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Research Article

Novel Development to the Theory of Dombi Exponential Aggregation Operators in Neutrosophic Cubic Hesitant Fuzzy Sets: Applications to Solid Waste Disposal Site Selection

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The neutrosophic cubic hesitant fuzzy set can efficiently handle the complex information in a decision-making problem because it combines the advantages of the neutrosophic cubic set and the hesitant fuzzy set. The algebraic operations based on Dombi norms and co-norms are more flexible than the usual algebraic operations as they involve an operational parameter. First, this paper establishes Dombi algebraic operational laws, score functions, and similarity measures in neutrosophic cubic hesitant fuzzy sets. Then, we proposed Dombi exponential operational laws in which the exponents are neutrosophic cubic hesitant fuzzy values and bases are positive real numbers. To use neutrosophic cubic hesitant fuzzy sets in decision-making, we are developing Dombi exponential aggregation operators in the current study. In the end, we present applications of exponential aggregation operators in multiattribute decision-making problems.

1. Introduction

Decision-making (DM) is one of the crucial problems in real life. Due to population increase and urbanization, the proper disposal of waste is a challenging problem nowadays. The proper disposal of waste is necessary for prevention of viral diseases like typhoid, dengue, and tuberculosis. Aggregation operators are fundamental tools in DM. Different sets and their generalizations like fuzzy set (FS), interval-valued fuzzy set (IVFS), intuitionistic fuzzy set (IFS), hesitant fuzzy set (HFS), neutrosophic set (NS), neutrosophic cubic set (NCS), and several aggregation operators have been defined so far [1–4]. Different researchers [5–8] established similarity measures and other important concepts and successfully applied their models to medical diagnosis and selection criteria. Krohling et al. established different useful techniques to sort out MADM problems [9–11]. Jun et al. combined IVFS and FS to form a cubic set (CS). The CS is a combination of FS and IVFS. CS has become a vital tool to deal with vague data. Several researchers explored algebraic aspects and apparently defined ideal theory in CS [12–15]. Smarandache initiated the concept of indeterminacy and described the notion of the neutrosophic set (NS) [16]. NS consist of three independent components, truth, indeterminacy, and falsehood. This characteristic of NS enables researchers to deal smoothly with inconsistent and vague data. For engineering purposes, the neutrosophic set is strict to [0, 1] and is called the single-valued neutrosophic set (SVNS) [17] presented by Wang et al. The NS was further extended to the interval neutrosophic set (INS) [18]. After the appearance of NS, researchers put their contributions to theoretical as well as technological developments of the set [19–22]. Several researchers use these sets to construct multiattribute decision-making MADM [23–27]. Zhan et al. define aggregation operators and furnished some applications in MADM [28]. Torra defines a hesitant fuzzy set (HFS) [29]. An HFS is basically a function set on X that, when applied to X, returns a subset of [0, 1]. Jun et al. introduced the concept of neutrosophic cubic set (NCS) [30] which consists of both interval-valued neutrosophic and neutrosophic set. These characteristics of neutrosophic cubic sets make it a powerful tool to deal the vague and inconsistent data more efficiently. Soon after its exploration, it attracted researchers to work in many fields like medicine, algebra, engineering, and decision-making theory. Later, the idea of a cubic hesitant fuzzy set was introduced by Mehmood et al. [31]. Ye [32] proposed the concept of the single-valued neutrosophic hesitant fuzzy set (SVNHFS) by combining the advantages of the NS and HFS, which permits its membership functions to have sets of possible values, known as truth, indeterminacy, and falsity membership hesitant functions. Liu and Shi [33] proposed hybrid geometric aggregation operators in INHFS and discuss its presentations in MADM. Zhu et al. [34] proposed the method of β-normalization to add some elements to an HFE, which is a useful technique in case of different cardinalities. Lu and Ye [35] introduced exponential laws in single-valued neutrosophic numbers. Later, the exponential aggregation operators were introduced and applied in typhoon disaster evaluation by Tan et al. [36]. Wang and Li proposed some aggregation operators in pictures hesitant fuzzy sets and compared these operators with some existing decision-making methods [37, 38]. Tan and Zhang introduced trapezoidal fuzzy neutrosophic number arithmetic averaging and hybrid arithmetic averaging for multicriteria decision-making [39]. Feng et al. define type-2 hesitant fuzzy sets and explore some important properties of these sets [40]. Turkarslan et al. proposed the similarity measures in fuzzy multiset with application in medical diagnosis [41].

Dombi [42] established a general class of fuzzy operators. The Dombi T-operators are based on an operational parameter, and hence, the operations based on Dombi operators are more flexible than the other algebraic operations. Shi and Ye [43] established Dombi arithmetic and geometric aggregation operators in neutrosophic cubic sets. Liu et al. [44] established Dombi Bonferroni mean operators of intuitionistic fuzzy sets and discuss their applications in MADM problems. Chen and Ye [45] proposed the Dombi weighted aggregation operators in SVNSs and discuss their applications in MADM problems. Bhattacharya et al. [46] proposed the machine learning model-based approach to discussing the demographic influence of risk behavior of urban investors. Biswas et al. [47] establish an ensemble approach for portfolio selection in a multicriteria decision-making framework. Shi and Ye [43] proposed Dombi aggregation operators in the neutrosophic cubic environments for MADM. Rehman et al. [48] established neutrosophic cubic hesitant fuzzy geometric aggregation operators for MADM problems. Ashraf et al. [49] proposed interval-valued the picture fuzzy Maclaurin symmetric mean operators for MADM problems. Riaz and Farid [50] established picture fuzzy aggregation approach and applied it to the third-party logistic provider selection process. Several researchers [51–54], proposed useful techniques and successfully applied their models to solve decision-making problems.

Motivation: the Dombi exponential laws and Dombi exponential aggregation operators are not defined in the intuitionistic fuzzy set, neutrosophic set, and their extensions. The Dombi T-operators are more flexible than the usual T-operators, and exponential operators are an essential family of aggregation operators. This motivates the researchers to define Dombi exponential aggregation operators so that the inconsistent, hesitant, and vague data can be efficiently handled in the complex work frame where the base is crisp value.

Organization: the rest of this paper is organized as follows: Section 2 deals with some basic definitions used later on. Section 3 discusses Dombi algebraic operational laws and score functions in NCHFSs. In Section 4, we introduced distance and similarity measures in NCHFSs. Exponential operating rules and some useful results in NCHFSs. Section 5 deals with establishing Dombi exponential aggregation operators as NCHFSs. In Section 6, we demonstrate a MADM method based on NCHFDEA operators and use this method in the MADM problem. Section 7 presents the conclusion of the proposed study.

2. Preliminaries

Definition 1 (see [1]). Let $E \neq \varnothing$ be a set. An FS on E is a function $\lambda : E \longrightarrow [0, 1]$.

Definition 2 (see [55]). Let $E \neq \varnothing$ be a set. The CS on E is defined by $\Psi = \{\langle \wp; I(\wp), \lambda(\wp) a\rangle | \wp \in E\}$, where $I(\wp)$ is an IVFS on E and $\lambda(\wp)$ is a FS on E.

Definition 3 (see [17]). An NS on nonempty set E is a set of the form $\{\wp; \langle \Gamma_T(\wp), \Gamma_I(\wp), \Gamma_F(\wp)\rangle | \wp \in E\}$, where $\Gamma_T, \Gamma_I, \Gamma_F$ are mappings from set E to closed interval [0, 1].

Definition 4 (see [30]). An NCS in $E \neq \varnothing$ is a pair $\beta = (B, \hbar)$ where $B = \{\langle \wp; B_T(\wp), B_I(\wp), B_F(\wp)\rangle / \wp \in E\}$ is an INS in E and $\hbar = \{\langle \wp; \hbar_T(\wp), \hbar_I(\wp), \hbar_F(\wp)\rangle / \wp \in E\}$ is an NS in E.

Definition 5 (see [32]). A neutrosophic hesitant fuzzy set (NHFS) on nonempty set E is described by $\hbar = \{\langle \wp; \hbar_T(\wp), \hbar_I(\wp), \hbar_F(\wp)\rangle / \wp \in E\}$, where $\hbar_T(\wp), \hbar_I(\wp), \hbar_F(\wp)$ are three hesitant fuzzy sets such that $\hbar_T(\wp) + \hbar_I(\wp), +\hbar_F(\wp) \leq 3$.

Definition 6 (see [33]). Consider a nonempty set E. An interval neutrosophic hesitant fuzzy set (INHFS) on E is given by the object $B = \{\langle \wp; B_T(\wp), B_I(\wp), B_F(\wp)\rangle / \wp \in E\}$, where $B_T(\wp), B_I(\wp), B_F(\wp)$ are intervalued hesitant fuzzy sets.

The following technique of β-normalization proposed by Zhu et al. is a useful technique in case of different cardinalities of two hesitant fuzzy sets.

Definition 7 (see [34]). Let ϖ^+ and ϖ^- be respectively the maximum and minimum values in a hesitant fuzzy set H and $\Theta\,(0 \le \Theta \le 1)$ be an optimized parameter; then, an element can be added to H as $\varpi = \varpi^+ + (1-\Theta)\varpi$.

Definition 8 (see [42]). The Dombi norms and dual conorms are given, respectively, as

$$D(s,t) = \frac{1}{1+\left((1-s/s)^\delta + (1-t/t)^\delta\right)^{(1/\delta)}},$$
$$D^*(s,t) = 1 - \frac{1}{1+\left((s/1-t)^\delta + (t/1-t)^\delta\right)^{(1/\delta)}}, \tag{1}$$

where $(a,b) \in (0,1)\times(0,1)$ and $\delta \in R$ is an operational parameter. Furthermore, the values of D and D^* for boundaries are defined as $D(1,0) = D(0,1) = D(0,0) = 0, D(1,1) = 1, D^*(1,1) = D^*(1,0) = D^*(0,1) = 1$ and $D^*(0,0) = 0$.

In the next section, first, we define Dombi algebraic operational laws by using Dombi T-norms and T-conorms in NCHFSs. Second, the score functions are defined for the comparison of two NCHFSs.

3. Dombi Operational Laws in Neutrosophic Cubic Hesitant Fuzzy Set

Definition 9. A neutrosophic cubic hesitant fuzzy set (NCHFS) on a nonempty set E is a pair $\alpha = (A, \hbar)$, where $A = \langle e; A_T(e), A_I(e), A_F(e)\rangle$ is an INHFS in E and $\hbar = \langle e; \hbar_T(e), \hbar_I(e), \hbar_F(e)\rangle$ is an NHFS in E. Furthermore $_T(e) = \left\{[^L_{j_T}(e), ^U_{j_T}(e)]; j = 1, \ldots, l_e\right\},_I(e) = \left\{[^L_{j_T}(e), ^U_{j_T}(e)]; j = 1, \ldots, m_e\right\}$, and $A_F(e) = \left\{[^L_{j_T}(e), ^U_{j_F}(e)]; i = 1, \ldots, n_e\right\}$ are some interval values in [0, 1] with the property that for each $j 0 \le ^U_{j_T}(x) + ^U_{j_I}(x) + ^U_{j_F}(x) \le 3$, and $_T(e) = \left\{_{j_T}(e); j = 1, \ldots, r_e\right\}, _I(e) = \left\{_{j_I}(e); j = 1, \ldots, s_e\right\}, _F(e) = \left\{_{j_F}(e); j = 1, \ldots, t_e\right\}$ are some values in [0, 1] satisfying $0 \le (e) +_{j_I}(e) +_{j_F}(e) \le 3$.

Example 1. Let $E = \{e_1, e_2, e_3\}$. The pair $\alpha = (A, \lambda)$ with

$$\begin{aligned}
&A_T(e_1) = \{[0.1, 0.5], [0.2, 0.7]\}, \hbar_T(e_1) = \{0.3, 0.5, 0.7\},\\
&A_I(e_1) = \{[0.2, 0.4], [0.3, 0.6]\}, \hbar_I(e_1) = \{0.1, 0.4, 0.7\},\\
&A_F(e_1) = \{[0.1, 0.4], [0, 0.3], [0.6, 0.8]\}, \hbar_F(e_1) = \{0.4, 0.6\},\\
&A_T(e_2) = \{[0.1, 0.5], [0.2, 0.7]\}, \hbar_T(e_2) = \{0.3, 0.5\},\\
&A_I(e_2) = \{[0.2, 0.3], [0.1, 0.6]\}, \hbar_I(e_2) = \{0.7, 0.8\},\\
&A_F(e_2) = \{[0.1, 0.4], [0, 0.3]\}, \hbar_F(e_2) = \{0.4, 0.6\},\\
&A_T(e_3) = \{[0.1, 0.5], [0.2, 0.7]\}, \hbar_T(e_3) = \{0.3, 0.5\},\\
&A_I(e_3) = \{[0.2, 0.3], [0.1, 0.6]\}, \hbar_I(e_3) = \{0.7, 0.8\},\\
&A_F(e_3) = \{[0.1, 0.4], [0, 0.3]\}, \hbar_F(e_3) = \{0.4, 0.6\},
\end{aligned} \tag{2}$$

is a NCHFS.

Definition 10. For two NCHF values $\alpha = (A, \lambda), \beta = (B, \mu)$ and scalars $q, \sigma > 0$, we define, respectively, the Dombi sum, Dombi product, Dombi scalar multiplication, and the Dombi qth power of α as

$$\alpha \oplus \beta = \left(\begin{array}{c}
\left\{\left[1-\left(1+\left(\left(\frac{A^L_{j_T}}{1-A^L_{j_T}}\right)^\sigma+\left(\frac{B^L_{j_T}}{1-B^L_{j_T}}\right)^\sigma\right)^{1/\sigma}\right)^{-1}, 1-\left(1+\left(\left(\frac{A^U_{j_T}}{1-A^U_{j_T}}\right)^\sigma+\left(\frac{B^U_{j_T}}{1-B^U_{j_T}}\right)^\sigma\right)1/\sigma\right)^{-1}\right]\right\},\\
\left\{\left[1-\left(1+\left(\left(\frac{A^L_{j_I}}{1-A^L_{j_I}}\right)^\sigma+\left(\frac{B^L_{j_I}}{1-B^L_{j_I}}\right)^\sigma\right)1/\sigma\right)^{-1}, 1-\left(1+\left(\left(\frac{A^U_{j_T}}{1-A^U_{j_T}}\right)^\sigma+\left(\frac{B^U_{j_T}}{1-B^U_{j_T}}\right)^\sigma\right)1/\sigma\right)^{-1}\right]\right\},\\
\left\{\left[\left(1+\left(\left(\frac{1-A^L_{j_F}}{A^L_{j_F}}\right)^\sigma+\left(\frac{1-B^L_{j_F}}{B^L_{j_F}}\right)^\sigma\right)1/\sigma\right)^{-1}, \left(1+\left(\left(\frac{1-A^U_{j_F}}{A^U_{j_F}}\right)^\sigma+\left(\frac{1-B^U_{j_F}}{B^U_{j_F}}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right]\right\},\\
\left\{1-\left(1+\left(\left(\frac{\lambda_{j_T}}{1-\lambda_{j_T}}\right)^\sigma+\left(\frac{\mu_{j_T}}{1-\mu_{j_T}}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right\},\\
\left\{1-\left(1+\left(\left(\frac{\lambda_{j_I}}{1-\lambda_{j_I}}\right)^\sigma+\left(\frac{\mu_{j_I}}{1-\mu_{j_I}}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right\}, \left\{\left(1+\left(\left(\frac{1-\lambda_{j_F}}{\lambda_{j_F}}\right)^\sigma+\left(\frac{1-\mu_{j_F}}{\mu_{j_F}}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right\}
\end{array}\right),$$

$$q\alpha=\left(\begin{array}{c}\left\{\left[1-\left(1+\left(q\left(\frac{A^L_{j_T}}{1-A^L_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},1-\left(1+\left(q\left(\frac{A^U_{j_T}}{1-A^U_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\},\\ \left\{\left[1-\left(1+\left(q\left(\frac{A^L_{j_I}}{1-A^L_{j_I}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},1-\left(1+\left(q\left(\frac{A^U_{j_T}}{1-A^U_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\},\\ \left\{\left[\left(1+\left(q\left(\frac{1-A^L_{j_F}}{A^L_{j_F}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},\left(1+\left(q\left(\frac{1-A^U_{j_F}}{A^U_{j_F}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\},\\ \left\{1-\left(1+\left(q\left(\frac{\lambda_{j_T}}{1-\lambda_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\},\left\{1-\left(1+\left(q\left(\frac{\lambda_{j_I}}{1-\lambda_{j_I}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\},\left\{\left(1+\left(q\left(\frac{1-\lambda_{j_F}}{\lambda_{j_F}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\}\end{array}\right),$$

$$\alpha^q=\left(\begin{array}{c}\left\{\left[\left(1+\left(q\left(\frac{1-A^L_{j_T}}{A^L_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},\left(1+\left(q\left(\frac{1-A^U_{j_T}}{A^U_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\},\\ \left\{\left[\left(1+\left(q\left(\frac{1-A^L_{j_I}}{A^L_{j_I}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},\left(1+\left(q\left(\frac{1-A^U_{j_T}}{A^U_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\},\\ \left\{\left[1-\left(1+\left(q\left(\frac{A^L_{j_F}}{1-A^L_{j_F}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},1-\left(1+\left(q\left(\frac{A^U_{j_F}}{1-A^U_{j_F}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\},\\ \left\{\left(1+\left(q\left(\frac{1-\lambda_{j_T}}{\lambda_{j_T}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\},\left\{\left(1+\left(q\left(\frac{1-\lambda_{j_I}}{\lambda_{j_I}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\},\left\{1-\left(1+\left(q\left(\frac{\lambda_{j_F}}{1-\lambda_{j_F}}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\}\end{array}\right). \quad (3)$$

Moreover, the β-normalization is used in case of different cardinalities.

Example 11. If $\alpha=(\{[0.1,0.2],[0.3,0.4]\},\{[0,.1,0.3],[0,0.2]\},\{[0.6,0.7],[0.3,0.5]\},\{0.1,0.2\},\{0.2,0.3\},\{0.6,0.7\})$ and $\beta=(\{[0.6,0.8],[0.5,0.7]\},\{[0.5,0.6],[0.6,0.7]\},\{[0.1,0.2],[0,0.2]\},\{0.6,0.7\},\{0.5,0.6\},\{0.2,0.3\})$, then for $\sigma=2$,

$$\begin{aligned}\alpha\oplus\beta&=\left(\begin{array}{c}\{[0.600656,0.800312],[0.521065,0.708174]\},\{[0.501534,0.609379],[0.6,0.701197]\},\\ \{[0.099754,0.199088],[0,0.296528]\},\\ \{0.600656,\ 0.701197\},\{0.507577,0.609379\},\{0.197817,0.296528\}\end{array}\right),\\ \alpha\otimes\beta&=\left(\begin{array}{c}\{[0.099754,0.199688],[0.282598,0.390621]\},\{[0.099449,0.291826],[0,0.199088]\},\\ \{[0.600656,0.701197],[0.3,0.507577]\},\\ \{0.099754,0.199088\},\{0.195194,0.291826\},\{0.603283,0.703472\},\end{array}\right),\\ 3\alpha&=\left(\begin{array}{c}\{[0.16139,0.302169],[0.426048,0.535898]\},\{[0.16139,0.426048],[0,0.302169]\},\\ \{[0.464102,0.573952],[0.198356,0.366025]\},\\ \{0.16139,0.302169\},\{0.302169,0.426048\},\{0.464102,0.573952\}\end{array}\right),\\ \alpha^3&=\left(\begin{array}{c}\{[0.060283,0.126132],[0.198356,0.277926]\},\{[0.060283,0.198356],[0,0.126132]\},\\ \{[0.722074,0.801644],[0.426048,0.633975]\},\\ \{0.060283,0.126132\},\{0.126132,0.198356\},\{0.722074,0.801644\}\end{array}\right).\end{aligned} \quad (4)$$

It is very important to discuss the comparison of NCHF values. For comparison of NCHF values, we propose score functions in next section.

Definition 12. Score, accuracy, and certainty.

The score, accuracy, and certainty of a NCHF value $\alpha = (A, \lambda)$, where $A = A_T, A_I, A_F, A_T = \{[A^L_{j_T}, A^U_{j_T}];\ j = 1, \ldots, l\}, A_I = \{[A^L_{j_I}, A^U_{j_I}];\ j = 1, \ldots, m\}, A_F = \{[A^L_{j_F}, A^U_{j_F}];\ i = 1, \ldots, n\}$ and $\lambda = \lambda_T, \lambda_I, \lambda_F, \lambda_T = \{\lambda_{j_T};\ j = 1, \ldots, r\}, \lambda_I = \{\lambda_{j_I};\ j = 1, \ldots, s\}, \lambda_F = \{\lambda_{j_F};\ j = 1, \ldots, t\}$ are defined as

$$S(\alpha) = \frac{1}{2}\left\{\begin{array}{c} \dfrac{1}{6}\left(\dfrac{1}{l}\displaystyle\sum_{j=1}^{l}\left(A^L_{j_T} + A^U_{j_T}\right) + \dfrac{1}{m}\sum_{j=1}^{m}\left(A^L_{j_I} + A^U_{j_I}\right) + \dfrac{1}{n}\sum_{j=1}^{n}\left(2 - \left(A^L_{j_F} + A^U_{j_F}\right)\right)\right) \\ + \dfrac{1}{3}\left(\dfrac{1}{r}\displaystyle\sum_{j=1}^{r}\lambda_{j_T} + \dfrac{1}{s}\sum_{j=1}^{s}\lambda_{j_I} + \dfrac{1}{t}\sum_{j=1}^{t}\left(1 - \lambda_{j_F}\right)\right) \end{array}\right\},$$

$$H(\alpha) = \frac{1}{9}\left\{\frac{1}{l}\sum_{j=1}^{l}\left(A^L_{j_T} + A^U_{j_T}\right) - \frac{1}{n}\sum_{j=1}^{n}\left(A^L_{j_F} + A^U_{j_F}\right) + \frac{1}{r}\sum_{j=1}^{r}\lambda_{j_T} - \frac{1}{t}\sum_{j=1}^{t}\lambda_{j_F}\right\}, \tag{5}$$

$$C(\alpha) = \frac{1}{3}\left\{\frac{1}{l}\sum_{j=1}^{l}\left(A^L_{j_T} + A^U_{j_T}\right) + \frac{1}{r}\sum_{j=1}^{r}\lambda_{j_T}\right\}.$$

The score function is a useful tool for ranking the NCHF values. An NCHF value is greater if the interval hesitant and hesitant degrees of truth and indeterminacy membership grades are bigger and the interval hesitant and hesitant degrees of falsity membership grades are smaller. The greater the difference between truth and falsity memberships, the accuracy is higher and the corresponding NCHF value becomes greater. The certainty of an NCHF value depends on the interval hesitant and hesitant degrees of truth membership grades. The bigger the interval between hesitant and hesitant degrees of truth and indeterminacy membership grades, the greater the NCHF value is.

The following technique is use for ranking the values.

If $\alpha = (A, \lambda)$, $\beta = (B, \mu)$ be two NCHF values then we say that $\alpha \succ \beta$ if and only if $S(\alpha) \succ S(\beta)$.

If $S(\alpha) = S(\beta)$, we compare them on the basis of accuracy. If they have same accuracies too, we define the above comparison on the basis of certainty.

Example 2. If $\alpha = (\{[0.1, 0.2], [0.3, 0.4]\}, \{[0, .1, 0.3], [0, 0.2]\}, \{[0.6, 0.7], [0.3, 0.5]\}, \{0.1, 0.2\}, \{0.2, 0.3\}, \{0.6, 0.7\})$ and $\beta = (\{[0.6, 0.8], [0.5, 0.7]\}, \{[0.5, 0.6], [0.6, 0.7]\}, \{[0.1, 0.2], [0, 0.2]\}, \{0.6, 0.7\}, \{0.5, 0.6\}, \{0.2, 0.3\})$, then

$$S(\alpha) = 0.270833, S(\beta) = 0.679167, H(\alpha) = 0.322222, H(\beta) = 0.466667, C(\alpha) = 0.216667, C(\beta) = 0.65. \tag{6}$$

The distances and similarities are important notions in theory of sets. These two concepts are important tools to describe the degree of closeness in sets. Keeping in mind the importance of these two notions, we define these in the neutrosophic cubic hesitant fuzzy structure.

4. Distance and Similarity Measures in Neutrosophic Cubic Hesitant Fuzzy Sets

Definition 13. A distance measure is a mapping such that

$$\begin{aligned} &(i)\ 0 \le d(\alpha, \beta) \le 1, \\ &(ii)\ d(\alpha, \beta) = 0 \Leftrightarrow \alpha = \beta, \\ &(iii)\ d(\beta, \alpha) = d(\alpha, \beta), \\ &(\text{iv})\ \text{If } \alpha \subseteq \beta \subset \gamma, \text{then } d(\alpha, \gamma) \\ &\qquad \ge d(\alpha, \beta) \text{ and } d(\alpha, \gamma) \ge d(\beta, \gamma). \end{aligned} \tag{7}$$

If $\alpha = (A, \lambda)$, $\beta = (B, \mu)$ are two NCHFS on a set $X = \{x_1, \ldots, x_t\}$, then we define the following.

Normalized Hamming Distance:

$$d_H(\alpha,\beta)=\frac{1}{9t}\left\{\sum_{i=1}^{t}\left(\begin{array}{c}\frac{1}{l_i}\sum_{j=1}^{l_i}\left(\left|A_{j_T}^U(x_i)-B_{j_T}^U(x_i)\right|+\left|A_{j_T}^L(x_i)-B_{j_T}^L(x_i)\right|\right)\\ +\frac{1}{m_i}\sum_{j=1}^{m_i}\left(\left|A_{j_I}^U(x_i)-B_{j_I}^U(x_i)\right|+\left|A_{j_I}^L(x_i)-B_{j_I}^L(x_i)\right|\right)\\ +\frac{1}{n_i}\sum_{j=1}^{n_i}\left(\left|A_{j_F}^U(x_i)-B_{j_F}^U(x_i)\right|+\left|A_{j_F}^L(x_i)-B_{j_F}^L(x_i)\right|\right)\\ +\frac{1}{p_i}\sum_{j=1}^{p_i}\left|\lambda_{j_T}(x_i)-\mu_{j_T}(x_i)\right|+\frac{1}{q_i}\sum_{j=1}^{q_i}\left|\lambda_{j_I}(x_i)-\mu_{j_I}(x_i)\right|+\frac{1}{r_i}\sum_{j=1}^{r_i}\left|\lambda_{j_F}(x_i)-\mu_{j_F}(x_i)\right|\end{array}\right)\right\}. \tag{8}$$

Normalized Euclidean Distance:

$$d_E(\alpha,\beta)=\left(\frac{1}{9t}\left\{\sum_{i=1}^{t}\left(\begin{array}{c}\frac{1}{l_i}\sum_{j=1}^{l_i}\left(\left|A_{j_T}^U(x_i)-B_{j_T}^U(x_i)\right|^2+\left|A_{j_T}^L(x_i)-B_{j_T}^L(x_i)\right|^2\right)\\ +\frac{1}{m_i}\sum_{j=1}^{m_i}\left(\left|A_{j_I}^U(x_i)-B_{j_I}^U(x_i)\right|^2+\left|A_{j_I}^L(x_i)-B_{j_I}^L(x_i)\right|^2\right)\\ +\frac{1}{n_i}\sum_{j=1}^{n_i}\left(\left|A_{j_F}^U(x_i)-B_{j_F}^U(x_i)\right|^2+\left|A_{j_F}^L(x_i)-B_{j_F}^L(x_i)\right|^2\right)\\ +\frac{1}{p_i}\sum_{j=1}^{p_i}\left|\lambda_{j_T}(x_i)-\mu_{j_T}(x_i)\right|^2+\frac{1}{q_i}\sum_{j=1}^{q_i}\left|\lambda_{j_I}(x_i)-\mu_{j_I}(x_i)\right|^2+\frac{1}{r_i}\sum_{j=1}^{r_i}\left|\lambda_{j_F}(x_i)-\mu_{j_F}(x_i)\right|^2\end{array}\right)\right\}\right)^{1/2}. \tag{9}$$

Normalized Generalized Distance:

$$d_G(\alpha,\beta)=\left(\frac{1}{9t}\left\{\sum_{i=1}^{t}\left(\begin{array}{c}\frac{1}{l_i}\sum_{j=1}^{l_i}\left(\left|A_{j_T}^U(x_i)-B_{j_T}^U(x_i)\right|^q+\left|A_{j_T}^L(x_i)-B_{j_T}^L(x_i)\right|^q\right)\\ +\frac{1}{m_i}\sum_{j=1}^{m_i}\left(\left|A_{j_I}^U(x_i)-B_{j_I}^U(x_i)\right|^q+\left|A_{j_I}^L(x_i)-B_{j_I}^L(x_i)\right|^q\right)\\ +\frac{1}{n_i}\sum_{j=1}^{n_i}\left(\left|A_{j_F}^U(x_i)-B_{j_F}^U(x_i)\right|^q+\left|A_{j_F}^L(x_i)-B_{j_F}^L(x_i)\right|^q\right)\\ +\frac{1}{p_i}\sum_{j=1}^{p_i}\left|\lambda_{j_T}(x_i)-\mu_{j_T}(x_i)\right|^q+\frac{1}{q_i}\sum_{j=1}^{q_i}\left|\lambda_{j_I}(x_i)-\mu_{j_I}(x_i)\right|^q+\frac{1}{r_i}\sum_{j=1}^{r_i}\left|\lambda_{j_F}(x_i)-\mu_{j_F}(x_i)\right|^q\end{array}\right)\right\}\right)^{1/q}, q \succ 0. \tag{10}$$

Definition 14. A similarity measure is mapping sm: $NCHFS \times NCHFS \longrightarrow [0,1]$, such that

$$\begin{aligned}&\text{(i) } 0 \leq sm(\alpha,\beta) \leq 1,\\&\text{(ii) } sm(\alpha,\beta) = 1 \Leftrightarrow \alpha = \beta,\\&\text{(iii) } sm(\alpha,\beta) = sm(\beta,\alpha),\\&\text{(iv) If } \alpha \subseteq \beta \subset \gamma, \text{ then } sm(\alpha,\gamma)\\&\leq sm(\alpha,\beta) \text{ and } sm(\alpha,\gamma) \leq sm(\beta,\gamma).\end{aligned} \tag{11}$$

Theorem 1. *If $d(\alpha,\beta)$ is a distance measure, then $sm(\alpha,\beta) = 1 - d(\alpha,\beta)$ is a similarity measure.*

Using the above theorem, the similarity measures based on distance measures defined in section 3.5.1 section 3.5.1 are as follows:

$$\begin{aligned}&sm_H(\alpha,\beta) = 1 - d_H(\alpha,\beta),\\&sm_E(\alpha,\beta) = 1 - d_E(\alpha,\beta), sm_G(\alpha,\beta) = 1 - d_G(\alpha,\beta).\end{aligned} \tag{12}$$

Remark 1. We can order the NCHF values by using similarity measures as follows:

Consider the NCHF values $\alpha = (A,\lambda)$, $\beta = (B,\mu)$ and corresponding maximum ideal NCHF value $\Omega = (\{[1,1]\}, \{1,1\}\{0,0\}, \{1\}, \{1\}, \{0\})$.

$$\begin{aligned}&\text{If } sm(\alpha,\Omega) \succ sm(\beta,\Omega), \text{ then } \alpha \succ \beta,\\&\text{If } sm(\alpha,\Omega) = sm(\beta,\Omega), \text{ then } \alpha = \beta.\end{aligned} \tag{13}$$

In the next section, we define novel Dombi exponential laws in which bases are crisp values and Table 1 exponents are NCHF values. Using these Dombi exponential laws, we define Dombi exponential aggregation operators in NCHFSs.

Example 3. If $\alpha = (\{[0.1,0.2],[0.3,0.4]\},\{[0,.1,0.3],[0,0.2]\}, \{[0.6,0.7],[0.3,0.5]\},\{0.1,0.2\},\{0.2,0.3\},\{0.6,0.7\})$ and $\beta = (\{[0.6,0.8],[0.5, \quad 0.7]\},\{[0.5,0.6],[0.6,0.7]\}, \{[0.1,0.2],[0,0.2]\},\{0.6,0.7\},\{0.5,0.6\},\{0.2,0.3\})$, then using (10) and (12), we have Table 1 of similarities:

5. Dombi Exponential Laws and Aggregation Operators

Definition 15. For an NCHFS $\alpha = (A, \lambda)$, scalar $q \in (0,1)$ and $\sigma > 0$, we define

$$q^{\alpha} = \begin{pmatrix} \left\{\left[\left(1+\left(\left(1-A^L_{j_T}\right)\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}, \left(1+\left(\left(1-A^U_{j_T}\right)\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[\left(1+\left(\left(1-A^L_{j_I}\right)\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}, \left(1+\left(\left(1-A^U_{j_T}\right)\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[1-\left(1+\left(A^L_{j_F}\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}, 1-\left(1+\left(A^U_{j_F}\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left(1+\left(\left(1-\lambda_{j_T}\right)\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\}, \left\{\left(1+\left(\left(1-\lambda_{j_I}\right)\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\}, \\ \left\{1-\left(1+\left(\lambda_{j_F}\left(\frac{1-q}{q}\right)\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\} \end{pmatrix}. \tag{14}$$

Note that if $\alpha > \beta$, then $q^{\alpha} > q^{\beta}$.

Table 1: Similarities of given values with maximum ideal.

	$sm_G(\alpha, \Omega)$	$sm_G(\beta, \Omega)$	Ranking (using Equation (12))
$q = 1$	0.277778 0	688889	$\beta > \alpha$
$q = 2$	0.25988	0.660065	$\beta > \alpha$
$q = 3$	0.244891	0.640545	$\beta > \alpha$
$q = 4$	**0.231861**	0.625411	$\beta > \alpha$
$q = 5$	0.220223	0.613145	$\beta > \alpha$

Theorem 2. *For an NCHFN $\alpha = (A, \lambda)$ and a scalar $q > 0$, q^α is a NCHFN.*

Theorem 3. *For two NCHFNs $\alpha = (A, \lambda)$, $\beta = (B, \mu)$ and a scalar $\varpi \in (0,1)$, the following commutative laws hold:*

$$\begin{aligned} &(\mathrm{i})\ \varpi^\alpha \oplus \varpi^\beta = \varpi^\beta \oplus \varpi^\alpha, \\ &(\mathrm{ii})\ \varpi^\alpha \otimes \varpi^\beta = \varpi^\beta \otimes \varpi^\alpha, \end{aligned} \tag{15}$$

Theorem 4. *For three NCHFNs $\alpha = (A, \lambda)$, $\beta = (B, \mu)$, $\gamma = (C, \nu)$ and a scalar $\varpi \in (0,1)$, the following associative laws hold:*

$$\begin{aligned} &(\mathrm{i})\ \left(\varpi^\alpha \oplus \varpi^\beta\right) \oplus \varpi^\gamma = \varpi^\beta \oplus \left(\varpi^\alpha \oplus \varpi^\gamma\right), \\ &(\mathrm{ii})\ \left(\varpi^\alpha \otimes \varpi^\beta\right) \otimes \varpi^\gamma = \varpi^\beta \otimes \left(\varpi^\alpha \otimes \varpi^\gamma\right). \end{aligned} \tag{16}$$

Definition 16. Dombi exponential aggregation operators

Let $\left\{\alpha_{(k)} = \left(A_{(k)}, \lambda_{(k)}\right)\right\}$ be a collection of NCHFNs and $q_k \in (0,1)$ be real numbers, then neutrosophic cubic hesitant fuzzy Dombi weighted exponential operator is defined as

$$NCHDWE(\alpha_1, \ldots, \alpha_n) = \bigotimes_{j=1}^{n} q^\alpha, \tag{17}$$

where $\alpha_{(k)} = \langle A_{(k)}, \lambda_{(k)} \rangle$ is the exponential weight of attribute values $q_k \in (0,1)$.

Theorem 5. *Let $\left\{\alpha_{(k)} = \left(A_{(k)}, \lambda_{(k)}\right)\right\}$ be a collection of NCHFS and $q_k \in (0,1)$ be real numbers, then neutrosophic cubic hesitant fuzzy Dombi weighted exponential operator is*

$$NCHFDWE(\alpha_1, \alpha_2, \ldots, \alpha_n) = \begin{pmatrix} \left\{\left[\left(1 + \left(\sum_{r=1}^{n}\left(1 - A^L_{j_{T_{(r)}}}\right)\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}, \left(1 + \left(\sum_{r=1}^{n}\left(1 - A^U_{j_{T_{(r)}}}\right)\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[\left(1 + \left(\sum_{r=1}^{n}\left(1 - A^L_{j_{I_{(r)}}}\right)\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}, \left(1 + \left(\sum_{r=1}^{n}\left(1 - A^U_{j_{I_{(r)}}}\right)\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[1 - \left(1 + \left(\sum_{r=1}^{n} A^L_{j_{F_{(r)}}}\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}, 1 - \left(1 + \left(\sum_{r=1}^{n} A^U_{j_{F_{(r)}}}\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left(1 + \left(\sum_{r=1}^{n}\left(1 - \lambda_{j_{T_{(r)}}}\right)\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right\}, \left\{\left(1 + \left(\sum_{r=1}^{n}\left(1 - \lambda_{j_{I_{(r)}}}\right)\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right\}, \\ \left\{1 - \left(1 + \left(\sum_{r=1}^{n} \lambda_{j_{F_{(r)}}}\left(\frac{1-q_r}{q_r}\right)^\sigma\right)^{1/\sigma}\right)^{-1}\right\} \end{pmatrix}, \tag{18}$$

and $\left\{\alpha_{(k)} = \langle A_{(k)}, \lambda_{(k)} \rangle\right\}$ are the exponential weights of $q_k \in (0,1)$. Furthermore, NCHF DW $E(\alpha_1, \alpha_2, \ldots, \alpha_m)$ is also a neutrosophic cubic hesitant fuzzy value.

Proof. We use induction. For $n=2$, we have

$$NCHFDWE(\alpha_1,\alpha_2)$$
$$=\begin{pmatrix} \left\{\left[\left(1+\left(\left(1-A^L_{j_{T(1)}}\right)\left(\frac{1-q_1}{q_1}\right)^{\sigma}+\left(1-A^L_{j_{T(2)}}\right)\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},\left(1+\left(\left(1-A^U_{j_{T(1)}}\right)\left(\frac{1-q_1}{q_1}\right)^{\sigma}+\left(1-A^U_{j_{T(2)}}\right)\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[\left(1+\left(\left(1-A^L_{j_{I(1)}}\right)\left(\frac{1-q_1}{q_1}\right)^{\sigma}+\left(1-A^L_{j_{I(2)}}\right)\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},\left(1+\left(\left(1-A^U_{j_{I(1)}}\right)\left(\frac{1-q_1}{q_1}\right)^{\sigma}+\left(1-A^U_{j_{I(2)}}\right)\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[1-\left(1+\left(A^L_{j_{F(1)}}\left(\frac{1-q_1}{q_1}\right)^{\sigma}+A^L_{j_{F(2)}}\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},1-\left(1+\left(A^L_{j_{F(1)}}\left(\frac{1-q_1}{q_1}\right)^{\sigma}+A^L_{j_{F(2)}}\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left(1+\left(\left(1-\lambda_{j_{T(1)}}\right)\left(\frac{1-q_1}{q_1}\right)^{\sigma}+\left(1-\lambda_{j_{T(2)}}\right)\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\}, \\ \left\{\left(1+\left(\left(1-\lambda_{j_{I(1)}}\right)\left(\frac{1-q_1}{q_1}\right)^{\sigma}+\left(1-\lambda_{j_{I(2)}}\right)\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\},\left\{1-\left(1+\left(\lambda_{j_{F(1)}}\left(\frac{1-q_1}{q_1}\right)^{\sigma}+\lambda_{j_{F(2)}}\left(\frac{1-q_2}{q_2}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\} \end{pmatrix}. \tag{19}$$

Considering the result to be true for $n=m$ we prove it for $n=m+1$ as follows:

$$NCHDWE(\alpha_1,\ldots,\alpha_{m+1})=\bigotimes_{j=1}^{m} q^{\alpha_j}\otimes q^{\alpha}. \tag{20}$$

Using (2) and (14), we have

$$NCHFDWE(\alpha_1,\alpha_2,\ldots,\alpha_{m+1})$$
$$=\begin{pmatrix} \left\{\left[\left(1+\left(\sum_{r=1}^{m+1}\left(1-A^L_{j_{T(r)}}\right)\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},\left(1+\left(\sum_{r=1}^{m+1}\left(1-A^U_{j_{T(r)}}\right)\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[\left(1+\left(\sum_{r=1}^{m+1}\left(1-A^L_{j_{I(r)}}\right)\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},\left(1+\left(\sum_{r=1}^{m+1}\left(1-A^U_{j_{I(r)}}\right)\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left[1-\left(1+\left(\sum_{r=1}^{m+1}A^L_{j_{F(r)}}\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1},1-\left(1+\left(\sum_{r=1}^{m+1}A^U_{j_{F(r)}}\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right]\right\}, \\ \left\{\left(1+\left(\sum_{r=1}^{m+1}\left(1-\lambda_{j_{T(r)}}\right)\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\},\left\{\left(1+\left(\sum_{r=1}^{m+1}\left(1-\lambda_{j_{T(r)}}\right)\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\}, \\ \left\{1-\left(1+\left(\sum_{r=1}^{m+1}\lambda_{j_{F(r)}}\left(\frac{1-q_r}{q_r}\right)^{\sigma}\right)^{1/\sigma}\right)^{-1}\right\} \end{pmatrix}. \tag{21}$$

□

6. Applications of Neutrosophic Cubic Hesitant Fuzzy Dombi Weighted Exponential Aggregation Operators to MADM Problems

Many methods in MADM ignore the uncertainty and hence yield the results that are unreliable. In this section, we construct algorithm using the Dombi exponential aggregation operators (NCHFDWEs) for MADM problems.

6.1. Solid Waste Disposal Site Selection. Due to population increase and urbanization, the proper disposal of waste is a challenging problem nowadays. The proper disposal of waste is necessary for the prevention of viral diseases like typhoid, dengue, and tuberculosis. Using the above algorithm to select the best alternative (solid waste disposal site) among the given alternatives (sites) $K_1, \ldots, K_5$ on the basis of attributes (i) F_1 water pollution; (ii) F_2 slope; (iii) F_3 distance from the residential area.

The decision matrix $D = (d_{ij})_{5\times 3}$, where entry d_{ij} represents the preference of alternative $K_i\,(i = 1, \ldots, 5)$ corresponding to attribute $F_j\,(j = 1, 2, 3)$, is constructed using fuzzy values with the help of the following scale.

Equal preference: 0.1, moderate preference: 0.3, strong preference: 0.5, demonstrated preference: 0.7, and extreme preference: 0.9. The mean values between the two assessments: 0.2, 0.4, 0.6, and 0.8.

$$D = \begin{bmatrix} 0.2 & 0.2 & 0.4 \\ 0.8 & 0.8 & 0.7 \\ 0.3 & 0.3 & 0.2 \\ 0.8 & 0.5 & 0.6 \\ 0.6 & 0.7 & 0.6 \end{bmatrix}. \tag{22}$$

The weights of the attributes are given as

$$w_1 = \begin{pmatrix} \{[0.3, 0.7], [0.2, 0.4]\}, \\ \{[0.2, 0.5], [0.1, 0.6]\}, \\ \{[0.2, 0.4], [0, 0.1]\}, \\ \{0.5, 0.6\}, \{0.2, 0.4\}, \{0.2, 0.3\} \end{pmatrix},$$
$$w_2 = \begin{pmatrix} \{[0.5, 0.7], [0.2, 0.5]\}, \\ \{[0.2, 0.3], [0.1, 0.6]\}, \\ \{[0.1, 0.4], [0, 0.3]\}, \\ \{0.4, 0.5\}, \{0.3, 0.4\}, \{0.2, 0.4\} \end{pmatrix}, \tag{23}$$
$$w_3 = \begin{pmatrix} \{[0.4, 0.5], [0.6, 0.7]\}, \\ \{[0.1, 0.3], [0.2, 0.5]\}, \\ \{[0.1, 0.2], [0.3, 0.4]\}, \\ \{0.3, 0.5\}, \{0.4, 0.6\}, \{0.3, 0.4\} \end{pmatrix}.$$

The explanation of weights is elaborated as follows.

In the case of w_1, {[0.3, 0.7], [0.2, 0.4]} is the interval hesitant degree of preference for attribute F_1, {[0.2, 0.5], [0.1, 0.6]} is the interval hesitant degree of indeterminacy (preference/nonpreference) for attribute F_1, {[0.2, 0.4], [0, 0.1]} is the interval hesitant degree of nonpreference for an attribute F_1, {0.1, 0.6}} is hesitant degree of preference for attribute F_1, {0.2, 0.4} is hesitant degree of indeterminacy (preference/nonpreference) for an attribute F_1, and {0.4, 0.6} is the hesitant degree of nonpreference for attribute F_1.

Aggregated values of alternatives using NCHFDWE operators ((17)) with $\sigma = 1$:

$$d_1 = \begin{pmatrix} \{[0.149254, 0.240964], [0.125, 0.17094]\}, \\ \{[0.114286, 0.145985], [0.106383, 0.20202]\}, \\ \{[0.574468, 0.777778], [0.310345, 0.6875]\}, \\ \{0.155039, 0.186916\}, \{0.126582, 0.15625\}, \\ \{0.672131, 0.75\} \end{pmatrix},$$

$$d_2 = \begin{pmatrix} \{[0.642202, 0.732984], [0.636364, 0.712468]\}, \\ \{[0.56, 0.625], [0.557769, 0.707071]\}, \\ \{[0.105431, 0.222222], [0.113924, 0.213483]\}, \\ \{0.634921, 0.694789\}, \{0.612691, 0.679612\}, \\ \{0.186047, 0.243243\} \end{pmatrix},$$

$$d_3 = \begin{pmatrix} \{[0.16129, 0.227273], [0.157895, 0.20979]\}, \\ \{[0.12, 0.151515], [0.119048, 0.205479]\}, \\ \{[0.52381, 0.727273], [0.545455, 0.716981]\}, \\ \{0.157068, 0.196078\}, \{0.144928, 0.185185\}, \\ \{0.680851, 0.75\} \end{pmatrix},$$

$$d_4 = \begin{pmatrix} \{[0.481928, 0.585366], [0.441176, 0.540541]\}, \\ \{[0.384615, 0.436364], [0.376176, 0.545455]\}, \\ \{[0.178082, 0.387755], [0.166667, 0.371728]\}, \\ \{0.456274, 0.517241\}, \{0.434783, 0.495868\}, \\ \{0.310345, 0.390863\} \end{pmatrix},$$

$$d_5 = \begin{pmatrix} \{[0.480549, 0.601719], [0.466667, 0.551181]\}, \\ \{[0.403846, 0.47619], [0.396975, 0.564516]\}, \\ \{[0.195402, 0.363636], [0.166667, 0.315961]\}, \\ \{0.486111, 0.551181\}, \{0.447761, 0.519802\}, \\ \{0.295302, 0.373134\} \end{pmatrix}. \tag{24}$$

Scores $S(d_i)$are given using equation (5).

$$S(d_1) = 0.2213, S(d_2) = 0.7043, S(d_3) = 0.2226, \\ S(d_4) = 0.5456, S(d_5) = 0.5655. \tag{25}$$

So $K_2 > K_5 > K_4 > K_3 > K_1$.

Similarity measures $sm_G(d_i)$ are given using (10) and (12).

Figure 1 is the graphical representation of score and similarity measures of aggregated values obtained in Table 2.

Aggregated values of alternatives using the NCHFDWE operators (equation (15)) with $\sigma = 2$:

$$d_1 = \begin{pmatrix} \{[0.180727, 0.233923], [0.162659, 0.189576]\}, \\ \{[0.159848, 0.179922], [0.153099, 0.211344]\}, \\ \{[0.691515, 0.784485], [0.451027, 0.729865]\}, \\ \{0.185911, 0.202422\}, \{0.165703, 0.182372\}, \\ \{0.726768, 0.764172\} \end{pmatrix},$$

$$d_2 = \begin{pmatrix} \{[0.699128, 0.735492], [0.705967, 0.739685]\}, \\ \{[0.660032, 0.68909], [0.662536, 0.726421]\}, \\ \{[0.161537, 0.227505], [0.190112, 0.238848]\}, \\ \{0.692421, 0.722115\}, \{0.688889, 0.721856\}, \\ \{0.220591, 0.24988\} \end{pmatrix},$$

$$d_3 = \begin{pmatrix} \{[0.199337, 0.229538], [0.204612, 0.233392]\}, \\ \{[0.172194, 0.191898], [0.173794, 0.221482]\}, \\ \{[0.406748, 0.764784], [0.382966, 0.688361]\}, \\ \{0.194328, 0.217786\}, \{0.191753, 0.217567\}, \\ \{0.648489, 0.799751\} \end{pmatrix},$$

$$d_4 = \begin{pmatrix} \{[0.526252, 0.5762], [0.496575, 0.549739]\}, \\ \{[0.472136, 0.494814], [0.466126, 0.55417]\}, \\ \{[0.283751, 0.417541], [0.267479, 0.41028]\}, \\ \{0.50742, 0.53636\}, \{0.497934, 0.525506\}, \\ \{0.370307, 0.413368\} \end{pmatrix},$$

$$d_5 = \begin{pmatrix} \{[0.549964, 0.609449], [0.548009, 0.587782]\}, \\ \{[0.512821, 0.551397], [0.510304, 0.592385]\}, \\ \{[0.28031, 0.368373], [0.267479, 0.344956]\}, \\ \{0.554875, 0.587782\}, \{0.535767, 0.573149\}, \\ \{0.337256, 0.377007\} \end{pmatrix}. \tag{26}$$

Scores $S(d_i)$ are given using equation (5):

$$S(d_1) = 0.2210, S(d_2) = 0.7296, \\ S(d_3) = 0.2290, S(d_4) = 0.5590, S(d_5) = 0.5948, \\ K_2 > K_5 > K_4 > K_3 > K_1. \tag{27}$$

Similarity measures $sm_G(d_i)$ are given using (10) and (12).

Figure 2 is the graphical representation of score and similarity measures of aggregated values obtained in Table 3.

Aggregated values of alternatives using NCHFDWE operators (equation (15)) with $\sigma = 3$:

Step 1: Identification of alternatives and attributes.
Let $\{K_1, K_2, \ldots, K_r\}$, be the set of r alternatives, $\{F_1, F_2, \ldots, F_s\}$ be s attributes. The NCHFS α_j is used as weight for the attribute F_j. A decision matrix is $D = (d_{ij})$ consisting fuzzy values, where d_{ij} represent the preference of alternative K_i corresponding to attribute F_j.
Step 2. Allocation of weights to attributes
The NCHF value α_j is used as weight assigned to attribute F_j.
Step 3. Computation of weighted aggregated values
Using NCHFDWE operators (equation (18)), we compute the aggregated values $d_j's$ $(j = 1, \ldots, r)$ of alternatives $K_j's$.
Step 4. Ranking of Alternatives
We calculate the scores and similarity measures $S(d_j), sm(d_j, \Omega)$; $j = 1, \ldots, r$ of the alternatives K_j; $i = 1, \ldots, r$ using equations (5), (10) and (14). Using scores and similarity measures, we rank the alternatives K_i; $i = 1, \ldots, n$. If scores of two alternatives are equal, then, we use accuracy function for ranking and if they have same accuracy, we use certainty.

ALGORITHM 1: Algorithm using the dombi exponential aggregation operators (NCHFDWE) for MADM problems.

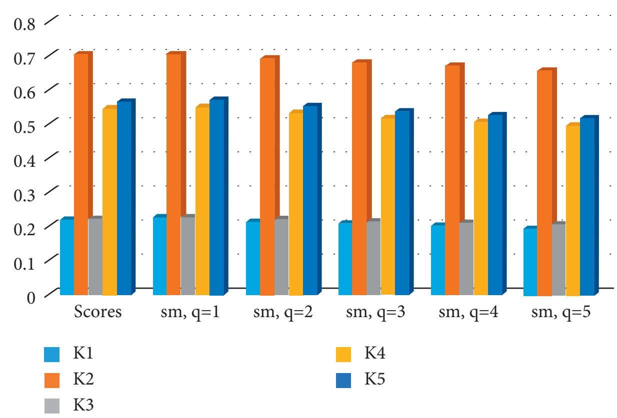

FIGURE 1: Graphical representation of scores and similarity measures with $\boldsymbol{\sigma} = 1$.

TABLE 2: Similarity measures of aggregated values using the NCHFDWE operators with $\boldsymbol{\sigma} = 1$.

	$sm_G(d_1, \Omega)$	$sm_G(d_2, \Omega)$	$sm_G(d_3, \Omega)$	$sm_G(d_3, \Omega)$	$sm_G(d_4, \Omega)$	Ranking
$q = 1$	0.228189	0.706196	0.227288	0.549464	0.568689	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 2$	0.215779	0.690722	0.220804	0.531238	0.551364	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 3$	0.206874	0.678439	0.21519	0.517484	0.537774	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 4$	0.200116	0.668399	0.210311	0.506618	0.526813	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 5$	0.19473	0.659948	0.206046	0.49767	0.51771	$K_2 > K_5 > K_4 > K_3 > K_1$

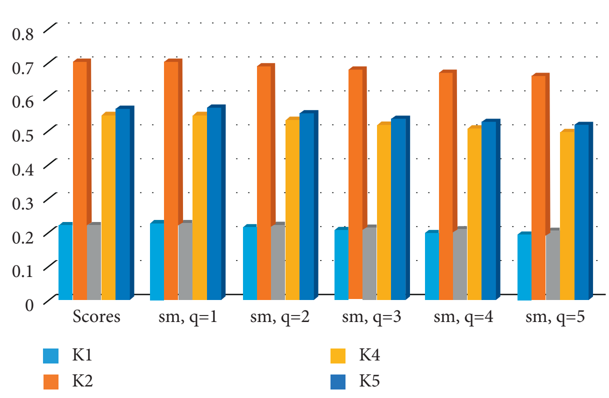

FIGURE 2: Graphical representation of scores and similarity measures with $\boldsymbol{\sigma} = 2$.

Table 3: Similarity measures of aggregated values using the NCHFDWE operators with $\sigma = 2$.

	$sm_G(d_1, \Omega)$	$sm_G(d_2, \Omega)$	$sm_G(d_3, \Omega)$	$sm_G(d_3, \Omega)$	$sm_G(d_4, \Omega)$	Ranking
$q = 1$	0.246277	0.730842	0.230984	0.557806	0.596572	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 2$	0.236448	0.726767	0.229542	0.551832	0.591733	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 3$	0.228268	0.723121	0.228174	0.546761	0.587395	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 4$	0.209457	0.719847	0.226876	0.542419	0.583509	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 5$	0.217899	0.716883	0.225643	0.538656	0.58002	$K_2 > K_5 > K_4 > K_3 > K_1$

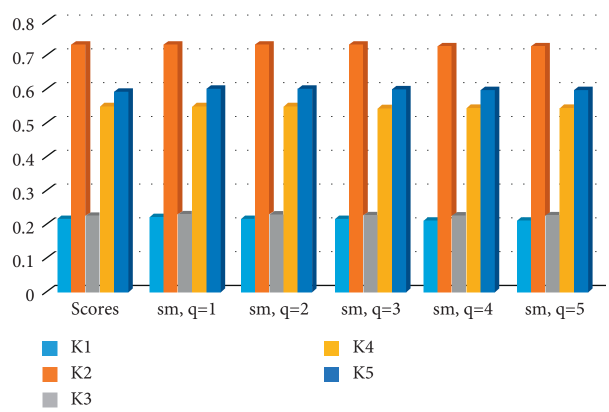

Figure 3: Graphical representation of scores and similarity measures with $\sigma = 3$.

$$
\begin{aligned}
d_1 &= \begin{pmatrix} \{[0.189119, 0.226119], [0.175473, 0.194219]\}, \\ \{[0.174696, 0.188901], [0.169392, 0.0.174524]\}, \\ \{[0.729242, \ 0.788562], [0.501035, 0.749884]\}, \\ \{0.193244, 0.20411\}, \{0.178227, 0.189559\}, \\ \{0.749095, 0.773387\} \end{pmatrix}, \\
d_2 &= \begin{pmatrix} \{[0.712208, 0.732458], [0.722704, 0.743899]\}, \\ \{[0.68604, 0.704447], [0.689676, 0.728379]\}, \\ \{[0.188605, 0.233444], [0.222939, 0.251172]\}, \\ \{0.705929, 0.726416\}, \{0.707496, 0.730469\}, \\ \{0.236791, 0.256193\} \end{pmatrix}, \\
d_3 &= \begin{pmatrix} \{[0.20958, 0.226801], [0.218287, 0.237351]\}, \\ \{[0.189706, 0.203424], [0.192323, 0.223189]\}, \\ \{[0.406748, 0.764784], [0.382966, 0.688361]\}, \\ \{0.204582, 0.221478\}, \{0.205815, 0.22503\}, \\ \{0.648489, 0.799751\} \end{pmatrix}, \\
d_4 &= \begin{pmatrix} \{[0.445183, 0.588428], [0.391659, 0.530621]\}, \\ \{[0.303378, 0.385061], [0.291323, 0.536908]\}, \\ \{[0.152128, 0.374173], [0.142083, 0.352469]\}, \\ \{0.412698, 0.500953\}, \{\, 0.379002 \,, 0.470432\}, \\ \{0.285738, 0.422203\} \end{pmatrix}, \\
d_5 &= \begin{pmatrix} \{[0.430671, 0.597105], [0.407249, 0.528306]\}, \\ \{[0.315454, 0.422037], [0.304401, 0.547489]\}, \\ \{[0.172143, 0.361839], [0.142083, 0.304007]\}, \\ \{0.437361, 0.528306\}, \{0.381049, 0.484407\}, \\ \{0.278734, 0.39362\} \end{pmatrix}.
\end{aligned}
\tag{28}
$$

Scores $S(d_i)$ are given using equation (5):

$$
\begin{aligned}
&S(d_1) = 0.2185, S(d_2) = 0.7324, S(d_3) = 0.2288, \\
&S(d_4) = 0.5497, S(d_5) = 0.5922, \\
&\quad K_2 > K_5 > K_4 > K_3 > K_1.
\end{aligned}
\tag{29}
$$

Similarity measures $sm_G(d_i)$ are given using (10) and (12):

Figure 3 is the graphical representation of score and similarity measures of aggregated values obtained in Table 4.

It can be seen from Figures 1–3, that we obtain the same ranking with respect to different choices of parameters q and σ. Hence the most desirable alternative is K_2.

6.2. Comparative Analysis. To check the validity of results, the above problem is solved by different existing techniques proposed in [36, 46]. The results obtained by these existing methods are similar to those obtained from this study in Tables 2–4.

7. Conclusion

First, the Dombi algebraic operations distance and similarity measures in neutrosophic cubic hesitant fuzzy sets are defined. Then the properties of these notions are discussed. Furthermore, the novel Dombi exponential laws and Dombi exponential aggregation operators are proposed. The Dombi exponential laws are more flexible than the usual exponential laws as they involve an operational parameter. Using these aggregation operators, we established a MADM method. The proper disposal of waste is necessary for the prevention of viral diseases like typhoid, dengue, and tuberculosis. The solid waste disposal site selection problem is solved using the proposed MADM method. To discuss the effectiveness of the

Table 4: Similarity measures of aggregated values using the NCHFDWE operators with $\sigma = 3$.

	$sm_G(d_1, \Omega)$	$sm_G(d_2, \Omega)$	$sm_G(d_3, \Omega)$	$sm_G(d_3, \Omega)$	$sm_G(d_4, \Omega)$	Ranking
$q = 1$	0.222346	0.733388	0.230072	0.551843	0.602963	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 2$	0.218951	0.731571	0.229472	0.548685	0.600807	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 3$	0.216225	0.729881	0.228891	0.545899	0.598787	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 4$	0.214005	0.728307	0.228328	0.543433	0.5969	$K_2 > K_5 > K_4 > K_3 > K_1$
$q = 5$	0.212168	0.726835	0.227783	0.541232	0.595128	$K_2 > K_5 > K_4 > K_3 > K_1$

proposed method, different values of parameters are considered to check the efficiency of the proposed method. At last, the comparative analysis is presented for validation of proposed operators and methods.

For further research, the neutrosophic cubic hesitant fuzzy Dombi arithmetic and geometric aggregation operators and multiexpert decision-making methods using these aggregation operators will be developed. The Dombi Bonferroni mean operators of neutrosophic cubic hesitant fuzzy sets will be established. The Dombi exponential laws and Dombi exponential aggregation operators for Pythagorean fuzzy sets and picture hesitant fuzzy sets will be established [51–54, 56–60].

7.1. Limitations of the Proposed Study. As Dombi operations and generalized similarity measures involve operational parameters, the manual calculation is very complicated and time-consuming. Hence there is a need to develop a software program for such computations. Furthermore, for MADM problems involving more alternatives and attributes handled by Dombi exponential aggregation operators, software programming is needed. We are trying to develop such programming using PYTHON.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, no. 3, pp. 338–353, 1965.
[2] L. A. Zadeh, "Outlines of New Approach to the Analysis of Complex System and Dicision Processes Interval Valued Fuzzy Sets," *IEEE Trans*, vol. 3, 1968.
[3] I. B. Turksen and T. Bilgic, "Interval valued strict preference with Zadeh triples," *Fuzzy Sets and Systems*, no. 2, pp. 183–195, 1996.
[4] S. J. Chen and C. L. Hwang, *Fuzzy multiple attribute decision-making, methods and applications Lecture Notes in Economics and Mathematical Systems*, Springer-, Berlin/Heidelberg, 1992.
[5] T. H. Chang and T. C. Wang, "Using the fuzzy multi-criteria decision making approach for measuring the possibility of successful knowledge management," *Information Sciences*, no. 4, pp. 355–370, 2009.
[6] S. P. Kalyan Mondal and K. Mondal, "Weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis," *International Journal of Innovative Research in Science, Engineering and Technology*, vol. 04, no. 02, pp. 158–164, 2015.
[7] C. T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, no. 1, pp. 1–9, 2000.
[8] G. Zhang and J. Lu, "An integrated group decision-making method dealing with fuzzy preferences for alternatives and individual judgments for selection criteria," *Group Decision and Negotiation*, no. 6, pp. 501–515, 2003.
[9] R. A. Krohling and V. C. Campanharo, "Fuzzy TOPSIS for group decision making: a case study for accidents with oil spill in the sea," *Expert Systems with Applications*, no. 4, pp. 4190–4197, 2011.
[10] M. Xia and Z. Xu, "A novel method for fuzzy multi-criteria decision making," *International Journal of Information Technology and Decision Making*, no. 03, pp. 497–519, 2014.
[11] M. K. Mehlawat and P. Gupta, "A new fuzzy group multi-criteria decision making method with an application to the critical path selection," *International Journal of Advanced Manufacturing Technology*, no. 5-8, pp. 1281–1296, 2016.
[12] Y. B. Jun, C. S. Kim, and K. O. Yang, "Cubic sets," *Ann. Fuzzy Math. Inform*, no. 1, pp. 83–98, 2012.
[13] Y. B. Jun, C. S. Kim, and M. S. Kang, "Cubic sub algebras and ideals of BCK/BCI-algebras, Far East," *Journal of Mathematical Sciences*, no. 44, pp. 239–250, 2010.
[14] Y. B. Jun, C. S. Kim, and J. G. Kang, "Cubic q-ideals of BCI-algebras," *Ann. Fuzzy Math. Inform.*, no. 1, pp. 25–34, 2011.
[15] Y. B. Jun, K. J. Lee, and M. S. Kang, "Cubic structures applied to ideals of BCI-algebras," *Computers & Mathematics with Applications*, no. 9, pp. 3334–3342, 2011.
[16] F. Smarandache, *A Unifying Field in Logics Neutrosophy: Neutrosophic Probability, Set and Logic*, American Research Press, Rehoboth, 1999.
[17] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, *Single Valued Neutrosophic Sets*, pp. 410–413, Multispace Multistructure, 2010.
[18] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, *Interval Neutrosophic Sets and Logics*, Hexis, Phoenix, USA, 2005.
[19] A. Kharal, "A neutrosophic multi-criteria decision making method," *New Mathematics and Natural Computation*, no. 02, pp. 143–162, 2014.
[20] J. Ye, "Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making,"

Journal of Intelligent and Fuzzy Systems, no. 1, pp. 165–172, 2014.

[21] J. Ye, "Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment," *Journal of Intelligent and Fuzzy Systems*, no. 6, pp. 2927–2935, 2014.

[22] K. Mondal and S. Pramanik, "Multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment," *Neutrosophic Sets Syst*, no. 6, pp. 28–34, 2014.

[23] P. Biswas, S. Pramanik, and B. C. Giri, "Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments," *Neutrosophic Sets Syst*, no. 2, pp. 102–110, 2014.

[24] P. Biswas, S. Pramanik, and B. C. Giri, "A new methodology for neutrosophic multi-attribute decision-making with unknown weight information," *Neutrosophic Sets Syst*, no. 3, pp. 44–54, 2014.

[25] P. Biswas, S. Pramanik, and B. C. Giri, "Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers," *Neutrosophic Sets Syst*, no. 8, pp. 46–56, 2014.

[26] K. Mondal and S. Pramanik, "Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis," *Neutrosophic Sets Syst*, no. 9, pp. 64–71, 2015.

[27] B. Li, J. Wang, L. Yang, and X. Li, "A novel generalized simplified neutrosophic number einstein aggregation operator," *International Journal of Applied mathematics*, no. 48, pp. 67–72, 2018.

[28] J. Zhan, M. Khan, M. Gulistan, and A. Ali, "Applications of neutrosophic cubic sets in multi-criteria decision-making," *International Journal for Uncertainty Quantification*, no. 5, pp. 377–394, 2017.

[29] V. Torra, "Hesitant fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, pp. 529–539, 2010.

[30] Y. B. Jun, F. Smarandache, and C. S. Kim, "Neutrosophic cubic sets," *New Mathematics and Natural Computation*, 2015.

[31] F. Mehmood, T. Mahmood, and Q. Khan, "Cubic hesitant fuzzy sets and their applications to multi criteria decision making," *International Journal of Algebra and Statistics*, vol. 5, pp. 19–51, 2016.

[32] J. Ye, "Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment," *Journal of Intelligent Systems*, vol. 24, no. 1, pp. 23–36, 2015.

[33] P. Liu and L. Shi, "The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making," *Neural Computing & Applications*, vol. 26, no. 2, pp. 457–471, 2015.

[34] B. Zhu, Z. S. Xu, and J. P. Xu, "Deriving a ranking from hesitant fuzzy preference relations under group decision making," *IEEE Transactions on Cybernetics*, vol. 44, no. 8, pp. 1328–1337, 2014.

[35] Z. Lu and J. Ye, "Exponential Operations and an Aggregation Method for Single-Valued Neutrosophic Numbers in Decision Making," *Information*, 2017.

[36] R. Tan, W. Zhang, and S. Chen, "Exponential aggregation operator of interval neutrosophic numbers and its application in typhoon disaster evaluation," *Symmetry*, 2018.

[37] R. Wang, Y. Li, and C. Chen, *Picture Hesitant Fuzzy Set and Its Application to Multiple Criteria Decision-Making, Symmetry*, vol. 10, 2018.

[38] N. H. Shah, K. Rabari, and E. Patel, "A deteriorating inventory model under overtime production and credit policy for stock- and price sensitive demand function Operational Research in Engineering Sciences: Theory and Applications," 2022.

[39] R. Tan and W. Zhang, "Multiple attribute group decision making methods based on trapezoidal fuzzy neutrosophic numbers," *Journal of Intelligent and Fuzzy Systems*, vol. 33, no. 4, pp. 2547–2564, 2017.

[40] L. Feng, F. Chuan-qiang, and X. Wei-he, "Type-2 hesitant fuzzy sets," *Fuzzy Information and Engineering*, vol. 10, no. 2, pp. 249–259, 2018.

[41] E. Turkarslan, J. Ye, M. Unver, and M. Olgun, "Consistency fuzzy sets and a cosine similarity measure in fuzzy multiset setting and application to medical diagnosis," *Mathematical Problems in Engineering*, vol. 2021, p. 1, 2021.

[42] J. Dombi, "A general class of fuzzy operators, the demorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators," *Fuzzy Sets and Systems*, vol. 8, no. 2, pp. 149–163, 1982.

[43] L. Shi and J. Ye, "Dombi Aggregation Operators of Neutrosophic Cubic Sets for Multiple Attribute Decision Making," *algorithms*, vol. 11, 2018.

[44] P. D Liu, J. L. Liu, and S. M. Chen, "Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making," *Journal of the Operational Research Society*, 2017.

[45] J. Q. Chen and J. Ye, "Some single-valued neutrosophic Dombi weighted aggregation operators for multiple attribute decision-making," *Symmetry*, vol. 9, no. 6, p. 82, 2017.

[46] A. Bhattacharya, A. Dutta, and S. Kar, "Does demographics influence the risk behaviour of urban investors? A machine learning model based approach," *Operational Research in Engineering Sciences: Theory and Applications*, 2022.

[47] S. Biswas, G. Bandyopadhyay, B. Guha, and M. Bhattacharjee, "An ensemble approach for portfolio selection in a multi-criteria decision making framework," *Decision Making: Applications in Management and Engineering*, vol. 2, no. 2, pp. 138–158, 2019.

[48] A. Rehman, M. Guistan, Z. Khan, and F. S. Al-Duas, "A study of neutrosophic cubic hesitant fuzzy hybrid geometric aggregation operators and its application to multi expert decision making system," *Neutrosophic Sets Syst*, vol. 50, pp. 83–110, 2022.

[49] A. Ashraf, K. Ullah, A. Hussain, and M. Bari, "Interval-valued picture fuzzy Maclaurin symmetric mean operator with application in multiple attribute decision-making," *Reports in Mechanical Engineering*, vol. 3, no. 1, pp. 301–317, 2022.

[50] M. Riaz and H. M. A. Farid, "Picture fuzzy aggregation approach with application to third-party logistic provider selection process," *Reports in Mechanical Engineering*, vol. 3, no. 1, pp. 318–327, 2022.

[51] M. I. Youssef and B. Webster, "A multi-criteria decision making approach to the new product development process in industry," *Reports in Mechanical Engineering*, vol. 3, no. 1, pp. 83–93.

[52] A. Mešić, S. Miškić, Ž. Stević, and Z. Mastilo, "Hybrid MCDM solutions for evaluation of the logistics performance

index of the western Balkan countries," *Economics - Innovative and Research Journal*, vol. 10, no. 1, pp. 13–34, 2022.

[53] B. F. Yildirim and S. K. Yıldırım, "Evaluating the satisfaction level of citizens in municipality services by using picture fuzzy VIKOR method: 2014-2019 period analysis," *Decision Making: Applications in Management and Engineering*, vol. 5, no. 1, pp. 50–66, 2022.

[54] S. Kumar, S. R. Maity, and L. Patnaik, "Optimization of wear parameters for duplex-tiain coated MDC-K tool steel using fuzzy MCDM techniques," *Operational Research in Engineering Sciences: Theory and Applications*, 2022.

[55] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, no. 1, pp. 87–96, 1986.

[56] K. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, no. 3, pp. 343–349, 1989.

[57] Z. Xu, "Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making," *Fuzzy Optimization and Decision Making*, no. 2, pp. 109–121, 2007.

[58] K. Mondal and S. Pramanik, "Intuitionistic fuzzy multi criteria group decision making approach to quality-brick selection problem," *J. Appl. Quant. Methods*, no. 9, pp. 35–50, 2014.

[59] P. P. Dey, S. Pramanik, and B. C. Giri, "Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in Khadi institution," *J. Appl. Quant. Methods*, no. 10, pp. 1–14, 2015.

[60] D. Pamucar, D. Bozanic, A. puska, and D. Marinkovic, "Application of Neuro-Fuzzy system for predicting the success of a company in public procurement," *Decision Making: Applications in Management and Engineering*, vol. 5, no. 1, pp. 135–153.