$\overline{OMSIF}^{\omega}_{\sum_{i=1}^{n} A_i}(X) = \{x_1, x_2, x_3, x_5, x_6\}$.

The positive region, boundary region, and negative region of the weighted optimistic multigranularity intuitionistic fuzzy rough set X are as follows:

$POS^{\omega}_{\sum_{i=1}^{n} A_i}(X)_O = \{x_1, x_2, x_3, x_5\}$,
$BND^{\omega}_{\sum_{i=1}^{n} A_i}(X)_O = \{x_6\}$,
$NEG^{\omega}_{\sum_{i=1}^{n} A_i}(X)_O = \{x_4, x_6\}$.

The approximation accuracy, roughness, and dependence of the weighted optimistic multigranularity intuitionistic fuzzy rough set X are as follows:

$\alpha^{\omega}_{OMSIF}(X) = (4/5)$,
$\rho^{\omega}_{OMSIF}(X) = 1 - (4/5) = (1/5)$,
$\gamma^{\omega}_{OMSIF}(X) = (4/6) = (2/3)$,

According to Definition 22, the lower approximation and upper approximation of the weighted pessimistic multigranularity intuitionistic fuzzy rough set X are obtained as follows:

$\underline{PMSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X) = \{x_1\}$,
$\overline{PMSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$.

The positive region, boundary region, and negative region of the weighted pessimistic multigranularity intuitionistic fuzzy rough set X are as follows:

$POS^{\omega}_{\sum_{i=1}^{3} A_i}(X)_P = \{x_1\}$,
$BND^{\omega}_{\sum_{i=1}^{3} A_i}(X)_P = \{x_1, x_2, x_3, x_4, x_5, x_6\}$,
$NEG^{\omega}_{\sum_{i=1}^{3} A_i}(X)_P = \varnothing$.

The approximation accuracy, roughness, and dependence of the weighted pessimistic multigranularity intuitionistic fuzzy rough set X are as follows:

$\alpha^{\omega}_{PMSIF}(X) = (1/6)$,
$\rho^{\omega}_{PMSIF}(X) = 1 - (1/6) = (5/6)$,
$\gamma^{\omega}_{PMSIF}(X) = (1/6)$.
Hence,
$\underline{PMSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X) \subseteq \underline{MSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X)_{\beta} \subseteq \underline{OMSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X)$,
$\overline{OMSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X) \subseteq \overline{MSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X)_{\beta} \subseteq \overline{PMSIF}^{\omega}_{\sum_{i=1}^{3} A_i}(X)$.

$\alpha^{\omega}_{PMSIF}(X) \le \alpha^{\omega}_{MSIF}(X)_{\beta} \le \alpha^{\omega}_{OMSIF}(X)_{\beta}$,
$\rho^{\omega}_{OMSIF}(X) \le \rho^{\omega}_{MSIF}(X)_{\beta} \le \rho^{\omega}_{PMSIF}(X)_{\beta}$,
$\gamma^{\omega}_{PMSIF}s(X) \le \gamma^{\omega}_{MSIF}(X)_{\beta} \le \gamma^{\omega}_{OMSIF}(X)$.

5. Experimental Evaluation and Analysis

In this paper, three weighted multigranulation intuitionistic fuzzy rough set models for MSIFIS are studied, and the uncertainty measurement methods of different models are discussed. In this section, we use an experiment to show the effectiveness of the models and methods proposed in this paper. We propose Algorithm 2 to calculate the uncertainty measure of the weighted generalized multigranulation intuitionistic fuzzy rough set in MSIFIS. Similarly, by changing the threshold β, a calculated weighted optimistic and pessimistic uncertainty measurement algorithm can be obtained.

The time complexity analysis of Algorithm 2 is as folloxws. From Steps 2 – 20, we calculate the dominant class of each object under each information source, and its time complexity is $O(|U|^2 * A * q)$ (q is the number of sources). From Steps 21 – 35, we calculate the support feature function of X and X^C under each information source, and the time complexity is $O(|U| * q)$. From Steps 36 – 45, we compute the upper and lower approximations of X in MSIFIS, and the time complexity is $O(|U|)$.

As we all know, we cannot get MSIFIS directly from UCI (http://archive.ics.uci.edu/ml/datasets.html), so we need to process the data. First, we need to download the "winequality-red" and "winequality-white" data sets from UCI and divide them by the maximum number of each column in the data table to make them fuzzy data tables. Then, we use the MATLAB software to randomly generate two fuzzy data sets. Finally, the method given in literature [23] is used to make them intuitionistic fuzzy data sets.

The details of the four data tables are shown in Table 4. We use the above four data sets as the original intuitionistic fuzzy information system. Finally, we randomly select 40% of the original data to add white noise and then randomly select 20% of the remaining data to add random noise, and the rest of the data remains unchanged to generate four MSIFIS*s* with ten sources. The entire experiment was run on a private computer. The specific operating environment, including hardware and software, is shown in Table 5.

We add white noise as follows: the q real numbers with a normal distribution $(n_1, n_2, \ldots, n_q)$ were first generated by MATLAB.

Table 4: Specific information about the operating environment.

Name	Model	Parameter
CPU	Intel® core™ i3-2370M	2.40 GHz
Platform	MATLAB	R2016b
System	Windows 7	64 bits
Memory	DDR3	4 GB; 1600 MHz
Hard disk	MQ01ABD050	500 GB

Table 5: Uncertainty measurement on "own-data 1."

		α			β			γ	
	G	O	P	G	O	P	G	O	P
1	0.842 1	1	0.082 4	0.157 9	0	0.917 6	0.752 9	0.8	0.082 4
2	0.722 6	1	0.047 1	0.277 4	0	0.952 9	0.658 8	0.8	0.047 1
3	0.727 8	1	0.070 6	0.272 2	0	0.929 4	0.676 5	0.8	0.070 6
4	0.784 0	1	0.111 8	0.216	0	0.888 2	0.747 1	0.8	0.111 8
5	0.726 1	1	0.052 9	0.273 9	0	0.947 1	0.670 6	0.8	0.052 9
6	0.516 1	0.992 7	0.052 9	0.483 9	0.007 3	0.947 1	0.470 6	0.8	0.052 9
7	0.783 8	0.797 3	0.052 9	0.216 2	0.202 7	0.947 1	0.682 4	0.694 1	0.052 9
8	0.682 4	0.802 7	0.052 9	0.317 6	0.197 3	0.947 1	0.594 1	0.694 1	0.052 9
9	0.478 0	0.992 7	0.023 5	0.522	0.007 3	0.976 5	0.447 1	0.8	0.023 5
10	0.337 6	0.992 7	$\leq$ 0.001	0.662 4	0.007 3	1	0.311 8	0.8	0

Table 6: Uncertainty measurement on "winequality-red."

		α			β			γ	
	G	O	P	G	O	P	G	O	P
1	0.746 1	0.985 3	0.531 6	0.253 9	0.014 7	0.468 4	0.657 9	0.794 9	0.500 3
2	0.742 2	0.936 9	0.636 7	0.257 8	0.063 1	0.363 3	0.653 5	0.770 5	0.585 4
3	0.745 2	0.970 0	0.638 3	0.254 8	0.03	0.361 7	0.656 7	0.789 9	0.586 0
4	0.730 7	0.969 4	0.638 3	0.269 3	0.030 6	0.361 7	0.649 8	0.793 0	0.586 0
5	0.735 6	0.953 5	0.637 2	0.264 4	0.046 5	0.362 8	0.656 0	0.781 7	0.585 4
6	0.731 1	0.962 5	0.502 3	0.268 9	0.037 5	0.497 7	0.652 9	0.785 5	0.471 5
7	0.730 9	0.940 9	0.637 2	0.269 1	0.059 1	0.362 8	0.652 3	0.776 7	0.585 4
8	0.743 5	0.996 1	0.629 8	0.256 5	0.003 9	0.370 2	0.657 9	0.803 0	0.579 7
9	0.728 4	0.939 3	0.585 4	0.271 6	0.060 7	0.414 6	0.647 3	0.774 2	0.585 4
10	0.739 8	0.931 1	0.586 6	0.260 2	0.068 9	0.413 4	0.656 0	0.769 2	0.586 6

Table 7: Uncertainty measurement on "own-data 2."

		α			β			γ	
	G	O	P	G	O	P	G	O	P
1	0.251 9	0.804 7	0.000 3	0.748 1	0.195 3	0.999 7	0.247	0.804 7	0.000 3
2	0.277 0	0.994 2	0.000 7	0.723	0.005 8	0.999 3	0.273 7	0.804	0.000 7
3	0.344 2	0.996 7	0.000 7	0.655 8	0.003 3	0.999 3	0.337 7	0.805 3	0.000 7
4	0.297 7	0.992 2	0.001 3	0.702 3	0.007 8	0.998 7	0.293	0.804 7	0.001 3
5	0.322 5	0.995 9	0.002 7	0.677 5	0.004 1	0.997 3	0.316 7	0.805	0.002 7
6	0.311 9	0.990 2	0.002 7	0.688 1	0.009 8	0.997 3	0.307 7	0.804 3	0.002 7
7	0.306 5	0.993 4	0.001	0.693 5	0.006 6	0.999	0.302 3	0.804 3	0.001
8	0.277 5	0.994 2	$\leq$ 0.001	0.722 5	0.005 8	1	0.272 3	0.805	$\leq$ 0.001
9	0.334 0	0.995 5	0.002	0.666	0.004 5	0.998	0.328	0.805 3	0.002
10	0.291 3	0.993 8	0.000 7	0.708 7	0.006 2	0.999 3	0.287	0.804 3	0.000 7

Table 8: Uncertainty measurement on “winequality-white.”

	α			β			γ		
	G	O	P	G	O	P	G	O	P
1	0.638 3	0.997 9	0.035 1	0.361 7	0.002 1	0.964 9	0.603 1	0.793 8	0.035 1
2	0.647 6	0.999 5	0.031 2	0.352 4	0.000 5	0.968 8	0.611 5	0.793 8	0.031 2
3	0.662 1	0.999 7	0.040 8	0.337 9	0.000 3	0.959 2	0.626 2	0.793 8	0.040 8
4	0.652 2	0.999 5	0.035 7	0.347 8	0.000 5	0.964 3	0.614 9	0.793 8	0.035 7
5	0.657 8	1	0.034 7	0.342 2	0	0.965 3	0.618 2	0.793 8	0.034 7
6	0.657 2	0.998 7	0.039 2	0.342 8	0.001 3	0.960 8	0.617 6	0.793 6	0.039 2
7	0.624 1	0.999 2	0.034 7	0.375 9	0.000 8	0.965 3	0.590 9	0.793 8	0.034 7
8	0.627 2	0.999 2	0.033 5	0.372 8	0.000 8	0.966 5	0.592 9	0.793 8	0.033 5
9	0.629 5	0.999 2	0.032 3	0.370 5	0.000 8	0.967 7	0.596 0	0.793 8	0.032 3
10	0.635 9	0.999 5	0.029 4	0.364 1	0.000 5	0.970 6	0.600 9	0.793 8	0.029 4

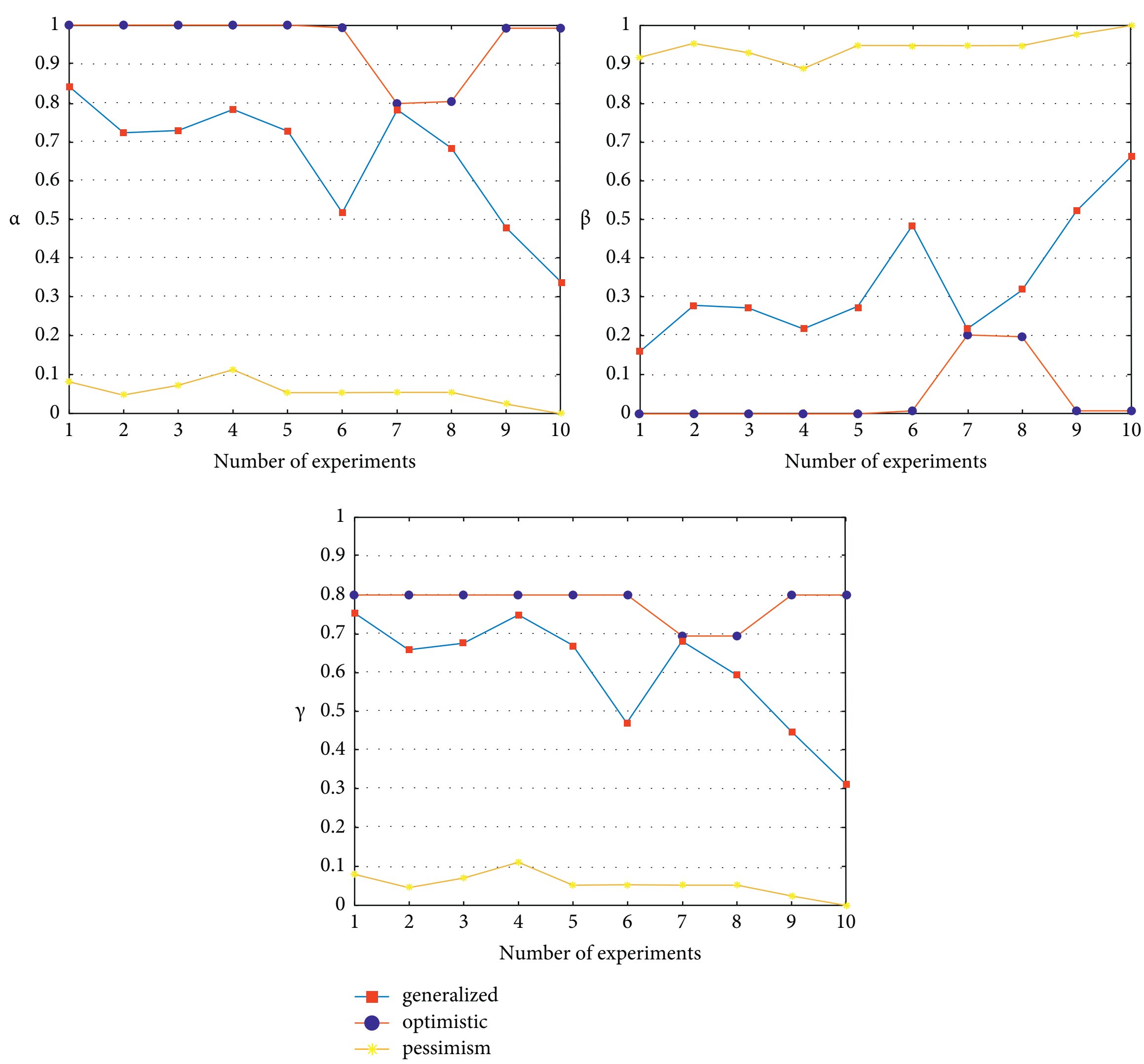

Figure 2: Uncertainty measurement of weighted multigranulation model for “own-data 1.”

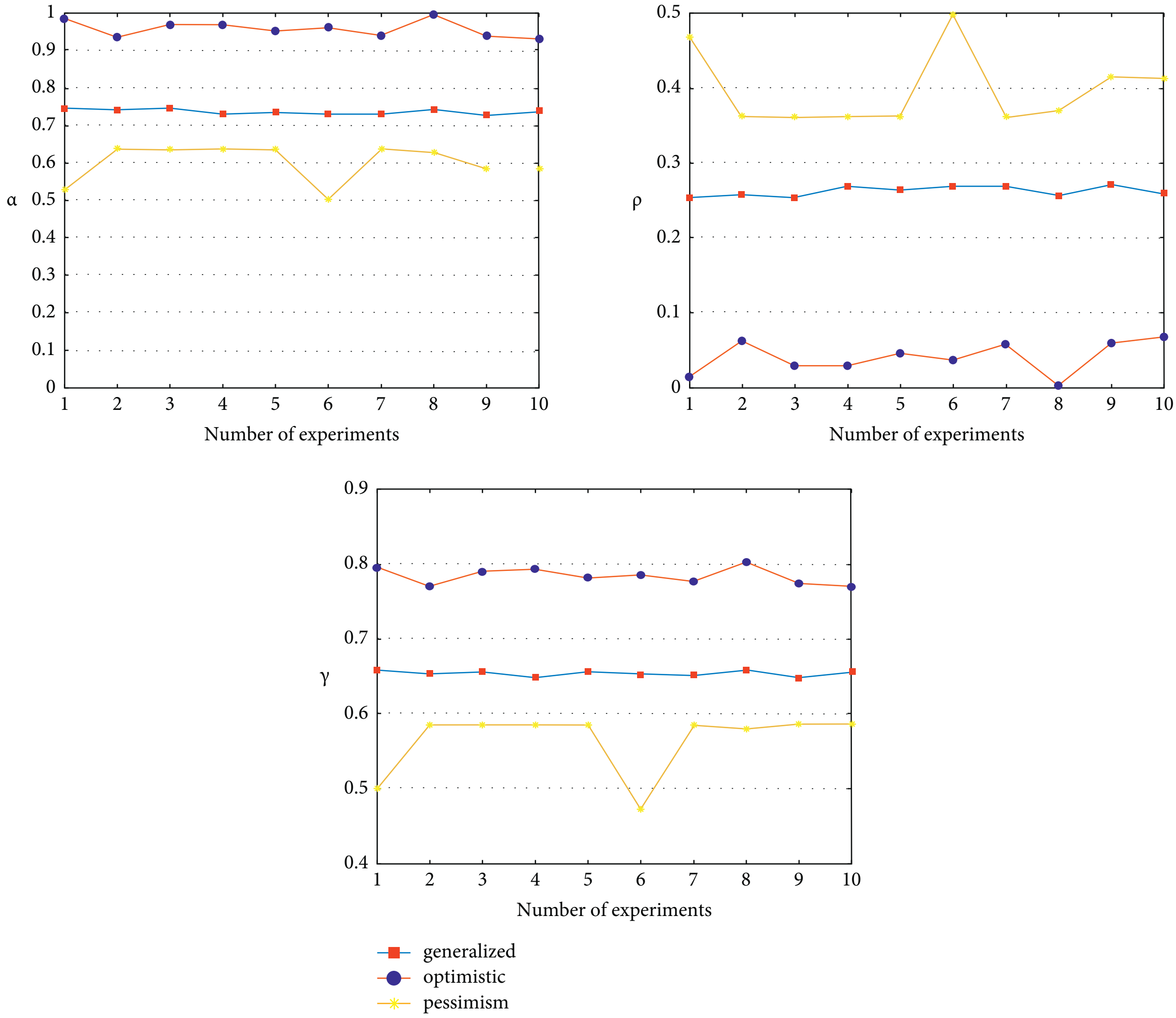

Figure 3: Uncertainty measurement of weighted multigranulation model for "winequality-red."

$$IFIS_i(\mu_x, \nu_x, a) = \begin{cases} IFIS_i(\mu_x + n_i, \nu_x + n_i, a), & 0 \le (\mu_x + n_i) + (\nu_x + n_i) \le 1, \\ IFIS_i(\mu_x, \nu_x, a), & \text{else}. \end{cases} \quad (35)$$

We add random noise as follows:

$$IFIS_i(\mu_x, \nu_x, a) = \begin{cases} IFIS_i(\mu_x + r_i, \nu_x + r_i, a), & 0 \le (\mu_x + r_i) + (\nu_x + r_i) \le 1, \\ IFIS_i(\mu_x, \nu_x, a), & \text{else}. \end{cases} \quad (36)$$

We conducted ten experiments on each data set, assuming that the granularity weights corresponding to the three information systems are assigned as $\omega = \{0.08, 0.04, 0.1, 0.03, 0.07, 0.02, 0.06, 0.2, 0.3, 0.1\}$, with

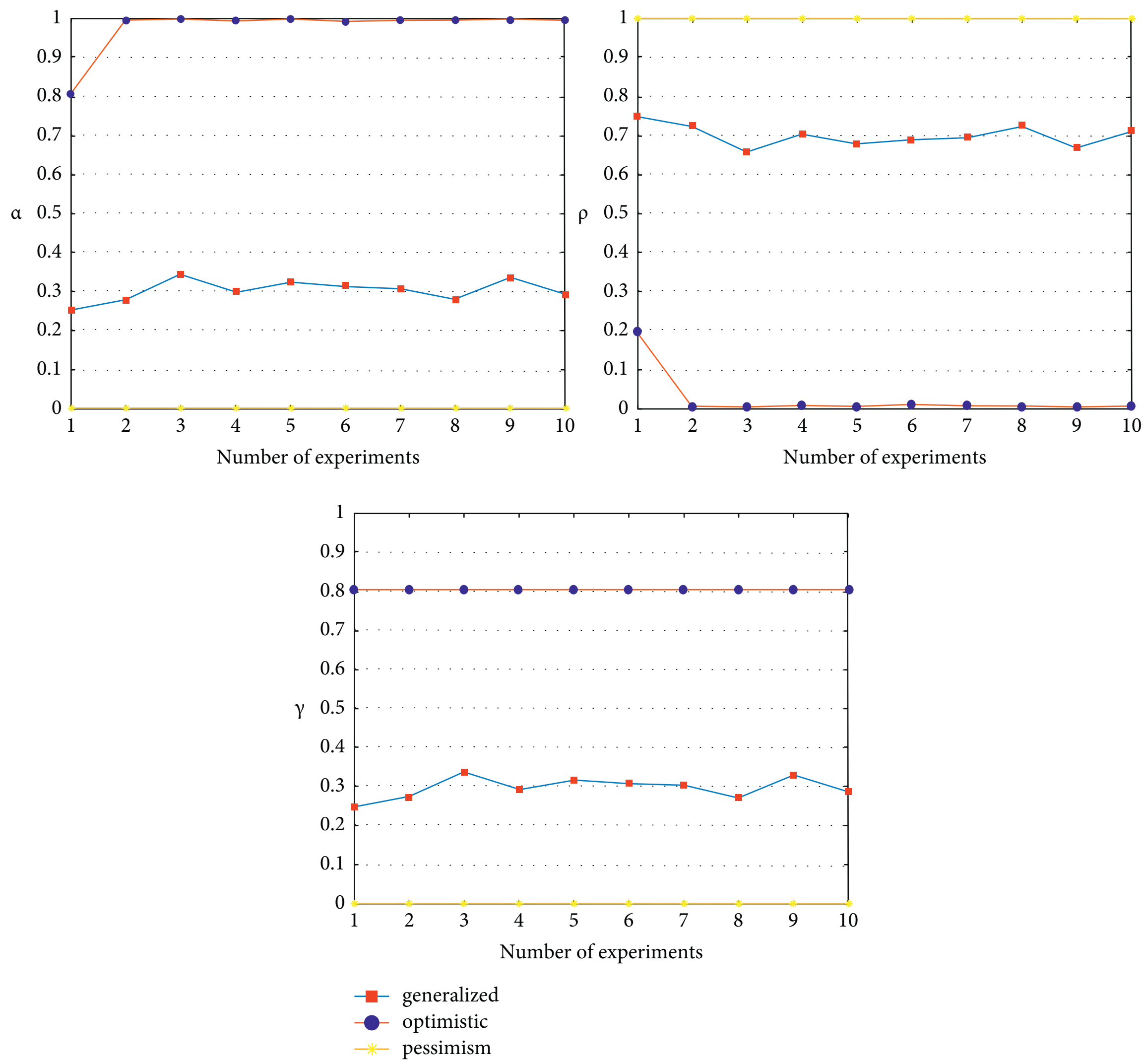

Figure 4: Uncertainty measurement of weighted multigranulation model for “own-data 2.”

threshold $\beta = 0.6$, and the concept set X is randomly generated from the target set. The uncertainty measurement of the three rough set models corresponding to each data set is shown in Tables 6–8, and the results are illustrated inFigure 2–5.

From Figures 2–5, we can find that the weighted pessimistic approximation accuracy is less than or equal to the weighted generalized approximation accuracy, which is also less than or equal to the weighted optimistic approximation accuracy; the weighted optimistic approximation roughness is less than or equal to the weighted generalized approximation roughness, which is also less than or equal to the weighted pessimistic approximation roughness. Similarly, the weighted pessimistic approximation dependence is less than or equal to the weighted generalized approximation dependence, which is also less than or equal to the weighted optimistic approximation dependence.

MSIFIS$'s$ optimistic multigranularity intuitionistic fuzzy rough set model requirements are too loose in selecting objects, but the pessimistic multigranularity intuitionistic fuzzy rough set model requirements are too strict in selecting objects. Therefore, in practical applications, the object selection can be completed by the weighted generalized multigranular intuitionistic fuzzy rough set model.

From Figures 2–5, in the same data set, MSIFIS has different calculation results in different multigranularity intuitionistic fuzzy rough set models. Therefore, in practical applications, we can complete the object selection by changing the threshold β.

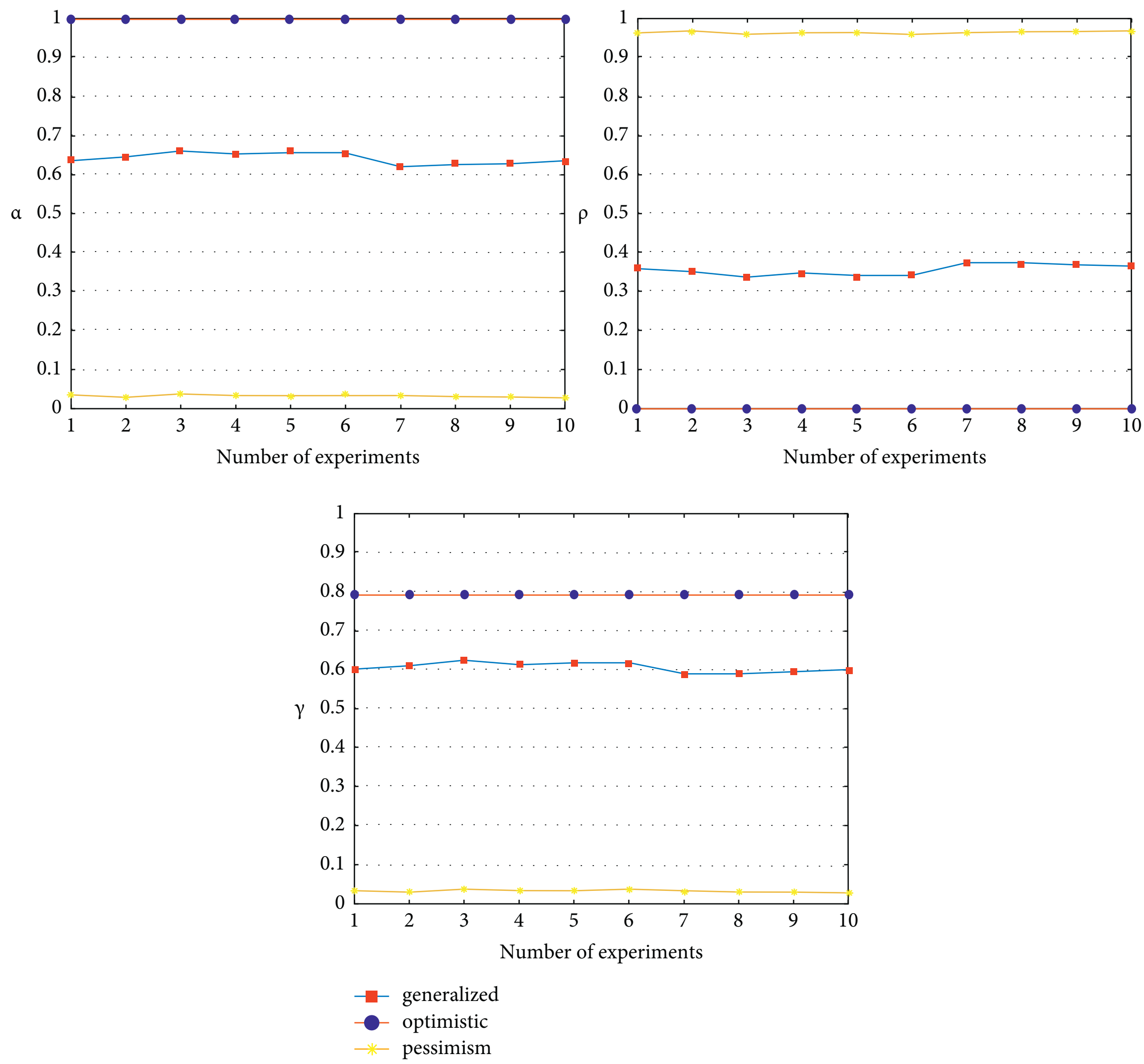

FIGURE 5: Uncertainty measurement of weighted multigranulation model for "winequality-white."

Input: ω, β, MSIFIS $= \{IFIS_1, IFIS_2, \ldots, IFIS_q\}$, $IFIS = \{U, A_i, V_i, IF_i\}$, $X \subseteq U$
Output: $\alpha^{\omega}_{MSIF}(X)_\beta$, $\rho^{\omega}_{MSIF}(X)_\beta$ and $\gamma^{\omega}_{MSIF}(X)_\beta$

```
begin
    for l = 1:q do
      for i = 1:n do
        [x_i]A_l = ∅;
        for j = 1:n do
          flag1 = 1;
          for k = 1:m do
            if μ_{A_l}(x_i, k) ≤ μ_{A_l}(x_j, k) ∧ ν_{A_l}(x_i, k) ≥ ∧ν_{A_l}(x_j, k) then
            flag1 = 1;
          else
                flag1 = 0;
                break;
            end
          end
          f flag1 == 1;
                [x_i]_{A_l} = [x_i]_{A_l} ∪ {x_j};
            end
          end
        end
    end
```

ALGORITHM 2: Continued.

for $i = 1$:n do
  $S_X^{A_l}(i) = 0, S_{X^C}^{A_l}(i) = 0$
  for $l = 1$:q do
    if $[x_i]_{A_l} \subseteq X$ then
    $S_X^{A_l}(i) = S_X^{A_l}(i) + 1;$
  else
    $S_X^{A_l}(i) = S_X^{A_l}(i);$
  end
  if $[x_i]_{A_l} \subseteq X^C$ then
    $S_{X^C}^{A_l}(i) = S_{X^C}^{A_l}(i) + 1;$
  else
    $S_{X^C}^{A_l}(i) = S_{X^C}^{A_l}(i);$
  end
 end
end
$\underline{MSIF}_{\sum_{i=1}^{q} A_l}(X)_\beta \leftarrow \emptyset$; $\overline{MSIF}_{\sum_{i=1}^{q} A_l}(X)_\beta \leftarrow \emptyset$;
for $i = 1$: n do
 if $\sum \omega_i S_X^{A_i}(x) \geq \beta$ then
  $\underline{MSIF}_{\sum_{i=1}^{q} A_l}(X)_\beta = \underline{MSIF}_{\sum_{i=1}^{q} A_l}(X)_\beta \cup x_i;$
 end
 if $\sum \omega_i (1 - S_X^{A_i}(x)) > 1 - \beta$ then
  $\overline{MSIF}_{\sum_{i=1}^{q} A_l}(X)_\beta = \overline{MSIF}_{\sum_{i=1}^{q} A_l}(X)_\beta \cup x_i;$
 end
end
return: $\alpha_{MSIF}^{\omega}(X)_\beta$, $\rho_{MSIF}^{\omega}(X)_\beta$ and $\gamma_{MSIF}^{\omega}(X)_\beta$
end

ALGORITHM 2: Uncertainty measurement of weighted generalized multigranulation intuitionistic fuzzy rough sets for MSIFIS.

6. Conclusions

In this paper, in order to solve the problem of knowledge discovery in the MSIFIS, the weighted multigranulation intuitionistic fuzzy rough set models, combined with the idea of multigranulation, are studied. We also studied the relationship between them. In order to further study the multigranulation intuitionistic fuzzy rough set model in the MSIFIS, the uncertainty measurement methods of different models are discussed. Finally, the effectiveness of the proposed models and methods is verified through an example. In the future, we need to continue to study the granularity weight distribution of multisource intuitionistic fuzzy information system and its application to decision-making.

Data Availability

The original data were obtained from UCI.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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