Research Article

Synchronization and Antisynchronization of Identical 4D Hyperchaotic Financial System with External Perturbation via Sliding Mode Control Technique

Fazal ur Rehman,1 Muhammad Rafiq Mufti,2 Muhammad Umar Farooq,3 Sami ud Din,4 Jawad Ali,1 and Nadir Mehmood1

1Department of Electrical Engineering, Capital University of Science and Technology, Islamabad 44000, Pakistan
2Department of Computer Science, COMSATS University Islamabad, Vehari Campus, Vehari 61100, Pakistan
3Department of Business Studies, Namal University Mianwali, Mianwali 42250, Pakistan
4Department of Electrical Engineering, Namal University Mianwali, Mianwali 42250, Pakistan

Correspondence should be addressed to Sami ud Din; sami.uddin@namal.edu.pk

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In this article, complete synchronization and antisynchronization in the identical financial chaotic system are presented. The proposed control strategies depend on first-order sliding mode and adaptive integral sliding mode for complete synchronization and antisynchronization of the identical financial chaotic system. In the primary case, the system parameters should be known, and first-order sliding mode control is utilized for synchronization and antisynchronization while in the second case, the system parameters are considered unknown. An adaptive integral sliding mode control strategy is utilized for synchronization and antisynchronization of the system considering the parameters unknown. The error system is changed into a particular structure containing a nominal part and several unknown terms to utilize the adaptive integral sliding mode control. Then, this error system is stabilized using integral sliding mode control. The stabilizing controller is usually developed based on the nominal part plus the compensator control part. To suppress the high-frequency oscillation (chattering) phenomenon, smooth continuous compensator control can be used rather than conventional discontinuous control. The compensator controller along with the adapted law is derived such that the time derivative of the Lyapunov function becomes strictly negative. The effectiveness of the proposed method was tested through computer simulations. The proposed control strategies are verified for that identical 4D hyperchaotic financial system to attain complete synchronization and antisynchronization along with the improved performance.

1. Introduction

Chaos theory plays an important role in nonlinear control theory. Chaotic synchronization and antisynchronization have gained considerable importance in the domain of biological systems, chemical reactions, secure communication, information sciences, plasma technology, and most important of all, financial systems [1–3]. The hyperchaotic systems are very sensitive towards initial conditions and possess at least two positive Lyapunov exponents. It has bounded trajectories in the phase space and exhibits more complex nonlinear behavior. Hyperchaos was first formalized in 1979 by Rossler [4]. Many other hyperchaotic systems were reported later on [5–7]. In 1990, Pecora and Carroll [8] introduced the synchronization of two identical chaotic systems with different initial conditions. Later, synchronization of chaotic systems was extensively studied in the last two decades, and many schemes were proposed for synchronization, which includes lag synchronization [9, 10], inverse lag synchronization [11], complete synchronization [12, 13], inverse π-lag synchronization [14], multiple chaotic systems synchronization [15, 16], partial
As said earlier, hyperchaotic systems possess complex dynamical behavior due to more than one positive Lyapunov exponent and their expansion in more than one direction. Considering the aforementioned scenario, it is very much mandatory for every government to take preventive measures before the outbreak of chaos. The dynamics of a financial system play a significant role in the growth and development of an economic system. However, a financial system’s dynamics depend on the multiple input variables in a highly complex and nonlinear fashion. A financial system, even if deterministic, can exhibit and initiate chaotic behavior. Since a chaotic system is more sensitive towards small changes and errors in parameters, their synchronization and antisynchronization are important from a control point of view.

As a control researcher, we know that the robust stabilization is one of the fundamental problems in control theory. It is considered to be more challenging because no such standard method exists for general nonlinear systems. In this regard, a control approach for complete synchronization and antisynchronization of an identical financial chaotic system is presented in this work. The sliding mode control (SMC) strategy is applied on the financial chaotic model when the system parameters are known; however, the adaptive integral sliding mode control (AISM C) approach is used for the case when the system’s parameters are unknown. Moreover, in conventional SMC, the problem of chattering and uncertainty in reaching phase may lead towards total system failure. Many researchers proposed several solutions like integrating SMC with back-stepping and using higher-order SMC to suppress the chattering. However, in order to solve the reaching phase problem, the proposed (AISM C) technique is quite effective. In the said approach, the reaching phase starts from the very beginning, resulting the closed loop system which becomes very robust since the first time instant. The proposed control strategy ensures the robust finite-time convergence in addition to the elimination of reaching phase with mitigation of the high-frequency phenomenon (known as chattering). In practical systems, robust convergence and mitigation of chattering are really appreciable. Numerical simulation results verify the effectiveness of the proposed methodologies.

The rest of this paper is organized as follows; Section 2 presents preliminaries regarding synchronization. Section 3 displays the dynamical model and proposed methodologies regarding sliding mode control (SMC) and adaptive integral sliding mode control (AISM C) design for the known and unknown parameters cases. The simulation results and discussion are posed in Section 4. Section 5 presents the conclusion. The references cited are listed at last.

2. Systems Description and Preliminaries

This section presents the dynamical model of an identical 4D hyperchaotic financial system along with the concepts of complete synchronization and antisynchronization. The financial chaotic systems have attracted a substantial amount of attraction from researchers in recent years. The financial systems are involved with the existence [29]. As we know,
The system parameters \( a, b, c, d, \) and \( k \) are selected as \( 0.9, 0.2, 1.5, 0.2, \) and \( 0.17 \), respectively, with these parameters system (2) exhibiting chaotic behavior.

### 3. Proposed Methodologies

#### 3.1. Sliding Mode Control for Identical 4D Hyperchaotic Financial System with Known Parameters

Hence, systems (1) and (2) with external perturbations are shown as follows:

\[
\begin{aligned}
\dot{x}_1 &= x_3 + (x_2 - a)x_1 + x_4, \\
\dot{x}_2 &= 1 - bx_2 - x_1^2, \\
\dot{x}_3 &= -x_1 - cx_3, \\
\dot{x}_4 &= -dx_1x_2 - kx_4,
\end{aligned}
\]  

(1)

\[
\begin{aligned}
\dot{y}_1 &= y_3 + (y_2 - a)y_1 + y_4 + u_1, \\
\dot{y}_2 &= 1 - by_2 - y_1^2 + u_2, \\
\dot{y}_3 &= -y_1 - cy_3 + u_3, \\
\dot{y}_4 &= -dy_1y_2 - ky_4 + u_4.
\end{aligned}
\]  

(2)

External disturbances are considered as follows:

\[
\begin{aligned}
\dot{v}_1 &= 2y_2y_3 - 0.4v_1, \\
\dot{v}_2 &= -2y_1y_4 - 0.8v_2, \\
\dot{v}_3 &= -1.2y_1y_2 - 0.5v_3, \\
\dot{v}_4 &= y_2y_3 - 0.5v_4.
\end{aligned}
\]  

(3)

By defining the error signals, 

\[
\begin{aligned}
\epsilon_1 &= y_1 - qx_1, \\
\epsilon_2 &= y_2 - qx_2, \\
\epsilon_3 &= y_3 - qx_3, \\
\epsilon_4 &= y_4 - qx_4.
\end{aligned}
\]  

(4)

For synchronization, select \( q = 1 \), and for antisynchronization, select \( q = -1 \). By taking derivative of error signals, error dynamics becomes

\[
\begin{aligned}
\dot{\epsilon}_1 &= \epsilon_1 - qx_1 = (y_3 + (y_2 - a)y_1 + y_4) + h_1v_1 + u_1 - q(x_3 + (x_2 - a)x_1 + x_4), \\
\dot{\epsilon}_2 &= \epsilon_2 - qx_2 = (1 - by_2 - y_1^2) + h_2v_2 + u_2 - q(1 - bx_2 - x_1^2), \\
\dot{\epsilon}_3 &= \epsilon_3 - qx_3 = (-y_1 - cy_3) + h_3v_3 + u_3 - q(-x_1 - cx_3), \\
\dot{\epsilon}_4 &= \epsilon_4 - qx_4 = (-dy_1y_2 - ky_4) + h_4v_4 + u_4 - q(-dx_1x_2 - kx_4).
\end{aligned}
\]  

(7)

Considering \( u_1, u_2, u_3 \) and \( u_4 \) displayed as follows for proceeding towards (9);
In (8), \( v \) represents the new input, which can be mentioned as follows:

\[
\begin{align*}
\dot{e}_1 &= e_2, \\
\dot{e}_2 &= e_3, \\
\dot{e}_3 &= e_4, \\
\dot{e}_4 &= v.
\end{align*}
\]  

(9)

Defining the Hurwitz sliding surface for (7) as follows:

\[
\sigma = \left(1 + \frac{d}{dt} \right)^3 e_1,
\]

(10)

\[
\sigma = e_1 + 3e_2 + 3e_3 + e_4.
\]

By taking time derivative, we have

\[
\dot{\sigma} = \dot{e}_1 + 3\dot{e}_2 + 3\dot{e}_3 + \dot{e}_4.
\]

(11)

By substituting the respective values in (11), we got the following:

\[
\dot{\sigma} = e_2 + 3e_3 + 3e_4 + v.
\]

(12)

The aforementioned system becomes stable by considering "\( v \)" displayed in the following:

\[
v = -e_2 - 3e_3 - 3e_4 - k\sigma.
\]

(13)

By putting \( v \) value \( \dot{\sigma} = -k\sigma \), the error system (7) is asymptotically stable, and following initial conditions and parametric values are used in the simulation of the aforementioned system:

\[
\begin{align*}
x(0) &= [3, 1, 2, -3]^T, \\
y(0) &= [-2, 3, -1, -4]^T, \\
a &= 0.9, \\
b &= 0.2, \\
c &= 1.5, \\
d &= 0.2, \\
k &= 0.17.
\end{align*}
\]

(14)

If we consider a Lyapunov function \( V = 0.5\sigma^2 \), then its derivative becomes \( \dot{V} = \sigma\dot{\sigma} \), moreover;

\[
\dot{V} = \sigma(-k\sigma) = -k\sigma^2.
\]

(15)

From this, we can say that \( \sigma \to 0; \) since \( \sigma \) is Hurwitz, then \( e_i \to 0, \) where \( i = 1, \ldots, 4. \) Therefore, the system shown in (9) is asymptotically stable.


In this section, AISMC is dispensed for synchronization and antisynchronization of identical 4D hyperchaotic financial systems. In this method, the parameters are expected to be unknown and are estimated using AISMC.

Let \( \bar{a}, \bar{b}, \bar{c}, \bar{d}, \) and \( \bar{k} \) be estimate value of \( a, b, c, d, \) and \( k, \) respectively, and let us consider \( a = \bar{a} - a, \) \( b = \bar{b} - b, \) \( c = c - \bar{c}, \) \( d = d - \bar{d}, \) \( k = k - \bar{k} \) be the errors.

Thus, systems (1) and (2) with external perturbations are displayed as follows:

\[
\begin{align*}
x_1 &= x_3 + x_2x_1 - \bar{a}x_1 + \bar{a}x_1 + x_4, \\
x_2 &= 1 - \bar{b}x_2 - \bar{b}x_2 - x_1, \\
x_3 &= -x_1 - \bar{c}x_1 - \bar{c}x_1, \\
x_4 &= \bar{a}x_1 - \bar{a}x_1 - \bar{k}x_4 - \bar{k}x_4.
\end{align*}
\]

(16)

\[
\begin{align*}
y_1 &= y_3 + y_2y_1 - \bar{a}y_1 + \bar{a}y_1 + y_4 + h_1y_1 + u_1, \\
y_2 &= 1 - \bar{b}y_2 - \bar{b}y_2 - y_1^2 + h_2y_2 + u_2, \\
y_3 &= -y_1 - \bar{c}y_1 - \bar{c}y_1 + h_3y_3 + u_3, \\
y_4 &= \bar{a}y_1 \bar{a}y_1 \bar{a}y_1 \bar{a}y_1 - \bar{k}y_4 - \bar{k}y_4 + h_4y_4 + u_4.
\end{align*}
\]

External disturbances are given as follows:

\[
\begin{align*}
\dot{v}_1 &= 2y_2y_3 - 0.4v_1, \\
\dot{v}_2 &= -2y_1y_4 - 0.8v_2, \\
\dot{v}_3 &= -1.2y_1y_2 - 0.5v_3, \\
\dot{v}_4 &= y_2y_3 - 0.5v_4.
\end{align*}
\]

(17)

By defining the error signals,

\[
\begin{align*}
e_1 &= y_1 - qx_1, \\
e_2 &= y_2 - qx_2, \\
e_3 &= y_3 - qx_3, \\
e_4 &= y_4 - qx_4.
\end{align*}
\]

(18)

For synchronization, select \( q = 1, \) and select \( q = -1 \) for antisynchronization. By taking derivative of error signals, we get error dynamics as follows:
Choose an integralsliding surface for system (21) as follows: [46]:

\[
\sigma_0 = \left(1 + \frac{d}{dt}\right)^3 e_1,
\]

where "\(v_0\)" the new input, and system (19) can be written as follows:

\[
\begin{align*}
\dot{e}_1 &= -\bar{a}y_1 + q(\bar{a}x_1) + e_2, \\
\dot{e}_2 &= -\bar{b}y_2 + q(\bar{b}x_2) + e_3, \\
\dot{e}_3 &= -\bar{c}y_3 + q(\bar{c}x_3) + e_4, \\
\dot{e}_4 &= -\bar{d}y_4 + q(\bar{d}x_4) + q(\bar{l}x_4) + v.
\end{align*}
\]

By using AISMC, the nominal system for (21) will be considered as follows:

\[
\begin{align*}
\dot{e}_1 &= e_2, \\
\dot{e}_2 &= e_3, \\
\dot{e}_3 &= e_4, \\
\dot{e}_4 &= v_0.
\end{align*}
\]  

Define the sliding surface (Hurwitz) for the nominal system displayed in (22) as

\[
\sigma = \sigma_0 + z,
\]

where

\[
\sigma_0 = e_1 + 3e_2 + 3e_3 + e_4.
\]
where $n(z)$ presents some integral term. To circumvent the reaching phase, choose $z(0)$ such that $\sigma(0) = 0$. Choose $v = v_0 + v_s$, where $v_0$ represents the nominal input, and the discontinuous term is displayed as $v_s$.

By taking derivative,

$$\dot{\sigma} = \dot{e}_1 + 3\dot{e}_2 + \dot{3}e_3 + \dot{e}_4 + \dot{z},$$

$$\dot{\sigma} = (-\bar{a}y_1 + q(\bar{a}x_1) + e_2) + 3(-\bar{b}y_2 + q(\bar{b}x_2) + e_3) + 3(-\bar{c}y_3 + q(\bar{c}x_3) + e_4) + (-\tilde{d}y_1y_2 - \bar{d}y_3 + q(dx_1x_2) + q(lx_4) + v) + \dot{z}. \quad (29)$$

By choosing Lyapunov function,

$$V = \frac{1}{2}a^2 + \frac{1}{2}(\bar{a}^2 + \bar{b}^2 + \bar{c}^2 + \bar{d}^2 + \bar{k}^2). \quad (30)$$

Design the adaptive laws for $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{k}, \bar{k}$, and $\nu_s$ is computed such that $V < 0$.

The Lyapunov function is considered as

$$\dot{\bar{a}} = \sigma e_1 - k_1\bar{a},$$

$$\dot{\tilde{a}} = 3\sigma e_2 - k_2\tilde{a},$$

$$\dot{\tilde{c}} = 3\sigma e_3 - k_3\tilde{c},$$

$$\dot{\tilde{d}} = \sigma x_1x_2 - \sigma y_1y_2 - k_4\tilde{d},$$

$$\dot{\tilde{k}} = \sigma e_4 - k_4\tilde{k}. \quad (32)$$

**Proof.** Since

$$V = \sigma \dot{\sigma} + \bar{a}\dot{\bar{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{k}\dot{\tilde{k}}. \quad (33)$$

by substituting the respective values displayed in (35) in following equation (34), we got (36);

$$V = \sigma(-\sigma e_1 - 3\sigma e_2 - 3\bar{e}_3 - \bar{d}x_1x_2 - \bar{k}e_4 - \bar{d}y_1y_2 - k\text{sign}(\sigma)) + \bar{a}\dot{\bar{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{k}\dot{\tilde{k}},$$

$$= \bar{a}(-\sigma e_1 + \bar{a}) + \tilde{b}(-3\sigma e_2 + \tilde{b}) + \tilde{c}(-3\sigma e_3 + \tilde{c}) + \tilde{d}(-\sigma x_1x_2 + \sigma y_1y_2 + \tilde{d}) + \tilde{k}(-\sigma e_4 + \tilde{k}) - k\sigma^2. \quad (34)$$

By putting
State trajectories of Slave system

Figure 1: Continued.
Figure 1: Continued.
Figure 1: Continued.
Figure 1: Synchronization of identical 4D hyperchaotic financial system. (a) Synchronization of interest rate corresponding to initial condition \([x_1(0), y_1(0) = (3, -2)]\). (b) Synchronization of investment demand corresponding to the initial condition \([x_2(0), y_2(0) = (1, 3)]\). (c) Synchronization of price index corresponding to initial condition \([x_3(0), y_3(0) = (2, -1)]\). (d) Synchronization of average profit margins corresponding to the initial condition \([x_4(0), y_4(0) = (-3, -4)]\). (e), (f) Time history of the errors \(e_1, e_2, e_3,\) and \(e_4\).
Figure 2: Synchronization of identical 4D hyperchaotic financial system. (a) Sliding manifold $\sigma$. (b) Control effort $v$. (c) $v_1$, $v_2$, $v_3$, and $v_4$ represent the time-varying disturbances.
Figure 3: Continued.
Figure 3: Continued.
Figure 3: Continued.
Figure 3: Antisynchronization of identical 4D hyperchaotic financial system. (a) Antisynchronization of interest rate corresponding to initial condition \([x_1(0), y_1(0) = (3, -2)]\). (b) Antisynchronization of investment demand corresponding to the initial condition \([x_2(0), y_2(0) = (1, 3)]\). (c) Antisynchronization of price index corresponding to initial condition \([x_3(0), y_3(0) = (2, -1)]\). (d) Antisynchronization of average profit margins corresponding to the initial condition \([x_4(0), y_4(0) = (-3, -4)]\). (e), (f) Time history of the errors \(e_1, e_2, e_3, \) and \(e_4\).
Figure 4: Antisynchronization of identical 4D hyperchaotic financial system. (a) Sliding manifold $\sigma$. (b) Control effort $v$. (c) $v_1, v_2, v_3$, and $v_4$ represent the time-varying disturbances.
(a)

(b)

(c)

**Figure 5**: Continued.
Figure 5: Synchronization of identical 4D hyperchaotic financial system with adaptation of parameters. (a) Synchronization of interest rate corresponding to the initial condition \([x_1(0), y_1(0) = (3, -2)]\). (b) Synchronization of investment demand corresponding to the initial condition \([x_2(0), y_2(0) = (1, 3)]\). (c) Synchronization of price index corresponding to initial condition \([x_3(0), y_3(0) = (2, -1)]\). (d) Synchronization of average profit margins corresponding to the initial condition \([x_4(0), y_4(0) = (-3, -4)]\). (e), (f) Time history of the errors \(e_1, e_2, e_3,\) and \(e_4\).
Figure 6: Synchronization of identical 4D hyperchaotic financial system. (a) $\hat{a}, \hat{b}, \hat{c}, \hat{d}$, and $\hat{k}$ represent the adaptation of unknown parameters. (b) $v_1, v_2, v_3$, and $v_4$ represent the time-varying disturbances.
\[
\begin{align*}
\dot{a} &= \sigma e_1 - k_1 \tilde{a}, \\
\dot{b} &= 3 \sigma e_2 - k_2 \tilde{b}, \\
\dot{c} &= 3 \sigma e_3 - k_3 \tilde{c}, \\
\dot{d} &= \sigma x_1 x_2 - \sigma y_1 y_2 - k_4 \tilde{d}, \\
\dot{\tilde{c}} &= \sigma e_4 - k_5 \tilde{c}, \\
\dot{\tilde{a}} &= -\sigma e_1 + k_1 \tilde{a}, \\
\dot{\tilde{b}} &= -3 \sigma e_2 + k_2 \tilde{b}, \\
\dot{\tilde{c}} &= -3 \sigma e_3 + k_3 \tilde{c}, \\
\dot{\tilde{d}} &= -\sigma x_1 x_2 + \sigma y_1 y_2 + k_4 \tilde{d}, \\
\dot{\tilde{\tilde{c}}} &= \sigma e_4 - k_5 \tilde{\tilde{c}}, \\
\end{align*}
\]
we have
\[
\dot{V} = -k_1 \sigma^2 - k_1 \tilde{a}^2 - k_2 \tilde{b}^2 - k_3 \tilde{c}^2 - k_4 \tilde{d}^2 - k_5 \tilde{\tilde{c}}^2 ,
\]
where \(\sigma, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{\tilde{c}} \rightarrow 0\). Since \(\sigma \rightarrow 0\), therefore \(e = (e_1, e_2, e_3, e_4) \rightarrow 0\).

Initial conditions for simulations are taken as follows:
\[
x(0) = [3, 1, 2, -3]^T, \\
y(0) = [-2, 3, -1, -4]^T.
\]

The actual value of the unknown parameters is chosen as follows:

\[\text{Figure 7: Synchronization of identical 4D hyperchaotic financial system with the adaptation of parameters. (a) Sliding manifold } \sigma.\]
Figure 8: Continued.
Figure 8: Continued.
Figure 8: Antisynchronization of identical 4D hyperchaotic financial system. (a) Antisynchronization of interest rate corresponding to initial condition \([x_1(0), y_1(0) = (3, -2)]\). (b) Antisynchronization of investment demand corresponding to the initial condition \([x_2(0), y_2(0) = (1, 3)]\). (c) Antisynchronization of price index corresponding to initial condition \([x_3(0), y_3(0) = (2, -1)]\). (d) Antisynchronization of average profit margins corresponding to the initial condition \([x_4(0), y_4(0) = (-3, -4)]\). (e), (f) Time history of the errors \(e_1, e_2, e_3, \) and \(e_4\).

Figure 9: Continued.
4. Results and Discussion

4.1. Synchronization of Identical 4D Hyperchaotic Financial System with Known Parameters: For Synchronization Set $q = 1$ in Equation (6). To achieve synchronization, $q = 1$ is employed in (6). This error signal profile is shown as (e) and (f) in Figure 1. Moreover, Figure 1(a) displays the interest rate synchronization, Figure 1(b) shows the synchronization among investment demands. Price index synchronization is portrayed as Figure 1(c). Figure 1(d) shows the average profit margin. However, the sliding surface profile and control effort are displayed in Figure 2 in this regard.

4.2. Antisynchronization of Identical 4D Hyperchaotic Financial System with Known Parameters: For Synchronization Set $q = -1$. This error signal profile is shown in Figures 3(e) and 3(f). Moreover, Figure 3(a) displays the interest rate antisynchronization, Figure 3(b) shows the antisynchronization among investment demands; price index antisynchronization is portrayed as Figure 3(c). Figure 3(d) shows the antisynchronization of average profit margin. However, the sliding surface profile and control effort are displayed in Figure 4 in this regard.

4.3. Antisynchronization of Identical 4D Hyperchaotic Financial System with Unknown Parameters: For Synchronization Set $q = -1$. A stable sliding manifold is mandatory to

4.4. Antisynchronization of Identical 4D Hyperchaotic Financial System with Unknown Parameters: For Synchronization Set $q = 1$. Figure 5 illustrates the synchronization of 4D hyperchaotic financial system with the adaption of the parameters; interest rate synchronization is placed as Figure 5(a); similarly investment demand is displayed as Figure 5(b); however, Figure 5(c) shows the synchronization of price index, and Figure 5(d) displays the synchronization of average profit margins. The time profile of errors is displayed as Figures 5(e) and 5(f). Moreover, the adapted unknown parameters are shown in Figure 6.

\[ \begin{align*}
    a &= 0.9, \\
    b &= 0.2, \\
    c &= 1.5, \\
    d &= 0.2, \\
    k &= 1.
\end{align*} \]
converge the system towards the origin in a finite-time instant. Figure 7 illustrates the stable sliding manifold and associated control efforts related to the sliding surface. Antisynchronization of identical 4D hyperchaotic financial system is displayed in Figure 8. Figure 8(a) reflects the antisynchronization of interest rate, where investment demand in this regard is displayed as Figure 8(b). Figure 8(c) portrays price indexing regarding antisynchronization of the 4D hyperchaotic financial system. Average profit margins are presented as Figure 8(d). Time history of errors is displayed as Figures 8(e) and 8(f), and it can be clearly observed that the error remains at zero.

Figure 9 displays the adaptation of unknown parameters; however, control effort and sliding surface stabilization are displayed in Figure 10. It can be observed from the figure that the sliding manifold converges toward origin in less than one second. In the upcoming section, the authors have concluded this work.

5. Conclusion

In the current era, we all know the importance of economic and financial stability. Stability of the financial system guaranties the social and economical development. Investments for the development of any financial/economic system purely depend on the stable financial system. However, after the financial globalization, financial stability becomes a highly complex nonlinear system. Minor parametric uncertainty may lead towards a chaos in any financial stable system, which may result turbulence in financial market and financial crises. The macroeconomic landscape of a country depends upon the interaction of demand, supply, discount rates, savings, and investments. The complex relationship of the abovementioned economic variables and taking account of all such variables open the door for the applications of 4D hyperchaotic fractional systems. According to the basic financial theory, this paper presents the controller for the
synchronization and antisynchronization of identical 4D hyperchaotic financial systems with external perturbations (which is quite complex to handle). Conventional SMC is employed for known parameters; however, to counter the aforementioned unknown parametric variation, AISMC control strategy is applied, which makes the system more responsive and robust (from the initial time instant) due to the absence of reaching phase. Moreover, it provides comprehensive suppression in the chattering phenomenon. The stability analysis is also performed, and the time derivative of the Lyapunov function proved strictly negative. The effectiveness of the proposed approaches can be confirmed via simulation results displayed in the previous section, which clearly indicated the adaptive integral sliding mode controller versatility over hyperchaotic financial system efficiently. Simulation results also proved the suggested technique outshines even in the presence of the system modelling uncertainties with unknown parameters.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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