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Research Article

Synchronization and Antisynchronization of Identical 4D Hyperchaotic Financial System with External Perturbation via Sliding Mode Control Technique

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In this article, complete synchronization and antisynchronization in the identical financial chaotic system are presented. The proposed control strategies depend on first-order sliding mode and adaptive integral sliding mode for complete synchronization and antisynchronization of the identical financial chaotic system. In the primary case, the system parameters should be known, and first-order sliding mode control is utilized for synchronization and antisynchronization while in the second case, the system parameters are considered unknown. An adaptive integral sliding mode control strategy is utilized for synchronization and antisynchronization of the system considering the parameters unknown. The error system is changed into a particular structure containing a nominal part and several unknown terms to utilize the adaptive integral sliding mode control. Then, this error system is stabilized using integral sliding mode control. The stabilizing controller is usually developed based on the nominal part plus the compensator control part. To suppress the high-frequency oscillation (chattering) phenomenon, smooth continuous compensator control can be used rather than conventional discontinuous control. The compensator controller along with the adapted law is derived such that the time derivative of the Lyapunov function becomes strictly negative. The effectiveness of the proposed method was tested through computer simulations. The proposed control strategies are verified for that Identical 4D hyperchaotic financial system to attain complete synchronization and antisynchronization along with the improved performance.

1. Introduction

Chaos theory plays an important role in nonlinear control theory. Chaotic synchronization and antisynchronization have gained considerable importance in the domain of biological systems, chemical reactions, secure communication, information sciences, plasma technology, and most important of all, financial systems [1–3]. The hyperchaotic systems are very sensitive towards initial conditions and possess at least two positive Lyapunov exponents. It has bounded trajectories in the phase space and exhibits more complex nonlinear behavior. Hyperchaos was first formalized in 1979 by Rossler [4]. Many other hyperchaotic systems were reported later on [5–7]. In 1990, Pecora and Carroll [8] introduced the synchronization of two identical chaotic systems with different initial conditions. Later, synchronization of chaotic systems was extensively studied in the last two decades, and many schemes were proposed for synchronization, which includes lag synchronization [9, 10], inverse lag synchronization [11], complete synchronization [12, 13], inverse π-lag synchronization [14], multiple chaotic systems synchronization [15, 16], partial synchronization [17, 18], generalized synchronization [19–21], projective synchronization [22], Q-S synchronization [20, 23], phase synchronization [24], anti-synchronization [25, 26], and fractional chaos synchronization [27].

In recent years, financial chaotic systems have been widely investigated due to their globally socioeconomically importance. Researchers are still exploring the new dimensions regarding synchronization and antisynchronization of financial chaotic systems, which is also considered an attractive idea of this era [27–30]. Since the fundamental chaotic financial models proposed in the literature from [31–37], like the Kaldorian model is listed at [31] and the IS-LM model as [32, 33], the hyperchaotic financial model is considered as [34]; moreover, the other nonlinear dynamical models are cited at [35–37]. External uncertainties arising from different environmental factors can cause the destabilization of chaotic financial systems and lead to undesirable effects [38–40]. The global stabilization and synchronization of chaotic financial systems in the presence of external uncertainty are necessary and have been the topic of excellent research works [41, 42]. As the financial systems are directly linked with our daily lives [29], chaos will occur when economic crises happen (like 2007). While the globalization and technology have opened new horizons of the international trade and economic cooperation among the states, these have also exposed the economic and financial systems to uncertainties and risks of unprecedented nature. One investment bank (Abraj Capital) filing bankruptcy can bring the whole system to a halt and hibernation stage. The economic crisis of 2007, which brought whole world trade to its knees, still echoes in the financial sector.

The macroeconomic landscape of a country depends upon the interaction of demand, supply, discount rates, savings, and investments. In the macroeconomic literature, the determinants of investments are not the same as the determinants of savings particularly, in the regions where capital mobility in the markets for loan-able funds is reasonably frictionless. Growth in the economy depends on investments which, on the other hand, depend upon the savings in the economy. It is important to note that investment and interest rates have a negative relationship while savings increase with the high-interest rates. The neoclassical economists consider that the growth-led monetary policy would focus upon the measures to increase the growth fueled by more investments, hence increasing the savings in the economy. They argue that measures to increase savings hamper economic expansion because it creates a demand deficit and pushes interest rates upwards, resulting in crowding out of the investments. Hence, investments bring down the demand deficit, increase growth, and result in higher savings.

The complex relationship of abovementioned economic variables and taking account of all such variables open the door for the applications of 3D and 4D hyperchaotic fractional systems. In this regard, stability and synchronization of the fractional chaotic model are proposed by Shahiri et al. in [43]. Kumar and Kumar proposed the stability analysis of the aforementioned model using the Lyapunov direct method in [44]. Xu and He studied the synchronization of the financial model by using an active control method [45].

As said earlier, hyperchaotic systems possess complex dynamical behavior due to more than one positive Lyapunov exponent and their expansion in more than one direction. Considering the aforementioned scenario, it is very much mandatory for every government to take preventive measures before the outbreak of chaos. The dynamics of a financial system play a significant role in the growth and development of an economic system. However, a financial system's dynamics depend on the multiple input variables in a highly complex and nonlinear fashion. A financial system, even if deterministic, can exhibit and initiate chaotic behavior. Since a chaotic system is more sensitive towards small changes and errors in parameters, their synchronization and antisynchronization are important from a control point of view.

As a control researcher, we know that the robust stabilization is one of the fundamental problem in control theory. It is considered to be more challenging because no such standard method exists for general nonlinear systems. In this regard, a control approach for complete synchronization and antisynchronization of an identical financial chaotic system is presented in this work. The sliding mode control (SMC) strategy is applied on the financial chaotic model when the system parameters are known; however, the adaptive integral sliding mode control (AISMC) approach is used for the case when the system's parameters are unknown. Moreover, in conventional SMC, the problem of chattering and uncertainty in reaching phase may lead towards total system failure. Many researchers proposed several solutions like integrating SMC with back-steeping and using higher-order SMC to suppress the chattering. However, in order to solve the reaching phase problem, the proposed (AISMC) technique is quite effective. In the said approach, the reaching phase starts from the very beginning, resulting the closed loop system which becomes very robust since the first time instant. The proposed control strategy ensures the robust finite-time convergence in addition to the elimination of reaching phase with mitigation of the high-frequency phenomenon (known as chattering). In practical systems, robust convergence and mitigation of chattering are really appreciable. Numerical simulation results verify the effectiveness of the proposed methodologies.

The rest of this paper is organized as follows; Section 2 presents preliminaries regarding synchronization. Section 3 displays the dynamical model and proposed methodologies regarding sliding mode control (SMC) and adaptive integral sliding mode control (AISMC) design for the known and unknown parameters cases. The simulation results and discussion are posed in Section 4. Section 5 presents the conclusion. The references cited are listed at last.

2. Systems Description and Preliminaries

This section presents the dynamical model of an identical 4D hyperchaotic financial system along with the concepts of complete synchronization and antisynchronization. The financial chaotic systems have attracted a substantial amount of attraction from researchers in recent years. The financial systems are involved with the existence [29]. As we know, economic crises result in chaos (like 2007). The dynamics of the financial system play a significant role while in the continuing build-out of the economic system. The dynamics of any financial system rely on multiple input variables in an incredibly complex and nonlinear fashion. The financial system, even though deterministic, can exhibit chaotic behavior. Since a chaotic system might be more responsive to minor errors and alterations in parameters, their synchronization is vital, originating from a control reason for view.

In this work, the authors have presented synchronization and antisynchronization of identical 4D hyperchaotic financial systems. The response strategy is taken like a perturbed system by some bounded external disturbances. Two cases are believed to be as follows:

(1) *System Parameters Are Known*. In this case, the synchronization and antisynchronization are achieved using first-order sliding mode control

(2) *System Parameters Are Unknown*. In this case, the adaptive integral sliding mode control is used to achieve synchronization and antisynchronization and estimate the system parameters

In 2012, a new hyperchaotic finance system was suggested. The model is expressed by the admirer's 4D hyperchaotic financial system [34]. Master system is given as follows:

$$\begin{aligned} \dot{x}_1 &= x_3 + (x_2 - a)x_1 + x_4, \\ \dot{x}_2 &= 1 - bx_2 - x_1^2, \\ \dot{x}_3 &= -x_1 - cx_3, \\ \dot{x}_4 &= -dx_1x_2 - kx_4, \end{aligned} \tag{1}$$

where x_1, x_2, x_3, and x_4 represent the interest rate, investment demand, price index, and average profit margins, respectively. Moreover, a, b, and c represent the saving amount, cost per investment, and elasticity of demand of commercial markets, respectively, where d and k represent some system's parameters. $a, b, c, d, and\, k$ are positive constants $(a, b, c, d, \text{and}\, k > 0)$. The slave system in this regard may be displayed as follows:

$$\begin{aligned} \dot{y}_1 &= y_3 + (y_2 - a)y_1 + y_4 + u_1, \\ \dot{y}_2 &= 1 - by_2 - y_1^2 + u_2, \\ \dot{y}_3 &= -y_1 - cy_3 + u_3, \\ \dot{y}_4 &= -dy_1y_2 - ky_4 + u_4. \end{aligned} \tag{2}$$

The system parameters a, b, c, d, and k are selected as 0.9, 0.2, 1.5, 0.2, and0.17, respectively, with these parameters system (2) exhibiting chaotic behavior.

3. Proposed Methodologies

3.1. Sliding Mode Control for Identical 4D Hyperchaotic Financial System with Known Parameters. Hence, systems (1) and (2) with external perturbations are shown as follows:

$$\begin{aligned} \dot{x}_1 &= x_3 + (x_2 - a)x_1 + x_4, \\ \dot{x}_2 &= 1 - bx_2 - x_1^2, \\ \dot{x}_3 &= -x_1 - cx_3, \\ \dot{x}_4 &= -dx_1x_2 - kx_4. \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{y}_1\sqrt{2} &= y_3 + (y_2 - a)y_1 + y_4 + h_1v_1 + u_1, \\ \dot{y}_2 &= 1 - by_2 - y_1^2 + h_2v_2 + u_2, \\ \dot{y}_3 &= -y_1 - cy_3 + h_3v_3 + u_3, \\ \dot{y}_4 &= -dy_1y_2 - ky_4 + h_4v_4 + u_4. \end{aligned} \tag{4}$$

External disturbances are considered as follows:

$$\begin{aligned} \dot{v}_1 &= 2y_2y_3 - 0.4v_1, \\ \dot{v}_2 &= -2y_1y_4 - 0.8v_2, \\ \dot{v}_3 &= -1.2y_1y_2 - 0.5v_3, \\ \dot{v}_4 &= y_2y_3 - 0.5v_4. \end{aligned} \tag{5}$$

By defining the error signals,

$$\begin{aligned} e_1 &= y_1 - qx_1, \\ e_2 &= y_2 - qx_2, \\ e_3 &= y_3 - qx_3, \\ e_4 &= y_4 - qx_4. \end{aligned} \tag{6}$$

For synchronization, select $q = 1$, and for antisynchronization, select $q = -1$. By taking derivative of error signals, error dynamics becomes

$$\begin{aligned} \dot{e}_1 &= \dot{y}_1 - q\dot{x}_1 = (y_3 + (y_2 - a)y_1 + y_4) + h_1v_1 + u_1 - q(x_3 + (x_2 - a)x_1 + x_4), \\ \dot{e}_2 &= \dot{y}_2 - q\dot{x}_2 = (1 - by_2 - y_1^2) + h_2v_2 + u_2 - q(1 - bx_2 - x_1^2), \\ \dot{e}_3 &= \dot{y}_3 - q\dot{x}_3 = (-y_1 - cy_3) + h_3v_3 + u_3 - q(-x_1 - cx_3), \\ \dot{e}_4 &= \dot{y}_4 - q\dot{x}_4 = (-dy_1y_2 - ky_4) + h_4v_4 + u_4 - q(-dx_1x_2 - kx_4). \end{aligned} \tag{7}$$

Considering u_1, u_2, u_3 and u_4 displayed as follows for proceeding towards (9);

$$
\begin{aligned}
u_1 &= -(y_3 + (y_2 - a)y_1 + y_4) - h_1 v_1 + q(x_3 + (x_2 - a)x_1 + x_4) + e_1,\\
u_2 &= \left(1 - by_2 - y_1^2\right) - h_2 v_2 + q\left(1 - bx_2 - x_1^2\right) + e_2,\\
u_3 &= -(-y_1 - cy_3) + h_3 v_3 + q(-x_1 - cx_3) + e_3,\\
u_4 &= -(-dy_1 y_2 - ky_4) + h_4 v_4 + q(-dx_1 x_2 - kx_4) + v.
\end{aligned}
\tag{8}
$$

In (8), v represents the new input, which can be mentioned as follows:

$$
\begin{aligned}
\dot{e}_1 &= e_2,\\
\dot{e}_2 &= e_3,\\
\dot{e}_3 &= e_4,\\
\dot{e}_4 &= v.
\end{aligned}
\tag{9}
$$

Defining the Hurwitz sliding surface for (7) as follows:

$$
\begin{aligned}
\sigma &= \left(1 + \frac{\mathrm{d}}{\mathrm{d}t}\right)^3 e_1,\\
\sigma &= e_1 + 3e_2 + 3e_3 + e_4.
\end{aligned}
\tag{10}
$$

By taking time derivative, we have

$$
\dot{\sigma} = \dot{e}_1 + 3\dot{e}_2 + 3\dot{e}_3 + \dot{e}_4. \tag{11}
$$

By substituting the respective values in (11), we got the following:

$$
\dot{\sigma} = e_2 + 3e_3 + 3e_4 + v. \tag{12}
$$

The aforementioned system becomes stable by considering "v" displayed in the following:

$$
v = -e_2 - 3e_3 - 3e_4 - k\sigma. \tag{13}
$$

By putting v value $\dot{\sigma} = -k\sigma$, the error system (7) is asymptotically stable, and following initial conditions and parametric values are used in the simulation of the aforementioned system:

$$
\begin{aligned}
x(0) &= [3, 1, 2, -3]^T,\\
y(0) &= [-2, 3, -1, -4]^T,\\
a &= 0.9,\\
b &= 0.2,\\
c &= 1.5,\\
d &= 0.2,\\
k &= 0.17.
\end{aligned}
\tag{14}
$$

If we consider a Lyapunov function $V = 0.5\sigma^2$, then its derivative becomes $\dot{V} = \sigma\dot{\sigma}$, moreover;

$$
\dot{V} = \sigma(-k\sigma) = -k\sigma^2. \tag{15}
$$

From this, we can say that $\sigma \longrightarrow 0$; since σ is Hurwitz, then $e_i \longrightarrow 0$, where $i = 1, .., 4$. Therefore, the system shown in (9) is asymptotically stable.

3.2. Adaptive Integral Sliding Mode Control for Identical 4D Hyperchaotic Financial System with Unknown Parameters. In this section, AISMC is dispensed for synchronization and antisynchronization of identical 4D hyperchaotic financial systems. In this method, the parameters are expected to be unknown and are estimated using AISMC.

Let $\widehat{a}, \widehat{b}, \widehat{c}, \widehat{d}$, and $\widehat{k}$ be estimate value of a, b, c, d, and k, respectively, and let us consider $\widetilde{a} = a - \widehat{a}, \widetilde{b} = b - \widehat{b}$, $\widetilde{c} = c - \widehat{c}, \widetilde{d} = d - \widehat{d}$, and $\widetilde{k} = k - \widehat{k}$ be the errors.

Thus, systems (1) and (2) with external perturbations are displayed as follows:

$$
\begin{aligned}
\dot{x}_1 &= x_3 + x_2 x_1 - \widehat{a}x_1 - \widetilde{a}x_1 + x_4,\\
\dot{x}_2 &= 1 - \widehat{b}x_2 - \widetilde{b}x_2 - x_1^2,\\
\dot{x}_3 &= -x_1 - \widehat{c}x_3 - \widetilde{c}x_3,\\
\dot{x}_4 &= -\widehat{d}x_1 x_2 - \widetilde{d}x_1 x_2 - \widehat{k}x_4 - \widetilde{k}x_4,\\
\dot{y}_1 &= y_3 + y_2 y_1 - \widehat{a}y_1 - \widetilde{a}y_1 + y_4 + h_1 v_1 + u_1,\\
\dot{y}_2 &= 1 - \widehat{b}y_2 - \widetilde{b}y_2 - y_1^2 + h_2 v_2 + u_2,\\
\dot{y}_3 &= -y_1 - \widehat{c}y_3 - \widetilde{c}y_3 + h_3 v_3 + u_3,\\
\dot{y}_4 &= -\widehat{d}y_1 y_2 - \widetilde{d}y_1 y_2 - \widehat{k}y_4 - \widetilde{k}y_4 + h_4 v_4 + u_4.
\end{aligned}
\tag{16}
$$

External disturbances are given as follows:

$$
\begin{aligned}
\dot{v}_1 &= 2y_2 y_3 - 0.4v_1,\\
\dot{v}_2 &= -2y_1 y_4 - 0.8v_2,\\
\dot{v}_3 &= -1.2y_1 y_2 - 0.5v_3,\\
\dot{v}_4 &= y_2 y_3 - 0.5v_4.
\end{aligned}
\tag{17}
$$

By defining the error signals,

$$
\begin{aligned}
e_1 &= y_1 - qx_1,\\
e_2 &= y_2 - qx_2,\\
e_3 &= y_3 - qx_3,\\
e_4 &= y_4 - qx_4.
\end{aligned}
\tag{18}
$$

For synchronization, select $q = 1$, and select $q = -1$ for antisynchronization. By taking derivative of error signals, we get error dynamics as follows:

$$
\begin{aligned}
\dot{e}_1 &= \dot{y}_1 - q\dot{x}_1,\\
\dot{e}_1 &= (y_3 + y_2y_1 - \widehat{a}y_1 - \widetilde{a}y_1 + y_4 + h_1v_1) + u_1 - q(x_3 + x_2x_1 - \widehat{a}x_1 - \widetilde{a}x_1 + x_4),\\
\dot{e}_2 &= \dot{y}_2 - q\dot{x}_2,\\
\dot{e}_2 &= \left(1 - \widehat{b}y_2 - \widetilde{b}y_2 - y_1^2 + h_2v_2\right) + u_2 - q\left(1 - \widehat{b}x_2 - \widetilde{b}x_2 - x_1^2\right),\\
\dot{e}_3 &= \dot{y}_3 - q\dot{x}_3,\\
\dot{e}_3 &= (-y_1 - \widehat{c}y_3 - \widetilde{c}y_3 + h_3v_3) + u_3 - q(-x_1 - \widehat{c}x_3 - \widetilde{c}x_3),\\
\dot{e}_4 &= \dot{y}_4 - q\dot{x}_4,\\
\dot{e}_4 &= \left(-\widehat{d}y_1y_2 - \widetilde{d}y_1y_2 - \widehat{k}y_4 - \widetilde{k}y_4 + h_4v_4\right) + u_4 - q\left(-\widehat{d}x_1x_2 - \widetilde{d}x_1x_2 - \widehat{k}x_4 - \widetilde{k}x_4\right).
\end{aligned}
\tag{19}
$$

Choose

$$
\begin{aligned}
u_1 &= -(y_3 + y_2y_1 - \widehat{a}y_1 - \widetilde{a}y_1 + y_4 + h_1v_1) + q(x_3 + x_2x_1 - \widehat{a}x_1 + x_4) + e_1,\\
u_2 &= -\left(1 - \widehat{b}y_2 - y_1^2 + h_2v_2\right) + q\left(1 - \widehat{b}x_2 - x_1^2\right) + e_2,\\
u_3 &= -(-y_1 - \widehat{c}y_3 + h_3v_3) + q(-x_1 - \widehat{c}x_3) + e_3,\\
u_4 &= -\left(-\widehat{d}y_1y_2 - \widehat{k}y_4 + h_4v_4\right) + q\left(-\widehat{d}x_1x_2 - \widehat{k}x_4\right) + v,
\end{aligned}
\tag{20}
$$

where $''v''$ the new input, and system (19) can be written as follows:

$$
\begin{aligned}
\dot{e}_1 &= -\widetilde{a}y_1 + q(\widetilde{a}x_1) + e_2,\\
\dot{e}_2 &= -\widetilde{b}y_2 + q\left(\widetilde{b}x_2\right) + e_3,\\
\dot{e}_3 &= -\widetilde{c}y_3 + q(\widetilde{c}x_3) + e_4,\\
\dot{e}_4 &= -\widetilde{d}y_1y_2 - \widetilde{l}y_4 + q\left(\widetilde{d}x_1x_2\right) + q\left(\widetilde{l}x_4\right) + v.
\end{aligned}
\tag{21}
$$

By using AISMC, the nominal system for (21) will be considered as follows:

$$
\begin{aligned}
\dot{e}_1 &= e_2,\\
\dot{e}_2 &= e_3,\\
\dot{e}_3 &= e_4,\\
\dot{e}_4 &= v_0.
\end{aligned}
\tag{22}
$$

Define the sliding surface (Hurwitz) for the nominal system displayed in (22) as

$$\sigma_0 = \left(1 + \frac{\mathrm{d}}{\mathrm{d}t}\right)^3 e_1, \tag{23}$$

$$\sigma_0 = e_1 + 3e_2 + 3e_3 + e_4. \tag{24}$$

By taking the derivative of sliding surface (24), we get

$$\dot{\sigma}_0 = \dot{e}_1 + 3\dot{e}_2 + 3\dot{e}_3 + \dot{e}_4, \tag{25}$$

$$\dot{\sigma}_0 = e_2 + 3e_3 + 3e_4 + v_0. \tag{26}$$

The aforementioned system becomes stable by considering “v0” displayed in (27)

$$v_0 = -e_2 - 3e_2 - 3e_4 - k\sigma_0, \text{for} > 0. \tag{27}$$

By substituting (26) in (27), we get $\dot{\sigma}_0 = -k\sigma_0$. So, we can say that the error system (22) is stable asymptotically. Now, choose an integral sliding surface for system (21) as follows [46]:

$$
\begin{aligned}
\sigma &= \sigma_0 + z,\\
\sigma &= e_1 + 3e_2 + 3e_3 + e_4 + z.
\end{aligned}
\tag{28}
$$

where "z" presents some integral term. To circumvent the reaching phase, choose $z(0)$ such that $\sigma(0) = 0$. Choose $v = v_0 + v_s$ where v_0 represents the nominal input, and the discontinuous term is displayed as v_s.

By taking derivative,

$$\dot{\sigma} = \dot{e}_1 + 3\dot{e}_2 + 3\dot{e}_3 + \dot{e}_4 + \dot{z},$$
$$\dot{\sigma} = (-\tilde{a}y_1 + q(\tilde{a}x_1) + e_2) + 3(-\tilde{b}y_2 + q(\tilde{b}x_2) + e_3) + 3(-\tilde{c}y_3 + q(\tilde{c}x_3) + e_4) + (-\tilde{d}y_1y_2 - \tilde{l}y_4 + q(\tilde{d}x_1x_2) + q(\tilde{l}x_4) + v) + \dot{z}. \tag{29}$$

By choosing Lyapunov function,

$$V = \frac{1}{2}\sigma^2 + \frac{1}{2}\left(\tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{k}^2\right). \tag{30}$$

Design the adaptive laws for $\tilde{a}, \hat{a}, \tilde{b}, \hat{b}, \tilde{c}, \hat{c}, \tilde{d}, \hat{d}, \tilde{k}, \hat{k}$, and v_s is computed such that $\dot{V} < 0$.

The Lyapunov function is considered as

$$V = \frac{1}{2}\sigma^2 + \frac{1}{2}\left(\tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{k}^2\right). \tag{31}$$

Afterward, considering the above adaptive laws for $\tilde{a}, \hat{a}, \tilde{b}, \hat{b}, \tilde{c}, \hat{c}, \tilde{d}, \hat{d}, \tilde{k}$, and $\hat{k}$ and adopting the given value of v_s lead towards $\dot{V} < 0$

$$\begin{aligned}
\dot{z} &= -e_2 - 3e_3 - 3e_4 - k\sigma_0\ ,\ k > 0 - v_0,\\
v_s &= -k\sigma - k\,\mathrm{sign}(\sigma_0),\\
\dot{\tilde{a}} &= \sigma e_1 - k_1\tilde{a},\\
\dot{\tilde{b}} &= 3\sigma e_2 - k_2\tilde{b},\\
\dot{\tilde{c}} &= 3\sigma e_3 - k_3\tilde{c},\\
\dot{\tilde{d}} &= \sigma x_1x_2 - \sigma y_1y_2 - k_4\tilde{d},\\
\dot{\tilde{k}} &= \sigma e_4 - k_5\tilde{k},\\
\dot{\hat{a}} &= -\sigma e_1 + k_1\tilde{a},\\
\dot{\hat{b}} &= -3\sigma e_2 + k_2\tilde{b},\\
\dot{\hat{c}} &= -3\sigma e_3 + k_3\tilde{c},\\
\dot{\hat{d}} &= -\sigma x_1x_2 + \sigma y_1y_2 + k_4\tilde{d},\\
\dot{\hat{k}} &= -\sigma e_4 - k_5\tilde{k}.
\end{aligned} \tag{32}$$

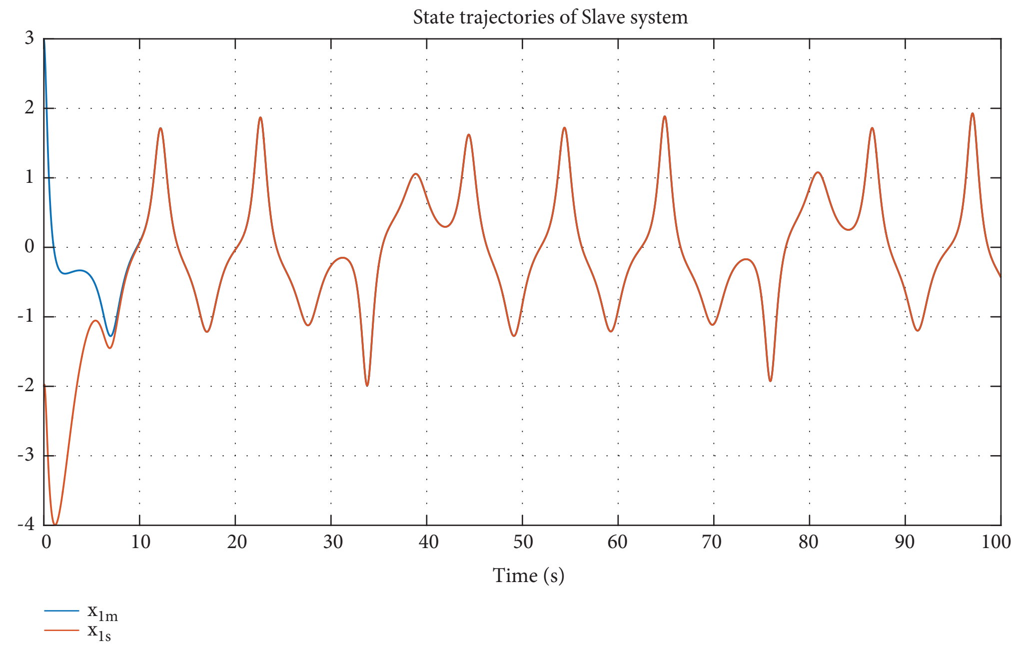
Proof. Since

$$\dot{V} = \sigma\dot{\sigma} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{k}\dot{\tilde{k}}, \tag{33}$$

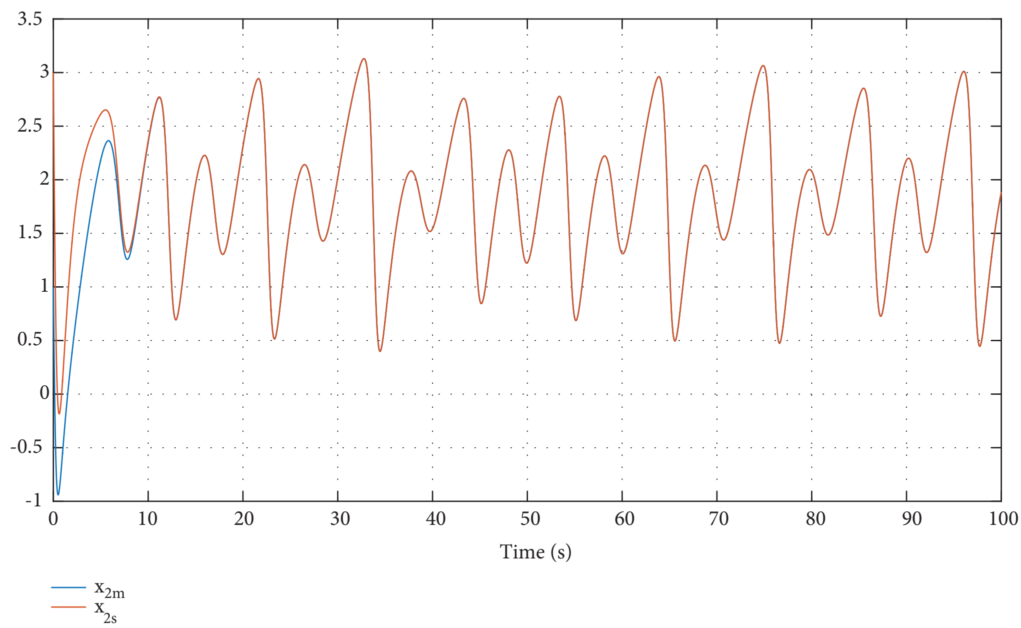
by substituting the respective values displayed in (35) in following equation (34), we got (36);

$$\begin{aligned}
&= \sigma\left(-\tilde{a}e_1 - 3\tilde{b}e_2 - 3\tilde{c}e_3 - \tilde{d}x_1x_2 - \tilde{k}e_4 - \tilde{d}y_1y_2 - k\,\mathrm{sign}(\sigma)\right) + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{k}\dot{\tilde{k}},\\
&= \tilde{a}\left(-\sigma e_1 + \dot{\tilde{a}}\right) + \tilde{b}\left(-3\sigma e_2 + \dot{\tilde{b}}\right) + \tilde{c}\left(-3\sigma e_3 + \dot{\tilde{c}}\right) + \tilde{d}\left(-\sigma x_1x_2 + \sigma y_1y_2 + \dot{\tilde{d}}\right) + \tilde{k}\left(-\sigma e_4 + \dot{\tilde{l}}\right) - k\sigma^2.
\end{aligned} \tag{34}$$

By putting

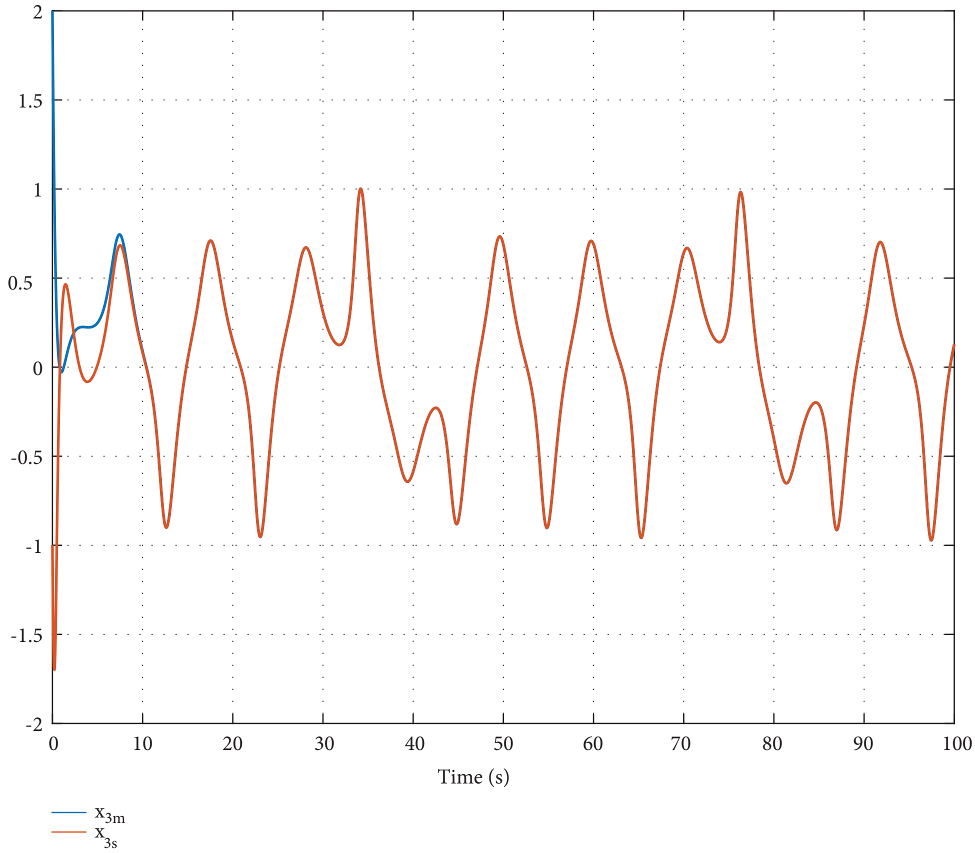


(a)

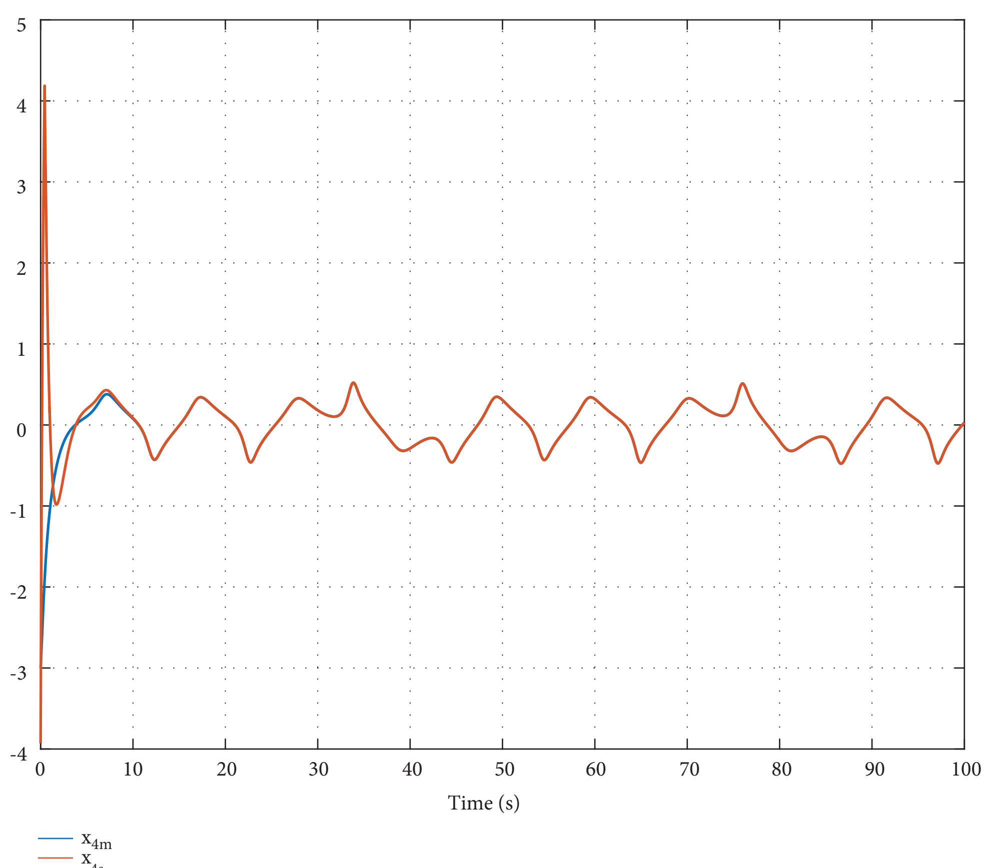


(b)

Figure 1: Continued.

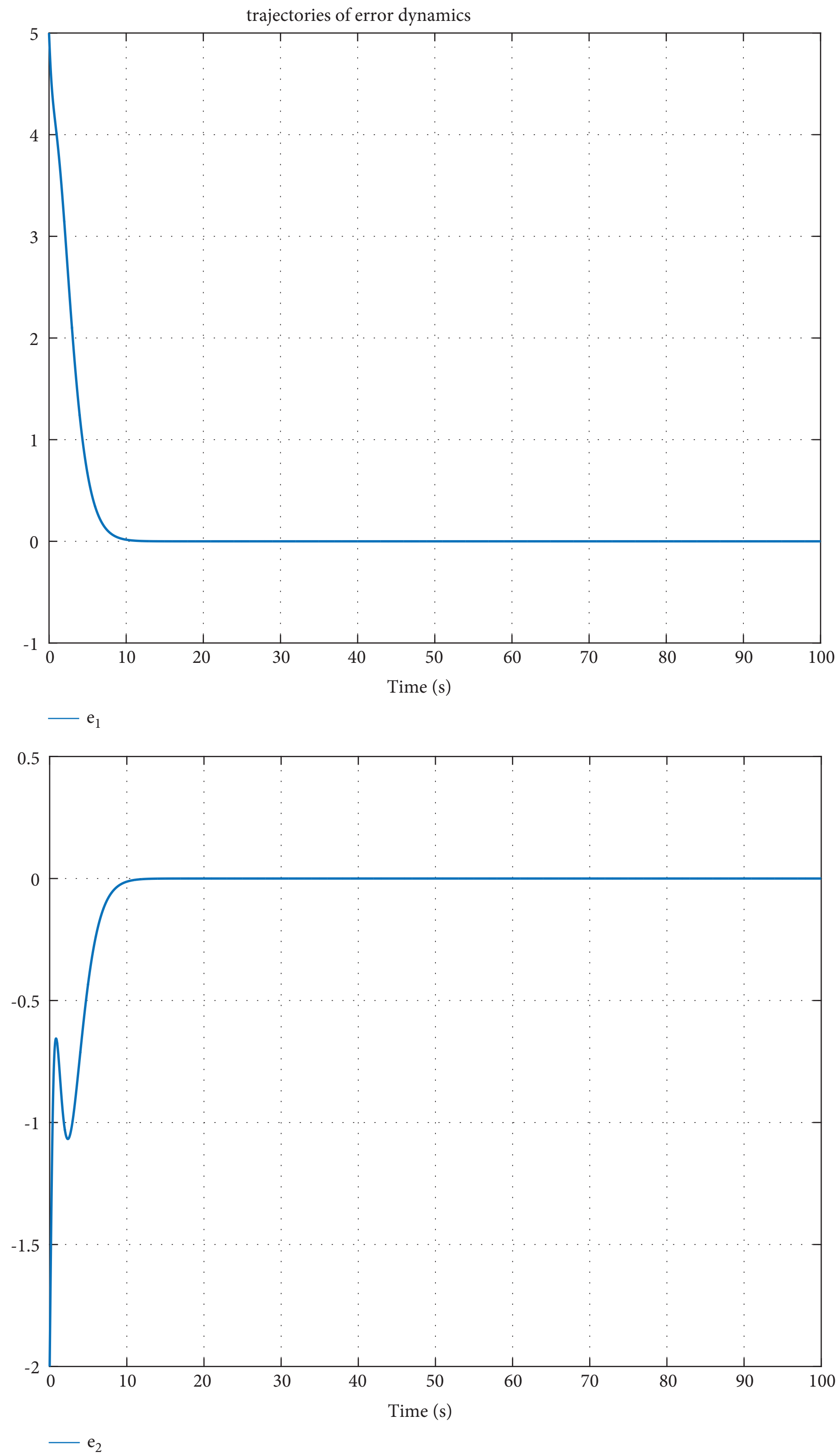


(c)



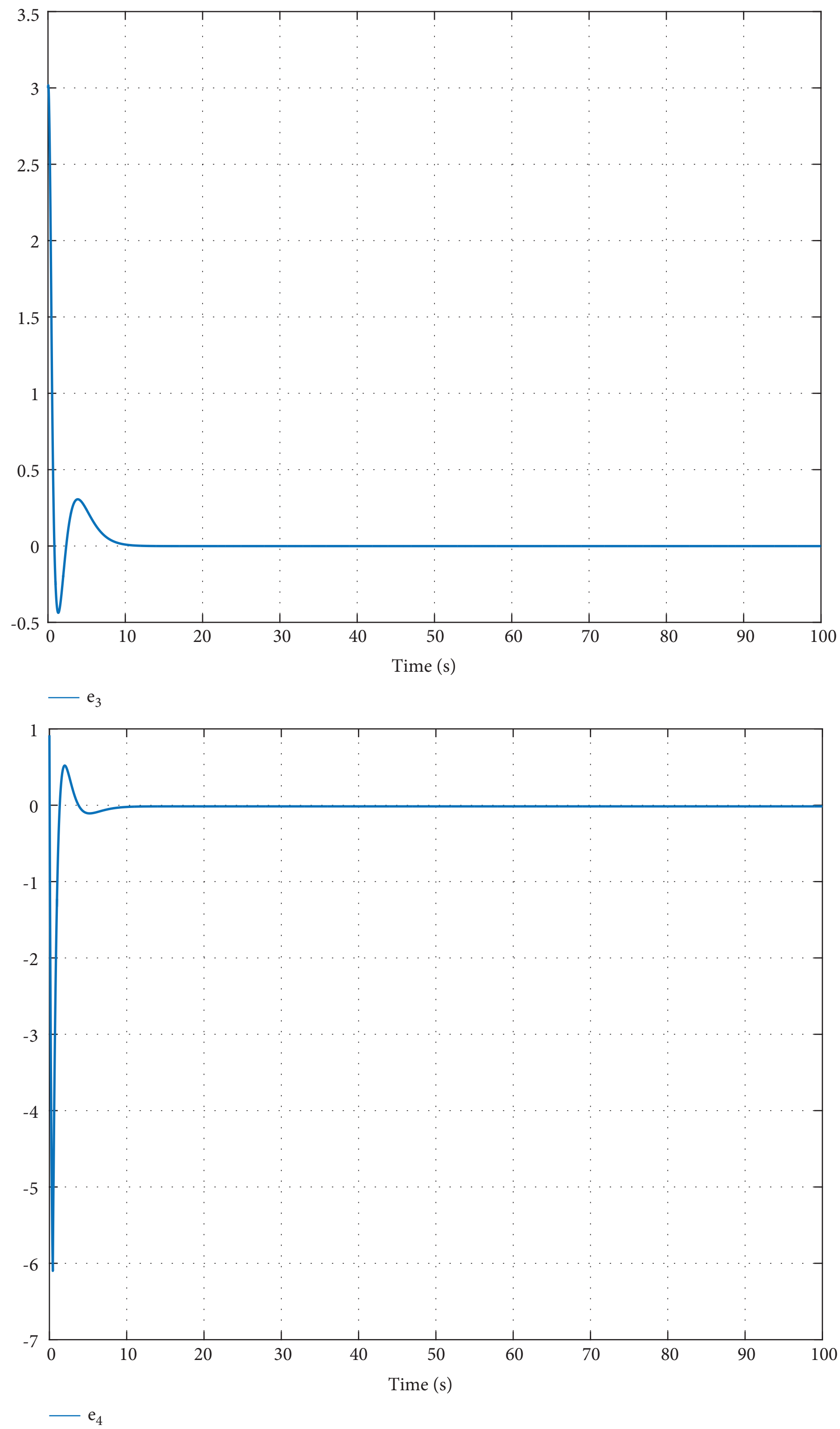
(d)

Figure 1: Continued.



(e)

Figure 1: Continued.



(f)

Figure 1: Synchronization of identical 4D hyperchaotic financial system. (a) Synchronization of interest rate corresponding to initial condition $[x_1(0), y_1(0) = (3, -2)]$. (b) Synchronization of investment demand corresponding to the initial condition $[x_2(0), y_2(0) = (1, 3)]$. (c) Synchronization of price index corresponding to initial condition $[x_3(0), y_3(0) = (2, -1)]$. (d) Synchronization of average profit margins corresponding to the initial condition $[x_4(0), y_4(0) = (-3, -4)]$. (e), (f) Time history of the errors e_1, e_2, e_3, and e_4.

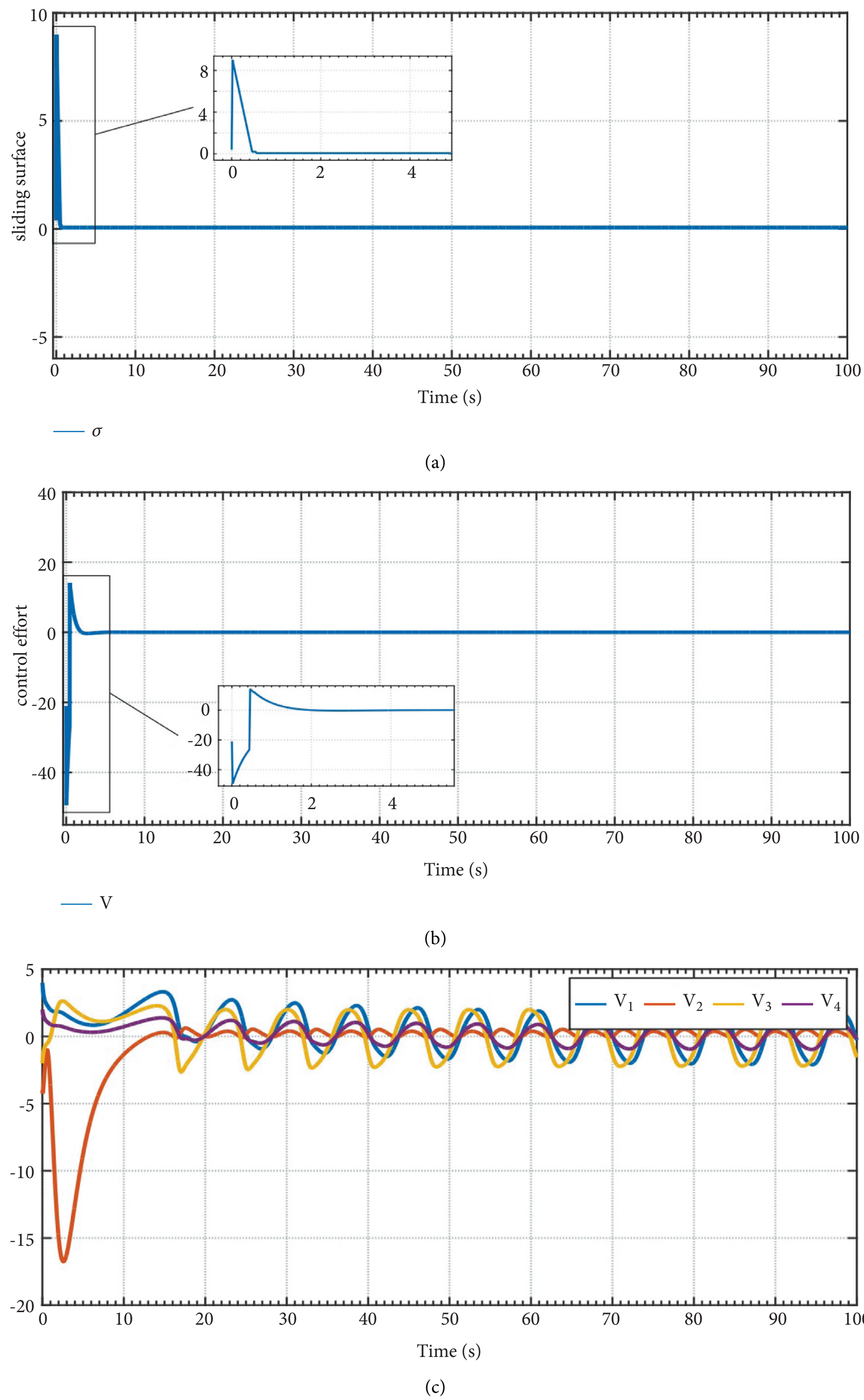
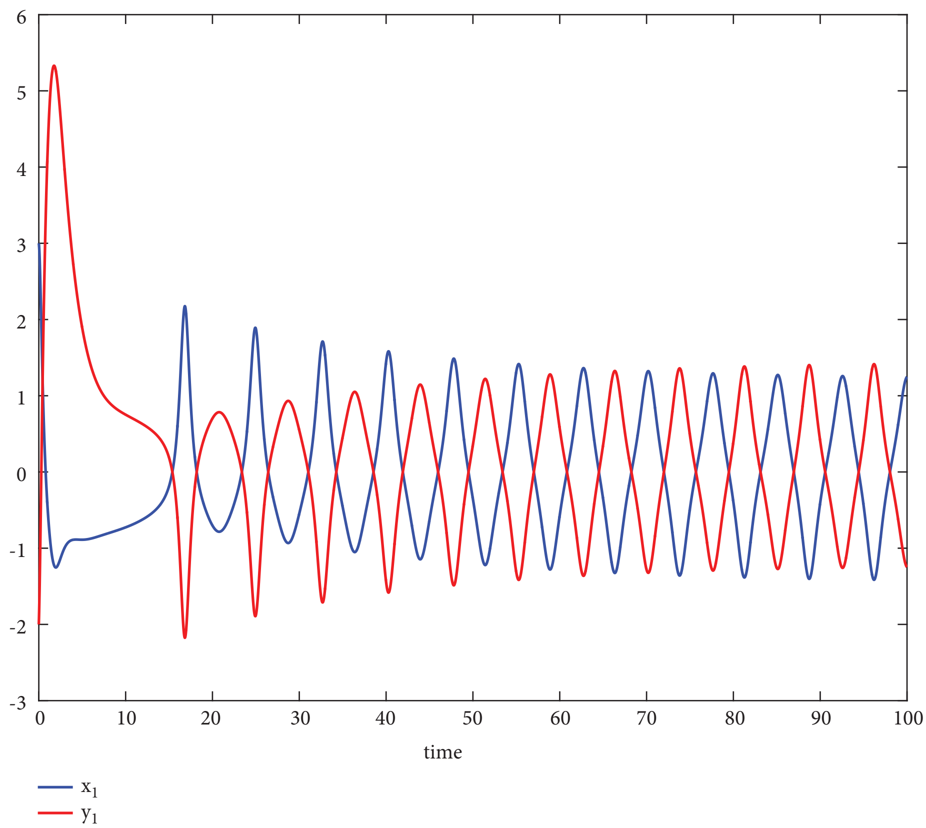
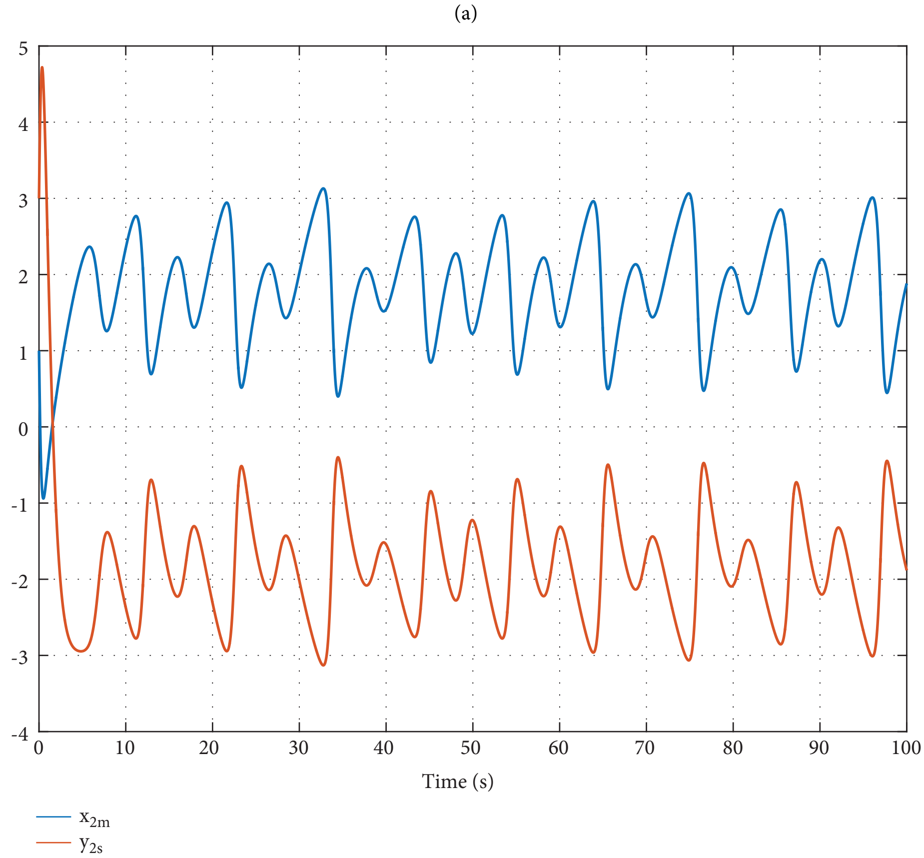


Figure 2: Synchronization of identical 4D hyperchaotic financial system. (a) Sliding manifold σ. (b) Control effort v. (c) $v_1, v_2, v_3,$ and v_4 represent the time-varying disturbances.

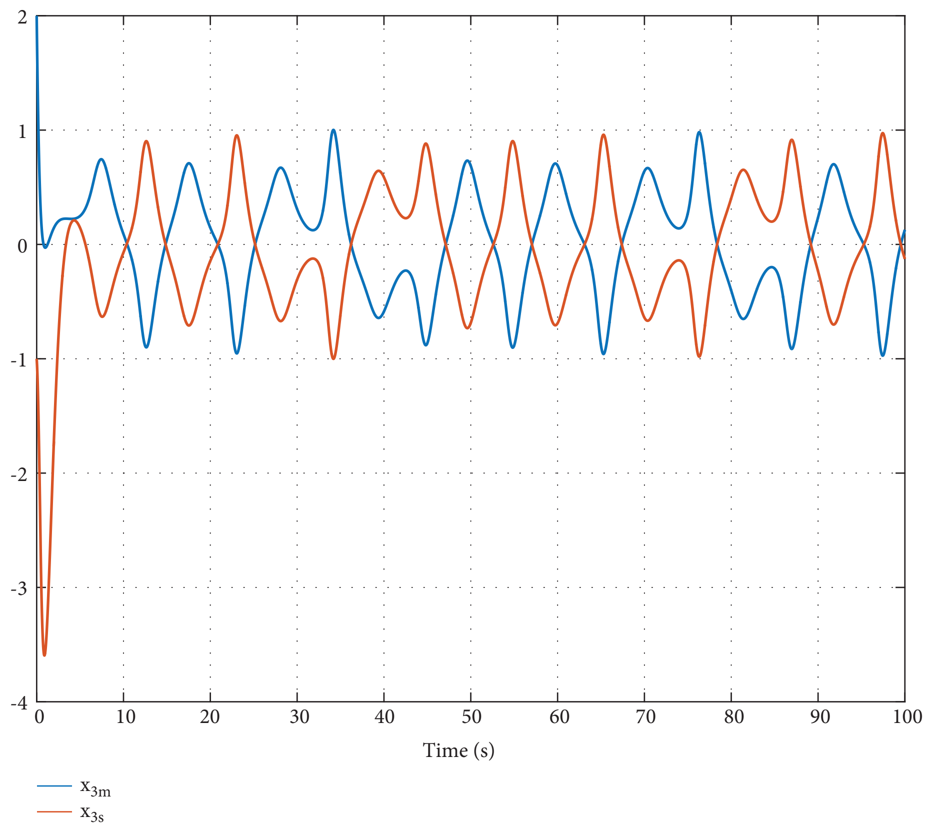


(a)

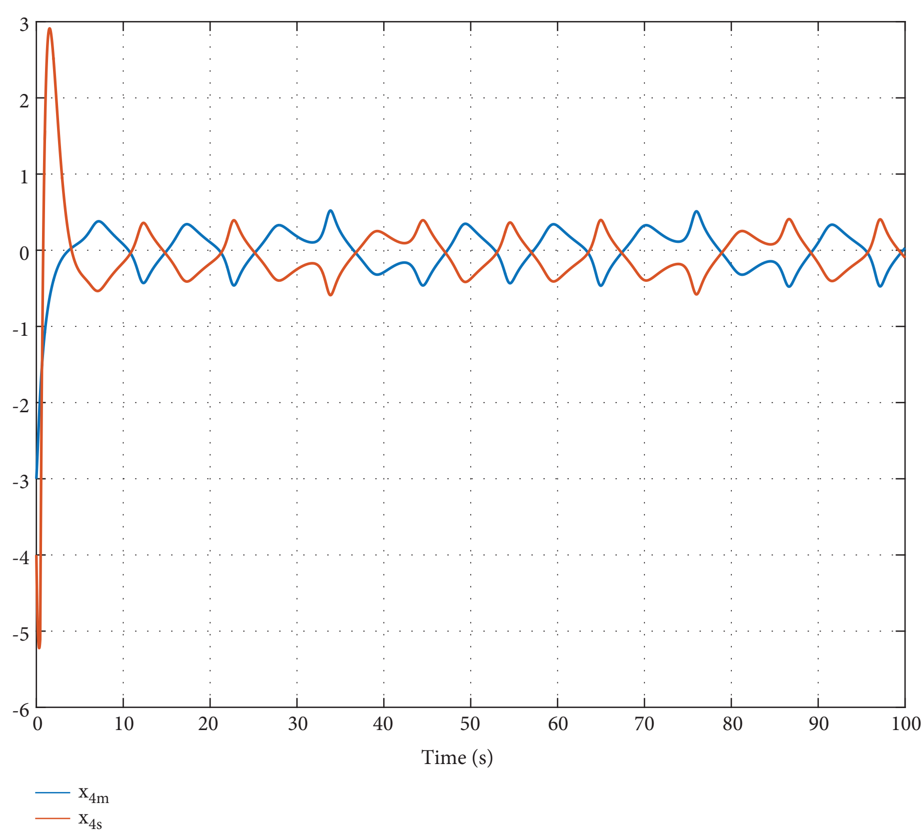


(b)

FIGURE 3: Continued.

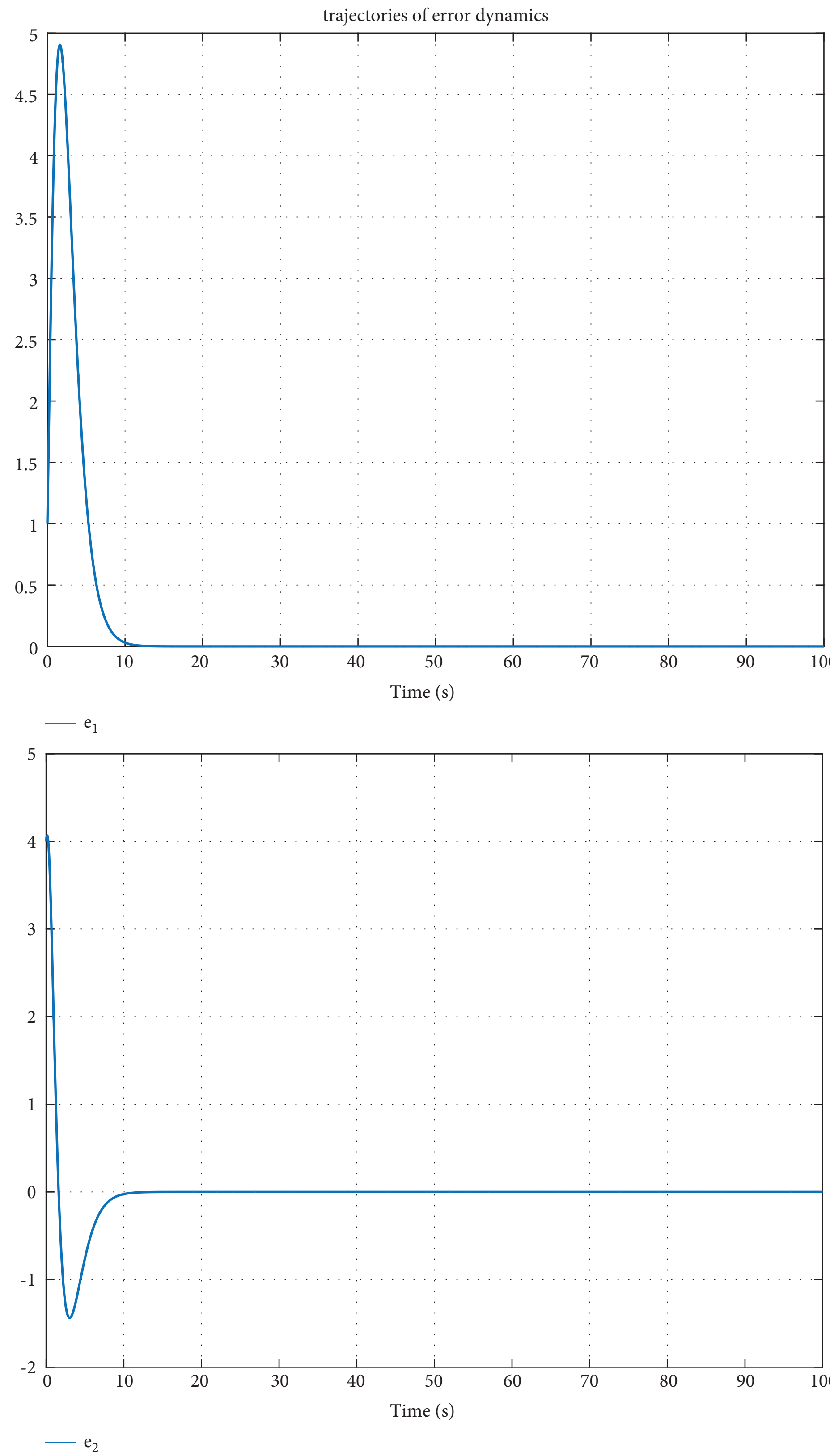


(c)



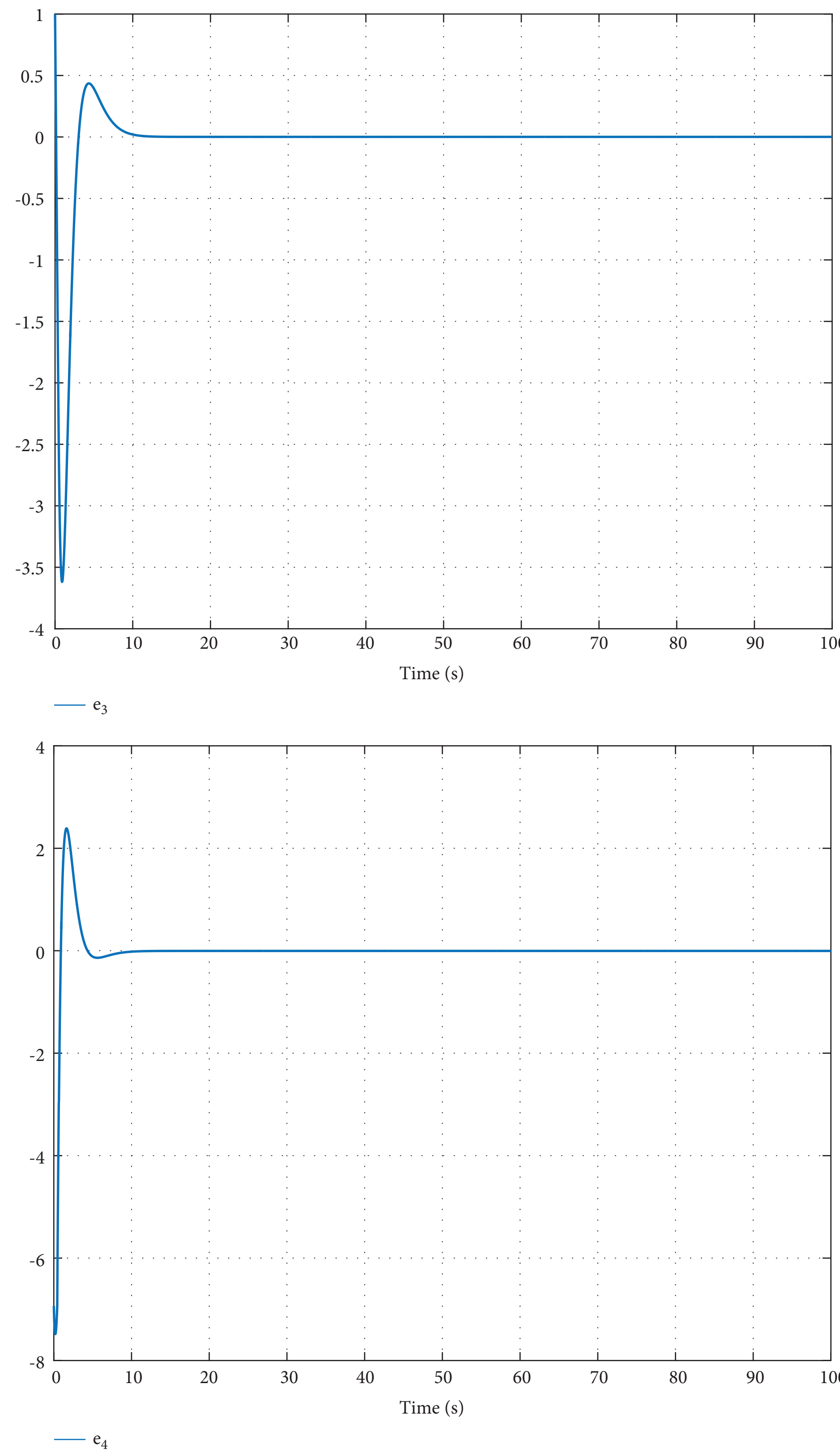
(d)

Figure 3: Continued.



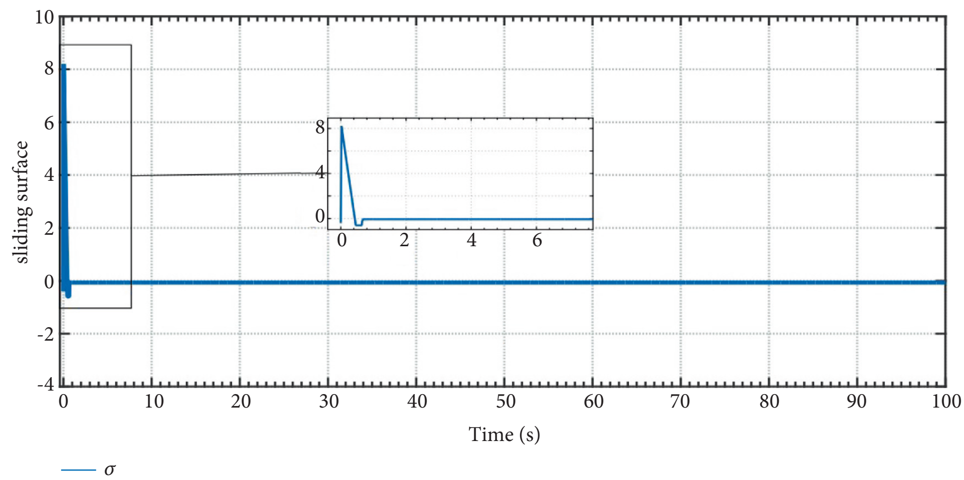
(e)

Figure 3: Continued.

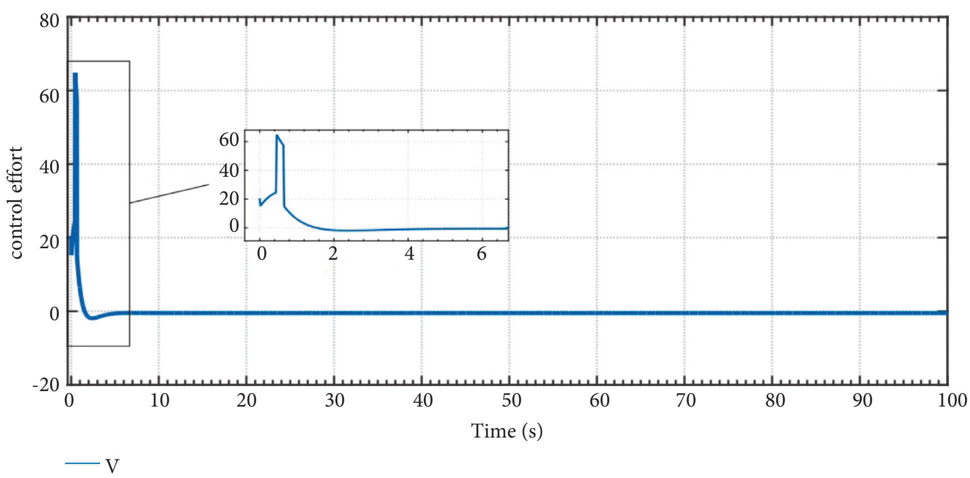


(f)

Figure 3: Antisynchronization of identical 4D hyperchaotic financial system. (a) Antisynchronization of interest rate corresponding to initial condition $[x_1(0), y_1(0) = (3, -2)]$. (b) Antisynchronization of investment demand corresponding to the initial condition $[x_2(0), y_2(0) = (1, 3)]$. (c) Antisynchronization of price index corresponding to initial condition $[x_3(0), y_3(0) = (2, -1)]$. (d) Antisynchronization of average profit margins corresponding to the initial condition $[x_4(0), y_4(0) = (-3, -4)]$. (e), (f) Time history of the errors e_1, e_2, e_3, and e_4.



(a)



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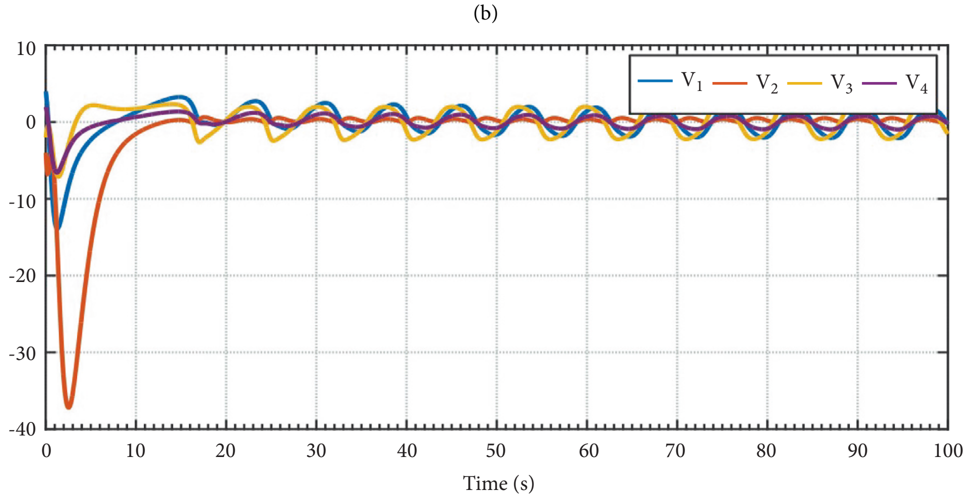
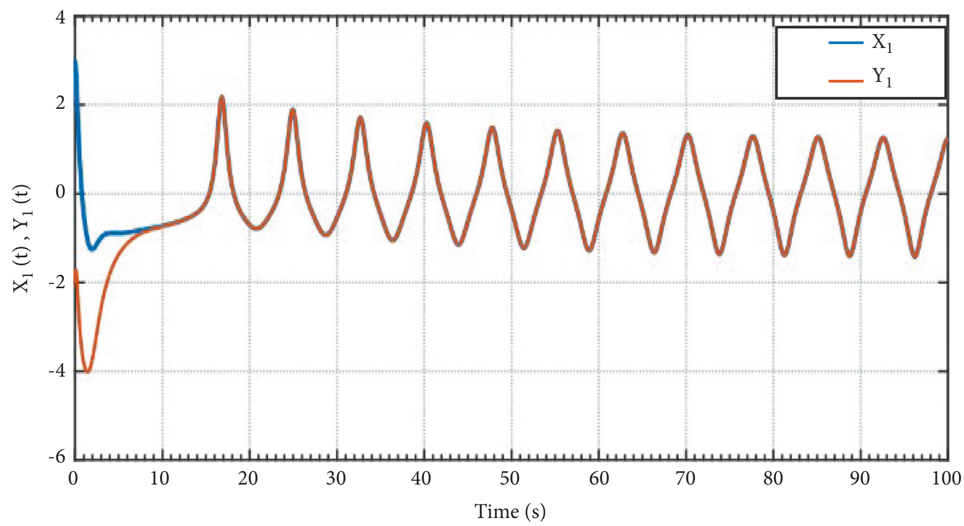
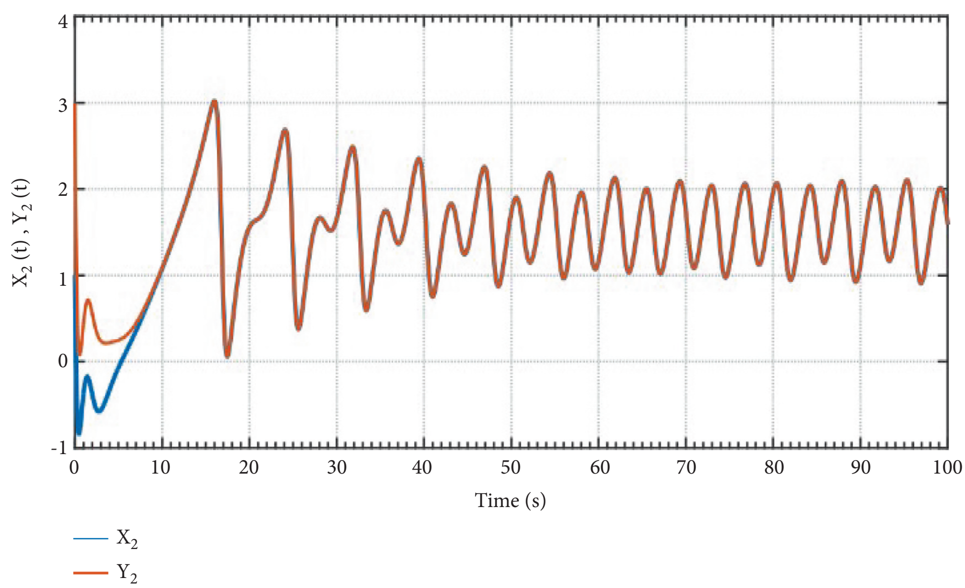


(c)

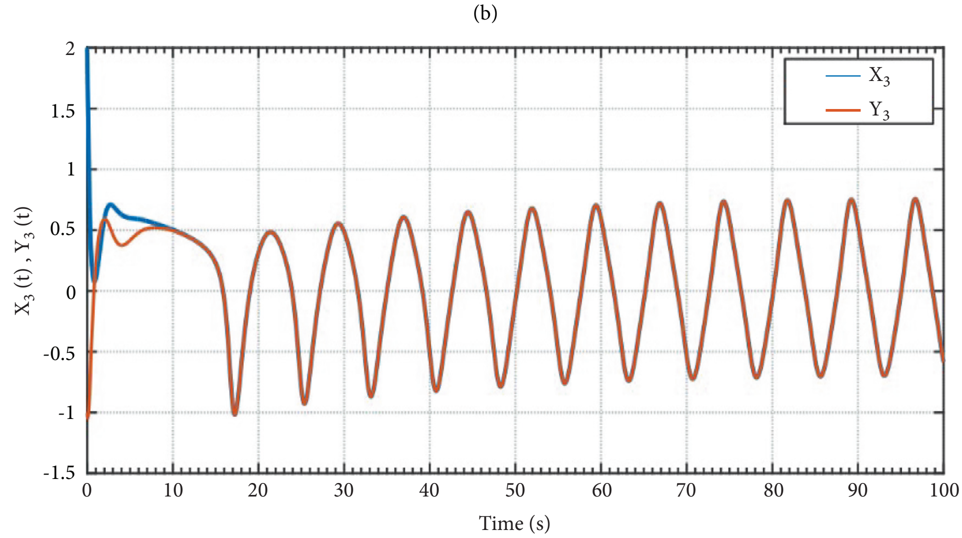
Figure 4: Antisynchronization of identical 4D hyperchaotic financial system. (a) Sliding manifold σ. (b) Control effort v. (c) v_1, v_2, v_3, and v_4 represent the time-varying disturbances.



(a)

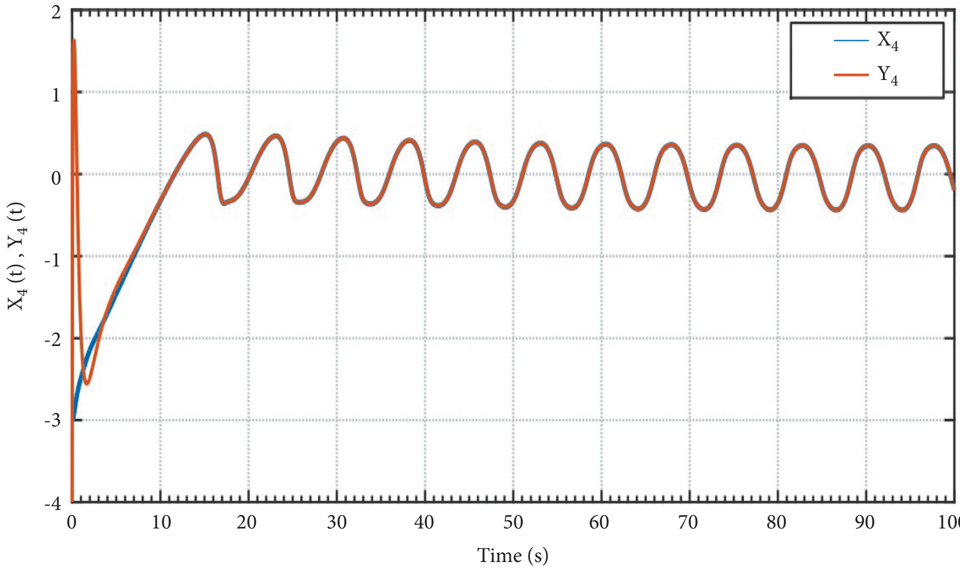


(b)

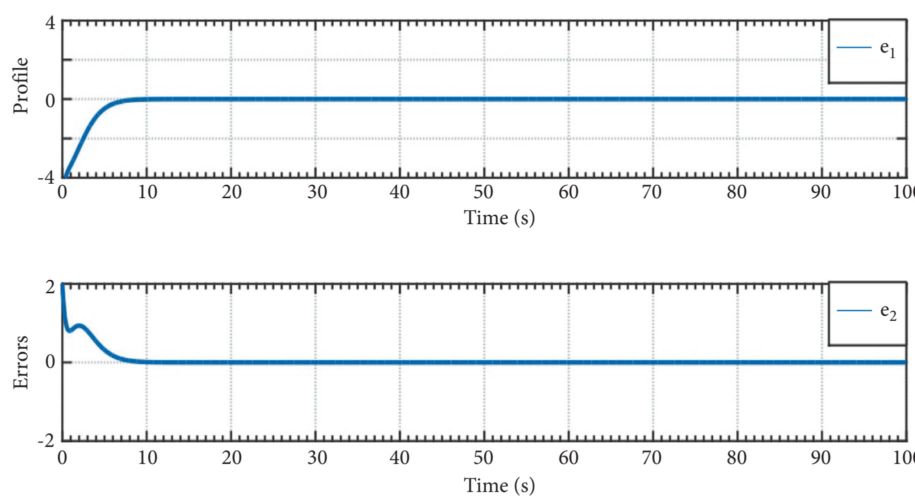


(c)

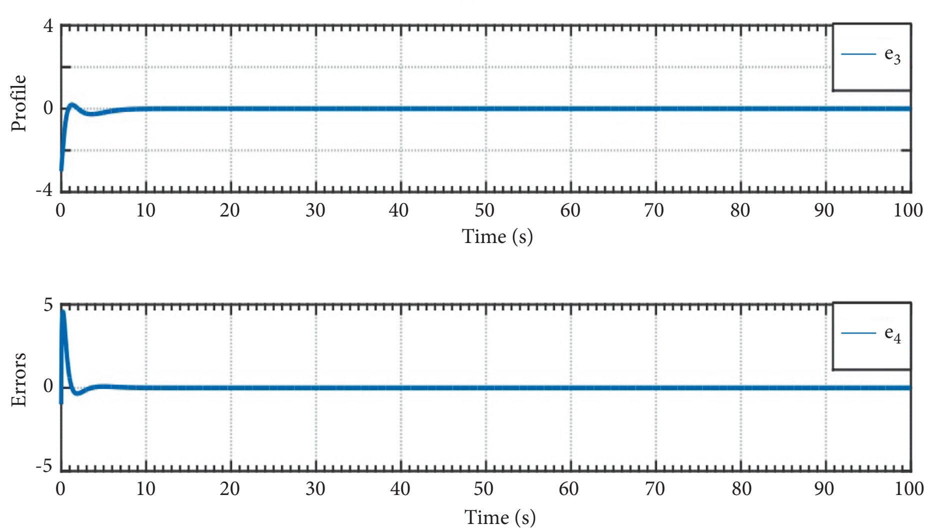
FIGURE 5: Continued.



(d)

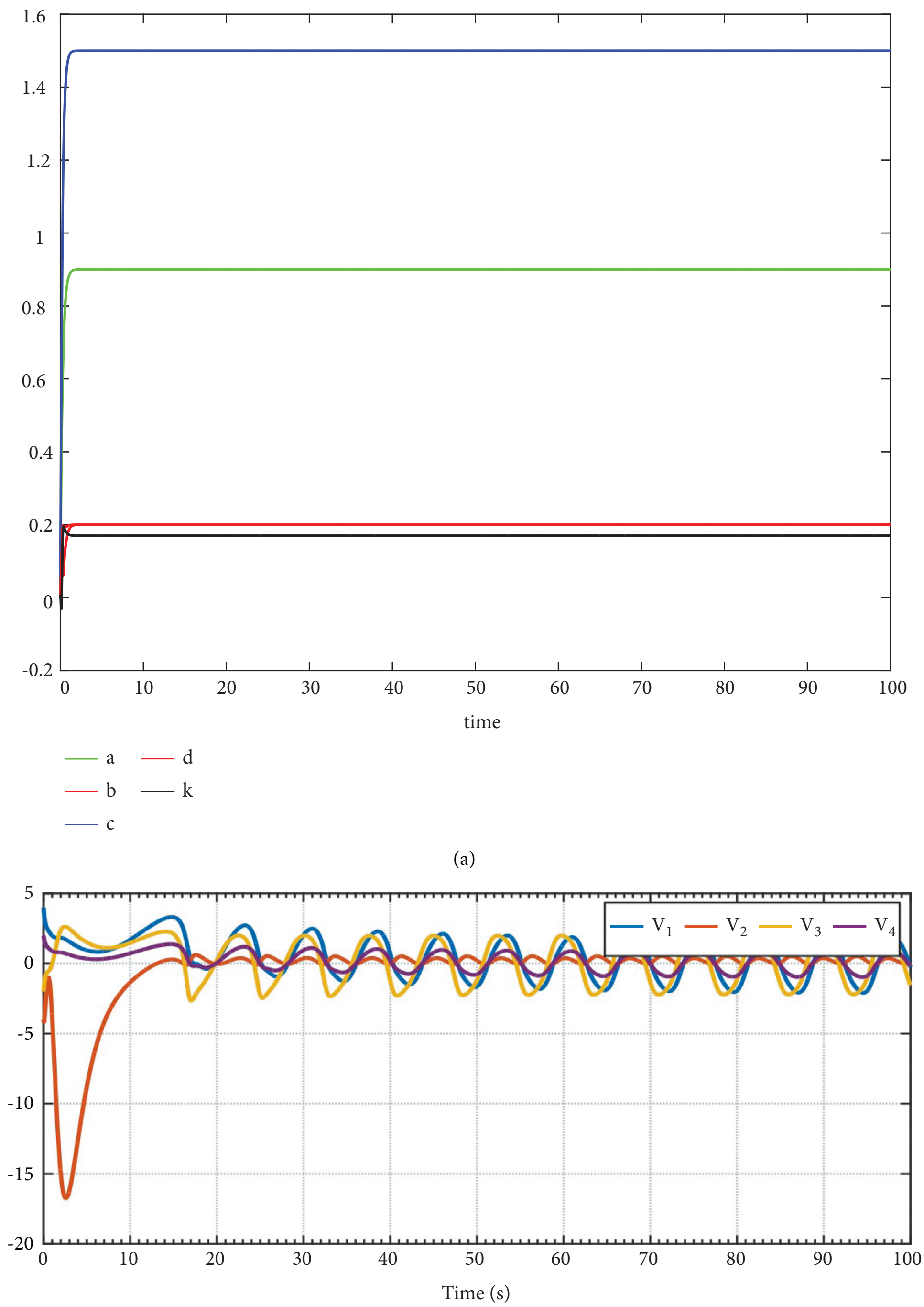


(e)



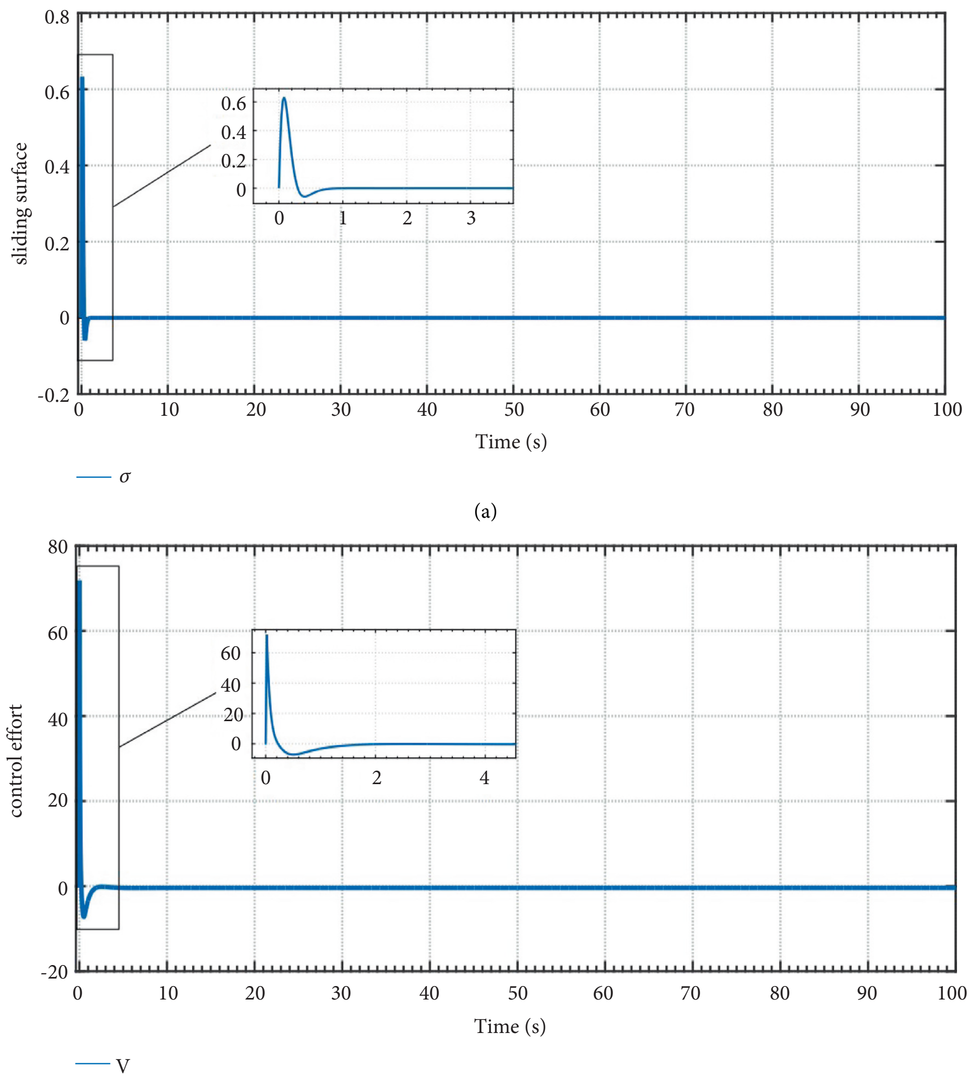
(f)

Figure 5: Synchronization of identical 4D hyperchaotic financial system with adaptation of parameters. (a) Synchronization of interest rate corresponding to the initial condition $[x_1(0), y_1(0) = (3, -2)]$. (b) Synchronization of investment demand corresponding to the initial condition $[x_2(0), y_2(0) = (1, 3)]$. (c) Synchronization of price index corresponding to initial condition $[x_3(0), y_3(0) = (2, -1)]$. (d) Synchronization of average profit margins corresponding to the initial condition $[x_4(0), y_4(0) = (-3, -4)]$. (e), (f) Time history of the errors e_1, e_2, e_3, and e_4.



(b)

Figure 6: Synchronization of identical 4D hyperchaotic financial system. (a) $\hat{a}, \hat{b}, \hat{c}, \hat{d}$, and $\hat{k}$ represent the adaptation of unknown parameters. (b) v_1, v_2, v_3, and v_4 represent the time-varying disturbances.



(a)

(b)

Figure 7: Synchronization of identical 4D hyperchaotic financial system with the adaptation of parameters. (a) Sliding manifold σ. (b) Control effort v.

$$
\begin{aligned}
\dot{\tilde{a}} &= \sigma e_1 - k_1\tilde{a},\\
\dot{\tilde{b}} &= 3\sigma e_2 - k_2\tilde{b},\\
\dot{\tilde{c}} &= 3\sigma e_3 - k_3\tilde{c},\\
\dot{\tilde{d}} &= \sigma x_1 x_2 - \sigma y_1 y_2 - k_4\tilde{d},\\
\dot{\tilde{k}} &= \sigma e_4 - k_5\tilde{k},\\
\dot{\hat{a}} &= -\sigma e_1 + k_1\tilde{a},\\
\dot{\hat{b}} &= -3\sigma e_2 + k_2\tilde{b},\\
\dot{\hat{c}} &= -3\sigma e_3 + k_3\tilde{c},\\
\dot{\hat{d}} &= -\sigma x_1 x_2 + \sigma y_1 y_2 + k_4\tilde{d},\\
\dot{\hat{k}} &= -\sigma e_4 - k_5\tilde{k},
\end{aligned}
\tag{35}
$$

we have

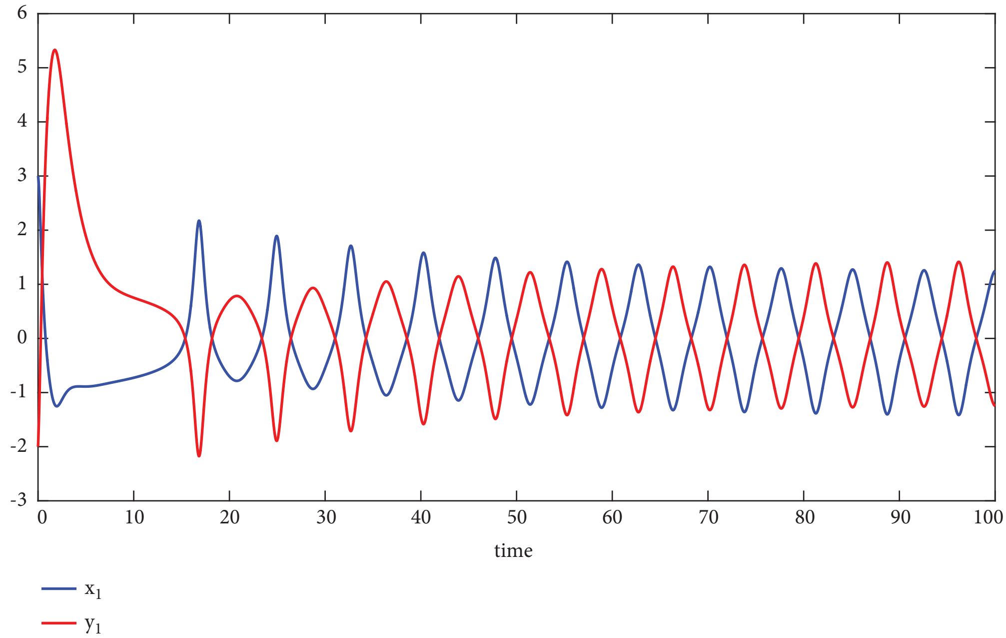
$$
\dot{V} = -k\sigma^2 - k_1\tilde{a}^2 - k_2\tilde{b}^2 - k_3\tilde{c}^2 - k_3\tilde{d}^2 - k_3\tilde{k}^2, \tag{36}
$$

where $\sigma, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{k} \longrightarrow 0$. Since $\sigma \longrightarrow 0$, therefore $e = (e_1, e_2, e_3, e_4) \longrightarrow 0$.

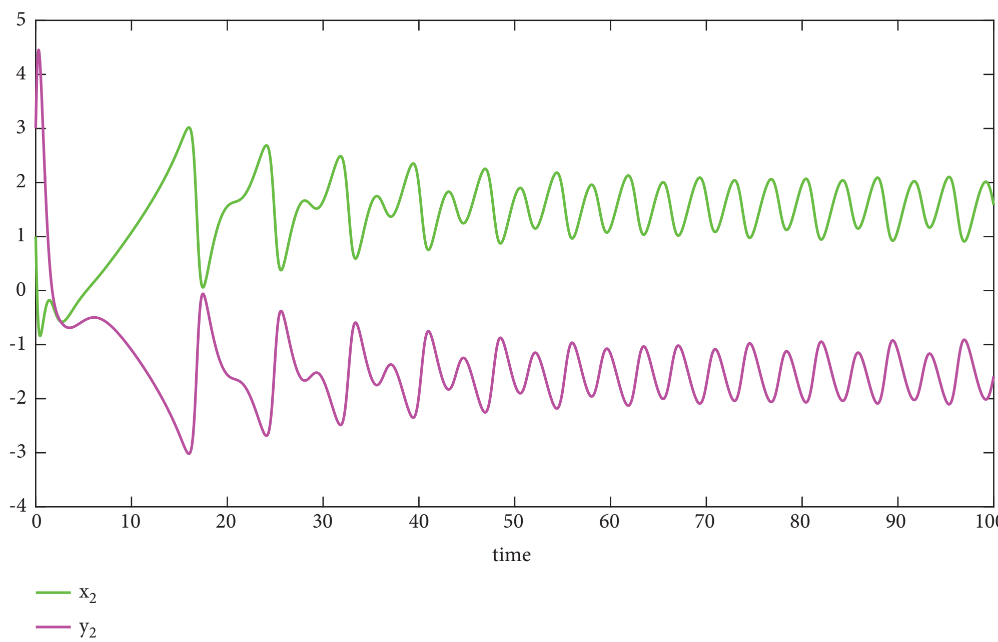
Initial conditions for simulations are taken as follows:

$$
\begin{aligned}
x(0) &= [3, 1, 2, -3]^T,\\
y(0) &= [-2, 3, -1, -4]^T.
\end{aligned}
\tag{37}
$$

The actual value of the unknown parameters is chosen as follows:

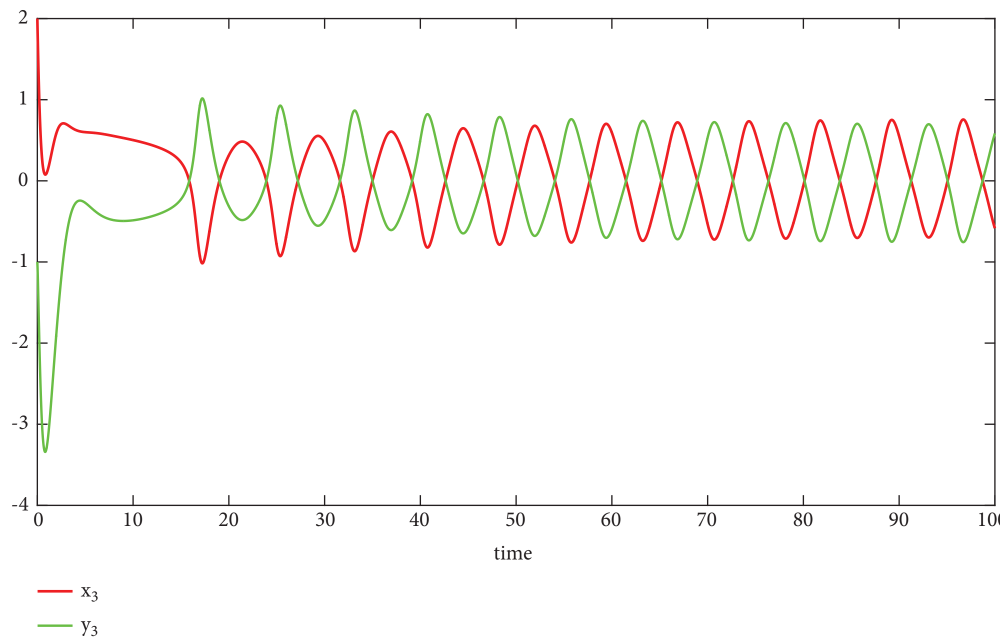


(a)

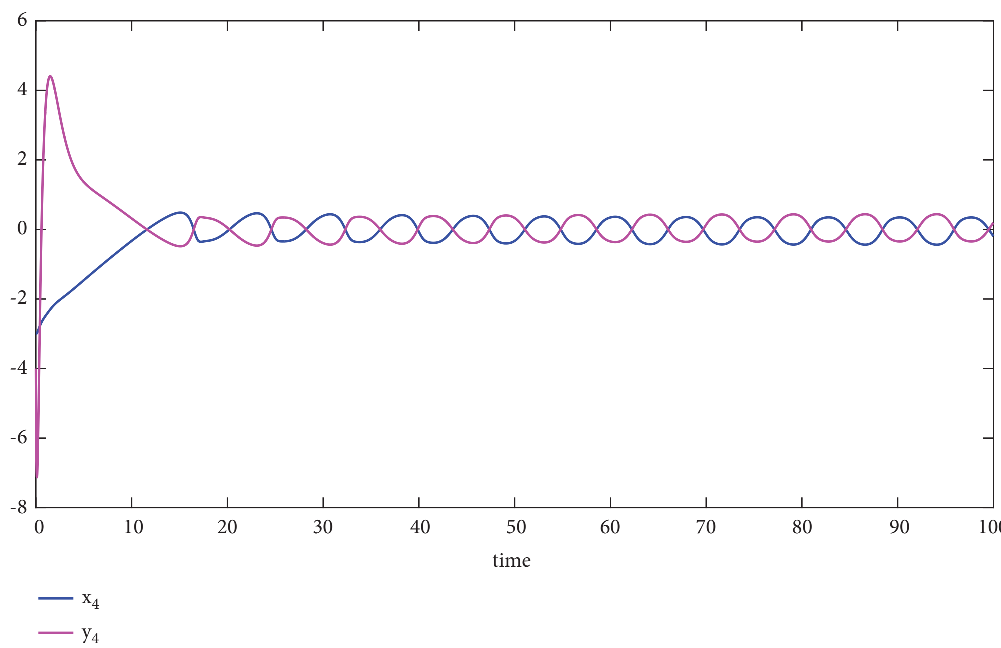


(b)

Figure 8: Continued.

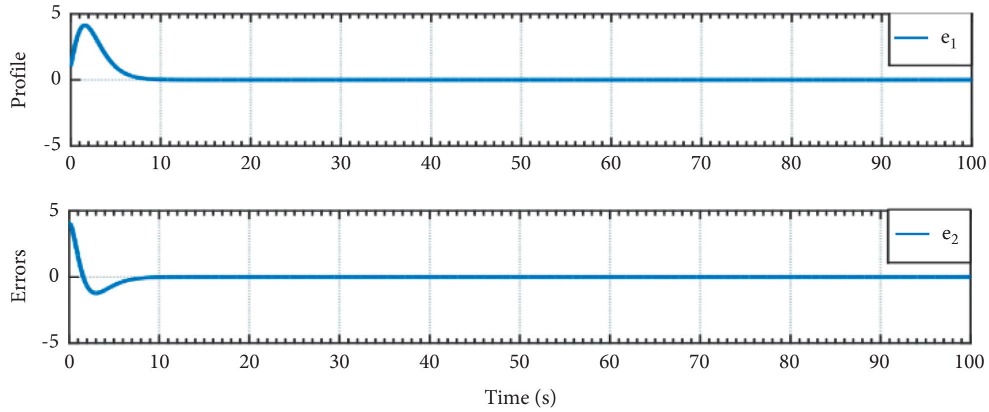


(c)

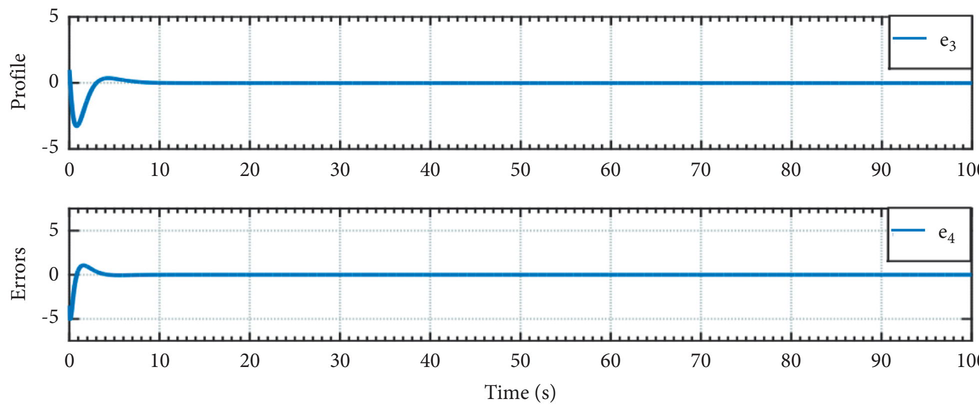


(d)

Figure 8: Continued.

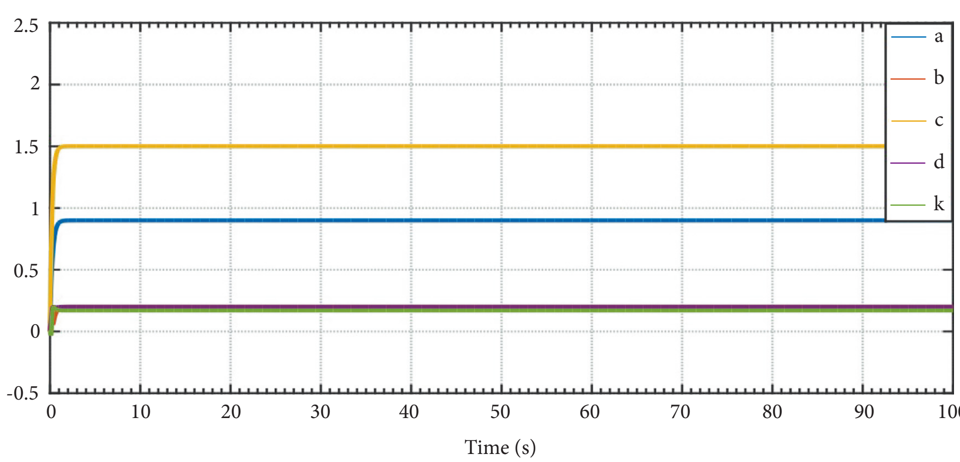


(e)



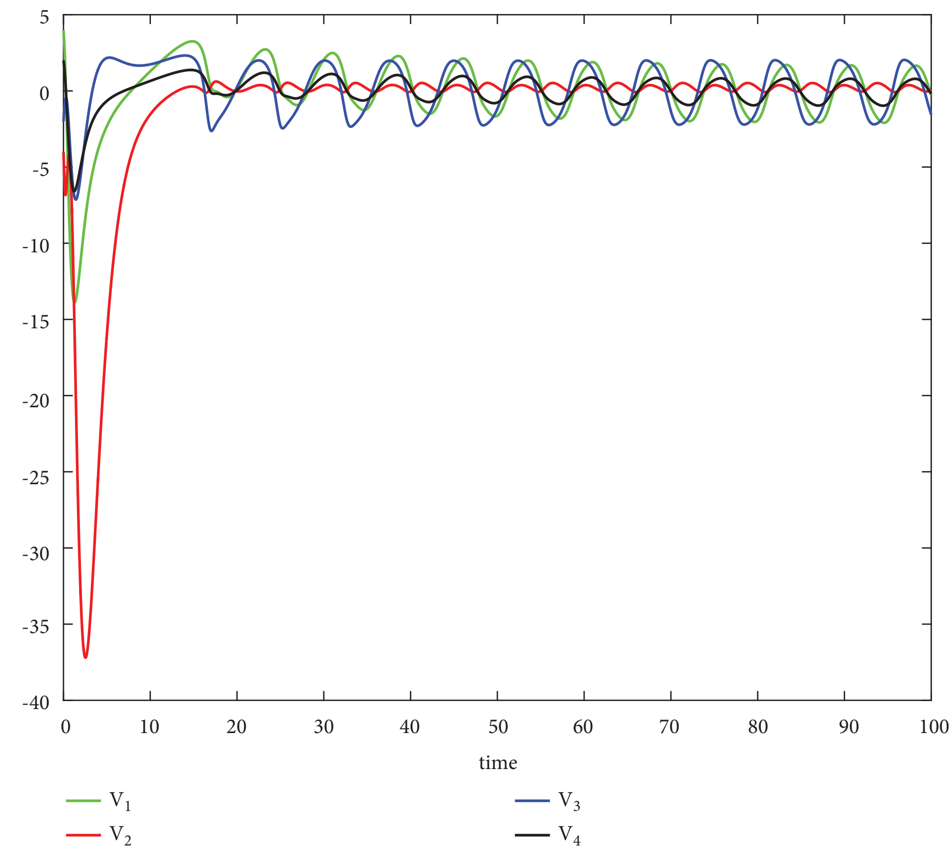
(f)

Figure 8: Antisynchronization of identical 4D hyperchaotic financial system. (a) Antisynchronization of interest rate corresponding to initial condition $[x_1(0), y_1(0) = (3, -2)]$. (b) Antisynchronization of investment demand corresponding to the initial condition $[x_2(0), y_2(0) = (1, 3)]$. (c) Antisynchronization of price index corresponding to initial condition $[x_3(0), y_3(0) = (2, -1)]$. (d) Antisynchronization of average profit margins corresponding to the initial condition $[x_4(0), y_4(0) = (-3, -4)]$. (e), (f) Time history of the errors $e_1, e_2, e_3,$ and e_4.



(a)

Figure 9: Continued.



(b)

FIGURE 9: Antisynchronization of identical 4D hyperchaotic financial system: (a) $\hat{a}, \hat{b}, \hat{c}, \hat{d}$, and $\hat{k}$ represent the adaptation of unknown parameters. (b) v_1, v_2, v_3, and v_4 represent the time-varying disturbances.

$$\begin{aligned} a &= 0.9, \\ b &= 0.2, \\ c &= 1.5, \\ d &= 0.2, \\ k &= 1. \end{aligned} \tag{38}$$

□

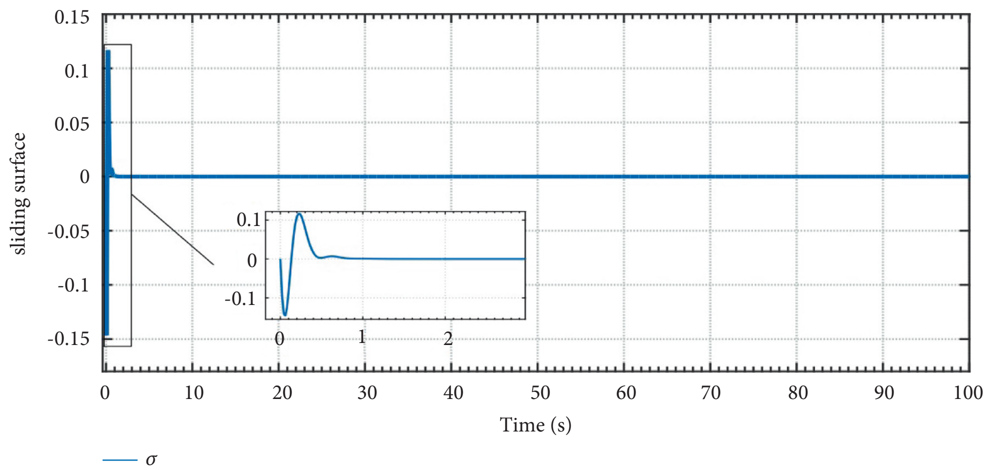
4. Results and Discussion

4.1. Synchronization of Identical 4D Hyperchaotic Financial System with Known Parameters: For Synchronization Set $q = 1$ in Equation (6). To achieve synchronization, $q = 1$ is employed in (6). This error signal profile is shown as (e) and (f) in Figure 1. Moreover, Figure 1(a) displays the interest rate synchronization, Figure 1(b) shows the synchronization among investment demands. Price index synchronization is portrayed as Figure 1(c). Figure 1(d) shows the average profit margin. However, the sliding surface profile and control effort are displayed in Figure 2 in this regard.

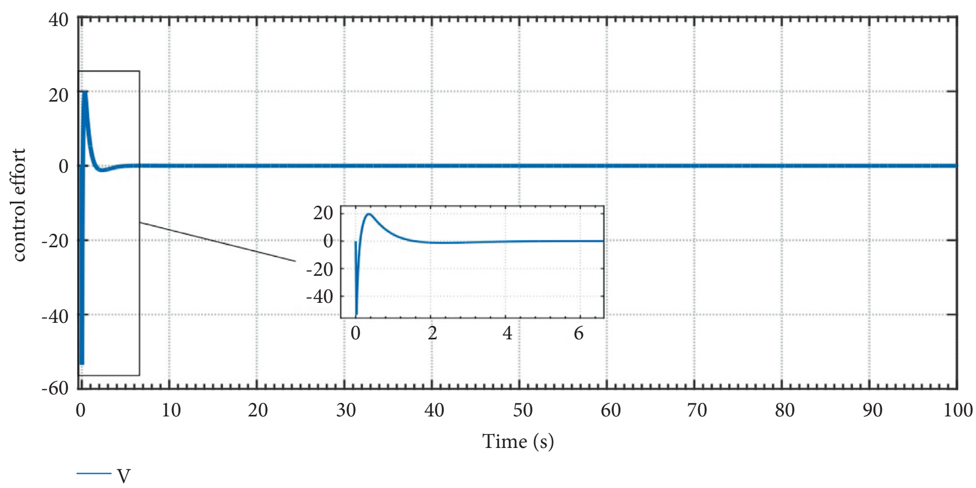
4.2. Antisynchronization of Identical 4D Hyperchaotic Financial System with Known Parameters: For Synchronization Set $\mathbf{q} = -1$ in Equation (6). To achieve antisynchronization, $q = -1$ is employed in (6). This error signal profile is shown in Figures 3(e) and 3(f). Moreover, Figure 3(a) displays the interest rate antisynchronization, Figure 3(b) shows the antisynchronization among investment demands; price index antisynchronization is portrayed as Figure 3(c). Figure 3(d) shows the antisynchronization of average profit margin. However, the sliding surface profile and control effort are displayed in Figure 4 in this regard.

4.3. Antisynchronization of Identical 4D Hyperchaotic Financial System with Unknown Parameters: For Synchronization Set $\mathbf{q} = 1$. Figure 5 illustrates the synchronization of 4D hyperchaotic financial system with the adaption of the parameters; interest rate synchronization is placed as Figure 5(a); similarly investment demand is displayed as Figure 5(b), however, Figure 5(c) shows the synchronization of price index, and Figure 5(d) displays the synchronization of average profit margins. The time profile of errors is displayed as Figures 5(e) and 5(f). Moreover, the adapted unknown parameters are shown in Figure 6.

4.4. Antisynchronization of Identical 4D Hyperchaotic Financial System with Unknown Parameters: For Synchronization Set $\mathbf{q} = -1$. A stable sliding manifold is mandatory to



(a)



(b)

Figure 10: Antisynchronization of identical 4D hyperchaotic financial system with the adaptation of parameters. (a) Sliding manifold σ. (b) Control effort v.

converge the system towards the origin in a finite-time instant. Figure 7 illustrates the stable sliding manifold and associated control efforts related to the sliding surface. Antisynchronization of identical 4D hyperchaotic financial system is displayed in Figure 8. Figure 8(a) reflects the antisynchronization of interest rate, where investment demand in this regard is displayed as Figure 8(b). Figure 8(c) portrays price indexing regarding antisynchronization of the 4D hyperchaotic financial system. Average profit margins are presented as Figure 8(d). Time history of errors is displayed as Figures 8(e) and 8(f), and it can be clearly observed that the error remains at zero.

Figure 9 displays the adaptation of unknown parameters; however, control effort and sliding surface stabilization are displayed in Figure 10. It can be observed from the figure that the sliding manifold converges toward origin in less than one second. In the upcoming section, the authors have concluded this work.

5. Conclusion

In the current era, we all know the importance of economic and financial stability. Stability of the financial system guaranties the social and economical development. Investments for the development of any financial/economic system purely depend on the stable financial system. However, after the financial globalization, financial stability becomes a highly complex nonlinear system. Minor parametric uncertainty may lead towards a chaos in any financial stable system, which may result turbulence in financial market and financial crises. The macroeconomic landscape of a country depends upon the interaction of demand, supply, discount rates, savings, and investments. The complex relationship of the abovementioned economic variables and taking account of all such variables open the door for the applications of 4D hyperchaotic fractional systems. According to the basic financial theory, this paper presents the controller for the synchronization and antisynchronization of identical 4D hyperchaotic financial systems with external perturbations (which is quite complex to handle). Conventional SMC is employed for known parameters; however, to counter the aforementioned unknown parametric variation, AISMC control strategy is applied, which makes the system more responsive and robust (from the initial time instant) due to the absence of reaching phase. Moreover, it provides comprehensive suppression in the chattering phenomenon. The stability analysis is also performed, and the time derivative of the Lyapunov function proved strictly negative. The effectiveness of the proposed approaches can be confirmed via simulation results displayed in the previous section, which clearly indicated the adaptive integral sliding mode controller versatility over hyperchaotic financial system efficiently. Simulation results also proved the suggested technique outshines even in the presence of the system modelling uncertainties with unknown parameters.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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