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Research Article

Information Propagation Influenced by Population Heterogeneity Behavioral Adoption on Weighted Network

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In the realistic world, various individuals have distinct personalities, preferences, and attitudes toward new information and behavior acceptance, called population heterogeneity. It is seldom taken into account and theoretically analyzed in information propagation on a weighted network. Therefore, we divide individuals into fashionable and conservative individuals according to their passion degree and willingness for novel behaviors acceptance. Then, we build two behavior adoption threshold models corresponding to fashionable and conservative individuals on the weighted network to explore the effect of population heterogeneity on information propagation. Next, a partition theory based on edge weight and population heterogeneity is proposed to qualitatively analyze the information propagation mechanism. The theoretical analyses and simulation results show that fashionable individuals promote information propagation and behavior adoption. More importantly, the crossover phenomena of phase transition appear. When the fraction of fashionable individuals is relatively large, the increasing pattern of the final adoption size shows a second-order continuous phase transition. In comparison, the increasing pattern alters to first-order discontinuous phase transition with the decrease of the fraction of fashionable individuals. Moreover, reducing weight distribution heterogeneity promotes information propagation and slightly accelerates the change of the phase transition pattern from the first-order discontinuous to the second-order continuous. Besides, increasing the degree distribution heterogeneity accelerates the change of the phase transition pattern. Finally, our theoretical analyses coincide well with the simulation results.

1. Introduction

With the rapid development of fifth-generation mobile communication technology and intelligent mobile terminals, social networks are increasingly significant in people's lives, such as Facebook, Twitter, WeChat, Microblog, and other social software [1–4]. The information propagation between users plays an increasingly significant role in social networks, which provides great convenience for people's work and life [5, 6]. The information propagation theory can be utilized to explain many behaviors in widespread reality application fields, such as social recommendation [7, 8], information fraud [9], health [10–12], and advertising marketing [13].

In recent years, researchers have conducted in-depth studies for information propagation models in terms of theoretical analyses and experimental models [14] and explored many potential factors influencing the information propagation mechanisms, such as node distribution structures [7, 15, 16], node contact preference [17], memory effects [18, 19], and heterogeneous adoption thresholds [20, 21].

Massive studies revealed that information propagation exhibits social affirmation or reinforcement due to the multiple confirmations of the credibility and legitimacy of the behavior information [17, 22]. To describe the social reinforcement, the threshold model based on the non-Markov process is one of the classic models, in which individual behavior adoption exhibits memory effects [17, 23]. Furthermore, various threshold models in line with the actual network have been put forward to verify their impact on information propagation, such as heterogeneous adoption thresholds [20, 21], binary adoption probability [24], truncated normal adoption probability [20, 25, 26], gate-like adoption probability [27], and sigmoid adoption probability [28]. Notably, the non-redundant feature cannot be ignored in information propagation, that is, repeated information propagation on the same edge is forbidden.

In addition, individual intimacy heterogeneity in realistic social networks significantly impacts information propagation. Individuals are more susceptible to receive information from those close to them, such as friends and families, rather than strangers. The researchers constructed the connective relationships between individuals as edges with heterogeneous weight distribution and verified the influence of intimacy heterogeneity on information propagation [29, 30].

Some existing works have suggested that the heterogeneity in populations such as individuals shows various attitudes toward the same behavior [31–35]. In the study of information propagation, population heterogeneity is seldom taken into account. In the realistic world, heterogeneity is widespread in populations. Various individuals have distinct personalities, willingness, and preferences for receiving new ideas and behaviors. In realistic social networks, some individuals are more susceptible to novel information, ideas, or behaviors. They have a strong willingness to accept novel thoughts or behaviors. For instance, some fashion-conscious individuals are interested in a new and stylish affair when it comes along, such as famous restaurants and emerging sightseeing spots. Nevertheless, traditional conservatives have little interest in novel thoughts or behaviors. They always hesitate to adopt the popular behavior and verify information many times before adopting it. Therefore, individuals can be divided into fashionable and conservative individuals according to their interest level and willingness to adopt novel ideas or behaviors. Fashionable individuals generate interest in behavior once they receive information. As they receive more information, they gradually increase their willingness to adopt behavior. But conservatives will verify the information many times before adopting behavior. They adopt behavior after receiving a certain amount of information. In conclusion, in the real world, the probability of individuals adopting a new behavior is related to their personalities.

Considering the above factors, we explore the effect of population heterogeneity on information propagation on the weighted network. A fraction of p individuals are fashionable, and others are conservative. Two adoption threshold probability functions corresponding to fashionable and conservative individuals are proposed to reflect the population heterogeneity. Furthermore, a partition theory based on edge weight and population heterogeneity is suggested to analyze the information propagation mechanism. Finally, we verify the information propagation with population heterogeneity behavioral adoption by simulated results, coinciding with theoretical analyses. The rest of this study is organized as follows: in Section 2, we build an information propagation model with population heterogeneity adoption on the weighted network. Section 3 exhibits a partition theory based on edge weight and population heterogeneity. In Section 4, the experimental results are discussed. Finally, Section 5 describes the conclusion.

2. Information Propagation Model with Population Heterogeneity Adoption

To explore the influence of population heterogeneity on the information propagation mechanism, we construct a weighted social network model with N individuals and degree distribution $P(k)$. The uncorrelated configuration model is used in our model to avoid intra-degree correlations. In the weighted social network model, a generalized SAR (susceptible-adopted-recovered) model is deployed to illustrate the information propagation mechanism, as shown in Figure 1(a). In the SAR model, at each moment, each node has one of the three different states: susceptible state (S-state), adopted state (A-state), and recovered state (R-state). The S-state node has not adopted the behavior and can receive information from its neighbors. The A-state nodes have adopted the behavior and are willing to propagate information to their neighbors. The R-state nodes become uninterested in the behavior and are no longer involved in information propagation.

Furthermore, the connection relationships between individuals are denoted as edges with weight distribution. Then we introduce the edge-weight distribution to reflect the edges heterogeneity. The edge weight between two adjacent nodes i and j is denoted by ω_{ij}, and the edge-weight distribution by a function $f(\omega)$. The probability that an S-state node j receives the information from its A-state neighbor node i is denoted by the following equation:

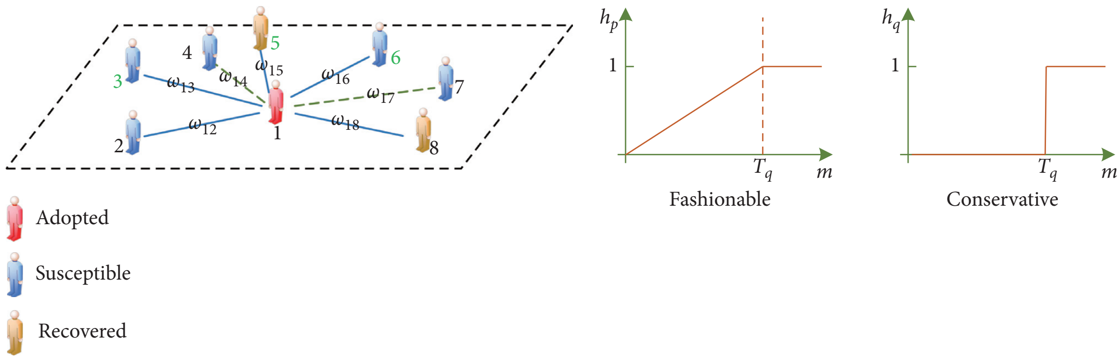
$$\lambda_\omega = \lambda\left(\omega_{ij}\right) = 1 - (1-\beta)^{\omega_{ij}}, \tag{1}$$

where β is the unit information propagation probability. If $\omega_{ij} = 1$, $\lambda_\omega = \beta$ implies that the edge weight does not influence the information propagation. Furthermore, with the increase of ω_{ij}, $\lambda(\omega_{ij})$ increases monotonically.

Let m be the aggregate pieces of information successfully received by the S-state node. Initially, there is no information propagation in the weighted social network, that is, $m_j = 0$ for the S-state node j. At each time step, once node j successfully receives information from an A-state neighbor i through the corresponding edge, and its aggregate information pieces will be increased by one, that is, $m_j \longrightarrow m_j + 1$.

In addition, in order to characterize the impact of population heterogeneity on information propagation, two functions are proposed to illustrate the individual behavior adoption threshold, as shown in Figure 1(b). For fashionable individuals, the function which illustrates the behavior adoption threshold is denoted by the following equation:

$$h_p\left(m, T_p\right) = \begin{cases} \dfrac{m}{T_p}, & 0 < m \le T_p, \\ 1, & m \ge T_p. \end{cases} \tag{2}$$



(a)

(b)

Figure 1: (a) The illustration of the SAR model on the weighted network. The green labels represent that the individuals are fashionable, such as individuals 3, 5, and 6. The black labels represent conservatives, such as individuals 1 and 2. The symbol ω is edge weight. At time t, A-state individual 1 propagates information to its neighbors. The green dashed lines imply that information cannot be transmitted via the edges. The blue solid lines indicate information has not been transmitted through the corresponding edges. (b) The left subgraph is the behavior adoption threshold model of fashionable individuals. The right subgraph is the behavior adoption threshold model of conservatives. m is the aggregate pieces of information that the S-state individual successfully received. p and q are the fractions of fashionable and conservative individuals, respectively. T_p and T_q are the behavior adoption thresholds of fashionable and conservative individuals, respectively.

For conservatives, the behavior adoption threshold function is denoted by the following equation:

$$h_q(m, T_q) = \begin{cases} 0, & 0 < m \leq T_q, \\ 1, & m \geq T_q, \end{cases} \tag{3}$$

where a fraction of p individuals are fashionable, and a fraction of q individuals are conservative. $p + q = 1$. In fact, for fashionable individuals, their interest in adopting the behavior increases as they receive more information. When receiving small pieces of information, they are relatively low willing to adopt the behavior. As they receive more information, their willingness for behavioral adoption gradually increases. Conservatives will adopt behavior only if the aggregate pieces of information they receive exceed the corresponding thresholds. Therefore, the adoption threshold function is relevant and meaningful.

The detailed information propagation in the weighted network is summarized as follows: Initially, we randomly choose a fraction of p nodes as fashionable nodes and all other nodes as conservative. Then we randomly choose a fraction of ρ_0 nodes as the A-state nodes (seeds) and all other nodes as S-state. The information is propagated from the A-state nodes to all its neighbors through the corresponding edges. The S-state node j successfully receives the information from its A-state neighbor node i with a probability $\lambda(\omega_{ij})$ through the corresponding edge of weight ω_{ij}. When the S-state node j successfully receives information, the aggregate pieces of information will become $m_j \longrightarrow m_j + 1$. Due to the nonredundancy of information propagation, the information will not be repropagated through the same edge. Moreover, if j is fashionable, it will adopt the behavior and alter to A-state with the probability $h_p(m, T_p)$. And if j is conservative, it will alter to A-state with the probability $h_p(m, T_p)$. After the information propagation, the A-state node loses interest in the behavior and alters to R-state with probability γ. Finally, the information propagation terminates once there is no A-state node in the weighted network.

3. The Analysis of Partition Theory Based on Edge Weight and Population Heterogeneity

Based on references [22, 33], we consider information propagation of non-redundant information memory with population heterogeneity on the weighted network. Then, a partition theory based on population heterogeneity is developed to analyze the individual information propagation mechanism. Assume that a node in a cavity state [36] implies it can accept information from its neighbors and is unable to deliver information outwards.

Due to the random distribution of the edge weight, a node has not accepted the information from its neighbors by time t with probability:

$$\theta(t) = \sum_{\omega} f(\omega)\theta_w(t), \tag{4}$$

where $\theta_w(t)$ is the probability that the edge of weight ω has not undergone the information propagation toward its S-state neighbors.

By time t, the probability that the S-state node i with degree k_i cumulatively accepts m pieces of information from its neighbors can be expressed as follows:

$$\phi(k_i, m, t) = \binom{k_i}{m}\theta(t)^{k_i - m}[1 - \theta(t)]^m. \tag{5}$$

Consider the population heterogeneity and the behavior adoption threshold functions. If the S-state node i is fashionable, after cumulatively accepting m pieces of information, it has not adopted the behavior and retains S-state by time t with a probability:

$$s_p(k_i,m,t) = \sum_{m=0}^{k_i} \phi(k_i,m,t) \prod_{l=0}^{m} \left[1 - h_p\left(l, T_p\right)\right] = \sum_{m=0}^{T_p-1} \phi(k_i,m,t) \prod_{l=0}^{m} \left(1 - \frac{l}{T_p}\right). \quad (6)$$

Then, for the randomly selected fashionable S-state node, the probability that the number of the aggregate information pieces is less than the corresponding adoption threshold by time t is calculated by the following equation:

$$\eta_p = \sum_{k_i} P(k_i) s_p(k_i,m,t). \quad (7)$$

If the S-state node i is conservative, after cumulatively receiving m pieces of information, it has not adopted the behavior and is still in S-state by time t with a probability:

$$s_q(k_i,m,t) = \sum_{m=0}^{k_i} \phi(k_i,m,t) \prod_{l=0}^{m} \left[1 - h_q\left(l, T_q\right)\right] = \sum_{m=0}^{T_q-1} \phi(k_i,m,t). \quad (8)$$

Then, the probability that the number of the aggregate information pieces of the randomly selected conservative S-state node is less than the corresponding adoption threshold by time t is calculated by the following equation:

$$\eta_q = \sum_{k_i} P(k_i) s_q(k_i,m,t). \quad (9)$$

Therefore, up to time t, the probability that the S-state node i receives m pieces of information and retains S-state can be expressed as follows:

$$s(k_i,t) = (1-\rho_0)\left[p s_p(k_i,m,t) + q s_q(k_i,m,t)\right]. \quad (10)$$

Thus the fraction of S-state nodes in the weighted network at time t can be expressed as follows:

$$S(t) = \sum_k P(k) s(k,t) = (1-\rho_0)\left[p\eta_p + q\eta_q\right]. \quad (11)$$

To calculate $S(t)$, we first consider $\theta_\omega(t)$. As there are only three states for all nodes in the network, $\theta_\omega(t)$ can be decomposed as follows:

$$\theta_\omega(t) = \psi_{S,\omega}(t) + \psi_{A,\omega}(t) + \psi_{R,\omega}(t), \quad (12)$$

where $\psi_{A,\omega}(t)$ is the probability that the S-state node i contacts an A-state neighbor j through the corresponding edge of weight ω and has not successfully accepted information from the A-state node j by time t. $\psi_{S,\omega}(t)$ (or $\psi_{R,\omega}(t)$) is the probability that the S-state node i contacts an S (or R)-state neighbor j via the corresponding edge with weight ω.

Then, we first calculate $\psi_{S,\omega}(t)$. Due to the theory of cavity, the node i in cavity state is unable to transmit information to its neighbors. Thus the S-state node j with degree k_j can get information from other $k_j - 1$ neighbors except node i. Therefore, the probability that the node j cumulatively receives n pieces of information from its neighbor nodes by time t is as follows:

$$\phi\left(k_j - 1, n, t\right) = \binom{k_j - 1}{n} \theta(t)^{k_j - 1 - n} \left[1 - \theta(t)\right]^n. \quad (13)$$

The population heterogeneity and the behavior adoption threshold functions are considered. If the S-state node j is fashionable, after cumulatively accepting n pieces of information, it has not adopted the behavior and retains S-state by time t with a probability as follows:

$$\Theta_p\left(k_j, t\right) = \sum_{n=0}^{k_j-1} \phi\left(k_j - 1, n, t\right) \prod_{l=0}^{n} \left[1 - h_p\left(l, T_p\right)\right] = \sum_{n=0}^{T_p-1} \phi\left(k_j - 1, n, t\right) \prod_{l=0}^{n} \left(1 - \frac{l}{T_p}\right). \quad (14)$$

If the S-state node j is conservative, after cumulatively accepting n pieces of information, it has not adopted the behavior and retains S-state by time t with a probability as follows:

$$\Theta_q\left(k_j, t\right) = \sum_{n=0}^{k_j-1} \phi\left(k_j - 1, n, t\right) \prod_{l=0}^{n} \left[1 - h_q\left(l, T_q\right)\right] = \sum_{n=0}^{T_q-1} \phi\left(k_j - 1, n, t\right). \quad (15)$$

Therefore, the probability that the node j remains S-state after cumulatively receiving n pieces of information is as follows:

$$\Theta\left(k_j, t\right) = p\Theta_p\left(k_j, t\right) + q\Theta_q\left(k_j, t\right). \quad (16)$$

Thereafter, the probability that the node i connects to the S-state node j through the corresponding edge of weight ω is as follows:

$$\psi_{S,\omega}(t) = (1-\rho_0) \frac{\sum_{k_j} k_j P\left(k_j\right) \Theta\left(k_j, t\right)}{\langle k \rangle}, \quad (17)$$

where $k_j P(k_j)/\langle k \rangle$ indicates the probability that node i connects to node j with degree k_j, and $\langle k \rangle$ is the average degree of the network.

Then, we analyze the evolutionary equation of $\psi_{R,\omega}(t)$ and $\psi_{A,\omega}(t)$. The S-state node i successfully accepts the information from its A-state neighbor j via the corresponding edge of weight ω with a probability λ_ω. Thus, the evolution of $\theta_\omega(t)$ can be expressed by the following equation:

$$\frac{d\theta_\omega(t)}{dt} = -\lambda_\omega \psi_{A,\omega}(t). \quad (18)$$

On the other hand, the A-state nodes may lose interest in information propagation and alter to R-state with a probability γ. So, the evolution of $\psi_{R,\omega}(t)$ can be calculated by the following equation:

$$\frac{d\psi_{R,\omega}(t)}{dt} = \gamma\psi_{A,\omega}(t)(1-\lambda_\omega). \tag{19}$$

Combining equations (18) and (19) and the initial conditions $\theta_\omega(0) = 1$ and $\psi_{R,\omega}(0) = 0$, we can obtain the evolution of $\psi_{R,\omega}(t)$ as follows:

$$\psi_{R,\omega}(t) = \gamma[1-\theta_\omega(t)]\left(\frac{1}{\lambda_\omega} - 1\right). \tag{20}$$

Substituting equations (17) and (20) into equation (12), we can obtain the following equation:

$$\begin{aligned}\psi_{A,\omega}(t) &= \theta_\omega(t) - \psi_{S,\omega}(t) - \psi_{R,\omega}(t)\\ &= \theta_\omega(t) - (1-\rho_0)\frac{\sum_{k_j} k_j P(k_j)\Theta(k_j,t)}{\langle k\rangle} - \gamma[1-\theta_\omega(t)]\left(\frac{1}{\lambda_\omega}-1\right).\end{aligned} \tag{21}$$

Substituting equation (21) into (18), the evolution of $\theta_\omega(t)$ can be rewritten as follows:

$$\begin{aligned}\frac{d\theta_\omega(t)}{dt} &= \left\{\theta_\omega(t) - (1-\rho_0)\frac{\sum_{k_j} k_j P(k_j)\Theta(k_j,t)}{\langle k^X\rangle} - \gamma[1-\theta_\omega(t)]\left(\frac{1}{\lambda_\omega}-1\right)\right\}\\ &= (1-\rho_0)\lambda_\omega\frac{\sum_{k_j} k_j P_X(k_j)\Theta(k_j,t)}{\langle k\rangle} + \gamma(1-\lambda_\omega) - [\gamma + \lambda_\omega(1-\gamma)]\theta_\omega(t).\end{aligned} \tag{22}$$

Throughout the network, the density variation of each state can be expressed as follows:

$$\frac{dR(t)}{dt} = \gamma A(t), \tag{23}$$

and

$$\frac{dA(t)}{dt} = -\frac{dS(t)}{dt} - \gamma A(t). \tag{24}$$

Therefore, by combining and iterating equations (11), (23), and (24), the values of $S(t)$, $A(t)$, and $R(t)$, that is, the density of each state at arbitrary time step, can be calculated.

While $t \longrightarrow \infty$, there are only S-state and R-state nodes in the whole network. $R(\infty)$ is the final behavior adoption size. Let $d\theta_\omega(t)/dt|_{t=\infty} \longrightarrow 0$. At this time, the probability that the edge of weight ω has not undergone the information propagation is as follows:

$$\theta_\omega(\infty) = \frac{(1-\rho_0)\lambda_\omega \sum_{k_j} k_j P(k_j)\Theta(k_j,\infty) + \langle k\rangle\gamma(1-\lambda_\omega)}{\langle k\rangle\gamma + (1-\gamma)\lambda_\omega\langle k\rangle}. \tag{25}$$

By combining and iterating equations (11) and (25), $A(\infty) = 0$, $S(\infty)$, and $R(\infty)$ can be obtained.

Then we focus on the critical unit propagation probability. Let

$$g[\theta_\omega(\infty), \rho_0, T_p, T_q, \gamma, \beta] = \frac{(1-\rho_0)\lambda_\omega \sum_{k_j} k_j P(k_j)\Theta(k_j,\infty)}{\langle k\rangle\gamma + (1-\gamma)\lambda_\omega\langle k\rangle} + \frac{\gamma(1-\lambda_\omega^X)}{\gamma + (1-\gamma)\lambda_\omega} - \theta_\omega(\infty). \tag{26}$$

The value of $\theta_\omega(\infty)$ is denoted by $\theta_\omega^c(\infty)$ that is the critical probability point. At the critical unit propagation probability, the information is not propagated to j through the corresponding edge when $t \longrightarrow \infty$. At the critical value of $\theta_\omega^c(\infty)$, $g[\theta_\omega(\infty), \rho_0, T_p, T_q, \gamma, \beta]$ is tangent to the horizontal axis. Therefore, the critical condition is as follows:

$$\frac{dg}{d\theta_\omega(\infty)}\Big|_{\theta_\omega^c(\infty)} = 0. \tag{27}$$

4. Results and Discussions

To verify the theoretical analysis above, we perform extensive numerical simulations and theoretical analyses on artificial weighted Erdos–Renyi (ER) networks [37] and weighted Scale-Free (SF) networks [38]. The network size is $N = 10^4$, that is, there are at lease 10^4 independent dynamical individuals in the network. The mean degree of the network is $\langle k\rangle = 10$. The weight distribution follows

$f_X(\omega) \sim \omega^{-\alpha_\omega}$ with $\omega^{\max} \sim 1/(\alpha_\omega - 1)$ and average wight $\langle\omega\rangle = 8$. In addition, the recovery probability is $\gamma = 1.0$.

To illustrate the critical unit propagation probability and critical condition in our simulation, we adopt the relative variance $\mathcal{X}$ [39, 40], which is expressed as follows:

$$\mathcal{X} = N \frac{\langle R(\infty)^2\rangle - \langle R(\infty)\rangle^2}{\langle R(\infty)\rangle}, \tag{28}$$

where $\langle\cdots\rangle$ represents the ensemble average. The peak values of the relative variance represent the critical point of information global propagation.

4.1. The Information Propagation on Weighted ER Network. At first, we explore the information propagation on weighted ER Network. The degree of nodes in ER network obeys Poisson distribution, that is, $P(k) = e^{-\langle k\rangle}\langle k\rangle^k/k!$.

Figure 2 explores the effects of unit propagation probability β on the final adaption size with various proportions of fashionable nodes p on weighted ER network. The initial fraction of seeds $\rho_0 = 0.005$. The adoption thresholds are $T_p = 1$ and $T_q = 3$. Figures 2(a1) and 2(b1) show that with the increase of β, the final adoption size $R(\infty)$ rises to global adoption. Also, with the increase of the fraction of fashionable individuals p, the final adoption size $R(\infty)$ rises. The behavior adoption can be promoted by p. There emerges a crossover phenomenon: the increasing pattern of $R(\infty)$ shows the second-order continuous phase transition with a relatively large fraction of p (i.e., $p = 0.5$ and $p = 0.9$). However, the increasing pattern of $R(\infty)$ shows the first-order discontinuous phase transition when p is small (i.e., $p = 0.1$). Meanwhile, Figures 2(a2) and 2(b2) show the relative variances of the theoretical analyses and the critical propagation probabilities of (a1) and (b1), respectively. At the critical point, the phase transition appears, and the global behavior adoption will occur. In addition, compared to (a) and (b), the information propagation outbreak earlier as the increase of weighted distribution exponent (reducing the weight distribution heterogeneity). But the weight distribution does not change the phase transition pattern. Moreover, our theoretically predicted values (lines) coincide well with the simulation results (symbols).

Figure 3 shows the effects of unit propagation probability β on the final adaption size with various adoption thresholds on weighted ER network. (a1) ($\alpha_\omega = 2$) and (b1) ($\alpha_\omega = 3$) show the influence of weight distribution heterogeneity over phase transition. Other parameters are $\rho_0 = 0.02$ and $p = 0.5$. Figure 3 shows that the adoption threshold accelerates the information propagation outbreak without altering the phase transition pattern. Also, compared to (a) and (b), decreasing the weight distribution heterogeneity accelerates the information propagation outbreak without altering the phase transition pattern. Moreover, our theoretical analyses (dotted lines) agree with the simulation values (symbols).

Figure 4 investigates the joint influence of unit propagation probability β and the fraction of fashionable individuals p on $R(\infty)$ for weighted ER network. Figure 4(a) ($\alpha_\omega = 2$) and (b) ($\alpha_\omega = 3$) display the effects of parameters plane (β, p) on $R(\infty)$ with robust and weak weight distribution heterogeneity, respectively. The initial fraction of seeds is $\rho_0 = 0.005$. The adoption thresholds are $T_p = 1$ and $T_q = 3$. As the increment of β, the crossover phenomenon emerges. The parameters plane (β, p) can be divided into three regions. In region I, the fraction of fashionable individuals p is extremely small, and there is no global behavior adoption outbreak. The reason is that the lack of sufficient fashionable individuals involved in information propagation in the initial stage hinders information propagation. In region II, with the increase of the fraction of fashionable individuals p, the increasing pattern of $R(\infty)$ exhibits the first-order discontinuous phase transition. In region III, the increasing pattern of $R(\infty)$ exhibits the second-order continuous phase transition. In fact, in region III, the fashionable individuals dominate the information propagation and stimulate the behavior adoption of conservatives. In addition, compared to (a) and (b), the white lines show that the weak weight distribution heterogeneity accelerates the information propagation and slightly promotes the change of the phase transition pattern from the first-order discontinuous to the second-order continuous.

4.2. The Information Propagation on Weighted SF Network. In SF networks, the nodes' degree distribution heterogeneity is negatively correlated with the degree exponent v. The degree of nodes obeys power-law distribution $P(k) = \xi k^{-v}$, where $\xi = 1/\sum_k k^{-v}$, and parameter v presents the degree exponent of the SF network. The minimum and maximum degrees are $k_{\min} = 4$ and $k_{\max} \approx 100$, respectively.

Figure 5 illustrates the effects of unit propagation probability β and the fraction of fashionable individuals p on the individual final adoption size for weighted SF network with degree distribution heterogeneity. The identical degree distribution exponent is utilized for the vertical subgraphs, that is, the subgraphs from first to third columns correspond to $v = 2.1, 3, 4$. The initial fraction of seeds is $\rho_0 = 0.005$. The weight distribution exponent is $\alpha_\omega = 2$. The adoption thresholds are $T_p = 1$ and $T_q = 3$. With the increment of β, $R(\infty)$ rises to global adoption. Also, the behavior adoption is promoted by p. With the increment of the fraction of fashionable individuals, the increasing pattern of the final adoption size changes from the first-order discontinuous phase transition to the second-order continuous phase transition.

In addition, with the increase of degree distribution heterogeneity, that is, using smaller values of the degree distribution exponent v, the network has a large number of individuals with very small degrees and more individuals with large degrees. Comparing subgraphs (a1), (b1), and (c1), for small values of β, increasing the degree distribution

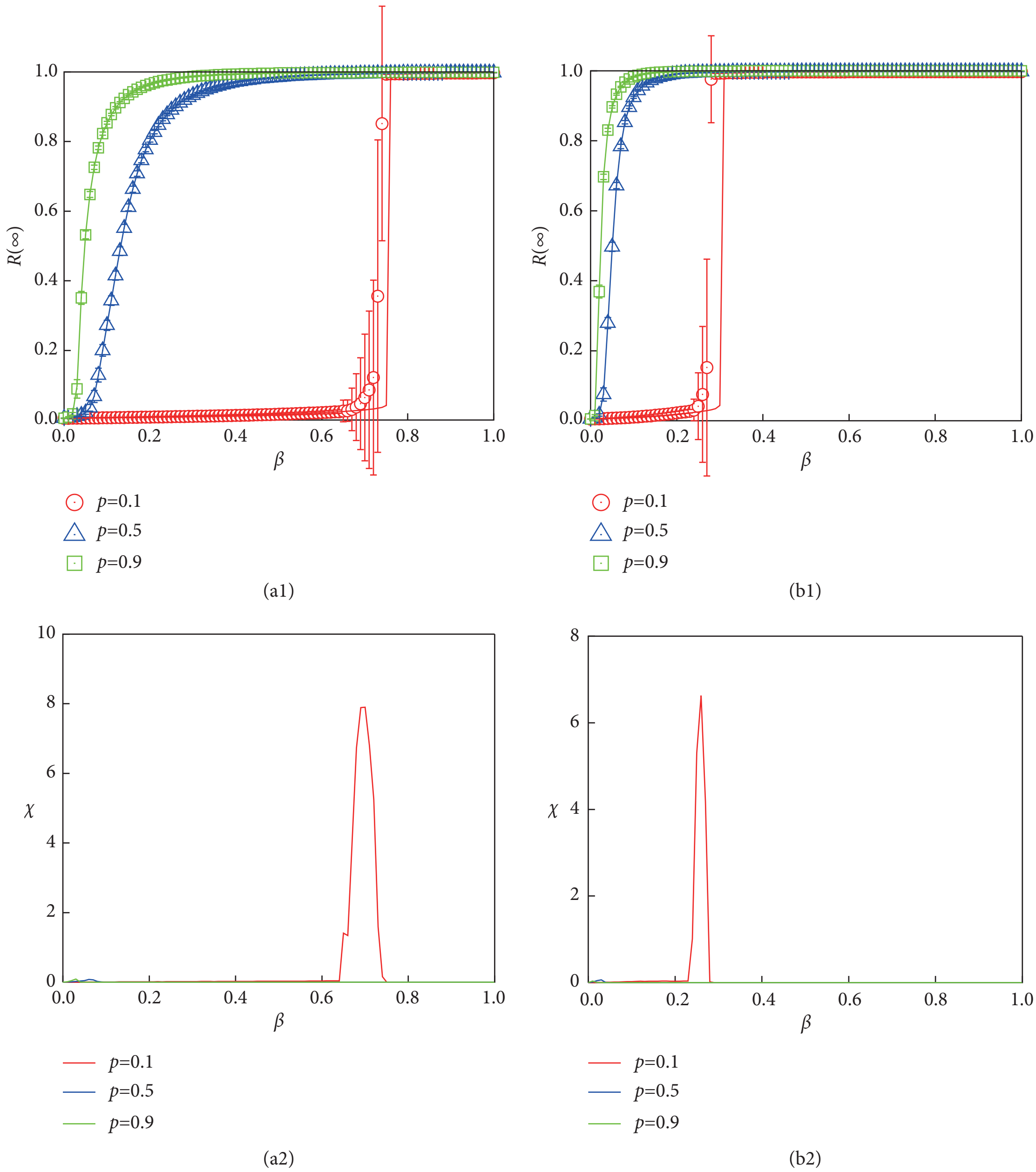


Figure 2: The effects of unit propagation probability β on the final adaption size with various proportions of fashionable nodes p on weighted ER network. (a1) ($\alpha_\omega = 2$) and (b1) ($\alpha_\omega = 3$) show the influence of weight distribution heterogeneity over phase transition. Subgraphs (a2) and (b2) represent the relative variances of the theoretical analyses and the critical points of (a1) and (b1), respectively. In addition, in subgraphs (a1) and (b1), the theoretically predicted values (dotted lines) coincide well with the simulation values (symbols). Other parameters are $\rho_0 = 0.005$, $T_p = 1$, and $T_q = 3$.

heterogeneity (i.e., using smaller values of the degree distribution exponent) promotes the information propagation. But for large values of β, decreasing the degree distribution heterogeneity promotes the global behavior outbreak. Besides, at a relatively small fraction of fashionable individuals, the second-order discontinuous phase transition becomes dramatic with the increment of the degree distribution exponent.

Figures 6(a)–6(c) investigate the variation of $R(\infty)$ on the information propagation parameter plane (β, p) for weighted SF network with $\nu = 2.1, 3, 4$, respectively. The initial fraction of seeds $\rho_0 = 0.005$. The weight heterogeneity exponent is $\alpha_\omega = 2$. The adoption thresholds are $T_p = 1$ and $T_q = 3$. As the increment of β, the crossover phenomena for the phase transitions emerge. More excitingly, when the degree distribution heterogeneity is strong, that is, $\nu = 2.1$

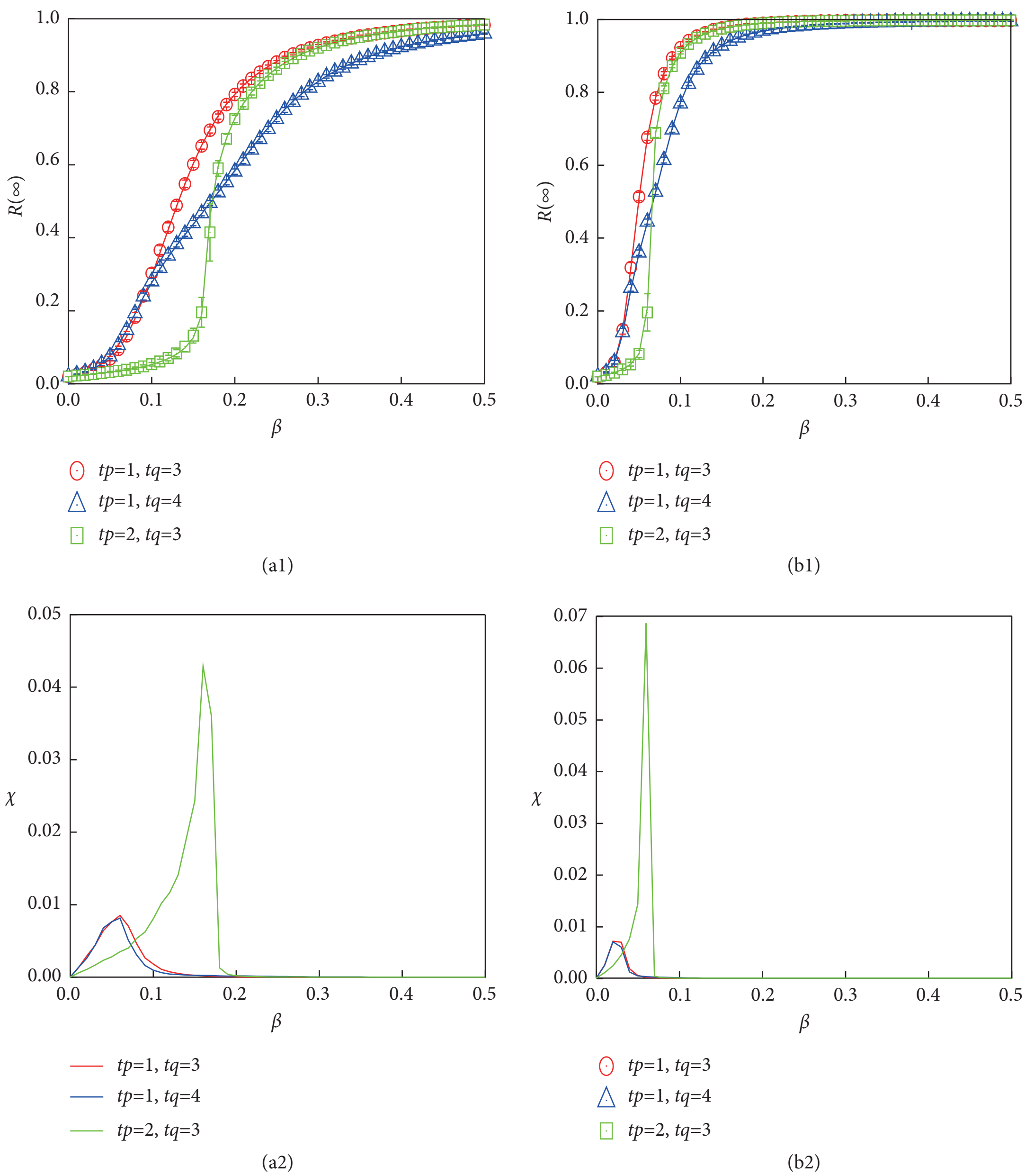


Figure 3: The effects of unit propagation probability β on the final adaption size with various adoption thresholds on weighted ER network. (a1) ($\alpha_\omega = 2$) and (b1) ($\alpha_\omega = 3$) show the influence of weight distribution heterogeneity over phase transition. Subgraphs (a2) and (b2) represent the relative variances of the theoretical analyses and the critical points of (a1) and (b1), respectively. The theoretically predicted values (dotted lines) coincide well with the simulation values (symbols). Other parameters are $\rho_0 = 0.02$ and $p = 0.5$.

and $\nu = 3$, the final individual adoption size can rise to global adoption at a small fraction of fashionable individuals. But global behavior adoption at a relatively small p disappears with the increment of the power exponent ν, that is, $\nu = 4$. Therefore, the parameters plane (β, p) can be divided into two regions for $\nu = 2.1$ and $\nu = 3$. In region I, the increasing pattern of $R(\infty)$ exhibits the first-order discontinuous phase transition with the increase of p. In region II, the increasing pattern of $R(\infty)$ exhibits the second-order continuous phase transition. With the increment of the power exponent ν, that is, $\nu = 4$, the parameters plane (β, p) is divided into three regions. In region I, there is no global behavior adoption outbreak. With the increment of p, the increasing pattern of $R(\infty)$ changes from discontinuous to continuous phase transition. In fact, in region II (Figures 6(a) and 6(b)) and region III (Figure 6(c)), the fashionable individuals

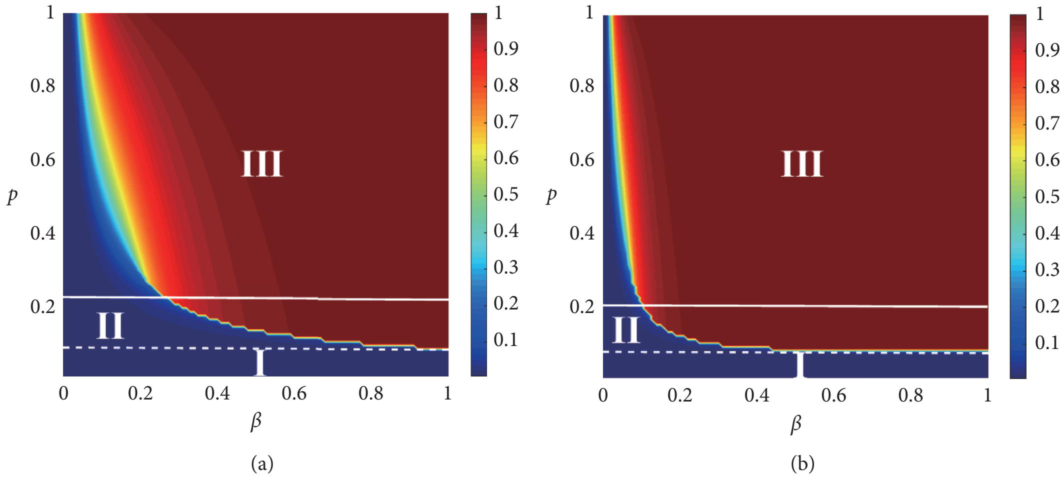


Figure 4: The combined effects of unit propagation probability β and the fraction of fashionable individuals p on the individual final adoption size for weighted ER network. In subgraphs (a) ($\alpha_\omega = 2$) and (b) ($\alpha_\omega = 3$), there emerge no global behavior outbreak, discontinuous phase transition, and continuous phase transition in regions I, II, and III, respectively. Other parameters are $\rho_0 = 0.005$, $T_p = 1$, and $T_q = 3$.

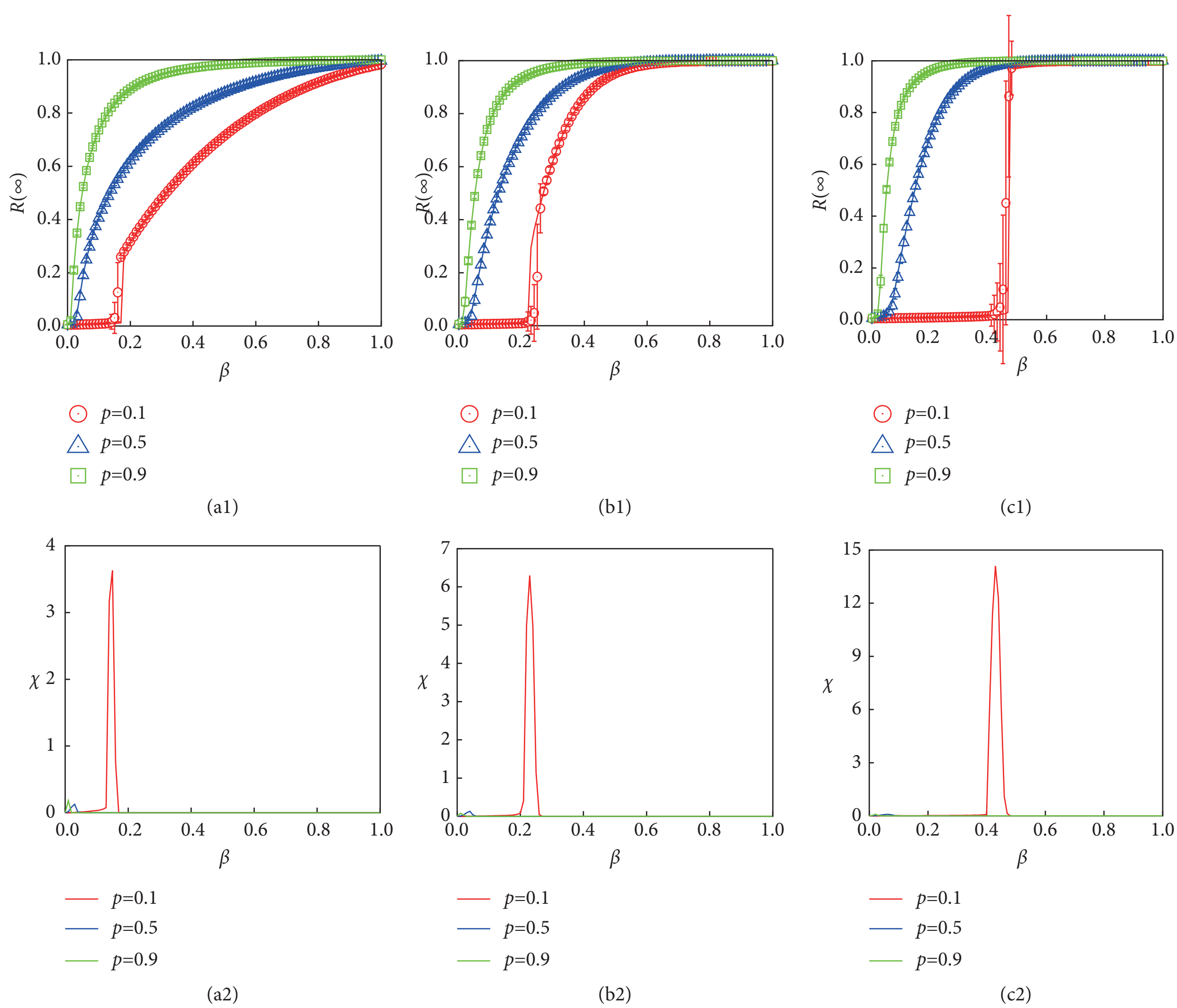


Figure 5: The effects of unit propagation probability β and the fraction of fashionable individuals p on the individual final adoption size for weighted SF network. The identical degree distribution exponent is utilized for the vertical subgraphs, that is, the subgraphs from first to third columns correspond to $v = 2.1, 3, 4$. Subgraphs (a1), (b1), and (c1) present the effects of β and p on the final individual adoption size with degree distribution heterogeneity. Subgraphs (a2), (b2), and (c2) present the relative variances of the theoretical analyses and the critical points of (a1), (b1), and (c1), respectively. The theoretical analyses (dotted lines) coincide well with the simulation values (symbols). Other parameters are $\rho_0 = 0.005$, $\alpha_\omega = 2$, $T_p = 1$, and $T_q = 3$.

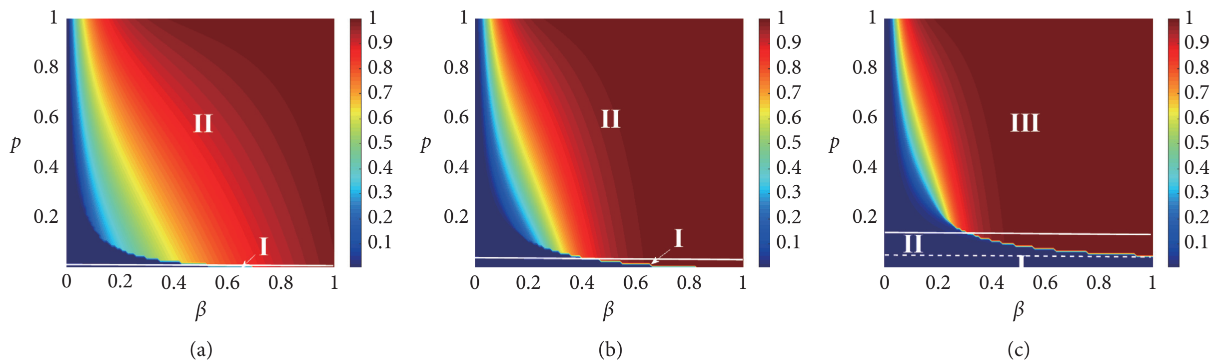


Figure 6: The joint effects of unit propagation probability β and the fraction of fashionable individuals p on the individual final adoption size for weighted SF network. Subgraphs (a, b, c) display the effects of (β, p) on the final adoption size with $v = 2.1, 3, 4$, respectively. There emerge two regions in (a) and (b): discontinuous phase transition in region I and continuous phase transition in region II. (c) Three regions: no global behavior outbreak in region I, discontinuous phase transition in region II, and continuous phase transition in region III. Other parameters are $\rho_0 = 0.005$, $\alpha_\omega = 2$, $T_p = 1$, and $T_q = 3$.

dominate the information propagation and stimulate the behavior adoption of conservatives. In addition, compared to (a), (b), and (c), the white lines show that the increment of the degree distribution heterogeneity accelerates the change of the phase transition pattern from the first-order discontinuous to the second-order continuous phase transition.

5. Conclusions

This study explores the effect of population heterogeneity on information propagation for the weighted network. We randomly select a fraction of p individuals as fashionable, and others are conservative. We also consider individual intimacy heterogeneity, which is modeled as the edge weight in the social network. Then, we propose two behavior adoption threshold functions to illustrate population heterogeneity. For fashionable individuals, the willingness of behavior adoption increases as they receive more information. But conservatives will adopt behavior only if they accept at least T_q pieces of information. To theoretically analyze the effect of population heterogeneity, we propose a partition theory based on edge weight and two behavior adoption threshold models. By theoretical analyses and simulation results, we find some exciting phenomena for information propagation. First, fashionable individuals promote information propagation and behavior adoption. Furthermore, the crossover phenomena of phase transition appear. With the increase of p, the increasing pattern of $R(\infty)$ changes from first-order discontinuous phase transition to second-order continuous phase transition. In addition, weak weight distribution heterogeneity promotes information propagation. Increasing degree distribution heterogeneity accelerates information propagation at small values of β and hinders information propagation at large values of β.

Population heterogeneity plays an essential role in information propagation, which lacks rigorous theoretical modeling and analysis. We model and analyze the impact of population heterogeneity qualitatively and quantitatively on weighted work. Our study reveals the underlying mechanism of information propagation by considering population heterogeneity on weighted network.

Data Availability

The datasets used in this study are available from the corresponding author upon request (fanss@bupt.edu.cn (Shaoshuai Fan)).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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