Hindawi
Complexity
Volume 2022, Article ID 4020266, 18 pages
https://doi.org/10.1155/2022/4020266

Research Article

Distributed Coordination for a Class of High-Order Multiagent Systems Subject to Actuator Saturations by Iterative Learning Control

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Received 30 December 2021; Revised 27 January 2022; Accepted 28 January 2022; Published 27 February 2022

Academic Editor: Xianming Zhang

This paper investigates a distributed coordination control for a class of high-order uncertain multiagent systems. Under the framework of iterative learning control, a novel fully distributed learning protocol is devised for the coordination problem of MASs including time-varying parameter uncertainties as well as actuator saturations. Meanwhile, the learning updating laws of various parameters are proposed. Utilizing Lyapunov theory and combining with Graph theory, the proposed algorithm can make each follower track a leader completely over a limited time interval even though each follower is without knowing the dynamics of the leader. Moreover, the extension to formation control is made. The validity and feasibility of the algorithm are verified conclusively by two examples.

1. Introduction

Coordination of multiagent systems (MASs) draws a lot of attention because of its wide applications in UAV, biological systems, sensor networks, and so forth. An important problem in coordination is to develop the distributed protocols, which specifies information exchange among agents, such that the group as a whole can agree on a common quantity. This kind of problem is called consensus [1–5]. For this consensus realization, a leader-follower consensus [6, 7] has been extensively studied by many scholars. Another problem of coordination is formation control; the fault-tolerant leader-following formation control and cluster formation control were discussed in [8, 9], respectively. However, in the real world, the dynamics of systems are usually uncertain. Therefore, the consensus or/and formation control for uncertain MASs become a hot topic in the field of control.

Generally speaking, an effective scheme to dispose parameter uncertainties is adaptive control [10–19]. An adaptive neural network approximation scheme was utilized to design the control protocols of the first-, second-, and high-order uncertain MASs in [10–12]. References [13, 14] used the adaptive idea to design the fully distributed control protocols by adjusting the protocols gains, so that the global information dependent on topology matrix can be avoided, and the promising technique is more and more popular among researchers for different purposes [15–19], where adaptive iterative learning control is utilized to handle the consensus for MASs over a finite time interval [20–24].

It is noteworthy that the abovementioned studies did not consider actuator saturations in the MASs. Actually, due to the limited actuator capability, actuator saturations may exist in most physical systems and cause the system instability. Consequently, it is useful to take into account actuator saturations in system analysis. Recently, a large number of work about MASs with actuator saturations emerge [25–32]. References [25–29] studied consensus algorithms under one-dimensional system framework when time goes to infinity and [30–32] tackled the consensus problem by iterative learning control (ILC). In [30], authors addressed the neural network consensus for the second-order MASs subject to saturation input; authors in [31] proposed the fully distributed learning scheme and made use of the properties of saturation function to solve actuator saturations; the leader-follower consensus for the first-order MASs with input saturation was researched in [32]. Thereinto, the dynamic of the leader was known to each follower in [31, 32].

Based on the aforementioned observations, we have successfully combined the fully distributed algorithm applying ILC over the finite time interval to address the coordination for a class of high-order MASs. The major contributions are highlighted as follows:

(i) Differing from the adaptive consensus of MASs with actuator saturations, the learning consensus researched in this paper can make each follower track the leader absolutely over, where the parameter uncertainties are time-varying

(ii) In contrast with the adaptive ILC consensus algorithm available, the algorithm in the paper is more complicated as well as each follower is without knowing the dynamic of the leader

(iii) The consensus problem is extended to the formation control for a class of high-order MASs

The rest is arranged as follows. Some useful preliminary results and problem formulation are presented in Section 2. Section 3 is the protocols design and consensus analysis. The extension to formation control is given in Section 4. Two examples for illustration are taken in Sections 5 and conclusion is drawn in Sections 6, respectively.

2. Problem Formulation

Here, the MASs with N followers and one leader are considered under a repetitive environment. The models are

$$\begin{aligned} &\text{followers}: \dot{x}_i^k(t) = Ax_i^k + \theta_i(t)\xi_i\left(x_i^k(t)\right) + \text{sat}\left(u_i^k(t), u^*\right) \quad i = 1,2,\ldots,N, \\ &\text{leader}: \dot{x}_0(t) = Ax_0(t) + f\left(x_0(t), t\right), \end{aligned} \tag{1}$$

where $x_i^k(t) \in R^n$ is the state vector of the i th follower and $u_i^k(t) \in R^n$ is the input vector of the i th follower; $\theta_i(t)$ is an unknown continuous time-varying parameter, which shows the uncertainty in system models for each follower agent; $\xi_i(x_i^k) \in R^n$ is a known smooth nonlinear vector valued function; $A \in R^{n\times n}$; $\text{sat}(u_{ij}^k, u_j{}^*)$ is the saturation function [31], $j = 1,2,\ldots,n$, and $\text{sat}(u_i^k, u^*) = [\text{sat}(u_{i1}^k, u_1^*), \text{sat}(u_{i2}^k, u_2^*), \ldots, \text{sat}(u_{in}^k, u_n^*)]^{\mathrm{T}}$ is a saturation vector function; $x_0(t) \in R^n$ is the state vector of the leader, and $f(x_0(t), t) = [f_{01}(x_0(t), t), f_{02}(x_0(t), t), \ldots, f_{0n}(x_0(t), t)]^{\mathrm{T}} \in R^n$ is an unknown but bounded vector valued nonlinear function.

Assumption 1. It is assumed that $|f_{0j}(x_0(t))| \leq \eta_j$ with η_j being an unknown positive constant. Denote $\overline{\eta} = [\eta_1, \eta_2, \ldots, \eta_n]^{\mathrm{T}}$.

Remark 1. From the above, it is known that each follower is without knowing the dynamic of the leader.

For the i th follower, the consensus error is

$$\delta_i^k = x_i^k - x_0. \tag{2}$$

In this paper, we aim to find suitable protocols $\{u_i^k, i = 1,2,\ldots,N, 0 \leq t \leq T\}$ and the updating laws of parametric uncertainties so that all the followers can uniformly track the leader over $[0,T]$ as k approaches to infinity, that is to say,

$$\lim_{k\longrightarrow\infty}\left\|\delta_i^k\right\| = \lim_{k\longrightarrow\infty}\left\|x_i^k - x_0(t)\right\| = 0, \quad i = 1,2,\ldots,N. \tag{3}$$

Assumption 2. The state vector of each follower and the leader satisfy $x_i^k(0) = x_i^{k-1}(t)$ and $x_0(0) = x_0(t)$.

Remark 2. From assumption 2, we have $\delta_i^k(0) = \delta_i^{k-1}(t)$.

To design the distributed protocols, the distribute error is

$$e_i^k(t) = \sum_{j=1}^N a_{ij}\left(x_j^k(t) - x_i^k(t)\right) + b_i\left(x_0(t) - x_i^k(t)\right). \tag{4}$$

The compact forms are

$$\begin{aligned} \delta^k &= x^k - 1_N \otimes x_0, \\ e^k &= -(L+B)\left(x^k - 1_N \otimes x_0\right) = -(H \otimes I_n)\delta^k, \end{aligned} \tag{5}$$

where $e^k = [(e_1^k)^{\mathrm{T}}, (e_2^k)^{\mathrm{T}}, \ldots, (e_N^k)^{\mathrm{T}}]^{\mathrm{T}} \in R^{Nn}$ and $\delta^k = [(\delta_1^k)^{\mathrm{T}}, (\delta_2^k)^{\mathrm{T}}, \ldots, (\delta_N^k)^{\mathrm{T}}]^{\mathrm{T}} \in R^{Nn}$; $H = L + B$ is a symmetric positive definite matrix, and the communication topology of the paper is the same as that in [31].

3. Learning Control Protocols Design and Consensus Analysis

The error dynamic can be calculated as

$$\dot{\delta}_i^k = A\delta_i^k + \theta_i(t)\xi_i^k + \text{sat}\left(u_i^k, u^*\right) - f\left(x_0, t\right). \tag{6}$$

Then, the learning protocol is devised as

$$u_i^k(t) = \widehat{c}_i^k(t)e_i^k - \widehat{\theta}_i^k(t)\xi_i\left(x_i^k\right) + \varpi\mu_i^k(t) + \upsilon_i^k(t), \tag{7}$$

where $\widehat{c}_i^k(t) \in R$ is a time-varying gain; $\widehat{\theta}_i^k(t) \in R$ is to estimate $\theta_i(t)$; $\varpi > 0$; $\mu_i^k(t)$ is to deal with the saturation term in (1); and $\upsilon_i^k(t) \in R^n$ is to offset $f(x_0(t), t)$.

$\upsilon_i^k(t)$, the learning adaptive laws of $\widehat{c}_i^k(t)$ and $\mu_i^k(t)$, is devised as

$$\upsilon_i^k(t) = \widehat{\eta}_i^k(t)\tanh\left(\frac{\left(e_i^k(t)\right)^{\mathrm{T}} P\widehat{\eta}_i^k(t)}{\Delta_{k+1}}\right), \tag{8}$$

$$\begin{cases} \dot{\widehat{c}}_i^k(t) = \rho_i\left(e_i^k(t)\right)^{\mathrm{T}} P e_i^k(t), \\ \widehat{c}_i^k(0) = \widehat{c}_i^{k-1}(T), \widehat{c}_i^0(0) = 0, \end{cases} \tag{9}$$

and

$$\begin{cases} \dot{\mu}_i^k(t) = \varpi e_i^k - \dfrac{\left(\delta u^k\right)^T \delta u^k}{2\left(\mu^k\right)^T (I_N \otimes P)\mu^k} \mu_i^k(t), \\ \\ \mu_i^k(0) = \mu_i^{k-1}(T), \end{cases} \tag{10}$$

where $\widehat{\eta}_i^k(t)$ is the estimation of $\overline{\eta}$ and P is symmetric positive definite; $\rho_i > 0$; $\delta u^k = u^k - \mathrm{sat}(u^k)$, $u^k = \mathrm{col}\{u_1^k, u_1^k, \ldots, u_N^k\} \in R^{Nn}$, $\mathrm{sat}(u^k) = \mathrm{col}\{\mathrm{sat}(u_1^k), \mathrm{sat}(u_2^k), \ldots, \mathrm{sat}(u_N^k)\} \in R^{Nn}$, and $\mu^k = \mathrm{col}\{\mu_1^k, \mu_2^k, \ldots, \mu_N^k\} \in R^{Nn}$.

Furthermore, the learning-based updating law of $\theta_i(t)$ is devised as

$$\begin{cases} \widehat{\theta}_i^k(t) = \mathrm{sat}\left(\widehat{\theta}_{i,*}^k(t)\right), \\ \widehat{\theta}_{i,*}^k(t) = \mathrm{sat}\left(\widehat{\theta}_{i,*}^{k-1}(t)\right) - m_i\left(e_i^k\right)^{\mathrm{T}} P \xi_i^k, \\ \widehat{\theta}_i^{-1}(t) = \mathrm{sat}\left(\widehat{\theta}_{i,*}^{-1}(t)\right) = 0, \quad t \in [0, T], \end{cases} \tag{11}$$

with $m_i > 0$.

Meanwhile,

$$\begin{aligned} \dot{\widehat{\eta}}_i^k(t) &= \Gamma_i\left|P e_i^k\right|, \\ \widehat{\eta}_i^k(0) &= \widehat{\eta}_i^{k-1}(T), \end{aligned} \tag{12}$$

where $\widehat{\eta}_i^k(t) = [\widehat{\eta}_{i1}^k(t), \widehat{\eta}_{i2}^k(t), \ldots, \widehat{\eta}_{in}^k(t)]^{\mathrm{T}}$; $\Gamma_i = \mathrm{diag}\{\gamma_{i1}, \gamma_{i2}, \ldots, \gamma_{in}\}$, $\gamma_{ij} > 0$, $i = 1, 2, \ldots, N$ and $j = 1, \ldots, n$; $\widehat{\eta}_i^0(0) = [\widehat{\eta}_{i1}^0(0), \widehat{\eta}_{i2}^0(0), \ldots, \widehat{\eta}_{in}^0(0)]^{\mathrm{T}}$, $\widehat{\eta}_{ij}^0(0) > 0$.

Remark 4. It is obvious that $\widehat{\eta}_i^k(t)$ is nonnegative, which is guaranteed from the updating law (12).

The i th error dynamic becomes

$$\dot{\delta}_i^k = A\delta_i^k + \widetilde{\theta}_i^k(t)\xi_i^k + \widehat{c}_i^k e_i^k + \varpi\mu_i^k(t) - \delta u_i^k + v_i^k - f(x_0, t), \tag{13}$$

where $\widetilde{\theta}_i^k(t) = \theta_i(t) - \widehat{\theta}_i^k(t)$ and $\delta u_i^k = u_i^k - \mathrm{sat}(u_i^k)$.

Thus,

$$\dot{\delta}^k = (I_N \otimes A)\delta^k + \left(\widetilde{\Theta}^k(t) \otimes I_n\right)\xi^k - \left(\widehat{C}^k(t) H \otimes I_n\right)\delta^k + \varpi\mu^k - \delta u^k + v^k - (1_N \otimes f(x_0, t)), \tag{14}$$

where

$$\begin{aligned} \widetilde{\Theta}^k(t) &= \mathrm{diag}\left\{\widetilde{\theta}_1^k(t), \widetilde{\theta}_2^k(t), \ldots, \widetilde{\theta}_N^k(t)\right\}, \\ \xi^k &= \left[\left(\xi_1^k\right)^{\mathrm{T}}, \left(\xi_2^k\right)^{\mathrm{T}}, \ldots, \left(\xi_N^k\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in R^{Nn}, \\ v^k(t) &= \mathrm{col}\left\{\left(v_1^k(t)\right), \left(v_2^k(t)\right), \ldots, \left(v_N^k(t)\right)\right\} \in R^{Nn} \text{ and } \widehat{C}^k(t) = \mathrm{diag}\left\{\widehat{c}_1^k(t), \widehat{c}_2^k(t), \ldots, \widehat{c}_N^k(t)\right\}. \end{aligned} \tag{15}$$

Theorem 1. *For the MASs (1), under assumptions 1 and 2, the protocols (7)-(8) as well as adaptive learning laws (9)-(12) guarantee that each follower can completely track the leader over* $[0, T]$ *along the iteration axis, i.e.,* $\lim_{k\longrightarrow\infty} \delta_i^k = 0$, $i = 1, \ldots, N$. *At the same time, the boundednesses of signals involved are obtained.*

Proof. The proof falls into three parts. At the k th iteration, a Lyapunov candidate is established as

$$\begin{aligned} V^k &= \left(\delta^k\right)^{\mathrm{T}} (H \otimes P)\delta^k + \sum_{i=1}^N \frac{1}{m_i} \int_0^t \left(\widetilde{\theta}_i^k(\tau)\right)^2 d\tau + \sum_{i=1}^N \frac{1}{\rho_i}\left(\widetilde{c}_i^k(t)\right)^2 \\ &\quad + \sum_{i=1}^N \left(\mu_i^k(t)\right)^{\mathrm{T}} P\mu_i^k(t) + \sum_{i=1}^N \left(\widetilde{\eta}_i^k(t)\right)^{\mathrm{T}} \Gamma_i^{-1}\widetilde{\eta}_i^k(t), \end{aligned} \tag{16}$$

where $\widetilde{c}_i^k = c_i - \widehat{c}_i^k(t)$ and $\widetilde{\eta}_i^k(t) = \overline{\eta} - \widehat{\eta}_i^k(t), i = 1, 2, \ldots, N$.

At first, the difference between V^k and V^{k-1} is

$$\Delta V^k(t) = V^k(t) - V^{k-1}(t) = \left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\delta^k - \left(\delta^{k-1}(t)\right)^{\mathrm{T}}(H\otimes P)\delta^{k-1}(t) + \sum_{i=1}^{N}\frac{1}{m_i}\int_0^t\left[\left(\tilde{\theta}_i^k\right)^2 - \left(\tilde{\theta}_i^{k-1}\right)^2\right]d\tau$$
$$+\sum_{i=1}^{N}\frac{1}{\rho_i}\left[\left(\tilde{c}_i^k\right)^2 - \left(\tilde{c}_i^{k-1}\right)^2\right] + \sum_{i=1}^{N}\left(\mu_i^k(t)\right)^{\mathrm{T}}\Gamma_i^{-1}\mu_i^k(t) - \sum_{i=1}^{N}\left(\mu_i^{k-1}(t)\right)^{\mathrm{T}}\Gamma_i^{-1}\mu_i^{k-1}(t) + \sum_{i=1}^{N}\left(\tilde{\eta}_i\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^k - \sum_{i=1}^{N}\left(\tilde{\eta}_i^{k-1}\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^{k-1}. \tag{17}$$

From the error dynamic (14), we have

$$\begin{aligned}\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\delta^k &= 2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\dot{\delta}^k d\tau + \left(\delta^k(0)\right)^{\mathrm{T}}(H\otimes P)\delta^k(0)\\ &= \int_0^t\left(\delta^k\right)^{\mathrm{T}}\left[H\otimes\left(PA + A^{\mathrm{T}}P\right)\right]\delta^k d\tau + 2\varpi\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\mu^k d\tau - 2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\delta u^k d\tau\\ &\quad - 2\int_0^t\left(\delta^k\right)^{\mathrm{T}}\left(HC^k(\tau)H\otimes I_n\right)\delta^k + 2\int_0^t\left(\delta^k\right)^{\mathrm{T}}\left(H\tilde{\Theta}^k(\tau)\otimes P\right)\xi^k d\tau\\ &\quad + 2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)v^k d\tau - 2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\left(1_N\otimes f(x_0)\right)d\tau\\ &\quad + \left(\delta^k(0)\right)^{\mathrm{T}}(H\otimes P)\delta^k(0),\end{aligned} \tag{18}$$

where

$$-2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\delta u^k d\tau \le \int_0^t\left(\delta^k\right)^{\mathrm{T}}\left(H^2\otimes P^2\right)\delta^k d\tau + \int_0^t\left(\delta u^k\right)^{\mathrm{T}}\delta u^k d\tau, \tag{19}$$

and

$$\begin{aligned}\sum_{i=1}^{N}\frac{1}{m_i}\int_0^t\left[\left(\tilde{\theta}_i^k\right)^2 - \left(\tilde{\theta}_i^{k-1}\right)^2\right]d\tau &= \sum_{i=1}^{N}\frac{1}{m_i}\int_0^t\left(\hat{\theta}_i^{k-1} - \hat{\theta}_i^k\right)\left[2\left(\theta_i - \hat{\theta}_i^k\right) + \left(\hat{\theta}_i^k - \hat{\theta}_i^{k-1}\right)\right]d\tau\\ &\leqslant \sum_{i=1}^{N}\frac{2}{m_i}\int_0^t\tilde{\theta}_i^k\left(\hat{\theta}_i^{k-1} - \hat{\theta}_i^k\right)d\tau \leqslant 2\sum_{i=1}^{N}\int_0^t\tilde{\theta}_i^k\left(e_i^k\right)^{\mathrm{T}}P\xi_i^k d\tau\\ &= -2\int_0^t\left(\delta^k\right)^{\mathrm{T}}\left(H\tilde{\Theta}^k\otimes P\right)\xi^k d\tau.\end{aligned} \tag{20}$$

Simultaneously,

$$\begin{aligned}\sum_{i=1}^{N}\frac{1}{\rho_i}\left[\left(\tilde{c}_i^k\right)^2 - \tilde{c}_i^{k-1}\right)^2\right] &= \sum_{i=1}^{N}\frac{2}{\rho_i}\int_0^t\tilde{c}_i^k\dot{\tilde{c}}_i^k d\tau + \sum_{i=1}^{N}\frac{1}{\rho_i}\left[\left(\tilde{c}_i^k(0)\right)^2 - \tilde{c}_i^{k-1}(t)\right)^2\\ &= -2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(HCH\otimes P)\delta^k d\tau + 2\int_0^t\left(\delta^k\right)^{\mathrm{T}}\left(H\hat{C}^k(\tau)H\otimes P\right)\delta^k d\tau + \sum_{i=1}^{N}\frac{1}{\rho_i}\left[\left(\tilde{c}_i^k(0)\right)^2 - \tilde{c}_i^{k-1}(t)\right)^2\right],\end{aligned} \tag{21}$$

where $C = \mathrm{diag}\{c_1, c_1, \ldots, c_N\}$.

In addition,

$$\begin{aligned}\sum_{i=1}^{N}\left(\mu_i^k(t)\right)^{\mathrm{T}} P\mu_i^k(t) - \sum_{i=1}^{N}\left(\mu_i^{k-1}(t)\right)^{\mathrm{T}} P\mu_i^{k-1}(t) &= 2\sum_{i=1}^{N}\int_0^t \left(\mu_i^k\right)^{\mathrm{T}} P\dot{\mu}_i^k d\tau - \sum_{i=1}^{N}\left(\mu_i^{k-1}(t)\right)^{\mathrm{T}} P\mu_i^{k-1}(t) + \sum_{i=1}^{N}\left(\mu_i^k(0)\right)^{\mathrm{T}} P\mu_i^k(0) \\ &= -2\varpi\int_0^t \left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\mu^k d\tau - \int_0^t \left(\delta u^k\right)^{\mathrm{T}}\delta u^k d\tau - \sum_{i=1}^{N}\left(\mu_i^{k-1}(t)\right)^{\mathrm{T}} P\mu_i^{k-1}(t) \\ &\quad + \sum_{i=1}^{N}\left(\mu_i^k(0)\right)^{\mathrm{T}} P\mu_i^k(0),\end{aligned} \tag{22}$$

and

$$\begin{aligned}\sum_{i=1}^{N}\left(\tilde{\eta}_i^k\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^k - \sum_{i=1}^{N}\left(\tilde{\eta}_i^{k-1}\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^{k-1} &= 2\sum_{i=1}^{N}\int_0^t \left(\tilde{\eta}_i^k\right)^{\mathrm{T}}\Gamma_i^{-1}\dot{\tilde{\eta}}_i^k d\tau - \sum_{i=1}^{N}\left(\tilde{\eta}_i^{k-1}(t)\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^{k-1}(t) + \sum_{i=1}^{N}\left(\tilde{\eta}_i^k(0)\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^k(0) \\ &= -2\sum_{i=1}^{N}\int_0^t \left(\tilde{\eta}_i^k\right)^{\mathrm{T}}\left|Pe_i^k\right| d\tau - \sum_{i=1}^{N}\left(\tilde{\eta}_i^{k-1}(t)\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^{k-1}(t) + \sum_{i=1}^{N}\left(\tilde{\eta}_i^k(0)\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^k(0).\end{aligned} \tag{23}$$

Substituting (18)–(23) into (17) yields

$$\begin{aligned}\Delta V^k(t) \leqslant{}& \int_0^t \left(\delta^k\right)^{\mathrm{T}}\left[H\otimes\left(PA + A^{\mathrm{T}}P\right) - \left(2HCH\otimes P - H^2\otimes P^2\right)\right]\delta^k d\tau + \left(\delta^k(0)\right)^{\mathrm{T}}(H\otimes P)\delta^k(0) - \left(\delta^{k-1}(t)\right)^{\mathrm{T}}(H\otimes P)\delta^{k-1}(t) \\ &+ \sum_{i=1}^{N}\frac{1}{\rho_i}\left[\left(\tilde{c}_i^k(0)\right)^2 - \tilde{c}_i^{k-1}(t)\right)^2 \\ &+ \sum_{i=1}^{N}\left(\mu_i^k(0)\right)^{\mathrm{T}} P\mu_i^k(0) - \sum_{i=1}^{N}\left(\mu_i^{k-1}(t)\right)^{\mathrm{T}} P\mu_i^{k-1}(t) + \sum_{i=1}^{N}\left(\tilde{\eta}_i^k(0)\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^k(0) - \sum_{i=1}^{N}\left(\tilde{\eta}_i^{k-1}(t)\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^{k-1}(t) \\ &+ 2\int_0^t \left(\delta^k\right)^{\mathrm{T}}(H\otimes P)v^k d\tau - 2\int_0^t \left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\left(1_N\otimes f(x_0)\right)d\tau - 2\sum_{i=1}^{N}\int_0^t \left(\tilde{\eta}_i^k\right)^{\mathrm{T}}\left|Pe_i^k\right| d\tau.\end{aligned} \tag{24}$$

Due to (8) and Property 1 in [20],

$$
\begin{aligned}
&2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)v^k d\tau-2\int_0^t\left(\delta^k\right)^{\mathrm{T}}(H\otimes P)\left(1_N\otimes f\left(x_0\right)d\tau-2\sum_{i=1}^N\int_0^t\left(\widetilde{\eta}_i^k\right)^{\mathrm{T}}\left|Pe_i^k\right|d\tau\right.\\
&=-2\sum_{i=1}^N\int_0^t\left(e_i^k\right)^{\mathrm{T}}Pv_i^k d\tau+2\sum_{i}^N\int_0^t\left(e_i^k\right)^{\mathrm{T}}Pf\left(x_0\right))d\tau-2\sum_{i=1}^N\int_0^t\left(\widetilde{\eta}_i^k\right)^{\mathrm{T}}\left|Pe_i^k\right|d\tau\\
&\leq-2\sum_{i=1}^N\int_0^t\left(e_i^k\right)^{\mathrm{T}}Pv_i^k d\tau+2\sum_{i}^N\int_0^t\left|\left(e_i^k\right)^{\mathrm{T}}P\right|\overline{\eta}d\tau-2\sum_{i=1}^N\int_0^t\left|\left(e_i^k\right)^{\mathrm{T}}P\right|\widetilde{\eta}_i^k d\tau\\
&=2\sum_{i=1}^N\int_0^t\left[\left|\left(e_i^k\right)^{\mathrm{T}}P\right|\widetilde{\eta}_i^k-\left(e_i^k\right)^{\mathrm{T}}P\widehat{\eta}_i^k\tanh\left(\frac{\left(e_i^k\right)^{\mathrm{T}}P\widehat{\eta}_i^k}{\Delta_{k+1}}\right)\right]d\tau\leqslant 2NT\varepsilon\Delta_{k+1}.
\end{aligned}
\tag{25}
$$

Therefore,

$$
\begin{aligned}
\Delta V^k(t)\leqslant&\int_0^t\left(\delta^k\right)^{\mathrm{T}}\left[H\otimes\left(PA+A^{\mathrm{T}}P\right)-\left(2HCH\otimes P-H^2\otimes P^2\right)\right]\delta^k d\tau\\
&+\left(\delta^k(0)\right)^{\mathrm{T}}(H\otimes P)\delta^k(0)-\left(\delta^{k-1}(t)\right)^{\mathrm{T}}(H\otimes P)\delta^{k-1}(t)+\sum_{i=1}^N\frac{1}{\rho_i}\left[\left(\widetilde{c}_i^k(0)\right)^2-\widetilde{c}_i^{k-1}(t)\right)^2\\
&+\sum_{i=1}^N\left(\mu_i^k(0)\right)^{\mathrm{T}}P\mu_i^k(0)-\sum_{i=1}^N\left(\mu_i^{k-1}(t)\right)^{\mathrm{T}}P\mu_i^{k-1}(t)\\
&+\sum_{i=1}^N\left(\widetilde{\eta}_i^k(0)\right)^{\mathrm{T}}\Gamma_i^{-1}\widetilde{\eta}_i^k(0)-\sum_{i=1}^N\left(\widetilde{\eta}_i^{k-1}(t)\right)^{\mathrm{T}}\Gamma_i^{-1}\widetilde{\eta}_i^{k-1}(t)+2N\varepsilon\Delta_{k+1}.
\end{aligned}
\tag{26}
$$

On account of the positiveness of H, C, and P, set $\widetilde{\delta}^k=(F^{\mathrm{T}}\otimes I_n)\delta^k$ and F is an orthogonal matrix:

$$
\begin{aligned}
\Delta V^k\leq&\sum_{i=1}^N\lambda_i(H)\int_0^t\left(\widetilde{\delta}_i^k\right)^{\mathrm{T}}\left[\left(PA+A^{\mathrm{T}}P\right)-2\left(c_{\min}\lambda_{\min}(H)\lambda_{\min}(P)-\lambda_{\max^2}(H)\lambda_{\max^2}(P)\right)I\right]\widetilde{\delta}_i^k d\tau+2NT\varepsilon\Delta_{k+1}\\
&+\left(\delta^k(0)\right)^{\mathrm{T}}(H\otimes P)\delta^k(0)-\left(\delta^{k-1}(t)\right)^{\mathrm{T}}(H\otimes P)\delta^{k-1}(t)+\sum_{i=1}^N\frac{1}{\rho_i}\left[\left(\widetilde{c}_i^k(0)\right)^2-\widetilde{c}_i^{k-1}(t)\right)^2\\
&+\sum_{i=1}^N\left(\mu_i^k(0)\right)^{\mathrm{T}}P\mu_i^k(0)-\sum_{i=1}^N\left(\mu_i^{k-1}(t)\right)^{\mathrm{T}}P\mu_i^{k-1}(t)+\sum_{i=1}^N\left(\widetilde{\eta}_i^k(0)\right)^{\mathrm{T}}\Gamma_i^{-1}\widetilde{\eta}_i^k(0)-\sum_{i=1}^N\left(\widetilde{\eta}_i^{k-1}(t)\right)^{\mathrm{T}}\Gamma_i^{-1}\widetilde{\eta}_i^{k-1}(t),
\end{aligned}
\tag{27}
$$

where $\lambda_i(H)>0$ shows the eigenvalue of H and $c_{\min}=\min_{1\leq i\leq N}\{c_i\}$. For the sufficient large constant $c_{\min}>0$, the inequality $PA+A^{\mathrm{T}}P-2(c_{\min}\lambda_{\min}(H)\lambda_{\min}(P)-\lambda_{\max^2}(H)\lambda_{\max^2}(P))\leq-\sigma I$ with $\sigma>0$ always holds.

When $t=T$, it follows that

$$
\Delta V^k(T)\leqslant-\sigma\lambda_{\min}(H)\sum_{i=1}^N\int_0^T\left(\delta_i^k\right)^{\mathrm{T}}\delta_i^k d\tau+2NT\varepsilon\Delta_{k+1},
\tag{28}
$$

which results in

$$
V^k(T)\leqslant V^{k-1}(T)+2NT\varepsilon\Delta_{k+1}.
\tag{29}
$$

In the second place, let us prove boundednesses of signals involved. On the basis of the definition of V^k results,

$$V^k = \Delta V^k + V^{k-1} \leqslant -\sigma\lambda_{\min}(H)\sum_{i=1}^{N}\int_0^t \left(\delta_i^k\right)^{\mathrm{T}}\delta_i^k d\tau + \left(\delta^k(0)\right)^{\mathrm{T}}(H\otimes P)\delta^k(0) - \left(\delta^{k-1}\right)^{\mathrm{T}}(H\otimes P)\delta^{k-1}$$
$$+\left(\delta^{k-1}\right)^{\mathrm{T}}(H\otimes P)\delta^{k-1} + \sum_{i=1}^{N}\frac{1}{m_i}\int_0^t\left(\tilde{\theta}_i^{k-1}\right)^2 d\tau + \sum_{i=1}^{N}\left(\mu_i^{k-1}\right)^{\mathrm{T}}P\mu_i^{k-1} + \sum_{i=1}^{N}\left(\mu_i^k(0)\right)^{\mathrm{T}}P\mu_i^k(0) - \sum_{i=1}^{N}\left(\mu_i^{k-1}\right)^{\mathrm{T}}P\mu_i^{k-1}$$
$$+\sum_{i=1}^{N}\left(\tilde{\eta}_i^{k-1}\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^{k-1} + \sum_{i=1}^{N}\left(\tilde{\eta}_i^k(0)\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^k(0) - \sum_{i=1}^{N}\left(\tilde{\eta}_i^{k-1}\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^{k-1} + \sum_{i=1}^{N}\frac{1}{\rho_i}\left(\tilde{c}_i^{k-1}\right)^2 + \sum_{i=1}^{N}\frac{1}{\rho_i}\left[\left(\tilde{c}_i^k(0)\right)^2 - \left(\tilde{c}_i^{k-1}\right)^2\right] + 2NT\varepsilon\Delta_{k+1}. \tag{30}$$

That is,

$$V^k \leqslant V^{k-1}(T) + 2NT\varepsilon\Delta_{k+1}. \tag{31}$$

It can be obtained from (29) and (31) that

$$\begin{aligned} V^k &\leqslant V^{k-1}(T) + 2NT\varepsilon\Delta_{k+1} \\ &\leqslant V^{k-2}(T) + 2NT\varepsilon\Delta_k + 2NT\varepsilon\Delta_{k+1} \\ &\vdots \\ &\leqslant V^0(T) + 2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l. \end{aligned} \tag{32}$$

As $\lim_{k\longrightarrow\infty} 2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l \leqslant 4NT\varepsilon a$ [20], $2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l$ is uniformly bounded, $\forall k\in Z^+$. Denote $2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l \leqslant S$ with $S>0$. Hence,

$$V^k \leqslant V^0(T) + S. \tag{33}$$

If the finiteness of $V^0(T)$ is attained, the uniform boundedness of $V^k(t)$ is followed. And then, we will show the finiteness of V^0. It obtained that

$$V^0(t) = \left(\delta^0\right)^{\mathrm{T}}(H\otimes P)\delta^0 + \sum_{i=1}^{N}\left(\mu_i^0(t)\right)^{\mathrm{T}}P\mu_i^0 + \sum_{i=1}^{N}\frac{1}{\rho_i}\left(\tilde{c}_i^0\right)^2 + \sum_{i=1}^{N}\frac{1}{m_i}\int_0^t\left(\tilde{\theta}_i^0\right)^2 d\tau + \sum_{i=1}^{N}\left(\tilde{\eta}_i^0\right)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\eta}_i^0, \tag{34}$$

and

$$\dot{V}^0 \leqslant \left(\delta^0\right)^{\mathrm{T}}\left[H\otimes\left(PA + A^{\mathrm{T}}P\right) - \left(2HCH\otimes P - H^2\otimes P^2\right)\right]\delta^0 + \sum_{i=1}^{N}\frac{1}{m_i}\theta_i^2(t) + 2\left(\delta^0\right)^{\mathrm{T}}(H\otimes P)v^0 - 2\left(\delta^0\right)^{\mathrm{T}}(H\otimes P)\left(1_N\otimes f(x_0)\right) - 2\sum_{i=1}^{N}\left(\tilde{\eta}_i^0\right)^{\mathrm{T}}\left|Pe_i^0\right|, \tag{35}$$

where

$$\begin{aligned} &2\left(\delta^0\right)^{\mathrm{T}}(H\otimes P)v^0 - 2\left(\delta^0\right)^{\mathrm{T}}(H\otimes P)\left(1_N\otimes f(x_0)\right) - 2\sum_{i=1}^{N}\left(\tilde{\eta}_i^0\right)^{\mathrm{T}}\left|Pe_i^0\right|d\tau \\ &\leqslant 2\sum_{i=1}^{N}\left[\left|\left(e_i^0\right)^{\mathrm{T}}P\right|\hat{\eta}_i^0 - \left(e_i^0\right)^{\mathrm{T}}P\hat{\eta}_i^0\tanh\left(\frac{\left(e_i^0\right)^{\mathrm{T}}P\hat{\eta}_i^0}{\Delta_1}\right)\right] \leqslant 2N\varepsilon a. \end{aligned} \tag{36}$$

Thus,

$$\dot{V}^0 \leqslant -\sigma\lambda_{\min}(H)\sum_{i=1}^{N}\left(\delta_i^0\right)^{\mathrm{T}}\delta_i^0 + \sum_{i=1}^{N}\frac{1}{m_i}\theta_i^2(t) + 2N\varepsilon a \leqslant \sum_{i=1}^{N}\frac{1}{m_i}\theta_i^2(t) + 2N\varepsilon a. \quad (37)$$

Since $\theta_i(t)$ is continuous, the boundedness of it obtained $[0, T]$. Then,

$$\dot{V}^0(t) \leqslant M_0', \quad (38)$$

where $M_0' = 1/m_{\min}\max_{t\in[0,T], 1\le i\le N}\left\{\theta_i^2(t)\right\} + 2N\varepsilon a$ and $q_{\min} = \min_{1\le i\le N}\{q_i\}$, $m_{\min} = \min_{1\le i\le N}\{m_i\}$. Meanwhile,

$$V^0 = \left|V^0(0)\right| + \int_0^t\left|\dot{V}^0\right|d\tau \leqslant \left(\delta^0(0)\right)^{\mathrm{T}}(H\otimes P)\delta^0(0) + \sum_{i=1}^{N}\frac{1}{\rho_i}c_i^2 + \sum_{i=1}^{N}\left(\mu_i^0(0)\right)^{\mathrm{T}}P\mu_i^0(0) + \sum_{i=1}^{N}\overline{\eta}^{\mathrm{T}}\Gamma_i^{-1}\overline{\eta} + TM_0' < \infty. \quad (39)$$

$V^0(t)$ is finite; it is followed that $V^0(T)$ is bounded. Therefore, the uniformly boundedness of $V^k(t)$ is obtained over $[0, T]$, $\forall k \in Z^+$. Furthermore, it is inspired by the definition of V^k; it obtained the uniform boundednesses of $\delta^k(t), \mu_i^k(t), \tilde{c}_i^k(t)$, and $\eta_i^k(t)$. The updating law (11) indicates that $\widehat{\theta}_i^k(t)$ is bounded. From (7), we can calculate the uniform boundedness of $u_i^k(t)$. So, the boundednesses of signals involved are gained.

At last, we prove the property of learning consensus. As we know,

$$V^k(T) = V^0(T) + \sum_{l=1}^{k}\Delta V^l(T). \quad (40)$$

From (28), it can yield

$$V^k(T) \leqslant V^0(T) - \sigma\lambda_{\min}(H)\sum_{l=1}^{k}\sum_{i=1}^{N}\int_0^T\left(\delta_i^l\right)^{\mathrm{T}}\delta_i^l d\tau + 2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l. \quad (41)$$

Since $V^k(T)$ is positive, $V^0(T)$ is bounded, and the series $2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l$ is convergent, and the series $\sum_{l=1}^{k}\sum_{i=1}^{N}\int_0^t(\delta_i^l)^{\mathrm{T}}\delta_i^l d\tau$ is convergent. Consequently, $\lim_{k\longrightarrow\infty}\sum_{i=1}^{N}\int_0^t(\delta_i^k)^{\mathrm{T}}\delta_i^k d\tau = 0$. It is easy to achieve $\lim_{k\longrightarrow\infty}\int_0^T(\delta_i^k)^{\mathrm{T}}\delta_i^k d\tau = 0$, $i = 1, 2, \ldots, N$. From (13), $\dot{\delta}_i^k(t)$ is bounded on $[0, T]$. At last, from Barbalat-like Lemma, $\lim_{k\longrightarrow\infty}\delta_i^k(t) = 0$ holds uniformly over $[0, T]$, i.e., $\lim_{k\longrightarrow\infty}(x_i^k(t) - x_0(t)) = 0$. In other words, each follower can completely track the leader over $[0, T]$. □

Remark 5. The condition $PA + A^{\mathrm{T}}P - 2(c_{\min}\lambda_{\min}(H)\lambda_{\min}(P) - \lambda_{\max^2}(H)\lambda_{\max^2}(P))I \le -\sigma I$ with $\sigma > 0$ is only for the analysis purpose; as a matter of fact, it is not utilized in the design of protocols. Accordingly, the distributed learning control protocols are fully distributed and the consensus for the MASs is solved faultlessly even if each follower is without knowing the dynamic of the leader.

4. Formation Control of the MASs

The formation control of the MASs (1) is concerned here. If the followers and leader form a formation at a certain distance over $[0, T]$, we can say that the formation control is achieved.

Let us define

$$\overline{x}_{i1}^k = x_{i1}^k - \Delta_i, \quad (42)$$

where Δ_i is the expected formation vector for the i th follower relative to the leader.

The formation error is

$$\delta_{i1}^k(t) = \overline{x}_{i1}^k(t) - x_{01}(t). \quad (43)$$

And, $\delta_{il}^k(t)$ is the same as $\delta_{il}^k(t)$ defined in (2), $l = 2, 3, \ldots, n$.

Like that, the problem of formation can be reformulated as the consensus problem, i.e., $\lim_{k\longrightarrow\infty}\|\delta_i^k\| = 0$.

Simultaneously, the neighborhood formation errors are

$$\begin{aligned} e_{i1}^k &= \sum_{j=1}^{N}a_{ij}\left(\overline{x}_{j1}^k - \overline{x}_{i1}^k\right) + b_i\left(x_{01} - \overline{x}_{i1}^k\right), \\ e_{il}^k &= \sum_{j=1}^{N}a_{ij}\left(x_{jl}^k - x_{il}^k\right) + b_i\left(x_{0l} - x_{il}^k\right), \quad l = 2, 3, \ldots, n. \end{aligned} \quad (44)$$

Assumption 3. For each follower, $\overline{x}_{i1}^k(0) = \overline{x}_{i1}^{k-1}(T), x_{il}^k(0) = x_{il}^{k-1}(T), i = 1, 2, \ldots, N$ and $l = 2, 3, \ldots, n$; for the leader, $x_0(0) = x_0(T)$. Then, it follows that $\delta_i^k(0) = \delta_i^{k-1}(T)$.

Theorem 2. *For the MASs with graph $\overline{G}$, under assumptions 1 and 3, N followers represented by (1) under the protocols (7)-(8) with learning-based updating laws (9)-(12) with the local neighborhood formation errors (44) can make the followers form the desired formation in the iteration domain on $t \in [0, T]$. The variables involved are bounded.*

5. Simulation

In this part, two examples are provided to validate the validity and practicability for the fully distributed learning protocol of this paper. As mentioned in [33], the LC

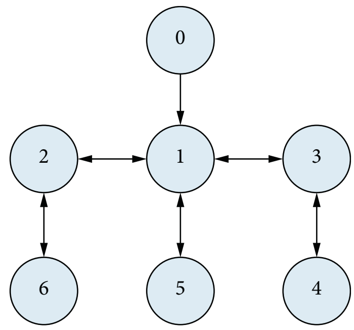

Figure 1: Topology graph (0 indicates the leader).

oscillator system can be considered as a MASs. Therefore, here, let us consider the MASs consisting of six followers as well as one leader. Figure 1 shows the communication graph.

Then, we have

$$L = \begin{pmatrix} 3 & -1 & -1 & 0 & -1 & 0 \\ -1 & 2 & 0 & 0 & 0 & -1 \\ -1 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix},$$
$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (45)$$

Example 1. Consider the MASs (1) with $A = [0, 1; -1, 0]$, $\theta_i(t) = \cos(2\pi i t)$, $\xi_i^k(t) = [(0.01 \sin x_{i1}^k)^3, (0.01 \sin (x_{i2}^k)^3)]^{\mathrm{T}}$, $i = 1, 2, 3, 4, 5, 6$. $x_0(t) = [\sin(t), \cos(t)]^{\mathrm{T}}$. In simulations, we choose $a = 1 \times 10^4$, $T = 2\pi$; $[-25, 25]$ is the lower and upper bounds of the saturation function.

Case 1. Consensus of the MASs

The designed parameters are chosen as $\varpi = 2$:

$$\begin{aligned} q_1 &= 0.2, \\ q_2 &= 0.3, \\ q_3 &= 0.1, \\ q_4 &= 0.3, \\ q_5 &= 0.5, \\ q_6 &= 0.1; \\ m_1 &= 0.1, \\ m_2 &= 0.3, \\ m_3 &= 0.1, \\ m_4 &= 0.1, \\ m_5 &= 0.1, \\ m_6 &= 0.2, \\ \rho_1 &= \rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = 0.1, \\ \Gamma_1 &= \mathrm{diag}\{0.2, 0.3\}, \\ \Gamma_2 &= \mathrm{diag}\{0.2, 0.4\}, \end{aligned}$$

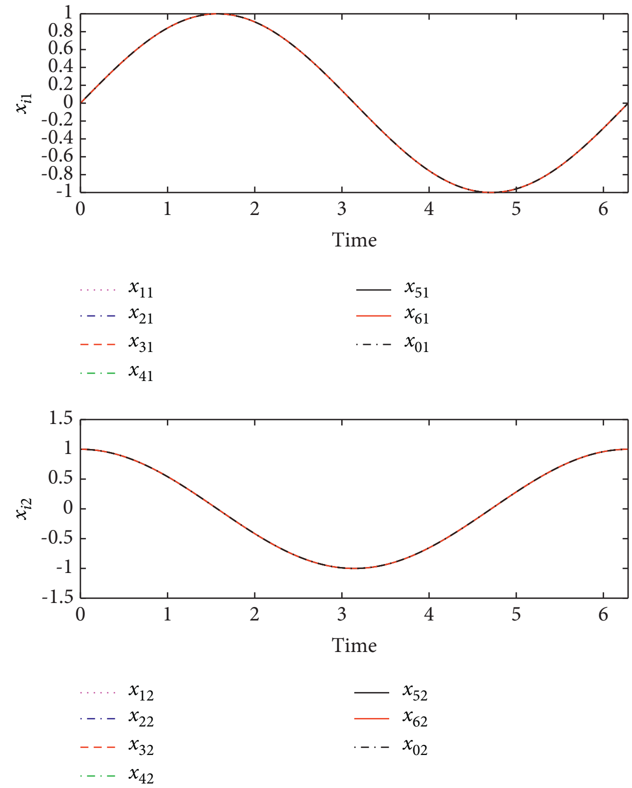

Figure 2: Consensus trajectories of six followers for Case 1 at 30th iteration.

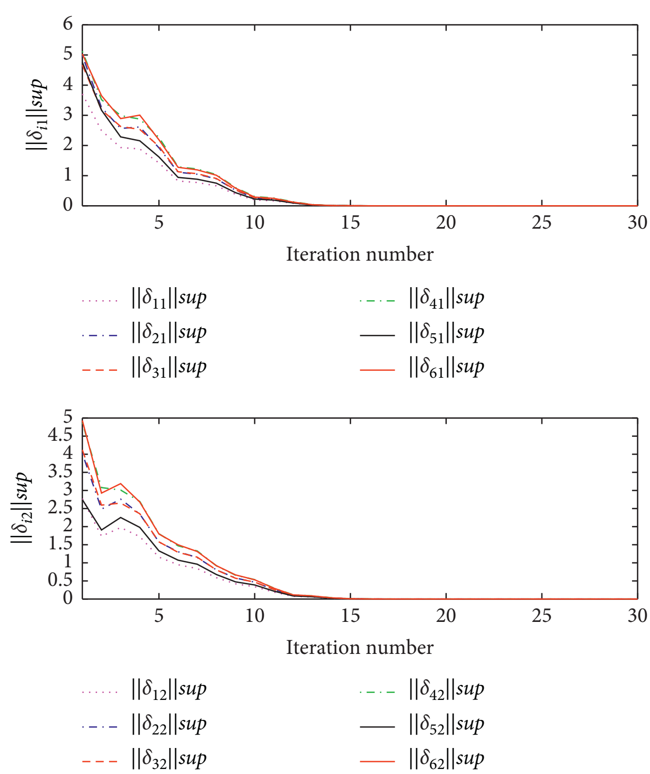

Figure 3: Consensus errors of six followers for Case 1.

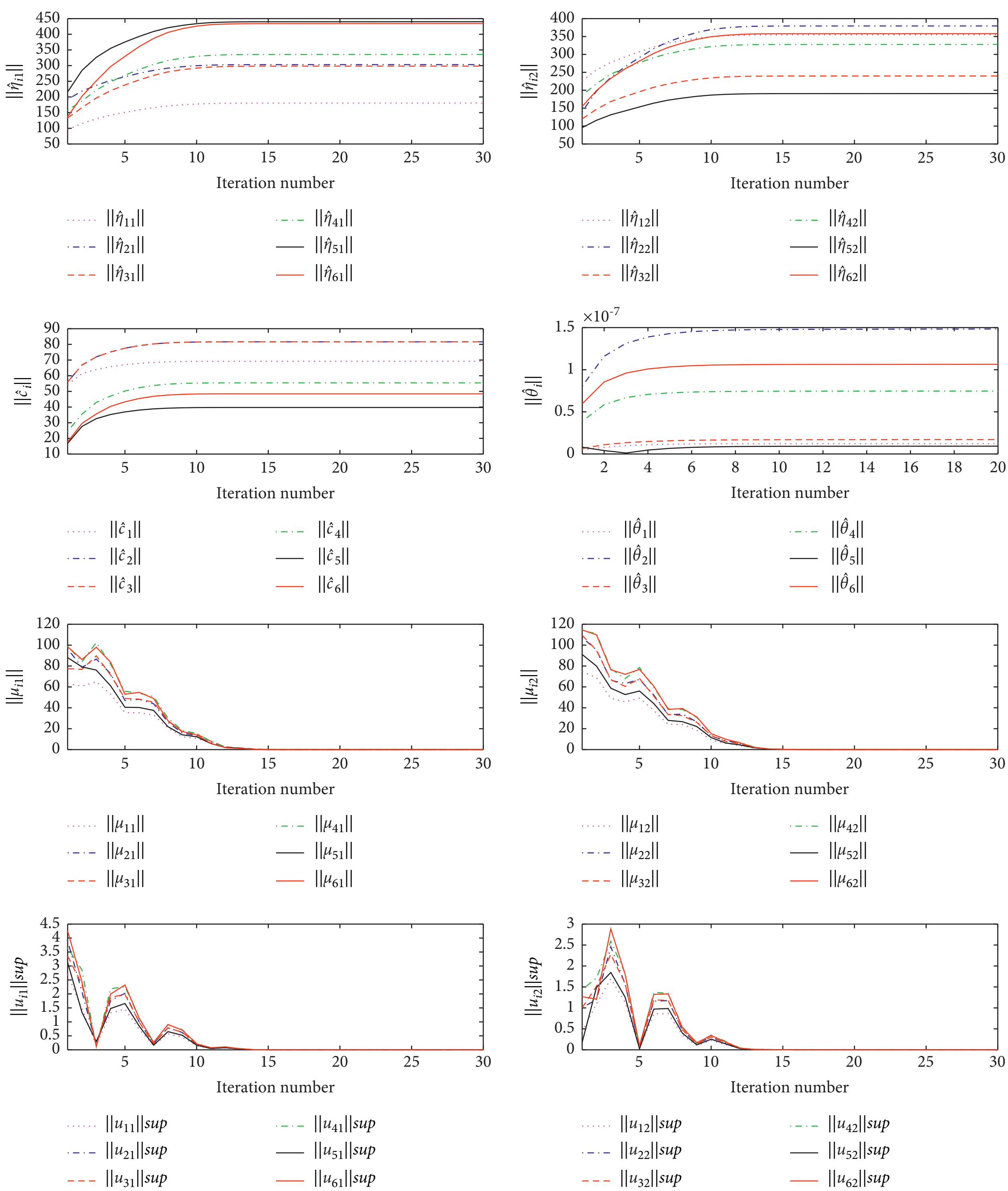

Figure 4: Responses of six followers for Case 1.

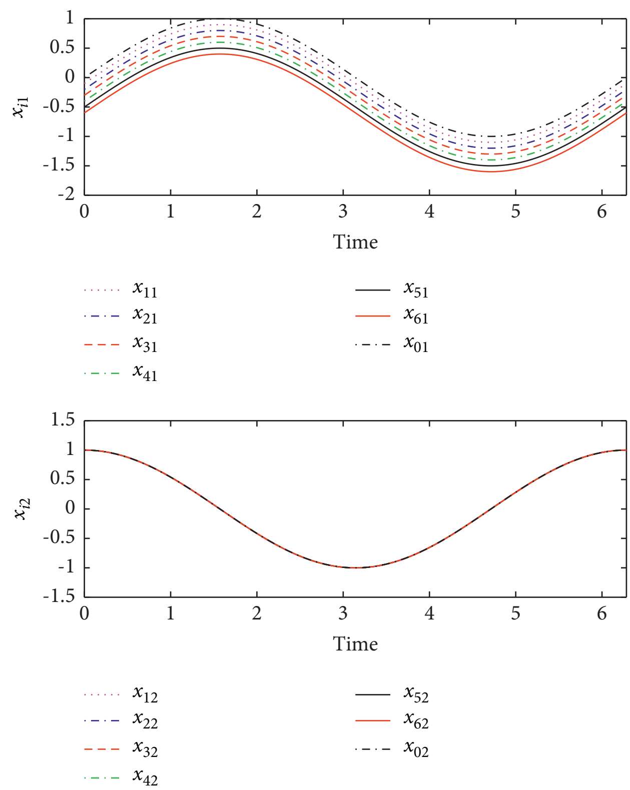

Figure 5: Formation trajectories of agents for Case 2 at 50th iteration.

$$\begin{aligned} \Gamma_3 &= \mathrm{diag}\{0.3, 0.2\}, \\ \Gamma_4 &= \mathrm{diag}\{0.3, 0.2\}, \\ \Gamma_5 &= \mathrm{diag}\{0.4, 0.2\}, \\ \Gamma_6 &= \mathrm{diag}\{0.5, 0.3\}, \\ P &= \mathrm{diag}\{1, 1\}, \\ c &= 3, \\ x_1(0) = [1,2]^{\mathrm{T}}, x_2(0) = [1,1.5]^{\mathrm{T}}, x_3(0) &= [3,1]^{\mathrm{T}}, x_4(0) = [1.5,2]^{\mathrm{T}}, x_5(0) = [1,1]^{\mathrm{T}}, x_6(0) = [2,3]^{\mathrm{T}}, \\ \hat{\eta}_1^0(0) = [1,2]^{\mathrm{T}}, \hat{\eta}_2^0(0) = [2,1]^{\mathrm{T}}, \hat{\eta}_3^0(0) &= [1,1]^{\mathrm{T}}, \hat{\eta}_4^0(0) = [1.5,2]^{\mathrm{T}}, \hat{\eta}_5^0(0) = [2,1]^{\mathrm{T}}, \hat{\eta}_6^0(0) = [1,1.5]^{\mathrm{T}}, \\ \mu_1^0(0) = [1,2]^{\mathrm{T}}, \mu_2^0(0) = [2,1]^{\mathrm{T}}, \mu_3^0(0) &= [1,1]^{\mathrm{T}}, \mu_4^0(0) = [1.5,2]^{\mathrm{T}}, \mu_5^0(0) = [2,1]^{\mathrm{T}}, \mu_6^0(0) = [1,1.5]^{\mathrm{T}}. \end{aligned} \tag{46}$$

After 30 cycles, Figures 2–4 show the simulation results. Even if the dynamic of the leader is unknown to each follower and there exit actuator saturations in the dynamic of system, it can be seen from Figures 2 and 3 that six followers can perfectly track the leader, and in Figure 4, it is evident the signals involved are bounded. The results fit into Theorem 1.

Case 2. Formation control of the MASs.

Select $\Delta_1 = -0.1, \Delta_2 = -0.2, \Delta_3 = -0.3, \Delta_4 = -0.4, \Delta_5 = -0.5, \Delta_6 = -0.6$. Other values involved are the same as in Case 1.

Figures 5–7 depict the results of formation control for the MASs (1). From Figure 5, we know that the agents form the desired formation. Figure 6 shows that the errors converge to zero over $[0, 2\pi]$, and Figure 7 illustrates that the signals involved in the closed-loop system are bounded. The results obtained align with Theorem 2.

Example 2. In the LC oscillator system [33] with six follower oscillators and one leader oscillator, each LC oscillator is governed by

$$\begin{cases} \dfrac{\mathrm{d}v_i(t)}{\mathrm{d}t} = \dfrac{1}{C} c_i(t), \\ \dfrac{\mathrm{d}c_i(t)}{\mathrm{d}t} = -\dfrac{1}{L} v_i(t), \end{cases} \tag{47}$$

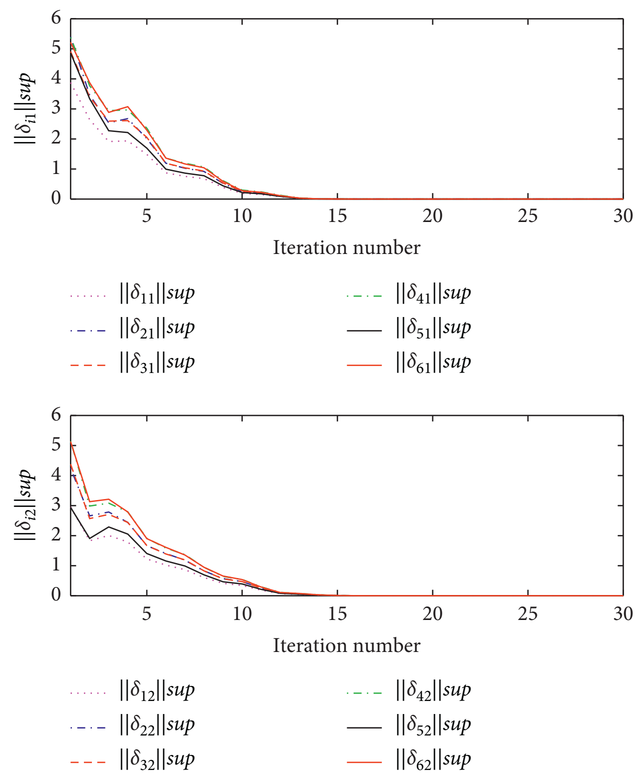

FIGURE 6: Formation errors of six followers for Case 2.

where L, C, $c_i(t)$, and $v_i(t)$ denote the inductance, capacitor, current, and voltage; $i = 1, 2, \ldots, 6$. Under the repetitive environment, we study the fully distributed adaptive ILC for the LC oscillator system (47); hence, the input is applied to each oscillator (47) and affected by actuator saturations. Furthermore, assume that that the disturbances are time-varying linearly parameterized uncertainties $\theta_i(t)\xi_i(x_i^k)$. Under these circumstances, system (47) is rewritten as

$$\dot{x}_i^k = Ax_i^k + \theta_i(t)\xi_i\left(x_i^k, t\right) + \operatorname{sat}\left(u_i^k(t)\right), \tag{48}$$

where $A = \begin{bmatrix} 0 & 1/C \\ -1/L & 0 \end{bmatrix}$, $x_i^k(t) = \begin{bmatrix} x_{i1}^k(t) \\ x_{i2}^k(t) \end{bmatrix} = \begin{bmatrix} v_i^k(t) \\ c_i^k(t) \end{bmatrix}$, $\xi_i^k(x_i^k, t) = \begin{bmatrix} 5x_{i2}^k \cos(x_{i1}^k) \\ 3x_{i1}^k \cos(x_{i2}^k) \end{bmatrix}$, $\theta_i(t) = 0.1 \sin(2\pi i t)$, and k is the iteration index. In addition, $x_0(t) = [\sin(\pi t), \cos(\pi t)]^{\mathrm{T}}$.

Select $a = 8 \times 10^6$, $T = 2$, $C = 1/2$, and $L = 1/4$; the bounds of saturation functions are $[-35, 35]$. The designed parameters are chosen as $\varpi = 1.3$:

$$\begin{aligned}
q_1 &= q_2 = q_3 = q_4 = q_5 = q_6 = 1, \\
m_1 &= m_2 = m_3 = m_4 = m_5 = m_6 = 2, \\
\rho_1 &= \rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = 1.5, \\
\Gamma_1 &= \Gamma_2 = \Gamma_3 = \Gamma_4 = \Gamma_5 = \Gamma_6 = \operatorname{diag}\{1, 0.2\}, \\
P &= \operatorname{diag}\{4, 5\}, \\
c &= 3, \\
x_1(0) &= x_2(0) = x_3(0) = x_4(0) = x_5(0) = x_6(0) = [-1, -1.5]^{\mathrm{T}}, \\
\widehat{\eta}_1^0(0) &= \widehat{\eta}_2^0(0) = \widehat{\eta}_3^0(0) = \widehat{\eta}_4^0(0) = \widehat{\eta}_5^0(0) = \widehat{\eta}_6^0(0) = [2, 3]^{\mathrm{T}}, \\
\mu_1^0(0) &= [1, 2]^{\mathrm{T}}, \\
\mu_2^0(0) &= [2, 1]^{\mathrm{T}},
\end{aligned}$$

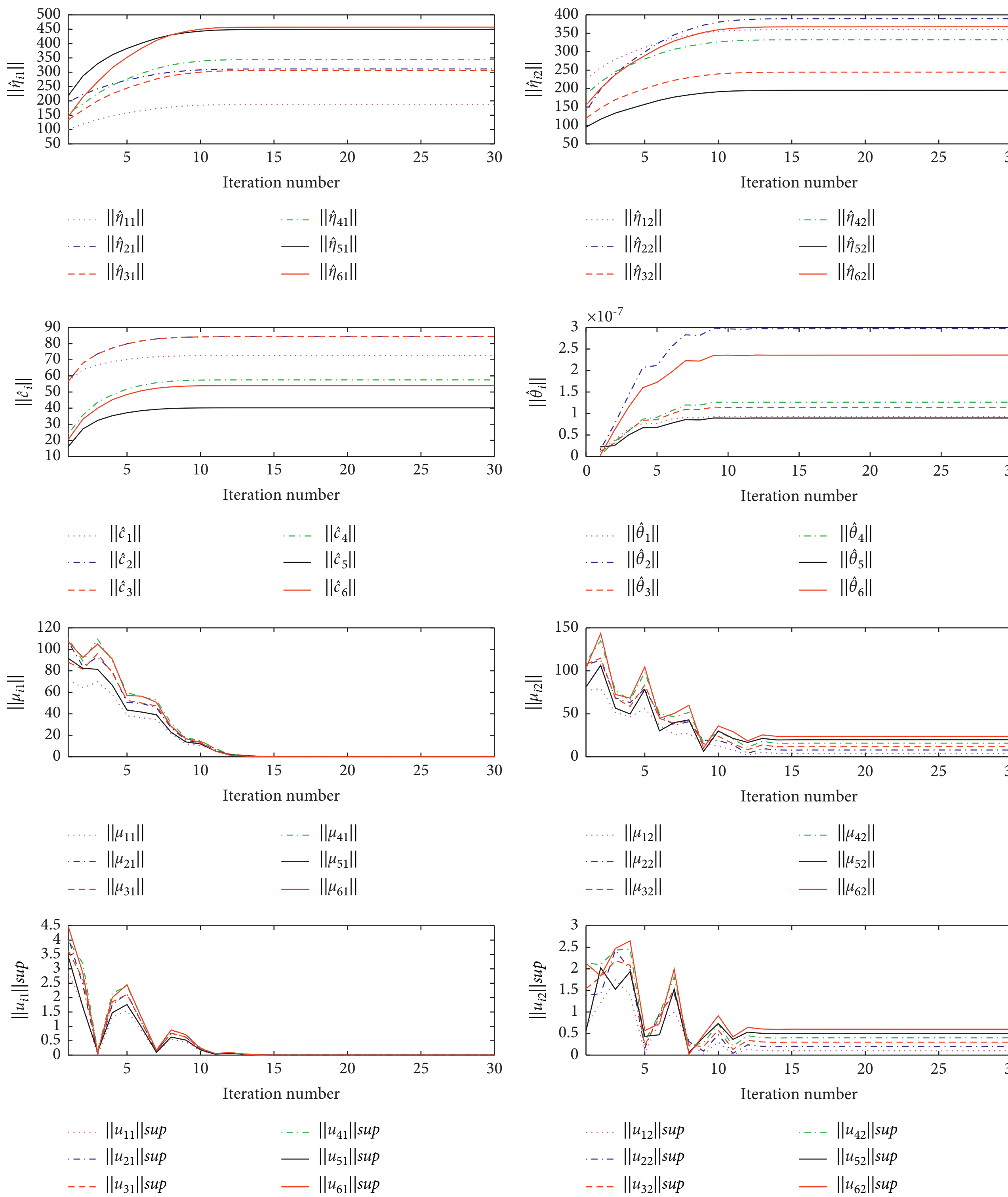

Figure 7: Responses of six followers for Case 2.

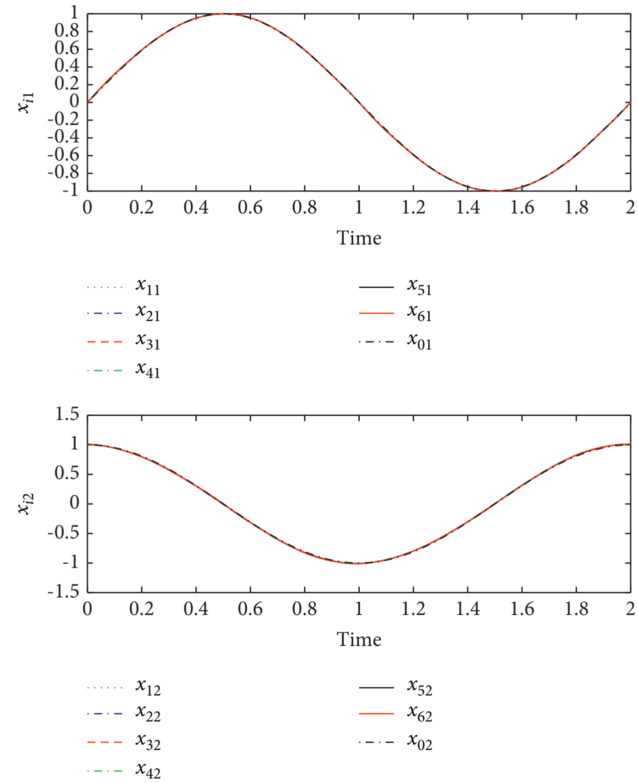

FIGURE 8: Consensus trajectories of six follower oscillators at 60th iteration.

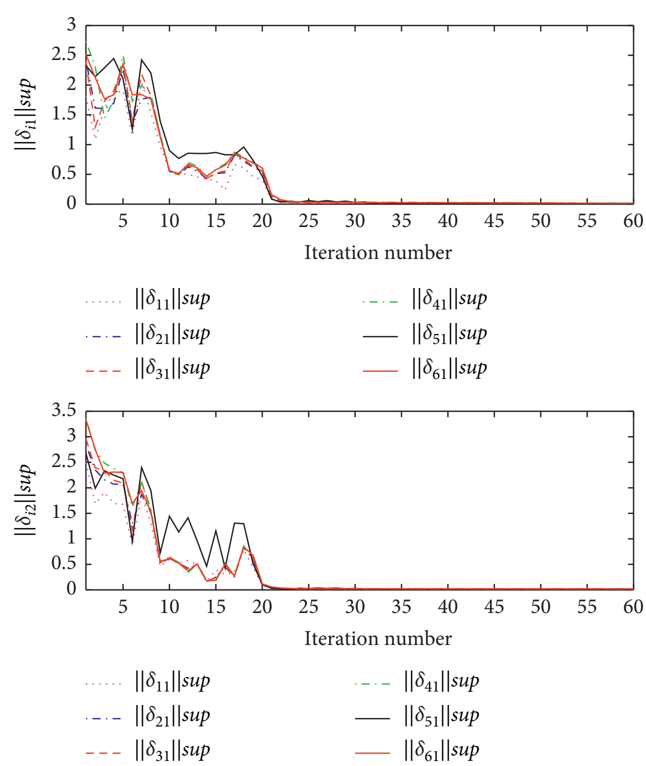

FIGURE 9: Consensus errors of six follower oscillators.

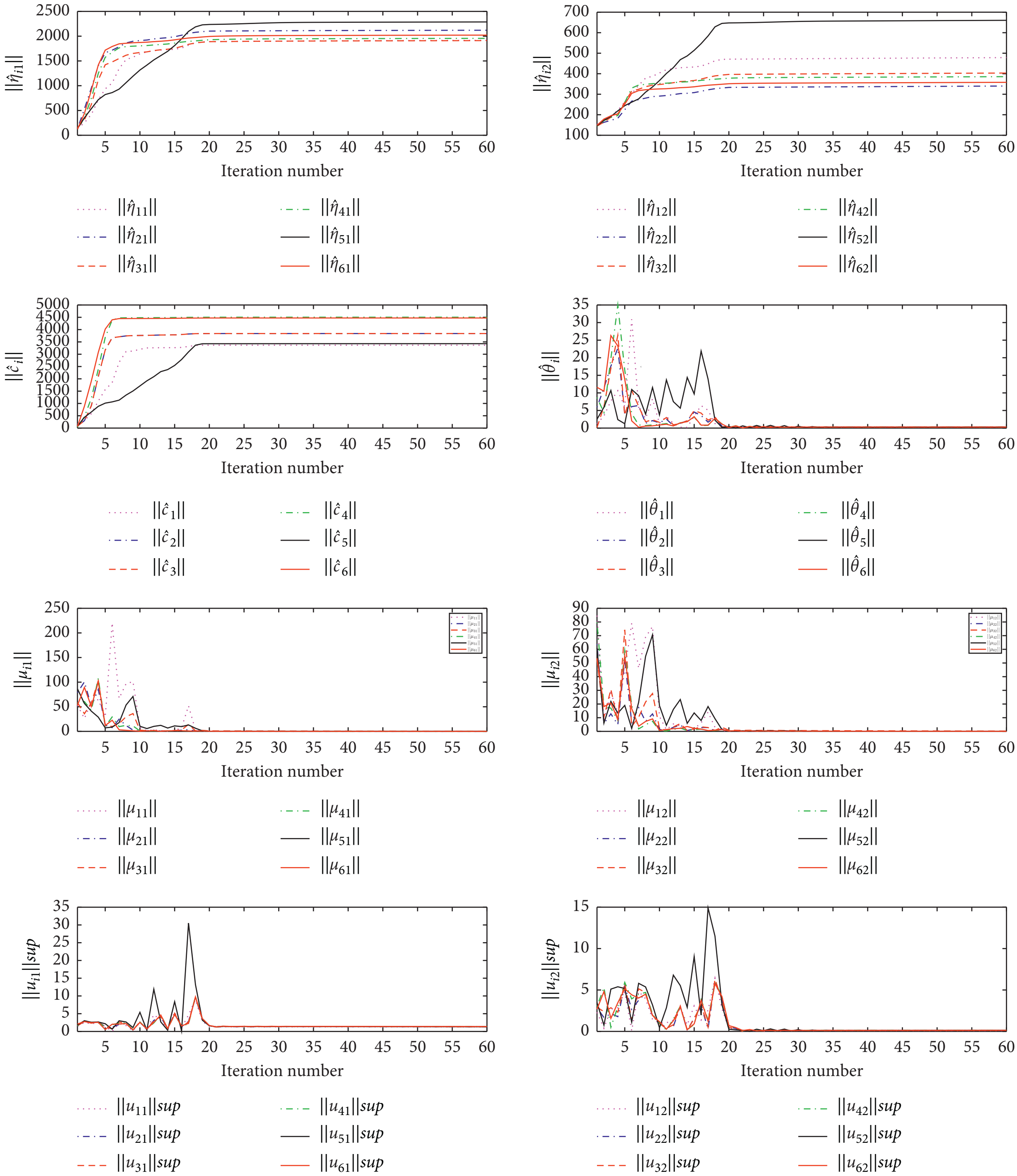

FIGURE 10: Responses of six follower oscillators.

$$\mu_3^0(0) = [1, 1]^T,$$
$$\mu_4^0(0) = [1.5, 2]^T,$$
$$\mu_5^0(0) = [2, 1]^T,$$
$$\mu_6^0(0) = [1, 1.5]^T. \quad (49)$$

Figures 8–10 demonstrate the results of consensus control for 60 iterations, respectively. We can see that six follower oscillators can perfectly track the leader oscillator from Figures 8 and 9, even if each follower oscillator is without knowing the dynamic of the leader oscillator, and the signals involved are bounded. The results fit into Theorem 1.

6. Conclusions

We have solved the fully distributed learning coordination problem of a class of high-order nonlinear MASs by adaptive ILC in this study. With the help of algebraic graph theory, Barlat-like lemma, and Lyapunov theory, the perfect consensus tracking as well as the formation control problem has been resolved over $[0, T]$. At last, two examples testify the effectiveness and efficiency of the algorithm devised in the paper.

Data Availability

No data were used to support this study.

Disclosure

This article has been submitted as a pre-print.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Innovation Fund Project of Gansu Provincial Department of Education (Grant No. 2021A-025), National Natural Science Foundation of China (Grant No. 61663024), and Hongliu First Class Discipline Development Project of Lanzhou University of Technology.

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